Statistics of energy flows, temperature fluctuations, and underlying techniques

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Outline

- Motivation(s)
- Review of general techniques
  - Extended Keldysh method
  - Pilgram-Kindermann
    - Examples: fluctuations of flows
  - Conservation laws
  - Coordinate-dependence
- Applications
  - Temperature fluctuations in an island
  - Temperature fluctuations in SET-transistor
    - “Gigantic” fluctuations
- Entropy and entanglement flows

Motivations

- Statistics beyond Gaussian
- Statistics of “Wonders”
- Experiment (Pekola)
  - Small, fast thermometry
  - of small objects
  - By unconventional methods

- Thermal transport: old and undrestood?
- Finkelstein
- Skvortsov
- Averin-Pekola

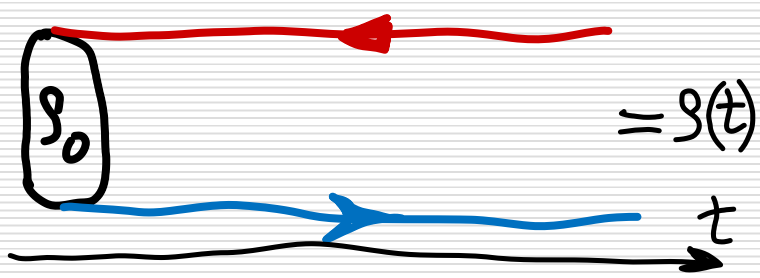
- Classical entropy flows
- Experiments on classical fluctuations (Ciliberto)
- Jarzynski and followers
- Quantum generalization

- Flows of unphysical vales
- Quantum information quantities
- Quantum quenches
- Relation of physical and unphysical

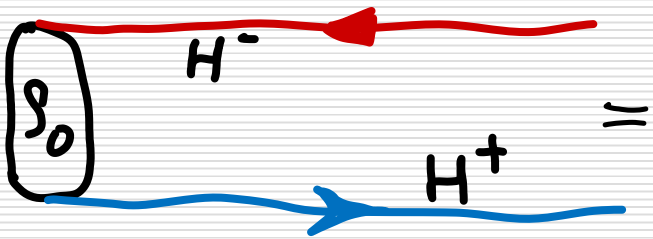
Extented Keldysh method

- Usually

$$\rho(t) = T e^{-i\int_0^t d\tau\, H(\tau)}\, \rho(0)\, \tilde{T} e^{i\int_0^t d\tau\, H(\tau)}$$



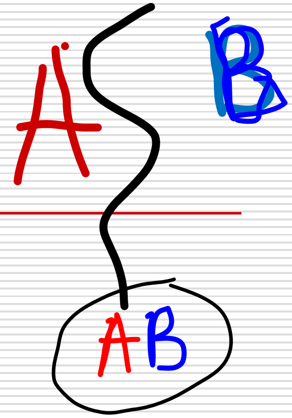
- Different H on different parts of the contour



$$= \tilde{\rho}(t)$$

- Action S: $$e^{-iS} = \mathrm{Tr}\{\tilde{\rho}(t)\}$$

Use for quantum variable separation



- Bipartition $H = H_A + H_{AB} + H_B$

$$H_{AB} = \sum_j \hat{A}_j \hat{B}_j$$

- Replace $\hat{A}_j$ with numbers:

$$\hat{A}_j \longrightarrow A_j^+(t),\ A_j^-(t)$$

- B: extended formalism

$$H^{\pm} = H_B + \sum_j \hat{B}_j A_j^{\pm}(t)$$

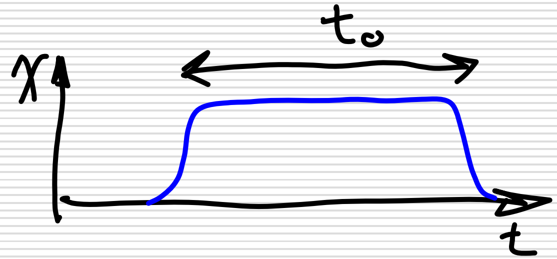
- Taking trace over B: action for A

$$\mathrm{Tr}_B\{\tilde{\rho}_B\} = e^{-iS(A_j^+,\, A_j^-)}$$

Its use for (counting) statistics

- Counting field χ

$$H^{\pm} = H \pm \chi(t)\,\hat{I}/2$$

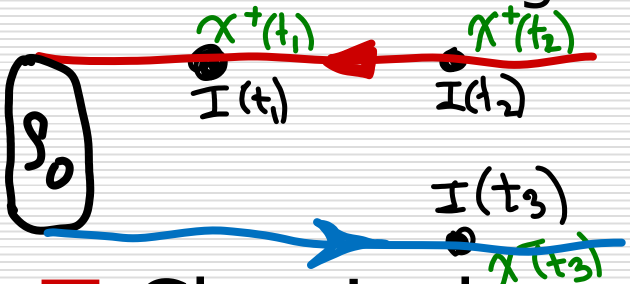


$$\dot{\hat{Q}} = \hat{I}$$

- The action => distribution of counts of $\hat{Q}$

$$\int e^{iS(\chi)}\, e^{-iQ\chi}\, d\chi = P_{t_0}(Q)$$

- Derivatives give (keldysh-ordered) correlators



$$\frac{\partial}{\partial\chi^+(t_1)}\frac{\partial}{\partial\chi^+(t_2)}\frac{\partial}{\partial\chi^-(t_3)}\, e^{-iS(\{\chi^+(t),\chi^-(t)\})}$$

- Classical counterpart: phase gauge field $\Rightarrow S \Rrightarrow S(\chi^+ - \chi^-)$

Interpretational difficulties

- Example: trasferred charge, charge detector

$$\rho_{det}(Q,Q') \xrightarrow[\text{representation}]{\text{conjugated}} \rho_{det}(\chi^+,\chi^-)$$

- Action: evolution of the d.m. of the detector

$$\rho_{after}(\chi^+,\chi^-) = e^{-iS(\chi^+,\chi^-)}\rho_{before}(\chi^+,\chi^-)$$

- Normal metal (gauge-invariant)

$$\Pi_{after}(Q) = \int dq\, P(q)\, \Pi_{before}(Q-q)$$

Pilgram-Kindermann, 2003: gauge field ξ

- New wavefunction: upon time shift

$$\tilde{\psi}(t) = \psi(t - \xi(t))$$

- Schrodinger equation:

$$i\hbar \frac{\partial \tilde{\psi}}{\partial t} = (1 - \dot{\xi}(t))\, \hat{H} \tilde{\psi}$$

- Make it practical: have it in a part of the whole system

$$H = H_A + H_{AB} + H_B$$

$$H_{AB} = \sum_j \hat{A}_j \hat{B}_j \qquad H_B \to (1 - \dot{\xi}(t)) H_B$$

- Interaction picture:

$$H_{AB}(t) = \sum_j \hat{A}_j(t)\, \hat{B}_j(t - \xi(t))$$

Pilgram-Kindermann: counting

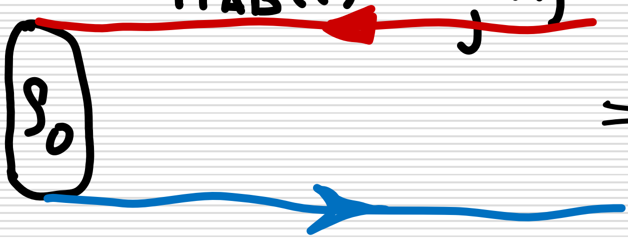
- Make ξ different on contours

$$H^{\pm} = H \pm \xi \dot{H}$$

field ξ counts energy

- Interaction picture: field at the boundary

$$H^{+}_{AB}(t) = \sum_j \hat{A}_j(t)\, \hat{B}_j(t - \xi(t))$$



$$= \tilde{\rho}(t)$$

That's it!

$$H^{-}_{AB}(t) = \sum_j \hat{A}_j(t)\, \hat{B}_j(t + \xi(t))$$

Example I:ext.force and bosons

- A-ext. Force, B- a linearly responding system

$$H_{AB} = -f(t)\hat{x}\ ; \ H^{\pm}_{AB} = -f(t)\,\hat{x}(t \mp \xi/2)$$

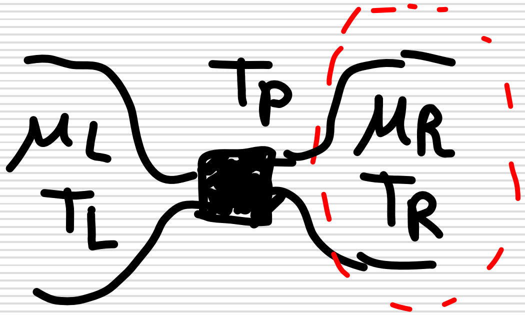
$$f = f_0 \cos\omega t$$

- Action in x linear part: $-it_0 f_0 \mathrm{Re}\{x^{+}_{\omega} e^{-i\omega\xi/2} - x^{-}_{\omega} e^{i\omega\xi/2}\}$
- Action in x quadratic: $it_0 (x^{+}_{\omega} - x^{-}_{\omega})\, \chi^{-1}(\omega) (x^{+}_{\omega} + x^{-}_{\omega})/2$
- Taking the path integral $-\frac{1}{2}|x^{-}_{\omega} - x^{+}_{\omega}|^2 B(\omega)$
- Action in ξ:

$$S(\xi) = t_0 f_0^2\, \mathrm{Im}(\chi)\,\omega \left[(e^{-i\xi\omega} - 1) n_B + (n_B + 1)(e^{i\xi\omega} - 1) \right]$$

Example II: quantum point contact: fermions

- *Transmissions T_p*



$$-iS = \frac{1}{2}\sum_p \mathrm{Tr}\ln\left\{1 + T_p\left(\left[\check{G}_L, \check{G}_R\right]_+ - 2\right)\right\}$$

- *Modification:*

$$G_R^{+-}(t-t') \Rightarrow G_R^{+-}(t-t'\underbrace{-\xi^+ + \xi^-}_{-\xi})$$

- *Answer:*

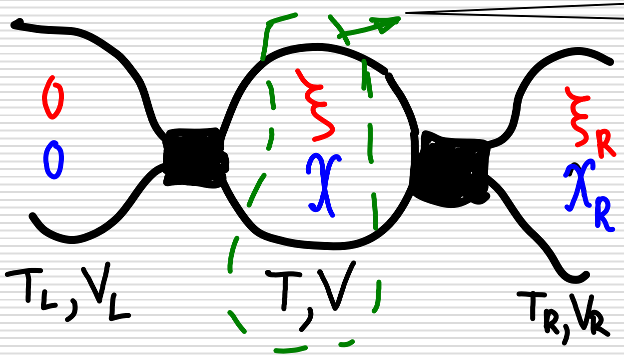
$$-iS = t_0\int\frac{d\varepsilon}{2\pi}\sum_p \ln\left\{1 + T_p f_L(1-f_R)e^{-i\xi(\varepsilon-\mu_R)} + T_p f_R(1-f_L)e^{i\xi(\varepsilon-\mu_R)}\right\}$$

Temperature fluctuations:motivation

- Physics: about most probable course?
- Least probable situation
  - Conditioned on improbable (hero must win)
  - Otherwise most natural
- Physics
  - Most probable laws
  - Least probable laws
- Calculation framework
  - Keldysh formalism
  - Extensions in the context of full counting statistics

Conservation laws (Pilgram 2004)

- Double junction: current and flux statistics



Dynamical variables

Introduce using conservation laws:

$$\delta\left(I_R - I_L + \frac{dQ}{dt}\right) \Rightarrow \int d\chi\, e^{i\chi(I_R - I_L + \dot{Q})}$$

$$\delta\left(I_R^{(E)} - I_L^{(E)} + \frac{dE}{dt}\right) \Rightarrow \int d\xi\, e^{-i\xi(I_R^{(E)} - I_L + \dot{E})}$$

- Action: sum of two contacts. Take saddle-point:

$$S(\xi_R, \chi_R) = \min_{\chi,\xi}\left[S^L(\chi,\xi) + S^R(\chi^R - \chi, \xi^R - \xi)\right]$$

Coordinate-dependent formulation

- Take a fine mesh, energy-charge conservation at each point

$$T_L \;\; \xi(t,x) \;\; \chi(t,x)$$

$$0,0$$

$$V_L \;\; \rho_Q(t,x),\ \rho_E(t,x)$$
$$T(t,x),\ V(t,x)$$

Conservation laws
+ gradient expansion
+"material relations"

$$T = F_1(\rho_E)\,;\ V = F_2(\rho_Q)$$

Action:

$$S = \int dx\,dt \left[\underbrace{\chi \dot{\rho}_Q + \xi \dot{\rho}_E}_{\text{storage}} + \underbrace{\begin{bmatrix} \chi' \\ \xi' \end{bmatrix} \check{A} \begin{bmatrix} T' \\ V' \end{bmatrix}}_{\text{responces}} + \underbrace{\begin{bmatrix} \chi' \\ \xi' \end{bmatrix} B \begin{bmatrix} \chi' \\ \xi' \end{bmatrix}}_{\text{noises}} \right]$$

FDT

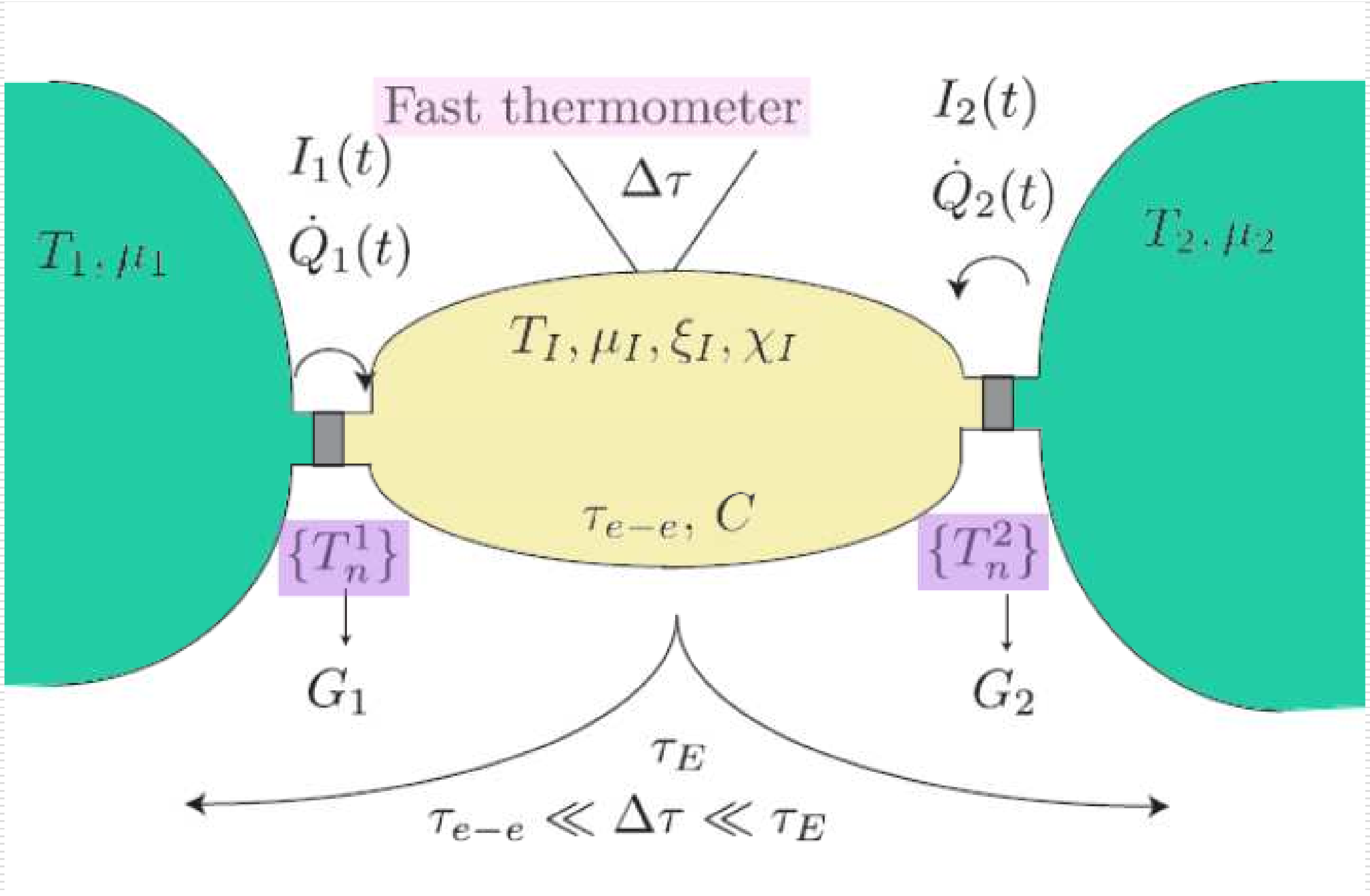
Summary

- Physics: about most probable course?
- Least probable situation
  - Conditioned on improbable (hero must win)
  - Otherwise most natural
- Physics
  - Most probable laws
  - Least probable laws
- Calculation framework
  - Keldysh formalism
  - Extensions in the context of full counting statistics

Formulation of the (exemplary) problem

- Island connected to two leads
- Out of equilibrium: leads are biased
- Fast relaxation of electron distribution
  - Full energy = temperature
- In comparison with: heat exchange with the leads
- Gaussian fluctuations: standard calculation
- Beyond Gaussian: Goal
- To compare with statistics of current
  - no infinite time interval
  - distribution of physical quantity

Setup



Fast thermometer
$\Delta\tau$
$I_1(t)$
$\dot{Q}_1(t)$
$I_2(t)$
$\dot{Q}_2(t)$
T_1, μ_1
T_2, μ_2
$T_I, \mu_I, \xi_I, \chi_I$
τ_{e-e}, C
$\{T_n^1\}$
$\{T_n^2\}$
G_1
G_2
τ_E
$\tau_{e-e} \ll \Delta\tau \ll \tau_E$

Keldysh action

- Green's function of the island
- Extended to include counting fields

$$\hat{G} = \exp\left(\frac{1}{2}(\chi(t) + E\xi(t))\hat{\tau}_3\right)\hat{G}_0(E)\exp\left(-\frac{1}{2}(\chi(t) + E\xi(t))\hat{\tau}_3\right)$$

$$\hat{G}_0(E) = \begin{pmatrix} 1-2f(E) & 2f(E) \\ 2-2f(E) & -1+2f(E) \end{pmatrix}$$

χ **counts charge**

ξ **counts energy transferred**

S. Pilgram, Phys. Rev. B 69, 115315 (2004).

Contacts contribute to the action =>

$$S_{c,\mathrm{el}} = \frac{1}{2}\sum_{\alpha}\sum_{n\in\alpha} \mathrm{Tr}\ln\left[1 + T_n^{\alpha}\frac{\{\check{G}_{\alpha}, \check{G}_I\} - 2}{4}\right].$$

Storage terms

Junction terms

Action

$$A = \int dt \left\{ Q\dot{\chi} + E\dot{\xi} + S_c(\xi(t), T(t), \chi(t), \mu(t)) \right\}$$

- example ballistic junctions

$$Q = C\mu / e;$$

$$E = \frac{\pi^2}{3\delta_I}(k_B T)^2$$

$$S_c = \sum_\alpha \frac{G_\alpha}{2}\left[\frac{2\mu_\alpha\chi + T_\alpha\chi^2 + [\pi^2 T_\alpha^2/3 + \mu_\alpha^2]\xi}{1 - T_\alpha\xi} - \frac{2\mu\ \chi - T\ \chi^2 + [\pi^2 T^2/3 + \mu^2]\xi}{1 + T\ \xi}\right].$$

μ, χ -fast variables, integrate out

$$A = \int dt \left\{ E\dot{\xi} + S_c(\xi(t), T(t)) \right\}$$

Pseudo-Hamiltonian form

$$A = \int dt \left\{ E\dot{\xi} + S_c(\xi(t), T(t)) \right\}$$

$$A = \frac{i}{\hbar} \int dt \left\{ p\dot{x} + \frac{p^2}{2m} + U(x) \right\}$$

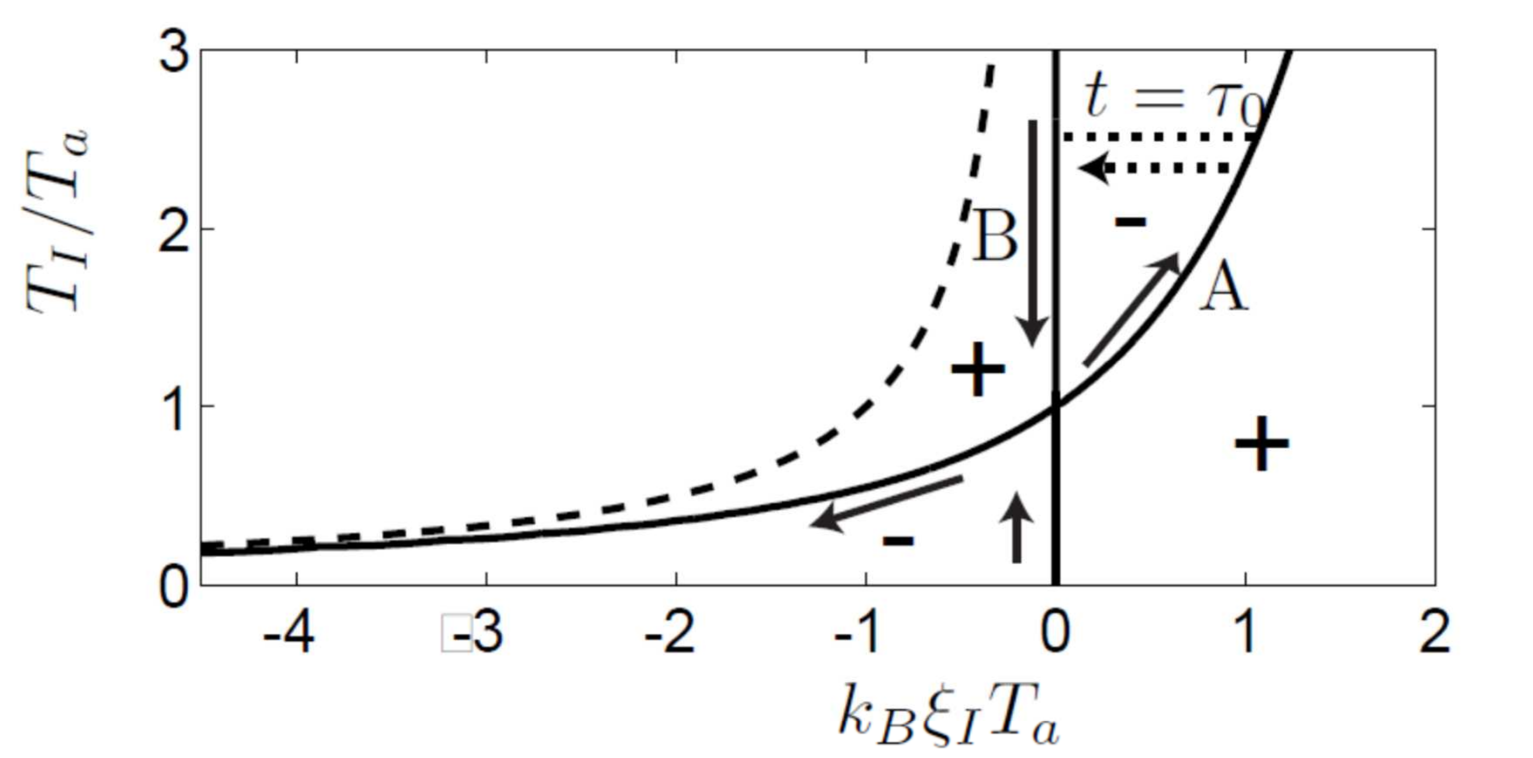
- extremum:

$$\dot{E} = \frac{\partial S_c}{\partial \xi}; \qquad \dot{\xi} = -\frac{\partial S_c}{\partial T}$$

$$S_c = const = 0$$

- "energy" conservation along the trajectories
- 0 - related to most or least probable physics

The essence of it



Relaxation and anti-relaxation

- B: relaxation: motion to the most probable point

ξ =0

$$\frac{dT}{dt} = f_B(T), \quad f_B(T \approx T_a) = -\frac{1}{\tau_R}(T - T_a)$$

- A: antirelaxation: motion away the most probable point

ξ implicit, n.e. 0

$$\frac{dT}{dt} = f_A(T), \quad f_A(T \approx T_a) = \frac{1}{\tau_R}(T - T_a)$$

Microscopic reversibility?

- Simplest hypothesis

$$\frac{dT}{dt} = f_B(T) = -f_A(T); ?$$

- Same trajectory, different directions of motion
  - near equilibrium
  - Generally for thermal equilibrium?
  - Langevin noise

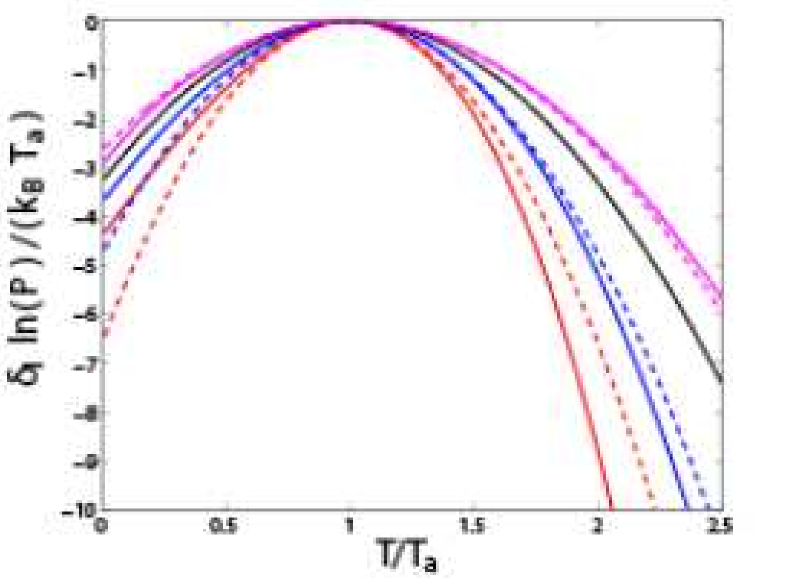
Results: distribution

- Simple equilibrium example

$$P = \exp\left(-\frac{E}{k_B T_a}\right)\exp\left(2\pi\sqrt{\frac{E}{3\delta_I}}\right) = \exp\left(\frac{k_B T_a}{\delta_I}\frac{\pi^2}{3}\left(1-\frac{T}{T_a}\right)^2\right)$$

- Sets the scale

$$\ln P = \frac{k_B T_a}{\delta_I} f(T/T_a)$$



Temperature bias, tunnel

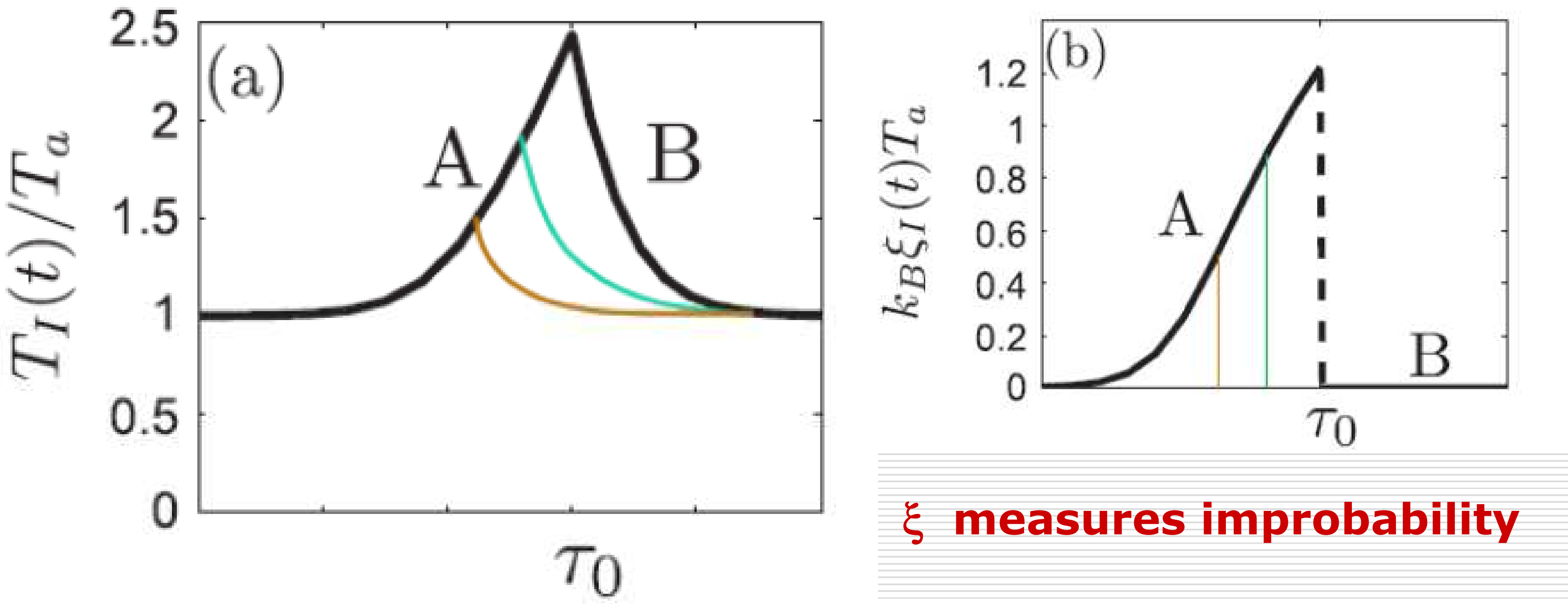
Equilibrium

Voltage bias, tunnel

Voltage bias, ballistic

Results: time-line

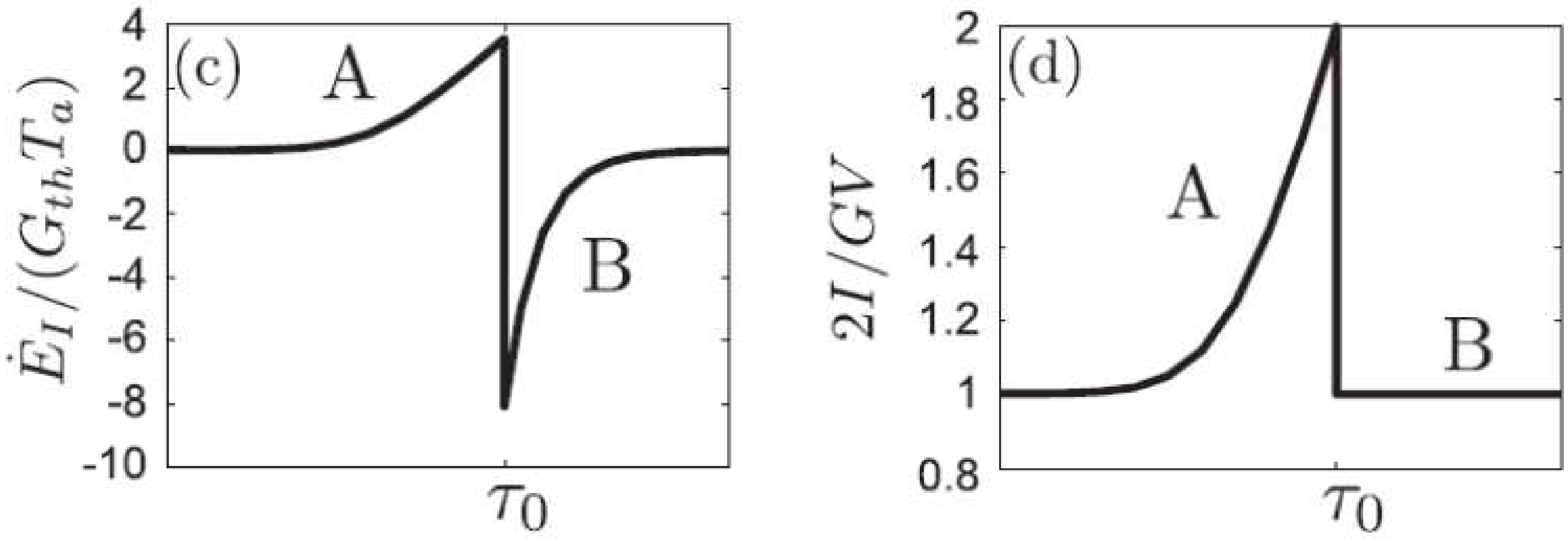
- ZeroT leads, voltage bias applied



ξ measures improbability

Temperature surge

Time-line currents



Heat current to the island

Electric current through the island

Conclusions

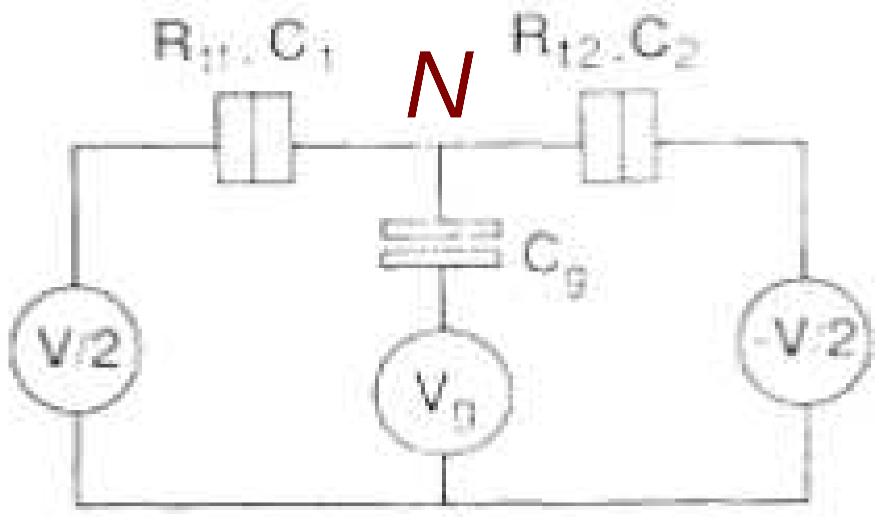
- Statististic of huge temperature fluctuations
- Exemplary system
- Simple anti-relaxation law governs
- That has to be a property of many systems
- Towards the theory of wonders:)

Fully overheated SE transistor

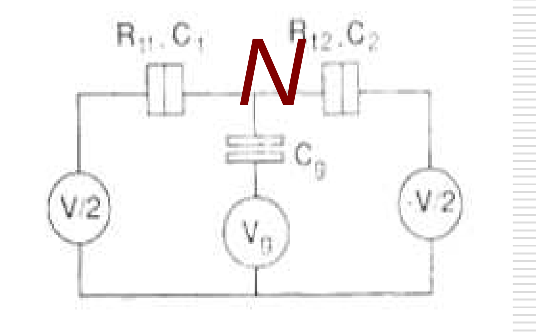
- **Introduction**
  - Coulomb blockade = history
  - SE tunneling and CO-tunneling
  - SE overheating
- **Regimes**
  - CO, CO+SE, crossover, SE
  - competition
- **Anomalous**
  - Temperature sensitivity
  - Temperature fluctuations
  - Current noise

Single Electron Transistor

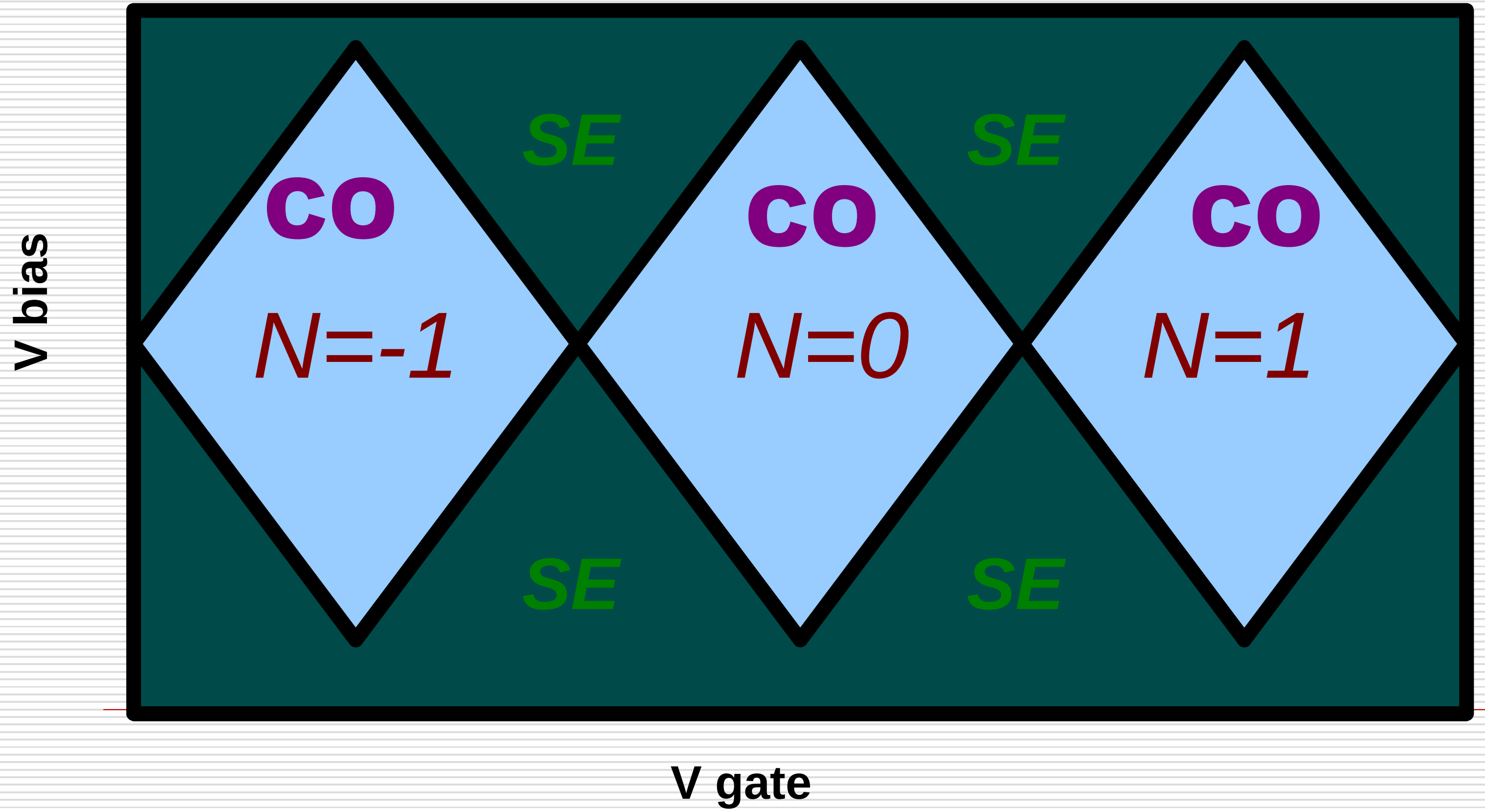
- Coulomb energy => fixes *N (1988)*
- Discrete charges but continuous levels
- Why don't make quantum dots?
- You can measure with...



Introduction: transport in SET



- Coulomb blockade => diamonds
- CO-tunneling => smuggles charge, weak

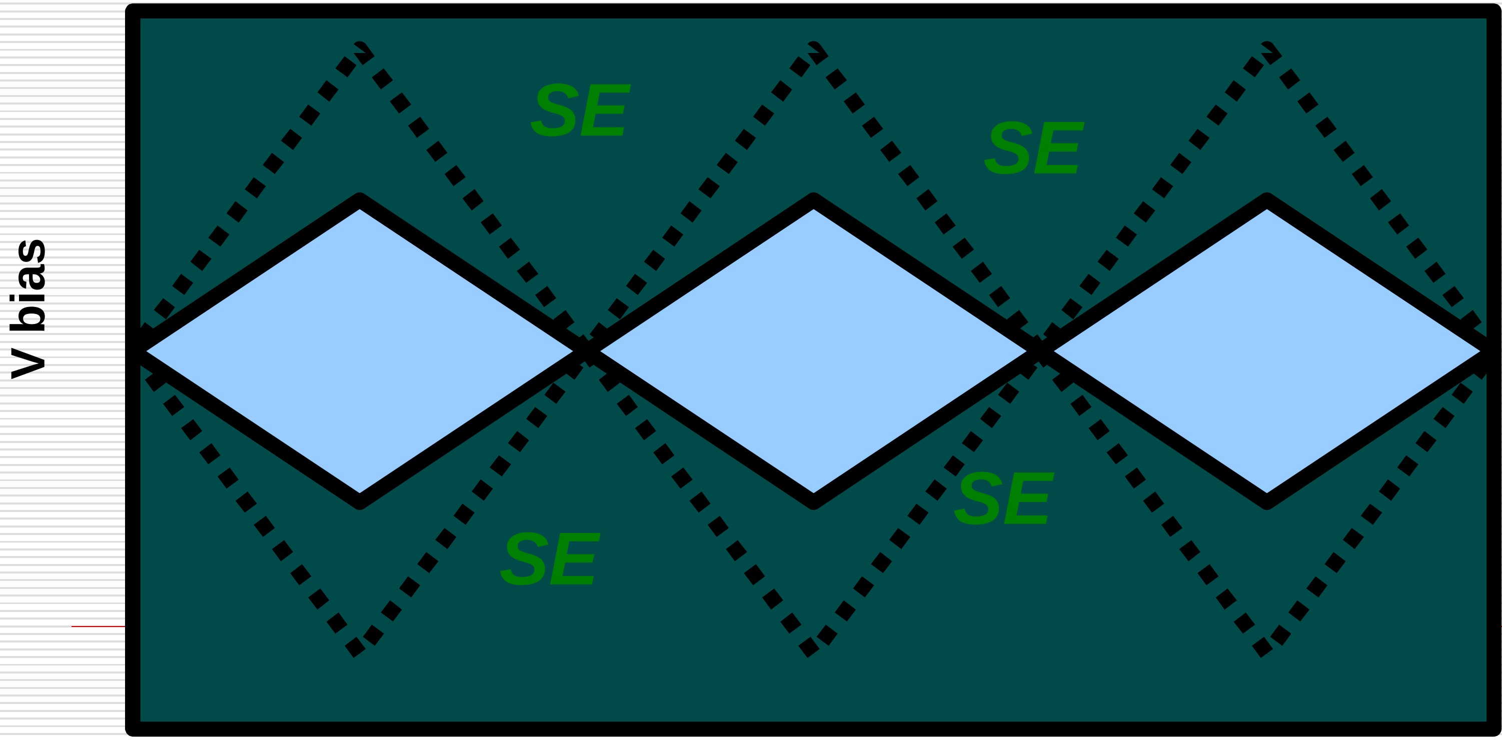


$$SE \propto (G_T/G_Q)$$
$$CO \propto (G_T/G_Q)^2$$

Do not compete except the diamond edge

Introduction: SE heating in SET

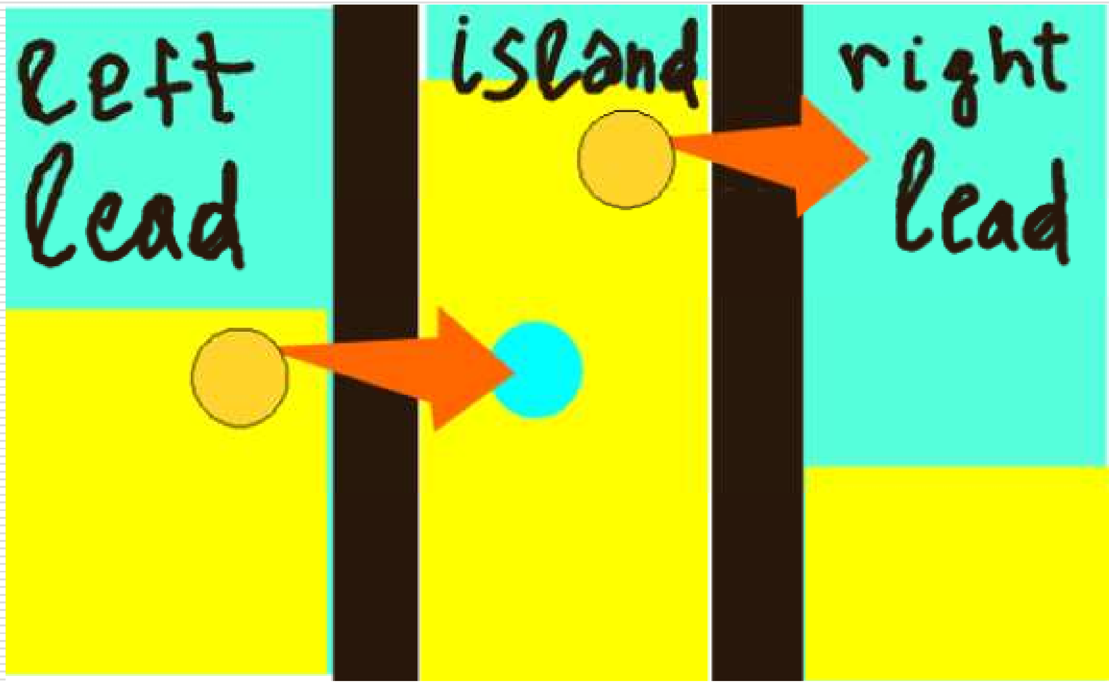
- Transport heats the island (fast equilibration)
- No external heat sink- overheating
- Korotkov et al. 1994 — *new threshold* (40%)



We: add CO

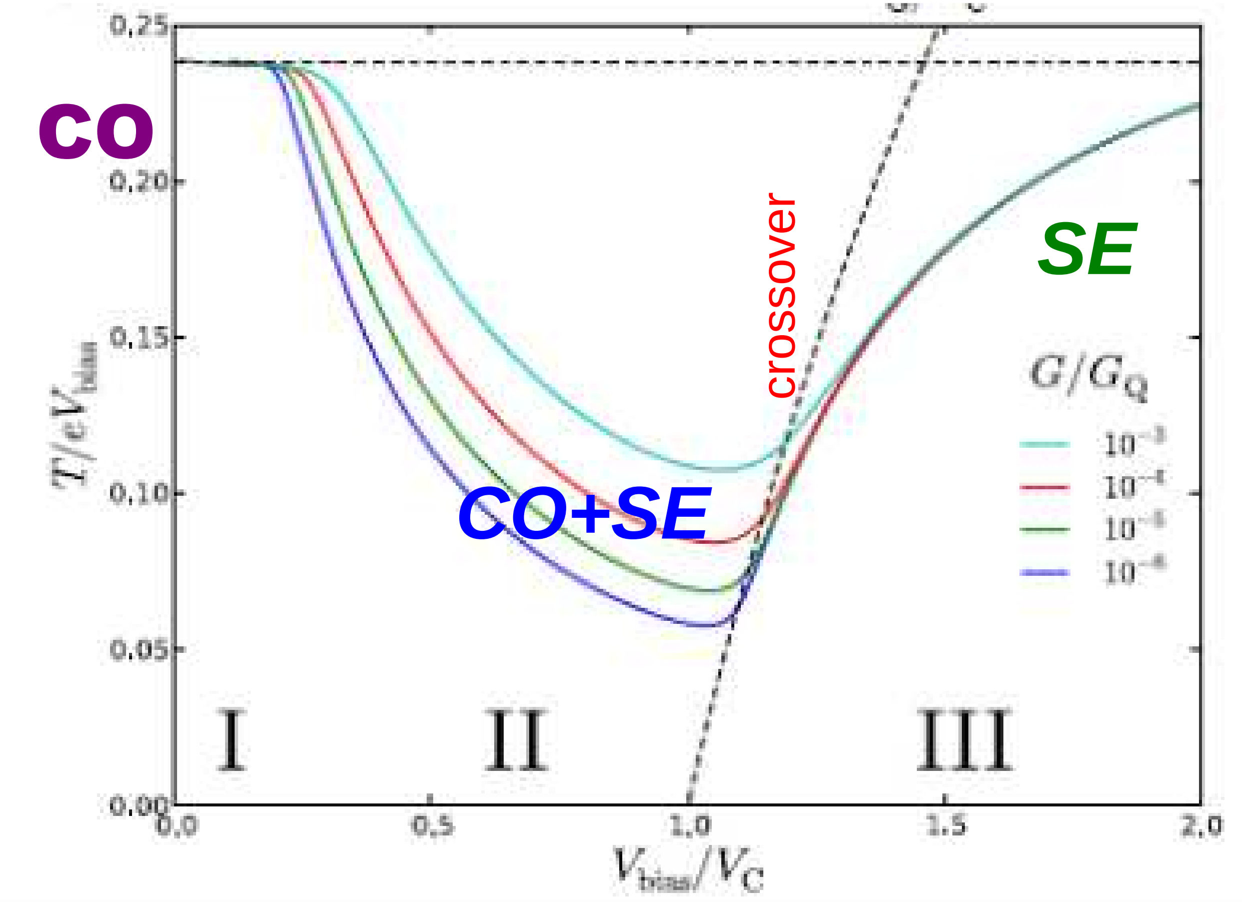
Heating or cooling?

- Thermally -activated SE can cool



CO — only heating (leaves e-h pair)

Regimes



- Log-physics
- CO=thermally activated SE

$$T_{min} \approx \frac{eV_C}{2\log(G_Q/G)}$$

Through regimes: cotunneling

- Low voltages $I_{cot} \propto V^3 + V T^2;$ $\Rightarrow \dfrac{I_{cot}^{hot}}{I_{cot}^{cold}} \approx 2$

  $T \simeq 0.238\, V$

- Pioneering experiment

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Observation of Macroscopic Quantum Tunneling through the Coulomb Energy Barrier

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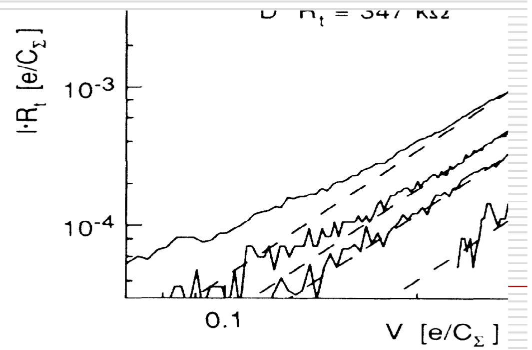
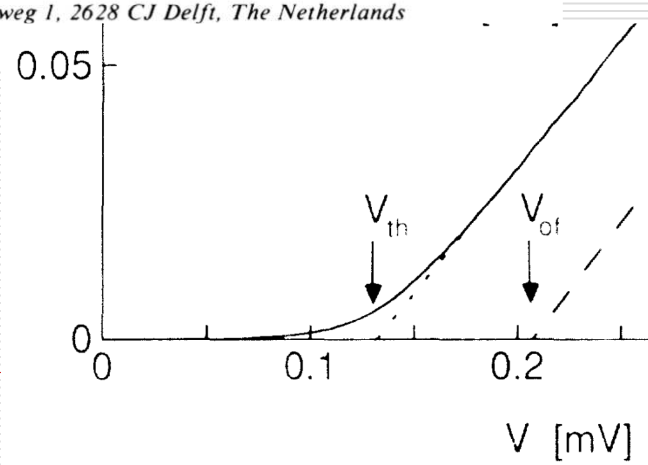
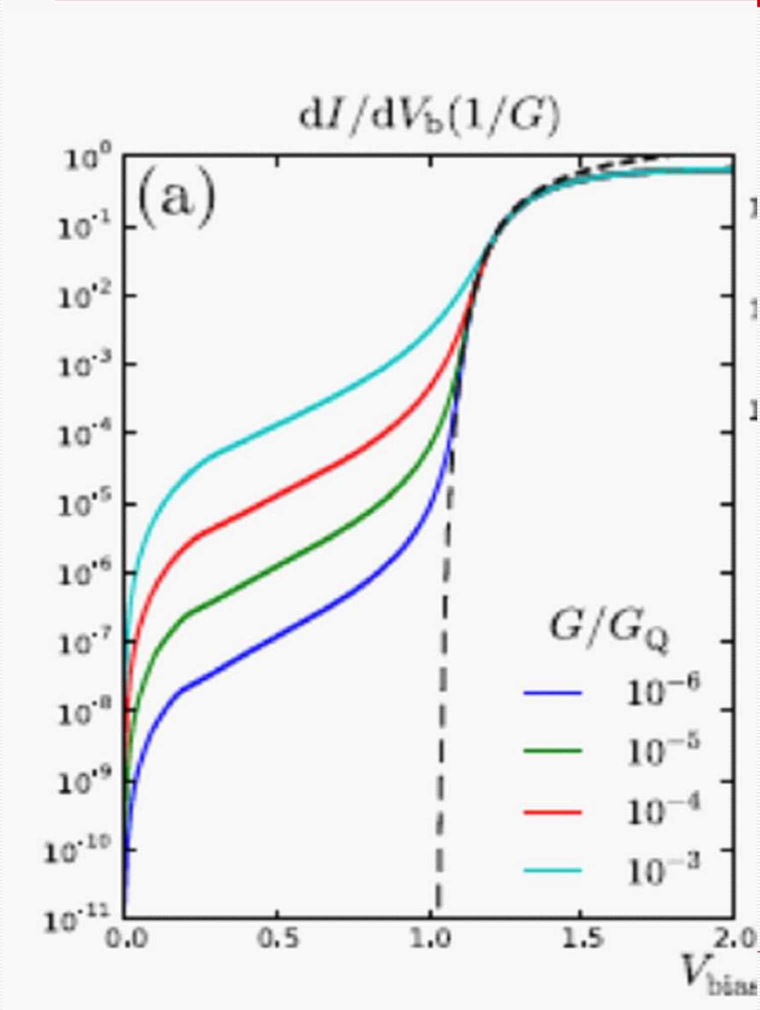
(Received 20 July 1990)

factor 2
mismatch
explained

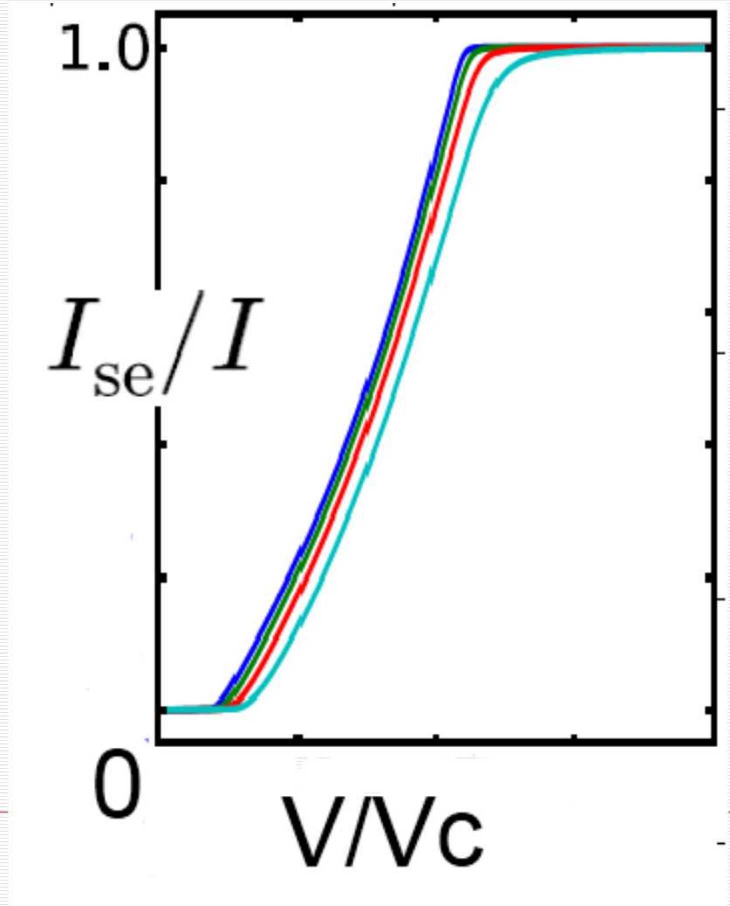




Details of competitions

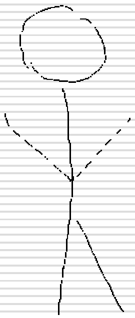


- Fraction of SE events in the flow



Anomalous temperature sensitivity

- Some biology



$T_0 = 300\,K$ – working temperature

δT – working range

Exp: $\delta T \ll T_0$ Why?

- Uses thermally activated rates

$\exp\left(-\frac{u}{T_0}\right) \simeq$ by a factor of 2

$\delta T \simeq T_0 \left(\frac{T_0}{u}\right)$; $\delta T \simeq 3K$

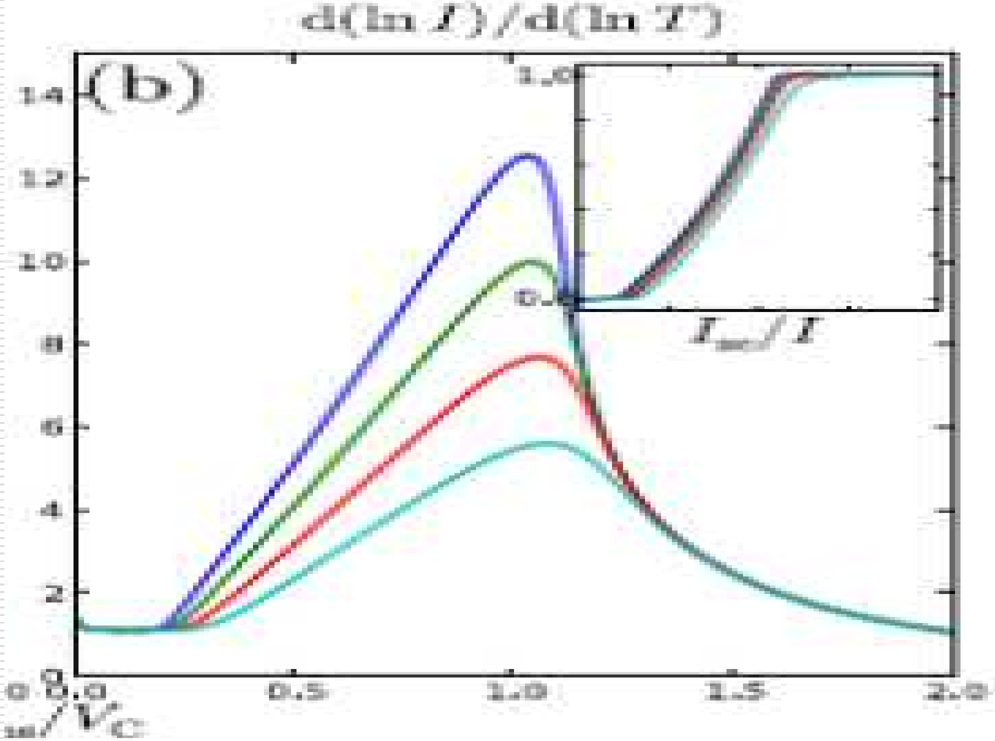
Anomalous: temperature sensitivity

- Overheated SET
- Normal

$$\frac{d(\ln I)}{d(\ln T)} \simeq 1$$

- Anomalous

$$\frac{d(\ln I)}{d(\ln T)} \simeq \ln\left(\frac{G_Q}{G}\right)$$



Self-detection of temperature fluctuations

- No equilibrium => temperature fluctuates
- Current depends on temperature
- => anomalous current noise $\delta I = (\partial I/\partial T)\delta T(t)$
- Fluctuation time-scale — big (volume)
- Low-frequency contribution(volume-independent)

$$S_{I,\mathrm{slow}} = \left(\frac{\partial I/\partial T}{\partial \dot{H}/\partial T}\right)^2 S_{\dot{H}}$$

“gigantic” Fano factor

- Normally,

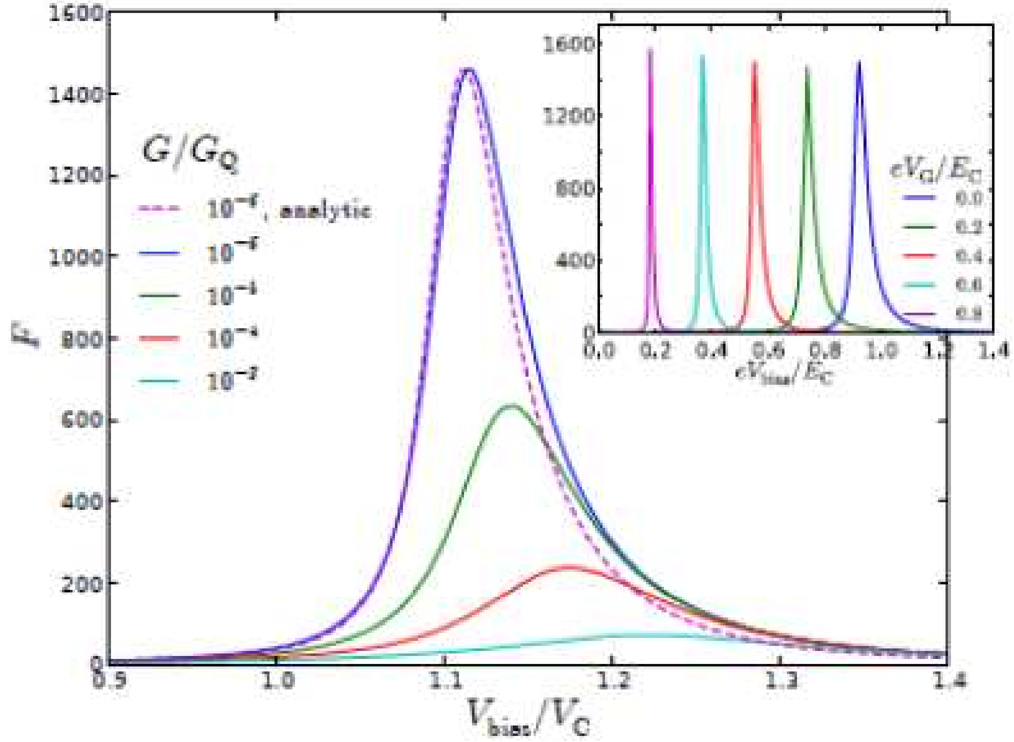
$$F = \frac{S_I}{Ie} ; F < 1$$

- Expected:

$$F_{extra} \simeq 1$$

- Actual

$$F_{extra} \simeq \ln^4(G_Q/G)$$



How about statistics?

- Yes we can do this
- ξ – dependence of the action

$$\mathcal{S}_{\rm se} = -\, g_T T e^{-W(T^{-1}+\xi)} \left\{ \left[2 - e^{(W+V)\xi} - e^{W\xi} \right] (\xi^{-1} - T) V_{\rm th}^{-1} + e^{W\xi} \right\},$$

$$\mathcal{S}_{\rm cot} = \dot{H}_{\rm cot}\xi$$

- Saddle point?

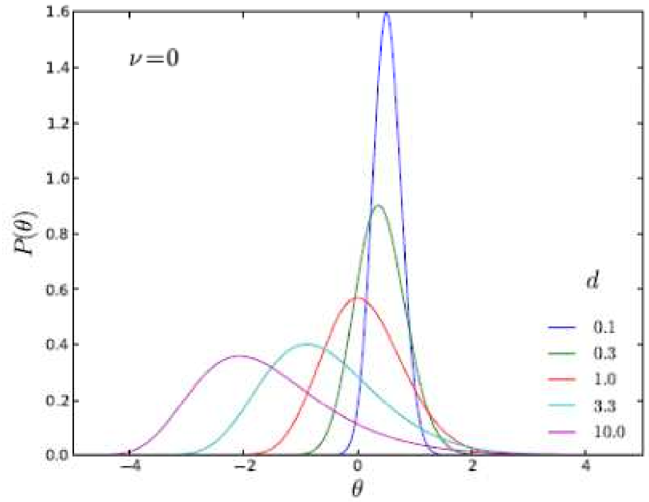
Fokker-Plank equation

- Expansion in ξ till second order
  - θ rescaled temperature

$$\frac{\pi^2 t_C^3}{3\delta_I \alpha g_T^2}\frac{\partial}{\partial \tau}\mathcal{P}(\theta,\tau) = -\frac{\partial}{\partial\theta}\left\{\left[-(\theta-\nu)e^{\theta}+1-\frac{\delta_I}{\pi^2 V_C t_C^5}e^{\theta}\right]\mathcal{P}(\theta,\tau) - \frac{\delta_I}{\pi^2 V_C t_C^5}e^{\theta}\frac{\partial}{\partial\theta}\mathcal{P}(\theta,\tau)\right\}.$$

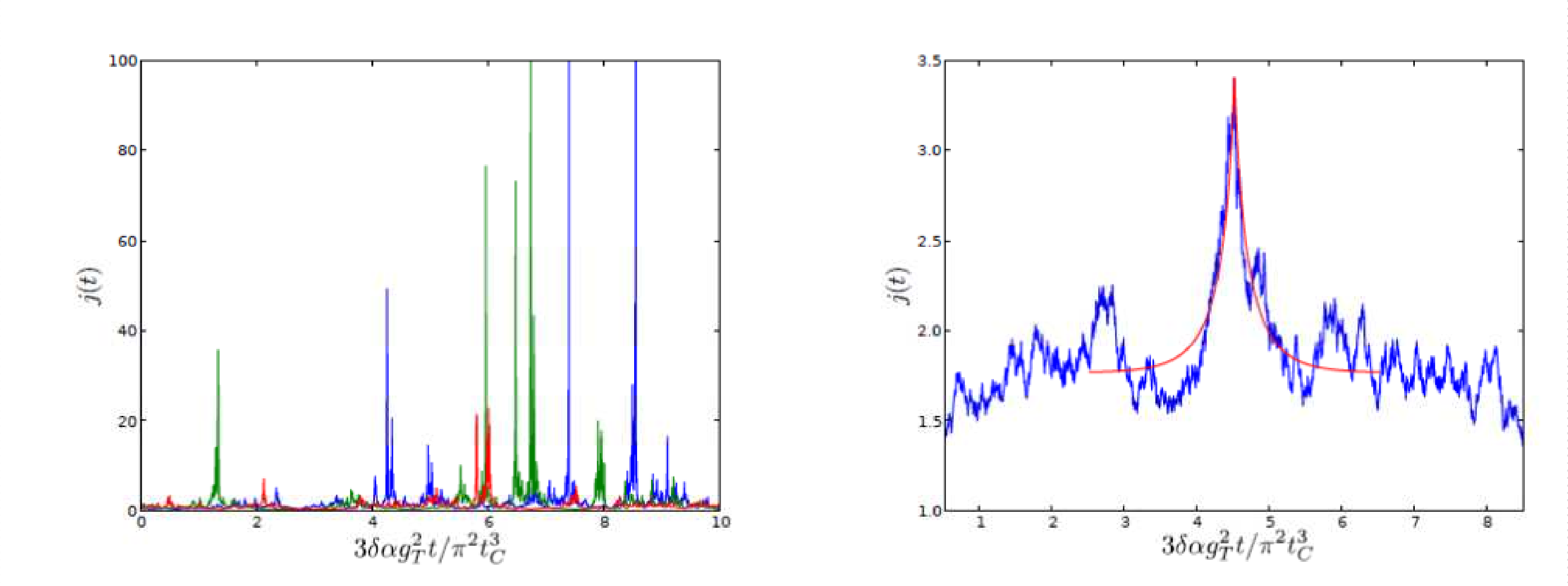
- Distribution function

$$\mathcal{P}_{\rm st}(\theta) \propto \exp\left\{-\frac{\pi^2 V_C t_C^5}{\delta_I}\left(\frac{1}{2}\theta^2 - \nu\theta + e^{-\theta}\right) - \theta\right\}$$



Gigantic fluctuations

- Anomalous senitivity
- Power-law distribution (tunable)



Conclusions, perspectives

- Funny interplay of co+se
- Thermometer?
- Counting statistics
- Towards gigantic fluctuations of temperature