Magnetoelectic effects in semiconductors with Rashba and Dresselhaus spin-orbit interaction

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Overview

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2. **Quantum-kinetic equation**
3. **Spin-remagnetization modes**
4. **Anomalous Hall effect**
5. **Spin Hall current**

Introduction

- **Spintronics: Generation and manipulation of nonequilibrium spin densities by exclusively electrical means in nonmagnetic semiconductors.**
- **Spin-orbit interaction (SOI) gives rise to an effective magnetic field $\longrightarrow$ magnetoelectric effect that profits from spin-charge coupling.**
- **Bulk and structural inversion asymmetry result in Dresselhaus and Rashba SOI, which depend on the in-plane k vector $\longrightarrow$ inhomogeneous broadening.**
- **Most important are electric-field mediated spin excitations in semiconductors.**

Quantum-kinetic equation (Hamiltonian)

Two-dimensional electron gas, SOI and in-plane electric field $\boldsymbol{E}$

$$H_0 = \sum_{k,s} a^\dagger_{ks}\left[\varepsilon_k - \varepsilon_F\right] a_{ks} - \sum_{k,s,s'} \left(\hbar\vec{\omega}_k \cdot \vec{\sigma}_{ss'}\right) a^\dagger_{ks} a_{ks'}$$

$$-ie\vec{E}\sum_{k,s} \nabla_\kappa a^\dagger_{k-\frac{\kappa}{2}s} a_{k+\frac{\kappa}{2}s}\Big|_{\kappa=0} + u\sum_{k,k'}\sum_{s} a^\dagger_{ks} a_{k's}$$

$\vec{\sigma}$ vector of Pauli matrices, ε_F Fermi energy, $\varepsilon_k = \hbar^2 k^2/2m^*$, general class of linear SOI: $\omega_i(k) = \alpha_{ij} k_j$, for instance Rashba-Dresselhaus model: $\alpha_{11} = -\alpha_{22} = \beta$, $\alpha_{12} = -\alpha_{21} = \alpha$.

Spin-density matrix:

$$f^s_{s'}(k, k' \mid t) = \langle a^\dagger_{ks} a_{k's'} \rangle_t$$

with the physical components $f = \mathrm{Tr}\widehat{f}$ (charge density) and $\vec{f} = \mathrm{Tr}(\vec{\sigma}\widehat{f})$ (spin density).

Quantum-kinetic equation (general form)

New vectors: $\boldsymbol{k} \to \boldsymbol{k} + \boldsymbol{\kappa}/2$, $\boldsymbol{k}' \to \boldsymbol{k} - \boldsymbol{\kappa}/2$, ($\boldsymbol{\kappa} \to$ inhomogeneity)

Equation for the **charge density**:

$$\frac{\partial}{\partial t} f(\boldsymbol{k}, \boldsymbol{\kappa}|t) - \frac{i\hbar}{m^*}(\boldsymbol{k} \cdot \boldsymbol{\kappa}) f - \frac{i}{\hbar}\vec{\omega}_{\boldsymbol{\kappa}} \cdot \vec{f} + \frac{e}{\hbar}\boldsymbol{E} \cdot \nabla_{\boldsymbol{k}} f$$
$$= \sum_{s,s_1,s_2} \sum_{\boldsymbol{k}'} \left\{ f_{s_2}^{s_1}(\boldsymbol{k}', \boldsymbol{\kappa} \mid t) W_{s_2 s}^{s_1 s}(\boldsymbol{k}', \boldsymbol{k}, \boldsymbol{\kappa}) - f_{s_2}^{s_1}(\boldsymbol{k}, \boldsymbol{\kappa} \mid t) W_{s_2 s}^{s_1 s}(\boldsymbol{k}, \boldsymbol{k}', \boldsymbol{\kappa}) \right\}$$

Equation for the **spin density**:

$$\frac{\partial}{\partial t} \vec{f}(\boldsymbol{k}, \boldsymbol{\kappa}|t) - \frac{i\hbar}{m^*}(\boldsymbol{k} \cdot \boldsymbol{\kappa}) \vec{f} - \frac{2}{\hbar}\vec{\omega}_{\boldsymbol{k}} \times \vec{f} - \frac{i}{\hbar}\vec{\omega}_{\boldsymbol{\kappa}} f + \frac{e}{\hbar}(\boldsymbol{E} \cdot \nabla_{\boldsymbol{k}})\vec{f}$$
$$= \sum_{\boldsymbol{k}', s_i} \left\{ f_{s_2}^{s_1}(\boldsymbol{k}', \boldsymbol{\kappa} \mid t) W_{s_2 s_4}^{s_1 s_3}(\boldsymbol{k}', \boldsymbol{k}, \boldsymbol{\kappa}) - f_{s_2}^{s_1}(\boldsymbol{k}, \boldsymbol{\kappa} \mid t) W_{s_2 s_4}^{s_1 s_3}(\boldsymbol{k}, \boldsymbol{k}', \boldsymbol{\kappa}) \right\} \vec{\sigma}_{s_3 s_4}$$

Kinetic equation (elastic scattering)

$$u\sum_{k,k'}\sum_{s} a^{\dagger}_{ks}a_{k's}, \quad \frac{1}{\tau} = \frac{2\pi u^2}{\hbar}\sum_{k'}\delta(\varepsilon(k)-\varepsilon(k'))$$

Approximations made in the calculation: No κ dependence in scattering, weak SOI ($\hbar^2 K^2/m^*\varepsilon_k \ll 1$, $K = m^*\alpha/\hbar^2$), no virtual transitions.

The final coupled set of **quantum-kinetic equations** for the spin-density matrix:

$$\frac{\partial}{\partial t}f(k,\kappa|t) - \frac{i\hbar}{m^*}\vec{k}\cdot\vec{\kappa}f - \frac{i}{\hbar}\vec{\omega}_\kappa\cdot\vec{f} + \frac{e}{\hbar}\vec{E}\cdot\nabla_k f = \frac{1}{\tau}(\overline{f} - f)$$

$$\frac{\partial}{\partial t}\vec{f}(k,\kappa|t) - \frac{i\hbar}{m^*}(\vec{k}\cdot\vec{\kappa})f - \frac{2}{\hbar}\vec{\omega}_k \times \vec{f} - \frac{i}{\hbar}\vec{\omega}_\kappa f + \frac{e}{\hbar}(\vec{E}\cdot\nabla_k)\vec{f}$$
$$= \frac{1}{\tau}(\overline{\vec{f}} - \vec{f}) + \frac{\vec{\omega}_k}{\tau}\frac{\partial}{\partial\varepsilon(k)}\overline{f} - \frac{1}{\tau}\frac{\partial}{\partial\varepsilon(k)}\overline{\vec{\omega}_k f}$$

$F = \overline{f}$ denotes an integration over the polar angle of the vector k.

Quantum-kinetic equation (solution)

Four methods are applied:

- **momentum method $\longrightarrow$ spin-remagnetization modes**
- **perturbation theory $\longrightarrow$ AHE, spin-Hall current**
- **drift-diffusion approach (strong SOI)**
- **drift-diffusion approach (weak SOI)**

Spin-remagnetization modes

$\kappa = 0$ so that there is no coupling between spin and charge components.

$$s\vec{f} - 2\vec{\omega}_k \times \vec{f} + \frac{e}{\hbar}\left(\vec{E}\cdot\nabla_k\right)\vec{f} = \frac{1}{\tau}(\vec{F} - \vec{f}) + \vec{f}^{(0)}$$

Momentum method: The decoupling $\overline{k_l k_m f_n} = (k^2/2)\delta_{l,m}F_n$ is exact at $E = 0$. We adopt this approximation also for weak fields.

$$F_z(s) = \frac{(\widehat{U}\mu\vec{E}\times\vec{f}^{(0)})_z}{s\left[(s+2/\tau_s)^2 + (eH_{\text{eff}}/mc)^2\right] + g^2\left[s + 2/\tau_s + (\mu E)^2/D\right]}$$

with the abbreviations:

$$\widehat{U} = \frac{2m^*}{\hbar^2}\left[s\widehat{\alpha} + g\widehat{\sigma_y}\widehat{\alpha}\widehat{\sigma_y}\right], \quad g = 4D\frac{m^2}{\hbar^4}|\widehat{\alpha}|, \quad H_{\text{eff}} = \frac{2m^{*2}c}{e\hbar^2}\widehat{\alpha}\mu E$$

$\frac{2}{\tau_s} = \frac{4Dm^2}{\hbar^4}\left(\alpha_{11}^2 + \alpha_{12}^2 + \alpha_{21}^2 + \alpha_{22}^2\right)$, D diffusion coefficient, μ mobility.

Spin-remagnetization modes

There are **three spin-remagnetization modes** (the zeros of the denominator). For instance for the Rashba model ($\beta = 0$, $E_y = 0$):

$$F_z(t) = \frac{\epsilon}{\sqrt{1-\epsilon^2}} e^{-3t/2\tau_s} \sinh\left(\sqrt{1-\epsilon^2}\frac{t}{2\tau_s}\right) f_x^{(0)}, \quad \epsilon = \frac{\mu E_x}{DK}$$

Damped out-of-plane spin rotation for $DK/\mu < E_x$ ($K = m^*\alpha/\hbar^2$) with the complex eigenfrequency:

$$\omega = 2DK^2\left(\sqrt{\epsilon^2-1}+3i\right), \quad 1/\tau_s = 4DK^2$$

Further excitation: persistent spin helix (as explained later).

Corresponding pseudospin excitations in graphene.

Perturbation approach (in E and κ)

Expansion with respect to E and κ by exploiting the formal exact solution

$$\vec{f}(k|s) = \frac{\sigma\vec{r} - 2\,\vec{\omega}_k \times \vec{r} + 4\,\vec{\omega}_k(\vec{\omega}_k \cdot \vec{r})/\sigma}{\sigma^2 + 4\omega_k^2}$$

with the abbreviations $\sigma = s + 1/\tau$, $\vec{r} = \vec{R} + \overline{\vec{f}}/\tau$ and

$$R = \frac{i\hbar}{m}(\kappa \cdot k)\vec{f} - i\vec{\omega}_\kappa f - \frac{e\vec{E}}{\hbar}\nabla_k \vec{f} + \frac{1}{\tau}\frac{\partial}{\partial \varepsilon_k}\overline{f\hbar\vec{\omega}_k} - \frac{\hbar\vec{\omega}_k}{\tau}\frac{\partial}{\partial \varepsilon_k}\overline{f}$$

To lowest order in E and κ:

$$f_{00} = \frac{n(\varepsilon_k)}{s} = \overline{f_{00}}, \quad \vec{f}_{00} = -\hbar\vec{\omega}_k\frac{n'}{s}, \quad \overline{\vec{f}_{00}} = 0, \quad \text{with} \quad n' = \frac{dn(\varepsilon_k)}{d\varepsilon_k}$$

Furthermore: The expression for $\vec{f}_{0E}$ is obtained from $\vec{f}_{\kappa 0}$ by the replacement $\kappa \longrightarrow -ieE\vec{e}_x\partial_\varepsilon$

Anomalous Hall effect (basic theory)

Nonanalytic behavior of the AHE in the limit $\omega \to 0$, $\tau \to \infty$ for the Rashba-Dresselhaus model.

$$\hbar\vec{\omega}_k = (-\alpha k_y + \beta k_x, \alpha k_x - \beta k_y, M)$$

(homogeneous out-of-plane magnetization M)

The charge current is given by the time derivative of the dipole moment:

$$j_\alpha(t) = \frac{\mathrm{d}e < \vec{r}_\alpha >}{\mathrm{d}t} = \frac{e}{\hbar}\sum_k \frac{\partial \varepsilon_k}{\partial k_\alpha} f(k \mid t) - \frac{e}{\hbar}\sum_k \frac{\partial \hbar\vec{\omega}_k}{\partial k_\alpha} \cdot \vec{f}(k \mid t)$$

$\sigma_{xy} = \sigma'_{xy} + \sigma''_{xy}$ results from an expansion with respect to the electric field. (pure Hall conductivity when $\sigma'_{xy} = 0$)

Anomalous Hall effect (exact result)

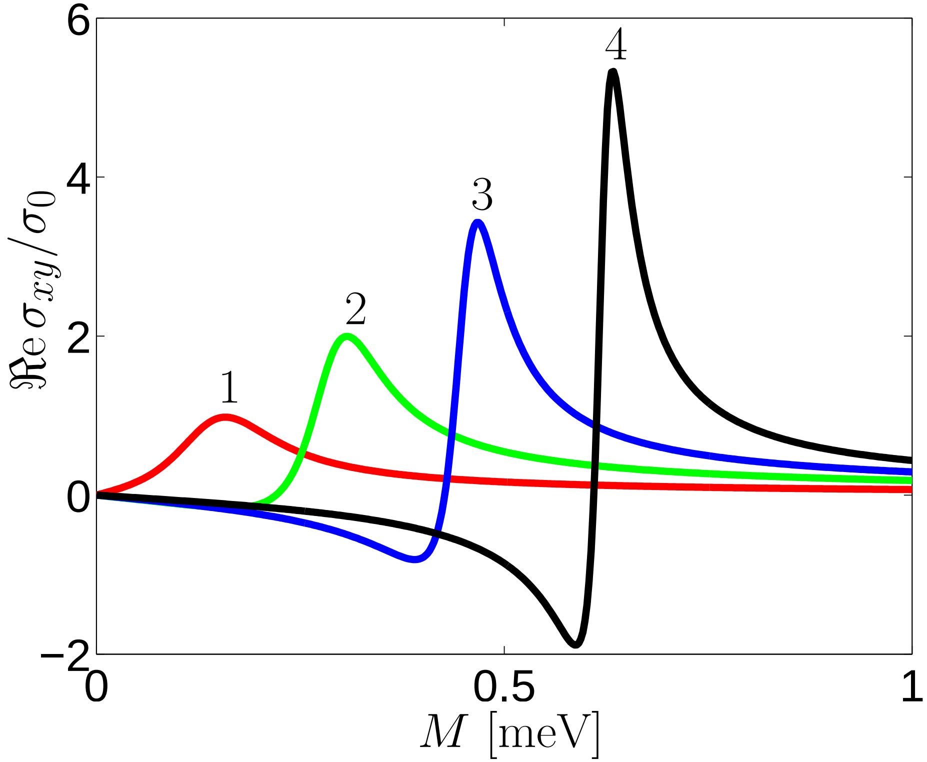
$$\sigma_{xy}(\omega) = -\frac{e^2}{\hbar^2}\frac{\omega\tau}{\omega + i/\tau}(\alpha^2 - \beta^2)\sum_k n'(\varepsilon_k)\frac{P_0(k \mid \omega)}{P_+(k \mid s)P_-(k \mid \omega) + P_0(k \mid \omega)^2}$$

$$P_\pm(k \mid \omega) = 1 - \frac{1}{2\sigma\tau}\left(1 + \sqrt{\frac{1 \pm \gamma}{1 \mp \gamma}}\right)\left[1 - \frac{\delta}{1 + \sqrt{1 - \gamma^2}}\right] + \frac{2}{\sigma}\frac{M}{\hbar}P_0(k \mid \omega)$$

$$P_0(k \mid \omega) = \frac{2}{\tau}\frac{M}{\hbar}\frac{1}{(\sigma^2 + 4\omega_k^2)\sqrt{1 - \gamma^2}},\ \gamma = \frac{2\alpha\beta}{\alpha^2 + \beta^2}\delta,\ \delta = \frac{4(\omega_k^2 - (M/\hbar)^2)}{\sigma^2 + 4\omega_k^2}$$

- **no AHE for the special Rashba-Dresselhaus model $\alpha = \beta$**
- **no $\omega = 0$ AHE for disordered samples**
- **nonanalytic behavior in the limit $\omega \to 0$ and $\tau \to \infty$**

Anomalous Hall effect (numerical results)



ac conductivity, Rashba model, resonances due to SOI. Parameters: $\omega\tau = 0.5$ (1), 1 (2), 1.5 (3), and 2 (4). σ_0 is given by $e^2 m\alpha^2\tau/(\pi\hbar^4)$. In addition: $\beta = 0$, $\alpha = 10^{-9}$ eVcm, $\tau = 1$ ps, and $n = 10^{10}$ cm^{-2}

Spin-Hall current

Two different definitions for the spin current (which does not enter Maxwells equations):

(1) "**conventional**": $$\widehat{j}^{s}(s) \sim \sum_{k} \nabla_{k}\varepsilon_{k} \otimes \vec{f}(k,\kappa \mid s)\,|_{\kappa=0}$$

(2) "**more physical**": $$\widehat{j}^{s}(s) \sim \sum_{k} \nabla_{\kappa} \otimes \vec{f}(k,\kappa \mid s)\,|_{\kappa=0}$$

According to (2): the spin current is given by the time derivative of the spin displacement. (physical motivation: J. Shi et al. Phys. Rev. Lett. **96**, 076604 (2006)).

"conventional" spin-Hall current

We treat the Rashba model and obtain a nonvanishing stationary spin-Hall current that is independent of the electric field!

$$j_y^x(s) = \frac{\hbar K}{2ms} \sum_k \varepsilon_k n'(\varepsilon_k)$$

which gives $j_y^x(\omega) = -K\varepsilon_F/(2\pi\hbar)$ at zero temperature. The field-induced spin-Hall current results from

$$\vec{\overline{f}}_{0E}(k|s) = \frac{\vec{\omega}_E}{\sigma s}\left\{(\varepsilon_k n')' - \frac{2\sigma\tau\omega_k^2 n'}{\sigma^2 s\tau + 2\omega_k^2(2s\tau+1)}\right\}, \quad \vec{\omega}_E = \frac{e\hbar}{m^*}(K \times \vec{E})$$

In clean samples $j_y^z(\omega = 0) = -eE/8\pi\hbar$ (universal dependence); in disordered samples no spin-Hall current j_y^z.

"physical" spin-Hall current

(1) there is also a contribution independent of the electric field:

$$j_y^x(s) = -\frac{\hbar K}{m} \sum_k n(\varepsilon_k) \frac{\omega_k^2 \tau}{\sigma^2 s\tau + 2\omega_k^2(2s\tau + 1)}$$

But it results from the initial spin accumulation according to $sf^\alpha(\varepsilon|s)n/2 = eE_\beta j_\beta^\alpha(\varepsilon|s)n'$. It disappears in the steady state.

(2) General transport in an infinite system: First the volume and afterwards the time goes to infinity (first $\kappa \longrightarrow 0$ and than $\omega \longrightarrow 0$). This procedure leads to the exact result (for a clean system):

$$j_y^z(\omega \longrightarrow 0) = -\frac{eE}{8\pi\hbar} \ln \frac{\omega}{2\omega_{k_F}}$$

Summary

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Summary

- We treated the influence of an in-plane electric field on a 2DEG with SOI and short-range elastic scattering. **Quantum-kinetic equations** were rigorously derived for the spin-density matrix. Analytic solutions were obtained by exploiting the momentum method and perturbation theory.
- Three **spin-remagnetization modes** were identified that are due to the electric field. These (resonant) rotations of the spin density have a simple origin.
- The **AHE** for a system with Rashba-Dresselhaus SOI was treated. The intrinsic AHE caused by the electric field is exactly cancelled by collisions (Smit (1955)). An ac electric field induces an AHE with a linear slope in M also for disordered samples.
- The "conventional" definition of the **spin-Hall current** has serious defects. The proper approach starts from the time derivative of the spin displacement. (In general, both definitions differ even for the charge transport!)

Overview

1. Introduction
2. Drift-diffusion approach 1
3. Drift-diffusion approach 2
4. Summary

Introduction

- Starting from the kinetic equations for the spin-density matrix, coupled spin-charge drift-diffusion equations are derived by two methods:
  1. rigorous expansion with respect to κ (arbitrary strength of SOI)
  2. approach for the drift-diffusion evolution period (weak SOI)
- Applications refer to the spin accumulation in a strongly confined two-dimensional hole gas and on spin-remagnetization waves on a cylindrical surface

Drift-diffusion approach ① (starting point)

Again, we start from the quantum-kinetic equations for the spin and charge degrees of freedom:

$$\frac{\partial}{\partial t} f(k,\kappa|t) - \frac{i\hbar}{m^*}(\vec{k}\cdot\vec{\kappa}) f - \frac{i}{\hbar}\vec{\omega}_{\kappa}\cdot\vec{f} + \frac{e}{\hbar}\vec{E}\cdot\nabla_k f = \frac{1}{\tau}(\overline{f} - f)$$

$$\frac{\partial}{\partial t}\vec{f}(k,\kappa|t) - \frac{i\hbar}{m^*}(\vec{k}\cdot\vec{\kappa})\vec{f} - \frac{2}{\hbar}\vec{\omega}_k \times \vec{f} - \frac{i}{\hbar}\vec{\omega}_{\kappa} f + \frac{e}{\hbar}(\vec{E}\cdot\nabla_k)\vec{f}$$
$$= \frac{1}{\tau}(\overline{\vec{f}} - \vec{f}) + \frac{\vec{\omega}_k}{\tau}\frac{\partial}{\partial\varepsilon(k)}\overline{f} - \frac{1}{\tau}\frac{\partial}{\partial\varepsilon(k)}\overline{\vec{\omega}_k f}$$

For the sake of simplicity, let us treat the Rashba model. These equations are nothing but a set of coupled linear equations, which have the following explicit form:

Drift-diffusion approach ① (method)

$$\sigma f + i\Omega(q_x f_y - q_y f_x) = R$$

$$\sigma f_x + 2\Omega\cos(\varphi) f_z - i\Omega q_y f = R_x + \Omega\sin(\varphi)\frac{\hbar}{\tau}\frac{\partial \overline{f}}{\partial \varepsilon_k}$$

$$\sigma f_y + 2\Omega\sin(\varphi) f_z + i\Omega q_x f = R_y - \Omega\cos(\varphi)\frac{\hbar}{\tau}\frac{\partial \overline{f}}{\partial \varepsilon_k}$$

$$\sigma f_z - 2\Omega\left[\cos(\varphi) f_x + \sin(\varphi) f_y\right] = R_z$$

with: $\Omega = \omega_k \tau$, $q_{x,y} = \kappa_{x,y}/k$, $\sigma = \sigma_0 - i\Omega k(q_x \cos\varphi + q_y \sin\varphi)/K$,

$$R = \overline{f} + \tau f_0, \quad R_x = \overline{f}_x + \tau f_{0x} - \frac{\hbar}{\tau}\frac{\partial}{\partial \varepsilon_k}\Omega\overline{f\sin(\varphi)}$$

$$R_z = \overline{f}_z + \tau f_{0z}, \quad R_y = \overline{f}_y + \tau f_{0y} + \frac{\hbar}{\tau}\frac{\partial}{\partial \varepsilon_k}\Omega\overline{f\cos(\varphi)}$$

- The exact solution is expanded up to second order in $q_{x,y}$. Integration over the angle φ; $\sigma_0 = s\tau + 1$
- For $t \gg \tau$ ($s\tau \ll 1$): $\overline{f}(\varepsilon_k, q|s) = n(\varepsilon_F)F(q|s)$

Drift-diffusion approach (1) (solution)

The final equations are given for $\overline{f}$, $\overline{f}_r = i(\vec{\kappa} \times \vec{\overline{f}})_z$, $\overline{f}_d = i\vec{\kappa}\vec{\overline{f}}$, $\overline{f}_z$

$$\left[s + D_0(s)\kappa^2\right]\overline{f} + V(s)\overline{f}_r = f_0$$

$$\left[s + \frac{1}{\tau_{s\perp}(s)} + D_r(s)\kappa^2\right]\overline{f}_r - V_r(s)\kappa^2\overline{f} = f_{r0}$$

$$\left[s + \frac{1}{\tau_{sz}(s)} + D_z(s)\kappa^2\right]\overline{f}_z - V_z(s)\overline{f}_d = f_{z0}$$

$$\left[s + \frac{1}{\tau_{s\perp}(s)} + D_d(s)\kappa^2\right]\overline{f}_d - V_d(s)\kappa^2\overline{f}_z = f_{d0}$$

with $\tau_{s\perp}(s) = \tau_s \dfrac{\sigma_0^2 + 2\Omega^2}{\sigma_0}$, $\tau_{sz}(s) = \tau_s \dfrac{\sigma_0}{2}$, $\dfrac{1}{\tau_s} = 4DK^2 = 2\dfrac{\Omega^2}{\tau}$ and

$$D_0(s) = \frac{D}{\sigma_0^2},\ D = v^2\tau/2, \quad D_z(s) = \frac{\sigma_0^2 - 12\Omega^2}{(\sigma_0^2 + 4\Omega^2)^2}D, \ldots$$

In the strong-coupling limit a non-Markovian behavior occurs.

But what is most astonishing?

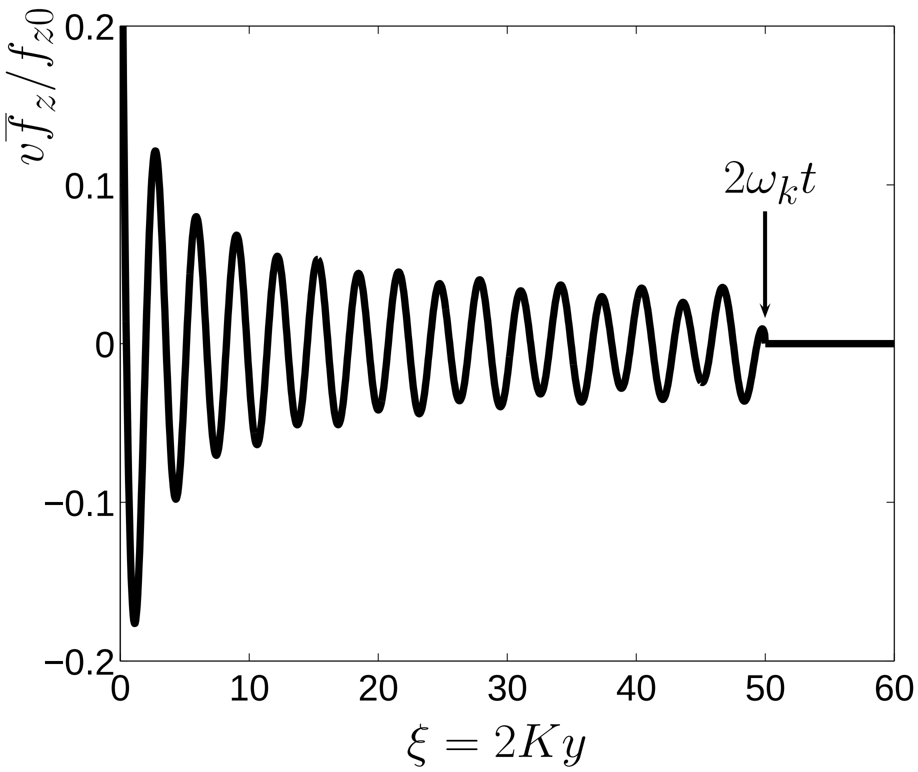
Drift-diffusion approach (1) (discussion)

- For strong coupling ($\Omega > 1/\sqrt{12}$) the diffusion coefficient D_z becomes negative $\longrightarrow$ instability in the spin system! (similar results by an alternative approach: T.D. Stanescu and V. Galitski Phys. Rev. B **75**, 125307 (2007))
- **However:** $\overline{f}_z$ and $\overline{f}_d$ rapidly change on the length scale of the mean-free path (q expansion?)
- **Nevertheless:** spin oscillations also appear from the exact solution of the kinetic equation in the ballistic regime

$$f_z(\kappa, s) = \frac{f_{z0}}{\beta(\kappa, s) + 4\omega_k^2/\beta(\kappa, s)}, \; \beta(\kappa, s) = s - i(\kappa \cdot v), \; v = \hbar k/m^*$$

Drift-diffusion approach ① (illustration)

Half plane $y > 0$ with a given spin polarization at the boundary:



Spin polarization $v\overline{f}_z/f_{z0}$ as a function of $\xi = 2Ky$ calculated for the half space $y > 0$.

Drift-diffusion approach ① (application)

We treat a two-dimensional hole gas and an in-plane electric field, which couples spin and charge degrees of freedom.

$$\left(s + i\frac{v_d}{\sigma_0}\kappa_x + D_0\kappa^2\right)\overline{f} + i\Gamma_z\kappa_y\overline{f}_z = f_0$$

$$\left(s + \frac{1}{\tau_{sz}} + i\frac{v_d}{\sigma_0}\kappa_x + D_z\kappa^2\right)\overline{f}_z + i\Gamma_0\kappa_y\overline{f} = 0$$

with the transport coefficients

$$D_0 = \frac{D}{\sigma_0^2}, \quad D_z = D\frac{\sigma_0^2 - 12\Omega^2}{(\sigma_0^2 + 4\Omega^2)^2}, \quad \sigma_0 = s\tau + 1$$

$$\Gamma_0 = v_d\frac{9\Omega^2}{2\gamma}\frac{3\sigma_0^2 - 4\Omega^2}{(\sigma_0^2 + 4\Omega^2)^2}, \quad \frac{1}{\tau_{sz}} = \frac{4\Omega^2}{\sigma_0\tau}$$

$$\Gamma_z = v_d\frac{9\Omega^2}{2\gamma}\sigma_0^2\frac{4\sigma_0\Omega^2 + 8\Omega^2 - 3\sigma_0^2 s\tau}{(\sigma_0 s\tau + 4\Omega^2)(\sigma_0^2 + 4\Omega^2)}$$

Coupling is due to the drift velocity $v_d = eE\tau/m^*$.

Drift-diffusion approach (1) (stripe)

As an example, a stripe geometry is treated ($-L_0 \leq y \leq L_0$, $\kappa_y \rightarrow i\partial/\partial y$). The electric field at the boundaries $E_y(y = \pm L_0) = \pm E_0$ is used to solve the Poisson equation $dE_y/dy = 4\pi e(\overline{f} - f_0)/\varepsilon$. We obtain the analytic solution

$$\boxed{\overline{f}_z(y) = -\frac{\varepsilon E_0 \Gamma_0}{4\pi e}\tau_{sz}\frac{\lambda_1^2\lambda_2^2}{\lambda_1^2 - \lambda_2^2}\left[\frac{\sinh(\lambda_1 y)}{\sinh(\lambda_1 L_0)} - \frac{\sinh(\lambda_2 y)}{\sinh(\lambda_2 L_0)}\right]}$$

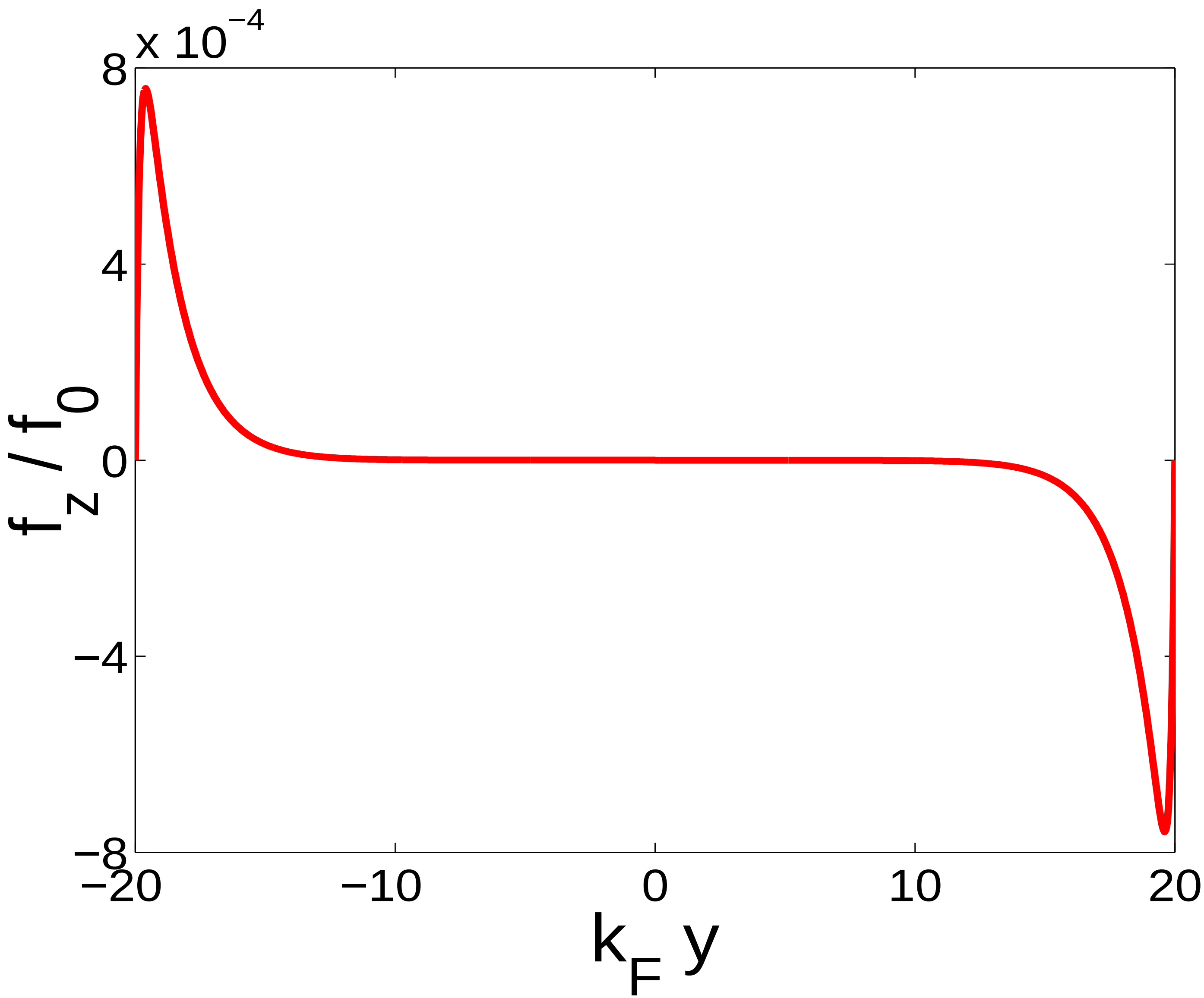
with $\lambda_{1,2}$ being the solution of the characteristic equation

$$\lambda^2\Gamma_0\Gamma_z - \left(\lambda^2 D_z - \frac{1}{\tau_{sz}}\right)\left[\lambda^2 D_0 + \frac{4\pi e}{\varepsilon}\mu f_0\right] = 0$$

Two different classes of solutions:

Drift-diffusion approach (1) (figure)

weak coupling



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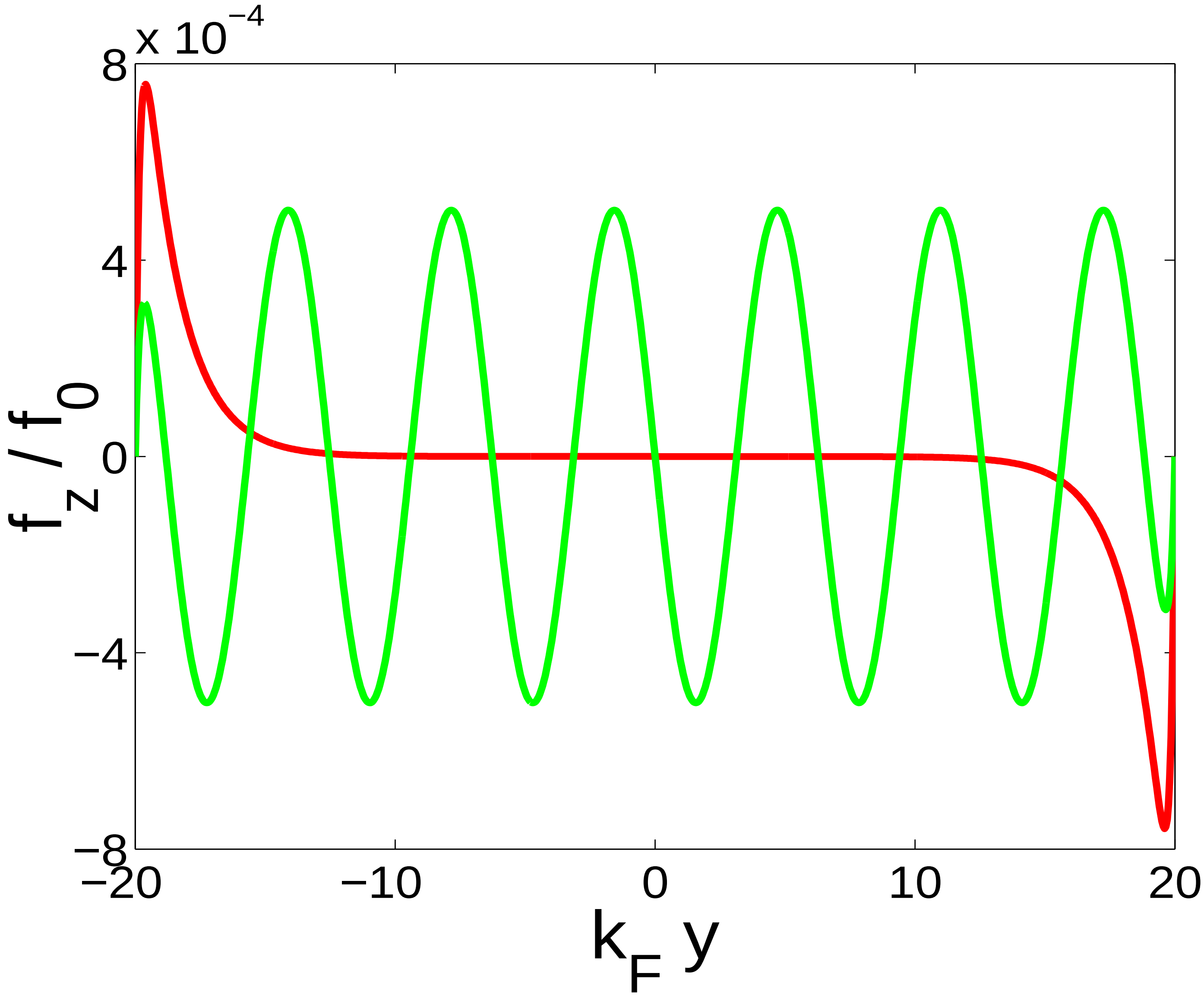


Drift-diffusion approach (2) (starting point)

- The second derivation of spin-charge coupled drift-diffusion equations applies for **weak SOI**. The procedure is illustrate for a 2DEG on a **cylindrical surface**. The model Hamiltonian [L. I. Magarill, D. A. Romanov, A. V. Chaplik, Zh. Eksp. Teor. Fiz. **86**, 771 (1998)]:

$$H_0 = \int_0^{2\pi} \frac{d\varphi}{2\pi} \sum_{k_z} \Big\{ \sum_s a^{\dagger}_{k_z s}(\varphi) \left[\frac{\hbar^2 k_z^2}{2m^*} + \frac{\widehat{p}_\varphi^2}{2m^*} \right] a_{k_z s}(\varphi)$$

$$+\alpha \sum_{s,s'} a^{\dagger}_{k_z s}(\varphi) \left[\sigma^z_{ss'} \widehat{p}_\varphi - \hbar k_z \Sigma_{ss'} \right] a_{k_z s'}(\varphi)$$

$$+\beta \sum_{s,s'} a^{\dagger}_{k_z s}(\varphi) \left[\frac{1}{2} \left(\Sigma_{ss'} \widehat{p}_\varphi + \widehat{p}_\varphi \Sigma_{ss'} \right) - \hbar k_z \sigma^z_{ss'} \right] a_{k_z s'}(\varphi) \Big\}$$

with the abbreviations:

$$\widehat{p}_\varphi = -\frac{i\hbar}{R} \frac{\partial}{\partial \varphi}, \quad \widehat{\Sigma} = \begin{pmatrix} 0 & -ie^{-i\varphi} \\ ie^{i\varphi} & 0 \end{pmatrix}$$

Drift-diffusion approach (2) (model)

The periodic boundary condition $\longrightarrow$ discrete Fourier transformation:

$$a_{k_z\uparrow}(\varphi) = \sum_{m=-\infty}^{\infty} e^{im\varphi} a_{k_z m\uparrow} \,, \quad a_{k_z\downarrow}(\varphi) = e^{i\varphi} \sum_{m=-\infty}^{\infty} e^{im\varphi} a_{k_z m\downarrow}$$

In addition: elastic scattering and electric field along the cylinder axis.

$$H = \sum_{k,s} \varepsilon(k) a^\dagger_{ks} a_{ks} + \sum_{k} \sum_{s,s'} (\hbar\vec{\omega}_1(k)\cdot\vec{\sigma}_{ss'}) a^\dagger_{ks} a_{ks'} + U \sum_{k,k'} \sum_{s} a^\dagger_{ks} a_{k's} - ie\vec{E}\cdot \sum_{k,s} \nabla_\kappa a^\dagger_{k-\frac{\kappa}{2}s} a_{k+\frac{\kappa}{2}s}\Big|_{\kappa=0}$$

$$\vec{k} = (k_\varphi, k_z, 0),\; k_\varphi = \left(m + \frac{1}{2}\right)/R,\; \varepsilon(k) = \frac{\hbar^2 k^2}{2m^*} - \frac{\hbar}{2R}\left(\alpha - \frac{\hbar}{4m^* R}\right)$$

$$\vec{\omega}_1(k) = \left(0, -(\alpha k_z - \beta k_\varphi), k_\varphi(\alpha - \frac{\hbar}{2m^* R}) - \beta k_z\right)$$

Drift-diffusion approach ② (kinetic eqs.)

Spin-density matrix: $\vec{f}(k, k'|s) = \sum_{s,s'} f^{s}_{s'} \vec{S}_{ss'}$

Shift of momentum vectors: $k \to k + \kappa/2$, $k' \to k - \kappa/2$

$$S^{\varphi} = \frac{1}{2}\begin{pmatrix} 0 & ie^{2i\varphi} \\ -ie^{-2i\varphi} & 0 \end{pmatrix}, \; S^{z} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \; S^{r} = \frac{1}{2}\begin{pmatrix} 0 & e^{2i\varphi} \\ e^{-2i\varphi} & 0 \end{pmatrix}$$

The derivation of kinetic equations proceeds as for a planar 2DEG.

The final result has the same form, however, with a specific vector:

$$\boxed{\vec{\omega}_{\kappa} = (\omega_{1y}(\kappa)\sin(2\varphi), \omega_{1y}(\kappa)\cos(2\varphi), -\omega_{1z}(\kappa))}$$

Again, an integration over the polar angle χ of the momentum vector $\vec{k} = k(\cos\chi, \sin\chi, 0)$ is necessary.

Drift-diffusion approach ② (Ansatz)

The main step: treat the evolution period, in which a non-equilibrium spin polarization and charge density still exist, whereas the energy of particles is already thermalized.
In this regime, the following Ansatz is justified:

$$\overline{f}(k,\kappa|t) = -F(\kappa|t)\frac{n'(\varepsilon(k))}{dn/d\varepsilon_F}, \quad \overline{\vec{f}}(k,\kappa|t) = -\vec{F}(\kappa|t)\frac{n'(\varepsilon(k))}{dn/d\varepsilon_F}$$

$n(\varepsilon(k))$ denotes the Fermi function and $n = \int d\varepsilon \rho(\varepsilon) n(\varepsilon)$.

Procedure: expand the exact solution with respect to $\vec{\kappa}$, integrate over α and finally over the energy $\varepsilon(k)$.

Drift-diffusion approach ② (result)

$$\left[\frac{\partial}{\partial t} - i\mu\vec{E}\cdot\vec{\kappa} + D\kappa^2\right] F + \frac{i}{\hbar\mu_B}\left[\vec{\omega}_\kappa - \vec{\Omega}_\kappa\right]\cdot\vec{M} = 0$$

$$\left[\frac{\partial}{\partial t} - i\mu E\cdot\kappa + D\kappa^2 + \widehat{\Gamma}\right]\vec{M} - \frac{e}{m^* c}\vec{M}\times\vec{H}_{\text{eff}} - \chi(\widehat{\Gamma}\vec{H}_{\text{eff}})\frac{F}{n} - \frac{i\mu}{2\tau c}\vec{\Omega}_\kappa F = \vec{G}$$

with: $\vec{M} = \mu_B\vec{F}$, $\mu_B = e\hbar/2m^*c$, $\chi = \mu_B^2 n'$, $\mu = eDn'/n$.

$$\vec{\Omega}_\kappa = \frac{2m^*\tau}{\hbar^2}(\vec{\omega}_\kappa\times\vec{\Lambda}),\ \ \vec{H}_{\text{eff}} = -\frac{2m^{*2}c}{e\hbar^2}(\vec{\Lambda} + 2iD\vec{\omega}_\kappa)$$

$$\vec{\Lambda} = (a_{21}\mu E_\varphi + a_{22}\mu E_z, a_{31}\mu E_\varphi + a_{32}\mu E_z, a_{11}\mu E_\varphi + a_{12}\mu E_z)$$

$$\widehat{\Gamma} = \frac{4Dm^{*2}}{\hbar^4}\begin{pmatrix} a_{11}^2 + a_{12}^2 + a_{31}^2 + a_{32}^2 & -(a_{22}a_{32} + a_{21}a_{31}) & -(a_{11}a_{21} + a_{22}a_{12}) \\ -(a_{22}a_{32} + a_{21}a_{31}) & a_{11}^2 + a_{12}^2 + a_{21}^2 + a_{22}^2 & -(a_{12}a_{32} + a_{11}a_{31}) \\ -(a_{11}a_{21} + a_{22}a_{12}) & -(a_{12}a_{32} + a_{11}a_{31}) & a_{21}^2 + a_{22}^2 + a_{31}^2 + a_{32}^2 \end{pmatrix}$$

$$a_{11} = \beta\sin(2\varphi),\quad a_{21} = \beta\cos(2\varphi),\quad a_{31} = -\left(\alpha - \frac{\hbar^2}{2m^* R}\right)$$

$$a_{12} = -\alpha\sin(2\varphi),\quad a_{22} = -\alpha\cos(2\varphi),\quad a_{32} = \beta$$

Drift-diffusion approach (2) (applications)

(1) **Solution of four linear equations for the charge density and the magnetization. The zeros of the determinant determine the eigenfrequencies of spin-remagnetization waves.**

(2) **Identify spin excitations by an appropriate experimental set up. Similar to physics of space-charge waves. New electric-field-induced modes appear due to the spin-charge coupling.**

① Persistent field-mediated spin mode

Eigenmodes $\omega = \omega(k)$ are solutions of the cubic equation:

$$\Sigma(\sigma^2 + \omega_H^2) + g_2\left(\sigma + \frac{(\mu E)^2}{D}\right) = 0$$

with $\sigma = \Sigma + g_1$, $\omega_H = (e/m^*c)H_{\text{eff}}$, and $\Sigma = i\omega - i\mu E \cdot k + Dk^2$.
Effective coupling constants are given by:

$$g_1 = 2\frac{4Dm^{*2}}{\hbar^2}\left[\alpha^2 + \beta^2 - \frac{\hbar}{2m^*R}\left(\alpha - \frac{\hbar}{4m^*R}\right)\right]$$

$$g_2 = \left(\frac{4Dm^{*2}}{\hbar^2}\right)^2\left[\beta^2 - \alpha\left(\alpha - \frac{\hbar}{2m^*R}\right)\right]^2$$

A long-lived spin excitation exists for the Rashba model ($\beta = 0$) and a given radius of the cylinder: $R = \hbar/2m^*\alpha$.

① Persistent field-mediated spin mode

The persistent spin mode at $k_z \to K = 2m^*\alpha/\hbar$ exists only on a cylindrical surface and has no counterpart in the planar Rashba model:

$$\boxed{\omega_{1,2}(k_z) = -\mu E_z \left(k_z \pm K\right) - iD \left(k_z \pm K\right)^2}$$

The excitation leads to spatial and temporal oscillations of the radial magnetization:

$$\begin{aligned} M_r(z,t) &= \frac{M_{r0}}{2}\Big\{ e^{-D(K_g+K)^2 t} \cos[K_g z + \mu E_z (K_g + K) t] \\ &\quad + e^{-D(K_g-K)^2 t} \cos[K_g z + \mu E_z (K_g - K) t] \Big\} \end{aligned}$$

To excite the spin wave, a regular pattern of spin polarization perpendicular to the cylinder surface can be used, which is provided by laser pulses.

② Spin-mediated space-charge waves

Under illumination, photogeneration of carriers along the cylinder axis:

$$g(z,t) = g_0 \left[1 + m \cos\left(K_g z + \Theta \cos(\Omega t)\right)\right]$$

Basic equations for the treatment of space-charge waves:

$$E(z,t) = E_0 + \delta E(z,t), \qquad \frac{\varepsilon}{4\pi}\frac{\partial E}{\partial t} + j(z,t) = I(t)$$

$$\frac{\partial n}{\partial t} + \frac{n - n_0}{\tau} + \frac{\varepsilon}{4\pi e}\frac{\partial^2 E}{\partial z \partial t} = g(z,t)$$

The spin enters by a modification of the charge current:

$$\boxed{j(z,t) = eD\frac{\partial n}{\partial z} + en\mu E + e\alpha\rho_{\varphi}}$$

② Spin-mediated space-charge waves

Without SOI the steady-state current response is given by the well-known result:

$$f_0 = \frac{\overline{I(t)}}{I_0} = \frac{1}{2}\left(\frac{mg_0}{n^{(0)}\tau_M}\right)^2 \sum_{l=-\infty}^{\infty} J_l^2(\Theta)\frac{1+\lambda l\omega}{|(\Omega-\Omega_1)(\Omega-\Omega_2)|^2}$$

$$\Omega_{1,2} = \frac{1}{2}\left(\frac{d}{\tau}+i\Gamma\right) \pm \sqrt{\frac{1}{4}\left(\frac{d}{\tau}+i\Gamma\right)^2 + \frac{1}{\tau\tau_M}},\ \Gamma = DK_g^2 + \frac{1}{\tau} + \frac{1}{\tau_M}$$

with $\tau_M = \varepsilon/(4\pi e\mu n_0)$ being the Maxwellian relaxation time, $d = \mu E_0 K_g \tau$, $\lambda = DK_g/\mu E_0$.

There are trap recharging waves ($\Omega \sim 1/K_g$) and oscillations of the free electron gas ($\Omega \sim K_g$).

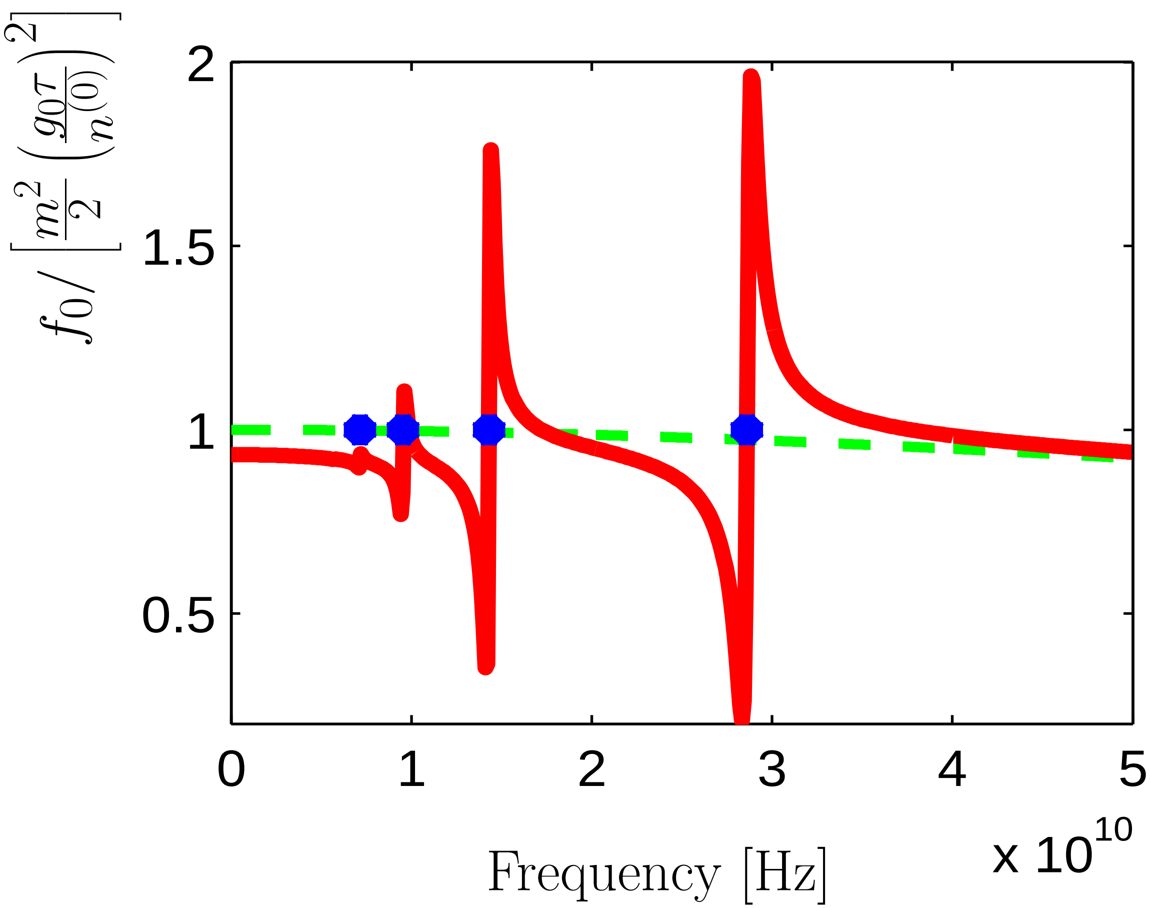
Are there any pronounced spin-mediated excitations?

Yes!

2 Spin-mediated space-charge waves

Under the condition $R \approx \hbar/2m^*\alpha$ sharp resonances appear at:

$$\Omega = \Omega_r/l, \quad \Omega_r = \mu E_0 K_g \left(1 + \frac{\tau}{2m^* D}\frac{\hbar^2 K^2}{2m^*}\right)$$



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Summary

- **Spin-charge coupled drift-diffusion equations are systematically derived from kinetic equations for the spin-density matrix.**
- **There are two complete different regimes: weak and strong SOI.**
- **Oscillations of the spin polarization occurs under non-Markovian conditions that develop at strong SOI.**
- **Numerous physical spin effects are described by the drift-diffusion equations. We focused on the study of spin-remagnetization waves.**

The end

Thank you for your attention