As a rep. theorist algebra A/ ass, over a.c. $\mathbb{K}$

x want $\operatorname{Irr} A$ – f.dim. simple modules.

modules expl. dimensions, ~~characters~~

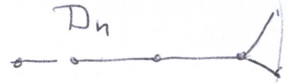
$\underline{\text{Ex}}$ $\mathfrak{g}$ – f.d. simple Lie alg./$\mathbb{C}$, $A = \mathcal{U}(\mathfrak{g})$.

W-algebra

$\operatorname{Irr} A \longleftrightarrow$ dominant int. weights

$L_\lambda = \Gamma(\mathcal{B}(\mathfrak{g}), \mathcal{O}_\lambda) \leftarrow \lambda$, $\mathcal{O}_\lambda$ – ~~lam~~ ample line bundle on $\mathcal{Fl}(\mathfrak{g})$ обильный пучок

$$\dim L_\lambda = \prod_{\alpha \in R^+} \frac{(\lambda+\rho, \alpha)}{(\rho, \alpha)}$$

~~in D_4  or A_n  or D_n ~~

~~$\exists$ external symmetry. S_3 or C_2~~

concrete $\mathcal{U}(\mathfrak{sl}_3)$ $\lambda = (x,y) \in \mathbb{N}^2$

$$\dim L_{(x,y)} = \frac{(x+1)(y+1)\frac{(x+y)}{2}}{2}$$

Let $G : A \circlearrowleft$ it ~~act~~ twists modules. скрученные модули

стабилизатор $M^{[g]} = M$, $a \cdot^{[g]} m = g(a)m$.

$M \in \operatorname{Irr} A$, stabiliser G_M acts on M projectively $g : M \xrightarrow{\cong} M^{[g]}$

$$G_M \longrightarrow PGL(M)$$

get $\pi_M \in H^2(G_M, \mathbb{K}^\times)$. – $\underline{\text{frill}}$

frill $\Leftarrow$ оборка translate to Russia

So, $\operatorname{Irr} A$ is a ~~decorated~~ украшенное G-set i.e

G-set X

\+ ~~frill~~ украшен $\pi_x \in H^2(G_x, \mathbb{K}^\times)$ at each point.

online today

$\boxed{\text{Th}}$ for f.dim s.s. ~~A, B – G-algebras.~~ T.F.A.E.

G-finite group, A, B – G-algebras.

(1) $\exists$ a non-deg. G-eq. Morita context between A and B

(2) A-mod $\sim$ B-mod as G-categories

(3) $A * G$-mod $\sim$ $B * G$-mod as module categories over G-Rep

(4) $\operatorname{Irr} A \cong \operatorname{Irr} B$ as decorated G-sets.

In $\boxed{\text{Out}}$

$\underline{\text{Question:}}$ Suppose $D^b(A) \sim D^b(B)$ as G-categories (A,B – any?) 1h

does $\operatorname{Irr} A \cong \operatorname{Irr} B$? fin. dimensional

2/3

2 лекции, что бы объяснить

Kostant, 78 (lynne) all. in RepTh.

$\mathfrak{g}$ - ss. Lie alg$/\mathbb{C}$, $x \in \mathfrak{g}^*_{nil} \rightsquigarrow I_x \trianglelefteq^{\ell} U(\mathfrak{g})$

$$W_x := \mathrm{End}_{U(\mathfrak{g})}(U(\mathfrak{g})/I_x) \xhookrightarrow{j} U(\mathfrak{g})/I_x$$

$$\varphi \longmapsto \varphi(1+I_x)$$

квантовые редукция

$$\mathrm{Im}\, j = \{a + I_x \mid I_x a \subseteq I_x\} \text{ — "quantum reduction"}$$

Факты:

Key ~~Facts~~ - $\mathrm{gr}(W_x) \cong \mathbb{C}\left[\begin{smallmatrix}\text{Slodowy}\\ \text{Slice}\end{smallmatrix}\right]$ (cf. $\mathrm{gr}(U(\mathfrak{g})) \cong \mathbb{C}[\mathfrak{g}^*]$)

Нётер область

- W_x is a noeth. domain acted upon by $A(x) := G_x/G_x^\circ$

- $Z(\mathfrak{g}) := Z(U(\mathfrak{g})) \longrightarrow W_x$ is an isom. with the centre.
  изоморфизм
  $\|$ Chevall
  $\mathbb{C}[\mathfrak{h}^*/w]$ — affine.
  Cartan (not himself)

Question: Let $W_x^\lambda = W_x \otimes_{Z(\mathfrak{g})} \mathbb{C}(\lambda)$ ① Compute $\mathfrak{A}_x = \{\lambda \mid \mathrm{Irr}\, W_x^\lambda \neq \phi\}$ only f.d.
② — " — $\mathrm{Irr}\, W_x^\lambda$
③ — " — find dimensions

Example: $x = 0 \Rightarrow W_x = U(\mathfrak{g}) \Rightarrow \mathfrak{A}_x$ - orbits of integral weights

x - principal $\Rightarrow W_x = \cancel{U(\mathfrak{g})}\ Z(\mathfrak{g}) \Rightarrow \mathfrak{A}_x = \mathfrak{h}^*/w$
регулярный

$A(x)$ ~~cacts~~ acts on Springer fibre $\mathcal{B}^x = \{\mathfrak{b} \leq \mathfrak{g} \mid x(\mathfrak{b}) = 0\}$

(Conjecture) Premet if right me if not

For a generic $[\lambda] \in \mathfrak{A}_x$ ~~в~~ общем положении

$$\mathrm{Irr}\, W_x^\lambda \cong \mathrm{Comp}\, \mathcal{B}^x \text{ as an extended } A(x)\text{-set}$$

украшенное

Evidence (from 2006)

Br-Kl - ok in type A

Lo - $\mathfrak{A}_x \neq \phi$

constructive Dodd
nonconstructive Et+Schedler $\Big\}\ \forall \lambda\ |\mathrm{Irr}\, W_x^\lambda| \leq |\mathrm{Comp}\, \mathcal{B}^x|$

+Goodwin $\mathrm{Irr}\, W_x^\lambda$ ~~is not ex~~ has trivial foills

— Positive char is better than Harish-Chandra bimodules

Suggest ~~эксперим~~ который докажет, что мир mental experiment to prove that the world has zero char, not large pos. char.

Pick $\mathbb{C} \supseteq A \twoheadrightarrow \underset{\uparrow \text{char } p.}{\mathbb{K}} \hookrightarrow \overline{\mathbb{K}}$

Reduce $W_\chi^A \otimes_A \overline{\mathbb{K}} \twoheadrightarrow H_\chi$ s.t $M_{p^{\operatorname{codim} \mathcal{B}_\chi}}(H_\chi) \cong U_\chi(\mathfrak{g}_{\mathbb{K}})$

Red. env. alg.

define.

λ integral $\Rightarrow H_\chi = \bigoplus_{\lambda\text{-integral weight}} H_\chi^\lambda$

отображение сдвигов

Dodd Cycle Map for integral λ:

$$K(W_\chi^\lambda\text{-Mod}) \to K(H_\chi^\lambda) \to K(U_\chi^\lambda(\mathfrak{g})) \xrightarrow{\text{BB-loc.}} K(T^*\mathcal{B}_{\mathbb{K}}\text{-Coh}_{\widetilde{\mathcal{B}}^\chi}) \xrightarrow{Ch} H^*(\mathcal{B}^\chi, \mathbb{C}) \twoheadrightarrow H^{top}(\mathcal{B}^\chi, \mathbb{C})$$

$$[M] \mapsto [M \otimes_A \mathbb{K}] \to [\widetilde{M}] \longmapsto [\mathcal{F}_{\widetilde{M}}] \mapsto Ch\, \mathcal{F}_{\widetilde{M}} \longmapsto Ch_{top} \mathcal{F}_{\widetilde{M}}$$

Notes: – $\widetilde{M}$ is irred in all but f. many char. p

– $\mathcal{F}_{\widetilde{M}}$ – complex of sheaves liftable into char 0

комплекс пучков.

Q What is about other cohomology?

Pick $\lambda_0 \in \lambda + \Lambda_{\mathbb{Z}}$ ~~[illegible]~~ st $H_\chi^{\lambda_0}$ has 1-dim rep.

Y := $\operatorname{Irr} H_\chi^{\lambda_0}$

$T_{\chi\,\lambda_0}^{\mu} : H_\chi^{\lambda_0}\text{-Mod} \xrightarrow{\sim} H_\chi^{\mu}\text{-Mod}$.

$\forall \mu \in$ the same p-alcove

p-альков

Th $\dim T_{\lambda_0}^{\mu}(M) = \sum_{k=0}^{\dim \mathcal{B}^\chi} f_{k,M}(\mu - \lambda_0)\, p^k \quad \forall p \gg 0$.

Скорость

Def Rate(M) := p-degree of $\dim T_{\lambda_0}^{\mu}(M)$ Rate = тариф.

$Y_k = \{ M \in Y \mid \text{Rate}(M) = k \}$

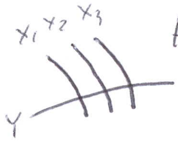
Clearly $Y_0 \equiv \operatorname{Irr} W_\chi^\lambda$

Cor

Cor $ch_k \mathcal{F}_M$, $M \in Y_{\dim \mathcal{B}^\chi - k}$, is a basis of $H^k(\mathcal{B}^\chi, \mathbb{C})$.

Examples:

(4a)



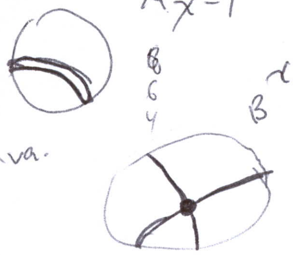
$A_\gamma = S_3$ sheaves

H^λ_X - dimension

$(\lambda + \rho) = (a,b)$.

a a a	$\longrightarrow$	$\mathcal{O}_{X_i}(-1)\,[1]$
b $p-2a-3b$		$\mathcal{O}_Y(-1)\,[1]$ $\mathcal{O}_{\beta^\times}$

(4b) $A_X = 1$



2-dim Schubert var.

8 6 4 β^λ

for $(\lambda+\rho) = (a,b)$

$$\boxed{\begin{aligned} &ap = \tfrac{1}{2}p^2 + \cancel{\tfrac{x-1}{2}p}\;(x-\tfrac{1}{2})p && xp \quad -2\\ &\tfrac{1}{2}b(p+2a+b) = \tfrac{1}{2}yp + \tfrac{1}{2}(2x+y)y && -1\\ &\tfrac{1}{2}(2p-b)(p+\cancel{2}(2a+b)) = \tfrac{1}{2}(2p-y)(\cancel{p}-2x-y) && -1\\ &\tfrac{1}{2}b(p-(2a+b)) = \tfrac{1}{2}y(-2x-y) && -0 \end{aligned}}$$

8 4 9 λ_0, $\boxed{b=1 \quad a=\frac{p-1}{2}}$ λ_0

9 5

11 7 5

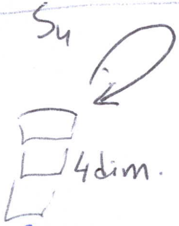
$x = a - \frac{p-1}{2}$, $y = p \cdot b - 1$ $\quad b = y+1$

$a = x + \frac{p-1}{2}$

$a = x + \frac{p}{2}$ $\quad b = y$

$F_4(4_3)$

S_4



$H^4 \quad 12\Sigma_4 + 9\Sigma_{3,1} + 6\Sigma_{2,2} + \Sigma_{2,1,1}$

$H^3 \quad 16\Sigma_4 + 8\Sigma_{3,1} + 4\Sigma_{2,2}$ λ_0

$H^2 \quad 9\Sigma_4 + 2\Sigma_{3,1}$

$H^1 \quad 4\Sigma_4$

$H^0 \quad \Sigma_4$

(12, 4, 4)

RX write.

10×8
$+ 12 + 12$

$\alpha_1, \alpha_2 \neq \beta_3, \beta_4$

X

$e_{\alpha_1} + e_{\alpha_2} \neq e_{\beta_3} + e_{\beta_4}$

Set Perfe

$Y = 21[S_4] + 11[S_3] + 3[D_8] + [C_2] + 6[K_1]$

on left 9 8 ; 3 3 0 5 6 ; $(1,2),(3,4)$

$Y_0 = [C_2] + 2[S_3] + [K_1] + 4$ clusters $\leftarrow$ get here (subject to bC)

$(1,2)$

$Y_1 = 4[S_3] + 4[S_4]$ + 4 clusters

$K_4 = \langle (1,2),(3,4) \rangle$

$Y_2 = 7[S_4] + 2[S_3]$

$Y_3 = 4[S_4]$

$Y_4 = [S_4]$

$(1,2)(3,4)$
$(1,3)(2,4)$

~~$[C_4] + [$~~

$[D_8] + [S_3] \sim [K_1] + [S_4]$

3 + 4 6 + 1