EMERGENT GEOMETRY FROM MATRIX MODELS

M.SIVAKUMAR
UNIVERSITY OF HYDERABAD

June 24, 2011
H.S.Yang and MS - (Phys Rev D 82-2010)
Lebedev Physical Institute

Organisation

- Emergent gravity

Organisation

- Emergent gravity
- Matrix model-emergent geometry

Organisation

- Emergent gravity
- Matrix model-emergent geometry
- Mass deformed IKKT model

Organisation

- Emergent gravity
- Matrix model-emergent geometry
- Mass deformed IKKT model
- Conclusion

Emergent gravity-motivations

- Sakharaov paradigm: "metrical elasticity of space-generalized forces which oppose the curving of space"- induced gravity

Emergent gravity-motivations

- Sakharaov paradigm: "metrical elasticity of space-generalized forces which oppose the curving of space"- induced gravity
- General Relativity -Thermodynamics nexus(Jacobson-Padmanabhan..)

Emergent gravity-motivations

- Sakharaov paradigm: "metrical elasticity of space-generalized forces which oppose the curving of space"- induced gravity
- General Relativity -Thermodynamics nexus(Jacobson-Padmanabhan..)
- Thermodynamics$->$ stat mech of atoms.

Emergent gravity-motivations

- Sakharaov paradigm: "metrical elasticity of space-generalized forces which oppose the curving of space"- induced gravity
- General Relativity -Thermodynamics nexus(Jacobson-Padmanabhan..)
- Thermodynamics$- >$ stat mech of atoms.
- Gen. Rel, dynamics of spacetime–$- >$ emergent spacetime

Emergent gravity-motivations

- Sakharaov paradigm: "metrical elasticity of space-generalized forces which oppose the curving of space"- induced gravity
- General Relativity -Thermodynamics nexus(Jacobson-Padmanabhan..)
- Thermodynamics$- >$ stat mech of atoms.
- Gen. Rel, dynamics of spacetime–$- >$ emergent spacetime
- Dark-energy, quantum gravity..
- Gravity not fundamental?

approaches

- No go theorem:Weinberg-Witten theorem

approaches

- No go theorem:Weinberg-Witten theorem
- Sakharov induced gravity:general relativity arises as an emergent property of quantized matter fields

approaches

- No go theorem:Weinberg-Witten theorem
- Sakharov induced gravity:general relativity arises as an emergent property of quantized matter fields
- AdS-CFT,

approaches

- No go theorem:Weinberg-Witten theorem
- Sakharov induced gravity:general relativity arises as an emergent property of quantized matter fields
- AdS-CFT,
- Horava-Lifshitz,(Gen Rel low energy -high energy spacetime non covariant-metric still fundamental)

approaches

- No go theorem:Weinberg-Witten theorem
- Sakharov induced gravity:general relativity arises as an emergent property of quantized matter fields
- AdS-CFT,
- Horava-Lifshitz,(Gen Rel low energy -high energy spacetime non covariant-metric still fundamental)
- Verlinde entropic gravity; spin foam...
- Non commutative(Moyal) spacetime and NC U(1)as emergent gravity?? Rivelles (2002),H.S.Yang (2005),also Steinacker (2008)

quantised spacetime-Moyal Plane

- Endow spacetime with a symplectic structure B_{ab}

quantised spacetime-Moyal Plane

- Endow spacetime with a symplectic structure B_{ab}
- quantize the spacetime y^a with its Poisson structure $\theta^{ab} \equiv (B^{-1})^{ab}$, described by $\{y^a, y^b\} = i\theta^{ab}$

quantised spacetime-Moyal Plane

- Endow spacetime with a symplectic structure B_{ab}
- quantize the spacetime y^a with its Poisson structure $\theta^{ab} \equiv (B^{-1})^{ab}$, described by $\{y^a, y^b\} = i\theta^{ab}$
- Deformation :$(f.g)(x) \rightarrow (f \star g)(x)$

quantised spacetime-Moyal Plane

- Endow spacetime with a symplectic structure B_{ab}
- quantize the spacetime y^a with its Poisson structure $\theta^{ab} \equiv (B^{-1})^{ab}$, described by $\{y^a, y^b\} = i\theta^{ab}$
- Deformation :$(f.g)(x) \rightarrow (f \star g)(x)$
- $[f, g]_\star - > \{f, g\}_\theta + ..$

NC EM as General Relativity ?

- $U(1)_*$ gauge transformation is translation: $\exp ik.y * f(x) * \exp -iky = f(x + \theta k)$. Infinitesmally $-iB_{ab}[y^b, f] = \partial_a f$

NC EM as General Relativity ?

- $U(1)_*$ gauge transformation is translation: $\exp ik.y * f(x) * \exp -iky = f(x + \theta k)$. Infinitesmally $-iB_{ab}[y^b, f] = \partial_a f$
- NC U(1): No local gauge invariant observables ,non- linearity :similarity to GTR.

NC EM as General Relativity ?

- $U(1)_*$ gauge transformation is translation: $\exp ik.y * f(x) * \exp -iky = f(x + \theta k)$. Infinitesmally $-iB_{ab}[y^b, f] = \partial_a f$
- NC U(1): No local gauge invariant observables ,non- linearity :similarity to GTR.
- NC scalar-U(1)coupling $\rightarrow$scalar- linear gravity coupling. (Rivelles):UV-IR mixing =effect of emergent gravity (Grosse-Steinacker)-NC U(N) =gravity + SU(N)(Steinacker)

NC EM as General Relativity ?

- $U(1)_*$ gauge transformation is translation: $\exp ik.y * f(x) * \exp -iky = f(x + \theta k)$. Infinitesmally $-iB_{ab}[y^b, f] = \partial_a f$
- NC U(1): No local gauge invariant observables ,non- linearity :similarity to GTR.
- NC scalar-U(1)coupling $\rightarrow$scalar- linear gravity coupling. (Rivelles):UV-IR mixing =effect of emergent gravity (Grosse-Steinacker)-NC U(N) =gravity + SU(N)(Steinacker)
- the gauge symmetry acting on $U(1)$ gauge fields as $A \rightarrow A + d\phi$ is a diffeomorphism symmetry generated by a vector field X_ϕ satisfying $\mathcal{L}_{X_\phi} B = 0$,

NC EM as General Relativity ?

- $U(1)_*$ gauge transformation is translation: $\exp ik.y * f(x) * \exp -iky = f(x + \theta k)$. Infinitesmally $-iB_{ab}[y^b, f] = \partial_a f$
- NC U(1): No local gauge invariant observables ,non- linearity :similarity to GTR.
- NC scalar-U(1)coupling $\rightarrow$scalar- linear gravity coupling. (Rivelles):UV-IR mixing =effect of emergent gravity (Grosse-Steinacker)-NC U(N) =gravity + SU(N)(Steinacker)
- the gauge symmetry acting on $U(1)$ gauge fields as $A \rightarrow A + d\phi$ is a diffeomorphism symmetry generated by a vector field X_ϕ satisfying $\mathcal{L}_{X_\phi} B = 0$,
- Darboux theorem of symplectic geometry as equivalence principle; can always locally eliminate EM by coordinate transformation .(H.S.Yang)
- Can $U(1)$ gauge theory on symplectic lead to gravity?

NC EM as General Relativity ?

- $U(1)_*$ gauge transformation is translation: $\exp ik.y * f(x) * \exp -iky = f(x + \theta k)$. Infinitesmally $-iB_{ab}[y^b, f] = \partial_a f$
- NC U(1): No local gauge invariant observables ,non- linearity :similarity to GTR.
- NC scalar-U(1)coupling $\rightarrow$scalar- linear gravity coupling. (Rivelles):UV-IR mixing =effect of emergent gravity (Grosse-Steinacker)-NC U(N) =gravity + SU(N)(Steinacker)
- the gauge symmetry acting on $U(1)$ gauge fields as $A \rightarrow A + d\phi$ is a diffeomorphism symmetry generated by a vector field X_ϕ satisfying $\mathcal{L}_{X_\phi} B = 0$,
- Darboux theorem of symplectic geometry as equivalence principle; can always locally eliminate EM by coordinate transformation .(H.S.Yang)

- Can $U(1)$ gauge theory on symplectic lead to gravity?
- Can quantised spacetime be emergent?

IKKT model (1996)

- Matrix models $\longrightarrow$ spacetime structure.
- $S_{IKKT} = -\frac{1}{4} Tr[X_a, X_b][X^a, X^b]$

IKKT model (1996)

- Matrix models $\longrightarrow$ spacetime structure.
- $S_{IKKT} = -\frac{1}{4} Tr[X_a, X_b][X^a, X^b]$
- Eqn of motion: $[X_a, [X^a, X^b]] = 0,$

IKKT model (1996)

- Matrix models $\longrightarrow$ spacetime structure.
- $S_{IKKT} = -\frac{1}{4} Tr[X_a, X_b][X^a, X^b]$
- Eqn of motion: $[X_a, [X^a, X^b]] = 0$,
- A classical solution is given by $X^a_{\mathrm{cl}} = y^a$, with $[y^a, y^b] = i\theta^{ab}$. (NC) spacetime is a solution and not given apriori.

IKKT model-fluctuation

- expand $X^a \equiv \theta^{ab}\widehat{D}_b$ around the Moyal vacuum .

IKKT model-fluctuation

- expand $X^a \equiv \theta^{ab}\widehat{D}_b$ around the Moyal vacuum .
- $\widehat{D}_a(y) = B_{ab}y^b + \widehat{A}_a(y)$

IKKT model-fluctuation

- expand $X^a \equiv \theta^{ab}\widehat{D}_b$ around the Moyal vacuum .
- $\widehat{D}_a(y) = B_{ab}y^b + \widehat{A}_a(y)$
- $UXU^\dagger$ symmetry leads to Gauge transform of A
- $-i[\widehat{D}_a(y), \widehat{D}_b(y)]_\star =$
  $\partial_a\widehat{A}_b(y) - \partial_b\widehat{A}_a(y) - i[\widehat{A}_a(y), \widehat{A}_b(y)]_\star - B_{ab}$
  $=\widehat{F}_{ab}(y) - B_{ab}.$

IKKT model-fluctuaion

- Then the IKKT matrix model becomes NC $U(1)$ gauge theory $S_{NC} = \frac{1}{4g_{YM}^2} \int d^{2n}y G^{ac} G^{bd} \left(\widehat{F} - B\right)_{ab} \star \left(\widehat{F} - B\right)_{cd}$ where $G^{ab} = \theta^{ac}\theta^{bc}$

IKKT model-fluctuaion

- Then the IKKT matrix model becomes NC $U(1)$ gauge theory $S_{NC} = \frac{1}{4g_{YM}^2} \int d^{2n}y G^{ac} G^{bd} \left(\widehat{F} - B\right)_{ab} \star \left(\widehat{F} - B\right)_{cd}$ where $G^{ab} = \theta^{ac}\theta^{bc}$
- equations of motion for NC gauge fields : $\widehat{D}_{[a}\widehat{F}_{bc]} = 0$

IKKT model-fluctuaion

- Then the IKKT matrix model becomes NC $U(1)$ gauge theory $S_{NC} = \frac{1}{4g_{YM}^2} \int d^{2n}y G^{ac} G^{bd} \left(\widehat{F} - B\right)_{ab} \star \left(\widehat{F} - B\right)_{cd}$ where $G^{ab} = \theta^{ac}\theta^{bc}$
- equations of motion for NC gauge fields : $\widehat{D}_{[a}\widehat{F}_{bc]} = 0$
- How metric is related to Gauge field?

metric from gauge field-HS Yang

- Poisson algebra provides $f \mapsto X_f$-Hamiltonian vector field $X_f(g) = \{g, f\}_\theta$

metric from gauge field-HS Yang

- Poisson algebra provides $f \mapsto X_f$-Hamiltonian vector field $X_f(g) = \{g, f\}_\theta$
- f and the corresponding Hamiltonian vector field X_f is the Lie algebra homomophism in the sense $X_{\{f,g\}_\theta} = -[X_f, X_g]$

metric from gauge field-HS Yang

- Poisson algebra provides $f \mapsto X_f$-Hamiltonian vector field $X_f(g) = \{g, f\}_\theta$
- f and the corresponding Hamiltonian vector field X_f is the Lie algebra homomophism in the sense $X_{\{f,g\}_\theta} = -[X_f, X_g]$
- Given D_a
  $-i[\widehat{D}_a(y), \widehat{f}(y)]_\star = -\theta^{\mu\nu}\frac{\partial D_a(y)}{\partial y^\nu}\frac{\partial f(y)}{\partial y^\mu} + \cdots$

metric from gauge field-HS Yang

- Poisson algebra provides $f \mapsto X_f$-Hamiltonian vector field $X_f(g) = \{g, f\}_\theta$
- f and the corresponding Hamiltonian vector field X_f is the Lie algebra homomophism in the sense $X_{\{f,g\}_\theta} = -[X_f, X_g]$
- Given D_a $-i[\widehat{D}_a(y), \widehat{f}(y)]_\star = -\theta^{\mu\nu} \frac{\partial D_a(y)}{\partial y^\nu} \frac{\partial f(y)}{\partial y^\mu} + \cdots$

metric from gauge field-HS Yang

- Poisson algebra provides $f \mapsto X_f$-Hamiltonian vector field $X_f(g) = \{g, f\}_\theta$
- f and the corresponding Hamiltonian vector field X_f is the Lie algebra homomophism in the sense $X_{\{f,g\}_\theta} = -[X_f, X_g]$
- Given D_a
  $-i[\widehat{D}_a(y), \widehat{f}(y)]_\star = -\theta^{\mu\nu}\frac{\partial D_a(y)}{\partial y^\nu}\frac{\partial f(y)}{\partial y^\mu} + \cdots$
- Vector fields $V_a[](y)$ to order θ is $\theta^{\mu\nu}\frac{\partial D_a(y)}{\partial y^\nu}\frac{\partial}{\partial y\mu}$ which form a set of vector fields. But V_a are not orthonormal.
- Orthonormal $E_a = (\lambda)^{-1}V_a$ where $\lambda^2 = \det V_a^\mu$
  $ds^2 = g_{ab}E^a \otimes E^b$

Emergent gravity and dark energy- HS Yang (JHEP)

- eqn of motion for gauge field $\longrightarrow$ Einstein eqn for the metric .

Emergent gravity and dark energy- HS Yang (JHEP)

- eqn of motion for gauge field $\longrightarrow$ Einstein eqn for the metric .
- Flat spacetime comes from $A_a^{(0)} = B_{ab} y^b$ whose field strength is constant $= B_{ab} \longrightarrow$ Flat spacetime condensation of gauge field

Emergent gravity and dark energy- HS Yang (JHEP)

- eqn of motion for gauge field $\longrightarrow$ Einstein eqn for the metric .
- Flat spacetime comes from $A_a^{(0)} = B_{ab} y^b$ whose field strength is constant $= B_{ab} \longrightarrow$ Flat spacetime condensation of gauge field
- Energy associated with vacuum is M_P^4

Emergent gravity and dark energy- HS Yang (JHEP)

- eqn of motion for gauge field $\longrightarrow$ Einstein eqn for the metric .
- Flat spacetime comes from $A_a^{(0)} = B_{ab}y^b$ whose field strength is constant $= B_{ab}\longrightarrow$ Flat spacetime condensation of gauge field
- Energy associated with vacuum is M_P^4
- Vacuum energy is used to make flat spacetime , hence does not gravitate! Soln to CC problem!

Emergent gravity and dark energy- HS Yang (JHEP)

- eqn of motion for gauge field $\longrightarrow$ Einstein eqn for the metric .
- Flat spacetime comes from $A_a^{(0)} = B_{ab}y^b$ whose field strength is constant $= B_{ab} \longrightarrow$ Flat spacetime condensation of gauge field
- Energy associated with vacuum is M_P^4
- Vacuum energy is used to make flat spacetime , hence does not gravitate! Soln to CC problem!
- Generalization to constant curvature space?

2d Constant curvature manifolds from Matrix models-HS Yang &MSK

- $S_{mCS} = \kappa Tr\Big(\frac{i}{3!}\varepsilon_{ABC}X^A[X^B, X^C] - \frac{\lambda}{2}X_A X^A\Big).$

2d Constant curvature manifolds from Matrix models-HS Yang &MSK

- $S_{mCS} = \kappa Tr\Big(\frac{i}{3!}\varepsilon_{ABC}X^A[X^B, X^C] - \frac{\lambda}{2}X_A X^A\Big).$
- Equation of motion : $[X^A, X^B] = -i\lambda\varepsilon^{AB}{}_C X^C$

2d Constant curvature manifolds from Matrix models-HS Yang &MSK

- $S_{mCS} = \kappa Tr\Big(\frac{i}{3!}\varepsilon_{ABC}X^A[X^B, X^C] - \frac{\lambda}{2}X_A X^A\Big).$
- Equation of motion : $[X^A, X^B] = -i\lambda\varepsilon^{AB}{}_C X^C$
- Invariant $(-)^\sharp R^2 \equiv g_{AB}X^A X^B$. They describe a two-dimensional manifold $\mathcal{M}_2$.

2d Constant curvature manifolds from Matrix models-HS Yang &MSK

- $S_{mCS} = \kappa Tr\Big(\frac{i}{3!}\varepsilon_{ABC}X^A[X^B, X^C] - \frac{\lambda}{2}X_A X^A\Big).$
- Equation of motion : $[X^A, X^B] = -i\lambda\varepsilon^{AB}{}_C X^C$
- Invariant $(-)^\sharp R^2 \equiv g_{AB}X^A X^B$. They describe a two-dimensional manifold $\mathcal{M}_2$.
- $\mathcal{M}_2$ is symplectic
  Local coordinates: $\frac{1}{2}B_{ab}dy^a \wedge dy^b = -dy^1 \wedge dy^2$.

2d Constant curvature manifolds from Matrix models-HS Yang &MSK

- $S_{mCS} = \kappa Tr\left(\frac{i}{3!}\varepsilon_{ABC}X^A[X^B, X^C] - \frac{\lambda}{2}X_A X^A\right)$.
- Equation of motion : $[X^A, X^B] = -i\lambda\varepsilon^{AB}{}_C X^C$
- Invariant $(-)^\sharp R^2 \equiv g_{AB}X^A X^B$. They describe a two-dimensional manifold $\mathcal{M}_2$.
- $\mathcal{M}_2$ is symplectic
  Local coordinates: $\frac{1}{2}B_{ab}dy^a \wedge dy^b = -dy^1 \wedge dy^2$.
- $\mathcal{M}_2$ as a hypersurface embedded in $\mathbb{R}^3$ or $\mathbb{R}^{2,1}$ and described by $L^A = L^A(y^a)$,
  $X^A = L^A$ is classical solution.

2d Constant curvature manifolds from Matrix models-HS Yang &MSK

- $S_{mCS} = \kappa Tr\Big(\frac{i}{3!}\varepsilon_{ABC}X^A[X^B, X^C] - \frac{\lambda}{2}X_A X^A\Big).$
- Equation of motion : $[X^A, X^B] = -i\lambda\varepsilon^{AB}{}_C X^C$
- Invariant $(-)^\sharp R^2 \equiv g_{AB}X^A X^B$. They describe a two-dimensional manifold $\mathcal{M}_2$.
- $\mathcal{M}_2$ is symplectic
  Local coordinates: $\frac{1}{2}B_{ab}dy^a \wedge dy^b = -dy^1 \wedge dy^2$.
- $\mathcal{M}_2$ as a hypersurface embedded in $\mathbb{R}^3$ or $\mathbb{R}^{2,1}$ and described by $L^A = L^A(y^a)$,
  $X^A = L^A$ is classical solution.
- For eg:dS_2 $y^a = (\sinh t, \varphi)$
  $L^1 = y, \quad L^2 = \sqrt{1+y^2}\sin\varphi, \quad L^3 = \sqrt{1+y^2}\cos\varphi$,where $y = \sinh t$.
- $V_A^{(0)} = \theta^{ab}\frac{\partial L_A}{\partial y^b}\frac{\partial}{\partial y^a}$.

d=2 Constant curvature manifolds from Matrix models

d=2 Constant curvature manifolds from Matrix models

d=2 Constant curvature manifolds from Matrix models

- fluctuation of the surface $\mathcal{M}_2$ around the vacuum geometry $X^A(y) = L^A(y) + A^A(y)$.

d=2 Constant curvature manifolds from Matrix models

- fluctuation of the surface $\mathcal{M}_2$ around the vacuum geometry $X^A(y) = L^A(y) + A^A(y)$.
- action for the fluctuations $\widehat{S}_{mCS} = -\frac{\kappa}{12\pi|\theta|}\int d^2y\left(\varepsilon_{ABC}\widehat{X}^A \star \widehat{F}^{BC} + \lambda\widehat{X}_A \star \widehat{X}^A\right)$.
  where $F^{AB} = \{L^A, A^B\}_\theta - \{L^B, A^A\}_\theta + \{A^A, A^B\}_\theta + \varepsilon^{AB}{}_C A^C$.

d=2 Constant curvature manifolds from Matrix models

- fluctuation of the surface $\mathcal{M}_2$ around the vacuum geometry $X^A(y) = L^A(y) + A^A(y)$.
- action for the fluctuations
$\widehat{S}_{mCS} = -\frac{\kappa}{12\pi|\theta|}\int d^2y\Big(\varepsilon_{ABC}\widehat{X}^A \star \widehat{F}^{BC} + \lambda\widehat{X}_A \star \widehat{X}^A\Big)$.
where $F^{AB} = \{L^A, A^B\}_\theta - \{L^B, A^A\}_\theta + \{A^A, A^B\}_\theta + \varepsilon^{AB}{}_C A^C$.
- The equations of motion derived from the variation with respect to $\widehat{A}^A$ is $\widehat{F}^{AB} = 0$,

d=2 Constant curvature manifolds from Matrix models

- fluctuation of the surface $\mathcal{M}_2$ around the vacuum geometry $X^A(y) = L^A(y) + A^A(y)$.
- action for the fluctuations
$\widehat{S}_{mCS} = -\frac{\kappa}{12\pi|\theta|}\int d^2y\Big(\varepsilon_{ABC}\widehat{X}^A \star \widehat{F}^{BC} + \lambda\widehat{X}_A \star \widehat{X}^A\Big)$.
where $F^{AB} = \{L^A, A^B\}_\theta - \{L^B, A^A\}_\theta + \{A^A, A^B\}_\theta + \varepsilon^{AB}{}_C A^C$.
- The equations of motion derived from the variation with respect to $\widehat{A}^A$ is $\widehat{F}^{AB} = 0$,
- The fluctuating coordinate system satisfies the following Poisson bracket relation $\{X^A, X^B\}_\theta = -\varepsilon^{AB}{}_C X^C + F^{AB}$ where

d=2 Constant curvature manifolds from Matrix models

- fluctuation of the surface $\mathcal{M}_2$ around the vacuum geometry $X^A(y) = L^A(y) + A^A(y)$.
- action for the fluctuations
  $\widehat{S}_{mCS} = -\frac{\kappa}{12\pi|\theta|} \int d^2y \Big(\varepsilon_{ABC} \widehat{X}^A \star \widehat{F}^{BC} + \lambda \widehat{X}_A \star \widehat{X}^A \Big)$.
  where $F^{AB} = \{L^A, A^B\}_\theta - \{L^B, A^A\}_\theta + \{A^A, A^B\}_\theta + \varepsilon^{AB}{}_C A^C$.
- The equations of motion derived from the variation with respect to $\widehat{A}^A$ is $\widehat{F}^{AB} = 0$,
- The fluctuating coordinate system satisfies the following Poisson bracket relation $\{X^A, X^B\}_\theta = -\varepsilon^{AB}{}_C X^C + F^{AB}$ where
- Note F^{AB} should satisfy Jacobi identity.This gives $\epsilon_{ABC}\{X^A, \{X^B, X^C\}_\theta\}_\theta = 0.$ This constraint can be solved by taking
  $F^{AB}(X) = \epsilon^{ABC} \frac{\partial F(X)}{\partial X^C}$

d=2 Constant curvature manifolds from Matrix models

- $\{X^A, X^B\}_\theta = \varepsilon^{ABC}\frac{\partial G(X)}{\partial X^C}$ where the polynomial $G(X)$ is defined in $\mathcal{M} = \mathbb{R}^{3-p,p}$ and given by $G(X) = F(X) - \frac{1}{2}g_{AB}X^A X^B + \rho$.

d=2 Constant curvature manifolds from Matrix models

- $\{X^A, X^B\}_\theta = \varepsilon^{ABC}\frac{\partial G(X)}{\partial X^C}$ where the polynomial $G(X)$ is defined in $\mathcal{M} = \mathbb{R}^{3-p,p}$ and given by $G(X) = F(X) - \frac{1}{2}g_{AB}X^AX^B + \rho$.
- function $F(X)$ is an arbitrary polynomial in 3 variables in M defines a two-dimensional surface $\mathcal{M}_2$

d=2 Constant curvature manifolds from Matrix models

- $\{X^A, X^B\}_\theta = \varepsilon^{ABC}\frac{\partial G(X)}{\partial X^C}$ where the polynomial $G(X)$ is defined in $\mathcal{M} = \mathbb{R}^{3-p,p}$ and given by $G(X) = F(X) - \frac{1}{2}g_{AB}X^A X^B + \rho$.
- function $F(X)$ is an arbitrary polynomial in 3 variables in M defines a two-dimensional surface $\mathcal{M}_2$
- determine the embedding coordinate by solving the polynomial equation $G(X) = 0$ .The metric of the two-dimensional surface given by the vector fields $\widehat{V}_A = \theta^{ab}\frac{\partial X_A(y)}{\partial y^b}\frac{\partial}{\partial y^a} + \ldots$.

d=2 Constant curvature manifolds from Matrix models

- $\{X^A, X^B\}_\theta = \varepsilon^{ABC} \frac{\partial G(X)}{\partial X^C}$ where the polynomial $G(X)$ is defined in $\mathcal{M} = \mathbb{R}^{3-p,p}$ and given by $G(X) = F(X) - \frac{1}{2} g_{AB} X^A X^B + \rho$.
- function $F(X)$ is an arbitrary polynomial in 3 variables in M defines a two-dimensional surface $\mathcal{M}_2$
- determine the embedding coordinate by solving the polynomial equation $G(X) = 0$ .The metric of the two-dimensional surface given by the vector fields $\widehat{V}_A = \theta^{ab} \frac{\partial X_A(y)}{\partial y^b} \frac{\partial}{\partial y^a} + \ldots$.
- $g_{AB} \frac{\partial X^A}{\partial y^a} \frac{\partial X^B}{\partial y^b}$ is the induced metric for arbitrary potential

Constant curvature Matrix model for d=2n

- Mass deformed IKKT model: $S_m = S_{IKKT} + m^2 Tr X^a X_a$
- "Linearized" form
  :$S_\kappa = Tr\left(\frac{1}{4} M_{ab} M^{ab} - \frac{1}{2\kappa} M_{ab}[X^a, X^b] + \frac{d-1}{2\kappa} X_a X^a\right)$

Constant curvature Matrix model for d=2n

- Mass deformed IKKT model: $S_m = S_{IKKT} + m^2 Tr X^a X_a$
- "Linearized" form
  $:S_\kappa = Tr\left(\frac{1}{4} M_{ab} M^{ab} - \frac{1}{2\kappa} M_{ab}[X^a, X^b] + \frac{d-1}{2\kappa} X_a X^a\right)$
- Symmetry: $(X^a, M_{ab}) \to U(X^a, M_{ab}) U^\dagger$

Constant curvature Matrix model for d=2n

- Mass deformed IKKT model: $S_m = S_{IKKT} + m^2 Tr X^a X_a$
- "Linearized" form
  $:S_\kappa = Tr\left(\frac{1}{4} M_{ab} M^{ab} - \frac{1}{2\kappa} M_{ab}[X^a, X^b] + \frac{d-1}{2\kappa} X_a X^a\right)$
- Symmetry: $(X^a, M_{ab}) \rightarrow U(X^a, M_{ab}) U^\dagger$
- Equation of motion: $[X^a, X^b] = \kappa M^{ab}$
  $[M^{ab}, X_b] + (d-1) X^a = 0.$

Constant curvature Matrix model for d=2n

- Mass deformed IKKT model: $S_m = S_{IKKT} + m^2 Tr X^a X_a$
- "Linearized" form
  $:S_\kappa = Tr\left(\frac{1}{4}M_{ab}M^{ab} - \frac{1}{2\kappa}M_{ab}[X^a, X^b] + \frac{d-1}{2\kappa}X_a X^a\right)$
- Symmetry: $(X^a, M_{ab}) \to U(X^a, M_{ab})U^\dagger$
- Equation of motion: $[X^a, X^b] = \kappa M^{ab}$
  $[M^{ab}, X_b] + (d-1)X^a = 0.$
- The above eqn is has Snyder algebra as solution:
  $[X^a, X^b] = \kappa M^{ab},$
  $[X^a, M^{bc}] = g^{ac}X^b - g^{ab}X^c,$
  $[M^{ab}, M^{cd}] = g^{ac}M^{bd} - g^{ad}M^{bc} - g^{bc}M^{ad} + g^{bd}M^{ac}$

Constant curvature Matrix model for d=2n

- Mass deformed IKKT model: $S_m = S_{IKKT} + m^2 Tr X^a X_a$
- "Linearized" form
  :$S_\kappa = Tr\left(\frac{1}{4} M_{ab} M^{ab} - \frac{1}{2\kappa} M_{ab}[X^a, X^b] + \frac{d-1}{2\kappa} X_a X^a\right)$
- Symmetry: $(X^a, M_{ab}) \to U(X^a, M_{ab})U^\dagger$
- Equation of motion: $[X^a, X^b] = \kappa M^{ab}$
  $[M^{ab}, X_b] + (d-1)X^a = 0.$
- The above eqn is has Snyder algebra as solution:
  $[X^a, X^b] = \kappa M^{ab},$
  $[X^a, M^{bc}] = g^{ac} X^b - g^{ab} X^c,$
  $[M^{ab}, M^{cd}] = g^{ac} M^{bd} - g^{ad} M^{bc} - g^{bc} M^{ad} + g^{bd} M^{ac}$
- Comments: Snyder algebra is Lorentz algebra in $d+1$ dim
  $\{M_{ab} \& X_a\} = M_{AB}$

Vaccuum geometry in mass-deformed IKKT

- To map Matrix algebra to NC * algebra:
  Snyder algebra from deformation quantisation of a Poisson manifold whose Poisson tensor :$\Pi = \frac{1}{2} L^{ab}(x) \frac{\partial}{\partial x^a} \wedge \frac{\partial}{\partial x^b}$
  $\{x^a, x^b\}_\Pi = L^{ab}(x)$

Vaccuum geometry in mass-deformed IKKT

- To map Matrix algebra to NC * algebra:
  Snyder algebra from deformation quantisation of a Poisson manifold whose Poisson tensor :$\Pi = \frac{1}{2} L^{ab}(x) \frac{\partial}{\partial x^a} \wedge \frac{\partial}{\partial x^b}$
  $\{x^a, x^b\}_\Pi = L^{ab}(x)$
- Need to define a vector field: Enough to have Poisson tensor
  $A \mapsto \Pi^\sharp(A) = L^{ab}(x) A_a(x) \frac{\partial}{\partial x^b}$Given a smooth Poisson manifold (M, Π), there is map $f \mapsto X_f = \Pi^\sharp(df)$ is a lie-algebra homomorphism

Vaccuum geometry in mass-deformed IKKT

- To map Matrix algebra to NC * algebra:
  Snyder algebra from deformation quantisation of a Poisson manifold whose Poisson tensor :$\Pi = \frac{1}{2} L^{ab}(x) \frac{\partial}{\partial x^a} \wedge \frac{\partial}{\partial x^b}$
  $\{x^a, x^b\}_\Pi = L^{ab}(x)$
- Need to define a vector field: Enough to have Poisson tensor $A \mapsto \Pi^\sharp(A) = L^{ab}(x) A_a(x) \frac{\partial}{\partial x^b}$Given a smooth Poisson manifold (M, Π), there is map $f \mapsto X_f = \Pi^\sharp(df)$ is a lie-algebra homomorphism
- the vector fields for the background $D_a^{(0)}(x)$ are given by
  $ad_{\widehat{D}_a^{(0)}}[\widehat{f}](x) = -i[\widehat{D}_a^{(0)}(x), \widehat{f}(x)]_\star = V_a^{(0)\mu}(x) \frac{\partial f(x)}{\partial x^\mu} + \cdots$

Vaccuum geometry in mass-deformed IKKT

- To map Matrix algebra to NC * algebra:
  Snyder algebra from deformation quantisation of a Poisson manifold whose Poisson tensor :$\Pi = \frac{1}{2} L^{ab}(x) \frac{\partial}{\partial x^a} \wedge \frac{\partial}{\partial x^b}$
  $\{x^a, x^b\}_\Pi = L^{ab}(x)$
- Need to define a vector field: Enough to have Poisson tensor $A \mapsto \Pi^\sharp(A) = L^{ab}(x) A_a(x) \frac{\partial}{\partial x^b}$Given a smooth Poisson manifold (M, Π), there is map $f \mapsto X_f = \Pi^\sharp(df)$ is a lie-algebra homomorphism
- the vector fields for the background $D_a^{(0)}(x)$ are given by $ad_{\widehat{D}_a^{(0)}}[\widehat{f}](x) = -i[\widehat{D}_a^{(0)}(x), \widehat{f}(x)]_\star = V_a^{(0)\mu}(x) \frac{\partial f(x)}{\partial x^\mu} + \cdots$
- The vector fields $(V_a^{(0)}, S_{ab}^{(0)}) \in \Gamma(TM_{\text{back}})$ as differential Lorentz generators of $SO(5-p,p)$ $S_{AB}^{(0)} = \kappa \left(x_B \frac{\partial}{\partial x^A} - x_A \frac{\partial}{\partial x^B} \right)$

Vaccuum geometry in mass-deformed IKKT

- To map Matrix algebra to NC * algebra:
  Snyder algebra from deformation quantisation of a Poisson manifold whose Poisson tensor :$\Pi = \frac{1}{2}L^{ab}(x)\frac{\partial}{\partial x^a} \wedge \frac{\partial}{\partial x^b}$
  $\{x^a, x^b\}_\Pi = L^{ab}(x)$
- Need to define a vector field: Enough to have Poisson tensor $A \mapsto \Pi^\sharp(A) = L^{ab}(x)A_a(x)\frac{\partial}{\partial x^b}$Given a smooth Poisson manifold (M, Π), there is map $f \mapsto X_f = \Pi^\sharp(df)$ is a lie-algebra homomorphism
- the vector fields for the background $D_a^{(0)}(x)$ are given by $ad_{\widehat{D}_a^{(0)}}[\widehat{f}](x) = -i[\widehat{D}_a^{(0)}(x), \widehat{f}(x)]_\star = V_a^{(0)\mu}(x)\frac{\partial f(x)}{\partial x^\mu} + \cdots$
- The vector fields $(V_a^{(0)}, S_{ab}^{(0)}) \in \Gamma(TM_{\rm back})$ as differential Lorentz generators of $SO(5-p,p)$ $S_{AB}^{(0)} = \kappa\left(x_B\frac{\partial}{\partial x^A} - x_A\frac{\partial}{\partial x^B}\right)$
- consider a invariant of the 5d Lorentz algebra $g_{AB}x^Ax^B = (-1)^\sharp R^2$.

Vaccuum geometry in mass-deformed IKKT

- To map Matrix algebra to NC * algebra:
  Snyder algebra from deformation quantisation of a Poisson manifold whose Poisson tensor :$\Pi = \frac{1}{2} L^{ab}(x) \frac{\partial}{\partial x^a} \wedge \frac{\partial}{\partial x^b}$
  $\{x^a, x^b\}_\Pi = L^{ab}(x)$
- Need to define a vector field: Enough to have Poisson tensor $A \mapsto \Pi^\sharp(A) = L^{ab}(x) A_a(x) \frac{\partial}{\partial x^b}$Given a smooth Poisson manifold (M, Π), there is map $f \mapsto X_f = \Pi^\sharp(df)$ is a lie-algebra homomorphism
- the vector fields for the background $D_a^{(0)}(x)$ are given by
  $ad_{\widehat{D}_a^{(0)}}[\widehat{f}](x) = -i[\widehat{D}_a^{(0)}(x), \widehat{f}(x)]_\star = V_a^{(0)\mu}(x) \frac{\partial f(x)}{\partial x^\mu} + \cdots$
- The vector fields $(V_a^{(0)}, S_{ab}^{(0)}) \in \Gamma(TM_{\rm back})$ as differential Lorentz generators of $SO(5-p,p)$ $S_{AB}^{(0)} = \kappa \left(x_B \frac{\partial}{\partial x^A} - x_A \frac{\partial}{\partial x^B} \right)$
- consider a invariant of the 5d Lorentz algebra
  $g_{AB} x^A x^B = (-1)^\sharp R^2$.

Vaccuum geometry in mass-deformed IKKT

- To map Matrix algebra to NC * algebra:
  Snyder algebra from deformation quantisation of a Poisson manifold whose Poisson tensor :$\Pi = \frac{1}{2}L^{ab}(x)\frac{\partial}{\partial x^a} \wedge \frac{\partial}{\partial x^b}$
  $\{x^a, x^b\}_\Pi = L^{ab}(x)$
- Need to define a vector field: Enough to have Poisson tensor $A \mapsto \Pi^\sharp(A) = L^{ab}(x)A_a(x)\frac{\partial}{\partial x^b}$Given a smooth Poisson manifold (M, Π), there is map $f \mapsto X_f = \Pi^\sharp(df)$ is a lie-algebra homomorphism
- the vector fields for the background $D_a^{(0)}(x)$ are given by $ad_{\widehat{D}_a^{(0)}}[\widehat{f}](x) = -i[\widehat{D}_a^{(0)}(x), \widehat{f}(x)]_\star = V_a^{(0)\mu}(x)\frac{\partial f(x)}{\partial x^\mu} + \cdots$
- The vector fields $(V_a^{(0)}, S_{ab}^{(0)}) \in \Gamma(TM_{\mathrm{back}})$ as differential Lorentz generators of $SO(5-p,p)$ $S_{AB}^{(0)} = \kappa\left(x_B\frac{\partial}{\partial x^A} - x_A\frac{\partial}{\partial x^B}\right)$
- consider a invariant of the 5d Lorentz algebra $g_{AB}x^Ax^B = (-1)^\sharp R^2$.
- $ds^2 = \mathfrak{G}_{ab}^{(0)}dx^a \otimes dx^b = (\det \mathfrak{G}_{ab}^{(0)})g_{AB}V_a^{(0)A}V_b^{(0)B}dx^a \otimes dx^b = (\det \mathfrak{G}_{ab}^{(0)})(g_{ab}x_5^2 + g_{55}x_a x_b)dx^a \otimes dx^b = g_{AB}\frac{\partial x^A}{\partial x^a}\frac{\partial x^B}{\partial x^b}dx^a \otimes dx^b$

fluctuation

- Emergent Vacuum geometry is Constant curvature space in d-dimensions such as $\mathbf{S}^d$, dS_d and AdS_d , depending on embedded flat Euclidean or Lorentzian spacetime in $(d+1)$-dimensions.

fluctuation

- Emergent Vacuum geometry is Constant curvature space in d-dimensions such as $\mathbf{S}^d$, dS_d and AdS_d , depending on embedded flat Euclidean or Lorentzian spacetime in $(d+1)$-dimensions.
- Consider fluctuations of $X^a \equiv \kappa g^{ab}\widehat{D}_b(x)$ around the vacuum solution as: $\widehat{D}_a(x) = \widehat{D}_a^{(0)}(x) + \widehat{A}_a(x)$ where $\widehat{D}_a^{(0)}(x) = \frac{g_{ab}}{\kappa} x^b$

fluctuation

- Emergent Vacuum geometry is Constant curvature space in d-dimensions such as $\mathbf{S}^d$, dS_d and AdS_d , depending on embedded flat Euclidean or Lorentzian spacetime in $(d+1)$-dimensions.
- Consider fluctuations of $X^a \equiv \kappa g^{ab} \widehat{D}_b(x)$ around the vacuum solution as: $\widehat{D}_a(x) = \widehat{D}_a^{(0)}(x) + \widehat{A}_a(x)$ where $\widehat{D}_a^{(0)}(x) = \frac{g_{ab}}{\kappa} x^b$

fluctuation

- Emergent Vacuum geometry is Constant curvature space in d-dimensions such as $\mathbf{S}^d$, dS_d and AdS_d , depending on embedded flat Euclidean or Lorentzian spacetime in $(d+1)$-dimensions.
- Consider fluctuations of $X^a \equiv \kappa g^{ab}\widehat{D}_b(x)$ around the vacuum solution as: $\widehat{D}_a(x) = \widehat{D}_a^{(0)}(x) + \widehat{A}_a(x)$ where $\widehat{D}_a^{(0)}(x) = \frac{g_{ab}}{\kappa}x^b$
- $[\widehat{D}_a, \widehat{D}_b]_\star =$ $i\kappa^{-1}\widehat{L}_{ab}(x) + [\widehat{D}_a^{(0)}, \widehat{A}_b]_\star - [\widehat{D}_b^{(0)}, \widehat{A}_a]_\star + [\widehat{A}_a, \widehat{A}_b]_\star \equiv i\widehat{F}_{ab}.$

fluctuation

- Emergent Vacuum geometry is Constant curvature space in d-dimensions such as $\mathbf{S}^d$, dS_d and AdS_d , depending on embedded flat Euclidean or Lorentzian spacetime in $(d+1)$-dimensions.
- Consider fluctuations of $X^a \equiv \kappa g^{ab}\widehat{D}_b(x)$ around the vacuum solution as: $\widehat{D}_a(x) = \widehat{D}_a^{(0)}(x) + \widehat{A}_a(x)$ where $\widehat{D}_a^{(0)}(x) = \frac{g_{ab}}{\kappa}x^b$
- $[\widehat{D}_a, \widehat{D}_b]_\star = i\kappa^{-1}\widehat{L}_{ab}(x) + [\widehat{D}_a^{(0)}, \widehat{A}_b]_\star - [\widehat{D}_b^{(0)}, \widehat{A}_a]_\star + [\widehat{A}_a, \widehat{A}_b]_\star \equiv i\widehat{F}_{ab}$.
- the action for the fluctuations in $\widehat{S}_\kappa = \frac{\kappa^2}{4} Tr_{\mathcal{H}} g^{ac}g^{bd}\widehat{F}_{ab} \star \widehat{F}_{cd} + \frac{(d-1)\kappa}{2} Tr_{\mathcal{H}} g^{ab}\widehat{D}_a \star \widehat{D}_b$

fluctuation

- Emergent Vacuum geometry is Constant curvature space in d-dimensions such as $\mathbf{S}^d$, dS_d and AdS_d , depending on embedded flat Euclidean or Lorentzian spacetime in $(d+1)$-dimensions.
- Consider fluctuations of $X^a \equiv \kappa g^{ab}\widehat{D}_b(x)$ around the vacuum solution as: $\widehat{D}_a(x) = \widehat{D}_a^{(0)}(x) + \widehat{A}_a(x)$ where $\widehat{D}_a^{(0)}(x) = \frac{g_{ab}}{\kappa}x^b$
- $[\widehat{D}_a, \widehat{D}_b]_\star =$
$i\kappa^{-1}\widehat{L}_{ab}(x) + [\widehat{D}_a^{(0)}, \widehat{A}_b]_\star - [\widehat{D}_b^{(0)}, \widehat{A}_a]_\star + [\widehat{A}_a, \widehat{A}_b]_\star \equiv i\widehat{F}_{ab}$.
- the action for the fluctuations in
$\widehat{S}_\kappa = \frac{\kappa^2}{4} Tr_{\mathcal{H}} g^{ac}g^{bd}\widehat{F}_{ab}\star\widehat{F}_{cd} + \frac{(d-1)\kappa}{2} Tr_{\mathcal{H}} g^{ab}\widehat{D}_a\star\widehat{D}_b$
- Respects NC $U(1)$ symmetry

Conclusion and open problems

Conclusion and open problems

- New Matrix model constructed and emergent fluctuating geometry studied. Spacetime satisfying Snyder algebra is classical solution. Spacetime again is emergent concept-background independence.

Conclusion and open problems

- New Matrix model constructed and emergent fluctuating geometry studied. Spacetime satisfying Snyder algebra is classical solution. Spacetime again is emergent concept-background independence.
- The correspondence between NC $\star$-algebra and,generalized vector fields extended to mass deformed IKKT model to get constant curvature space. Fluctuation about vacuum geometry is NC massive gauge theory.

Conclusion and open problems

- New Matrix model constructed and emergent fluctuating geometry studied. Spacetime satisfying Snyder algebra is classical solution. Spacetime again is emergent concept-background independence.
- The correspondence between NC $\star$-algebra and,generalized vector fields extended to mass deformed IKKT model to get constant curvature space. Fluctuation about vacuum geometry is NC massive gauge theory.
- Einstein gravity with CC from massive gauge theory?

Conclusion and open problems

- New Matrix model constructed and emergent fluctuating geometry studied. Spacetime satisfying Snyder algebra is classical solution. Spacetime again is emergent concept-background independence.
- The correspondence between NC $\star$-algebra and,generalized vector fields extended to mass deformed IKKT model to get constant curvature space. Fluctuation about vacuum geometry is NC massive gauge theory.
- Einstein gravity with CC from massive gauge theory?
- Nonlinear Snyder algebra? $S_G = Tr\left(\frac{1}{4}M_{ab}M^{ab} - \frac{1}{2\kappa}M_{ab}[X^a, X^b] + \frac{1}{2\kappa}G(X)\right)$. This has as eqn of motion:$[M^{ab}, X_b] + \left[\frac{\partial G(X)}{\partial X_a}\right] = 0$ If $G(X)$ is given $[M^{ab}, M^{cd}]$ can be calculated by applying the Jacobi identity.

Conclusion and open problems

- New Matrix model constructed and emergent fluctuating geometry studied. Spacetime satisfying Snyder algebra is classical solution. Spacetime again is emergent concept-background independence.
- The correspondence between NC $\star$-algebra and,generalized vector fields extended to mass deformed IKKT model to get constant curvature space. Fluctuation about vacuum geometry is NC massive gauge theory.
- Einstein gravity with CC from massive gauge theory?
- Nonlinear Snyder algebra? $S_G = Tr\left(\frac{1}{4}M_{ab}M^{ab} - \frac{1}{2\kappa}M_{ab}[X^a, X^b] + \frac{1}{2\kappa}G(X)\right)$. This has as eqn of motion:$[M^{ab}, X_b] + \left[\frac{\partial G(X)}{\partial X_a}\right] = 0$ If $G(X)$ is given $[M^{ab}, M^{cd}]$ can be calculated by applying the Jacobi identity.
- $G(X)$ is quadratic$->$ usual Snyder algebra $->$ Constant curvature.If $G(X)$ cubic and above $->$what geometry?

Broad picture

- Matrix model $\rightarrow$NC spacetime$\rightarrow$NC algebra fluctuating coordinates-¿ NC $U(1)$ gauge field$\rightarrow$ fluctuating geometry.

Broad picture

- Matrix model →NC spacetime→NC algebra
  fluctuating coordinates-¿ NC $U(1)$ gauge field→ fluctuating geometry.
- Pure gauge fluctuation is Killing symmetry of vacuum geometry
- Dynamics of $NCU(1)$ gauge field → dynamics of gravity
  Opposite to KK!

Broad picture

- Matrix model $\rightarrow$NC spacetime$\rightarrow$NC algebra
  fluctuating coordinates-¿ NC $U(1)$ gauge field$\rightarrow$ fluctuating geometry.
- Pure gauge fluctuation is Killing symmetry of vacuum geometry
- Dynamics of $NCU(1)$ gauge field $\rightarrow$ dynamics of gravity
  Opposite to KK!
- A link between symplective geometry and Riemann geometry!
  Symmetries of Matrix model$\rightarrow$ symmetries of NC spacetime $\rightarrow$ symmetries of general relativity

Broad picture

- Matrix model $\rightarrow$NC spacetime$\rightarrow$NC algebra
  fluctuating coordinates-¿ NC $U(1)$ gauge field$\rightarrow$ fluctuating geometry.
- Pure gauge fluctuation is Killing symmetry of vacuum geometry
- Dynamics of $NCU(1)$ gauge field $\rightarrow$ dynamics of gravity
  Opposite to KK!
- A link between symplective geometry and Riemann geometry!
  Symmetries of Matrix model$\rightarrow$ symmetries of NC spacetime $\rightarrow$ symmetries of general relativity

Broad picture

- Matrix model →NC spacetime→NC algebra
  fluctuating coordinates-¿ NC $U(1)$ gauge field→ fluctuating geometry.
- Pure gauge fluctuation is Killing symmetry of vacuum geometry
- Dynamics of $NCU(1)$ gauge field → dynamics of gravity
  Opposite to KK!
- A link between symplective geometry and Riemann geometry!
  Symmetries of Matrix model→ symmetries of NC spacetime → symmetries of general relativity
- Emergent time ?Emergent matter?

Broad picture

- Matrix model →NC spacetime→NC algebra
  fluctuating coordinates-¿ NC $U(1)$ gauge field→ fluctuating geometry.
- Pure gauge fluctuation is Killing symmetry of vacuum geometry
- Dynamics of $NCU(1)$ gauge field → dynamics of gravity
  Opposite to KK!
- A link between symplective geometry and Riemann geometry!
  Symmetries of Matrix model→ symmetries of NC spacetime
  → symmetries of general relativity
- Emergent time ?Emergent matter?
- THANKS!