Thermal transport in the disordered electron liquid

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Part II

Outline

Part I - General overview

1. Traditional RG scheme for the disordered electron liquid
2. Thermal transport and the Wiedemann Franz law
3. Sigma model with Luttinger's gravitational potential
4. Specific heat
5. RG and fixed point
6. Sub-thermal temperatures
7. Metallic side of the MIT in Si MOSFETs

Part II - Details …

1. **Structure of the density and heat-density correlation functions**
2. **Static parts of the correlation functions**
3. **Dynamical parts of the correlation functions**
4. **Heat density in the Coulomb problem**
5. **Correlation function in the subthermal regime**

Structure of the correlation functions RG-regime

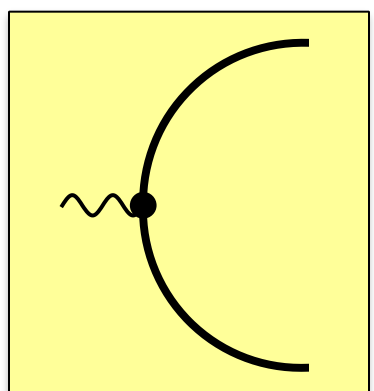
NLσM for the calculation of the density correlation function

$$S[Q] \sim \int d\mathbf{r}\ \mathrm{tr}[D(\nabla Q)^2 + 4iz\hat{\varepsilon}Q] - Q(\hat{\Gamma}_0 + \hat{\Gamma}_1 - \hat{\Gamma}_2)Q$$

$$S_{sc}[Q] \sim \int \underbrace{\mathrm{tr}\left[(\gamma^{\rho}_{\triangleleft}\varphi + \gamma^{\sigma}_{\triangleleft}\boldsymbol{\varphi}\boldsymbol{\sigma})Q\right]}_{\text{static part + dynamical part}} + \int \underbrace{(\varphi\gamma^{\rho}_{\bullet}\varphi + \boldsymbol{\varphi}^T\gamma^{\sigma}_{\bullet}\boldsymbol{\varphi})}_{\text{static part}}$$

static part + dynamical part **static part**

$\varphi, \boldsymbol{\varphi}$

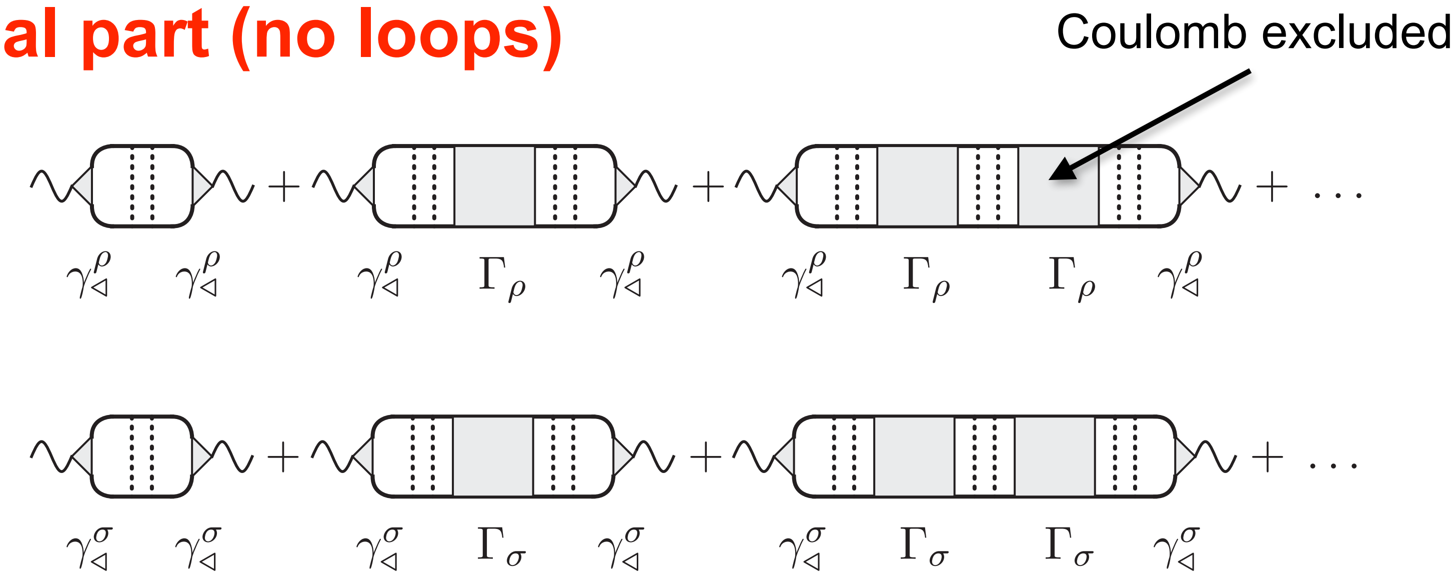


Dynamical parts

dynamical part (no loops)

χ^{dyn}_{nn} :

$\chi^{dyn}_{s^i s^i}$:



$$\bar{\chi}^R_{nn}(\mathbf{q},\omega) = -2\nu\gamma^\rho_\bullet - 2\nu(\gamma^\rho_\triangleleft)^2 \frac{i\omega}{D\mathbf{q}^2 - iz_1\omega}$$

$$= -2\nu\gamma^\rho_\bullet \frac{D\mathbf{q}^2 - i\omega\left(z_1 - \frac{(\gamma^\rho_\triangleleft)^2}{\gamma^\rho_\bullet}\right)}{D\mathbf{q}^2 - iz_1\omega}$$

$$z_1 = z - 2\Gamma_1 + \Gamma_2 = z - \Gamma_\rho$$

Particle number conservation

$$\bar{\chi}^R_{nn}(\mathbf{q}=0,\omega\to 0) = 0$$

$$\boxed{z_1 = \frac{(\gamma^\rho_\triangleleft)^2}{\gamma^\rho_\bullet}}$$

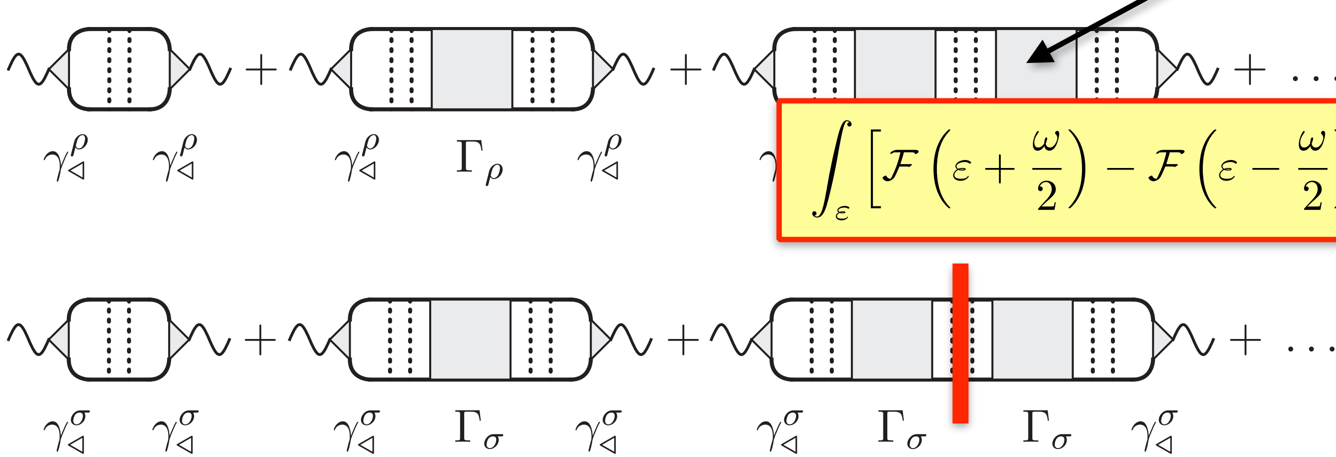
Dynamical parts

dynamical part (no loops)

Coulomb excluded

χ_{nn}^{dyn} :

$\chi_{s^i s^i}^{dyn}$:



$$\int_\varepsilon \left[\mathcal{F}\left(\varepsilon+\frac{\omega}{2}\right)-\mathcal{F}\left(\varepsilon-\frac{\omega}{2}\right)\right] \propto \omega$$

$$\bar{\chi}_{nn}^R(\mathbf{q},\omega) = -2\nu\gamma_\bullet^\rho - 2\nu(\gamma_\triangleleft^\rho)^2 \frac{i\omega}{D\mathbf{q}^2 - iz_1\omega}$$

$$= -2\nu\gamma_\bullet^\rho \frac{D\mathbf{q}^2 - i\omega\left(z_1 - \frac{(\gamma_\triangleleft^\rho)^2}{\gamma_\bullet^\rho}\right)}{D\mathbf{q}^2 - iz_1\omega}$$

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$$\bar{\chi}_{nn}^R(\mathbf{q}=0, \omega\to 0) = 0$$

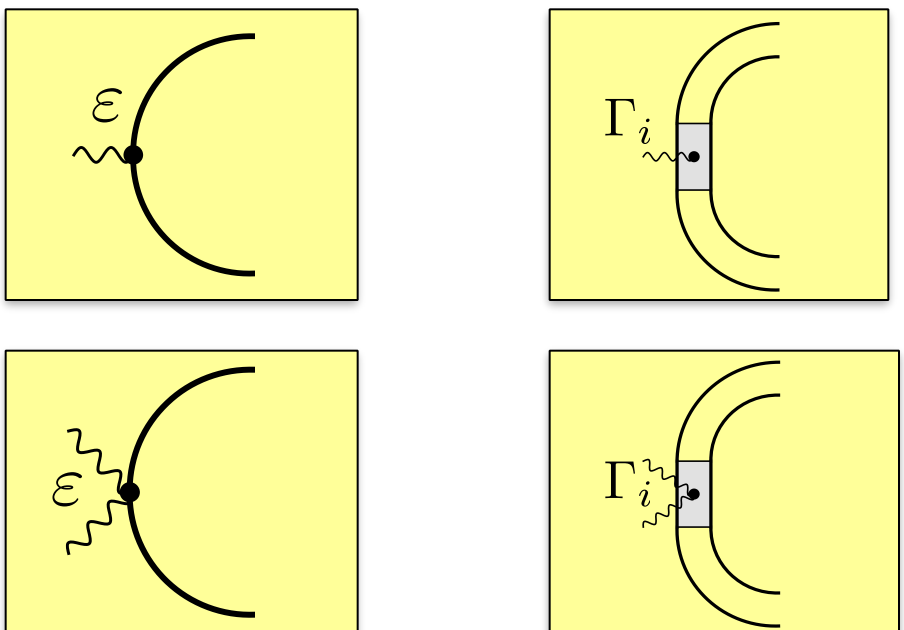
$$\boxed{z_1 = \frac{(\gamma_\triangleleft^\rho)^2}{\gamma_\bullet^\rho}}$$

NLσM for the calculation of the heat density correlation function

$$S[Q] \sim \int d\mathbf{r}\ \mathrm{tr}[D(\nabla Q)^2 + 4iz\hat{\varepsilon}Q] - Q(\hat{\Gamma}_0 + \hat{\Gamma}_1 - \hat{\Gamma}_2)Q$$

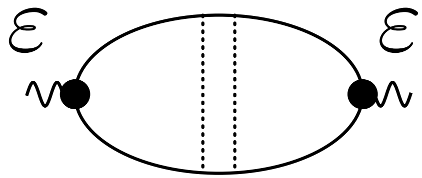
$$S_{sc} \sim \underbrace{\int \mathrm{tr}\left[z\{\varepsilon, \delta\lambda\}\delta Q\right] - \int Q\delta\lambda(\hat{\Gamma}_0 + \hat{\Gamma}_1 - \hat{\Gamma}_2)Q}_{\text{static part + dynamical part}} + \underbrace{(Tc_0)\int \eta\gamma^z_{\bullet}\eta}_{\text{static part}}$$

static part + dynamical part **static part**



dynamical part (no loops)

χ_{kk}^{dyn} :



$$\chi^R_{kk}(\mathbf{q},\omega) = -c_0 T\gamma^z_\bullet - c_0 T(\gamma^z_\triangleleft)^2 \frac{i\omega}{D\mathbf{q}^2 - iz\omega}$$

$$= -c_0 T\gamma^z_\bullet \frac{D\mathbf{q}^2 - i\omega\left(z - \frac{(\gamma^z_\triangleleft)^2}{\gamma^z_\bullet}\right)}{D\mathbf{q}^2 - iz\omega}$$

Particle number conservation

$$\chi^R_{kk}(\mathbf{q}=0,\omega\to 0) = 0$$

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dynamical part (no loops)

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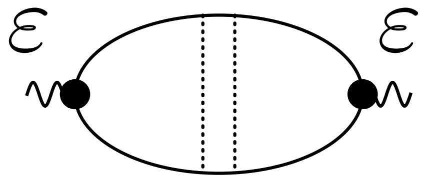
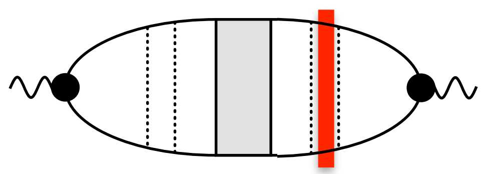


no rescattering:



$$\boxed{\int_\varepsilon \varepsilon \left[\mathcal{F}\left(\varepsilon + \frac{\omega}{2}\right) - \mathcal{F}\left(\varepsilon - \frac{\omega}{2}\right)\right] = 0}$$

Particle number conservation

$$\chi_{kk}^R(\mathbf{q}, \omega) = -c_0 T \gamma_\bullet^z - c_0 T (\gamma_\triangleleft^z)^2 \frac{i\omega}{D\mathbf{q}^2 - iz\omega}$$

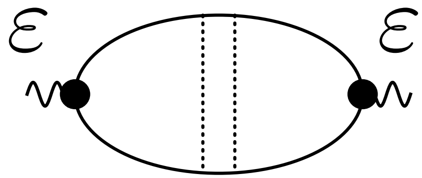
$$= -c_0 T \gamma_\bullet^z \frac{D\mathbf{q}^2 - i\omega\left(z - \frac{(\gamma_\triangleleft^z)^2}{\gamma_\bullet^z}\right)}{D\mathbf{q}^2 - iz\omega}$$

$$\chi_{kk}^R(\mathbf{q} = 0, \omega \to 0) = 0$$

$$\boxed{z = \frac{(\gamma_\triangleleft)^2}{\gamma_\bullet^z}}$$

dynamical part (no loops)

$$\chi^{dyn}_{kk}:$$



$$\chi^R_{kk}(\mathbf{q},\omega) = -c_0 T\gamma^z_\bullet - c_0 T(\gamma^z_\triangleleft)^2 \frac{i\omega}{D\mathbf{q}^2 - iz\omega}$$

$$= -c_0 T\gamma^z_\bullet \frac{D\mathbf{q}^2 - i\omega\left(z - \frac{(\gamma^z_\triangleleft)^2}{\gamma^z_\bullet}\right)}{D\mathbf{q}^2 - iz\omega}$$

Particle number conservation

$$\chi^R_{kk}(\mathbf{q}=0, \omega\to 0) = 0$$

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dynamical part (no loops)

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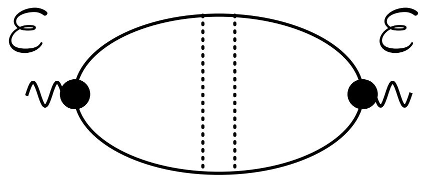
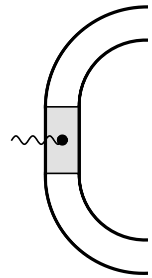


no interaction vertex



leads to additional loops

$$\chi_{kk}^{R}(\mathbf{q},\omega) = -c_0 T\gamma_{\bullet}^{z} - c_0 T(\gamma_{\triangleleft}^{z})^2 \frac{i\omega}{D\mathbf{q}^2 - iz\omega}$$

$$= -c_0 T\gamma_{\bullet}^{z} \frac{D\mathbf{q}^2 - i\omega\left(z - \frac{(\gamma_{\triangleleft}^{z})^2}{\gamma_{\bullet}^{z}}\right)}{D\mathbf{q}^2 - iz\omega}$$

Particle number conservation

$$\chi_{kk}^{R}(\mathbf{q}=0,\omega\to 0) = 0$$

$$\boxed{z = \frac{(\gamma_{\triangleleft})^2}{\gamma_{\bullet}^{z}}}$$

General structure of the correlation functions, NLσM approach

Density-density correlation function:

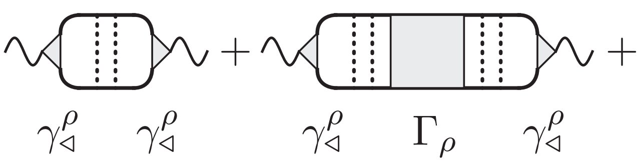
$$\bar{\chi}^R_{nn} = -\frac{\partial n}{\partial \mu} - 2\nu(\gamma^\rho_\triangleleft)^2 \frac{i\omega}{D\mathbf{q}^2 - i(z - \Gamma_\rho)\omega}$$

Heat density-heat density correlation function:

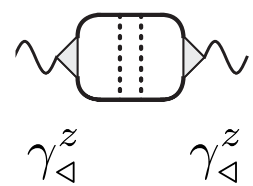
$$\chi^R_{kk} = -cT - c_0 T(\gamma^z_\triangleleft)^2 \frac{i\omega}{D\mathbf{q}^2 - iz\omega}$$

Castellani et al. (1987)

Dynamical part:



Dynamical part:

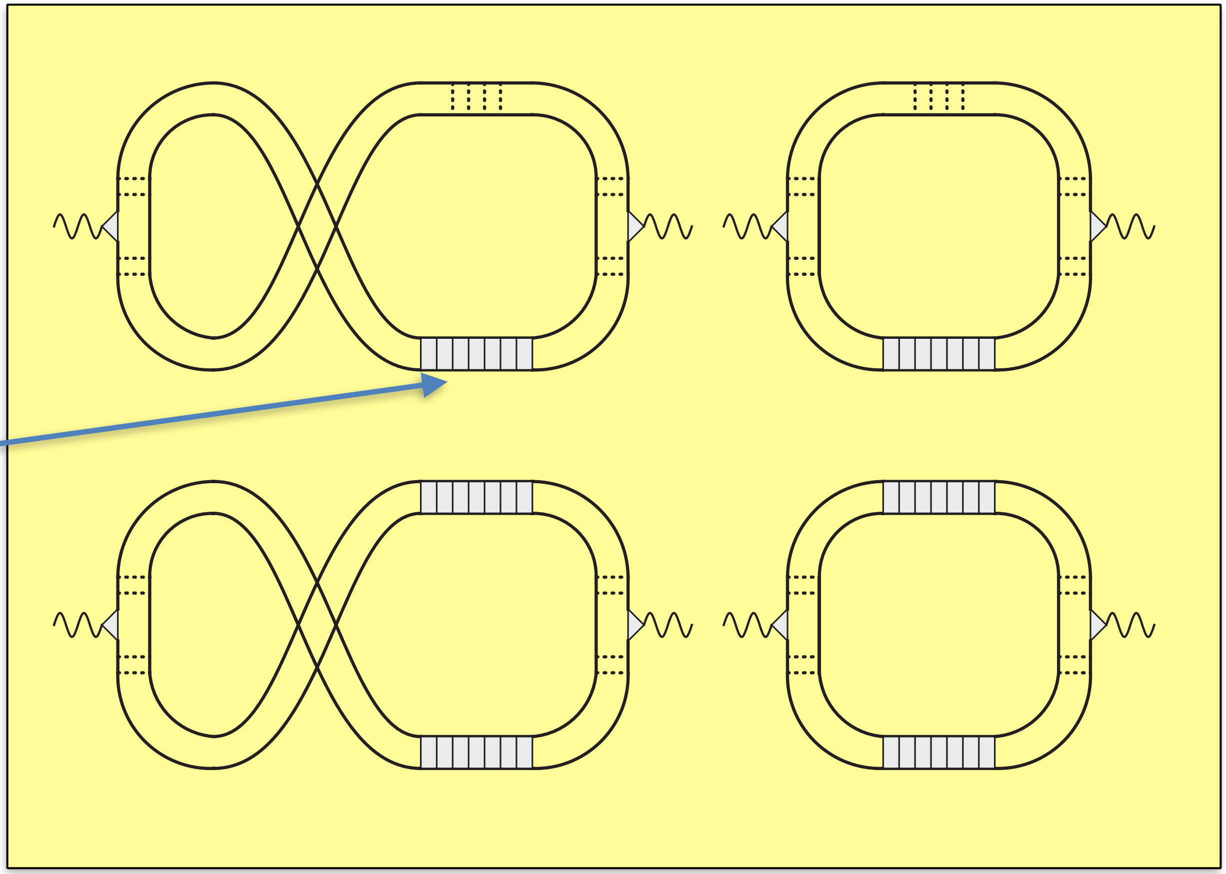


Static parts

Corrections to the static part of the density correlation function

$$\int \left(\varphi \gamma^{\rho}_{\bullet} \varphi + \boldsymbol{\varphi}^{T} \gamma^{\sigma}_{\bullet} \boldsymbol{\varphi}\right)$$

Screened interaction



Specific heat

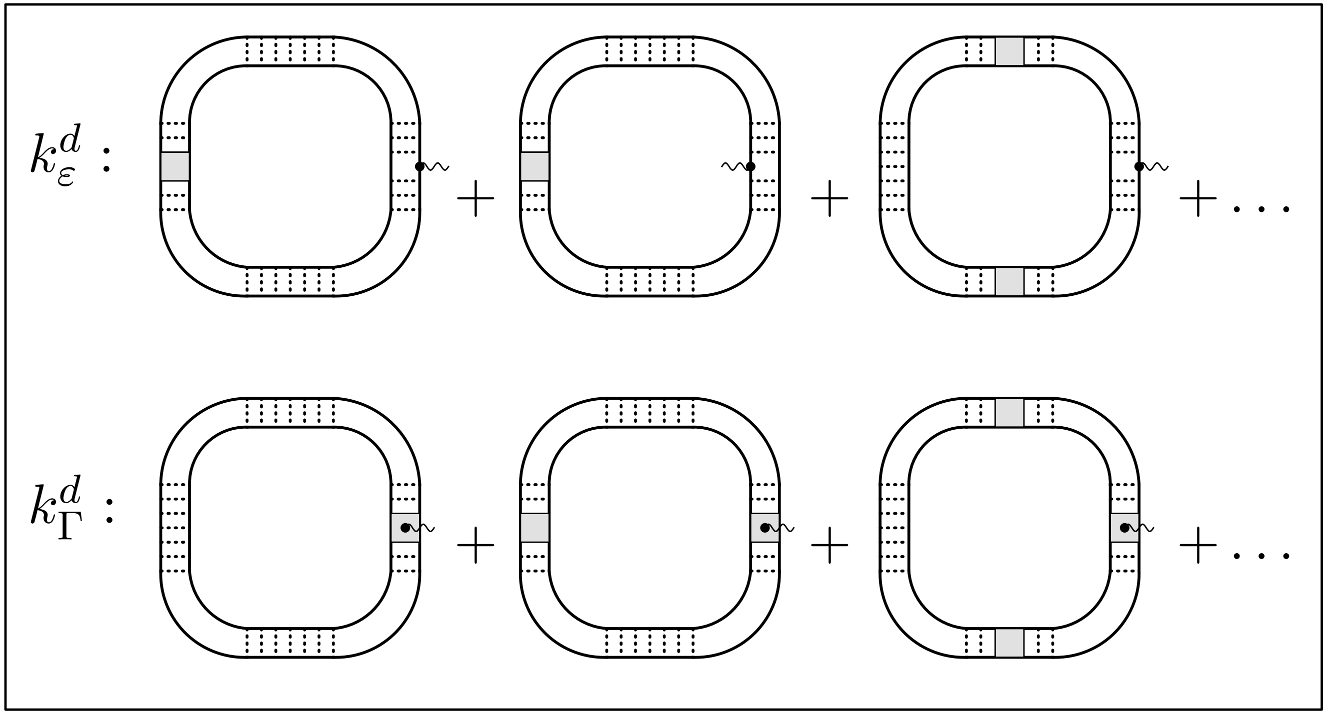
1. Direct calculation: Heat density - specific heat
(linear terms in η)
2. Static part of the correlation function
(quadratic terms in η)

Heat density

$$S[Q] \sim \int d\mathbf{r} \mathrm{tr}[D(\nabla Q)^2 - 2z\{\hat{\varepsilon}, 1 - \eta + \eta^2\}Q] + Q(1 - \eta + \eta^2)(\Gamma_1 + \Gamma_2)Q$$

Heat density

$$k^d_{\eta=0} = \frac{i}{2} \frac{\delta \mathcal{Z}}{\delta \eta}\bigg|_{\eta=0}$$



Heat density and specific heat

$$S[Q] \sim \int d\mathbf{r}\,\mathrm{tr}[D(\nabla Q)^2 - 2z\{\hat{\varepsilon}, 1-\eta+\eta^2\}Q] + Q(1-\eta+\eta^2)(\Gamma_1+\Gamma_2)Q$$

$$k^d_{\eta=0} = \frac{i}{2}\,\frac{\delta \mathcal{Z}}{\delta\eta}\bigg|_{\eta=0} = \frac{1}{2}\int_{\mathbf{q},\omega} \underline{\mathcal{B}_\omega} D\mathbf{q}^2(z_1\mathcal{D}_1\overline{\mathcal{D}}_1 + 3z_2\mathcal{D}_2\overline{\mathcal{D}}_2 - 4z\mathcal{D}\overline{\mathcal{D}})\omega$$

Diffusion with rescattering in singlet and triplet channel

$$\mathcal{D}_{1,2} = \frac{1}{D\mathbf{q}^2 - iz_{1,2}\omega} \qquad \mathcal{D} = \frac{1}{D\mathbf{q}^2 - iz\omega} \qquad \begin{aligned} z_1 &= z - 2\Gamma_1 + \Gamma_2 \\ z_2 &= z + \Gamma_2 \end{aligned}$$

Specific heat

$$\delta c = \partial_T k^d_{\eta=0} = \frac{1}{2}\int_{\mathbf{q},\omega} \partial_T \underline{\mathcal{B}_\omega}\, D\mathbf{q}^2(z_1\mathcal{D}_1\overline{\mathcal{D}}_1 + 3\mathcal{D}_2\overline{\mathcal{D}}_2 - 4z\mathcal{D}\overline{\mathcal{D}})\omega = zc_{FL}$$

Castellani, Di Castro (1986)

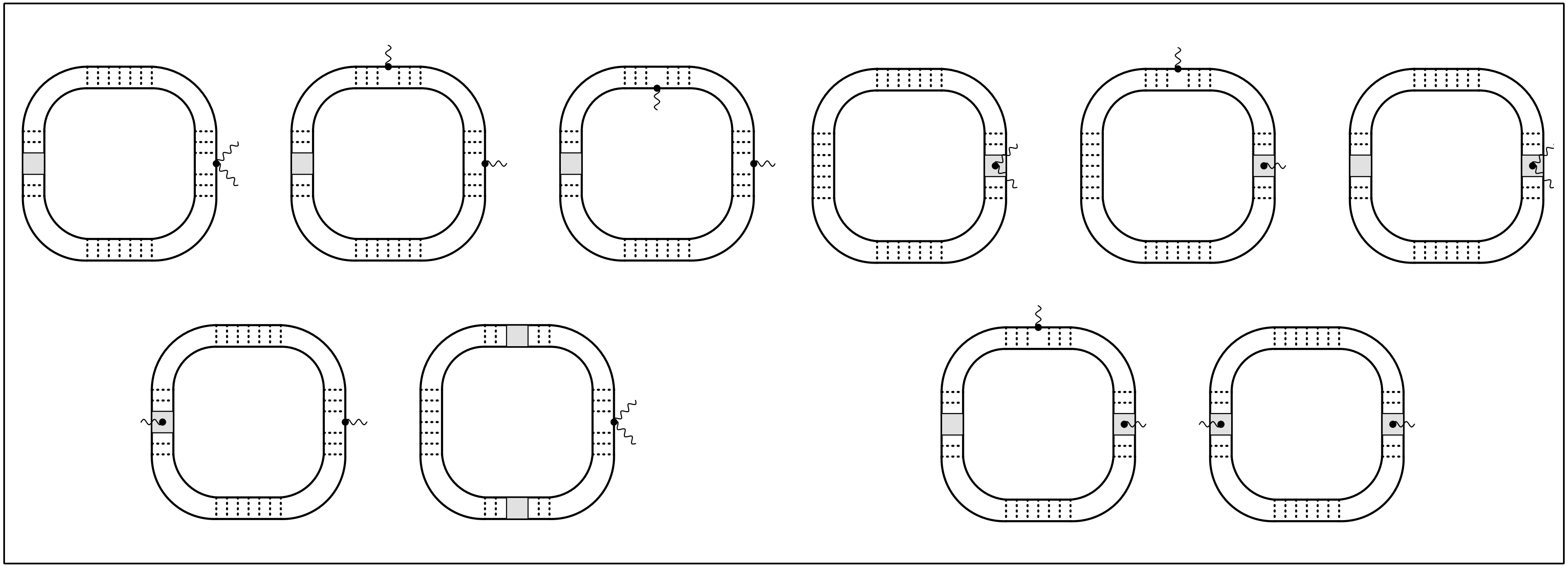
Specific heat and the static part of the correlation function

$$S[Q] \sim \int d\mathbf{r}\,\mathrm{tr}[D(\nabla Q)^2 - 2z\{\varepsilon, 1-\eta+\eta^2\}Q] + Q(1-\eta+\eta^2)(\Gamma_1+\Gamma_2)Q$$

$$\chi^{st}_{kk} = \frac{i}{2}\frac{\delta^2 \mathcal{Z}}{\delta\eta^2} = -Tc$$

$$c = zc_{FL}$$

Castellani, Di Castro (1986)



Specific heat and the static part of the correlation function

$$\boxed{S[Q] \sim \int d\mathbf{r}\ \mathrm{tr}[D(\nabla Q)^2 - 2z\{\hat{\varepsilon}, 1-\eta+\eta^2\}Q] + Q(1-\eta+\eta^2)(\Gamma_1+\Gamma_2)Q}$$

$$\chi_{kk}^{st} = \frac{i}{2}\frac{\delta^2 \mathcal{Z}}{\delta\eta\delta\eta} = -\frac{1}{2}\int_{\mathbf{q},\omega} \omega \mathcal{B}_\omega D\mathbf{q}^2(\omega\partial_\omega + 2)[z_1\mathcal{D}_1\overline{\mathcal{D}}_1 + 3z_2\mathcal{D}_2\overline{\mathcal{D}}_2 - 4z\mathcal{D}\overline{\mathcal{D}}]$$

1. $\omega(\omega\partial_\omega + 2)f(\omega) = \partial_\omega(\omega^2 f(\omega))$

2. Partial integration in ω

3. $\omega\partial_\omega\mathcal{B}_\omega = -T\partial_T\mathcal{B}_\omega$

$$\boxed{\chi_{kk}^{st} = -\frac{T}{2}\int_{\mathbf{q},\omega} \partial_T\mathcal{B}_\omega\ D\mathbf{q}^2(z_1\mathcal{D}_1\overline{\mathcal{D}}_1 + 3z_2\mathcal{D}_2\overline{\mathcal{D}}_2 - 4z\mathcal{D}\overline{\mathcal{D}})\omega = -T\partial_T k^d = -Tc}$$

Dynamical parts

RG - dynamical part of the density correlation function

For the renormalization of the parameters $D, z, \Gamma_1, \Gamma_2, \gamma_\triangleleft^\rho$ use

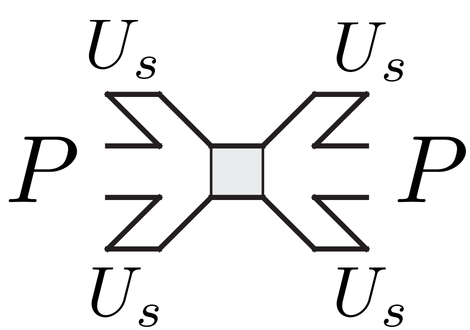
Parameterization: $$Q = u\, U_s Q_f U_s^{-1}\, u^{-1} \qquad Q^2 = 1$$

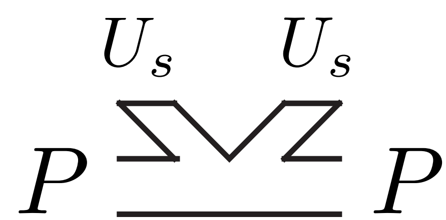
$Q_f = U_f \Lambda U_f^{-1}$ fast

$Q_s = U_s \Lambda U_s^{-1}$ slow

u slowest: distribution function

$$U_f = \exp(-P/2) \qquad \langle PP \rangle \propto \frac{1}{D\mathbf{q}^2 - iz\omega}$$





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Example: Renormalization of the interaction amplitudes

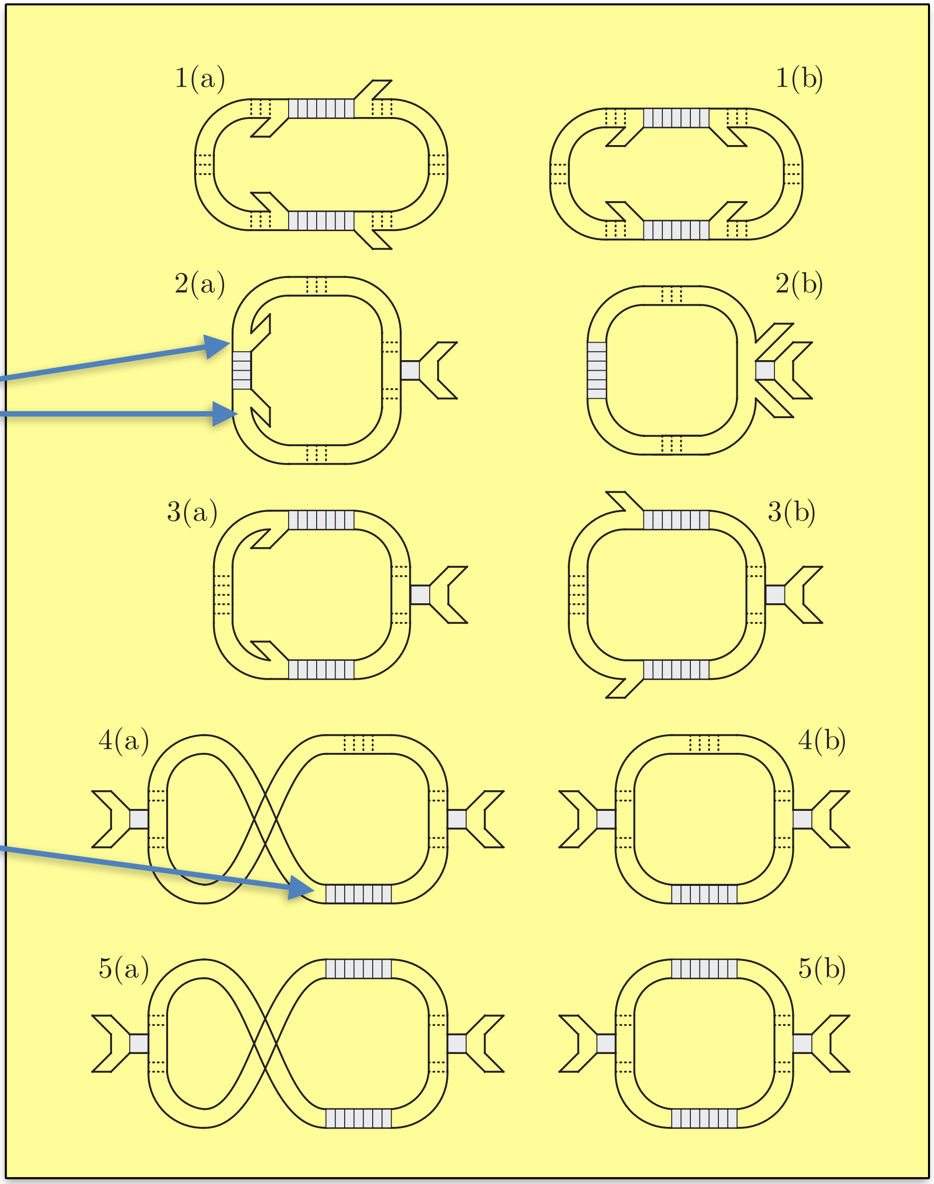
$$\int Q\hat{\Gamma}_i Q$$

4 slow modes U_S

one loop - first order in ρ

all orders in Γ_i
(rescattering)

Coulomb interaction amplitude
Γ_0 is unrenormalized

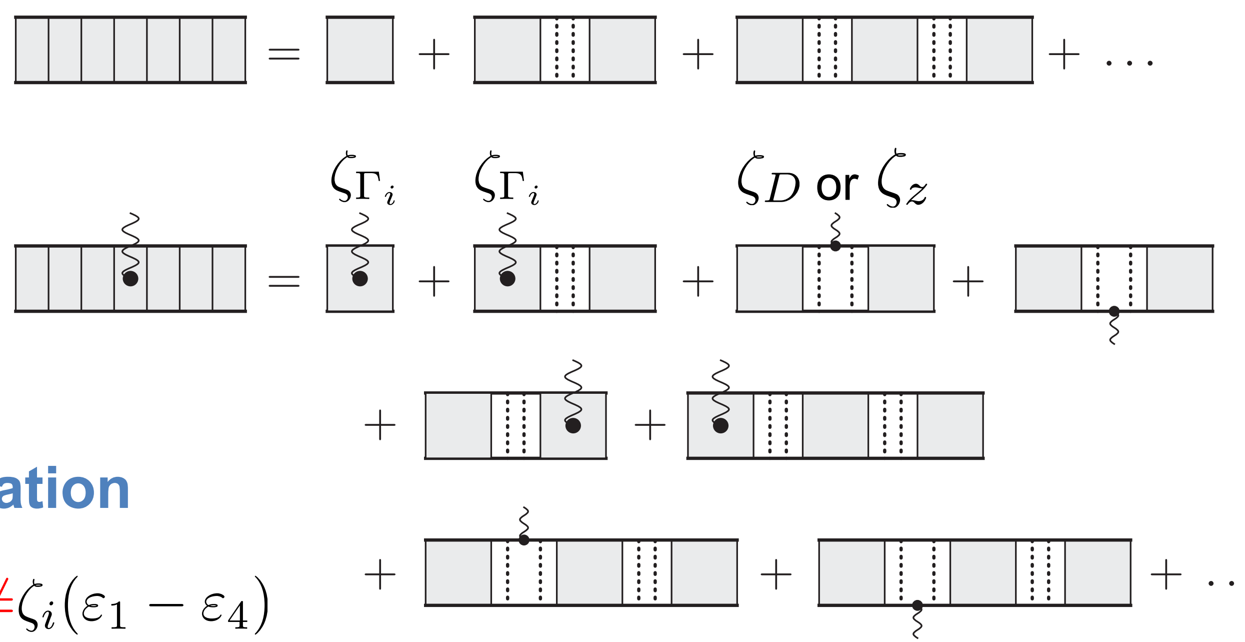


Why differentiation?

$$\Delta(D\zeta_D) = \left(\zeta_D D\frac{\partial}{\partial D} + \zeta_z z\frac{\partial}{\partial z} + \zeta_{\Gamma_1}\Gamma_1\frac{\partial}{\partial \Gamma_1} + \zeta_{\Gamma_2}\Gamma_2\frac{\partial}{\partial \Gamma_2}\right)\Delta D$$

$$\Delta(z\zeta_z) = \left(\zeta_D D\frac{\partial}{\partial D} + \zeta_z z\frac{\partial}{\partial z} + \zeta_{\Gamma_1}\Gamma_1\frac{\partial}{\partial \Gamma_1} + \zeta_{\Gamma_2}\Gamma_2\frac{\partial}{\partial \Gamma_2}\right)\Delta z$$

$$\vdots$$

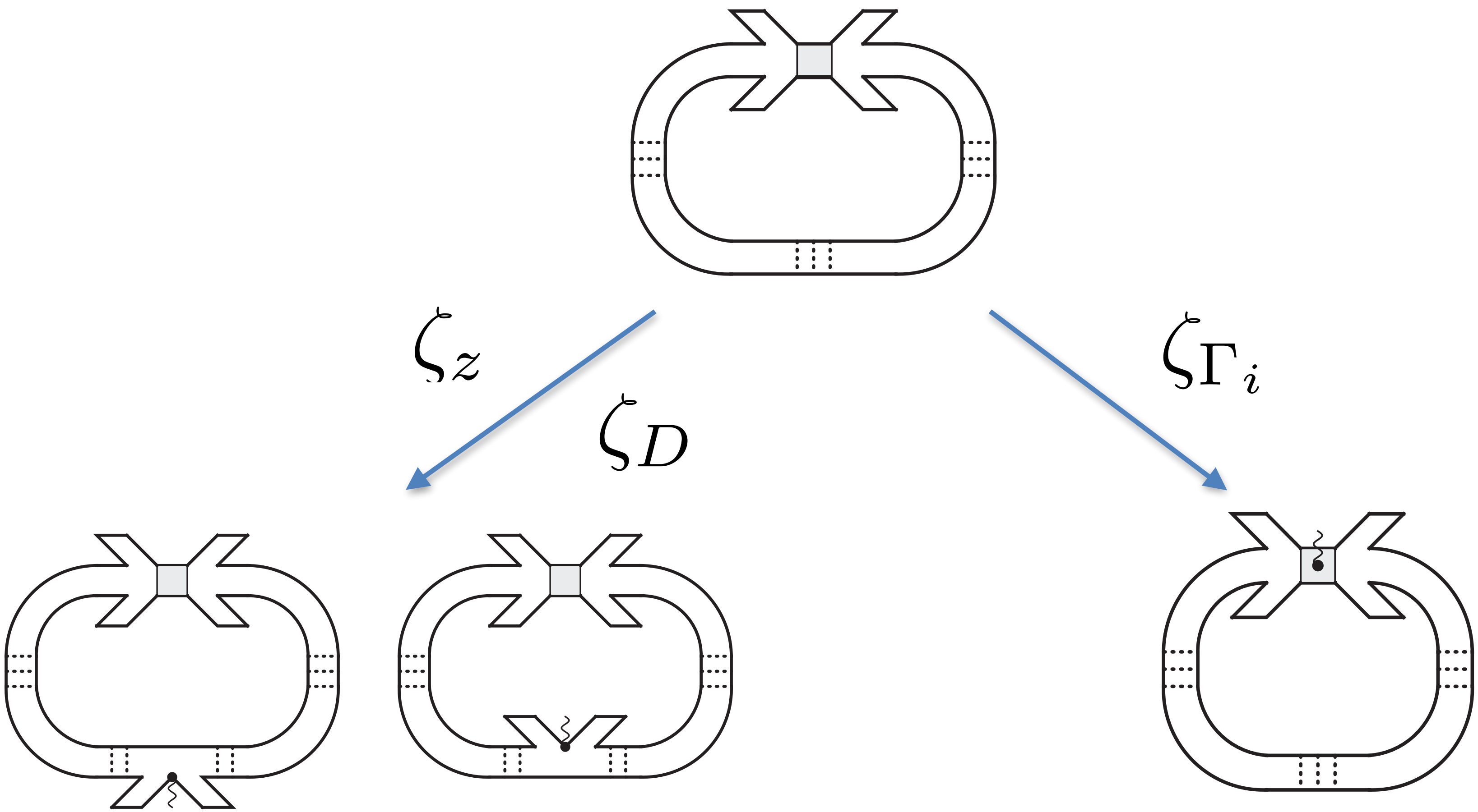


Major complication

$$U^{-1}_{\varepsilon_1\varepsilon_2}\zeta_i(\varepsilon_2-\varepsilon_3)U_{\varepsilon_3\varepsilon_4}\neq\zeta_i(\varepsilon_1-\varepsilon_4)$$

Example for a vertex correction

Bare diagram
- Renormalization of the interaction amplitudes



Heat density for the Coulomb problem

The Coulomb problem

For the long-range Coulomb interaction the definition of the energy density and associated current requires some care.

Our approach:

1. Find the energy momentum tensor and read off the energy density.

$$\boxed{T^{00} = T^{00}(x, y, z, t)}$$

2. "Project" the density to the plane by integrating in the perpendicular direction.

$$\boxed{k(x, y, t) = \int dz\, T^{00}(x, y, z, t)}$$

3. Introduce the source term

$$\boxed{S_\eta = \int dt\, d^2r\, \eta(\mathbf{r}, t) k(\mathbf{r}, t)}$$

The canonical energy-momentum tensor

Construction of the canonical energy momentum tensor

Schrödinger field:

$$\boxed{\mathcal{L}_S = \frac{i}{2}[\psi^* \partial_t \psi - \partial_t \psi^* \psi] - \frac{1}{2m}(i\nabla - q\mathbf{A})\psi^*(-i\nabla - q\mathbf{A})\psi - q\phi\psi^*\psi}$$

Electromagnetic field, free part:

$$\boxed{\mathcal{L}_{em} = -\frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu}} \qquad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \qquad A^\mu = (\phi, \mathbf{A})$$

Noether construction for invariance under translation of coordinates gives

$$\Theta^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)} \partial^\nu \psi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi^*)} \partial^\nu \psi^* + \frac{\partial \mathcal{L}}{\partial(\partial_\mu A^\sigma)} \partial^\nu A^\sigma - g^{\mu\nu} \mathcal{L}$$

$$\boxed{\partial_\mu \Theta^{\mu 0} = 0}$$

Conservation law

$$\boxed{\Theta^{00} = u_\psi + q\phi\rho - \frac{1}{8\pi}\mathbf{E}^2 + \ldots}$$

Energy density?

The Belinfante tensor

Belinfante Energy Momentum Tensor

$$T^{\mu\nu} = \Theta^{\mu\nu} + \frac{1}{4\pi}\partial_\sigma(F^{\mu\sigma}A^\nu) \qquad \partial_\mu T^{\mu\nu} = 0$$

$$\boxed{T^{00} = u_\psi + \frac{1}{8\pi}\mathbf{E}^2 + \ldots}$$

same form obtained in Catelani, Aleiner (2005)

Coulomb Gauge and non-relativistic limit

$$\boxed{T^{00} = \frac{1}{2m}\nabla\psi^*\nabla\psi + \frac{1}{8\pi}\left[\mathbf{E}^{\|}\right]^2} \qquad \mathbf{E}^{\|} = -\frac{1}{4\pi}\nabla\int d\mathbf{r}'\,\frac{\rho(\mathbf{r}',t)}{|\mathbf{r}-\mathbf{r}'|}$$

2d energy density: $k(x,y,t) = \int dz\, T^{00}(x,y,z,t)$

`Coulomb interaction in the presence of η

$$\mathcal{V}_\eta = \frac{1}{2}\{1+\eta, V_0\} + \frac{1}{8\pi e^2}V_0(\nabla^2\eta)V_0$$

Screening $\longrightarrow$

$$\boxed{\mathcal{V}^s_\eta = \frac{1}{2}\{1+\eta, V^s_0\} + \frac{1}{8\pi e^2}V^s_0(\nabla^2\eta)V^s_0 + \mathcal{O}(\eta^2)}$$

The Belinfante tensor

Belinfante Energy Momentum Tensor

$$T^{\mu\nu} = \Theta^{\mu\nu} + \frac{1}{4\pi}\partial_\sigma(F^{\mu\sigma}A^\nu) \qquad \partial_\mu T^{\mu\nu} = 0$$

$$\boxed{T^{00} = u_\psi + \frac{1}{8\pi}\mathbf{E}^2 + \dots}$$

same form obtained
in Catelani, Aleiner (2005)

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Screening $\longrightarrow$

$$\boxed{\mathcal{V}_\eta^s = \frac{1}{2}\{1+\eta, V_0^s\} + \cancel{\frac{1}{8\pi e^2}V_0^s(\nabla^2\eta)V_0^s} + \mathcal{O}(\eta^2)}$$

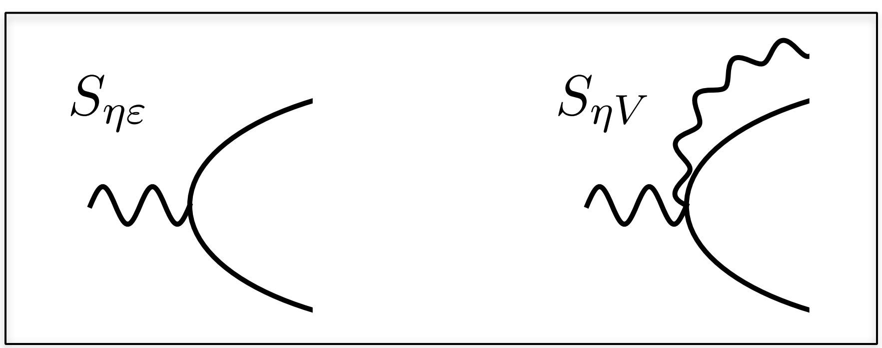
Correlation function in the subthermal regime

Perturbation theory - Coulomb-only problem

Perturbative calculation of the heat-density correlation function

- Calculate leading quantum corrections in the parameter $1/E_F\tau$.
- Coulomb interaction only - FL corrections later
- Reproduce RG-type corrections from energies $T<E<1/\tau$
- **Identify additional corrections from sub-temperature energies $E<T$.**

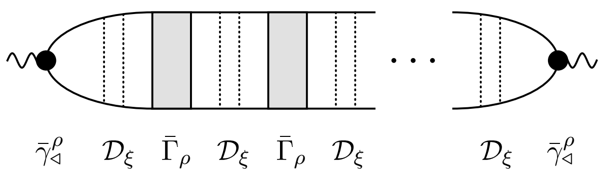
Relevant vertices



Perturbation theory - Coulomb only problem

$$\chi^{dyn}_{nn}(\mathbf{q},\omega) = -2\nu_0(\bar{\gamma}^{\rho}_{\triangleleft})^2 \frac{i\omega}{\mathcal{D}_{\xi}^{-1}(\mathbf{q},\omega) + i\bar{\Gamma}_{\rho}\omega}$$

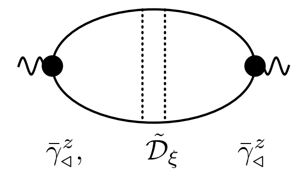
$$\mathcal{D}_{\xi}(\mathbf{q},\omega) = \frac{\xi^2}{D\mathbf{q}^2 - iz\omega}$$



$$\chi^{dyn}_{nn}(\mathbf{q},\omega) = -2\nu_0 \frac{i\omega(1+\delta\xi^2+2\delta\bar{\gamma}^{\rho}_{\triangleleft})}{(D_0+\delta D)\mathbf{q}^2 - i\omega(1+\delta z - \delta\bar{\Gamma}_{\rho})}$$

$$\chi^{dyn}_{kk}(\mathbf{q},\omega) = -c_0 T(\gamma^{z}_{\triangleleft})^2 \frac{i\omega}{\tilde{\mathcal{D}}_{\xi}^{-1}(\mathbf{q},\omega)}$$

$$\tilde{\mathcal{D}}_{\xi}(\mathbf{q},\omega) = \frac{\tilde{\xi}^2}{\tilde{D}\mathbf{q}^2 - i\tilde{z}\omega}$$



$$\chi^{dyn}_{kk}(\mathbf{q},\omega) = -c_0 T \frac{i\omega(1+\delta\tilde{\xi}^2+2\delta\bar{\gamma}^{z}_{\triangleleft})}{(D_0+\delta\tilde{D})\mathbf{q}^2 - i\omega(1+\delta\tilde{z})}$$

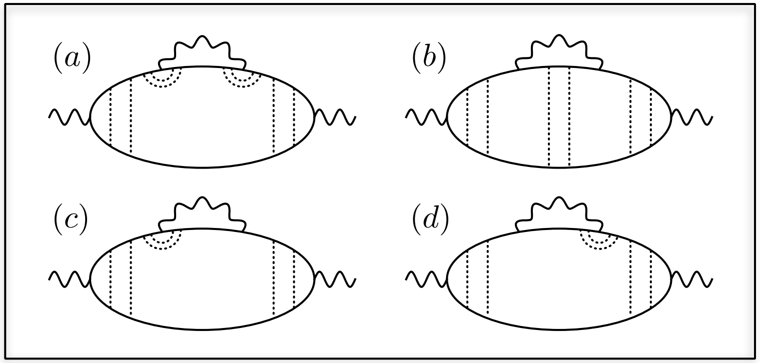
$$\frac{\kappa}{\sigma T} = \frac{z\tilde{D}}{\tilde{z}D}\mathcal{L}_0$$

RG: $z = \tilde{z} \quad D = \tilde{D}$

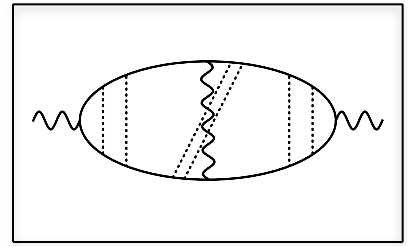
WFL

Perturbation theory - Coulomb only problem

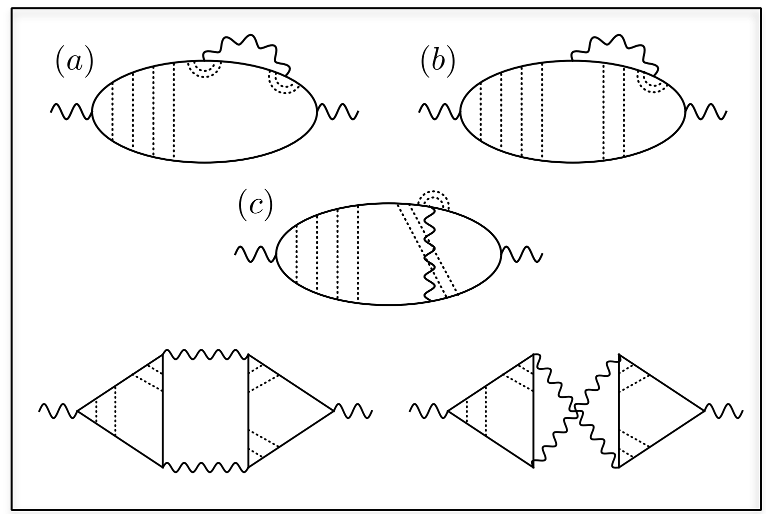
horizontal diagrams



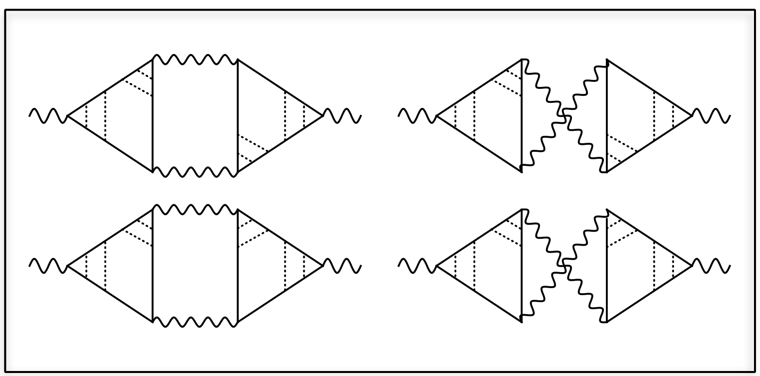
vertical diagrams



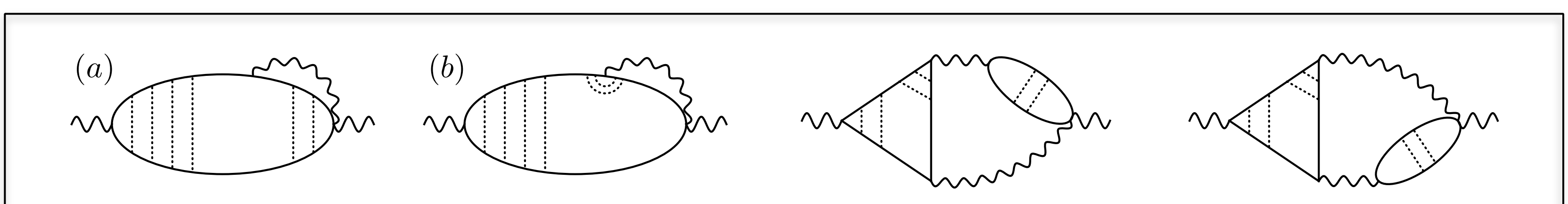
regular vertex corrections



drag diagrams

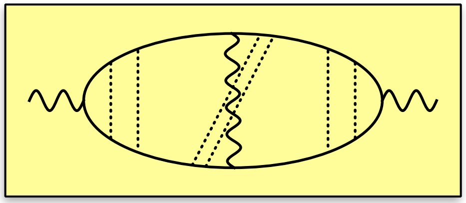


anomalous vertex corrections

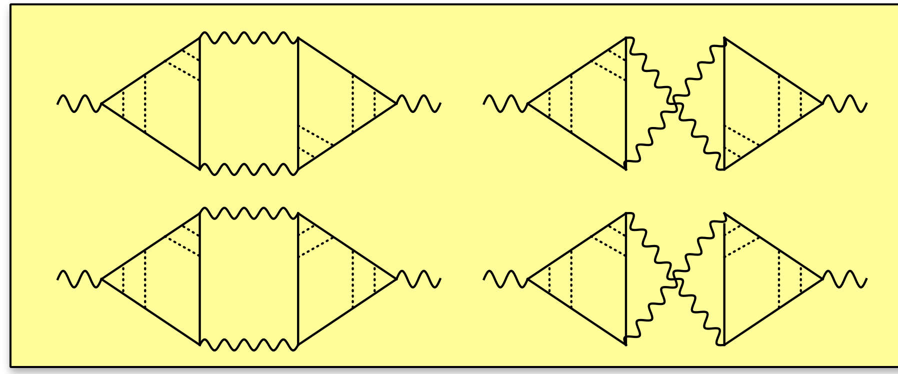


Coulomb only problem - Violation of the WFL

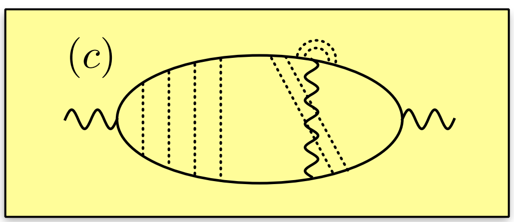
vertical diagrams



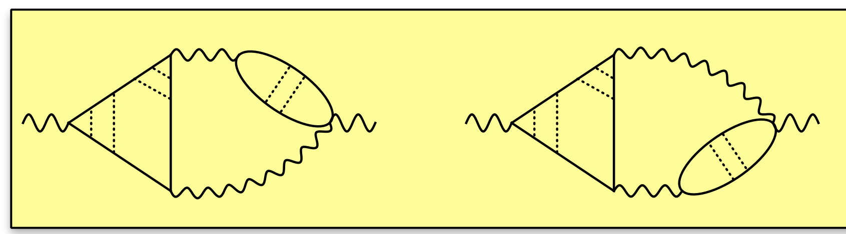
drag diagrams



regular vertex corrections



anomalous vertex corrections



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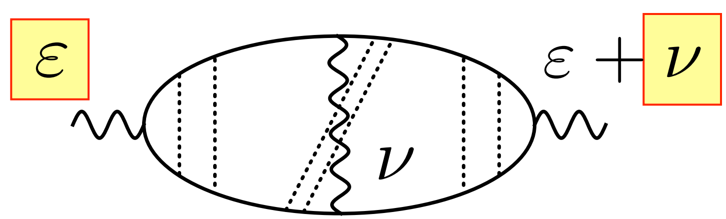
why vertical?

$\varepsilon - \varepsilon$ **WFL**

$\varepsilon - \nu$ ~~**WFL**~~



How the logarithm arises

$$\delta\chi_{kk} \propto \omega D \mathbf{q}^2 \mathcal{D}(\mathbf{q},\omega) \int_{\mathbf{k},\varepsilon,\nu} \varepsilon\nu\partial_\varepsilon \mathcal{F}_\varepsilon (\mathcal{F}_{\varepsilon+\nu} + \mathcal{F}_{\varepsilon-\nu}) \mathrm{Re}\mathcal{D}^2(\mathbf{k},\nu) \mathrm{Im} V^R(\mathbf{k},\nu)$$

$$\int_\nu (\mathcal{F}_{\varepsilon+\nu} + \mathcal{F}_{\varepsilon-\nu}) \sim \varepsilon$$

$$\frac{1}{\nu^2}$$

$$\mathrm{Im} V^R_{\mathbf{k},\nu} \approx -\frac{1}{2\nu_0} \frac{\nu}{D_0 \mathbf{k}^2}$$

$$\frac{|\nu|}{D\kappa_s} < k < \sqrt{\frac{|\nu|}{D}}$$

$$\delta\chi_{kk} \propto \omega D \mathbf{q}^2 \mathcal{D}^2(\mathbf{q},\omega) \times \int_\varepsilon \varepsilon^2 \partial_\varepsilon \mathcal{F}_\varepsilon \times \log \frac{D\kappa_s^2}{T}$$

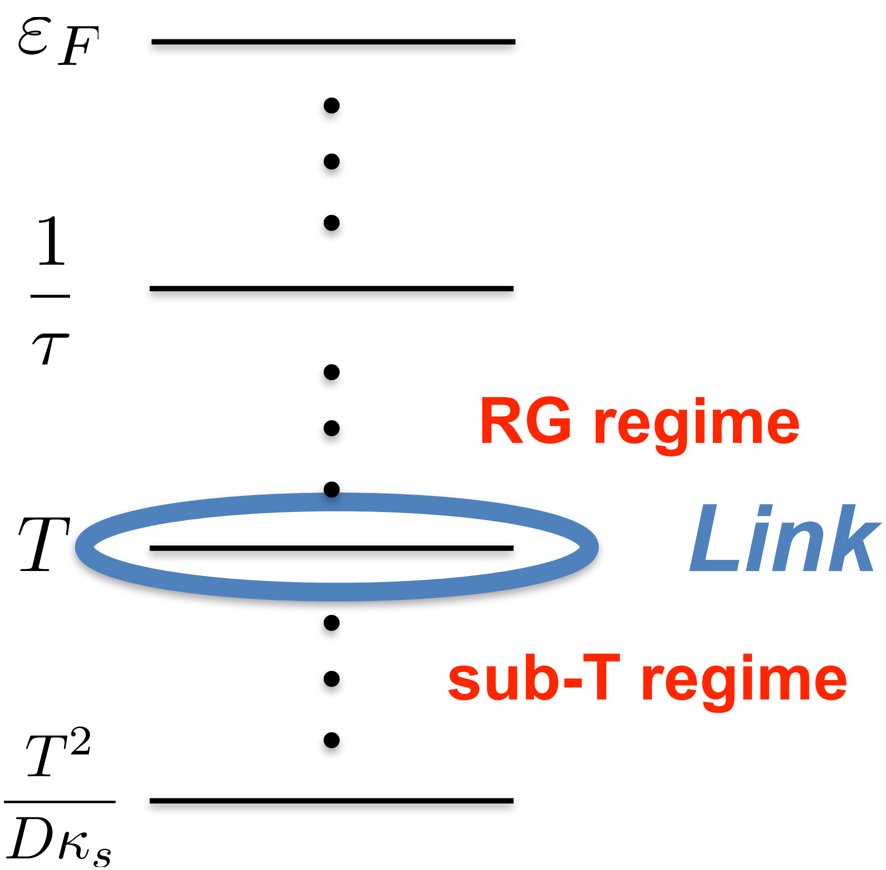
$$\delta\tilde{D}^h = \frac{1}{2}\rho \log \frac{D\kappa_s^2}{T}$$

$$\delta\kappa = \frac{T}{12} \log \frac{D\kappa_s^2}{T}$$

Final step - how does the renormalization affect δκ ?

So far:

- RG corrections from the interval T<E<1/τ
- calculation for E<T without RG corrections or FL renormalization



We checked that

- all Fermi-liquid renormalizations
- the parameter z and all RG renormalizations

drop out when calculating δκ.

We thereby expect that the answer obtained for δκ is final.

Thank you