Thermal transport in the disordered electron liquid

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Outline

Part I - General overview

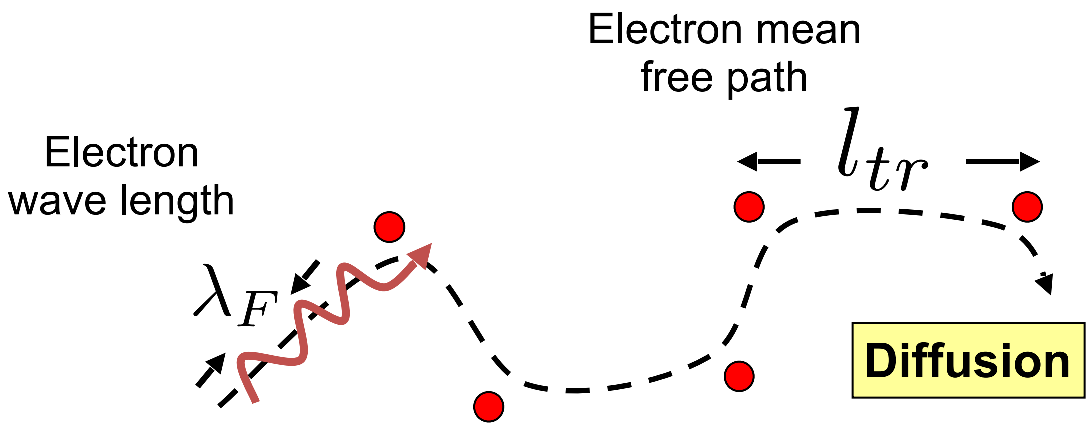
1. Traditional RG scheme for the disordered electron liquid
2. Thermal transport and the Wiedemann Franz law
3. Sigma model with Luttinger's gravitational potential
4. Specific heat
5. RG and fixed point
6. Subthermal regime
7. Metallic side of the MIT in Si MOSFETs

Part II - Details …

1. Structure of the density and heat-density correlation functions
2. Static parts of the correlation functions
3. Dynamical parts of the correlation functions
4. Heat density in the Coulomb problem
5. Correlation function in the subthermal regime

Part I

Drude conductivity



Metallic regime

$$\lambda_F \ll l_{tr} \quad \left[\varepsilon_F \gg \frac{1}{\tau_{tr}}\right]$$

Drude-Boltzmann

$$\sigma_D = \frac{ne^2\tau_{tr}}{m}$$

For high temperatures!

Quantum corrections

The wave nature of the electrons leads to quantum corrections at low T

Most effective in the limit

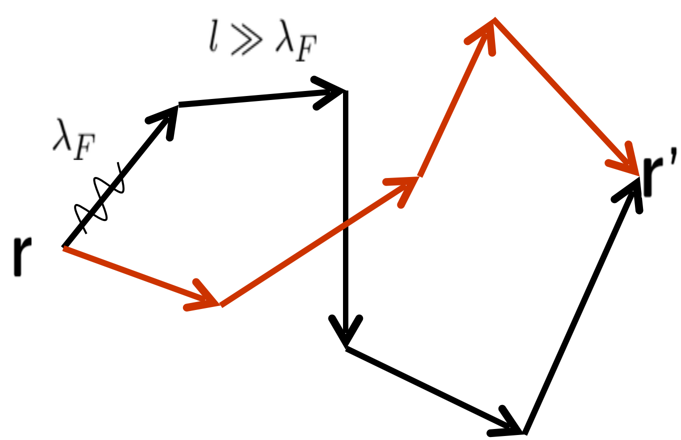
$$T \ll \frac{1}{\tau_{tr}} \ll \varepsilon_F$$

Typical magnitude

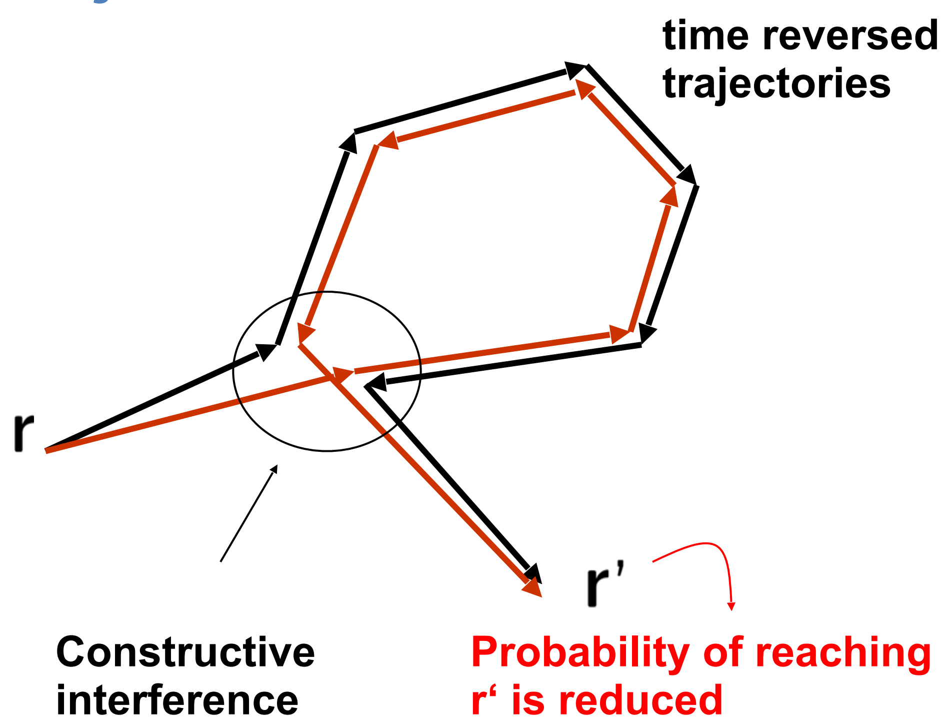
$$\frac{e^2}{h} \approx \frac{1}{26k\Omega}$$

Weak localization

Sum over trajectories



$$|\psi_1 + \psi_2 + \cdots|^2 = |\psi_1|^2 + |\psi_2|^2 + \cdots$$



Suppression of conductivity

$$\sigma = \sigma_0 - \frac{e^2}{\pi h} \log \frac{\tau_\phi(T)}{\tau}$$

Origin of the common wisdom
“no metal in 2D“

Abrahams, Anderson, Licciardello and Ramakrishnan (1979)

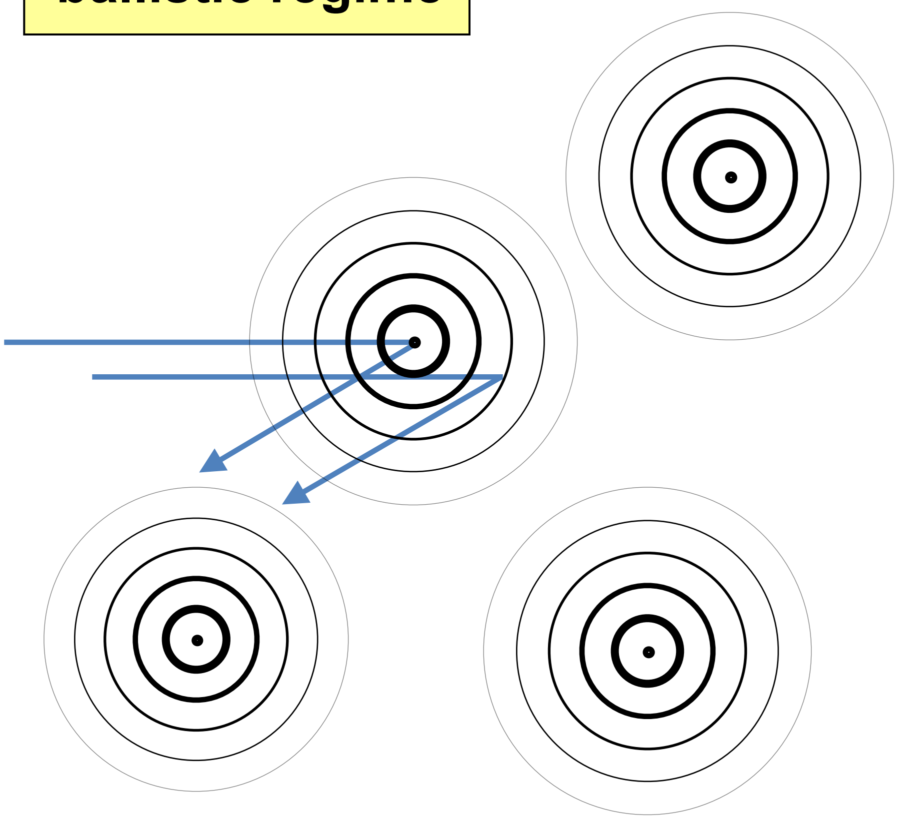
Altshuler - Aronov interaction corrections

Friedel oscillations caused by a single impurity:

$$\delta\rho(r) \sim U_0 \frac{T^2}{\sinh^2(rT/v_F)} \sin(2k_F r)$$

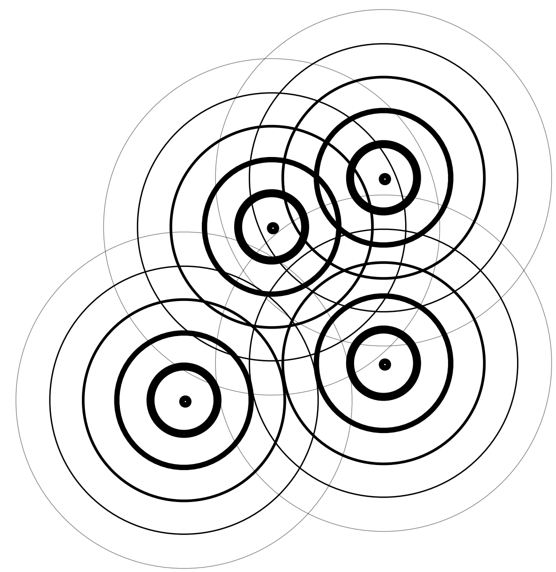
$T > 1/\tau$

ballistic regime



$T < 1/\tau$

diffusive regime



Zala, Narozhny, Aleiner (2001)

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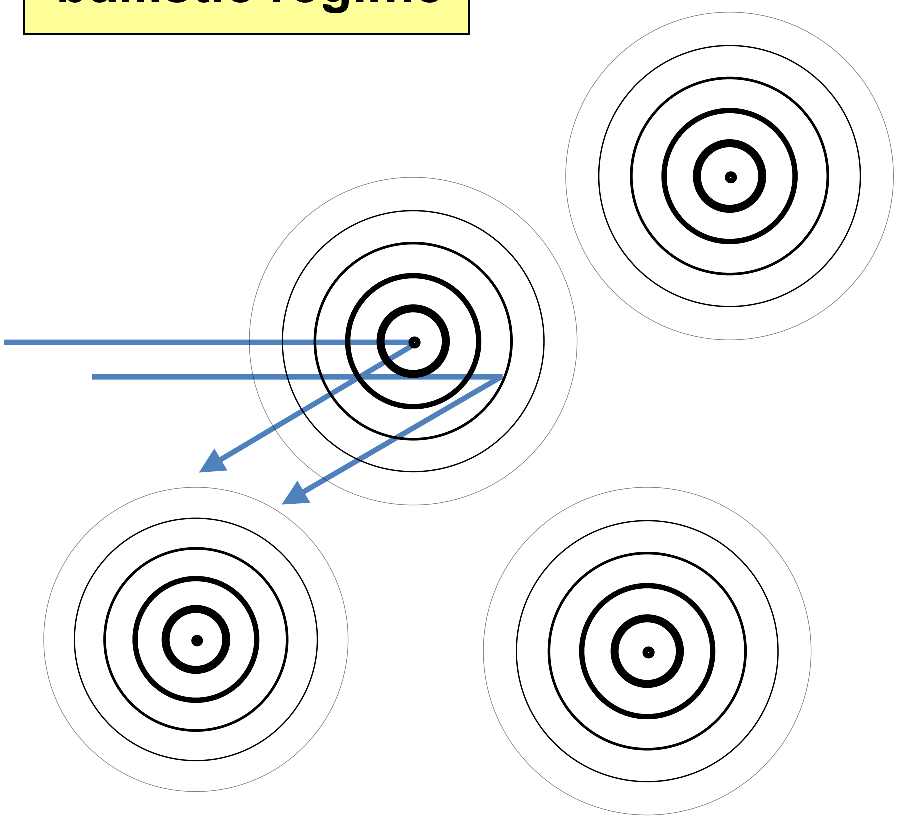

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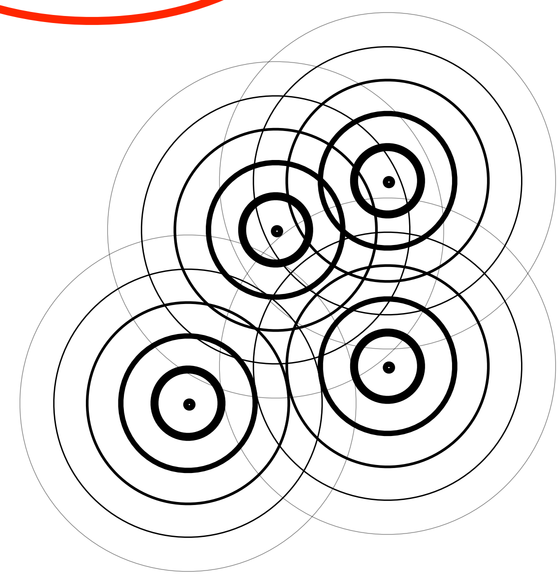
diffusive regime



Zala, Narozhny, Aleiner (2001)

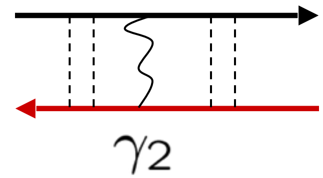
Perturbation theory – interacting systems

$$\delta\sigma(T) = -\frac{e^2}{\pi h}\left[1 + \left(1 - \frac{3}{2}\gamma_2\right)\right]\log(1/T\tau)$$

WL correction (Gang of 4)

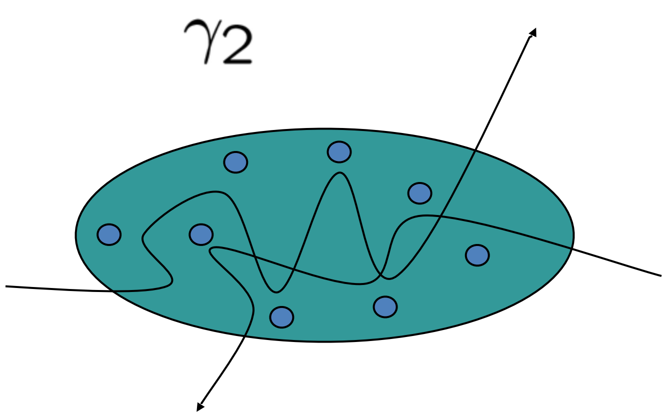
1-singlet contribution

3-triplet contribution



$$\boxed{\text{For}\quad \gamma_2 \ll 1, \quad \delta\sigma(T) < 0}$$ **(insulating)**

Altshuler, Aronov and Lee (1980), Finkel'stein (1983)



Finkel'stein (1983)

Disorder makes the interaction scale-dependent

$$\boxed{\delta\gamma_2(T) \sim \rho\log(1/T\tau) > 0}$$

In the presence of disorder
γ_2 is considerably enhanced at low T

Non-linear Sigma model (NLσM)

Non-linear Sigma model: Effective low energy ($T < 1/\tau < E_F$) action for the disordered Fermi/electron liquid- *Finkel'stein (1983)*
[noninteracting case: Wegner, Efetov Larkin Khmelnitskii,… (1979-)]

$$\boxed{S[Q] \sim \int d\mathbf{r}\ \mathrm{tr}\left[D(\nabla Q)^2 + 4iz\hat{\epsilon}Q\right] + Q(\Gamma_1 + \Gamma_2 + \Gamma_c)Q}$$

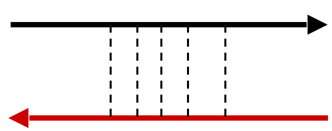
$$\rho \sim 1/D$$

frequency renormalization

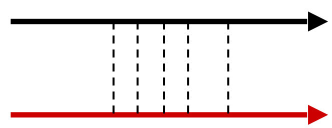
$$\Gamma_2 = -\frac{F_0^\sigma}{1+F_0^\sigma}$$

triplet-channel

$$\langle QQ \rangle \sim \frac{1}{D\mathbf{q}^2 - iz\omega}$$

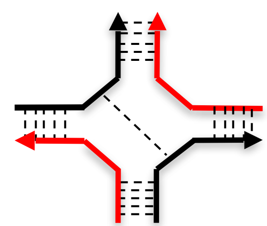


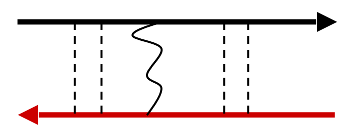
diffuson



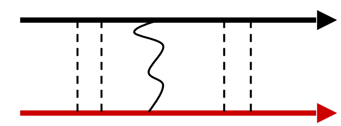
cooperon

$$Q^2(\mathbf{r}) = 1$$





Γ_2



Γ_c

Different methods: Replica/Keldysh

Structure of the RG equations

The interplay of disorder and interactions is captured by a set of coupled Renormalization Group (scaling) equations for ρ and γ_2

$$\frac{d \ln \rho}{d\xi} = \beta_\rho(\rho, \gamma_2)$$

$$\frac{d\gamma_2}{d\xi} = \beta_2(\rho, \gamma_2)$$

$$\xi = \ln(1/T\tau)$$

$$\gamma_2 = \frac{\Gamma_2}{z}$$

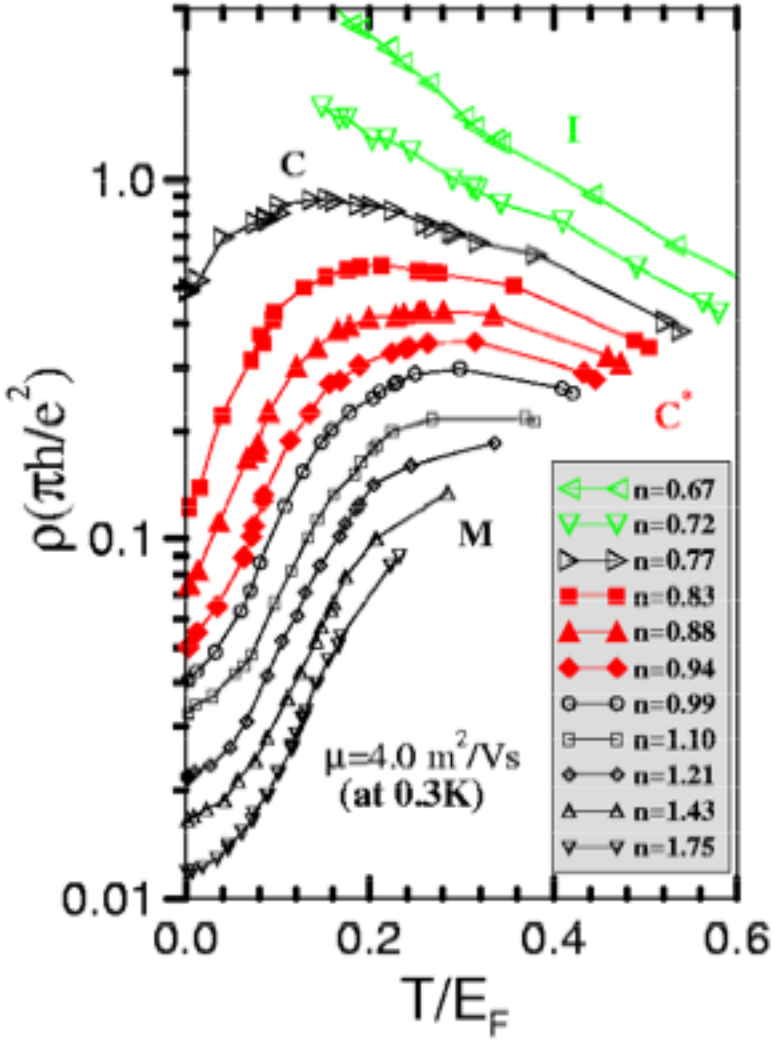
One more equation:

$$\frac{d \ln z}{d\xi} = \beta_z(\rho, \gamma_2)$$

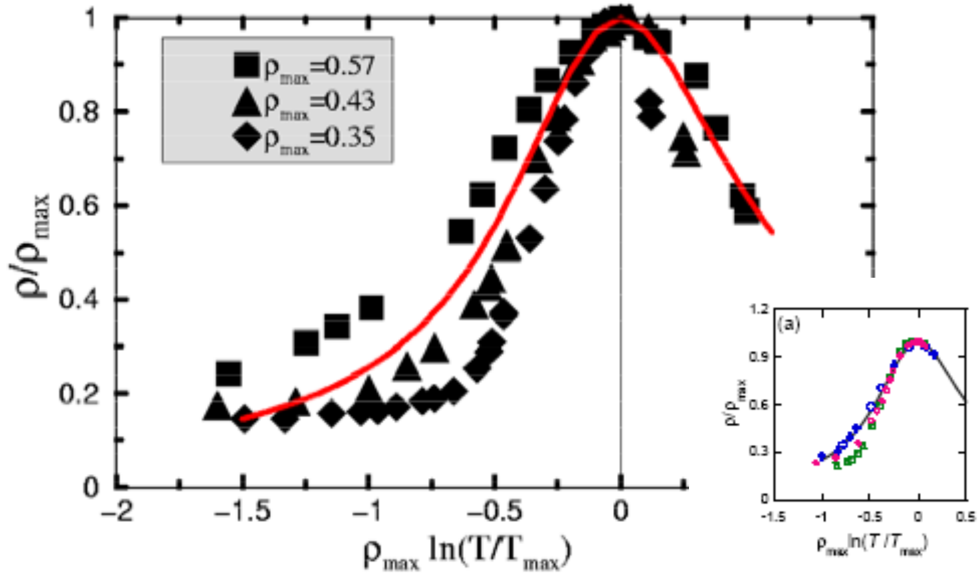
1-loop: leading order in ρ, all orders in the interaction.

does not affect the flow of ρ and γ_2,
important to understand thermodynamic properties

Analysis of high-mobility sample with RG for **two valleys**



Pudalov, et al., ('98)



Data from the region **C*** in a high-mobility sample. **No adjustable parameters** are used.

A.Punnoose and A. Finkelstein, PRL (2002)
S. Anissimova et al., Nature Physics (2007)

Thermal transport and the Wiedemann Franz law

Transport coefficients

$$\begin{pmatrix} \mathbf{j}_e \\ \mathbf{j}_k \end{pmatrix} = \begin{pmatrix} \sigma & \alpha\sigma \\ \Pi\sigma & \tilde{\kappa} \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ -\nabla T \end{pmatrix}$$

Electric current

$$\mathbf{j}_e$$

Heat current

$$\mathbf{j}_k = \mathbf{j}_\varepsilon - \mu \mathbf{j}_n$$

Energy current

Particle current

Electric conductivity σ

Seebeck coefficient α

Peltier coefficient Π

Thermal conductivity $\tilde{\kappa}$

Onsager relation

$$\Pi = T\alpha$$

The Wiedemann-Franz law

Electric conductivity σ | α Seebeck coefficient

Peltier coefficient Π | $\tilde{\kappa}$ Thermal conductivity

$\mathbf{j}_e = 0$

$$\kappa = \tilde{\kappa} + \mathcal{O}\left((T/\varepsilon_F)^2\right)$$

$\mathbf{E} = 0$

The Wiedemann-Franz "law"

$$\kappa = \mathcal{L}_0 \sigma T$$

Lorenz number

$$\mathcal{L}_0 = \frac{\pi^2}{3}\frac{k_B^2}{e^2} = \frac{c_{FL}}{2\nu e^2 T}$$

The Wiedemann Franz law is an approximate low-temperature relation for itinerant electron systems. What is the range of validity?

Heat transport and the Wiedemann-Franz law in disordered electron systems - History of the problem

- **Wiedemann-Franz law ($\kappa/\sigma T$=const.) holds**
  for noninteracting disordered electron systems - *Chester, Thellung (1961)*.

- **Wiedemann-Franz law holds**
  for a Fermi liquid - *Langer (1962).*

After the development of the scaling theory of localization for interacting electrons [Finkel'stein 83, Castellani et al. 84]:

- **Wiedemann-Franz law holds**
  for the disordered electron liquid (renormalized perturbation theory, Ward Identities)
  - *Castellani, di Castro, Kotliar, Lee, Strinati (1987-).*

- **Wiedemann-Franz law violated**
  for the disordered electron liquid (perturbation theory)

  Kubo-formula – *Arfi (1992), Niven, Smith (2005).*

  Kinetic equation approaches - *Livanov et al. (1991), Raimondi et al. (2004), Catelani, Aleiner (2005), Michaeli, Finkelstein (2009)*.

While approaches differ, the result is common: Additional corrections, Wiedemann-Franz law violated

Can one resolve the contradiction and construct a comprehensive theory (including RG and additional log corrections) ?

Can one generalize the RG approach to thermal transport?

- How is heat transported through the system?
- What are the consequences of replacing the electric field by a temperature gradient

How to approach the problem?
How to do RG including a temperature gradient?

Perturbative calculations for κ were (mostly) based on kinetic equation approaches. Including a temperature gradient is straightforward, but how to do RG?

The scaling theory for σ was developed on the basis of a field theory (NlσM) with source fields. How to account for a temperature gradient?

$$\mathrm{Nl}\sigma\mathrm{M} \quad \overset{source\ \varphi}{\longrightarrow} \quad \langle nn \rangle \quad \overset{Einstein}{\longrightarrow} \quad \sigma$$

Our approach: Renormalize the NlσM with source fields (Luttinger's „gravitational potential" mimics temperature variation).

$$\mathrm{Nl}\sigma\mathrm{M} \quad \overset{source\ ??}{\longrightarrow} \quad \langle kk \rangle \quad \overset{Einstein}{\longrightarrow} \quad \kappa$$

Source fields for the heat density correlation function

Action:

$$S[\psi^*,\psi] = \int_{\mathbf{r},t} \left(\psi^* i\partial_t \psi - k[\psi^*,\psi]\right)$$

$$\mathcal{Z} = \int D(\psi,\psi^*)\, \mathrm{e}^{iS} \qquad k = h_0 + h_{int} - \mu n$$

$$S[\psi^*,\psi] = \int_{\mathbf{r},t} \left(\psi^* i\partial_t \psi - (1+\eta) k[\psi^*,\psi]\right)$$

$$\chi_{kk} = \frac{i}{2} \frac{\delta^2 \mathcal{Z}}{\delta\eta_{\mathbf{r}_1 t_1} \delta\eta_{\mathbf{r}_2 t_2}}$$

Gravitational potential

Luttinger (1964)

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Gravitational potential

Luttinger (1964)

Source fields for the heat density correlation function

$$S[\psi^*, \psi] = \int_{\mathbf{r},t} (\psi^* i\partial_t \psi - (1+\eta) k[\psi^*, \psi])$$

Problem: $$S_{dis} = - \int_{\mathbf{r},t} (1+\eta) \psi^* u_{dis} \psi$$

Change of variables: $$\psi \to \frac{1}{\sqrt{1+\eta}} \psi \qquad \psi^* \to \psi^* \frac{1}{\sqrt{1+\eta}}$$

After this transformation, the derivation of the NLσM is straightforward:

$$S[Q] \sim \int d\mathbf{r} \; \mathrm{tr} \left[D(\nabla Q)^2 + 2iz\{\hat{\epsilon}, \lambda\} Q \right] + Q\lambda(\Gamma_1 + \Gamma_2) Q$$

$$\lambda = \frac{1}{1+\eta} \approx 1 - \eta + \eta^2 + \ldots$$

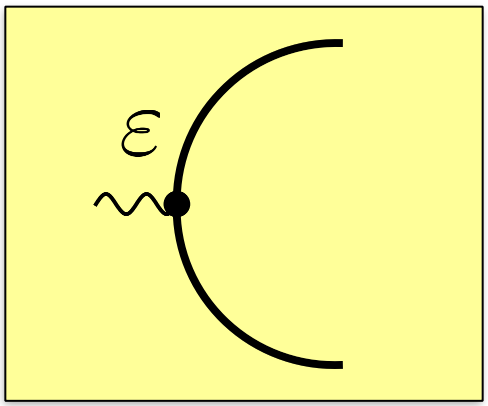
nonlinear in η!

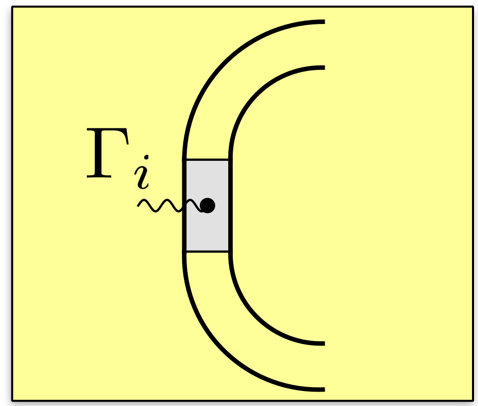
NlσM with "gravitational potentials"

$$S[Q] \sim \int d\mathbf{r}\ \mathrm{tr}\left[D(\nabla Q)^2 + 2iz\{\hat{\epsilon}, \lambda\}Q\right] + Q\lambda(\Gamma_1 + \Gamma_2)Q$$

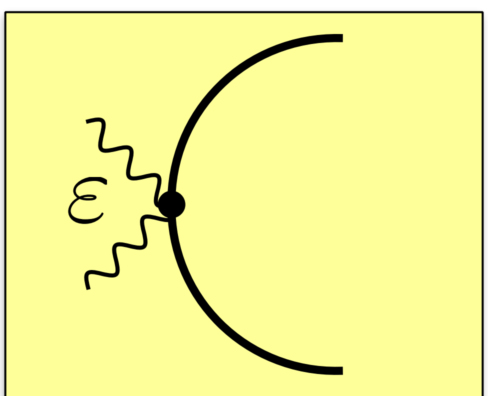
$$\lambda \approx 1 - \eta + \eta^2$$

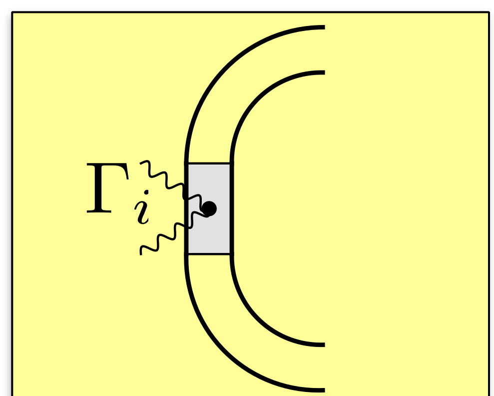
η





η^2





The correlation function - phenomenology

Heat density correlation function in the diffusive limit

$$\chi_{kk} = -cT \frac{D_k \mathbf{q}^2}{D_k \mathbf{q}^2 - i\omega}$$

Static limit

$$\chi_{kk}(\mathbf{q} \to 0, \omega = 0) = -cT$$

Conservation law

$$\chi_{kk}(\mathbf{q} = 0, \omega \to 0) = 0$$

Thermal conductivity

$$\kappa = cD_k$$

The correlation function - phenomenology

Heat density correlation function in the diffusive limit

$$\chi_{kk} = -cT \frac{D_k \mathbf{q}^2}{D_k \mathbf{q}^2 - i\omega}$$

$$\chi_{nn} = -\frac{\partial n}{\partial \mu} \frac{D_n \mathbf{q}^2}{D_n \mathbf{q}^2 - i\omega}$$

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$$\chi_{nn}(\mathbf{q} = 0, \omega \to 0) = 0$$

Thermal conductivity

$$\kappa = cD_k$$

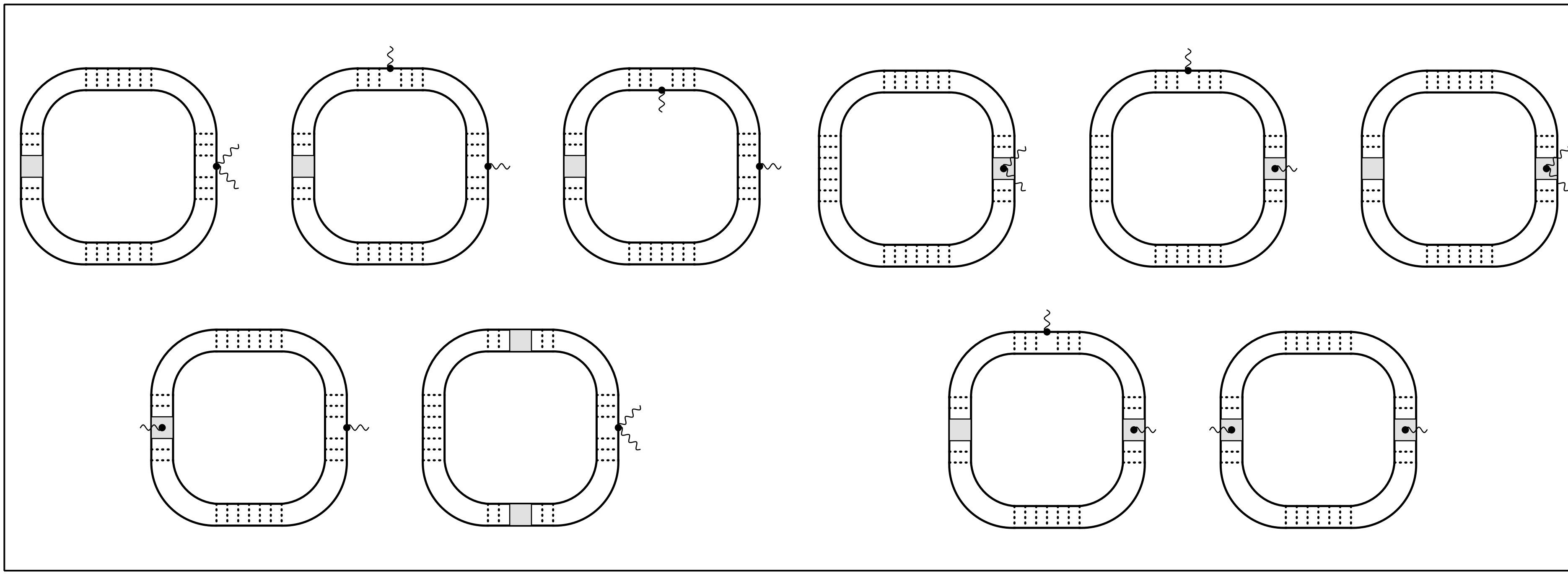
$$\sigma = e^2\frac{\partial n}{\partial \mu}D_n$$

Specific heat and the static part of the correlation function

$$\chi^{st}_{kk} = \frac{i}{2}\frac{\delta^2 Z}{\delta\eta^2} = -Tc$$

$$c = z c_{FL}$$

Castellani, Di Castro (1986)



RG and the dynamical part of the correlation function

RG and the dynamical part of the correlation function

$$S = \int \mathrm{tr}[D(1+\zeta_D)(\nabla Q)^2 + 2iz\{\hat{\varepsilon}, 1+\zeta_z\}Q] + \sum_{i=1,2} Q(1+\zeta_{\Gamma_i})\Gamma_i Q$$

Initial conditions: $\zeta_D = 0 \quad \zeta_z = \zeta_{\Gamma_1} = \zeta_{\Gamma_2} = -\eta$

Parameterization: $Q = u\, U_s Q_f U_s^{-1}\, u^{-1} \qquad Q^2 = 1$

$Q_f = U_f \Lambda U_f^{-1}$ fast

$Q_s = U_s \Lambda U_s^{-1}$ slow

u slowest: distribution function

$$U^{-1}_{\varepsilon_1\varepsilon_2}\zeta_i(\varepsilon_2 - \varepsilon_3)U_{\varepsilon_3\varepsilon_4} \neq \zeta_i(\varepsilon_1 - \varepsilon_4)$$

$$\Delta(D\zeta_D) = \left(\zeta_D D\frac{\partial}{\partial D} + \zeta_z z\frac{\partial}{\partial z} + \zeta_{\Gamma_1}\Gamma_1\frac{\partial}{\partial \Gamma_1} + \zeta_{\Gamma_2}\Gamma_2\frac{\partial}{\partial \Gamma_2}\right)\Delta D$$

$$\Delta(z\zeta_z) = \left(\zeta_D D\frac{\partial}{\partial D} + \zeta_z z\frac{\partial}{\partial z} + \zeta_{\Gamma_1}\Gamma_1\frac{\partial}{\partial \Gamma_1} + \zeta_{\Gamma_2}\Gamma_2\frac{\partial}{\partial \Gamma_2}\right)\Delta z$$

$$\vdots$$

RG and the dynamical part of the correlation function

$$S = \int \mathrm{tr}[D(1+\zeta_D)(\nabla Q)^2 + 2iz\{\hat{\varepsilon}, 1+\zeta_z\}Q] + \sum_{i=1,2} Q(1+\zeta_{\Gamma_i})\Gamma_i Q$$

Result: Fixed point

$$\Delta\zeta_D = \Delta\zeta_z = \Delta\zeta_{\Gamma_1} = \Delta\zeta_{\Gamma_2} = 0$$

$$\zeta_D = 0 \qquad \zeta_z = \zeta_{\Gamma_1} = \zeta_{\Gamma_2} = -\eta$$

$$S = \int \mathrm{tr}[D(\nabla Q)^2 + 2iz[\{\hat{\varepsilon}, 1-\eta\}Q] + Q(1-\eta)(\Gamma_1 + \Gamma_2)Q$$

$$\chi_{kk} = -cT\frac{D_k\mathbf{q}^2}{D_k\mathbf{q}^2 - i\omega} \qquad D_k = \frac{D}{z}$$

Conductivities and the Wiedemann Franz law

(Generalized) Einstein relations:

$$\boxed{\sigma = e^2 \frac{\partial n}{\partial \mu} D_n = 2\nu e^2 D}$$

$$\boxed{\kappa = c D_k = c_{FL} D}$$

$$D_n = \frac{D}{\frac{\partial n}{\partial \mu}/2\nu}$$

$$D_k = \frac{D}{z} = \frac{D}{c/c_{FL}}$$

$$\boxed{\kappa = \frac{c_{FL}}{2\nu e^2}\sigma}$$

The structure immediately implies:

Wiedemann Franz law is not violated within the RG regime (T<ε<1/τ),

neither for short-range nor for long-range (Coulomb) interaction.

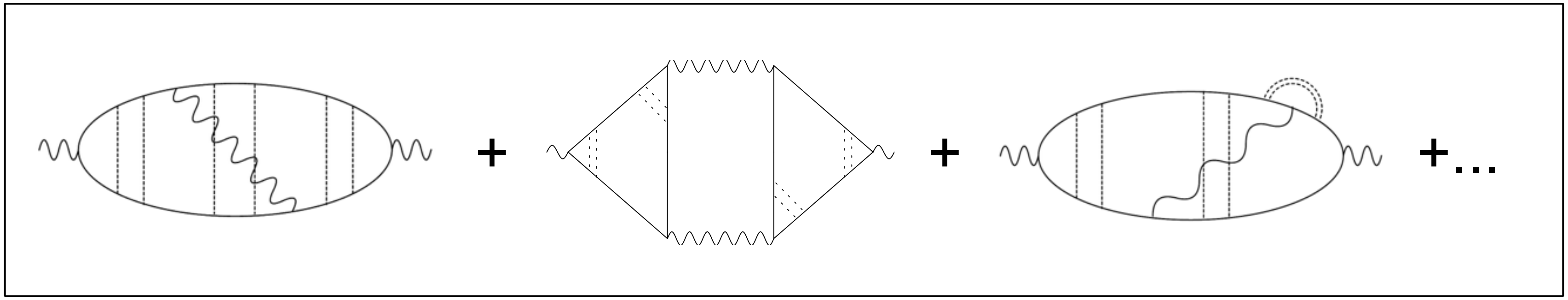
Something is missing in this treatment!

Beyond RG – the sub-temperature regime

Additional logarithms

For **short-range interactions** no additional (log) corrections

For **long-range Coulomb interactions**
additional logarithmic corrections
from scattering processes with sub-T frequency transfer.



All contributions are proportional to Im(V^R):
Decay into particle-hole pairs or drag-processes

Example:

$$\delta\chi_{kk} \propto \int_{\mathbf{k},\varepsilon,\nu} \varepsilon\nu\partial_\varepsilon\mathcal{F}_\varepsilon(\mathcal{F}_{\varepsilon+\nu}+\mathcal{F}_{\varepsilon-\nu})\mathrm{Re}\mathcal{D}^2(\mathbf{k},\nu)\mathrm{Im}V^R(\mathbf{k},\nu)$$

Corrections to heat conductivity

Additional logarithmic correction (not related to c!):

$$\chi_{kk} = -cT\frac{D_k\mathbf{q}^2}{D_k\mathbf{q}^2 - i\omega}$$

$$D_k = \frac{1}{z}\left(D_n + \delta D^h\right)$$

Consistent with conservation law!

Additional logarithmic correction to κ:

$$\delta\kappa = \frac{T}{12}\log\frac{D\kappa_s^2}{T}$$

κ_s: screening radius

WF law is violated!

Agrees with the result of (recent) kinetic equation approaches

From the regime:

$$\boxed{\frac{T^2}{D\kappa^2} < D\mathbf{k}^2 < T}$$

Thermal conductivity in the ballistic regime.

$$\kappa \propto \frac{T\varepsilon_F}{\Gamma_\varepsilon}$$

$$\Gamma = \Gamma_{imp} + \Gamma_{e-e}$$

$$\Gamma_{imp} = \frac{1}{\tau}$$

$$\kappa \propto T\tau\varepsilon_F$$

Drude

$$\Gamma_{e-e} = a\frac{T^2}{\varepsilon_F}\ln\frac{\varepsilon_F}{T}$$

$$\kappa = \frac{\varepsilon_F^2}{T\ln\frac{\varepsilon_F}{T}}$$

Lyakov, Mishchenko (2003)

$$\boxed{\kappa \propto T\tau\varepsilon_F - aT(T\tau)^2\ln\frac{\varepsilon_F}{T}}$$

“Localizing”

Catelani, Aleiner (2005)

Results: Thermal transport and the WFL

$$S[Q] \sim \int d\mathbf{r}\ \mathrm{tr}[D(\nabla Q)^2 + 2iz\{\hat{\varepsilon}, \lambda\}Q] + \sum_i Q\lambda\Gamma_i Q$$

$$\lambda \approx 1 - \eta + \eta^2$$

Energy scales

GS & Finkel'stein, with Keldysh NLσM
PRB **89** (2014); PRB **90** (2014)(R); PRB **90** (2014); arXiv:1509.02519, arXiv:1510.06529

ε_F

$\vdots$

$\frac{1}{\tau}$

$\vdots$

RG regime

WFL

-disordered Fermi liquid

-disordered electron liquid

despite strong renormalizations

T

$\vdots$

sub-T regime

WFL -disordered Fermi liquid

~~WFL~~ -disordered electron liquid

$\frac{T^2}{D\kappa_s}$

Additional (delocalizing) logarithmic corrections!

The metallic side of the metal-insulator transition in Si-MOSFETS

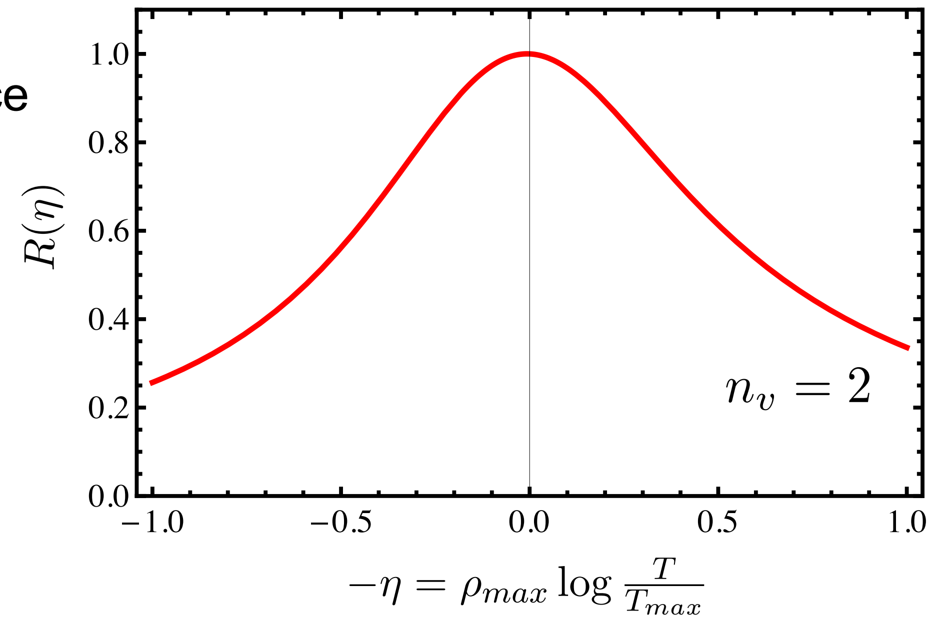
One-loop result:

Universal behavior of the resistance

$$R(\eta) = \rho(\eta)/\rho_{max}$$

$$\eta = \rho_{max} \ln(T_{max}/T)$$

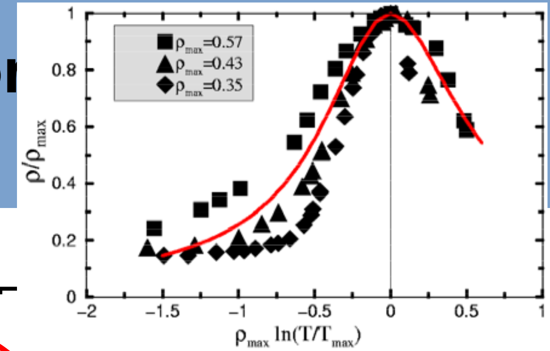
n_v : Number of valleys



$$\frac{\partial \rho}{\partial \xi} = \rho^2 \left[n_v + 1 - (4n_v^2 - 1)\left(\frac{1+w_2}{w_2} \ln(1+w_2) - 1 \right) \right]$$

$$\frac{\partial w_2}{\partial \xi} = \rho \frac{(1+w_2)^2}{2} \qquad w_2 = \frac{\Gamma_2}{z}$$

The metallic side of the metal-insulator transition in Si-MOSFETS



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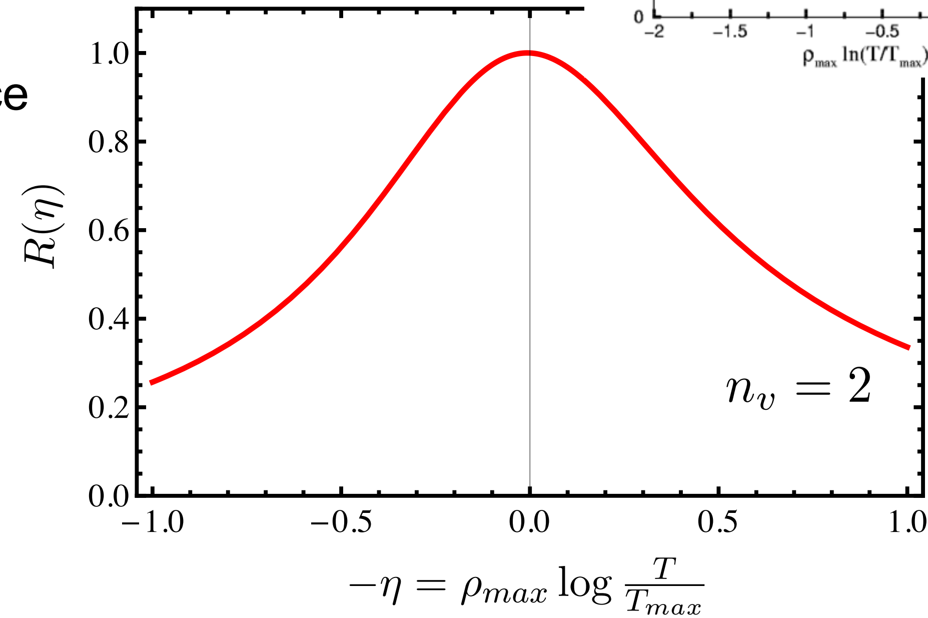

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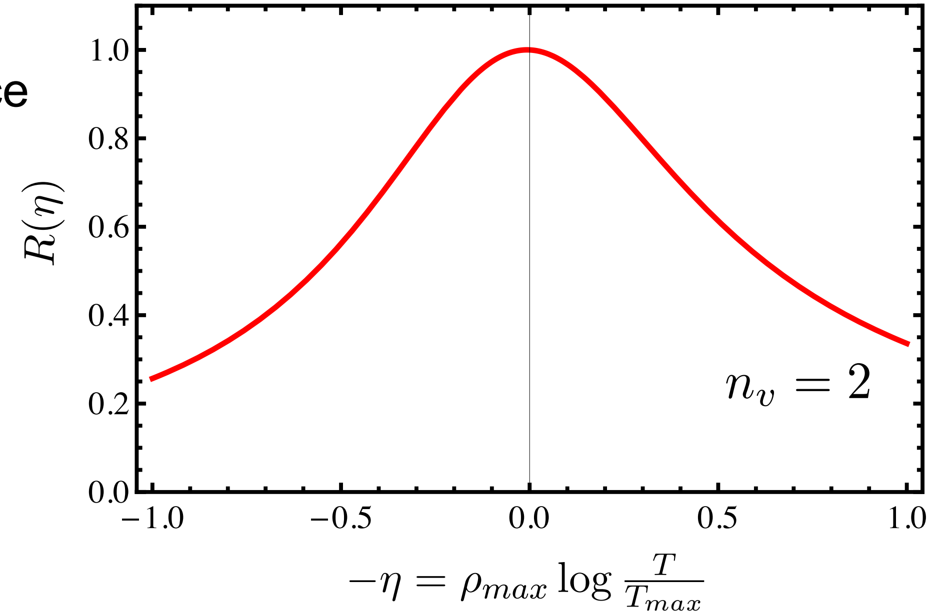

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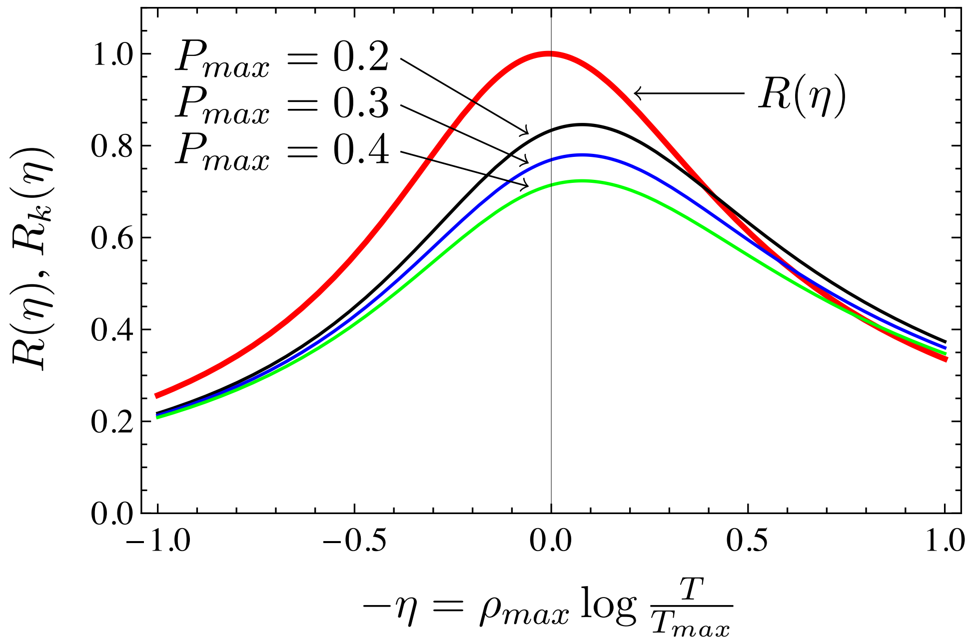
$$\frac{\partial w_2}{\partial \xi} = \rho \frac{(1 + w_2)^2}{2} \qquad w_2 = \frac{\Gamma_2}{z}$$

Application: Thermal transport on the metallic side of the metal-insulator transition in Si MOSFETs

$$R_k(\eta) = \rho_k(\eta)/\rho_{max}$$

$$\rho_k = \frac{e^2}{2\pi^2}\frac{\mathcal{L}_0 T}{\kappa}$$



Wiedemann-Franz law:

$$\rho_k = \rho, \; R_k(\eta) = R(\eta)$$

Violation parametrized by:

$$P_{max} = \frac{\rho_{max} - \rho_k(0)}{\rho_k(0)}$$

Maximum in R_k at universal (P_{max} independent) value η=-0.0785.

Summary - Part I

- We developed a field theoretic model with „gravitational potentials" suitable for the analysis of heat density correlation function in the disordered electron liquid.
- For short range interactions the renormalization of κ and of σ are linked through the WF law.
- For long-range (Coulomb) interaction there are additional logarithmic corrections originating from outside of the RG regime. They lead to a violation of the WF law.
- As an application, we considered the metallic side of the MIT in Si-MOSFETS.

Georg Schwiete & Alexander Finkel'stein,
Phys. Rev. B **89** (2014); RG with Keldysh NLσM;
Phys. Rev. B **90** (2014)(R); Wiedemann Franz law
Phys. Rev. B **90** (2014); RG for Keldysh NLσM with grav. potentials
arXiv:1509.02519: Analysis of low temperature regime, electron gas
arXiv:1510.06529: Analysis of low temperature regime, electron liquid

Thank you!