Small Cosmological Constant from Running Gravitational Coupling

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Andrei Frolov

Department of Physics
Simon Fraser University

Jun-Qi Guo (SFU)

Sternberg Astronomical Institute

MSU, Moscow, Russia
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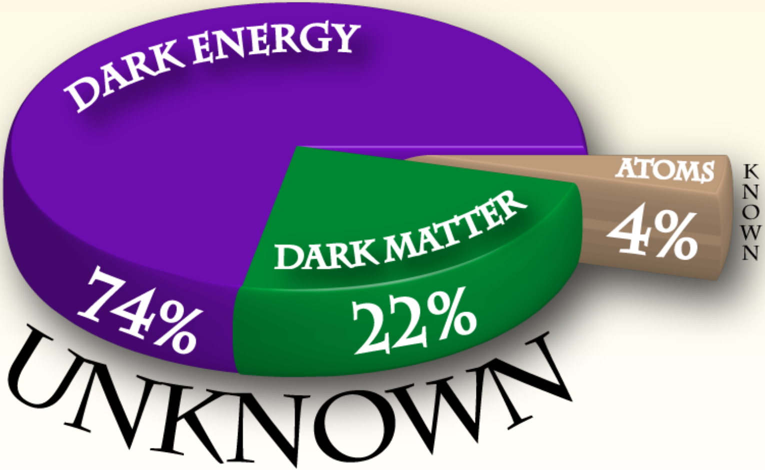
What's the Matter with Cosmology?



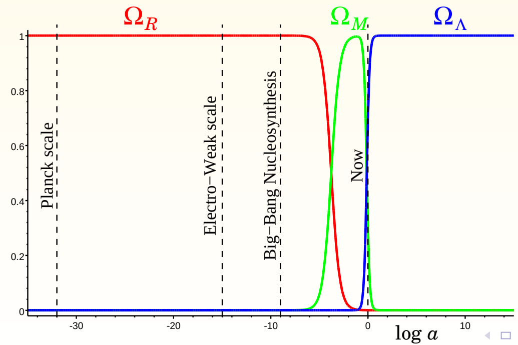
Value problem:

expected $\rho_\Lambda \sim m_{\text{pl}}^4 \longleftarrow 10^{120} \longrightarrow \rho_\Lambda \sim \rho_M \neq 0$ observed

Coincidence problem:

$\Omega_\Lambda = 0.7 \qquad \Omega_M = 0.3 \qquad \Omega_R = 5 \cdot 10^{-5}$

$\rho_\Lambda \propto a^0 \qquad \rho_M \propto a^{-3} \qquad \rho_R \propto a^{-4}$



Gravity is an Effective Field Theory! Maybe...

$$S[\lambda] = \int \left\{ \sum_{n=0}^{\infty} \lambda^{4-2n} g^{(n)}(\lambda)\, \mathscr{R}^{(n)} + \ldots \right\} \sqrt{-g}\, d^4x$$

$$8\pi G = \alpha\, m_{\text{pl}}^{-2}$$

$$\mu \frac{d\alpha}{d\mu} = \beta(\alpha)$$

$$\int \frac{d\alpha}{\beta(\alpha)} = \int \frac{d\mu}{\mu}$$

$$\mathscr{L}_{\text{GR}} = \frac{R}{16\pi G} \mapsto \frac{m_{\text{pl}}^2}{2} \frac{R}{\alpha} = \mathscr{L}_{f(R)}$$

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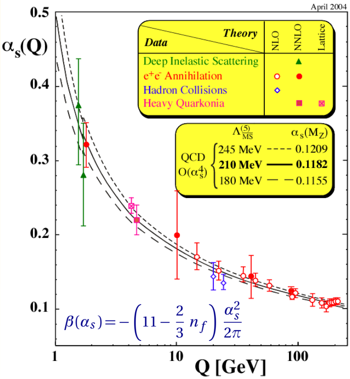

This Has Been Tried Before...

What if instead of curvature in Einstein-Hilbert action we had

$$S = \int \left\{ \frac{f(R)}{16\pi G} + \mathscr{L}_\mathrm{m} \right\} \sqrt{-g}\, d^4x$$

UV modification:

$$f(R) = R + \frac{R^2}{M^2}$$

Starobinsky (1980)

IR modification:

$$f(R) = R - \frac{\mu^4}{R}$$

Capozziello et. al. [astro-ph/0303041]
Carroll et. al. [astro-ph/0306438]

For F(R) theory to make sense we need:

- $f' > 0$ – otherwise gravity is a ghost
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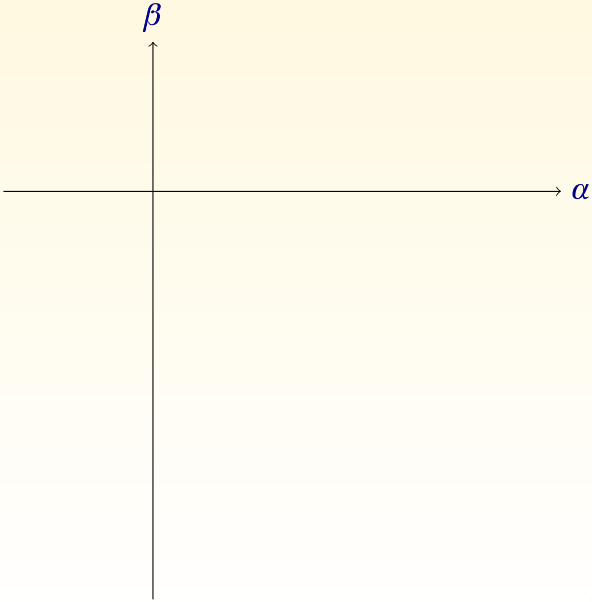
... BUT NOT QUITE IN THIS WAY!



stable?

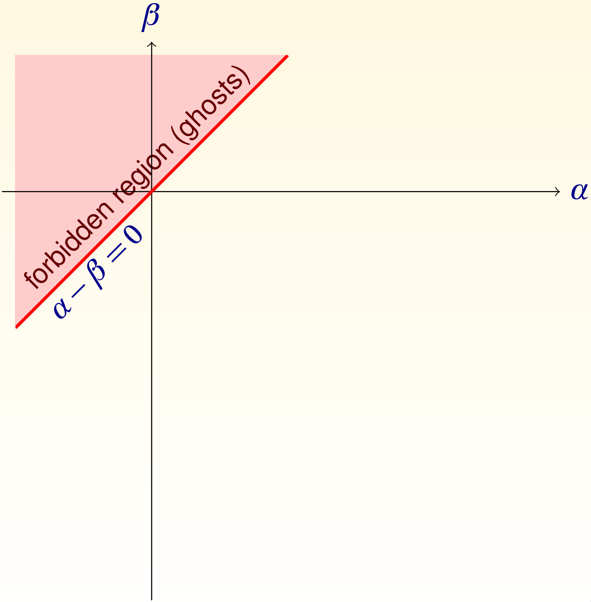
$$f' = \frac{\alpha - \beta}{\alpha^2} > 0$$

$$R f'' = -\left(1 - 2\frac{\beta}{\alpha} + \beta'\right)\frac{\beta}{\alpha^2} > 0$$

de Sitter?

$$2f - f'R = \frac{\alpha + \beta}{\alpha^2}\, R = 0$$

Accelerated RG Flows of Newton's Constant



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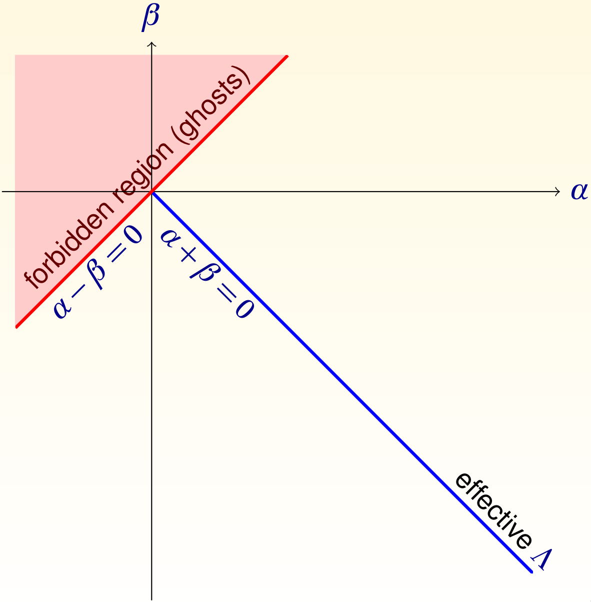

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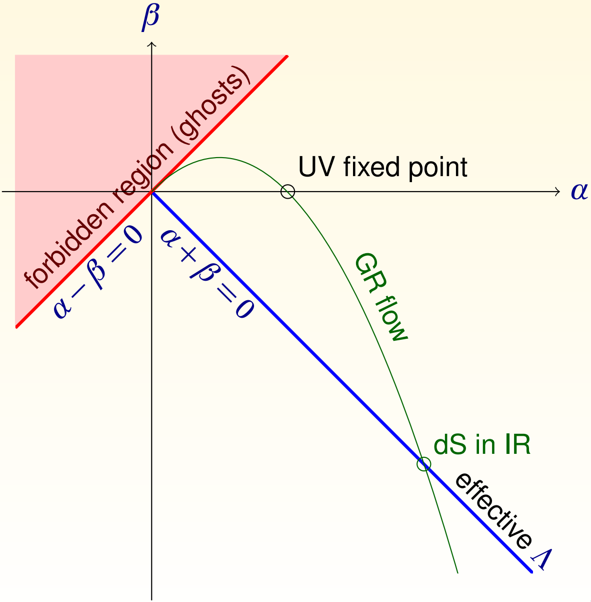

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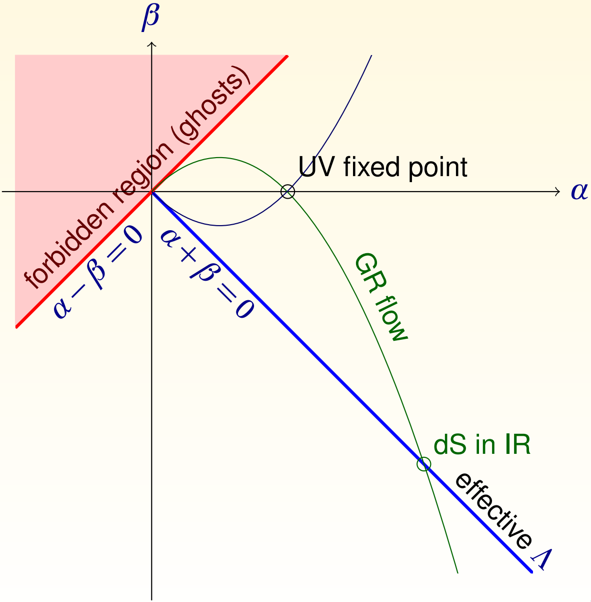
$$\beta(\alpha) = -\alpha(\alpha - 1)$$

$$f(R) = R - 2\Lambda$$

Einstein's gravity

ACCELERATED RG FLOWS OF NEWTON'S CONSTANT



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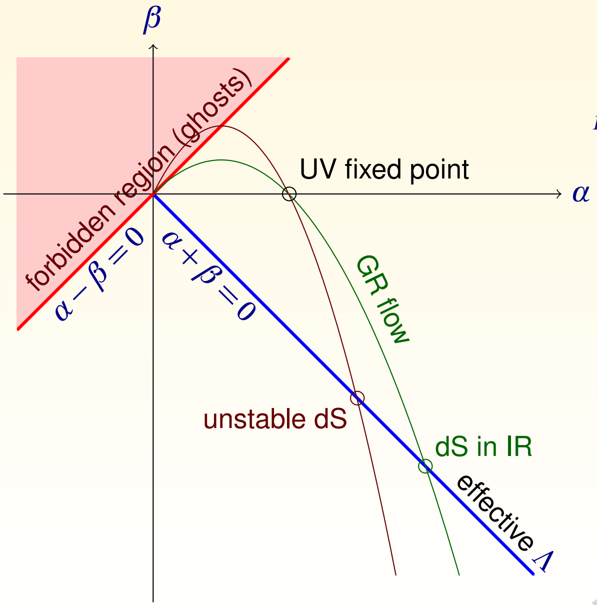

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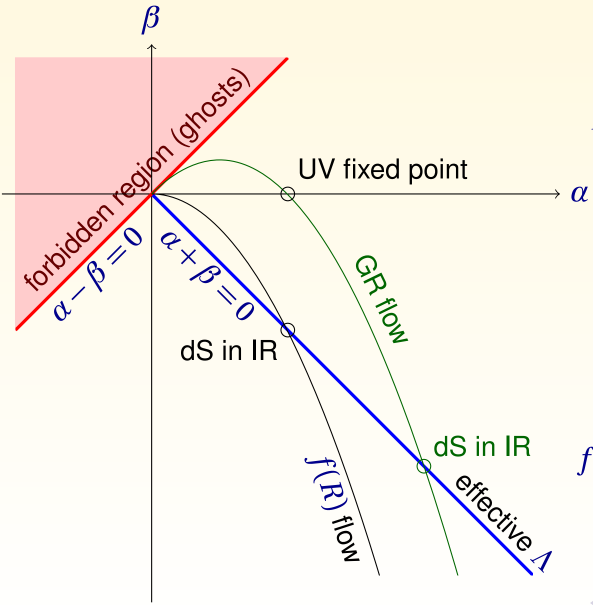

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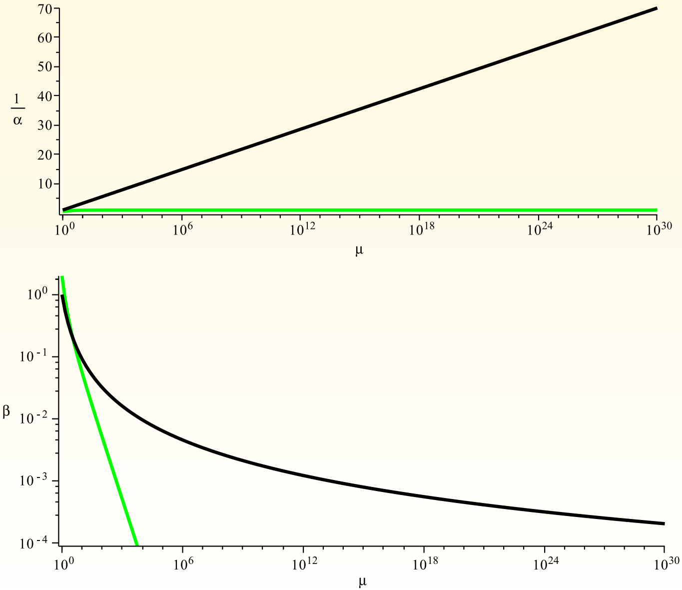
$$\beta(\alpha) = -\alpha^2$$

$$f(R) = \frac{R}{\alpha_0}\left(1 + \alpha_0 \ln \frac{R}{R_0}\right)$$

something new?

So What Does This Mean for Hierarchy?..



$f(R)$ Gravity is a Scalar-Tensor Theory

- Einstein equations turn into a fourth-order equation:

$$f' G_{\mu\nu} - f'_{;\mu\nu} + \left[\Box f' - \frac{1}{2}(f - f'R)\right] g_{\mu\nu} = m_{\text{pl}}^{-2}\, T_{\mu\nu}$$

- A new scalar degree of freedom $\phi \equiv f' - 2$ appears:

$$\Box f' = \frac{1}{3}(2f - f'R) + m_{\text{pl}}^{-2}\,\frac{T}{3}$$

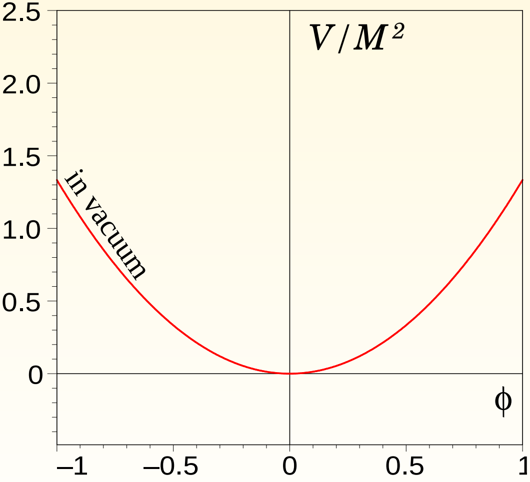
- Can rewrite fourth-order field equation as two second order ones!

$$G_{\mu\nu} = m_{\text{pl}}^{-2}\, T_{\mu\nu} + \mathcal{Q}_{\mu\nu}, \quad \Box\phi = V'(\phi) - \mathscr{F}$$

$$\mathcal{Q}_{\mu\nu} = -(1+\phi)G_{\mu\nu} + \phi_{;\mu\nu} - \left[\Box\phi - 3P(\phi)\right] g_{\mu\nu}$$

$$P(\phi) \equiv \frac{1}{6}(f - f'R), \quad V'(\phi) \equiv \frac{1}{3}(2f - f'R), \quad \mathscr{F} \equiv \frac{m_{\text{pl}}^{-2}}{3}(\rho - 3p)$$

Example I: (Safe) UV Modification



$$f(R) = R + \frac{R^2}{M^2}$$

$$\phi = \frac{2R}{M^2}$$

$$V = \frac{1}{3}\frac{R^2}{M^2} = \frac{M^2}{12}\phi^2$$

massive scalar field!

scalar degree of freedom ϕ
is heavy and hard to excite

$$U(\phi) = V(\phi) + \mathscr{F}(\phi_* - \phi)$$

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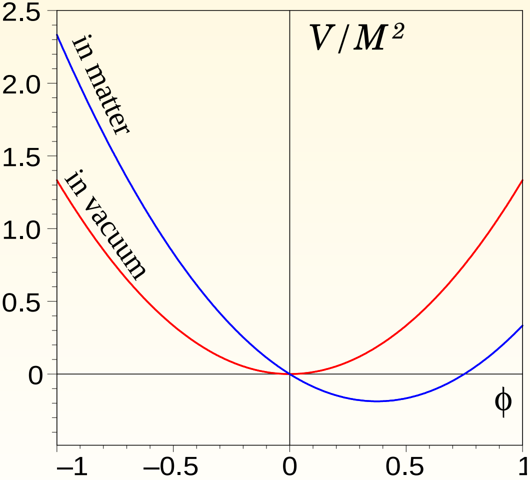

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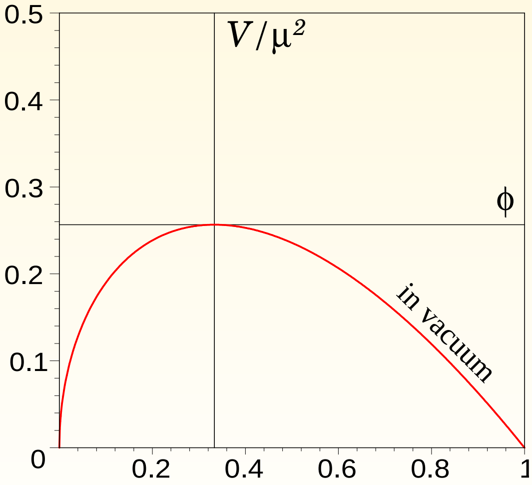

$$f(R) = R - \frac{\mu^4}{R}$$

$$\phi = \frac{\mu^4}{R^2}$$

$$V = \frac{2}{3}\left(\frac{\mu^4}{R} - \frac{\mu^8}{R^3}\right) = \frac{2}{3}\mu^2\left(\phi^{\frac{1}{2}} - \phi^{\frac{3}{2}}\right)$$

field ϕ is unstable!

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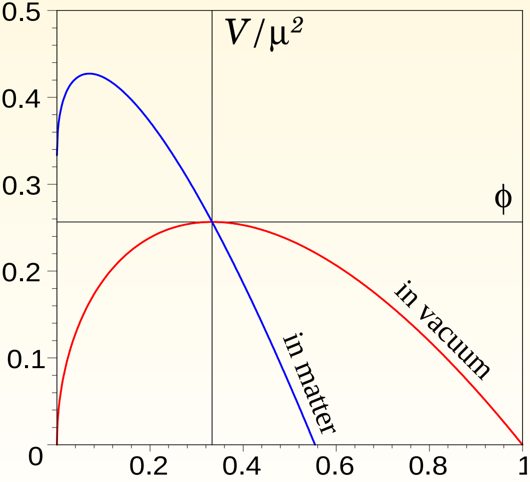
Dolgov & Kawasaki (astro-ph/0307285)

Example II: (Failed) IR Modification



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Can We Come Up With Something Better?..

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$$\phi = \ln \frac{R}{R_0} + \frac{1-\alpha_0}{\alpha_0} = \ln \frac{R}{R_*}$$

$$R_* \equiv 4\Lambda = R_0 \exp\left[\frac{\alpha_0 - 1}{\alpha_0}\right]$$

$$P(\phi) = -\frac{2}{3}\Lambda e^{\phi}, \quad V'(\phi) = \frac{4}{3}\Lambda e^{\phi}\phi$$

$$V(\phi) = \frac{4}{3}\Lambda e^{\phi}(\phi - 1)$$

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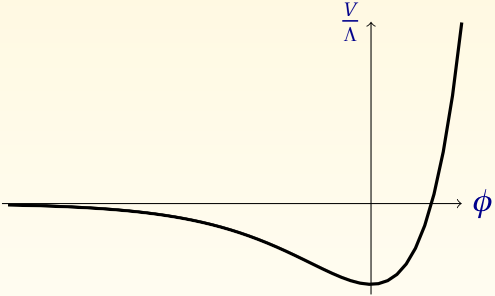

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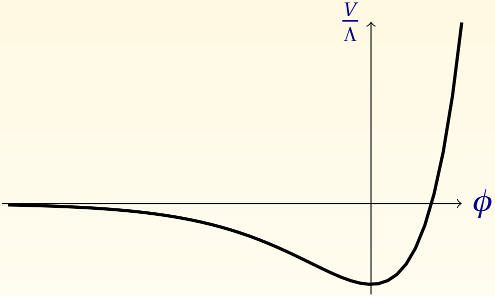

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2 scalar DoF, 4D phase space

$$\{\phi, \pi, a, H\}$$

dynamical system equations

$$\dot{\phi} = \pi, \quad \dot{a} = aH$$

$$\dot{H} = \frac{R}{6} - 2H^2$$

$$\dot{\pi} + 3H\pi + V'(\phi) = m_{\text{pl}}^{-2}\,\frac{\rho - 3p}{3}$$

subject to a constraint

$$H^2(\phi+2) + H\pi + P(\phi) = m_{\text{pl}}^{-2}\,\frac{\rho}{3}$$

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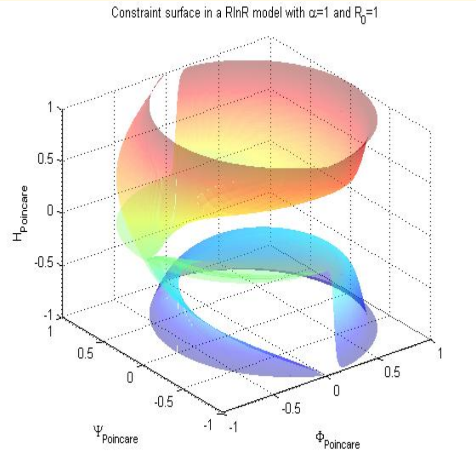

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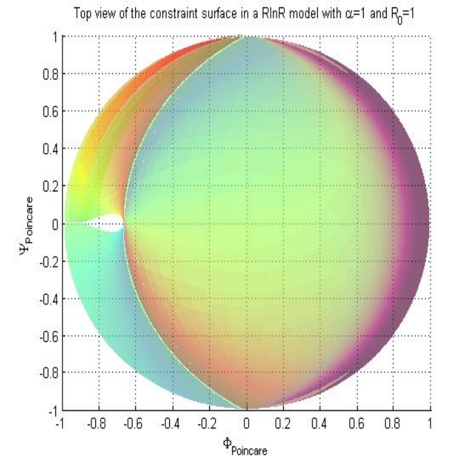

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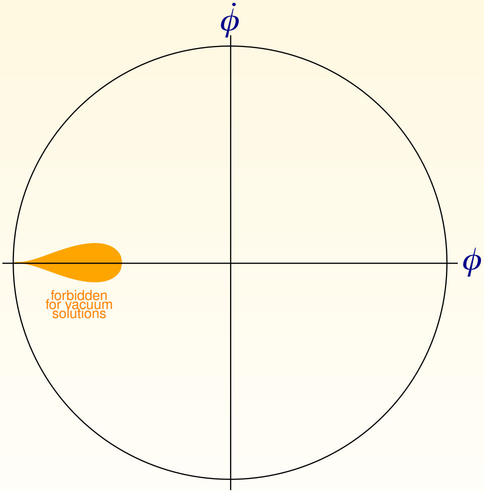

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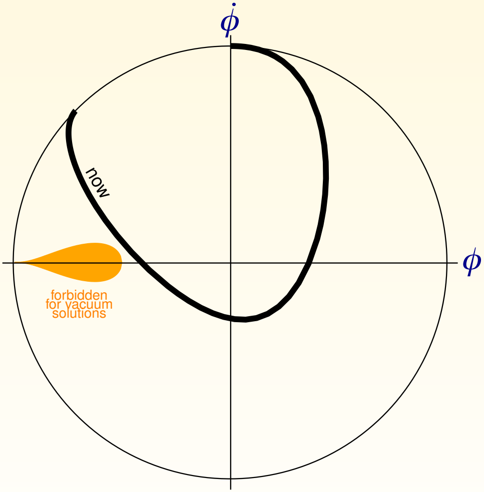

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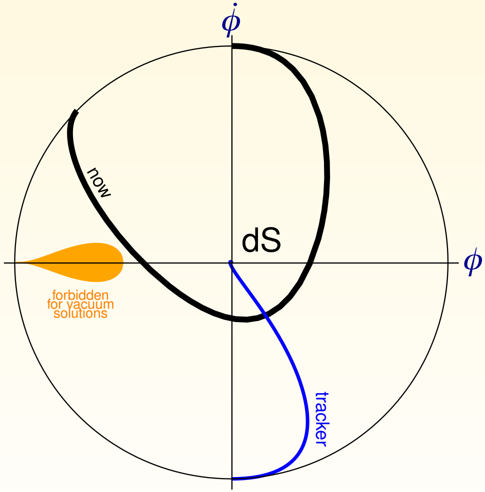

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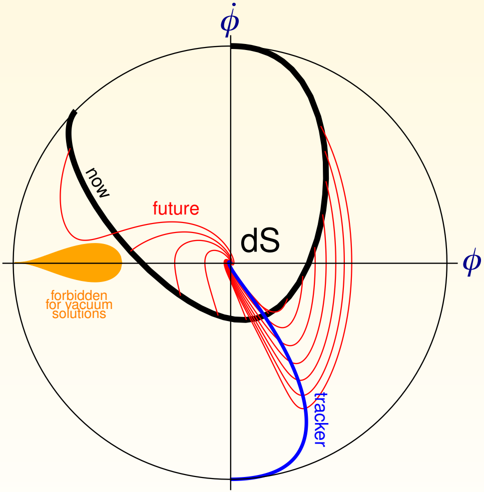
subject to a constraint

$$H^2(\phi+2) + H\pi + P(\phi) = m_{\text{pl}}^{-2}\,\frac{\rho}{3}$$

$$P(\phi) \equiv \frac{1}{6}(f - f'R) = -\frac{2}{3}\Lambda e^{\phi}$$

$$V'(\phi) \equiv \frac{1}{3}(2f - f'R) = \frac{4}{3}\Lambda e^{\phi}\phi$$



$$\Omega_{M,0} = 0.278$$

$$\Omega_{\mathcal{Q},0} = 0.397$$

2 scalar DoF, 4D phase space

$$\{\phi, \pi, a, H\}$$

dynamical system equations

$$\dot{\phi} = \pi, \quad \dot{a} = aH$$

$$\dot{H} = \frac{R}{6} - 2H^2$$

$$\dot{\pi} + 3H\pi + V'(\phi) = m_{\rm pl}^{-2}\,\frac{\rho - 3p}{3}$$

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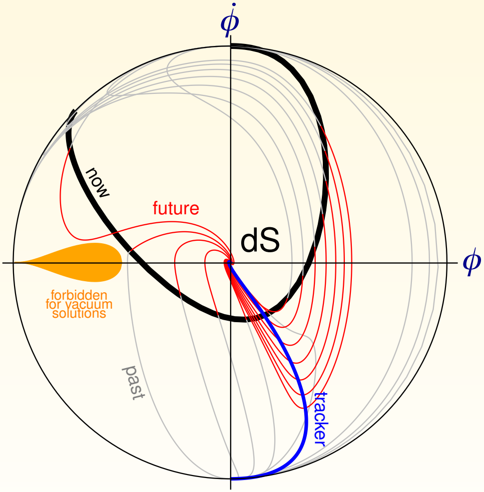

$$\Omega_{M,0} = 0.278$$

$$\Omega_{\mathscr{Q},0} = 0.397$$

Universe as a Dynamical System

2 scalar DoF, 4D phase space

$$\{\phi, \pi, a, H\}$$

dynamical system equations

$$\dot{\phi} = \pi, \quad \dot{a} = aH$$

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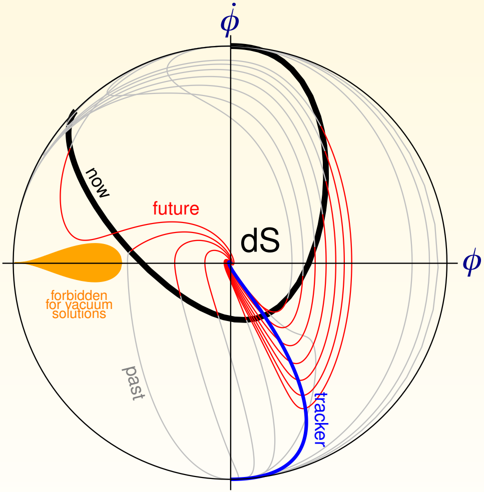

$$\Omega_{M,0} = 0.278$$

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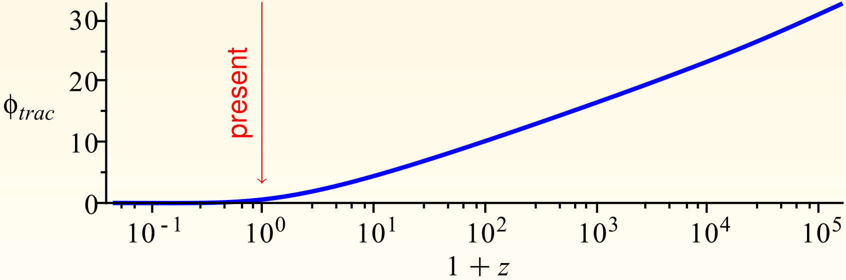
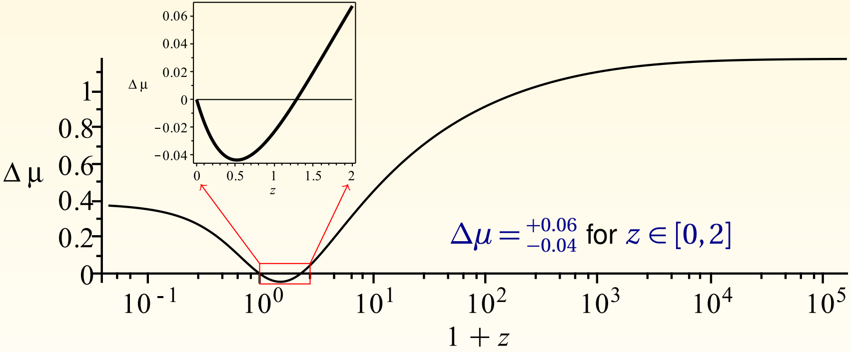
Cosmological Evolution: Tracker Solution



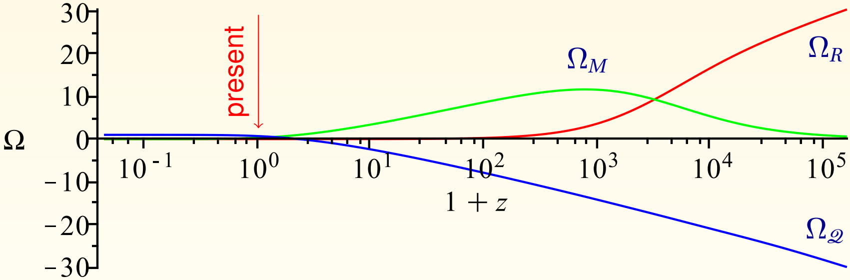
$$m_{\rm pl}^{-2}\frac{\rho}{3} = \left(\frac{\Omega_{M,0}}{a^3} + \frac{\Omega_{R,0}}{a^4}\right) H_0^2, \qquad \phi_{\rm trac} \simeq W\left(\frac{3H_0^2}{4\Lambda}\frac{\Omega_{M,0}}{a^3}\right)$$



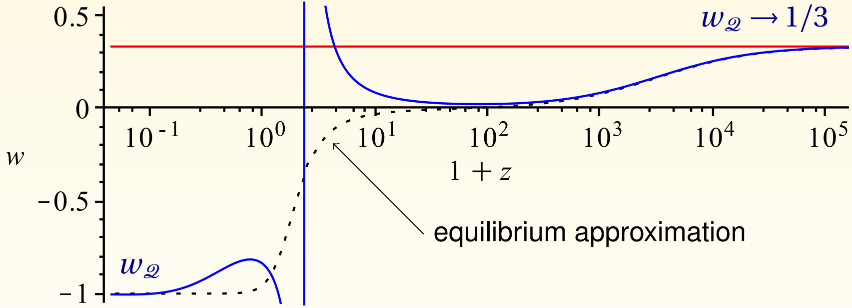
$\Delta\mu = {}^{+0.06}_{-0.04}$ for $z \in [0,2]$

$$\Delta\mu = 5\log_{10}\frac{d_{L,f(R)}}{d_{L,\Lambda\text{CDM}}}, \qquad d_L = \frac{1}{a}\int_a^1 \frac{da}{a^2 H}$$

Cosmological Evolution: Effective Ω & w



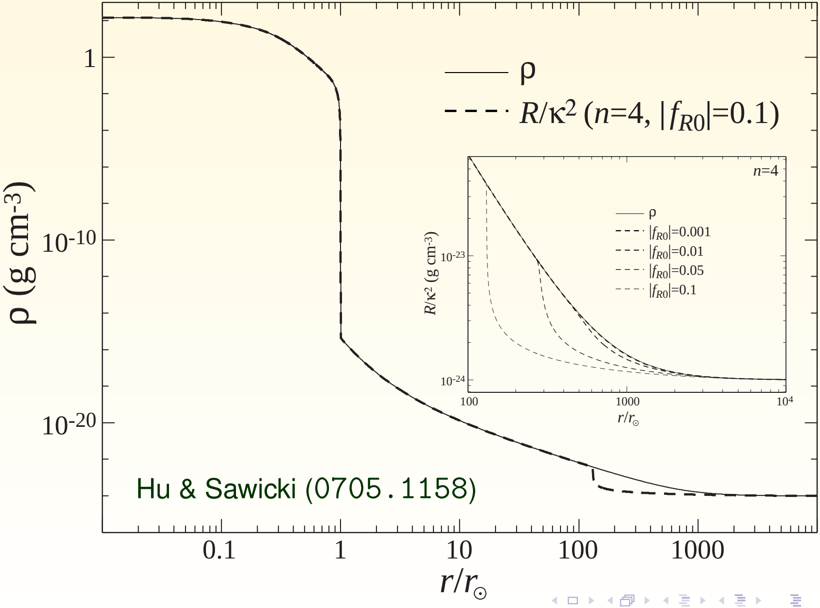
$$\Omega_{\mathscr{Q}} = \frac{\rho_{\mathscr{Q}}}{3m_{\text{pl}}^2 H^2}, \qquad \begin{aligned} \rho_{\mathscr{Q}} &= 3m_{\text{pl}}^2 H^2 - \rho \\ p_{\mathscr{Q}} &= m_{\text{pl}}^2 (H^2 - R/3) - p \end{aligned}$$



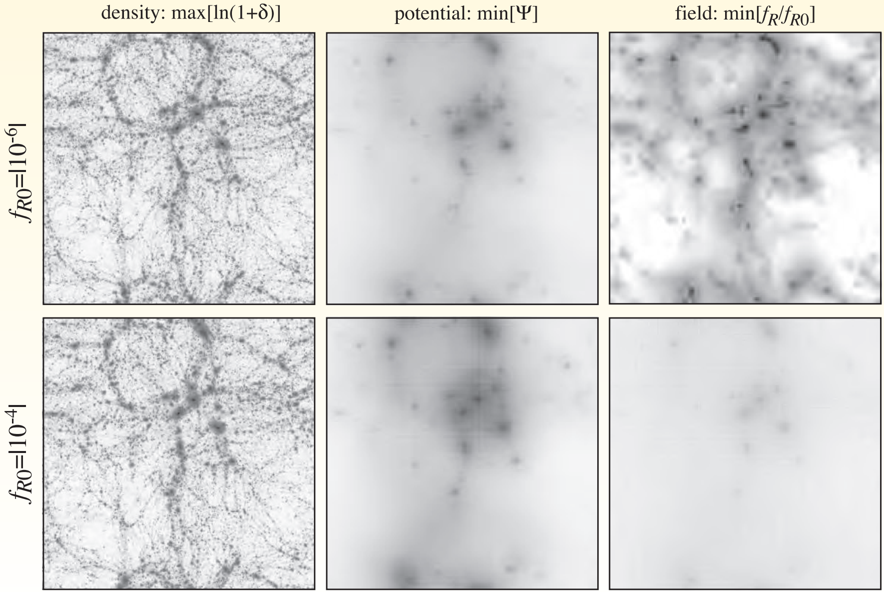
$$w_{\mathscr{Q}} = \frac{p_{\mathscr{Q}}}{\rho_{\mathscr{Q}}}, \qquad \rho_{\mathscr{Q}} = 3m_{\text{pl}}^2 H^2 - \rho$$

$$p_{\mathscr{Q}} = m_{\text{pl}}^2 (H^2 - R/3) - p$$

INVOKE CHAMELEON TO SATISFY LOCAL TESTS



N-Body Simulations with F(R) Dark Energy



Oyaizu, Lima & Hu (0807.2462)

Longitudinal Gravity Waves from Collapse!

Harada, Chiba, Nakao, & Nakamura (`gr-qc/9611031`)