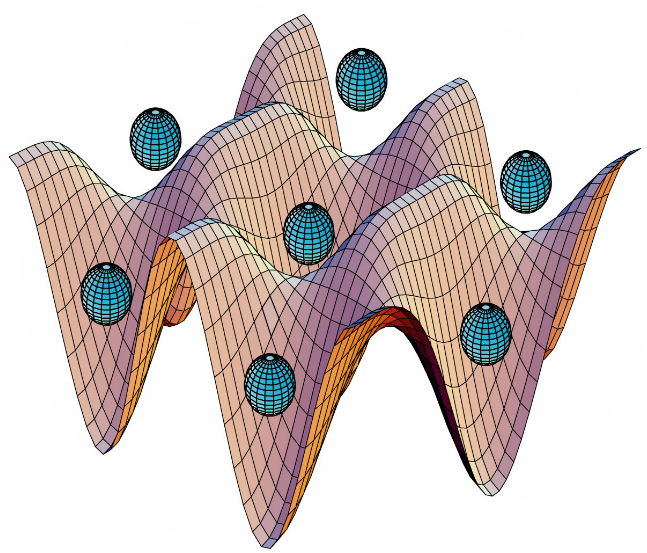
Петербургский электротехнический университет
"ЛЭТИ"
Факультет электроники

The World of Quantum Information



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Outline

- Quantum Information: fundamental principles

  (and how it is different from the classical one).
  - Bits & Qubits
  - Quantum weirdness: entanglement, superposition & measurement
  - Logic gates & Quantum circuits
- Cryptography & quantum information
- A brief introduction to quantum computing
- Real world: what do we need to build a quantum computer/quantum network?
- Current status & future roadmap

Why quantum information?

Information is physical!

Any processing of information

is always performed by physical means

Bits of information obey laws of classical physics.

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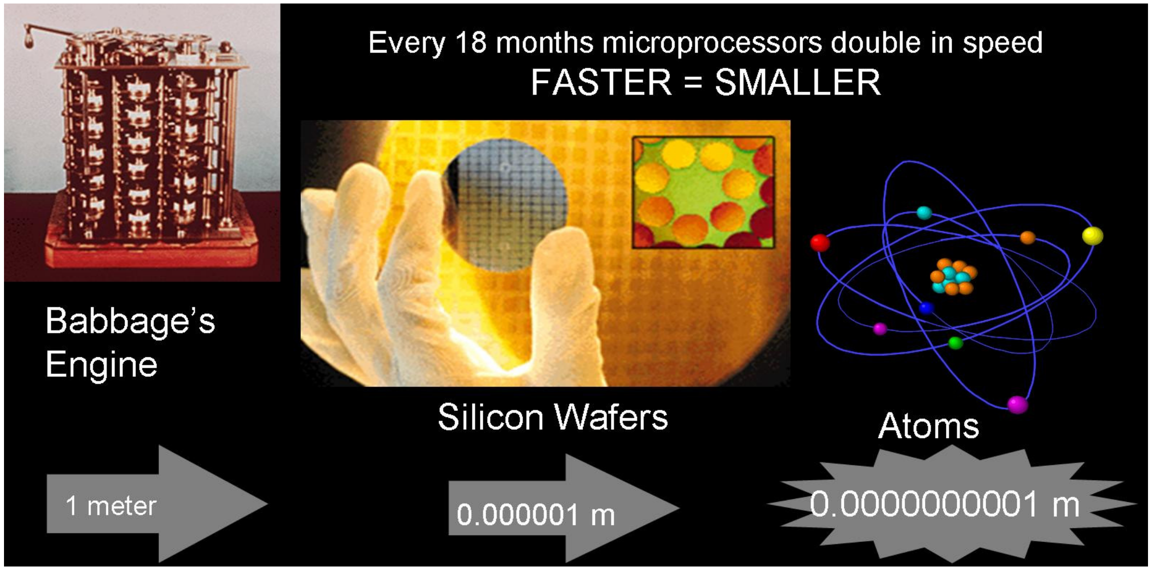
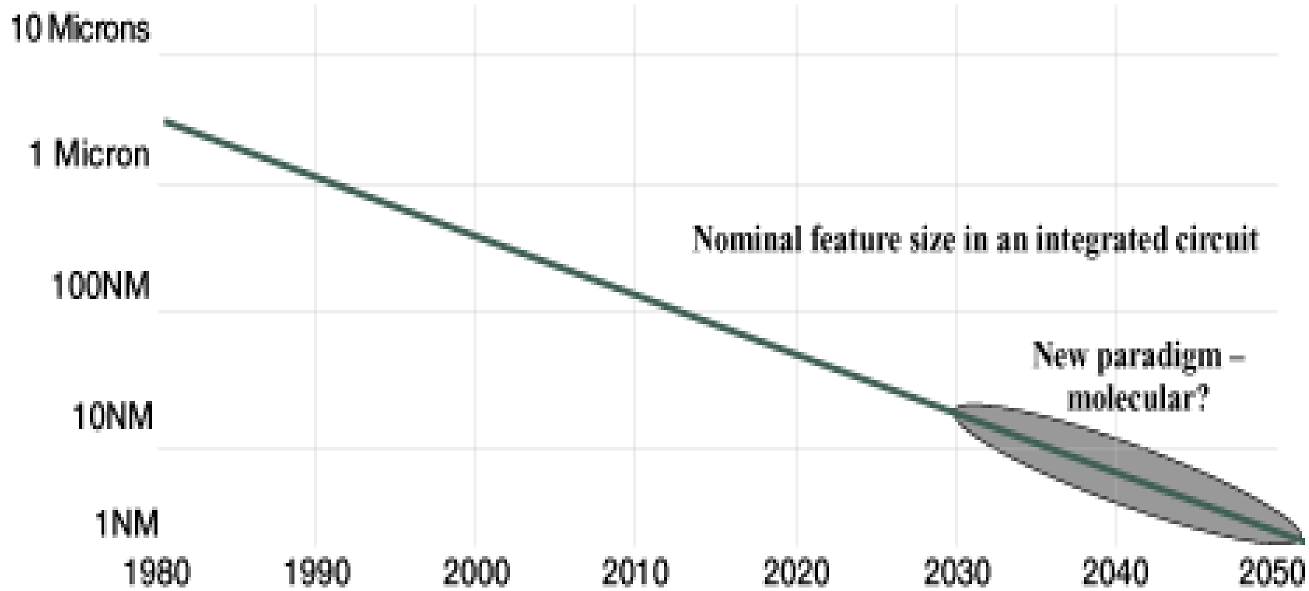


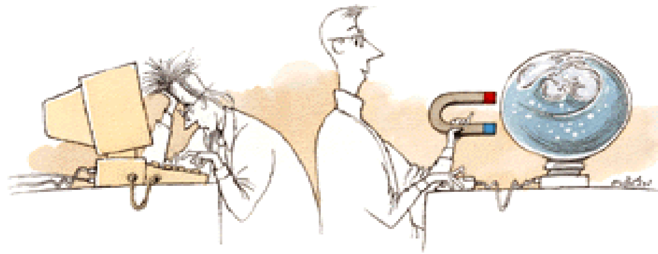
Why Quantum Computers?



Computer technology is making devices smaller and smaller…

…reaching a point where classical physics is no longer a suitable model for the laws of physics.

Bits & Qubits



Fundamental building blocks of classical computers:

BITS

STATE:

Definitely

0 or 1

Fundamental building blocks of quantum computers:

Quantum bits

or

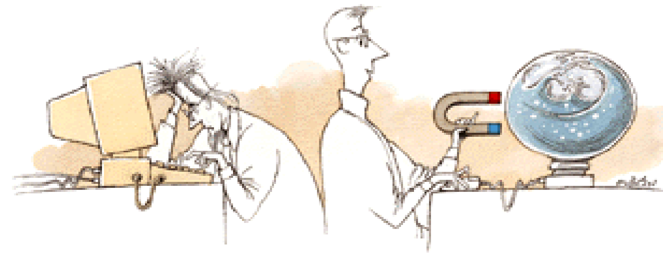
QUBITS

Basis states: $|0\rangle$ and $|1\rangle$

Superposition:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Bits & Qubits



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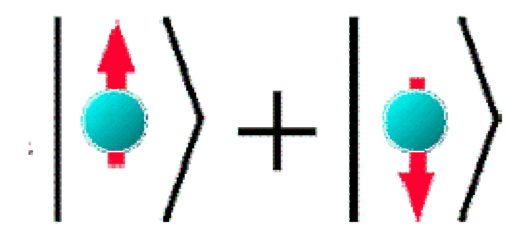
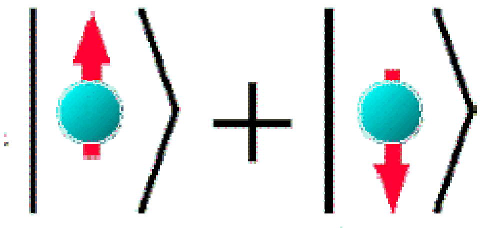
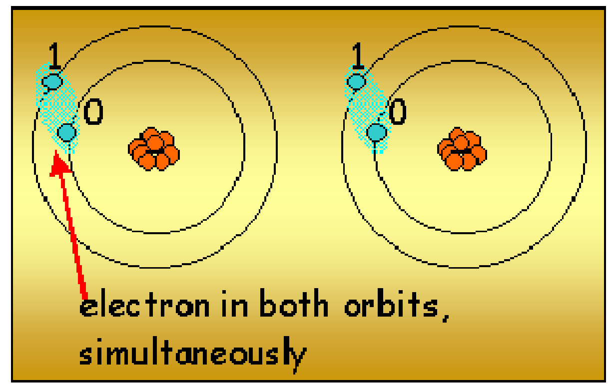
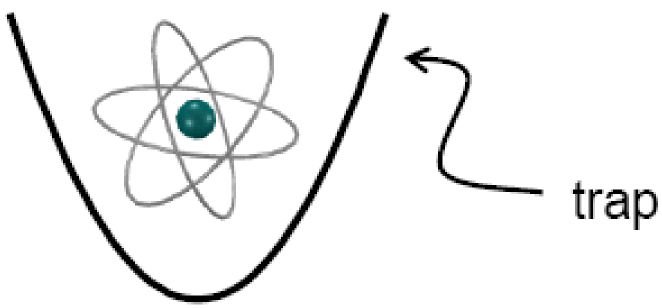


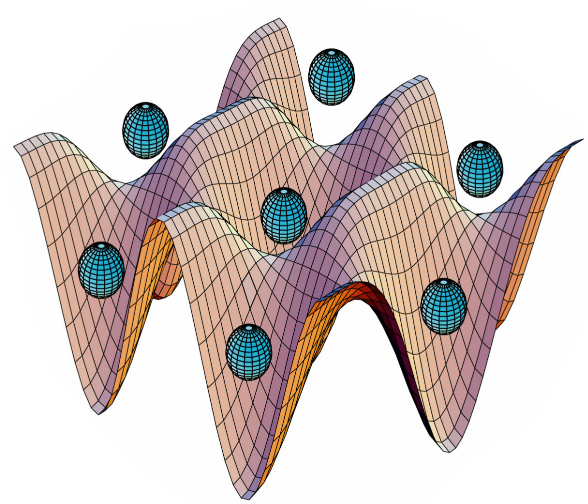
Qubit: any suitable two-level quantum system





single trapped atom:

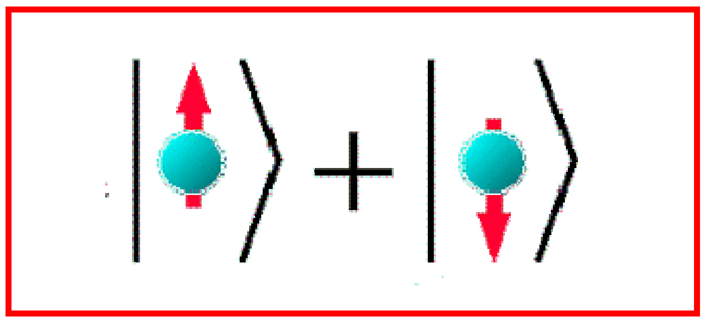




Bits & Qubits: primary differences

Superposition

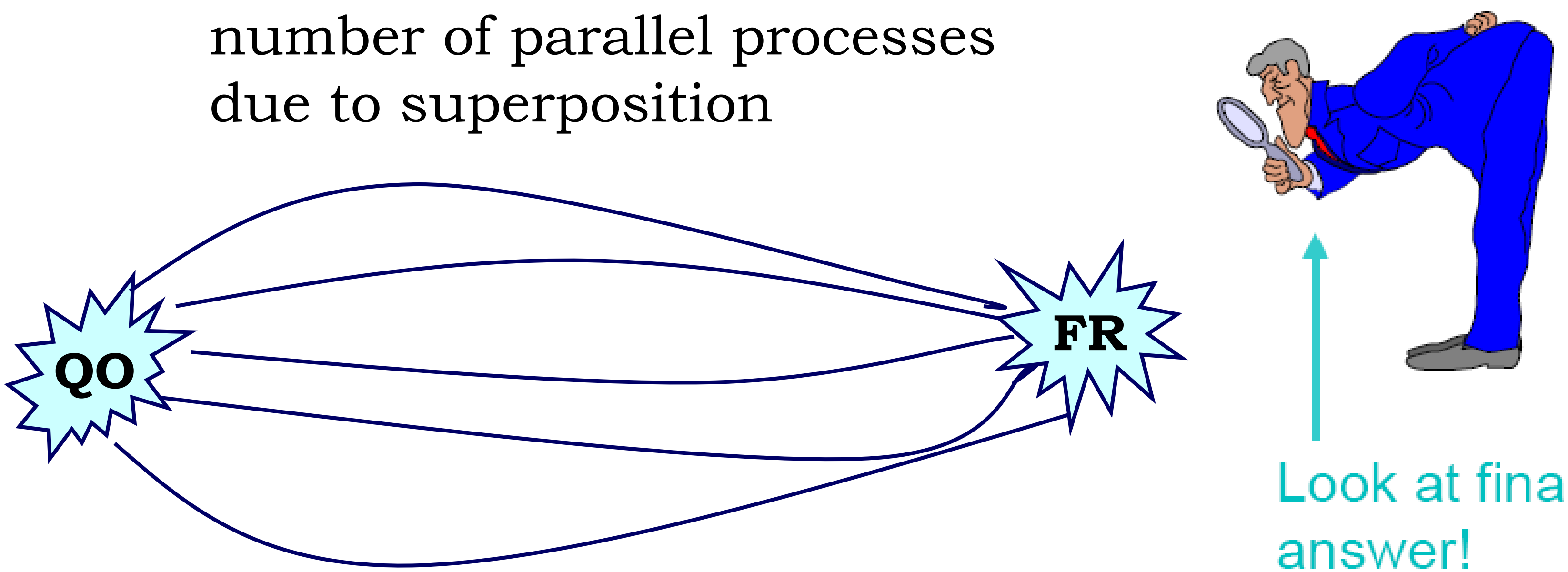
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$



Bits & Qubits: primary differences

Measurement

- Classical bit: we can find out if it is in state 0 or 1 and the measurement will not change the state of the bit.
- Qubit: Quantum calculation: number of parallel processes due to superposition



Bits & Qubits: primary differences

- Superposition
- Measurement

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

- Classical bit: we can find out if it is in state 0 or 1 and the measurement will not change the state of the bit.
- Qubit: we cannot just measure α and β and thus determine its state! We get either $|0\rangle$ or $|1\rangle$ with corresponding probabilities $|\alpha|^2$ and $|\beta|^2$.

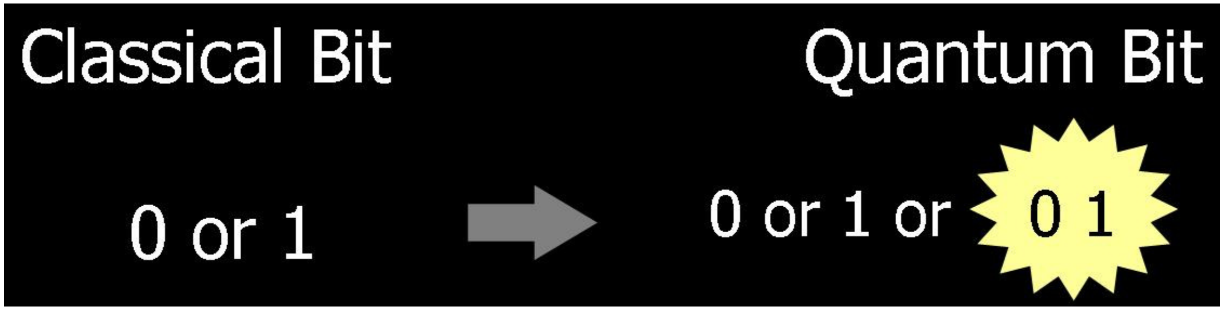
$$|\alpha|^2 + |\beta|^2 = 1$$

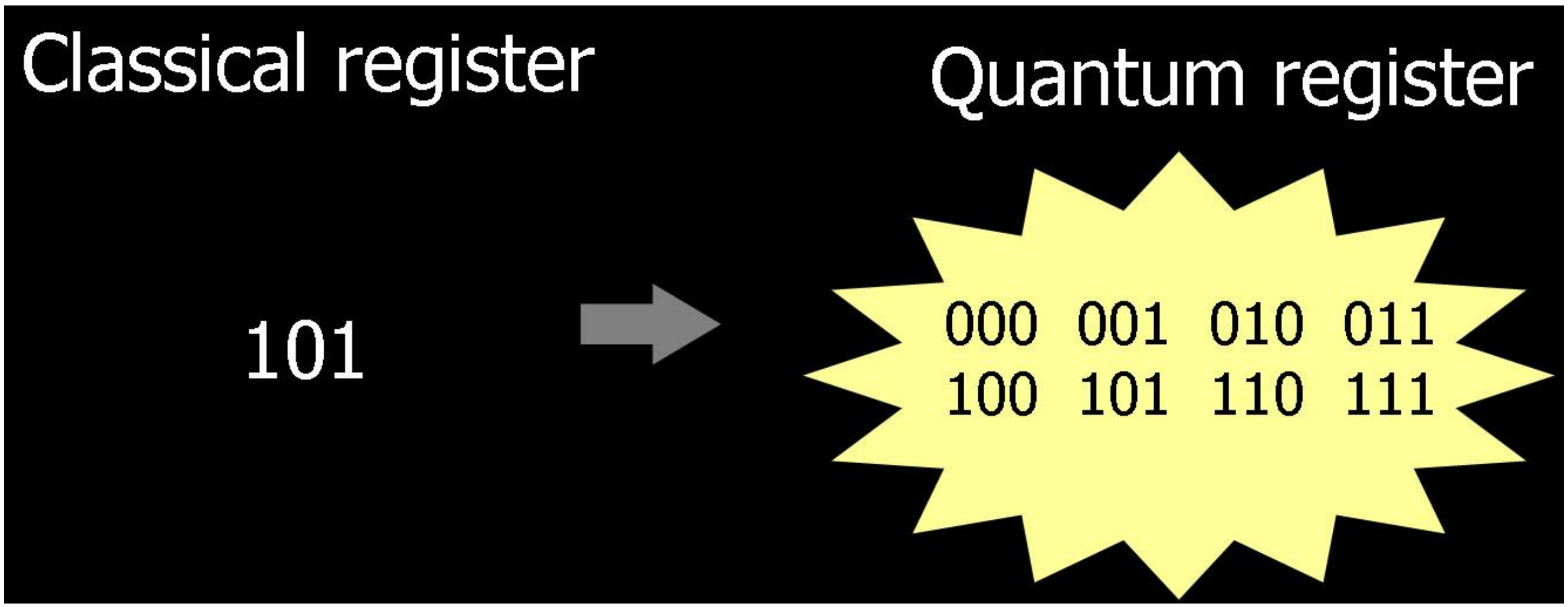
- The measurement changes the state of the qubit!

Hilbert space is a big place!

- Carlton Caves

Multiple qubits





Hilbert space is a big place!

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Multiple qubits

- Two bits with states 0 and 1 form four definite states 00, 01, 10, and 11.
- Two qubits: can be in superposition of four computational basis set states.

$$|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$$

2 qubits	4 amplitudes
3 qubits	8 amplitudes
10 qubits	1024 amplitudes
20 qubits	1 048 576 amplitudes
30 qubits	1 073 741 824 amplitudes
500 qubits	**More amplitudes than our estimate of number of atoms in the Universe!!!**

Entanglement

$$|\psi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

Results of the measurement

First qubit	0	1
Second qubit	0	1

$$|\psi\rangle \neq |\alpha\rangle \otimes |\beta\rangle \longrightarrow \boxed{\text{Entangled states}}$$

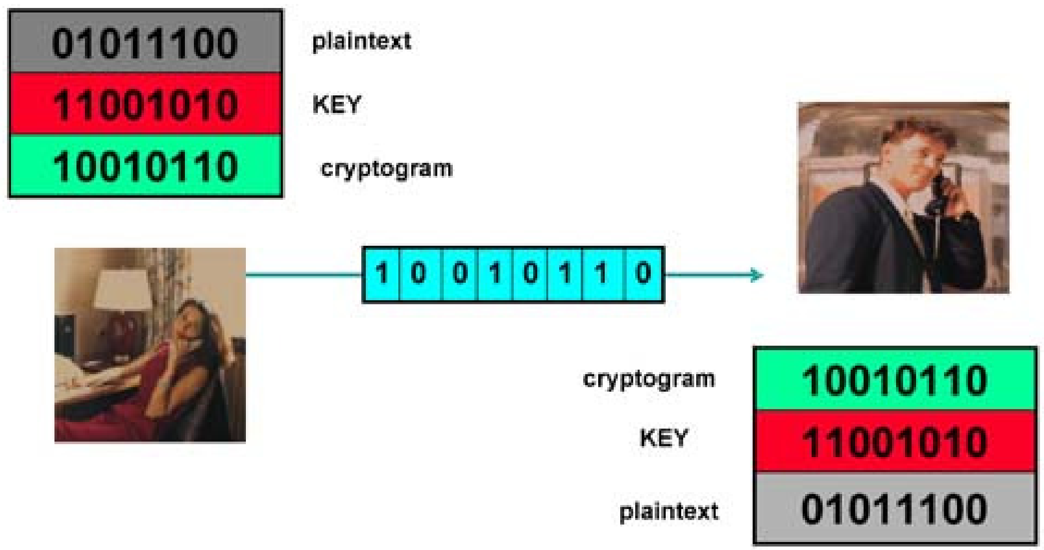
Quantum cryptography

Classical cryptography

Scytale - the first known mechanical device to implement permutation of characters for cryptographic purposes

Classical cryptography

Private key cryptography



How to securely transmit a private key?

Key distribution

A central problem in cryptography:
the key distribution problem.

1) Mathematics solution: public key cryptography.
2) Physics solution: quantum cryptography.

Public-key cryptography relies on the computational difficulty of certain hard mathematical problems (computational security)

Quantum cryptography relies on the laws of quantum mechanics (information-theoretical security).

Quantum key distribution

- Quantum mechanics: quantum bits cannot be copied or monitored.

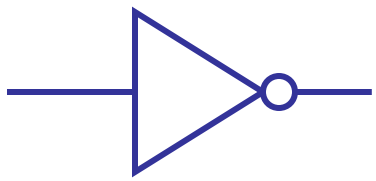
- Any attempt to do so will result in altering it that can not be corrected.

- Problems
  - Authentication
  - Noisy channels

Quantum logic gates

Logic gates

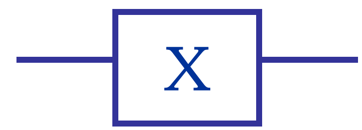
Classical NOT gate

A  NOT A

A	NOT A
0	1
1	0

The only non-trivial single bit gate

Quantum NOT gate (X gate)

$\alpha|0\rangle + \beta|1\rangle$  $\alpha|1\rangle + \beta|0\rangle$

Matrix form representation

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$X \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \beta \\ \alpha \end{bmatrix}$$

More single qubit gates

Any unitary matrix U will produce a quantum gate!

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\alpha|0\rangle + \beta|1\rangle \;—\boxed{Z}—\; \alpha|0\rangle - \beta|1\rangle$$

Hadamard gate:

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

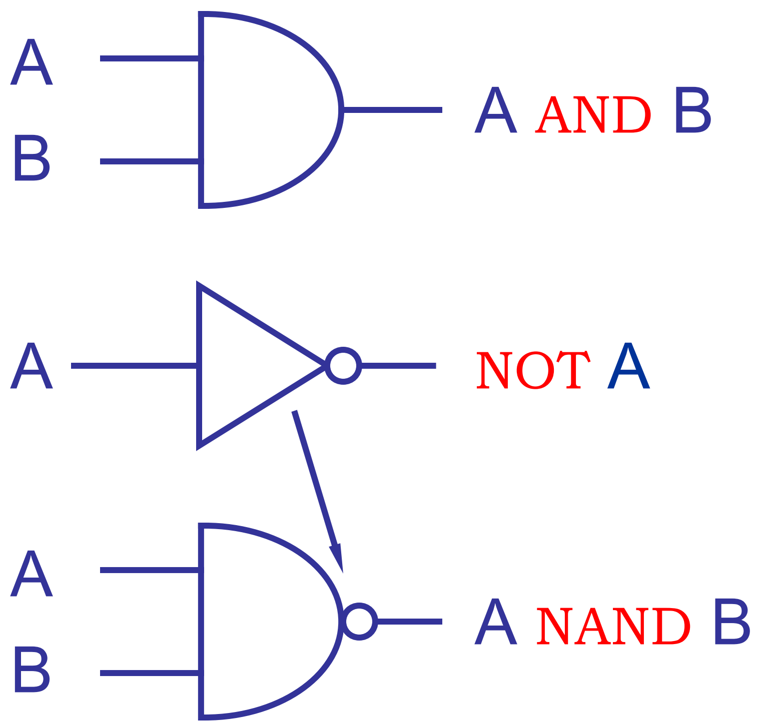
$$\alpha|0\rangle + \beta|1\rangle \;—\boxed{H}—\; \alpha\frac{|0\rangle + |1\rangle}{\sqrt{2}} + \beta\frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

Single qubit gates, two-qubit gates, three-qubit gates …

- How many gates do we need to make?
- Do we need three-qubit and four-qubit gates?
- Where do we find such physical interactions?
- Coming up with one suitable controlled interaction for physical system is already a problem!

Universality:
classical computation

Only one classical gate (NAND) is needed to compute any function on bits!



A	B	A AND B	A NAND B
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

Universality: quantum computation

How many quantum gates do we need to build any quantum gate?

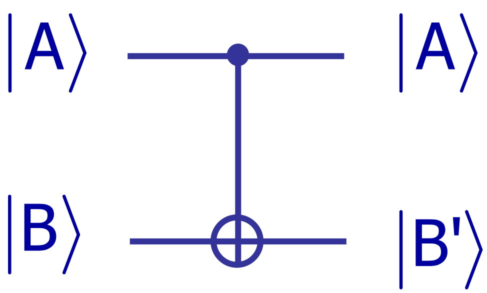
Any n-qubit gate can be made from 2-qubit gates.
(Since any unitary nxn matrix can be decomposed to product of two-level matrices.)

Only one two-qubit gate is needed!

Example: CNOT gate

Universal set of gates

CNOT



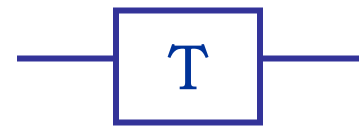
$\vert AB\rangle$	$\vert AB'\rangle$
$\vert 00\rangle$	$\vert 00\rangle$
$\vert 01\rangle$	$\vert 01\rangle$
$\vert 10\rangle$	$\vert 11\rangle$
$\vert 11\rangle$	$\vert 10\rangle$

Hadamard gate:

$\vert A\rangle$	$\vert A'\rangle$
$\vert 0\rangle$	$\frac{\vert 0\rangle+\vert 1\rangle}{\sqrt{2}}$
$\vert 1\rangle$	$\frac{\vert 0\rangle-\vert 1\rangle}{\sqrt{2}}$

$$H=(X+Z)/\sqrt{2}$$

π/8 gate:



$$T=\begin{bmatrix}1 & 0\\ 0 & e^{i\pi/4}\end{bmatrix}$$

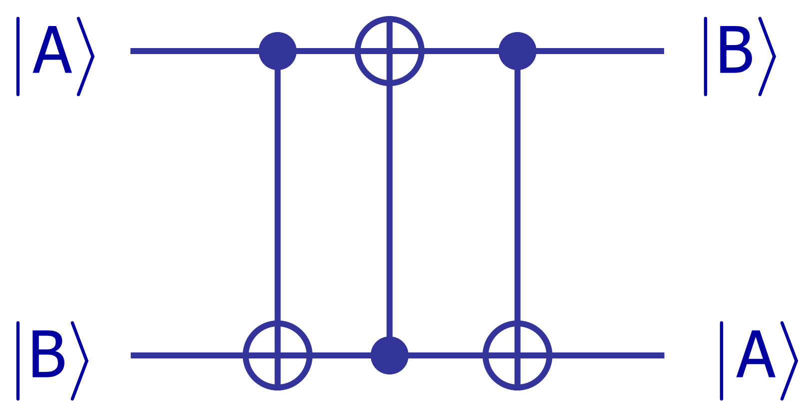
Phase gate S:

$$S=\begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}$$

$$S=T^2$$

From gates to circuits

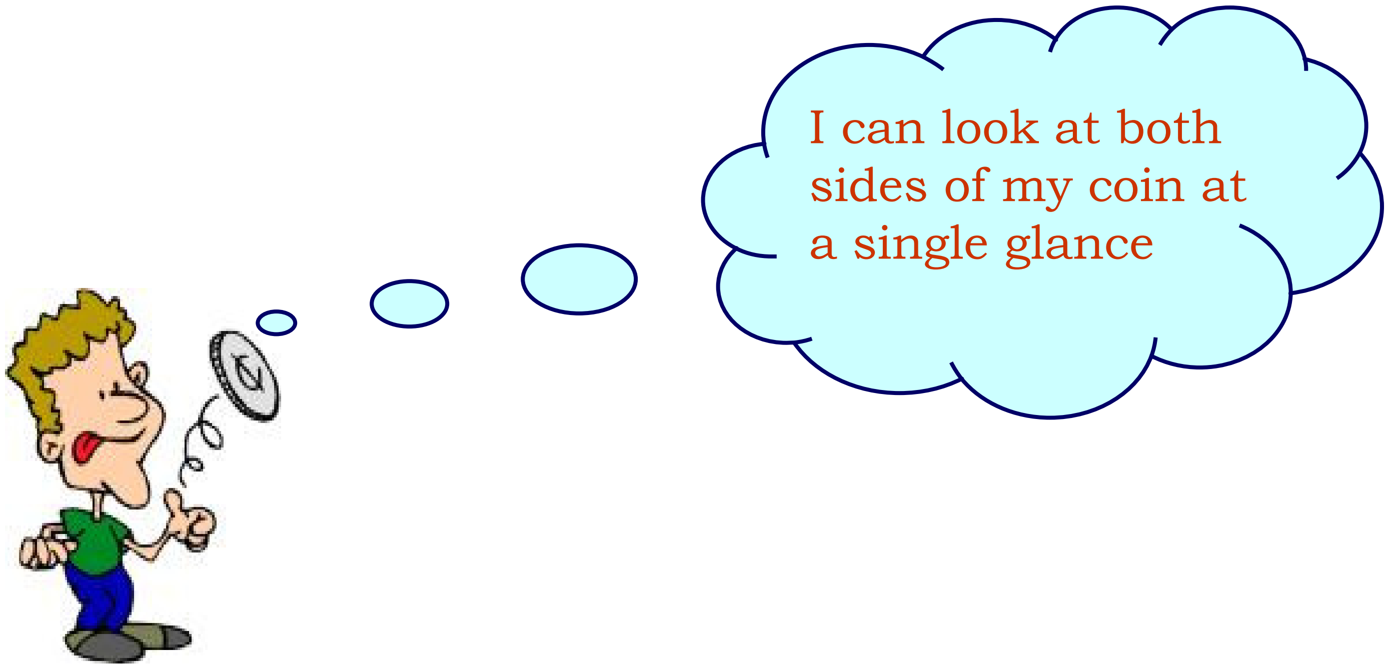
Example: swap circuit



Differences with classical circuits

- No loops - no feedback from one part of circuit to another.
- No wires joined together since it is not reversible.
- No "copy a qubit" operation (forbidden by quantum mechanics).

Quantum parallelism



Quantum parallelism

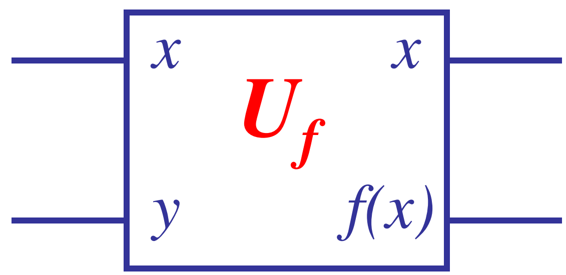
$$f(x):\{0,1\}\rightarrow\{0,1\}$$

$$|x,y\rangle \xrightarrow{U_f} |x, y\oplus f(x)\rangle$$

Superposition

$$\frac{|0\rangle+|1\rangle}{\sqrt{2}}$$

$$|0\rangle$$

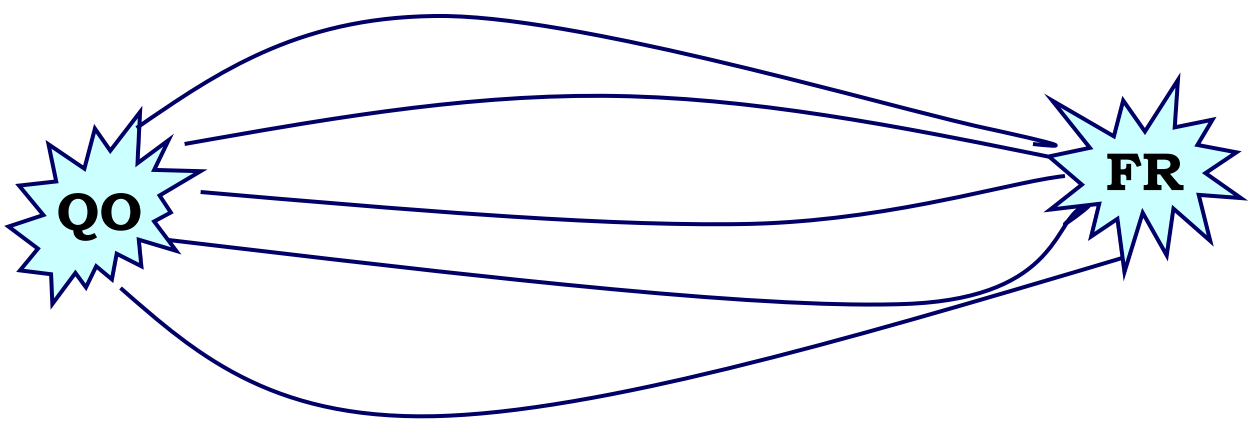


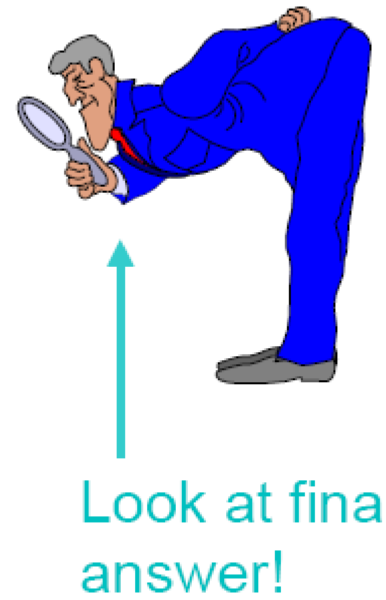
$$\frac{|0,f(0)\rangle+|1,f(1)\rangle}{\sqrt{2}}$$

Single circuit just evaluated ***f(x)*** for **both** x=0 and 1 simultaneously!

Quantum parallelism:
a major problem

- So we can evaluate functions for all values of x at the same time using just one circuit!
- Need only n+1 qubits to evaluate 2^n values of x.
- But we still get only one answer when we measure the result: it collapses to x,f(x)!!!





Quantum algorithms

Unique features of quantum computation

- Superposition: n qubits can represent 2^n integers.
- Problem: if we read the outcome we lose the superposition and we can't know with certainty which one of the values we will obtain.

- Entanglement: measurements of states of different qubits may be highly correlated.

Current advantages of quantum computation

- Shor's quantum Fourier transform provides exponential speedup over known classical algorithms.
- Applications: solving discrete logarithm and factoring problems which enables a quantum computer to break public key cryptosystems such as RSA.
- Quantum searching (Grover's algorithm) allows quadratic speedup over classical computers.
- Simulations of quantum systems.

How to factor 15?

- Pick a number less then 15: 7
- Calculate 7^n mod 15:

n	7^n	15×A	7^n mod 15
1	7	1	7
2	49	45	4
3	343	330	13
4	2401	2400	1

R=4

- Calculate $\gcd\left\{7^{R/2} \pm 1, 15\right\}$
- $\gcd\{48,15\} = 3,\ \gcd\{50,15\} = 5$

Shor's algorithm for N=15

- Choose n such as 2^n<15: n=4
- Choose y: y=7
- Initialize two four-qubit register $|\psi_0\rangle = |0000\rangle|0000\rangle$
- Create a superposition of states of the first register
- Compute the function $f(k)=7^k$ mod 15 on the second register.
- Operate on the first register by a Fourier transform
- Measure the state of the first register: u=0, 4, 8, 12 are only non-zero results.
- Two cases give period R=4, therefore the procedure succeeds with probability 1/2 after one run.

Back to the real world:

What do we need to build a quantum computer?

- **Qubits** which retain their properties.

  Scalable array of qubits.

- **Initialization:** ability to prepare one certain state repeatedly on demand. Need continuous supply of $|0\rangle$.

- **Universal set of quantum gates.** A system in which qubits can be made to evolve as desired.

- **Long relevant decoherence times.**

- Ability to efficiently **read out the result.**

The Mid-Level Quantum Computation Roadmap: Promise Criteria

QC Approach	The DiVincenzo Criteria						
	Quantum Computation					QC Networkability	
	#1	#2	#3	#4	#5	#6	#7
NMR	🔴	🟡	🟡	🟢	🟡	🔴	🔴
Trapped Ion	🟡	🟢	🟡	🟢	🟢	🟡	🟡
Neutral Atom	🟡	🟢	🟡	🟡	🟡	🟡	🟡
Cavity QED	🟡	🟢	🟡	🟡	🟢	🟡	🟡
Optical	🟡	🟡	🟢	🟡	🟡	🟡	🟢
Solid State	🟡	🟡	🟡	🟡	🟡	🔴	🔴
Superconducting	🟡	🟢	🟡	🟡	🟡	🔴	🔴
Unique Qubits	This field is so diverse that it is not feasible to label the criteria with "Promise" symbols.						

Legend: 🟢 = a potentially viable approach has achieved sufficient proof of principle

🟡 = a potentially viable approach has been proposed, but there has not been sufficient proof of principle

🔴 = no viable approach is known