

Петербургский институт ядерной физики

ATOMIC CALCULATIONS FOR STUDIES
OF FUNDAMENTAL SYMMETRIES AND
ATOMIC CLOCK RESEARCH

MARIANNA SAFRONOVA



UNIVERSITY OF
DELAWARE®

May 17, 2012



National Institute of Standards and Technology
Technology Administration, U.S. Department of Commerce



Outline

1. Atomic parity violation

- Present status
- Implications for high-energy and nuclear physics
- Future prospects

2. High-precision atomic methodologies

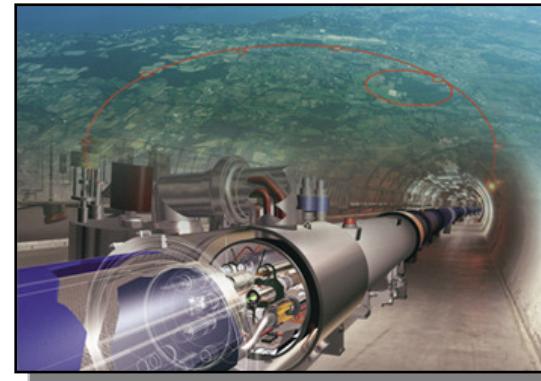
- Coupled-cluster method for monovalent systems
- Development of hybrid configuration interaction + coupled - cluster approach

3. Applications to atomic clocks – BBR shifts

4. TI EDM enhancement factor controversy

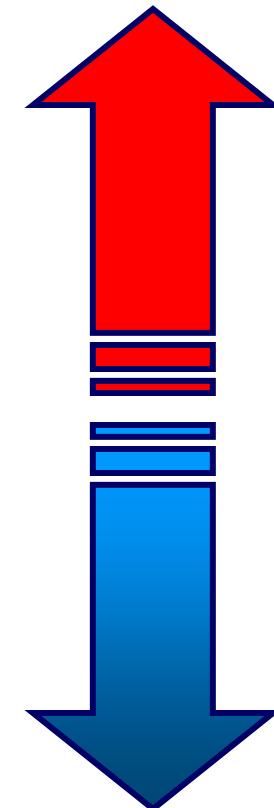
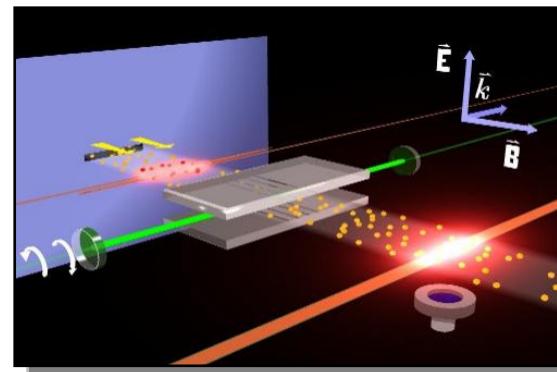
Atomic Parity Violation

Instead of search for new processes or particles directly



High energies

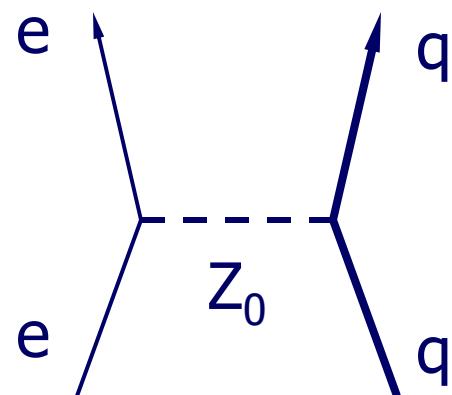
Determine **weak charge Q_W** from atomic parity violation studies and compare the result with Standard Model prediction



Low energies

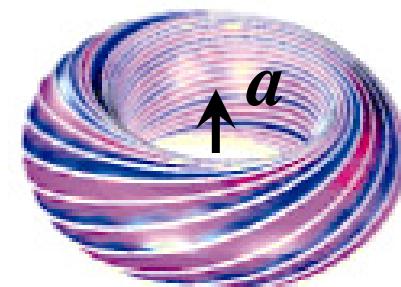
Two sides of the atomic parity violation

NUCLEAR
SPIN-INDEPENDENT
PNC:
SEARCHES FOR NEW
PHYSICS
BEYOND THE
STANDARD MODEL



Weak Charge Q_W

NUCLEAR
SPIN-DEPENDENT
PNC:
STUDY OF PNC
IN THE NUCLEUS



Nuclear anapole moment

PERIODIC TABLE
Atomic Properties of the Elements

NIST
 National Institute of Standards and Technology
 Technology Administration, U.S. Department of Commerce

EXPERIMENTAL

PNC

STUDIES

Group																			
1 IA	2 IIA																		
1 H Hydrogen 1.00794 1s 13.984	2 Be Beryllium 9.012182 1s ² 2s ² 9.3227																		
3 Li Lithium 6.941 1s ² 2s ¹ 5.3917	4 Be Beryllium 9.012182 1s ² 2s ² 9.3227																		
11 Na Sodium 22.989770 [Ne]3s ¹ 5.1391	12 Mg Magnesium 24.3050 [Ne]3s ² 7.6462																		
3 IIIIB	4 IVB	5 VB	6 VIB	7 VIIB	8 VIII														
19 K Potassium 39.0983 [Ar]4s ¹ 4.3407	20 Ca Calcium 40.078 [Ar]4s ² 6.1132	21 Sc Scandium 44.955910 [Ar]3d ¹ 6.5615	22 Ti Titanium 47.867 [Ar]3d ² 6.8281	23 V Vanadium 50.9415 [Ar]3d ³ 6.7462	24 Cr Chromium 51.9945 [Ar]3d ⁵ 6.7665	25 Mn Manganese 54.938049 [Ar]3d ⁵ 7.4340	26 Fe Iron 55.845 [Ar]3d ⁶ 7.9024	27 Co Cobalt 58.933200 [Ar]3d ⁷ 7.8810	28 Ni Nickel 58.6934 [Ar]3d ⁸ 7.6398	29 Cu Copper 63.546 [Ar]3d ¹⁰ 7.7264	30 Zn Zinc 65.409 [Ar]3d ¹⁰ 7.3942	31 Ga Gallium 69.723 [Ar]3d ¹⁰ 5.9985	32 Ge Germanium 72.64 [Ar]3d ¹⁰ 7.8984	33 As Arsenic 74.92160 [Ar]3d ¹⁰ 9.7886	34 Se Selenium 78.96 [Ar]3d ¹⁰ 9.7524	35 Br Bromine 79.904 [Ar]3d ¹⁰ 11.8138	36 Kr Krypton 83.798 [Ar]3d ¹⁰ 13.9996		
37 Rb Rubidium 85.4678 [Kr]5s ¹ 4.1771	38 Sr Strontium 87.62 [Kr]5s ² 5.6949	39 Y Yttrium 88.90585 [Kr]4d ⁵ 6.2173	40 Zr Zirconium 91.224 [Kr]4d ⁵ 6.6339	41 Nb Niobium 92.90638 [Kr]4d ⁵ 6.7589	42 Mo Molybdenum 95.94 [Kr]4d ⁵ 7.0924	43 Tc Technetium (98) 96.107 [Kr]4d ⁵ 7.28	44 Ru Ruthenium 102.9050 [Kr]4d ⁷ 7.3605	45 Rh Rhodium 106.42 [Kr]4d ⁹ 7.4589	46 Pd Palladium 107.8682 [Kr]4d ¹⁰ 7.5762	47 Ag Silver 112.411 [Kr]4d ¹⁰ 8.3369	48 Cd Cadmium 114.818 [Kr]4d ¹⁰ 8.7864	49 In Indium 118.710 [Kr]4d ¹⁰ 9.0096	50 Sn Tin 121.760 [Kr]4d ¹⁰ 10.4513	51 Sb Antimony 126.90447 [Kr]4d ¹⁰ 12.1298	52 Te Tellurium 127.60 [Kr]4d ¹⁰ 14.513	53 I Iodine 131.293 [Kr]4d ¹⁰ 16.2298			
55 Cs Cesium 132.90545 [Xe]6s ¹	56 Ba Barium 137.327 [Xe]6s ²	72 Hf Hafnium 178.49 [Xe]4f ¹⁴ 6.8251	73 Ta Tantalum 180.9479 [Xe]4f ¹⁴ 7.5496	74 W Tungsten 183.84 [Xe]4f ¹⁴ 7.8335	75 Re Rhenium 186.207 [Xe]4f ¹⁴ 7.8640	76 Os Osmium 190.23 [Xe]4f ¹⁴ 8.4382	77 Ir Iridium 192.217 [Xe]4f ¹⁴ 8.9570	78 Pt Platinum 195.078 [Xe]4f ¹⁴ 9.2255	79 Au Gold 196.96555 [Xe]4f ¹⁴ 10.4375	80 Hg Mercury 200.59 [Hg]6p ²	81 Tl Thallium 204.3833 [Hg]6p ²	82 Pb Lead 207.2 [Hg]6p ²	83 Bi Bismuth (209) 208.98038 [Hg]6p ²	84 Po Polonium (210) [Hg]6p ⁵	85 At Astatine (222) [Hg]6p ⁵				
87 Fr Francium (223) [Rn]7s ² 4.0727	88 Ra Radium (226) [Rn]7s ² 5.2784	104 Rf Rutherfordium (261) [Rn]7s ² 6.0?	105 Db Dubnium (262)	106 Sg Seaborgium (266)	107 Bh Bohrium (264)	108 Hs Hassium (277)	109 Mt Meitnerium (268)	110 Uun Ununnilium (281)	111 Uuu Unununium (272)	112 Uub Ununbium (285)	114 Uuq Ununquadium (289)	116 Uuh Ununhexium (292)	118 Lu Lutetium 174.967 [Xe]4f ¹⁴ 5.4259						

Atomic Number	Ground-state Level
Symbol	58^{1G_4}
Name	Ce
Atomic Weight [†]	140.116 [Xe]4f ⁵ 5d ⁶ s ² 5.5387
Ground-state Configuration	[Xe]4f ⁵ 5d ⁶ s ²
Ionization Energy (eV)	5.5387

Lanthanides	Actinides
57 La Lanthanum 138.9055 [Xe]5d ¹ 5.5387	58 Ce Cerium 140.116 [Xe]4f ⁵ 6s ² 5.473
59 Pr Praseodymium 140.90765 [Xe]4f ⁵ 6s ² 5.5250	60 Nd Neodymium 144.24 [Xe]4f ⁵ 6s ² 5.582
61 Pm Promethium (145) 150.36 [Xe]4f ⁵ 6s ² 5.6437	62 Sm Samarium 151.964 [Xe]4f ⁵ 6s ² 5.6704
63 Eu Europium 151.964 [Xe]4f ⁵ 6s ² 5.6704	64 Gd Gadolinium 162.500 [Xe]4f ⁷ 6s ² 6.1498
65 Tb Terbium 164.93032 [Xe]4f ⁹ 6s ² 5.9389	66 Dy Dysprosium 165.2534 [Xe]4f ¹⁰ 6s ² 5.9389
67 Ho Holmium 167.259 [Xe]4f ¹¹ 6s ² 6.0215	68 Er Erbium 173.04 [Xe]4f ¹² 6s ² 6.1077
69 Tm Thulium 188.93421 [Xe]4f ¹³ 6s ² 6.1843	70 Yb Ytterbium 174.967 [Xe]4f ¹⁴ 6s ² 6.2542
71 Lu Lutetium 174.967 [Xe]4f ¹⁴ 5.4259	72 Hf Hafnium 178.49 [Xe]4f ¹⁴ 6.8251

Solids

Liquids

Gases

Artificially Prepared

Physics Laboratory
physics.nist.gov

Standard Reference Data Group
www.nist.gov/srd

18
VIIIA

He
Helium
4.002602
1s²
24.5874

2
He
Helium
4.002602
1s²
24.5874

Ne
Neon
20.1797
1s²2s²2p⁶
21.5645

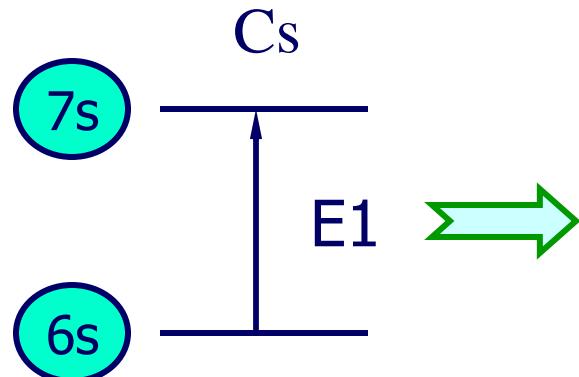
[†]Based upon ¹²C. () indicates the mass number of the most stable isotope.

For a description of the data, visit physics.nist.gov/data

NIST SP 966 (September 2003)

Atomic Parity Violation

$$\vec{r} \rightarrow -\vec{r}$$



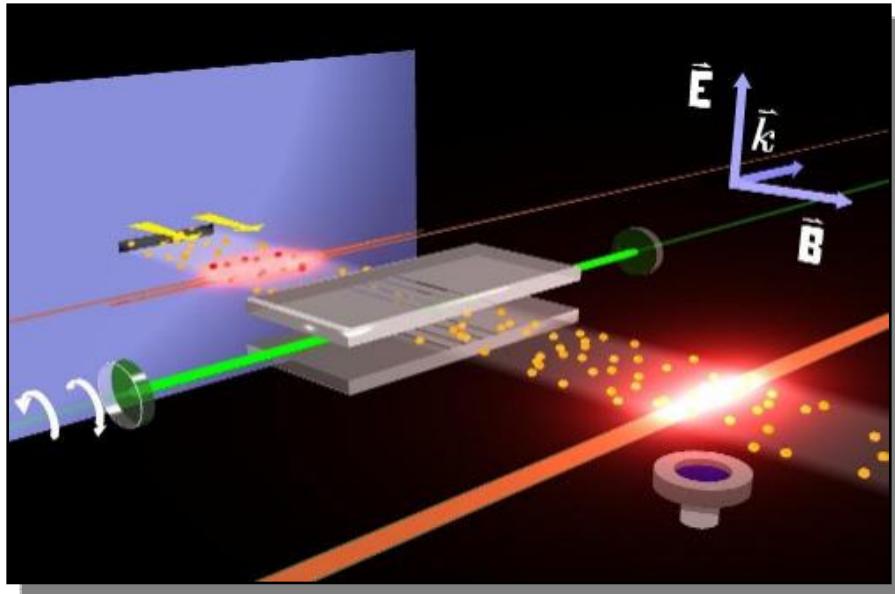
Both 6s and 7s states acquire an opposite-parity ($np_{1/2}$) admixture



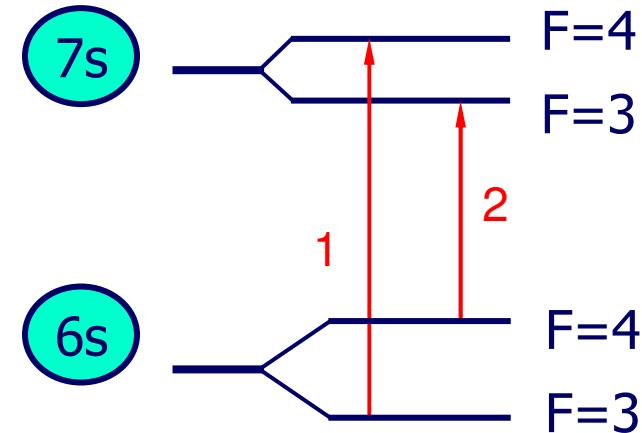
Note: it is really tiny effect !!! $E_{PNC} \sim 10^{-11}$ atomic units
E1 amplitude for 6s – 6p transition is **4.5** atomic units

The most precise measurement of PNC amplitude (cesium)

C.S. Wood et al. Science 275, 1759 (1997)



0.35% accuracy

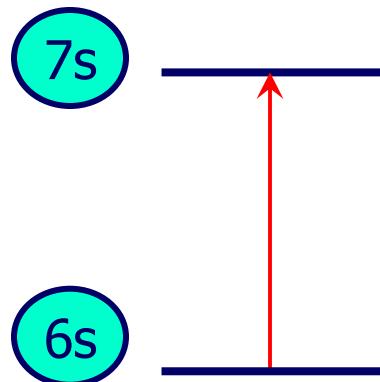


$$\frac{\text{Im}(E_{\text{PNC}})}{\beta} = \begin{cases} -1.6349(80) \text{ mV/cm} & 1 \\ -1.5576(77) \text{ mV/cm} & 2 \end{cases}$$

Stark interference scheme to measure ratio of the PNC amplitude and the Stark-induced amplitude β

Analysis of Cs PNC experiment

NUCLEAR
SPIN-INDEPENDENT
PNC

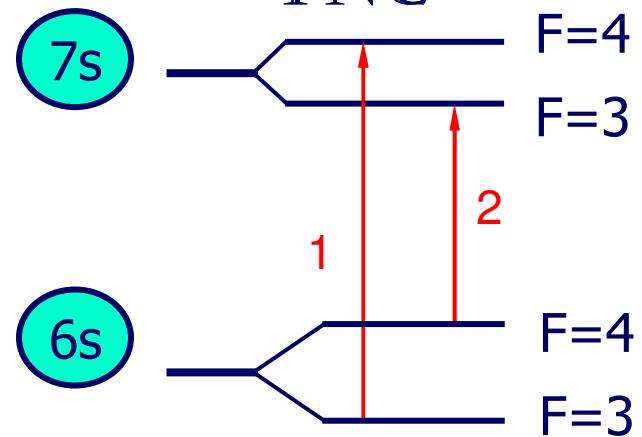


Average of 1 & 2

$$\frac{\text{Im}(E_{\text{PNC}}^{\text{si}})}{\beta} = -1.5935(56) \text{ mV/cm}$$

Weak Charge Q_w

NUCLEAR
SPIN-DEPENDENT
PNC



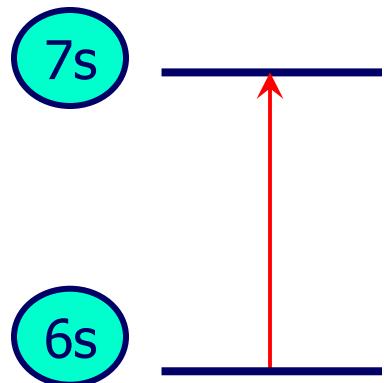
Difference of 1 & 2

$$\Delta \left[\frac{\text{Im}(E_{\text{PNC}}^{\text{sd}})}{\beta} \right]_{34-43} = -0.077(11) \text{ mV/cm}$$

Nuclear anapole moment

Analysis of Cs PNC experiment

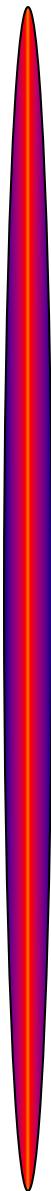
NUCLEAR
SPIN-INDEPENDENT
PNC



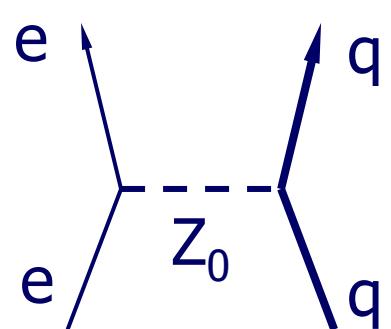
Average of 1 & 2

$$\frac{\text{Im}(E_{\text{PNC}}^{\text{si}})}{\beta} = -1.5935(56) \text{ mV/cm}$$

Weak Charge Q_w



How to extract weak charge Q_w from Cs experiment?



Electron-quark parity violating interaction
(exchange of virtual Z_0 boson)

$$H_W = \frac{G_F}{\sqrt{2}} (\bar{e} \gamma_\mu \gamma_5 e) \{ C_{1u} \bar{u} \gamma^\mu u + C_{1d} \bar{d} \gamma^\mu d \} + \dots$$

Electronic sector: $H_{PNC}^{(1)} = \frac{G_F}{2\sqrt{2}} Q_w \gamma_5 \rho(r)$

Neutron density function

Extraction of weak the charge:

Measured value

$$\longrightarrow E_{PNC} = E_{PNC}^{\text{theory}} Q_w^{\text{inferred}}$$

Theoretical calculation of
PNC amplitude

Calculation of PNC amplitude

1. Main part – Coulomb interactions

$$E_{\text{PNC}}^{\text{theory}} = \sum_{n=2}^{\infty} \frac{\langle 7s | d | np_{1/2} \rangle \langle np_{1/2} | H_{\text{PNC}} | 6s \rangle}{E_{6s} - E_{np_{1/2}}} + \sum_{n=2}^{\infty} \frac{\langle 7s | H_{\text{PNC}} | np_{1/2} \rangle \langle np_{1/2} | d | 6s \rangle}{E_{7s} - E_{np_{1/2}}}$$

Electric-dipole matrix elements

Energies

PNC matrix elements

Sum is separated to main part, $n = 6 - 9$ and the tail

2. Other small corrections:

Breit, QED, Neutron skin, e – e weak interaction

Cs PNC: Comparison with the standard model

Standard Model [1] : $Q_W^{\text{SM}} = -73.16(3)$

Most current result for Cs PNC Expt/Theory:

Atomic physics [2] :

$Q_W^{\text{inferred}} = -73.16(29)_{\text{expt}}(20)_{\text{theory}}$

No deviation from the Standard Model

[1] C. Amsler et al. (Partical Data Group), Phys. Lett. B 667, 1 (2008)

[2] S. G. Porsev, K. Beloy and A. Derevianko, PRL 102, 181601 (2009),
Phys. Rev. D 82, 036008 (2010)

IMPLICATIONS FOR PARTICLE PHYSICS

Confirms fundamental “running” (energy dependence) of the electroweak force over energy span 10 MeV → 100 GeV

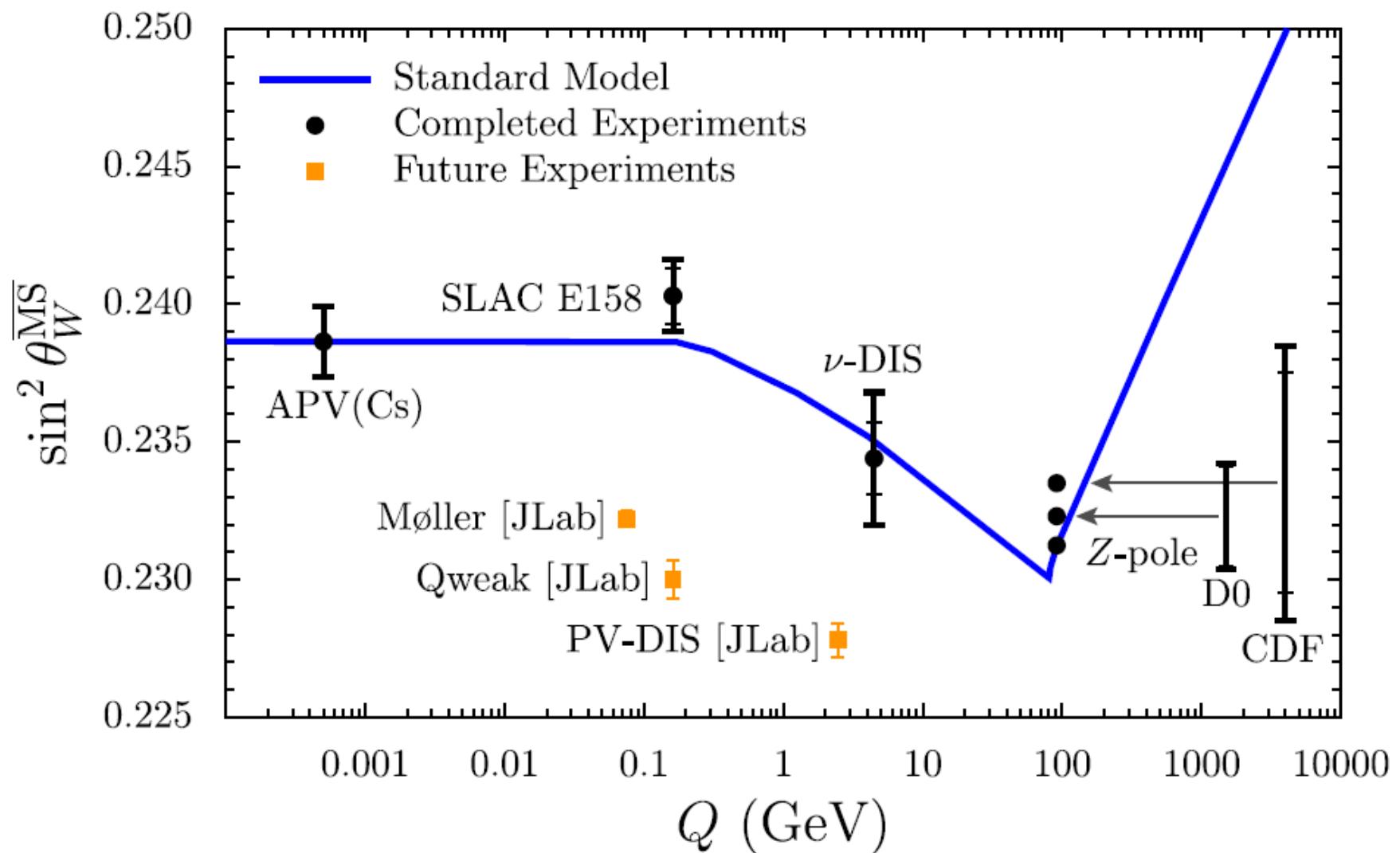


Figure is from Bentz *et al.* *Phys. Lett. B693*, 462 (2010).

Probing new physics

New physics can be phenomenologically described by weak isospin - conserving S and isospin - breaking T parameters [1].

$$\Delta Q = Q_W^{\text{inferred}} - Q_W^{\text{SM}} = -0.800S - 0.007T$$

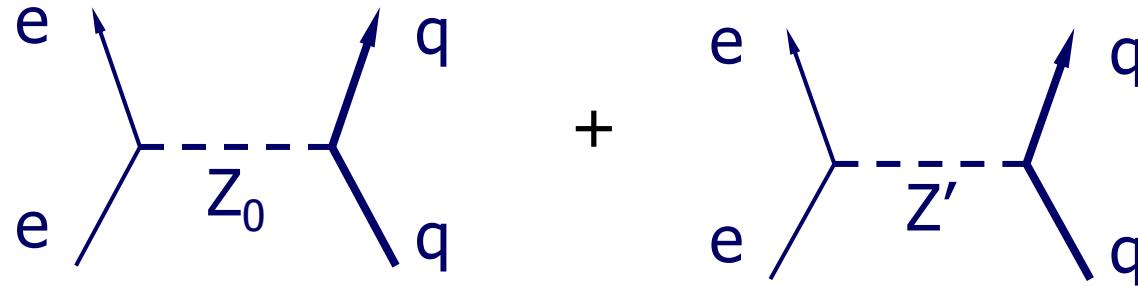
Present result [2]: $|S| < 0.45$

Parameter S is important for indirect constraint on the mass of Higgs particle [1].

[1] J.L. Rosner, PRD 65, 073026 (2002)

[2] S. G. Porsev, K. Beloy and A. Derevianko, PRL 102, 181601 (2009),
Phys. Rev. D 82, 036008 (2010)

Probing new physics: extra Z bosons



Atomic parity violation is uniquely sensitive to Z'

Z'_x in SO(10) GUT, Marciano & Rosner

$$\Delta Q = Q_W^{\text{inferred}} - Q_W^{\text{SM}} \approx \left(\frac{0.736 \text{ TeV} / c^2}{M_{Z'_x}} \right)^2$$

Probing new physics: extra Z bosons

Z'_x in SO(10) GUT, Marciano & Rosner

$$\Delta Q = Q_W^{\text{inferred}} - Q_W^{\text{SM}} \approx \left(\frac{0.736 \text{ TeV} / c^2}{M_{Z'_x}} \right)^2$$

Cs result [1] implies $M_{Z'_x} > 1.3 \text{TeV} / c^2$

Direct search at Tevatron collider [2]

$$M_{Z'_x} > 0.82 \text{TeV} / c^2$$

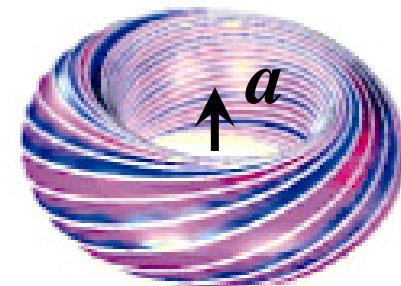
[1] S. G. Porsev, K. Beloy and A. Derevianko, PRL 102, 181601 (2009)

[2] T. Aaltonen et al., Phys. Rev. Lett. 99, 171802 (2007)

Parity violation in atoms

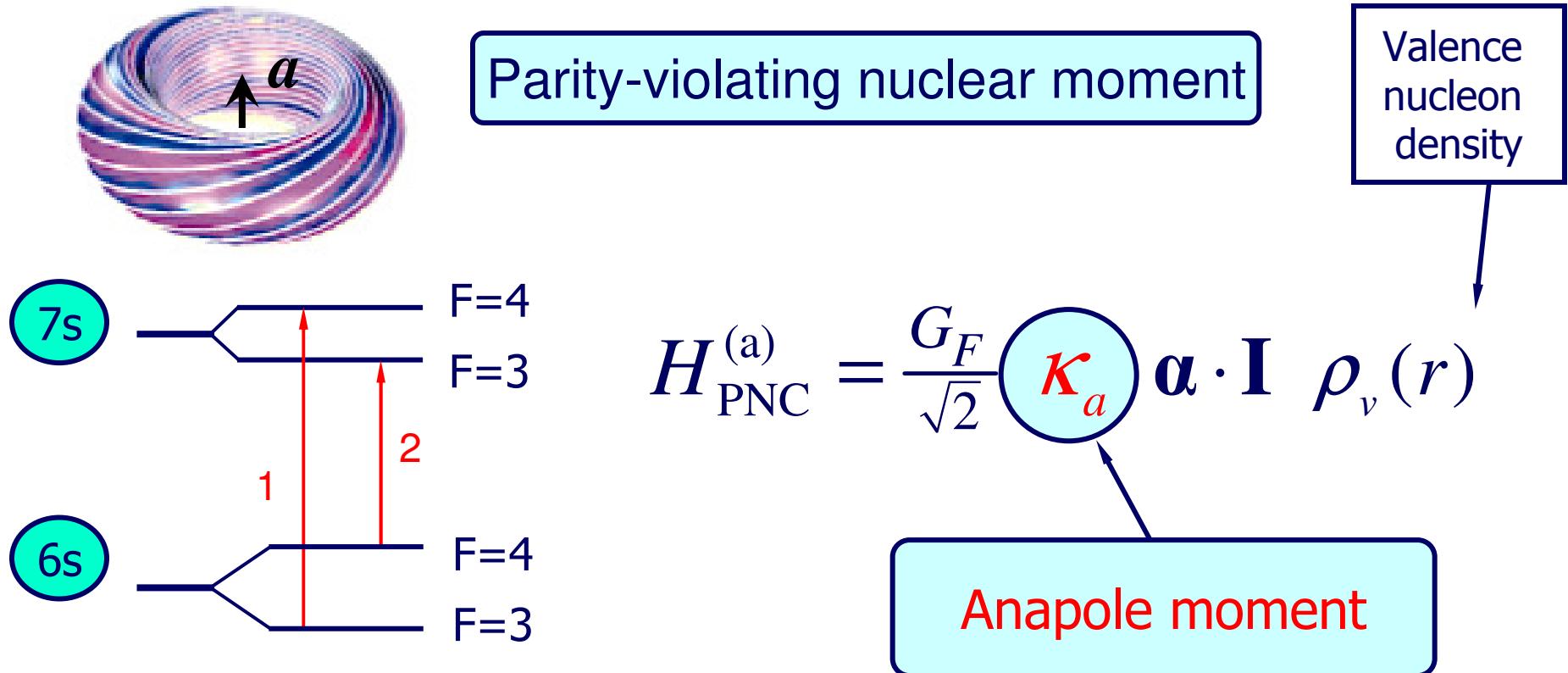
The other part
of the story

NUCLEAR
SPIN-DEPENDENT
PNC:
STUDY OF PNC
IN THE NUCLEUS



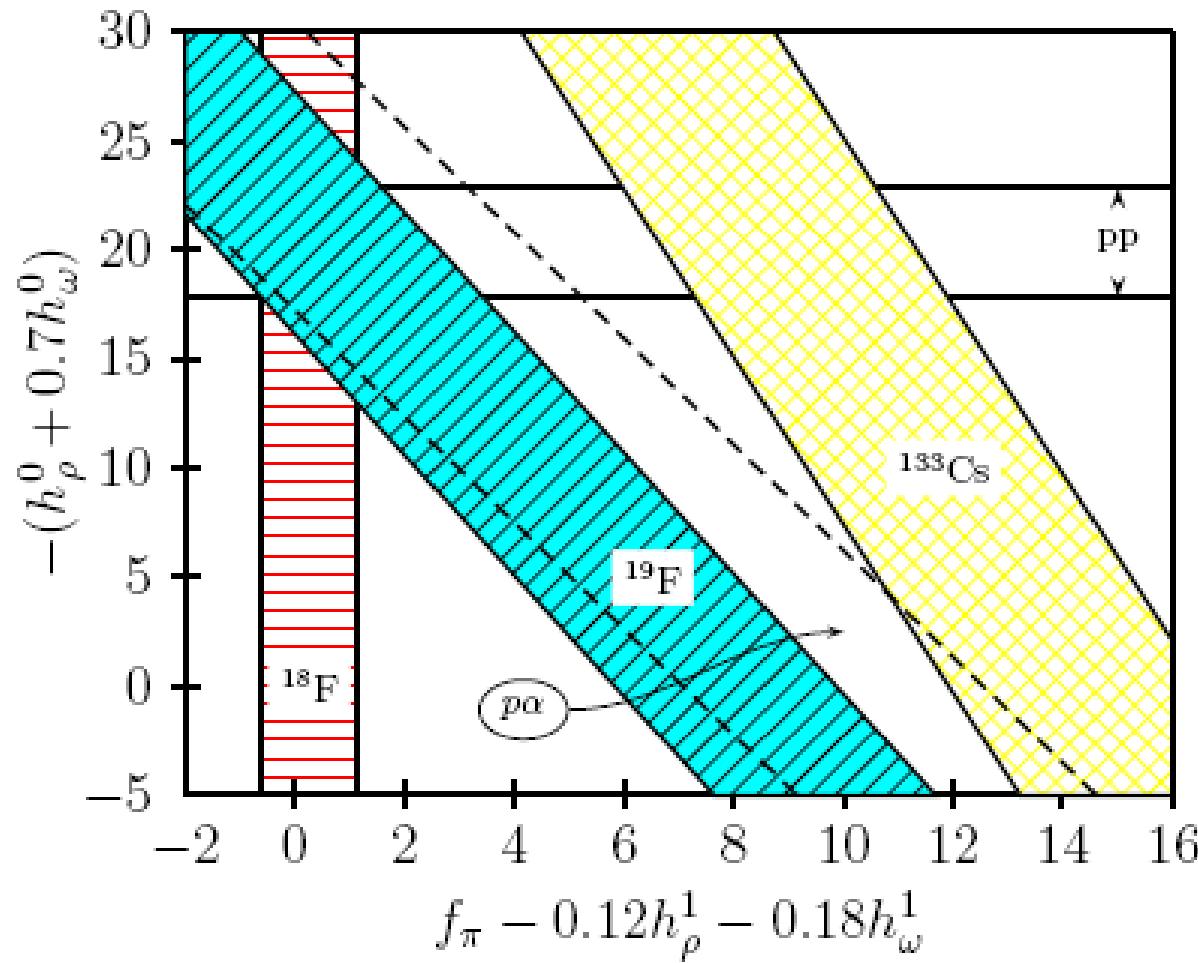
Nuclear anapole
moment

Spin-dependent parity violation: Nuclear anapole moment



Nuclear anapole moment is parity-odd, time-reversal-even E1 moment of the electromagnetic current operator.

Constraints on nuclear weak coupling constants



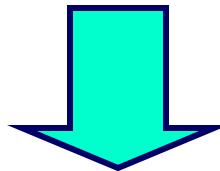
W. C. Haxton and C. E. Wieman, Ann. Rev. Nucl. Part. Sci. 51, 261 (2001)

Nuclear anapole moment?

The constraints obtained from the Cs experiment were found to be ***inconsistent*** with constraints from other nuclear PNC measurements, which favor a smaller value of the ^{133}Cs anapole moment.

Possible atomic calculation solution?

$$K = 0.117(16)$$



Incomplete correlation calculation of spin-dependent PNC amplitude?

Nuclear anapole moment: Test of hadronic weak interactions

The constraints obtained from the Cs experiment were found to be **inconsistent** with constraints from other nuclear PNC measurements, which favor a smaller value of the ^{133}Cs anapole moment.

All-order (LCCSD) calculation of spin-dependent PNC amplitude:

$$k = 0.107(16)^* \text{ [1\% theory accuracy]}$$

No significant difference with previous value $k = 0.112(16)$ is found.

NEED NEW EXPERIMENTS!!!

Fr, Yb, Ra⁺

*M.S. Safronova, Rupsi Pal, Dansha Jiang, M.G. Kozlov, W.R. Johnson, and U.I. Safronova, Nuclear Physics A 827 (2009) 411c

Summary: Atomic Parity Violation

A: New analysis of atomic PNC experiment is Cs: Nuclear spin-independent part:

- (1) Provided most accurate to-date test of the low-energy electroweak sector of the SM.
- (2) Confirmed fundamental “running” (energy dependence) of the electroweak force.
- (3) Placed constraints are on a variety of new physics scenarios beyond the SM.

B: New analysis of atomic PNC experiment is Cs: Nuclear spin-independent part (anapole moment)

- (1) New calculations, accurate to 1% - essentially the same result.
- (2) Constraints on nuclear weak coupling constants are still inconsistent with nuclear physics experiments.

NEED NEW
EXPERIMENTAL
PNC
STUDIES

WANTED!

PROSPECTS FOR THEORY IMPROVEMENT

Fr and Ra⁺: 1-3% at present

0.5% possible with CCSDT (same as Cs)

Yb: 9% at present

Significant improvement should be possible with CI + coupled-cluster, especially if triples are implemented

Dy: ? Previous: no signal within 2 orders of magnitude from prediction. The problems appears to be somewhat understood at this time. Significant improvement possible if CI+MBPT could be implemented.

Tl, Bi, Pb – improvements to 1% should be possible

2. HIGH-PRECISION ATOMIC METHODOLOGIES

SUMMARY OF THEORY METHODS FOR PNC CALCULATIONS

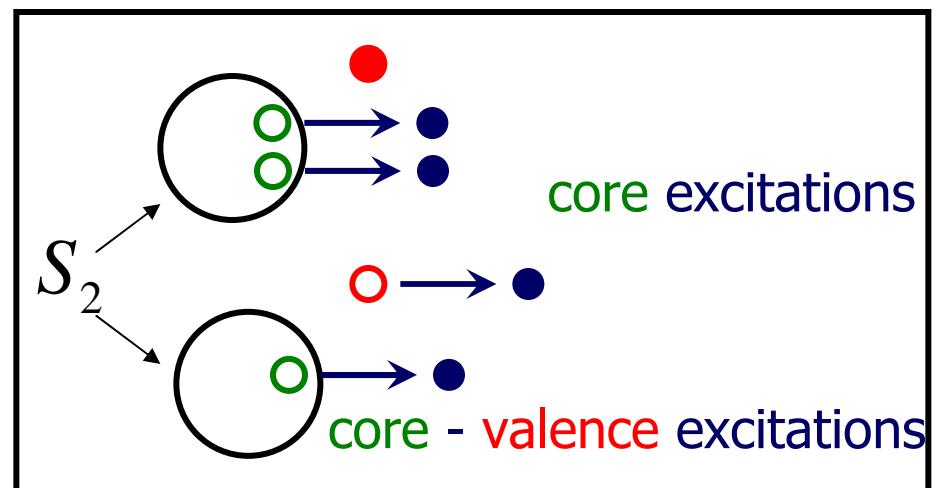
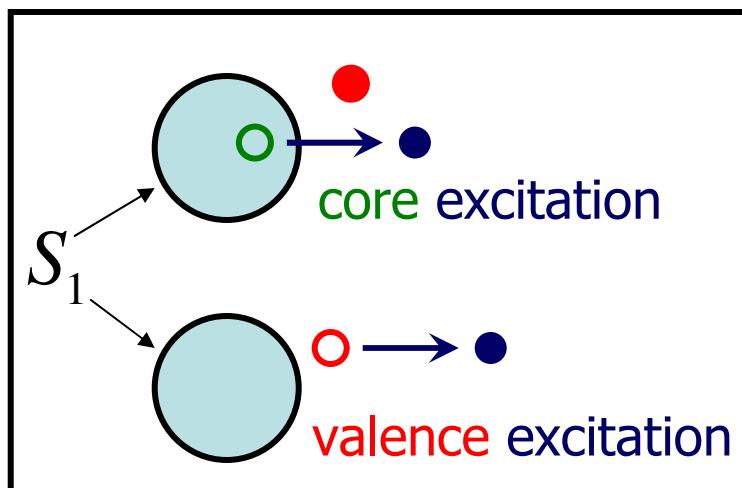
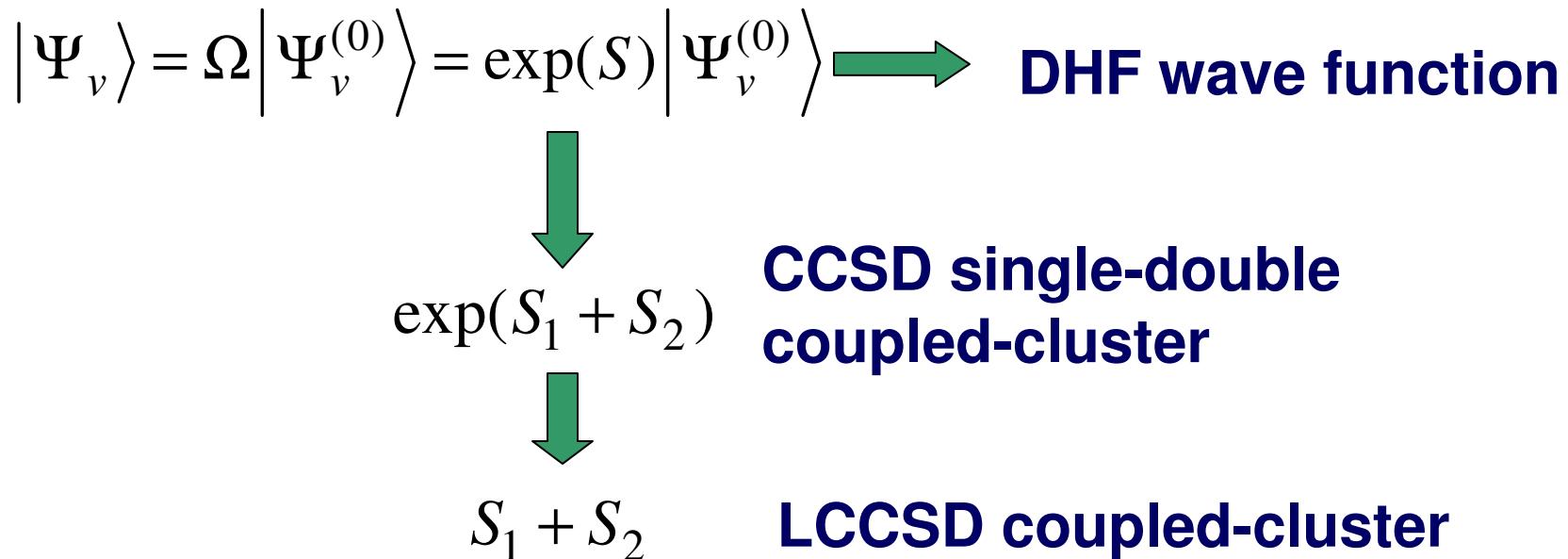
- Configuration interaction (CI)
- Many-body perturbation theory
- Relativistic all-order method (coupled-cluster)
- Perturbation theory in the screened Coulomb interaction (PTSCI), all-order approach
- Configuration interaction + second-order MBPT
- Configuration interaction + coupled-cluster method*

*under development

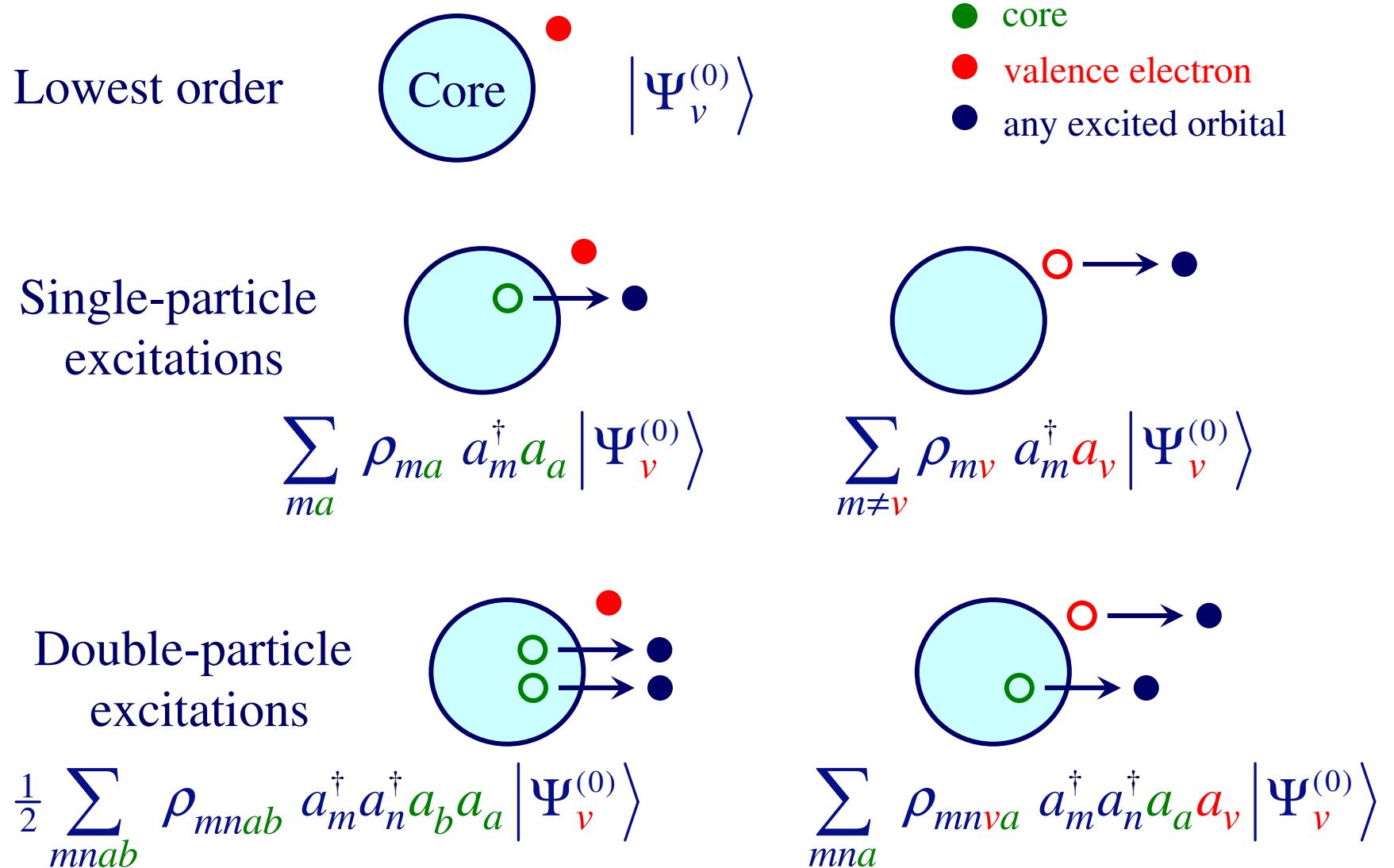
SUMMARY OF THEORY METHODS FOR PNC CALCULATIONS

- Configuration interaction (CI)
 - Many-body perturbation theory
 - **Relativistic all-order method (coupled-cluster)**
 - Perturbation theory in the screened Coulomb interaction (PTSCI), all-order approach
-
- Configuration interaction + second-order MBPT
 - Configuration interaction + coupled-cluster method

Coupled-cluster method (CCSD)



ALL-ORDER ATOMIC WAVE FUNCTION (SD)



ALL-ORDER MATRIX ELEMENTS

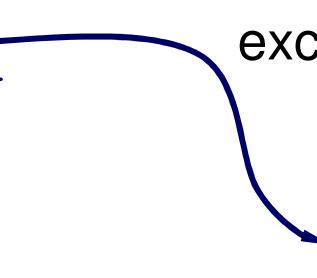
So we calculated all ρ_{ma} , ρ_{mnab} , ρ_{mv} , ρ_{mnva} .

We now have a really large file with ρ .

How do we calculate matrix elements?

$$Z_{wv} = \frac{\langle \Psi_w | Z_{(E1, E2, PNC, etc.)} | \Psi_v \rangle}{\sqrt{\langle \Psi_w | \Psi_w \rangle \langle \Psi_v | \Psi_v \rangle}}$$

Linear or quadratic functions of
excitations coefficients


$$z_{wv}^{(0)} + z_{wv}^{(a)} + \dots + z_{wv}^{(t)}$$

This method works for any one-body operator: ok to calculate E1, E2, E3, M1, M2, PNC, spin-dep. PNC, hyperfine A and B, etc.

Polarizabilities, PNC amplitudes, etc.

Sum-over-states in a finite basis set

Dynamic polarizability:

$$\alpha_v^0(\omega) = \frac{1}{3(2j_v + 1)} \sum_n \frac{(E_n - E_v) |\langle n | D | v \rangle|^2}{(E_n - E_v)^2 - \omega^2}$$

PNC amplitude in Cs:

$$E_{\text{PNC}}^{\text{theory}} = \sum_{n=2}^{\infty} \frac{\langle 7s | d | np_{1/2} \rangle \langle np_{1/2} | H_{\text{PNC}} | 6s \rangle}{E_{6s} - E_{np_{1/2}}} + \sum_{n=2}^{\infty} \frac{\langle 7s | H_{\text{PNC}} | np_{1/2} \rangle \langle np_{1/2} | d | 6s \rangle}{E_{7s} - E_{np_{1/2}}}$$

Actual implementation

1. Need very accurate (large) basis sets, up to $70 \times 13 = 910$ orbitals

ρ_{mnab}

Even with 35 orbitals for each partial wave:

Cs: $a, b = 1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 5s^2 5p^6$

$m, n : \text{finite basis set} = (35 \times 13) \times (35 \times 13)$

Total actually $15412 \times 35 \times 35 \sim 19\,000\,000$ equations

to be solved iteratively!

Our implementation of the coupled-cluster is different from quantum chemistry – new sets of codes were developed.

Actual implementation

These are really complicated equations !!!

- “Quadruple” term:

$$\sum_{rs} g_{mnrs} \rho_{rsab}$$

a,b core
(17 shells)

Indices *mnr*s can be ANY orbitals

Basis set: $n_{\max}=35, l_{\max}=6$

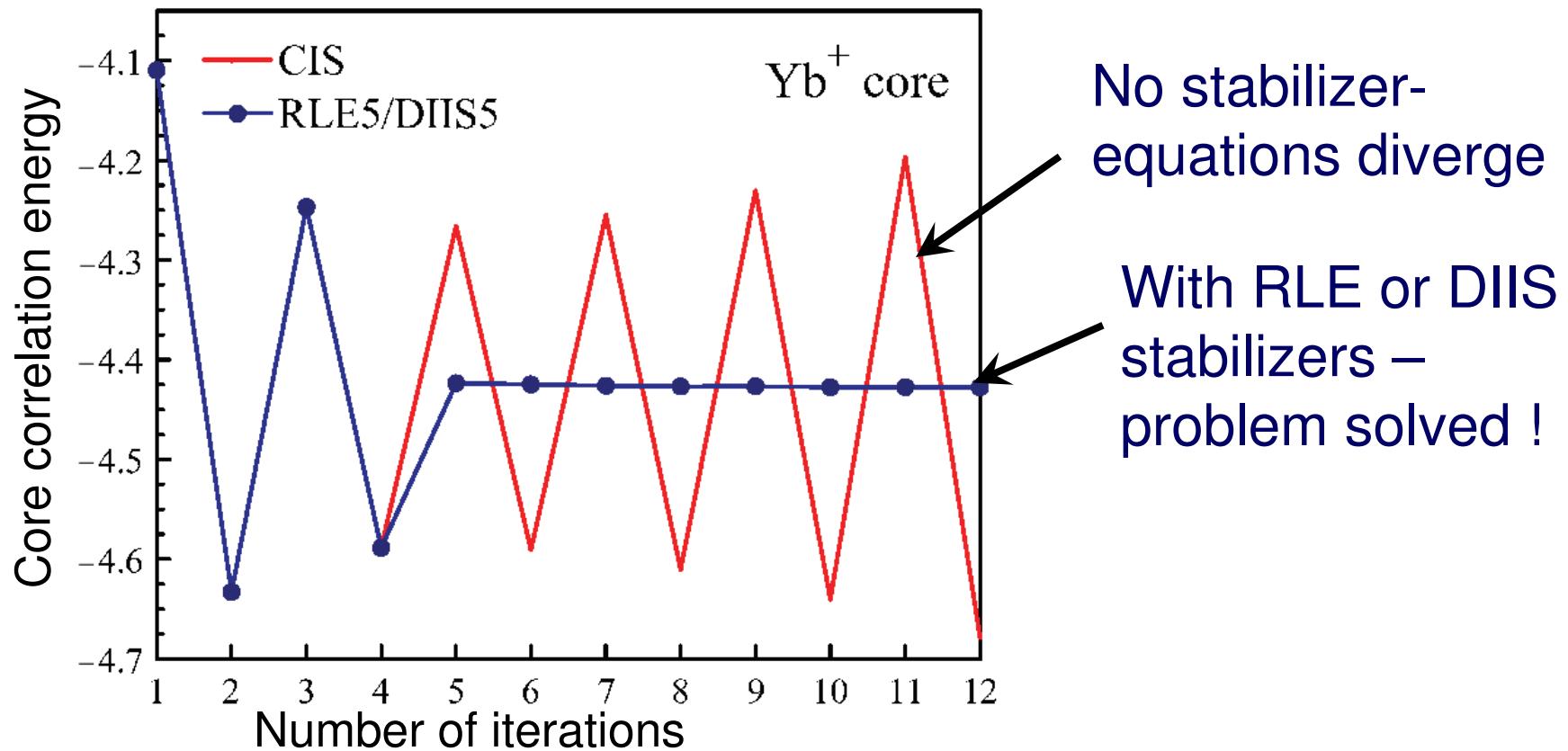
$17 \times 17 \times (35 \times 13)^4 = 5 \times 10^{12}!$

- Program has to be exceptionally efficient!

Actual implementation

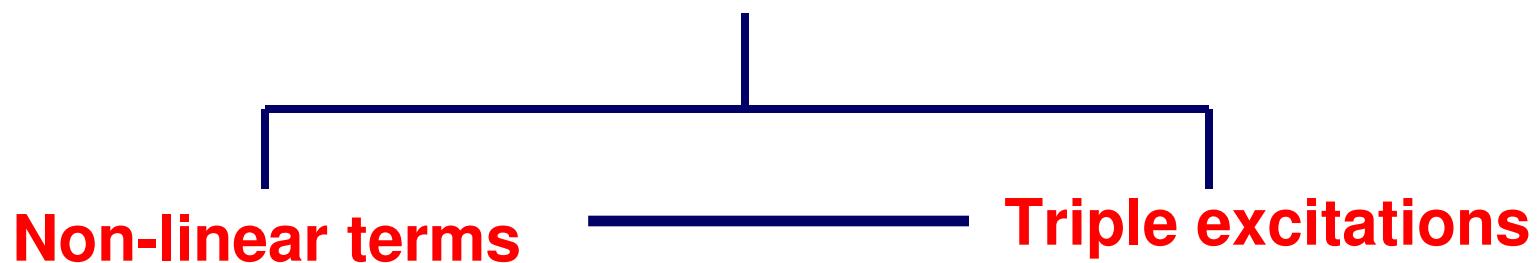
Equations for excitation coefficients ρ may diverge!

Example: iterations for Yb^+ core



Extensions of LCCSD

Add more terms to the all order wave-function



$$\frac{1}{2}(S_1^2 + 2S_1S_2 + S_2^2)$$

$$S_3$$

$$+\frac{1}{6}(S_1^3 + 3S_1^2S_2) + \frac{1}{24}S_1^4$$

Non-linear terms

$$H \frac{1}{2} S_2^2 |\Psi_v^{(0)}\rangle \rightarrow :a_i^+ a_j^+ a_l a_k : a_m^+ a_n^+ a_r^+ a_s^+ a_d a_c a_b a_a a_v^+ : |0_c\rangle$$

Contract operators by Wick's theorem

$\frac{1}{2} S_2^2$

$H \frac{1}{2} S_2^2 |\Psi_v^{(0)}\rangle \rightarrow :a_i^+ a_j^+ a_l a_k : a_m^+ a_n^+ a_r^+ a_s^+ a_d a_c a_b a_a a_v^+ : |0_c\rangle$

Contract operators by Wick's theorem

Non-linear terms

$$H \frac{1}{2} S_2^2 |\Psi_v^{(0)}\rangle \rightarrow :a_i^+ a_j^+ a_l a_k : a_m^+ a_n^+ a_r^+ a_s^+ a_d a_c a_b a_a a_v^+ : |0_c\rangle$$

Contract operators by Wick's theorem

800 TERMS!

The diagram illustrates the application of Wick's theorem to a non-linear term. At the top left, a blue circle contains the term $\frac{1}{2} S_2^2$. A black arrow points downwards from this circle to the mathematical expression below. To the right of the arrow, the text "Contract operators by Wick's theorem" is written in bold black font, with a brace above the expression indicating the scope of the contraction. The mathematical expression itself is $H \frac{1}{2} S_2^2 |\Psi_v^{(0)}\rangle \rightarrow :a_i^+ a_j^+ a_l a_k : a_m^+ a_n^+ a_r^+ a_s^+ a_d a_c a_b a_a a_v^+ : |0_c\rangle$. Below this expression, a large blue starburst shape contains the text "800 TERMS!" in red, emphasizing the complexity of the resulting terms.

Codes that write formulas

The derivation gets **really complicated** if you add triples and non-linear terms!

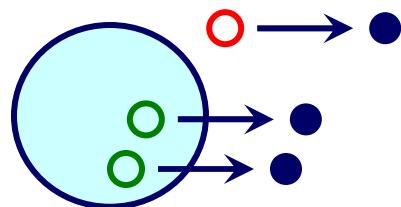
Solution: develop analytical codes that do all the work for you!

Input: ASCII input of terms of the type

$$\sum_{mnrab} \sum_{ijkl} g_{ijkl} \rho_{mnrvab} : a_i^\dagger a_j^\dagger a_l a_k : : a_m^\dagger a_n^\dagger a_r^\dagger a_b a_a a_v : | \Psi_v^{(0)} \rangle$$

Output: final simplified formula in LATEX to be used in the all-order equation

Triple excitations



- core
- valence electron
- any excited orbital

$$\sum_{mnrvab} \rho_{mnrvab} a_m^\dagger a_n^\dagger a_r^\dagger a_a a_b a_v | \Psi_v^{(0)} \rangle$$

Problem 1: too many excitation coefficients ρ_{mnrvab}

Problem 2: increased complexity of equations .

Triple excitations

Problem: too many excitation coefficients $\rho_{mnr}vab$.

Doubles:

ρ_{mnab} Cs: $a, b = 1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 5s^2 5p^6$
 $m, n : \text{finite basis set} = (35 \times 13) \times (35 \times 13)$

Smallest required basis set:

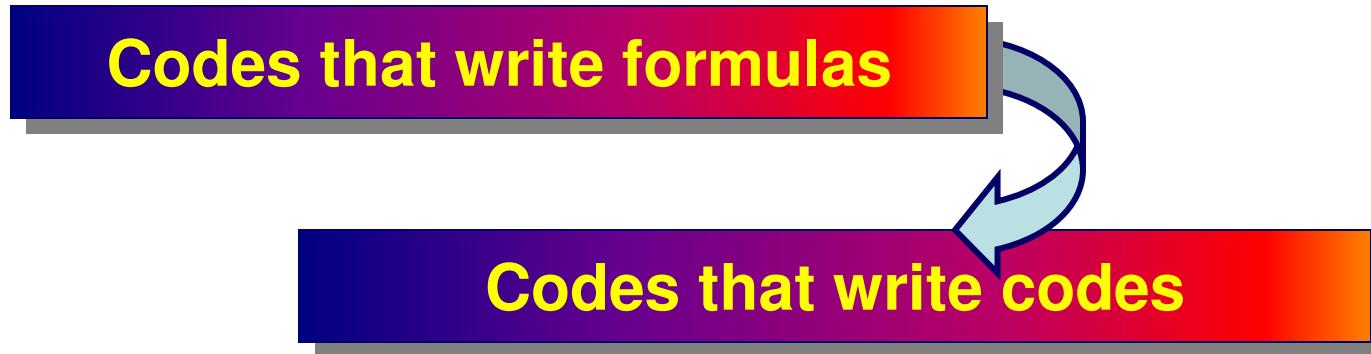
Need total about 300 MB (+extra 150MB file)

Extra index r gives at least a factor (35×13) : over 130GB!

The complexity of the equations also increases.

Problem with all-order extensions: TOO MANY TERMS

Solution: automated code generation !



Input: list of formulas to be programmed

Output: final code (need to be put into a main shell)

Features: simple input, essentially just type in a formula!

Monovalent systems: very brief summary of what we calculated with all-order method

Properties

- Energies
- Transition matrix elements (E1, E2, E3, M1)
- Static and dynamic polarizabilities & applications
 - Dipole (scalar and tensor)
 - Quadrupole, Octupole
 - Light shifts
 - Black-body radiation shifts
 - Magic wavelengths
- Hyperfine constants
- C_3 and C_6 coefficients
- Parity-nonconserving amplitudes (derived weak charge and anapole moment)
- EDM enhancement factors
- Isotope shifts (field shift and one-body part of specific mass shift)
- Atomic quadrupole moments
- Nuclear magnetic moment (Fr), from hyperfine data

Systems

Li, Na, Mg II, Al III,
Si IV, P V, S VI, K,
Ca II, In, In-like ions,
Ga, Ga-like ions, Rb,
Cs, Ba II, Tl, Fr, Th IV,
U V, other Fr-like ions,
Ra II

GOALS OF THE OUR PRESENT PROJECT:

1. CALCULATE PROPERTIES OF DIVALENT AND TRIVALENT ATOMS WITH PRECISION COMPARABLE TO ALKALI-METAL ATOMS
2. EXTEND THIS APPROACH TO MORE COMPLICATED SYSTEMS

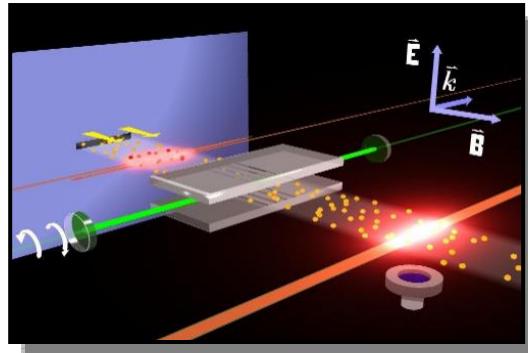
M. S. Safronova, M. G. Kozlov, S. G. Porsev

SUMMARY OF THEORY METHODS FOR PNC CALCULATIONS

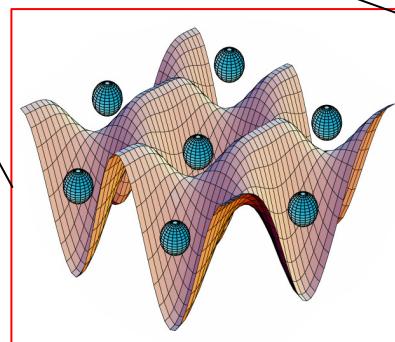
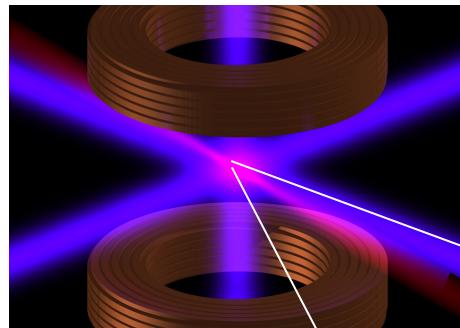
- Configuration interaction (CI)
- Many-body perturbation theory
- Relativistic all-order method (coupled-cluster)
- Perturbation theory in the screened Coulomb interaction (PTSCI), all-order approach
- Configuration interaction + second-order MBPT
- Configuration interaction + coupled-cluster method*

*under development

MOTIVATION: STUDY OF GROUP II – TYPE SYSTEMS



- Study of parity violation
- Search for EDM
- Atomic clocks
- Degenerate quantum gases, alkali-group II mixtures
- Quantum information
- Variation of fundamental constants



Divalent ions: Al^+ , In^+ , etc.

Mg
Ca
Sr
Ba
Ra
Zn
Cd
Hg
Yb

CONFIGURATION INTERACTION METHOD

$$\Psi = \sum_i c_i \Phi_i \leftarrow \text{Single-electron valence basis states}$$

$$(H^{eff} - E)\Psi = 0$$

Example: two particle system:

$$H^{eff} = \underbrace{h_1(r_1) + h_1(r_2)}_{\text{one-body part}} + \underbrace{h_2(r_1, r_2)}_{\text{two-body part}}$$

$$\frac{1}{|\mathbf{r}_1 - \mathbf{r}_2|}$$

CONFIGURATION INTERACTION + MANY-BODY PERTURBATION THEORY

CI works for systems with many valence electrons but can not accurately account for core-valence and core-core correlations.

MBPT can not accurately describe valence-valence correlation for large systems but accounts well for core-core and core-valence correlations.

Therefore, two methods are combined to acquire benefits from both approaches.

CONFIGURATION INTERACTION METHOD + MBPT

H_{eff} is modified using perturbation theory expressions

$$h_1 \rightarrow h_1 + \Sigma_1 \quad || \longrightarrow \quad (H^{\text{eff}} - E) \Psi = 0$$
$$h_2 \rightarrow h_2 + \Sigma_2$$

Σ_1, Σ_2 are obtained using perturbation theory

- V. A. Dzuba, V. V. Flambaum, and M. G. Kozlov , Phys. Rev. A **54**, 3948 (1996)
V. A. Dzuba and W. R. Johnson , Phys. Rev. A **57**, 2459 (1998)
V. A. Dzuba, V. V. Flambaum, and J. S. Ginges , Phys. Rev. A **61**, 062509 (2000)
S. G. Porsev, M. G. Kozlov, Yu. G. Rakhlina, and A. Derevianko, Phys. Rev. A **64**, 012508 (2001)
M. G. Kozlov, S. G. Porsev, and W. R. Johnson, Phys. Rev. A **64**, 052107 (2001)
I. M. Savukov and W. R. Johnson, Phys. Rev. A **65**, 042503 (2002)
Sergey G. Porsev, Andrei Derevianko, and E. N. Fortson, Phys. Rev. A **69**, 021403 (2004)
V. A. Dzuba and J. S. Ginges, Phys. Rev. A **73**, 032503 (2006)
V. A. Dzuba and V. V. Flambaum , Phys. Rev. A **75**, 052504 (2007)

CONFIGURATION INTERACTION METHOD + MBPT

H_{eff} is modified using perturbation theory expressions

$$h_1 \rightarrow h_1 + \Sigma_1 \quad || \longrightarrow \quad (H^{\text{eff}} - E) \Psi = 0$$
$$h_2 \rightarrow h_2 + \Sigma_2$$

Σ_1, Σ_2 are obtained using perturbation theory

Problem: accuracy deteriorates for heavier systems owing to larger correlation corrections

CONFIGURATION INTERACTION + COUPLED-CLUSTER METHOD

H_{eff} is modified using all-order excitation coefficients

$$(\Sigma_1)_{mn} = (\tilde{\varepsilon}_n - \varepsilon_m) \rho_{mn}$$

$$(\Sigma_2)_{mnkl}^L = (\tilde{\varepsilon}_k + \tilde{\varepsilon}_l - \varepsilon_m - \varepsilon_n) \rho_{mnkl}^L$$

Advantages: most complete treatment of the correlations and applicable for many-valence electron systems

CI + ALL-ORDER RESULTS

Two-electron binding energies, differences with experiment

Atom	CI	CI + MBPT	CI + All-order
Mg	1.9%	0.11%	0.03%
Ca	4.1%	0.7%	0.3%
Zn	8.0%	0.7%	0.4 %
Sr	5.2%	1.0%	0.4%
Cd	9.6%	1.4%	0.2%
Ba	6.4%	1.9%	0.6%
Hg	11.8%	2.5%	0.5%
Ra	7.3%	2.3%	0.67%

Development of a configuration-interaction plus all-order method for atomic calculations, M.S. Safronova, M. G. Kozlov, W.R. Johnson, Dansha Jiang, Phys. Rev. A 80, 012516 (2009).

3. APPLICATIONS TO ATOMIC CLOCKS – BBR SHIFTS

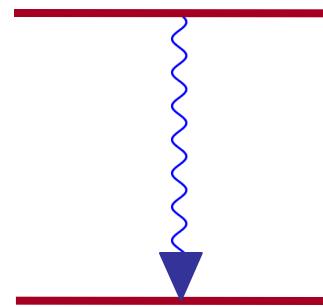
**Precision Calculation of Blackbody Radiation
Shifts for Optical Frequency Metrology**

M.S. Safronova, M. G. Kozlov and C. W. Clark

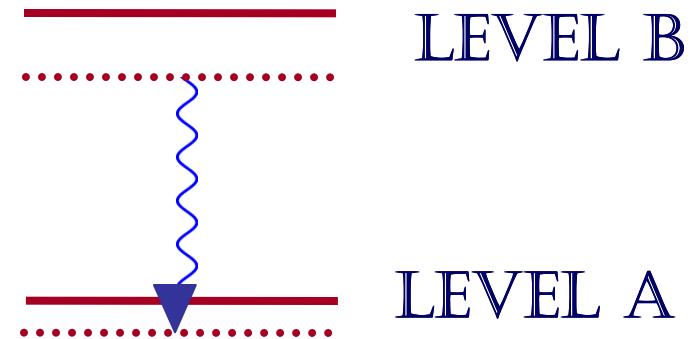
Physical Review Letters, 107, 143006 (2011) .

BLACKBODY RADIATION SHIFTS

CLOCK
TRANSITION



$$T = 0 \text{ K}$$



$$T = 300 \text{ K}$$

$$\Delta_{\text{BBR}}$$

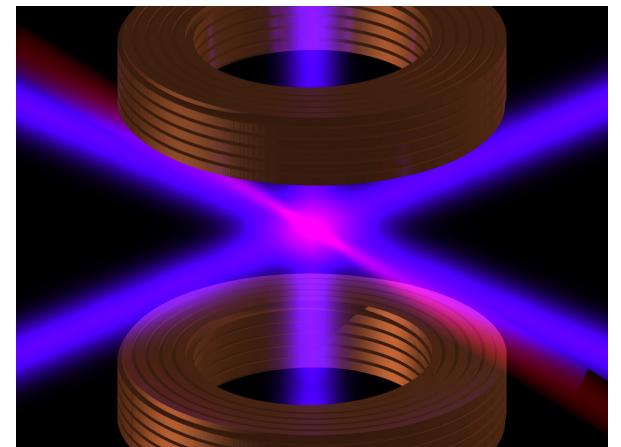
Transition frequency should be corrected to account for the effect of the black body radiation at $T=300\text{K}$.

ATOMIC CLOCKS BLACK-BODY RADIATION (BBR) SHIFT

Motivation:

BBR shift gives large contribution into uncertainty budget for some of the atomic clock schemes.

Accurate calculations (or measurements) are needed to achieve ultimate precision goals at room temperature.



Summary of the fractional uncertainties $\Delta v/v_0$ due to BBR shift and the fractional error in the absolute transition frequency induced by the BBR shift uncertainty at T = 300 K in various frequency standards

Atom	Clock transition	$\Delta v/v_0$	Uncertainty	Reference
Rb	5s (F=2 - F=1)	-1.25×10^{-14}	4×10^{-17}	Safronova et al. 2010
Cs	6s (F=4 - F=3)	-1.7×10^{-14}	3×10^{-17}	Simon et al. 1998
Ca ⁺	4s - 3d _{5/2}	9.2×10^{-16}	1×10^{-17}	Safronova et al. 2011
Sr ⁺	5s - 4d _{5/2}	5.6×10^{-16}	2×10^{-17}	Jiang et al. 2009
Yb ⁺	6s - 5d 2D _{3/2}	-5.3×10^{-16}	1×10^{-16}	Tamm et al. 2007
Yb ⁺	6s - 4f ¹³ 6s ² 2F _{7/2}	-5.7×10^{-17}	1×10^{-17}	Hosaka et al 2009
Mg	3s ² 1S ₀ - 3s3p 3P ₀	-3.9×10^{-16}	1×10^{-17}	Porsev et al. 2006
Al ⁺	3s ² 1S ₀ - 3s3p 3P ₀	-7×10^{-18}	3×10^{-18}	Rosenband et al. 2006
Sr	5s ² 1S ₀ - 5s5p 3P ₀	-5.5×10^{-15}	7×10^{-17}	Porsev et al. 2006
Yb	6s ² 1S ₀ - 6s6p 3P ₀	-2.6×10^{-15}	3×10^{-16}	Porsev et al. 2006
Hg	6s ² 1S ₀ - 6s6p 3P ₀	-1.6×10^{-16}		Hachisu et al. 2008

TOPICAL REVIEW: Theory and applications of atomic and ionic polarizabilities,
J. Mitroy, M.S. Safronova, and C.W. Clark, J. Phys. B: At. Mol. Opt. Phys. 43 (2010) 202001

M. S. Safronova et al., IEEE - TUFFC 57, 94 (2010)

BBR SHIFT OF A LEVEL

- The temperature-dependent electric field created by the blackbody radiation is described by (in a.u.) :

$$E^2(\omega) = \frac{8\alpha}{\pi} \frac{\omega^3 d\omega}{\exp(\omega/kT) - 1}$$

- Frequency shift caused by this electric field is:

$$\Delta\nu_{\text{BBR}} = -A \times \int \alpha(\omega) E^2(\omega) d\omega$$

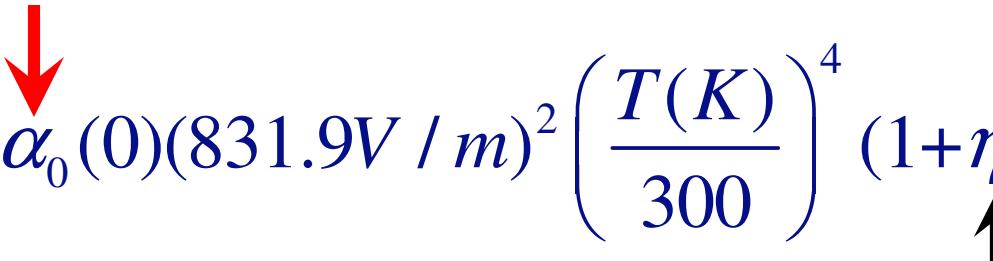


Dynamic polarizability

BBR SHIFT AND POLARIZABILITY

BBR shift of atomic level can be expressed in terms of a **scalar static polarizability** to a good approximation [1]:

$$\Delta\nu_{\text{BBR}} = -\frac{1}{2} \alpha_0(0) (831.9 V/m)^2 \left(\frac{T(K)}{300} \right)^4 (1+\eta)$$


Dynamic correction

Dynamic correction is generally small.

Multipolar corrections (M1 and E2) are suppressed by α^2 [1].

VECTOR & TENSOR POLARIZABILITY AVERAGE
OUT DUE TO THE ISOTROPIC NATURE OF FIELD.

[1] Sergey Porsev and Andrei Derevianko, Physical Review A 74, 020502R (2006)

Al^+ energy levels, differences with experiment

Level		CI	CI+MBPT	CI+All
$3s^2$	1S_0	1.2%	0.043%	0.006%
$3p^2$	1D_2	2.3%	0.07%	-0.022%
$3s4s$	3S_1	1.4%	0.07%	0.015%
$3p^2$	3P_0	1.6%	0.04%	0.008%
$3p^2$	3P_1	1.6%	0.03%	0.004%
$3p^2$	3P_2	1.6%	0.02%	-0.004%
$3s4s$	1S_0	1.4%	0.05%	0.003%
$3s3p$	3P_0	3.1%	0.15%	0.007%
$3s3p$	3P_1	3.1%	0.14%	0.008%
$3s3p$	3P_2	3.1%	0.12%	-0.017%
$3s3p$	1P_1	0.4%	-0.17%	-0.14%

Al⁺ POLARIZABILITIES (A.U.)

	CI	CI+MBPT	CI + All-order
α ($3s^2$ 1S_0)	24.405	24.030	24.408
α ($3s3p$ 3P_0)	24.874	24.523	24.543
$\Delta\alpha$ (3P_0 - 1S_0)	0.469	0.493	0.495

Al^+ POLARIZABILITIES (A.U.)

	CI	CI+MBPT	CI + All-order
$\alpha (3s^2 \ ^1S_0)$	24.405	24.030	24.408
$\alpha (3s3p \ ^3P_0)$	24.874	24.523	24.543
$\Delta\alpha(^3P_0 - ^1S_0)$	0.469	0.493	0.495

Accuracy of $\Delta\alpha (^3P_0 - ^1S_0)$?

Difference (CI+MBPT \rightarrow CI + all-order) = 0.4%

Difference (CI \rightarrow CI + all-order) = 2.6 %

Other uncertainties: 1.4% (Breit) and 2% (core)

Estimate: 10%

Sr POLARIZABILITIES (a.u.)

Sr	CI +MBPT	CI+all-order	Recomm.*
$5s^2 \ ^1S_0$	195.6	198.0	197.2(2)
$5s5p \ ^3P_0$	483.6	459.4	458.3(3.6)

*From expt. matrix elements, S. G. Porsev and A. Derevianko,
PRA 74, 020502R (2006).

Summary of the fractional uncertainties $\Delta\nu/\nu_0$ due to BBR shift and the fractional error in the absolute transition frequency induced by the BBR shift uncertainty at T = 300 K in various frequency standards

Atom	Clock transition	$\Delta\nu/\nu_0$	Uncertainty	Reference
Rb	5s (F=2 - F=1)	-1.25×10^{-14}	4×10^{-17}	Safronova et al. 2010
Cs	6s (F=4 - F=3)	-1.7×10^{-14}	3×10^{-17}	Simon et al. 1998
Ca ⁺	4s - 3d _{5/2}	9.2×10^{-16}	1×10^{-17}	Safronova et. Al, 2011
Sr ⁺	5s - 4d _{5/2}	5.6×10^{-16}	2×10^{-17}	Jiang et al. 2009
Yb ⁺	6s - 5d 2D _{3/2}	-5.3×10^{-16}	1×10^{-16}	Tamm et al. 2007
Yb ⁺	6s - 4f ¹³ 6s ² 2F _{7/2}	-5.7×10^{-17}	1×10^{-17}	Hosaka et al 2009
Mg	3s ² 1S ₀ - 3s3p 3P ₀	-3.9×10^{-16}	1×10^{-17}	Porsev et al. 2006
B ⁺	2s ² 1S ₀ - 2s2p 3P ₀	1.42×10^{-17}	1×10^{-18}	This work
Al ⁺	3s ² 1S ₀ - 3s3p 3P ₀	-3.8×10^{-18}	4×10^{-19}	This work
In ⁺	5s ² 1S ₀ - 5s5p 3P ₀	-1.36×10^{-17}	1×10^{-18}	This work
Sr	5s ² 1S ₀ - 5s5p 3P ₀	-5.5×10^{-15}	7×10^{-17}	Porsev et al. 2006
Yb	6s ² 1S ₀ - 6s6p 3P ₀	-2.6×10^{-15}	3×10^{-16}	Porsev et al. 2006
Hg	6s ² 1S ₀ - 6s6p 3P ₀	-1.6×10^{-16}		Hachisu et al. 2008

SUMMARY: BBR SHIFTS

We have performed the calculations of the BBR shifts in B^+ , Al^+ , and In^+ that allowed to reduce the ultimate uncertainties due to this effect at room temperature to 10^{-18} level for B^+ and In^+ and to 4×10^{-19} for Al^+ .

These uncertainties approach recent estimates of the feasible precision of currently proposed optical atomic clocks.

Future: addition of more all-order correlation corrections for further improvement of accuracy, BBR in Yb , Hg , and Sr .

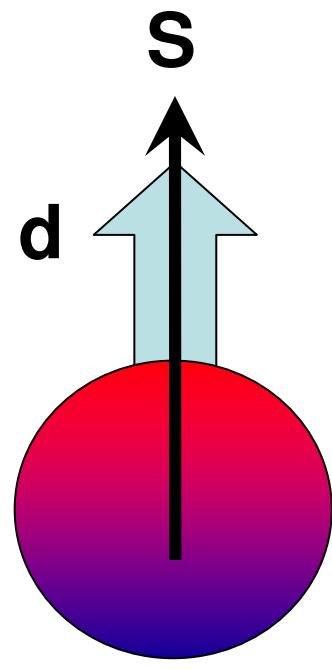
4. TL EDM ENHANCEMENT FACTOR CONTROVERSY

Electric Dipole Moment Enhancement Factor of Thallium

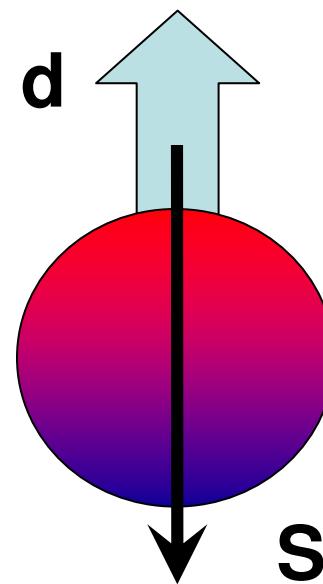
S. G. Porsev, M. S. Safronova, and M. G. Kozlov,
Phys. Rev. Lett. 108, 173001 (2012)

Permanent electric-dipole moment (EDM)

Time-reversal invariance must be violated for an elementary particle or atom to possess a permanent EDM.



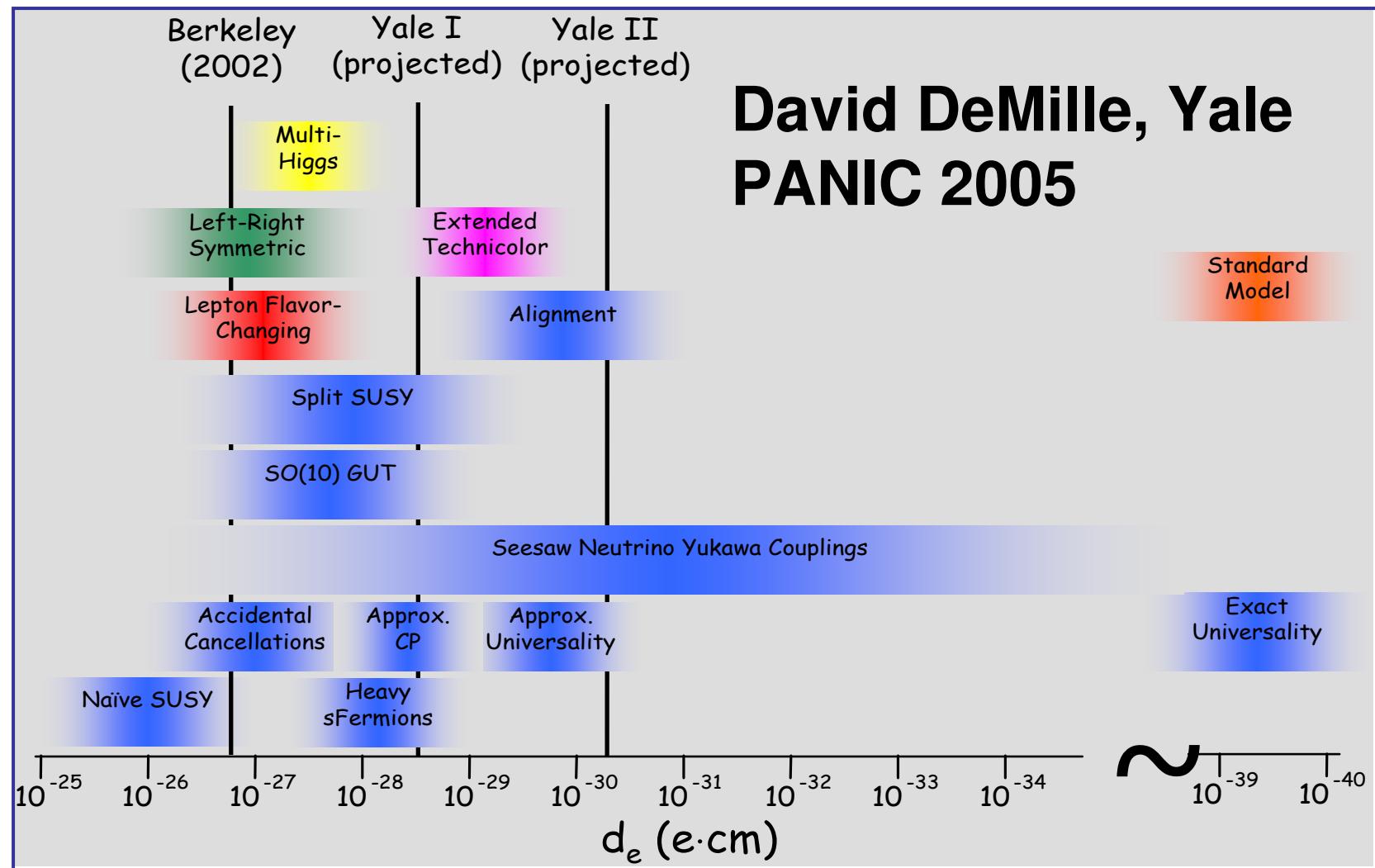
$$\begin{aligned}t &\rightarrow -t \\ \vec{S} &\rightarrow -\vec{S} \\ \vec{d} &\rightarrow \vec{d}\end{aligned}$$



$$\boxed{\vec{d} = d \frac{\vec{S}}{S}}$$
$$d = 0$$

EDM and new physics

Many theories beyond the Standard Model predict electron EDM within or just beyond the present experimental capabilities.



Atomic calculations and search for EDM

EDM effects are enhanced in some heavy atoms and molecules.

Limits on the electron EDM

Tl atom: $|d_e| < 1.6 \times 10^{-27} \text{ e cm}$

Regan et al., Phys. Rev. Lett. 88, 071805 (2002)

YbF molecule: $|d_e| < 1.05 \times 10^{-27} \text{ e cm}$

Hudson et al., Nature 473, 493 (2011)

Both results crucially depend on the calculated values of the effective electric field on the valence electron. In the case of Tl this effective field is proportional to the applied field E_0 , $E = K E_0$ and $d(^{205}\text{Tl}) = K d_e$, **$K = -585$**

TI EDM enhancement factor controversy

Z. W. Liu and H. P. Kelly, PRA 45, R4210 (1992).

K = - 585 (30-60)

V.A. Dzuba and V. V. Flambaum, PRA 80, 062509 (2009)

K = - 582 (20)

TI EDM enhancement factor controversy

Z. W. Liu and H. P. Kelly, PRA 45, R4210 (1992).

K = - 585 (30-60)

V.A. Dzuba and V. V. Flambaum, PRA 80, 062509 (2009)

K = - 582 (20)

H. S. Nataraj, B. K. Sahoo, B. P. Das, and D. Mukherjee,
PRL 106, 200403 (2011)

K = - 466 (10)

TI EDM enhancement factor

[1] V.A. Dzuba and V. V. Flambaum, PRA 80, 062509 (2009)

[2] H. S. Nataraj, B. K. Sahoo, B. P. Das, and D. Mukherjee, PRL 106, 200403 (2011)

	Ref. [1]	Ref. [2]	Difference
Result	- 582(20)	- 466(10)	25% or 5σ
Method	CI+MBPT	CCSD(T)	
Potential	V^{N-3} fully three particle treatment $5d^{10}$ core	V^{N-1} treats TI as Cs $5d^{10}6s^2$ core	
Correlations			
Core	S - all-order D – second-order	SD all-order	
Valence	all-order (via CI)	all-order	
Numerical Issues	CI basis restriction MBPT part restriction	Basis 38s, 34p, 34d, 30f, 20g CCSD basis 14s, 13p, 13d, 9f, 8g	
Other issues		Drastic dif. with CCSD of Liu and Kelly	

Present work: Calculation # 1

- 1a.** We have repeated the CI+MBPT calculation of Dzuba and Flambaum in V^{N-3} potential and ensured that we agree with their value at the same level of approximation (-578 vs. -582).
- 1b.** We have calculated a number of corrections that were omitted in 2009, including structure radiation, core-Brueckner, two-particle, selected three-particle, and normalization corrections.

We found that some of these corrections are large, 5 - 7% percent, but partially canceling, causing lower accuracy of V^{N-3} CI+MBPT results than was previously expected.

Present work: Calculation # 2

2. We have carried out the **CI + all-order** calculation in the same V^{N-3} potential – all previous corrections included.

This calculation allowed us to evaluate the effect of higher-order core-core and core-valence corrections to the EDM (omitted in CI+MBPT).

We found that the effect of these corrections to the enhancement factor is small, 0.7%.

Result in V^{N-3} : $K = -538(46)$ instead of $K = -582(20)$ in [1]

[1] V.A. Dzuba and V. V. Flambaum, PRA 80, 062509 (2009)

Present work: Calculation # 3

3. We have carried out the **CI + MBPT** calculation in the V^{N-1} potential used in [2]. We still treat all three valence electrons using CI. Therefore, we can accurately treat the contributions of the $6s6p^2$ states to K on the same footing as the $6s^2ns$ states in contrast with the work of Nataraj et al., (2011).

Final result in V^{N-1} : $K = -573(20)$

Uncertainty estimate:

(a) Accuracy of relevant hyperfine constants and E1 matrix elements is 3% or better.

(b) CI value is $K = -594$. All other corrections contribute 3.7%.

Present work: Calculation # 4

4. In an attempt to compare with coupled-cluster calculations of [2], we carried out full relativistic coupled-cluster calculations in V^{N-1} potential.

Contribution to K	CI+MBPT	CC (present)	CC [2]
$6s^2 7s \ ^2S_{1/2}$	-216.6	-212.2	-188
$6s^2 8s \ ^2S_{1/2}$	-22.4	-22.8	
$6s6p^2 \ ^4P_{1/2}$	-74.6		
$6s6p^2 \ ^2P_{1/2}$	-179.2		
$6s6p^2 \ ^4D_{1/2}$?	1.3		
Other	-81.1		
Total (present work)	-572.5		

Present work: Calculation # 4b

4b. Comparison with Ref. [2]: basis set issues.

Basis in [2]: **38s, 34p, 34d, 30f, 20g**

Basis used in coupled-cluster calculation of [2]: **14s, 13p, 13d, 9f**

We have conducted another coupled-cluster calculations with a truncated basis that we have constructed using parameters of [2].

The truncation of the basis set to the size of the one used in Ref. [2] produced very large reduction (by 18%) in the value of the $6p_{1/2}$ - $7s$ EDM matrix element.

Other corrections:

Breit interaction effect on the EDM: negligible (0.36%).

Summary: TI EDM enhancement factor

Z. W. Liu and H. P. Kelly, PRA 45, R4210 (1992).

K = - 585 (30-60)

V.A. Dzuba and V. V. Flambaum, PRA 80, 062509 (2009)

K = - 582 (20)

H. S. Nataraj, B. K. Sahoo, B. P. Das, and D. Mukherjee,
PRL 106, 200403 (2011)

K = - 466 (10)

Summary: TI EDM enhancement factor

Z. W. Liu and H. P. Kelly, PRA 45, R4210 (1992).

K = - 585 (30-60)

V.A. Dzuba and V. V. Flambaum, PRA 80, 062509 (2009)

K = - 582 (20)

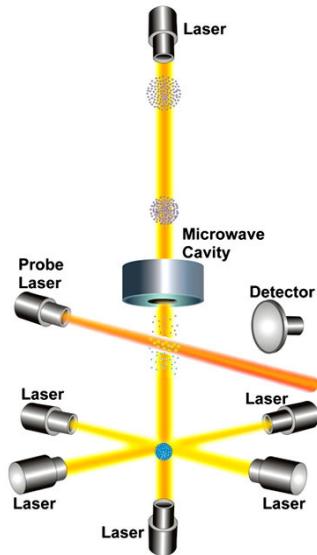
H. S. Nataraj, B. K. Sahoo, B. P. Das, and D. Mukherjee,
PRL 106, 200403 (2011)

K = - 466 (10)

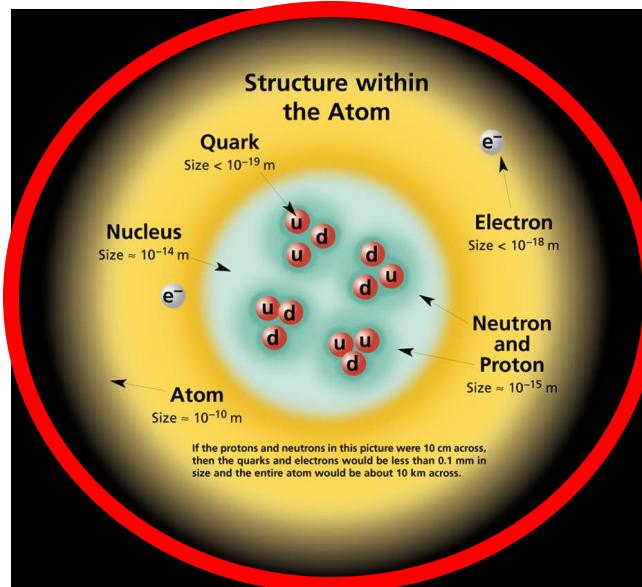
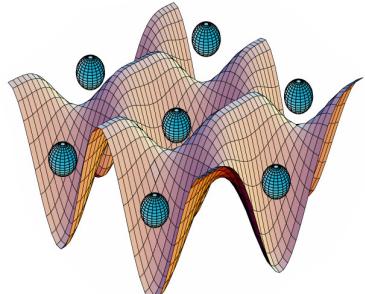
S. G. Porsev, M. S. Safronova, and M. G. Kozlov,
arXiv:1201.5615, Phys. Rev. Lett, in press, April 2012

K = - 573 (20) - several calculations carried out

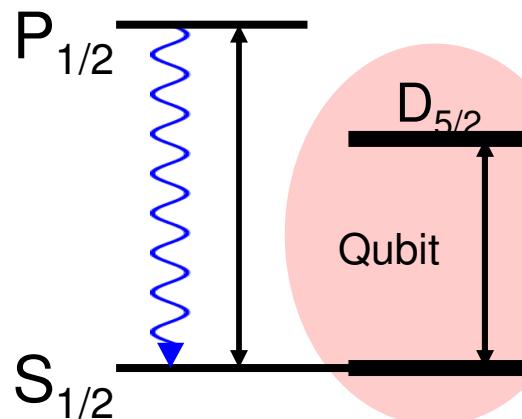
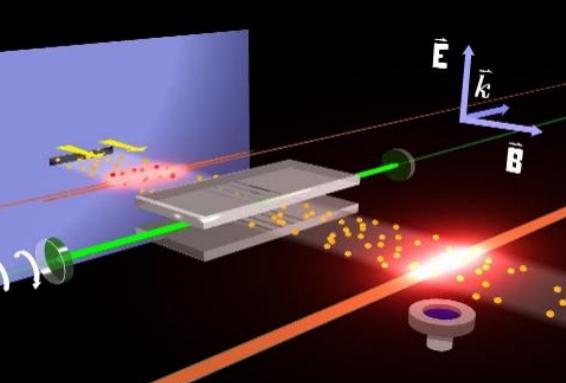
CONCLUSION



Atomic
Clocks



**Future:
New Systems
New Methods,
New Problems**



Quantum information



COLLABORATORS

Michael Kozlov, PNPI, Russia

Sergei Porsev, PNPI, Russia

Charles Clark, NIST

Ulyana Safronova, University of Nevada-Reno

Andrei Derevianko, University of Nevada-Reno

Ephraim Eliav, Tel Aviv University, Israel

Walter Johnson, University of Notre Dame

GRADUATE STUDENTS:

Rupsi Pal*

Dansha Jiang*

Bindiya Arora*

Jenny Tchoukova*

Z. Zhuriadna

Matt Simmons

*Graduated with a Ph.D.