When ExaBytes are not Enough: An Efficient Quantum Algorithm for Dynamics of a Spin System Coupled to Specific Spin Baths

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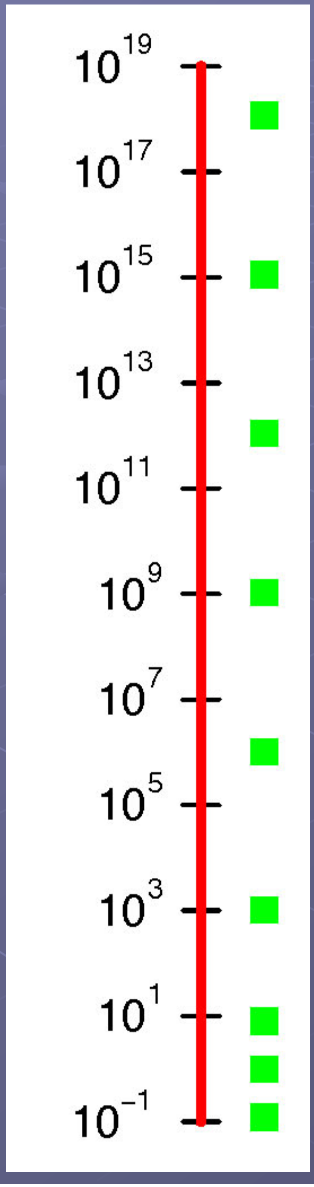
③ Jülich Supercomputing Center
④ RWTH Aachen
⑤ Mississippi U. for Women

2012 Landau Days

How Big/Small is an Exabyte?

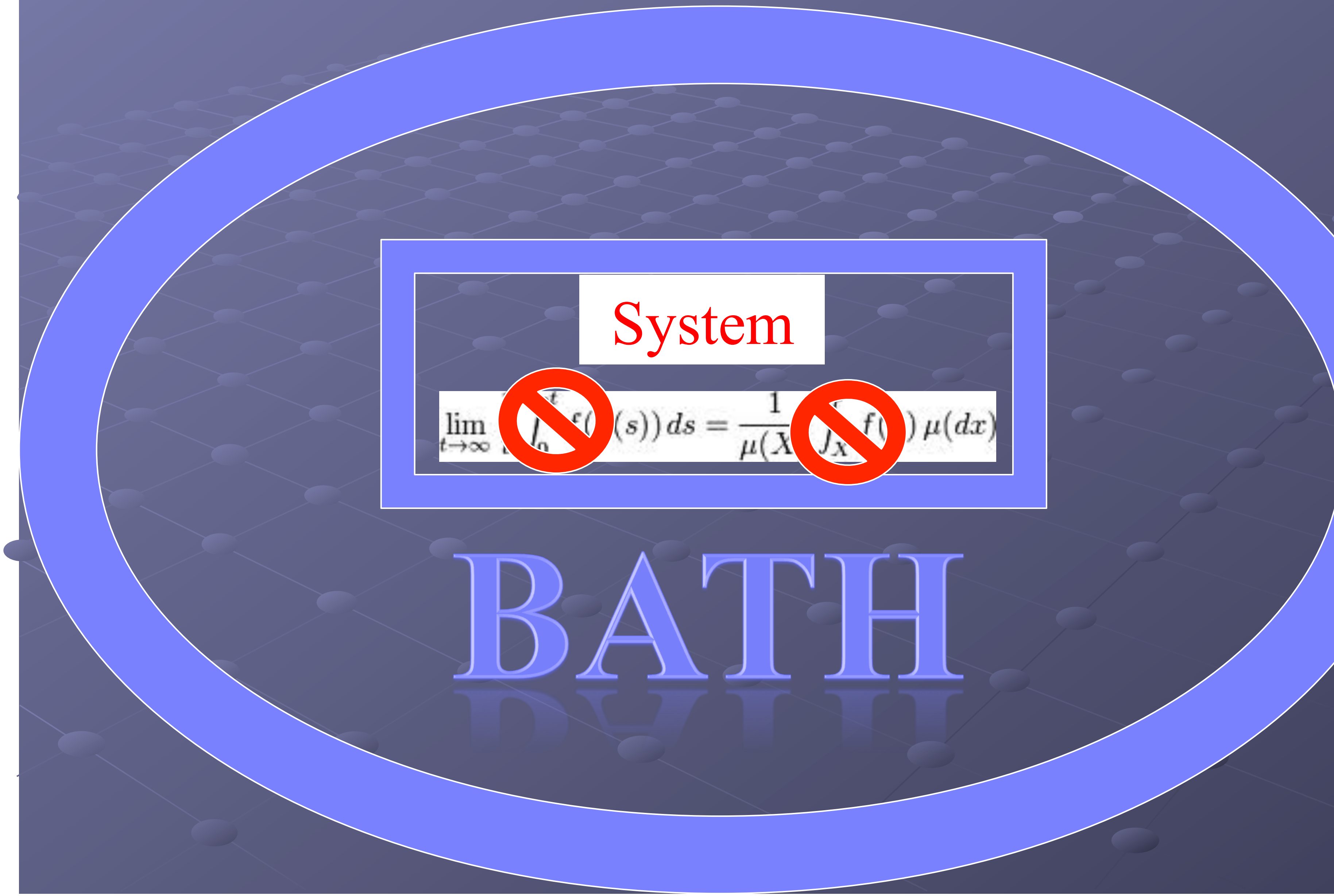


Exabyte = 2^{60}	All words spoken by humans
Petabyte = 2^{50}	All US research libraries
Terabyte = 2^{40}	Academic research library
Gigabyte = 2^{30}	10 meters of books on shelf
	1 picture
Megabyte =2^{20}	1 computer 3.5” diskette
Kilobyte = 2^{10}	
Byte = 2^{0}	1 computer card
Bit = 2^{-3}	

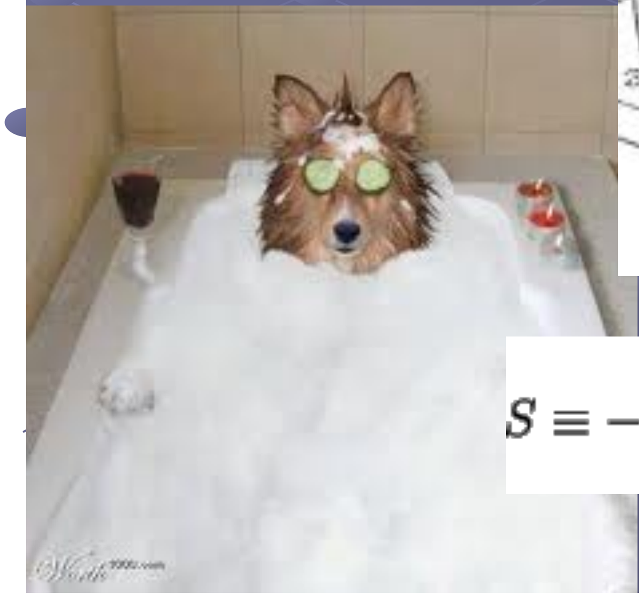
Quantum Statistical Mechanics

- ❖Discrete quantum system can be thermalized by a finite bath
  - ❖Glemer and Michel, Europhysics Letters 2006
- ❖ Prove reaching equilibrium is a universal property of quantum systems: almost any subsystem in interaction with a large enough bath will reach an equilibrium state and remain close to it for almost all times
  - ❖ Linden, Popescu, Short, Winger, PRE 2009
- ❖ For sufficiently large times the ensemble is for all practical purpose indistinguishable from a canonical density operator
  - ❖Reimann, New J. Physics 2010
- ❖ Different processes coupled to the same noise source can have different decoherence rates (stronger noise can have faster decoherence)
  - ❖Zhao, Wang, Liu, Phys. Rev. Lett. 2011
- ❖ As long as finite bath drives system toward a quantum-typicality state, decoherence for spin ½ bath goes as $\sigma \approx 2^{-Nbath/2}$
  - ❖Jin, De Raedt, Novotny, Michielsen, Miyashita, to be submitted

Quantum Statistical Mechanics



Are All Baths Equal?



$$S \equiv -k_{\mathrm{B}} \sum_{i=1}^{\Omega} P_i \ln P_i \qquad \left(\frac{\text{joules}}{\text{K} \cdot \text{microstate}}\right)$$

MOTIVATION : *PHYSICS*

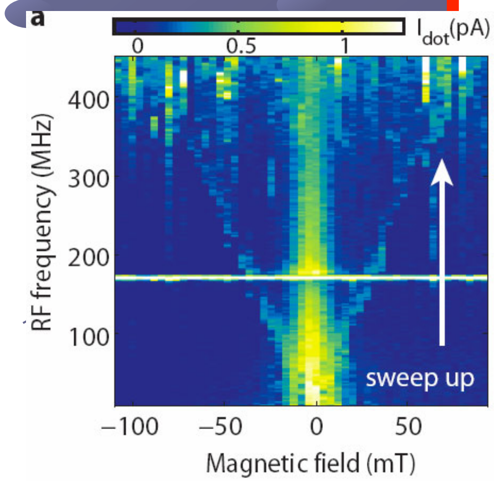
Locking electron spins into magnetic resonance by electron-nuclear feedback

Ivo T Vink, Katja C Nowack, Frank H L Koppens, Jeroen Danon, Yuli V Nazarov and Lieven M K Vandersypen Nat Phys 5(10):764-768 (2009)

PRL 103, 046601 (2009) PHYSICAL REVIEW LETTERS week ending 24 JULY 2009

Multiple Nuclear Polarization States in a Double Quantum Dot

J. Danon, I. T. Vink, F. H. L. Koppens, K. C. Nowack, L. M. K. Vandersypen, and Yu. V. Nazarov
Kavli Institute of NanoScience, Delft University of Technology, 2628 CJ Delft, The Netherlands



electron spin

nuclear spin bath

10^9 spins

MOTIVATION :
Computational Science

$$|\Psi(t)\rangle = \mathrm{e}^{-\mathrm{i}tH}|\Psi(0)\rangle$$

$\hbar = 1$

2 difficulties:

① *Minus sign problem (phase)*

② *Size of vector* 2^{M+N}

M spins

spin bath (N)

MOTIVATION :
Exascale Supercomputers

$$|\Psi(t)\rangle = \mathrm{e}^{-\mathrm{i}tH}|\Psi(0)\rangle$$

$\hbar = 1$

2 difficulties:

① *Minus sign problem (phase)*

② *Size of matrix 2^{M+N}*

2011	1 PB	$M+N=50$
2015	5 PB	$M+N=52$
2018	16 PB	$M+N=54$

M spins

spin bath (N)

Journal of the Physical Society of Japan
Vol. 79, No. 12, December, 2010, 124005

Approach to Equilibrium in Nano-scale Systems at Finite Temperature

Fengping JIN*, Hans DE RAEDT†, Shengjun YUAN[1‡], Mikhail I. KATSNELSON[1§], Seiji MIYASHITA[2,3¶], and Kristel MICHIELSEN[4‖]

System

$N_S=4 \quad N_B<=32$

BATH

$$H_S = -\sum_{i=1}^{n_S-1}\sum_{j=i+1}^{n_S}\sum_{\alpha=x,y,z} J_{i,j}^{\alpha} S_i^{\alpha} S_j^{\alpha},$$

$$H_E = -\sum_{i=1}^{n_E-1}\sum_{j=i+1}^{n_E}\sum_{\alpha=x,y,z} \Omega_{i,j}^{\alpha} I_i^{\alpha} I_j^{\alpha},$$

$$H_{SE} = -\sum_{i=1}^{n_S}\sum_{j=1}^{n_E}\sum_{\alpha=x,y,z} \Delta_{i,j}^{\alpha} S_i^{\alpha} I_j^{\alpha}.$$

$$H = H_S + H_E + H_{SE}$$

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$$\widetilde{\rho}(t) \equiv \mathrm{Tr}_E\, \rho(t)$$

$$|\Psi(t)\rangle = \mathrm{e}^{-\mathrm{i}tH}|\Psi(0)\rangle = \sum_{i=1}^{D_S}\sum_{p=1}^{D_E} c(i,p,t)|i,p\rangle,$$

System

$$\sigma(t) = \sqrt{\sum_{i=1}^{D_S-1}\sum_{j=i+1}^{D_S} |\widetilde{\rho}_{ij}(t)|^2},$$

$$\mathrm{var}(t) \equiv \sqrt{\sum_{i=1}^{D_S}[\lambda_i(t)-\lambda_i(t_\mathrm{f})]^2},$$

$$\delta(t) = \sqrt{\sum_{i=1}^{D_S}\left(\widetilde{\rho}_{ii}(t) - \frac{\mathrm{e}^{-b(t)E_i}}{\sum_{i=1}^{D_S}\mathrm{e}^{-b(t)E_i}}\right)^2},$$

BAT

$$b(t) = \frac{\sum_{i<j,E_i\neq E_j}[\ln\widetilde{\rho}_{ii}(t) - \ln\widetilde{\rho}_{jj}(t)]/(E_j-E_i)}{\sum_{i<j,E_i\neq E_j} 1},$$

$$S(t) \equiv -\mathrm{Tr}\,\widetilde{\rho}(t)\ln\widetilde{\rho}(t) = -\sum_{i=1}^{D_S}\lambda_i(t)\ln\lambda_i(t)$$

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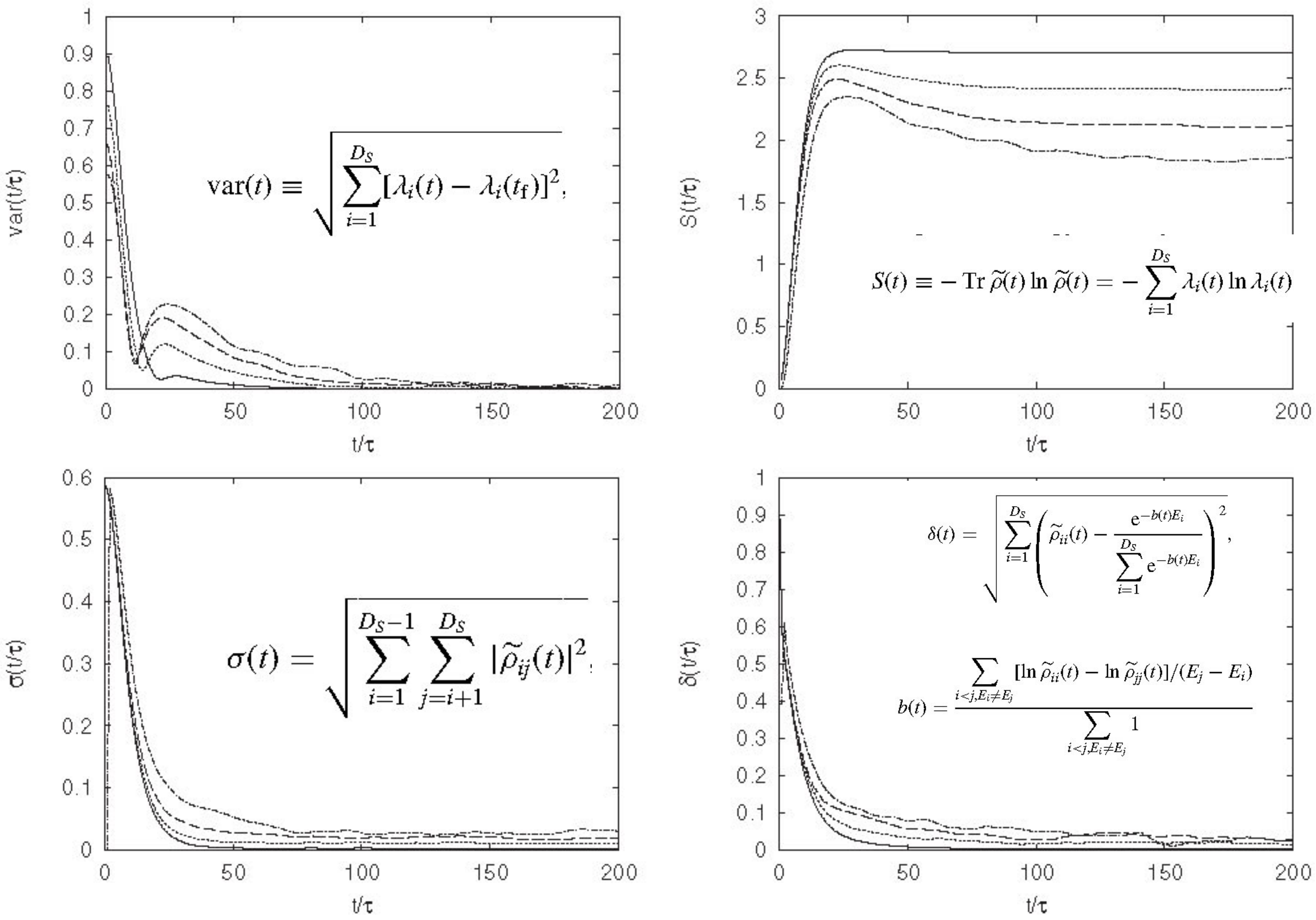


$$\mathrm{var}(t) \equiv \sqrt{\sum_{i=1}^{D_S}[\lambda_i(t)-\lambda_i(t_\mathrm{f})]^2},$$

$$S(t) \equiv -\mathrm{Tr}\,\widetilde{\rho}(t)\ln\widetilde{\rho}(t) = -\sum_{i=1}^{D_S}\lambda_i(t)\ln\lambda_i(t)$$

$$\sigma(t) = \sqrt{\sum_{i=1}^{D_S-1}\sum_{j=i+1}^{D_S}|\widetilde{\rho}_{ij}(t)|^2},$$

$$\delta(t) = \sqrt{\sum_{i=1}^{D_S}\left(\widetilde{\rho}_{ii}(t)-\frac{e^{-b(t)E_i}}{\sum_{i=1}^{D_S}e^{-b(t)E_i}}\right)^2},$$

$$b(t) = \frac{\sum_{i<j,E_i\neq E_j}[\ln\widetilde{\rho}_{ii}(t)-\ln\widetilde{\rho}_{jj}(t)]/(E_j-E_i)}{\sum_{i<j,E_i\neq E_j}1}$$

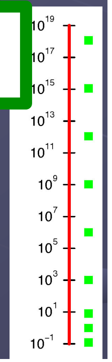
Outl

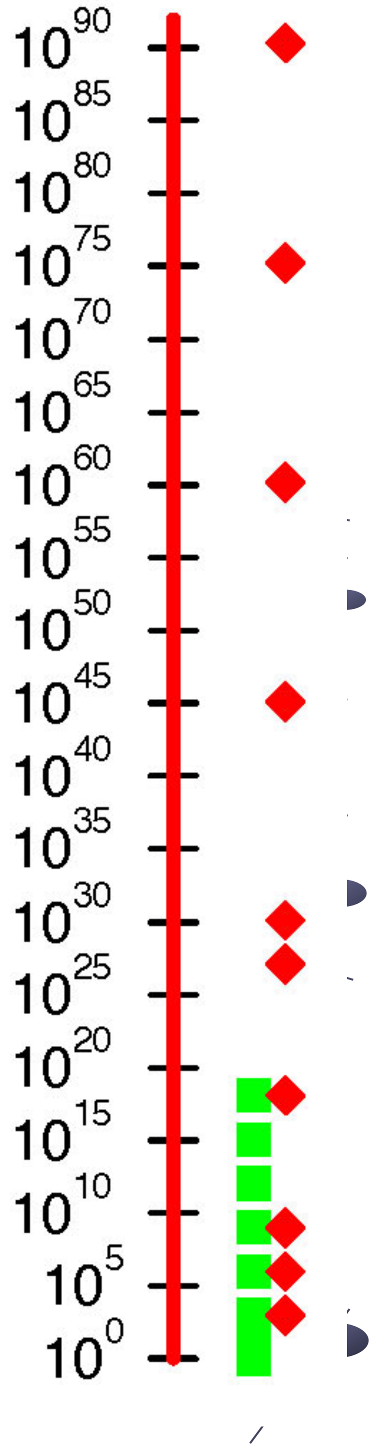
‘ExaGiga’byte 2^{300}

- Motivation
  - Desire for real-time simulations of quantum syst
- Method
  - Efficient real-time dynamic algorithm for spin sys
    - V.V. Dobrovitski and H. De Raedt, Phys. Rev. E **67**, 05
  - Measure expectation values of spin comp
  - Measure quantum purity: $P(t) = \mathrm{Tr}(\rho^2)$
  - Measure entropy: $S(t)=-\rho(t)\ \ln\rho(t)$
  - Measure thermalization: $\sigma(t)=|$
- Spin ½ coupled to special spin baths
  - **New algorithm**

$= 2^{128}$

Exabyte $= 2^{60}$





Spin-1/2 coupled to spin bath

$$\mathcal{H} = \mathcal{H}_S + \mathcal{H}_B + \mathcal{H}_{SB},$$

$$\mathcal{H}_S = \omega_0 S_0^z,$$

$$\mathcal{H} |\Psi(t)\rangle = -\frac{\hbar}{i}\frac{\partial}{\partial t} |\Psi(t)\rangle$$

$$|\Psi(t)\rangle = \exp\left(-\frac{i\mathcal{H}t}{\hbar}\right)|\Psi(0)\rangle$$

$$\rho(t) = \mathrm{Tr}_B\left[\rho_{S+B}(t)\right],$$

$$\left\langle S_0^\ell(t)\right\rangle = \mathrm{Tr}\left[s^\ell \rho(t)\right]$$

Method: Set up

$$S_m^\ell = \mathbf{I}_2 \otimes \mathbf{I}_2 \otimes \cdots \otimes \mathbf{I}_2 \otimes s^\ell \otimes \mathbf{I}_2 \otimes \cdots \otimes \mathbf{I}_2$$

$$|\Psi(t)\rangle = \mathrm{e}^{-itH}|\Psi(0)\rangle$$

$$\ell \in \{x, y, z\}$$

$$\begin{aligned}
\mathcal{H} = {} & \omega_0 S^z_{N_x+N_y+1} \\
& + S^x_{N_x+N_y+1}\left[\sum_{i_1=0}^{1}\sum_{i_2=0}^{1}\cdots\sum_{i_{N_x}=0}^{1} J_{i_1,i_2,\cdots,i_{N_x}} \left(S^x_1\right)^{i_1}\left(S^x_2\right)^{i_2}\cdots\left(S^x_{N_x}\right)^{i_{N_x}}\right] \\
& + \sum_{i_1=0}^{1}\sum_{i_2=0}^{1}\cdots\sum_{i_{N_x}=0}^{1} \tilde{J}_{i_1,i_2,\cdots,i_{N_x}} \left(S^x_1\right)^{i_1}\left(S^x_2\right)^{i_2}\cdots\left(S^x_{N_x}\right)^{i_{N_x}} \\
& + S^y_{N_x+N_y+1}\left[\sum_{j_1=0}^{1}\sum_{j_2=0}^{1}\cdots\sum_{j_{N_y}=0}^{1} K_{j_1,j_2,\cdots,j_{N_y}} \left(S^y_{N_x+1}\right)^{j_1}\left(S^y_{N_x+2}\right)^{j_2}\cdots\left(S^y_{N_x+N_y}\right)^{j_{N_y}}\right] \\
& + \sum_{j_1=0}^{1}\sum_{j_2=0}^{1}\cdots\sum_{j_{N_x}=0}^{1} \tilde{K}_{j_1,j_2,\cdots,j_{N_y}} \left(S^y_{N_x+1}\right)^{j_1}\left(S^y_{N_x+2}\right)^{j_2}\cdots\left(S^y_{N_x+N_y}\right)^{j_{N_y}} .
\end{aligned}$$

x - bath

y - bath

M=1 spins

spin bath (*N*)

Method: Transform

$$|\Psi(t)\rangle = \mathrm{e}^{-\mathrm{i}tH}|\Psi(0)\rangle$$

Unitary transformation

$$P_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$P_y = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}$$

$$P_x^\dagger S^x P_x = \frac{1}{2}\frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = S^z$$

$$P_p = P_x \otimes P_x \otimes \cdots P_x \otimes P_y \otimes P_y \otimes \cdots \otimes P_y \otimes I_2$$

$$\tilde{\mathcal{H}} = P_p^\dagger \mathcal{H} P_p$$

M=1 spins

spin bath (*N*)

Method: Block Diagonalize

$$|\Psi(t)\rangle = \mathrm{e}^{-\mathrm{i}tH}|\Psi(0)\rangle$$

$$\begin{aligned}
\tilde{\mathcal{H}} &= \omega_0 S^z_{N_x+N_y+1} \\
&+ S^x_{N_x+N_y+1}\left[\sum_{i_1=0}^{1}\sum_{i_2=0}^{1}\cdots\sum_{i_{N_x}=0}^{1} J_{i_1,i_2,\cdots,i_{N_x}} \left(S^z_1\right)^{i_1}\left(S^z_2\right)^{i_2}\cdots\left(S^z_{N_x}\right)^{i_{N_x}}\right] \\
&+ \sum_{i_1=0}^{1}\sum_{i_2=0}^{1}\cdots\sum_{i_{N_x}=0}^{1} \tilde{J}_{i_1,i_2,\cdots,i_{N_x}} \left(S^z_1\right)^{i_1}\left(S^z_2\right)^{i_2}\cdots\left(S^z_{N_x}\right)^{i_{N_x}} \\
&+ S^y_{N_x+N_y+1}\left[\sum_{j_1=0}^{1}\sum_{j_2=0}^{1}\cdots\sum_{j_{N_y}=0}^{1} K_{j_1,j_2,\cdots,j_{N_y}} \left(S^z_{N_x+1}\right)^{j_1}\left(S^z_{N_x+2}\right)^{j_2}\cdots\left(S^z_{N_x+N_y}\right)^{j_{N_y}}\right] \\
&+ \sum_{j_1=0}^{1}\sum_{j_2=0}^{1}\cdots\sum_{j_{N_x}=0}^{1} \tilde{K}_{j_1,j_2,\cdots,j_{N_y}} \left(S^z_{N_x+1}\right)^{j_1}\left(S^z_{N_x+2}\right)^{j_2}\cdots\left(S^z_{N_x+N_y}\right)^{j_{N_y}}
\end{aligned}$$

$$\tilde{\mathcal{H}} = P_p^\dagger \mathcal{H} P_p$$

x - bath

y - bath

$$\tilde{\mathcal{H}}_{\mathrm{B}} = \begin{pmatrix} \Omega_z + \omega_0 & \Omega_x - i\Omega_y \\ \Omega_x + i\Omega_y & \Omega_z - \omega_0 \end{pmatrix}$$

Block Diagonal

M=1 spins

spin bath (*N*)

Bring together spin-1/2 and bath at *t=0*

$$|\psi(0)\rangle = \begin{pmatrix} \sin\alpha_0 \\ \cos\alpha_0 \end{pmatrix}$$

$$|\widetilde{\Phi}(0)\rangle = \sum_{j=1}^{2^{N_B}} c_j |\varphi_j\rangle \qquad \text{with} \qquad \sum_{j=1}^{2^{N_B}} |c_j|^2 = 1,$$

$$|\Psi(0)\rangle = |\Phi(0)\rangle \otimes |\psi(0)\rangle$$

$$\begin{aligned}
\rho(t) &= \mathrm{Tr}_B\left[e^{-i\mathcal{H}t}\left(\rho(0)\otimes\rho_B(0)\right)e^{i\mathcal{H}t}\right] \\
&= \mathrm{Tr}_B\left[P_p e^{-i\widetilde{\mathcal{H}}t} P_p^\dagger \left(\rho(0)\otimes\rho_B(0)\right) P_p e^{i\widetilde{\mathcal{H}}t} P_p^\dagger\right] \\
&= \mathrm{Tr}_B\left[e^{-i\widetilde{\mathcal{H}}t}\left(\rho(0)\otimes\widetilde{\rho}_B(0)\right)e^{i\widetilde{\mathcal{H}}t}\right].
\end{aligned}$$

$$\begin{aligned}
\rho(t) &= \left(|c_1|^2 e^{-i\widetilde{\mathcal{H}}_1 t}\rho(0)e^{i\widetilde{\mathcal{H}}_1 t}\right) + \left(|c_2|^2 e^{-i\widetilde{\mathcal{H}}_2 t}\rho(0)e^{i\widetilde{\mathcal{H}}_2 t}\right) + \cdots + \left(|c_{2^{N_B}}|^2 e^{-i\widetilde{\mathcal{H}}_{2^{N_B}} t}\rho(0)e^{i\widetilde{\mathcal{H}}_{2^{N_B}} t}\right) \\
&= |c_1|^2\rho_1(t) + |c_2|^2\rho_2(t) + \cdots + |c_{2^{N_B}}|^2\rho_{2^{N_B}}(t).
\end{aligned}$$

Main Equations

$$\begin{aligned}\rho(t) &= \left(|c_1|^2\, e^{-i\widetilde{\mathcal{H}}_1 t}\rho(0)e^{i\widetilde{\mathcal{H}}_1 t}\right) + \left(|c_2|^2\, e^{-i\widetilde{\mathcal{H}}_2 t}\rho(0)e^{i\widetilde{\mathcal{H}}_2 t}\right) + \cdots + \left(|c_{2^{N_B}}|^2\, e^{-i\widetilde{\mathcal{H}}_{2^{N_B}} t}\rho(0)e^{i\widetilde{\mathcal{H}}_{2^{N_B}} t}\right) \\ &= |c_1|^2 \rho_1(t) + |c_2|^2 \rho_2(t) + \cdots + |c_{2^{N_B}}|^2 \rho_{2^{N_B}}(t)\,.\end{aligned}$$

$$\left\langle S_0^\ell(t)\right\rangle = \sum_{j=1}^{2^{N_B}} \left|c_j\right|^2 \mathrm{Tr}\left[s^\ell \rho_j(t)\right]$$

- Only requires storage for *2x2* matrices, for any N_B
- Requires solution of 2^{N_B} different TDSE to get solution

Main Equations

$$\begin{aligned}\rho(t) &= \left(|c_1|^2\, e^{-i\widetilde{\mathcal{H}}_1 t}\rho(0)e^{i\widetilde{\mathcal{H}}_1 t}\right) + \left(|c_2|^2\, e^{-i\widetilde{\mathcal{H}}_2 t}\rho(0)e^{i\widetilde{\mathcal{H}}_2 t}\right) + \cdots + \left(|c_{2^{N_B}}|^2\, e^{-i\widetilde{\mathcal{H}}_{2^{N_B}} t}\rho(0)e^{i\widetilde{\mathcal{H}}_{2^{N_B}} t}\right) \\ &= |c_1|^2 \rho_1(t) + |c_2|^2 \rho_2(t) + \cdots + |c_{2^{N_B}}|^2 \rho_{2^{N_B}}(t)\,.\end{aligned}$$

$$\left\langle S_0^{\ell}(t)\right\rangle = \sum_{j=1}^{2^{N_B}} |c_j|^2\, \mathrm{Tr}\left[s^{\ell}\rho_j(t)\right]$$

If symmetry in Hamiltonian (not necessiaryly in initial bath vector)

$\rho_j(t)$ are identical for $j \in \{i, \cdots, i+k\}$

$$\left\langle S_0^{\ell}(t)\right\rangle = \cdots + \left\{\left(\sum_{j=i}^{i+k} |c_j|^2\right) \mathrm{Tr}\left[s^{\ell}\rho_{\mathrm{sym}}(t)\right]\right\} + \cdots$$

If all interactions identical and 2-body only

$$\vec{\Omega}_{m_x m_y} = \left(m_x J_2,\ m_y K_2,\ \omega_0\right),$$

$$\Omega_{m_x m_y} = \sqrt{m_x^2 J_2^2 + m_y^2 K_2^2 + \omega_0^2}$$

$$\lambda_{N_x, m_x} = \begin{pmatrix} N_x \\ N_x/2 - m_x \end{pmatrix} = \frac{N_x!}{(N_x/2 - m_x)!\,(N_x/2 + m_x)!}$$

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \exp\left[i\widetilde{\mathcal{H}}_j t\right] \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \exp\left[-i\widetilde{\mathcal{H}}_j t\right] \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{2} \frac{\omega_0^2 + \left(\Omega_{xj}^2 + \Omega_{yj}^2\right) \cos\left[\sqrt{\Omega_{xj}^2 + \Omega_{yj}^2 + \omega_0^2}\, t\right]}{\Omega_{xj}^2 + \Omega_{yj}^2 + \omega_0^2}.$$

$$\boxed{\left\langle S_0^z(t) \right\rangle = \frac{1}{2^{N_x + N_y + 1}} \sum_{m_x = -N_x/2}^{N_x/2} \sum_{m_y = -N_y/2}^{N_y/2} \lambda_{N_x, m_x} \lambda_{N_y, m_y} \frac{\left[\left(m_x^2 J_2^2 + m_y^2 K_2^2\right) \cos\left(\Omega_{m_x m_y} t\right)\right] + \omega_0^2}{\Omega_{m_x m_y}^2}}$$

Sum only over N_b+1 for each x-bath or y-bath

Theory:

no interactions among bath spins; all spins start down

New Journal of Physics
The open-access journal for physics

Quantum frustration of dissipation by a spin bath

D D Bhaktavatsala Rao[1], Heiner Kohler[2] and Fernando Sols[3]

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[2] Department of Physics, University of Duisburg-Essen, D-47057 Duisburg, Germany
[3] Departamento de Física de Materiales, Universidad Complutense de Madrid, E-28040 Madrid, Spain
E-mail: f.sols@fis.ucm.es

New Journal of Physics **10** (2008) 115017 (17pp)

$$H = H_S + H_{SB}, \qquad (1)$$

$$H_S = \omega_0 S^z, \qquad (2)$$

$$H_{SB} = g_1 S^x \sum_{k=1}^{N} I_k^x + g_2 S^y \sum_{l=1}^{N} J_l^y, \qquad (3)$$

No bath dynamics: I^x_{Tot} and J^y_{Tot} conserved

$$\rho(t) = \oplus_{m_1,m_2=-N/2}^{N/2} \rho_{m_1 m_2}(t)$$

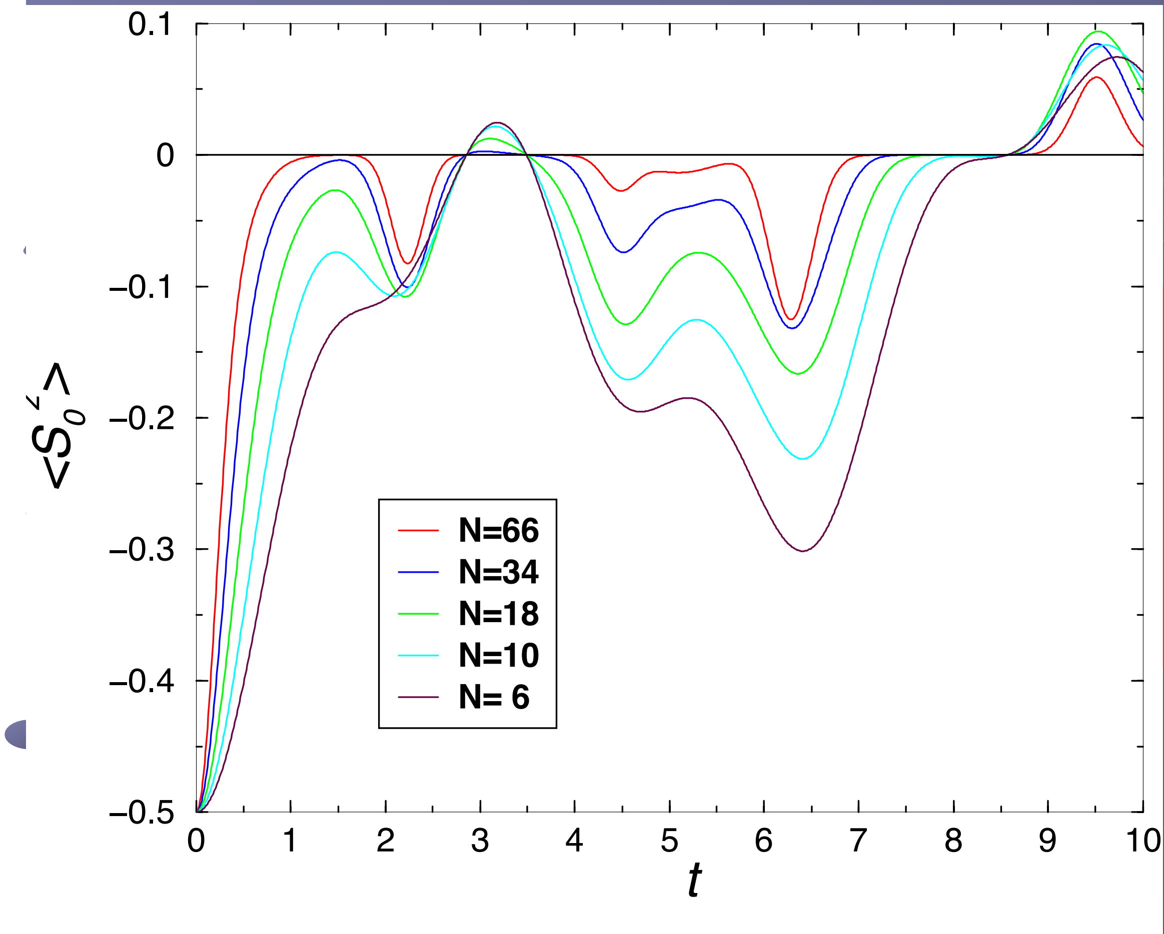
$$\mathcal{P} = \mathrm{Tr}\rho^2$$

Final result

$$\mathcal{P}(t) - \tfrac{1}{2} = \tfrac{1}{2}\exp(-Ng^2t^2/8) \qquad \text{all } t \quad \text{One Bath}$$

$$\simeq \frac{8}{N^2 g^4 t^4} \qquad t \to \infty \qquad \text{Two Baths}$$

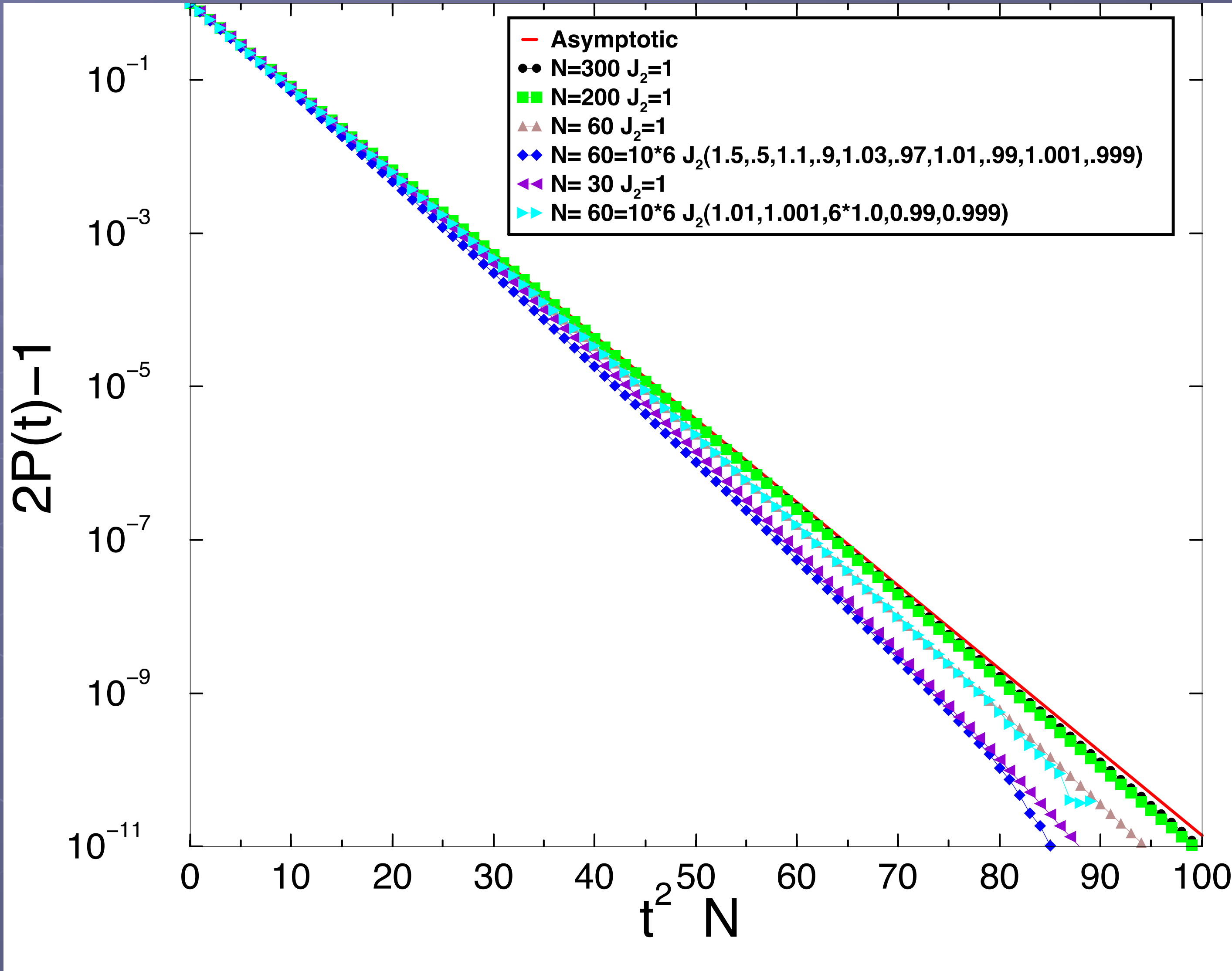
1-bath: x-bath



2-body, 3-body, N-body interactions

$2^{67} \approx 10^{20}$

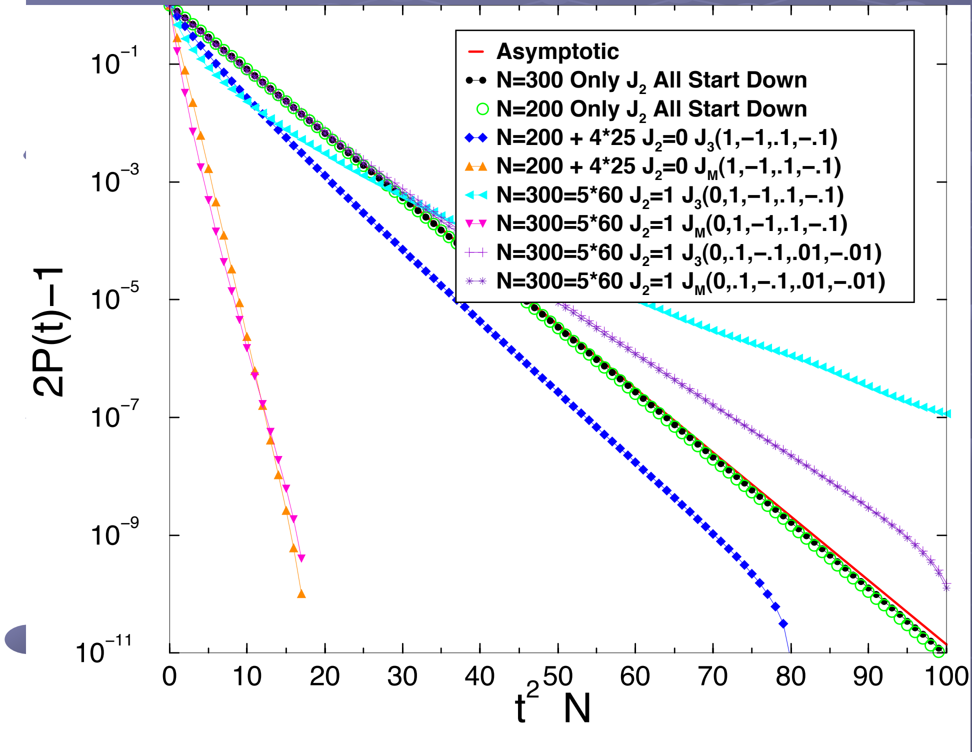
1-bath: x-bath

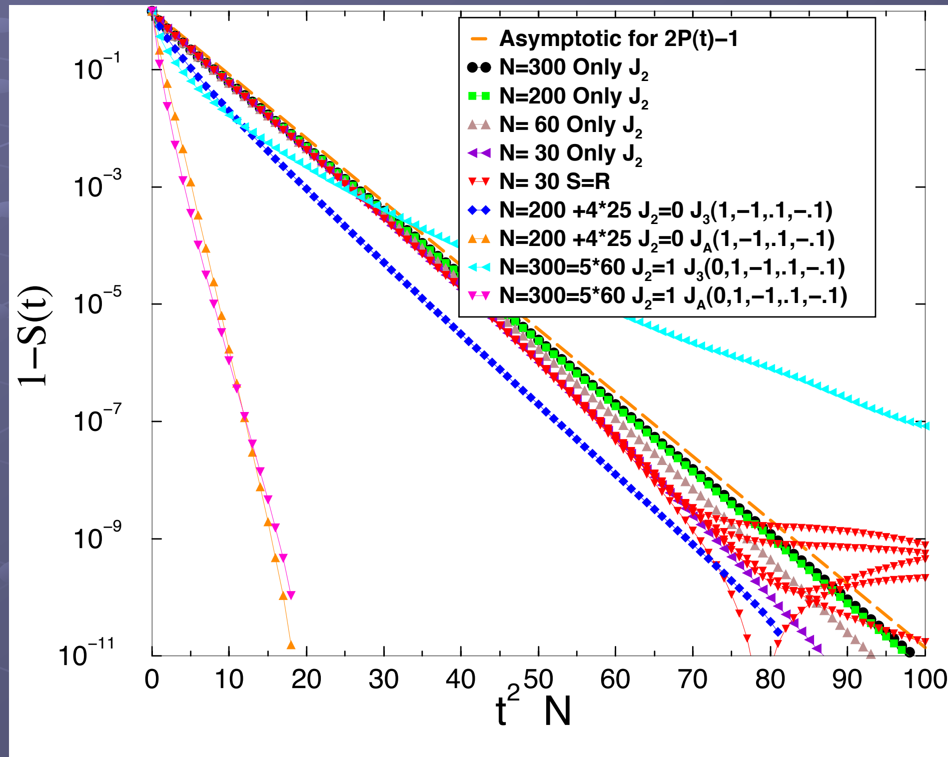


$2^{300} \approx 10^{90}$

Different Randomness

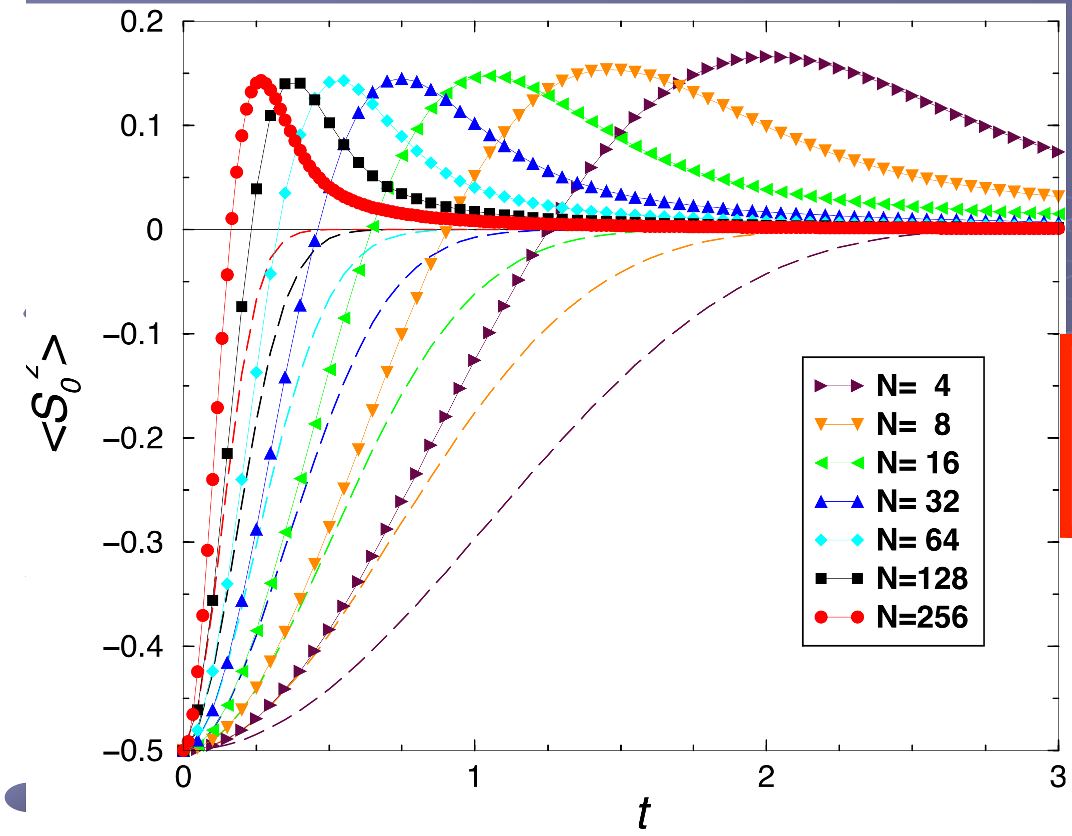
1-bath: x-bath



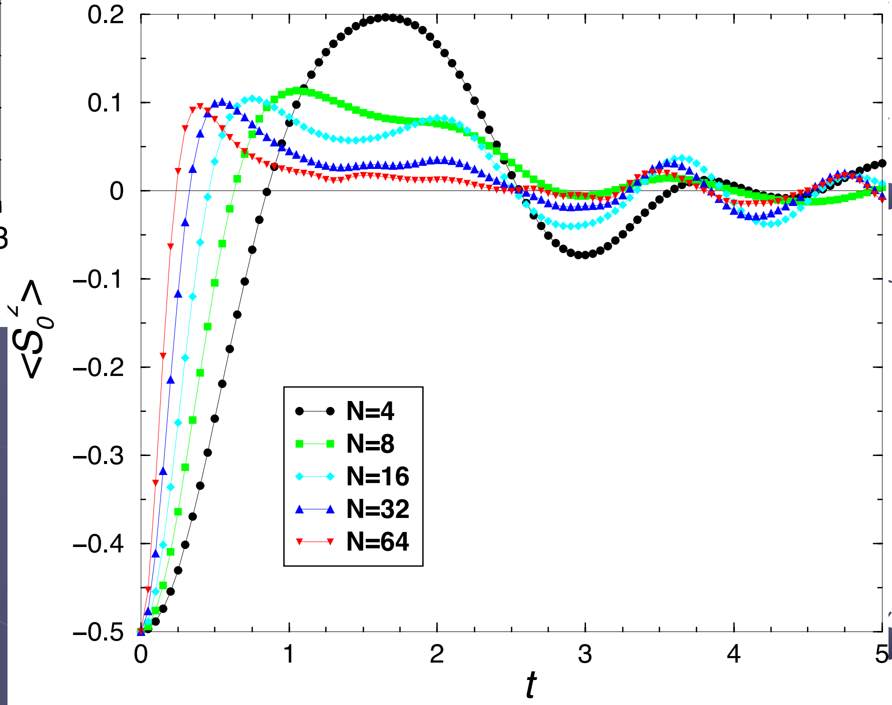


small *t* One Bath: Exponential

2-bath: x,y-bath



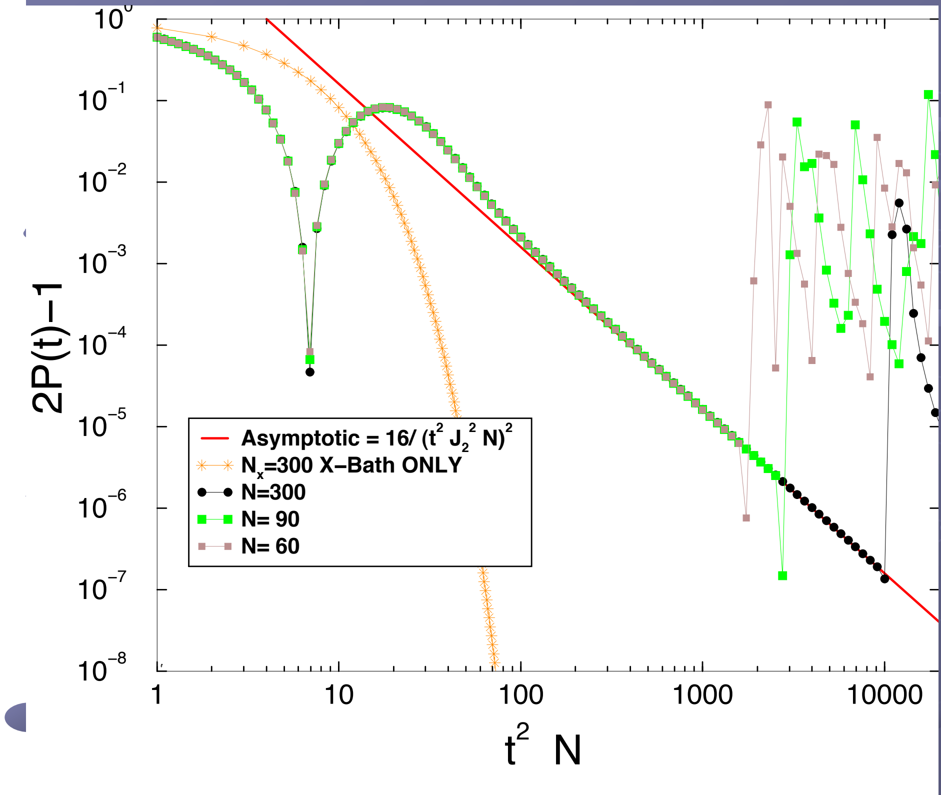
2-body, 3-body, N-body interactions

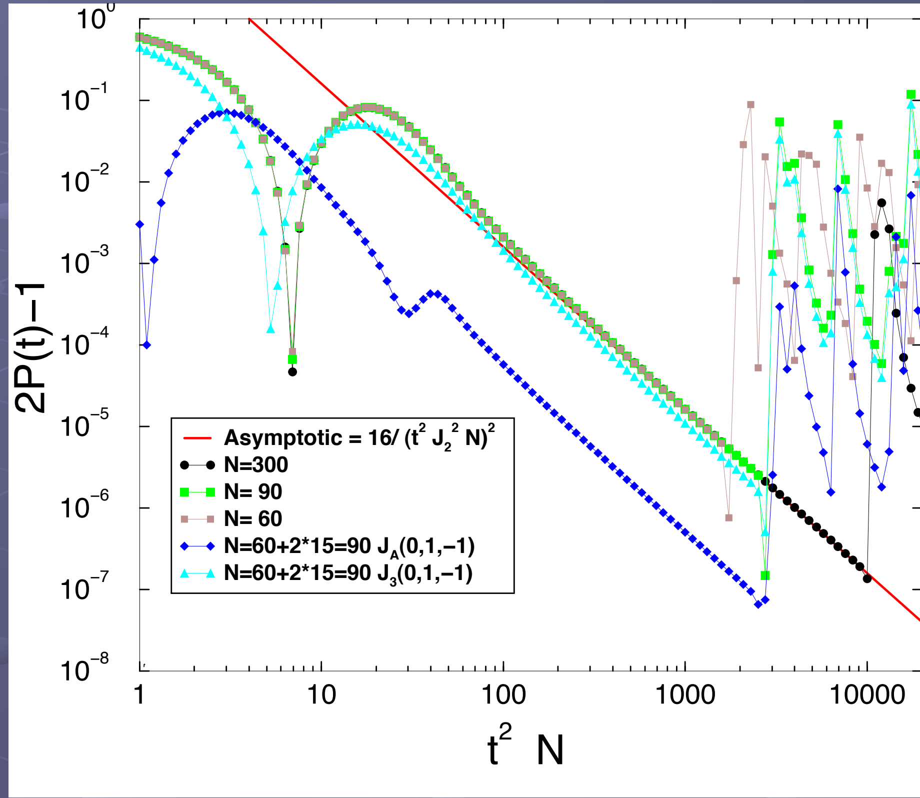


2-body interactions

$2^{512} \approx 10^{154}$

2-bath: x,y-bath





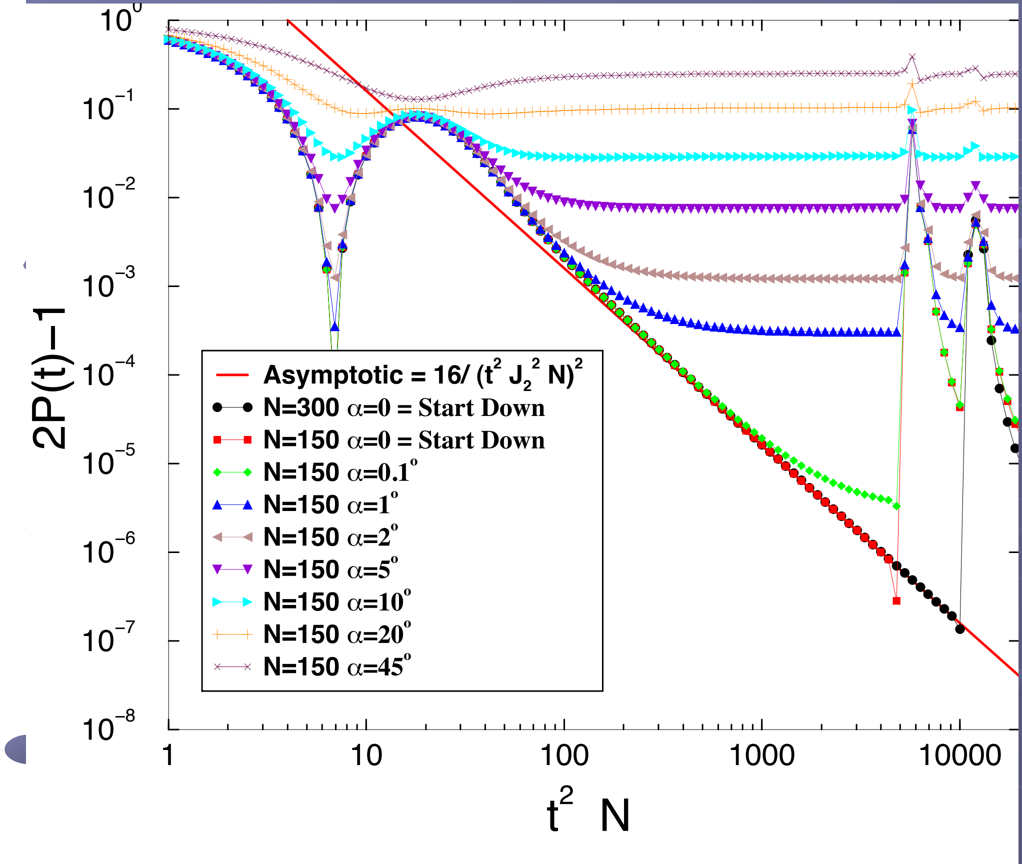
$$\mathcal{P}(t) - \frac{1}{2} = \frac{1}{2}\exp(-Ng^2t^2/8)$$

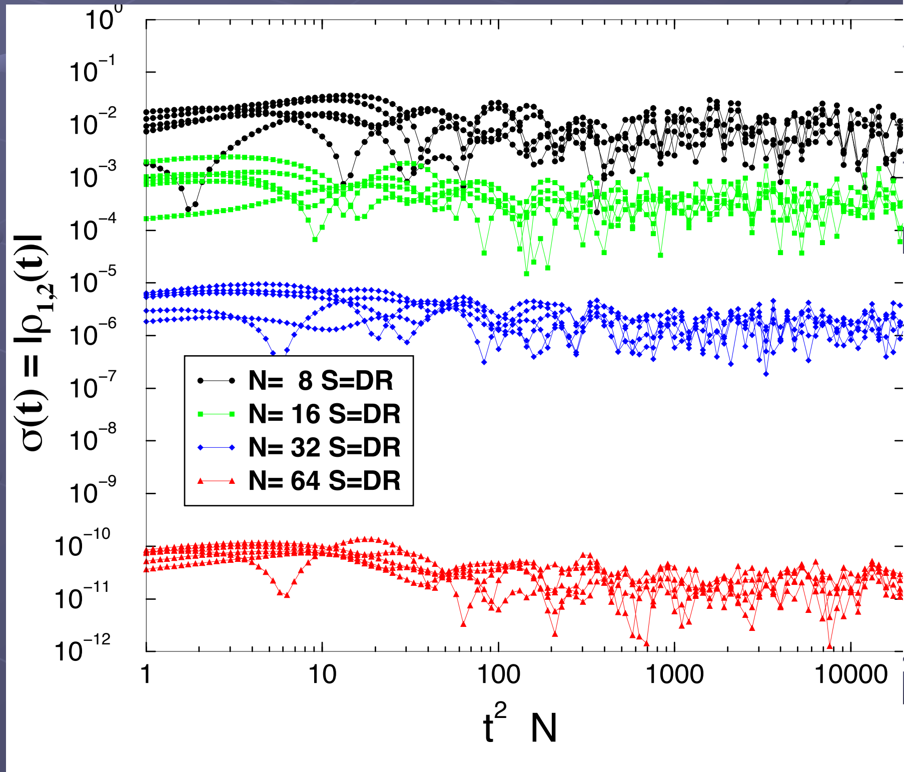
all t One Bath

$$\simeq \frac{8}{N^2g^4t^4} \qquad t \to \infty$$

Two Baths

2-bath: x,y-bath: problems



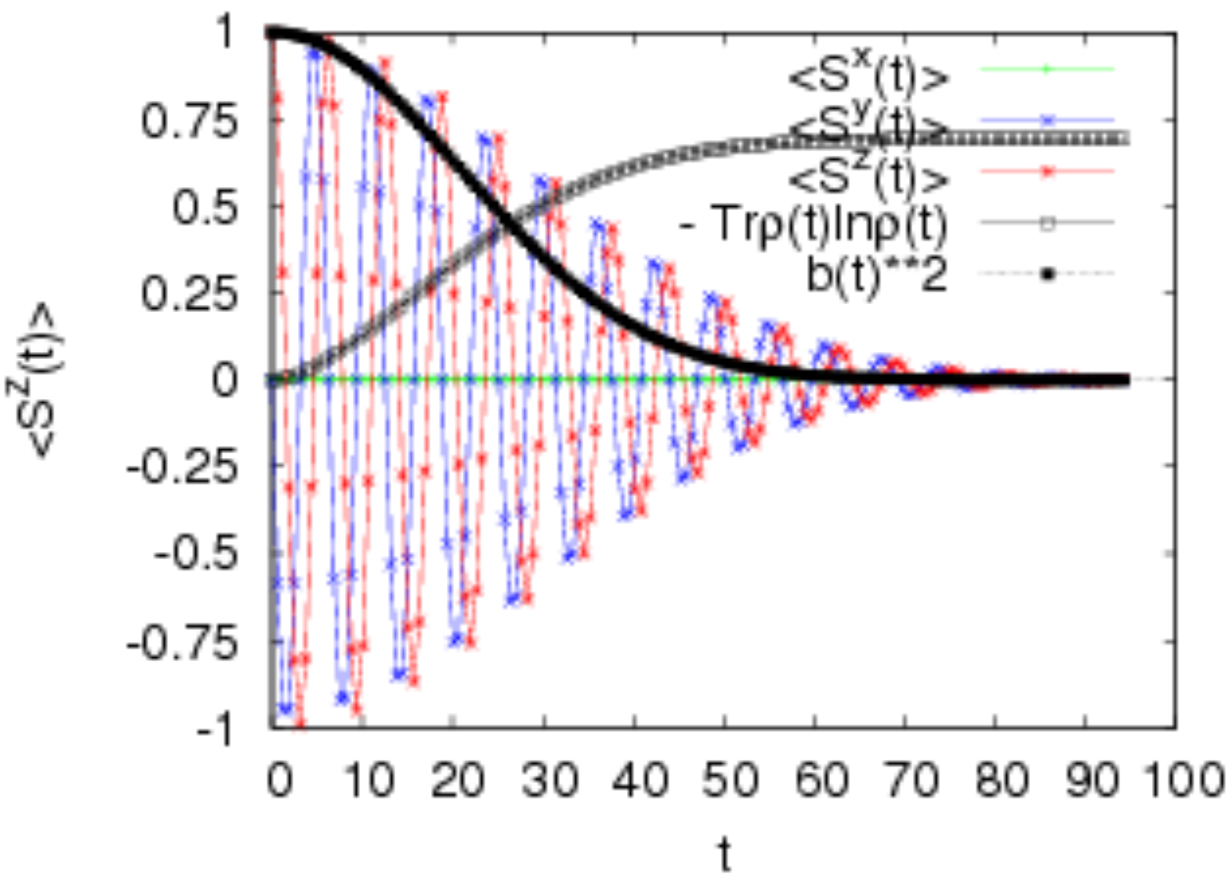


$$|\psi(0)\rangle = \begin{pmatrix} \sin\alpha_0 \\ \cos\alpha_0 \end{pmatrix}$$

New Algorithms (2 baths)

$$|\Psi(t)\rangle = \mathrm{e}^{-itH}|\Psi(0)\rangle$$

Decoherence and Thermalization



$$S(t) \equiv -\operatorname{Tr}\widetilde{\rho}(t)\ln\widetilde{\rho}(t) = -\sum_{i=1}^{D_S}\lambda_i(t)\ln\lambda_i(t)$$

$$\delta(t) = \sqrt{\sum_{i=1}^{D_S}\left(\widetilde{\rho}_{ii}(t) - \frac{\mathrm{e}^{-b(t)E_i}}{\sum_{i=1}^{D_S}\mathrm{e}^{-b(t)E_i}}\right)^2}$$

$$b(t) = \frac{\sum_{i<j,E_i\neq E_j}[\ln\widetilde{\rho}_{ii}(t) - \ln\widetilde{\rho}_{jj}(t)]/(E_j - E_i)}{\sum_{i<j,E_i\neq E_j} 1}$$

M=4

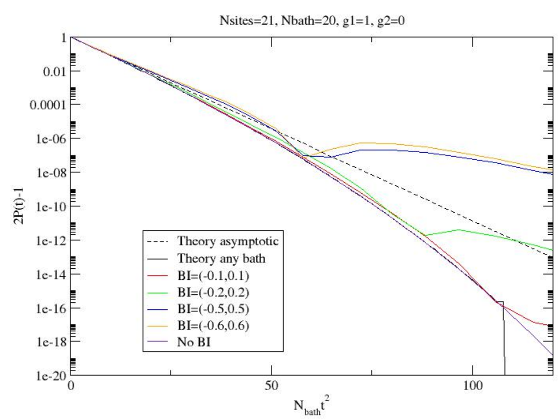
N=35

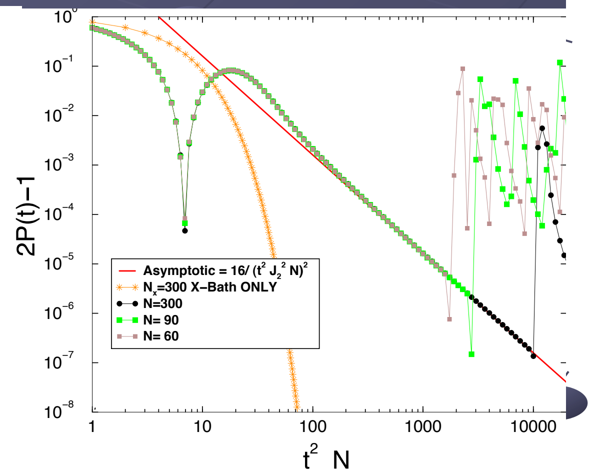
Summary and Conclusion

- Different baths (any interactions) have different asymptotic forms for the quantum purity
  - One –bath : exponential in $N_{bath}\ t^2$
  - Two-baths : power law in $N_{bath}\ t^2$
- **Algorithm requires storage 2^M, rather than 2^{M+N} (sum over 2^N different 2^M systems)**

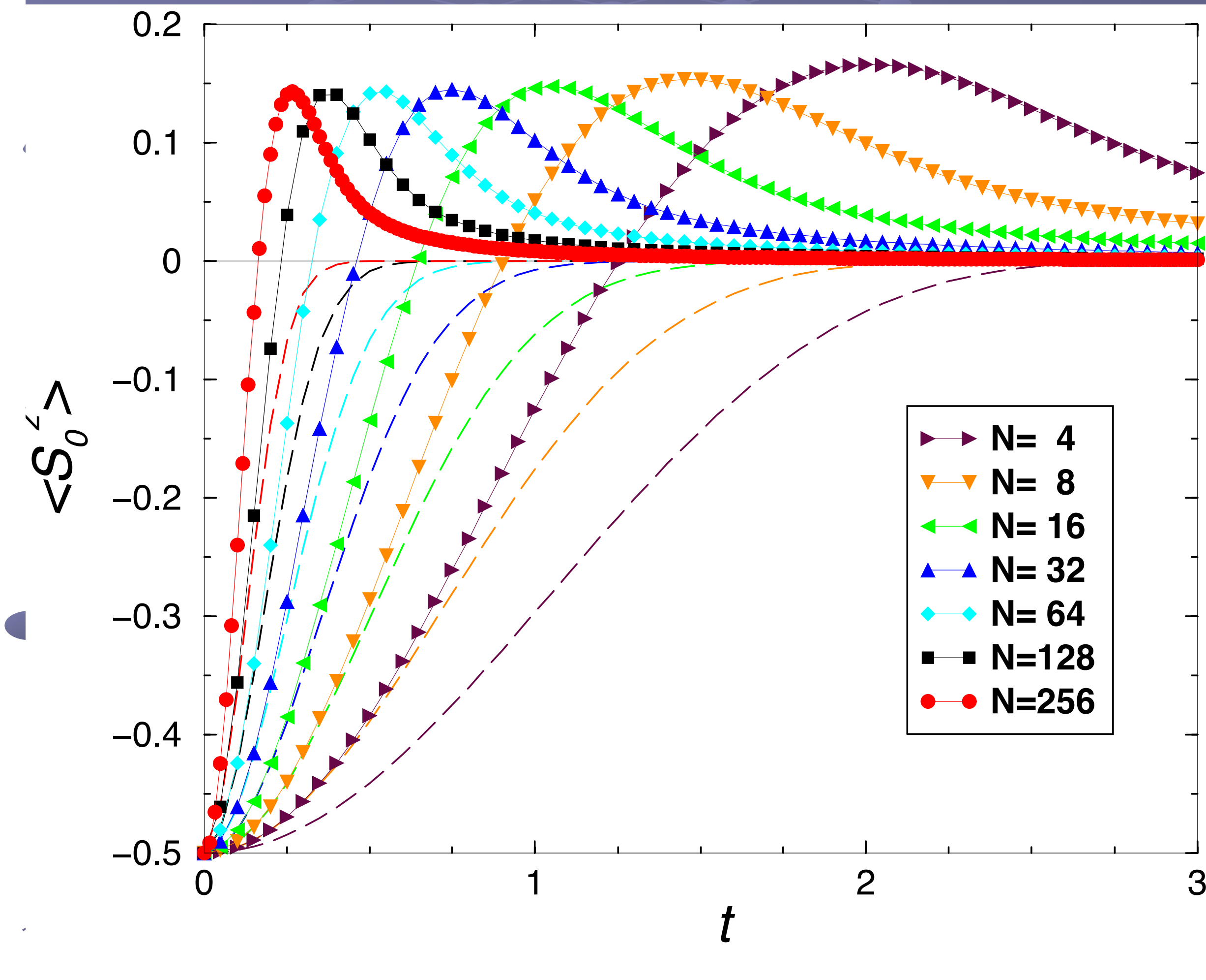
All Baths are NOT the same

$$\left\langle S_0^{\ell}(t)\right\rangle = \sum_{j=1}^{2^{N_B}} \left|c_j\right|^2 \mathrm{Tr}\left[s^{\ell}\rho_j(t)\right]$$

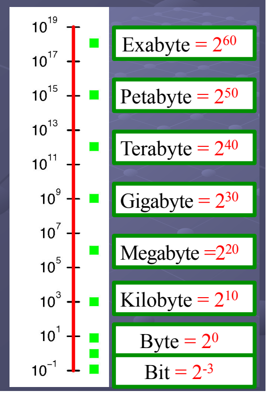




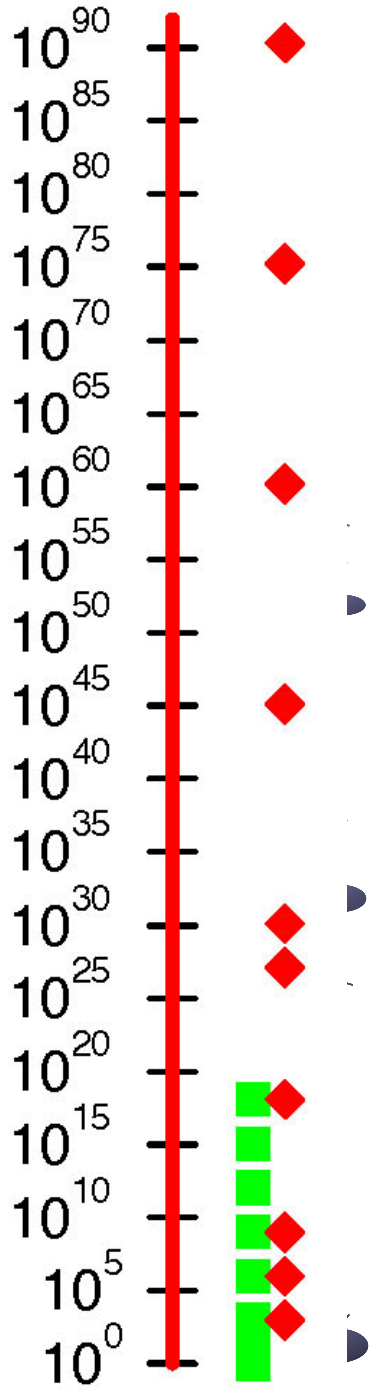
THANK YOU



0.2
0.1
0
−0.1
−0.2
−0.3
−0.4
−0.5
0
1
2
3
t
<S0ᶻ>
N= 4
N= 8
N= 16
N= 32
N= 64
N=128
N=256



Exabyte = 2^60
Petabyte = 2^50
Terabyte = 2^40
Gigabyte = 2^30
Megabyte = 2^20
Kilobyte = 2^10
Byte = 2^0
Bit = 2^-3



10^90
10^85
10^80
10^75
10^70
10^65
10^60
10^55
10^50
10^45
10^40
10^35
10^30
10^25
10^20
10^15
10^10
10^5
10^0