

A cohomological equation over a dynamics codes the possibility of transforming a general map (dynamics) into a simpler one. For instance, the most basic example is that over a circle diffeomorphism: solving this equation allows conjugating it to a rotation.

In general, it is impossible to solve the equation, but in many cases finding approximate solutions is possible. We will see that this is related to almost reducing the dynamics into a simpler one.

More importantly, we will show a nonlinear geometric framework in which several classical results do persist (roughly, isometries of nonpositively curved spaces). Among concrete results/applications, we will show that:

- 1) Circle diffeomorphisms of irrational rotation number have no invariant 1-distributions (joint with M. Triestino).
- 2) The space of C^1 actions of a finitely generated nilpotent group by either circle or interval diffeomorphisms is path connected.
- 3) Every linear cocycle having only zero Lyapunov exponents is C^0 close to a cocycle conjugated to a rotations cocycle (joint with J. Bochi).