Torus Actions via Combinatorics

Survey of work by Altmann ....

Intro

- T-Varieties, Complexity, Toric Varieties
  - T-Var $X \leftrightarrow X/\!\!/T$ + combinatorics
- Ex $\mathbb{A}^2$: $(\mathbb{C}^*)^2$ action ; $\mathbb{C}^*$ action w/ wts (a,b)
- Outline: I Polyhedral Geometry
  - II Toric Varieties
  - III T-Varieties
  - IV Constructing p-divisors
  - V ~~Examples~~ Motivation

I Polyhedral Geometry

M, N

- Polyhedra, Cones, Polytopes
  - Tail Cone, Minkowski Sum
  - Faces, Fans, Dual Cones, Normal Fan

II Toric Varieties

- $TV(\sigma)$ (-4) example
- $TV(\Delta)$ Ex: $\Delta =$ 
- $\tau \prec \sigma \rightsquigarrow TV(\tau) \overset{\text{open}}{\hookrightarrow} TV(\Sigma)$
- $TV(\Sigma)$ Ex: $\mathbb{P}^2$ 

- $\widetilde{\mathbb{P}}(\nabla)$

III. T- Varieties

- X affine
- Imaginary situation: $\pi: X \longrightarrow X/\!\!/_T = Y$
  - Fibers of $\pi \cong$ toric bouquets $\cong \Delta_y$

  $\rightsquigarrow X \cong (Y, \Delta_y)$

- Polyhedral Divisors, p-divisors, etc.

  $X(\mathcal{D})$, $\widetilde{X}(\mathcal{D})$ • Big Thm

  - Ex $[0,1] \times 0 / \mathbb{A}^1$ $[0,\infty) \times 0 / \mathbb{A}^1$ $[1, \infty) \times 0 / \mathbb{A}^1$

- $\Delta_y \rightsquigarrow$ orbits of $\widetilde{X}(\mathcal{D})$

- $\varphi: Y' \longrightarrow Y$ bir. proper $\rightsquigarrow X(\varphi^* \mathcal{D}) = X(\mathcal{D})$
  - "Minimal Y" = Chow quotient $X/\!\!/_T = \lim_n X/\!\!/_T^n$

IV. Constructing p-divisors

- Toric Downgrade Ex $\mathbb{A}^2$ w/ diag action
- General Case

V. Motivation

- Homogeneous Spaces
- Deformations