INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

Cosmological implications of Horava-Lifshitz gravity

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ref. Horava-Lifshitz Cosmology: A Review
arXiv: 1007.5199 [hep-th]
also arXiv: 1105.0246 [hep-th] with K.Izumi
arXiv: 1104.2087 [hep-th] with E.Gumrukcuoglu

Power counting

$$I \supset \int dt dx^3 \dot{\phi}^2$$

$$\int dt dx^3 \phi^n$$

$$\propto E^{-(1+3+ns)}$$

- **Scaling dim of ϕ**

  t → b t (E → b^{-1}E)

  x → b x

  ϕ → b^s ϕ

  1+3-2+2s = 0

  s = -1

- Renormalizability

  $$n \leq 4$$

- Gravity is highly non-linear and thus non-renormalizable

Abandon Lorentz symmetry?

$$I \supset \int dt dx^3 \dot{\phi}^2$$

$$\int dt dx^3 \phi^n$$

$$\propto E^{-(z+3+ns)/z}$$

- Anisotropic scaling
  t → b^z t (E→b^{-z}E)
  x → b x
  ϕ → b^s ϕ
  z+3-2z+2s = 0
  s = -(3-z)/2
- s = 0 if z = 3

- For z = 3, any nonlinear interactions are renormalizable!
- **Gravity becomes renormalizable!?**

Cosmological implications

- Higher curvature terms lead to regular bounce (Calcagni 2009, Brandenberger 2009).
- Higher curvature terms ($1/a^6$, $1/a^4$) might make the flatness problem milder (Kiritsis&Kofinas 2009).
- The z=3 scaling solves the horizon problem and leads to scale-invariant cosmological perturbations without inflation (Mukohyama 2009).
- Absence of local Hamiltonian constraint leads to CDM as integration “constant” (Mukohyama 2009).
- New mechanism for generation of primordial magnetic seed field (S.Maeda, Mukohyama, Shiromizu 2009).

Scale-invariant cosmological perturbations from Horava-Lifshitz gravity without inflation

arXiv:0904.2190 [hep-th]

c.f. Basic mechanism is common for “Primordial magnetic field from non-inflationary cosmic expansion in Horava-Lifshitz gravity”, arXiv:0909.2149 [astro-th.CO] with S.Maeda and T.Shiromizu.

Usual story with z=1

- $\omega^2 >> H^2$: oscillate

  $\omega^2 << H^2$: freeze

  oscillation → freeze-out iff $d(H^2/\omega^2)/t > 0$

  $\omega^2 = k^2/a^2$ leads to $d^2a/dt^2 > 0$

  Generation of super-horizon fluctuations requires accelerated expansion, i.e. inflation.

- Scaling law

  $t \rightarrow b\, t$ ($E \rightarrow b^{-1} E$)

  $x \rightarrow b\, x$ $\Longrightarrow$ $\delta\phi \propto E \sim H$

  $\phi \rightarrow b^{-1} \phi$

  Scale-invariance requires almost const. H, i.e. inflation.

UV fixed point with z=3

- oscillation → freeze-out iff $d(H^2/\omega^2)/t > 0$

  $\omega^2 = M^{-4}k^6/a^6$ leads to $d^2(a^3)/dt^2 > 0$

  OK for $a \sim t^p$ with $p > 1/3$

- Scaling law

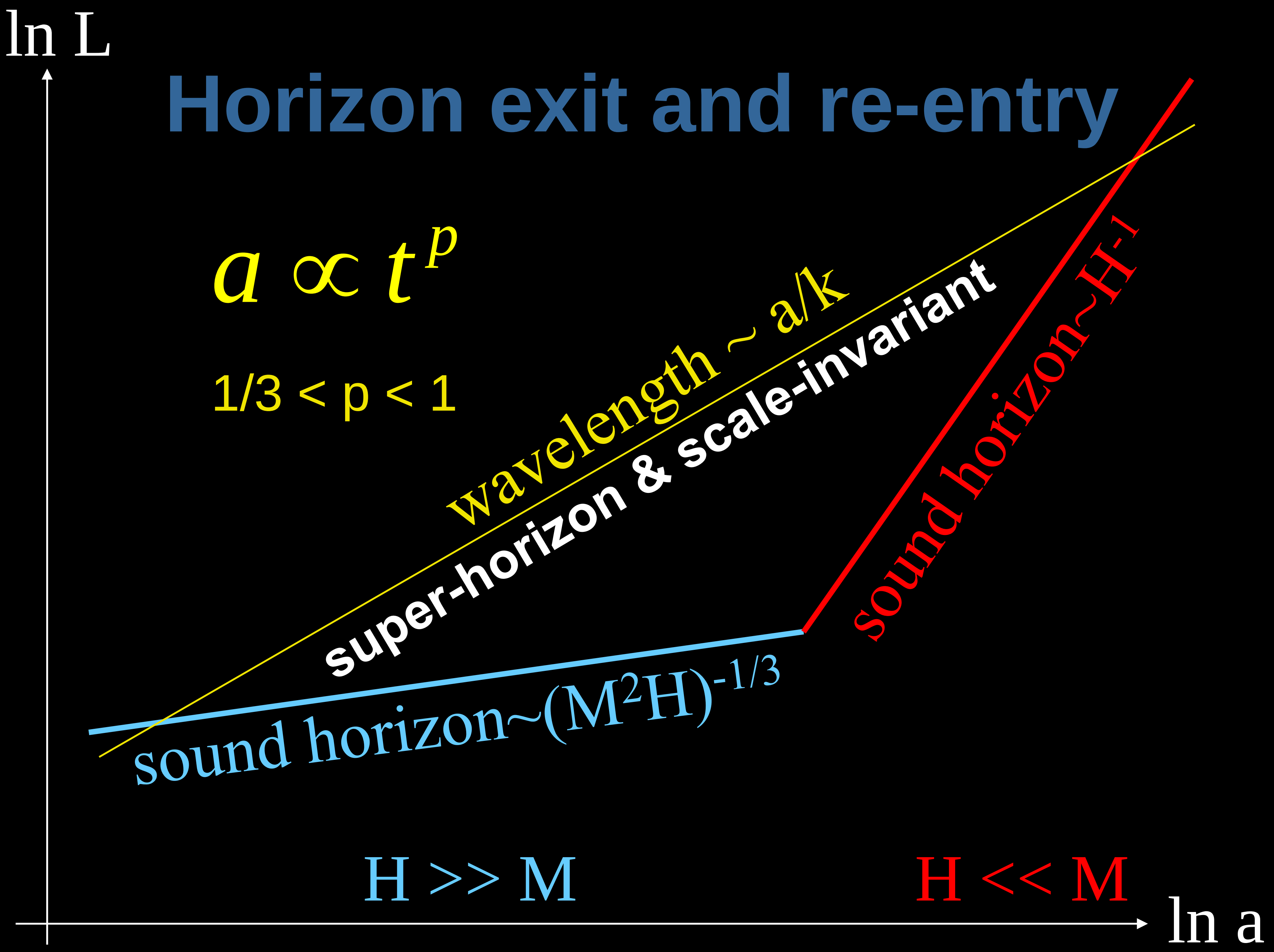
  $t \to b^3 t$ ($E \to b^{-3}E$)

  $x \to b\, x$

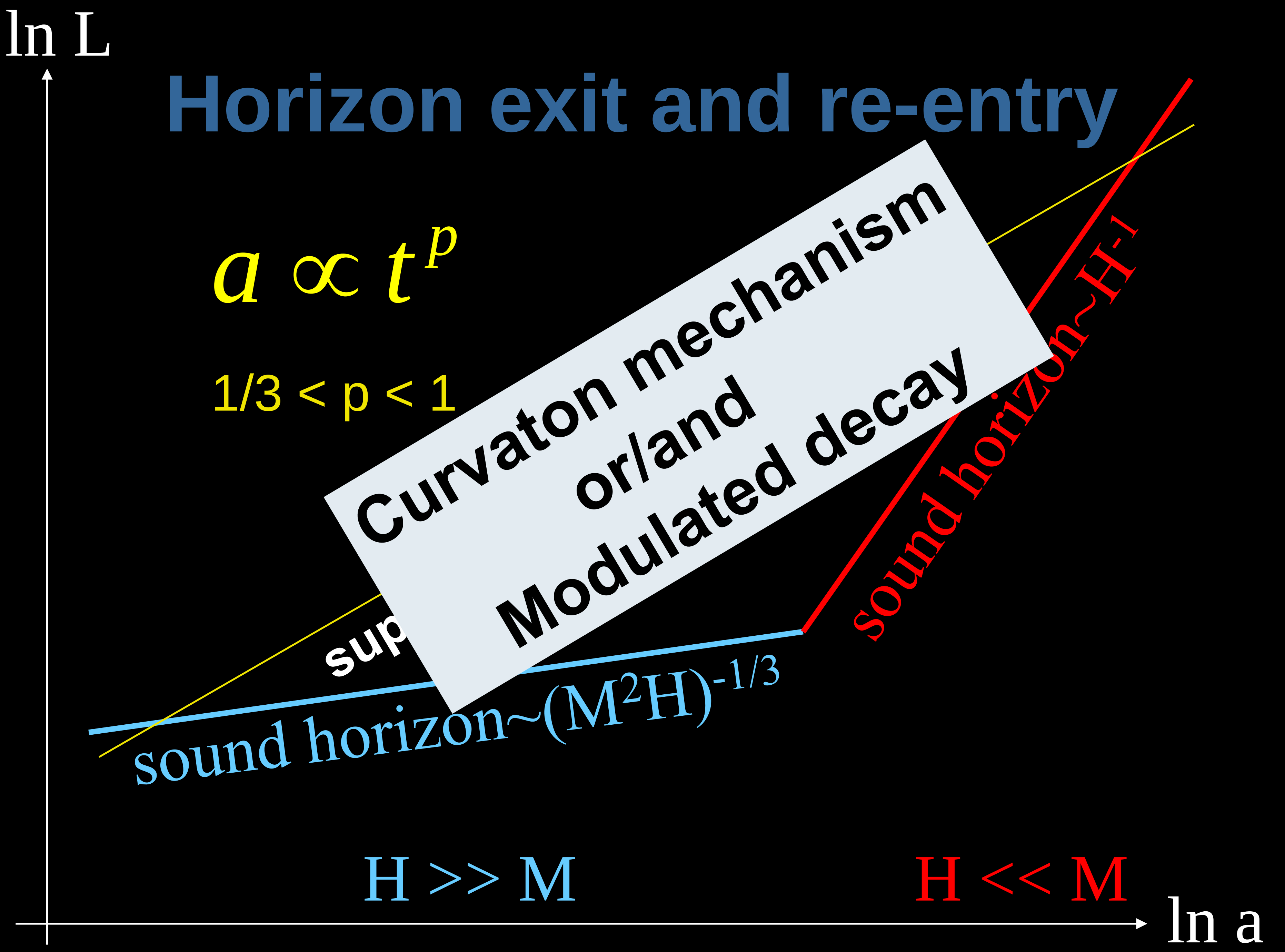
  $\phi \to b^0 \phi$ ⇒ $\delta\phi \propto E^0 \sim H^0$

  Scale-invariant fluctuations!

Horizon exit and re-entry



Horizon exit and re-entry



GOING BACK TO HORAVA‘S IDEA

Horava-Lifshitz gravity

Horava (2009)

- Basic quantities:
  lapse $N(t)$, shift $N^i(t,x)$, 3d spatial metric $g_{ij}(t,x)$
- ADM metric (emergent in the IR)
  $ds^2 = -N^2dt^2 + g_{ij}\,(dx^i + N^i dt)(dx^j + N^j dt)$
- Foliation-preserving deffeomorphism
  $t \rightarrow t'(t), \quad x^i \rightarrow x'^i(t,x^j)$
- Anisotropic scaling with z=3 in UV
  $t \rightarrow b^z\, t, \quad x^i \rightarrow b\, x^i$
- Ingredients in the action

$$Ndt \qquad \sqrt{g}\,d^3x \qquad g_{ij} \qquad D_i \qquad R_{ij}$$

$$K_{ij} = \frac{1}{2N}\left(\partial_t g_{ij} - D_i N_j - D_j N_i\right)$$

(C_{ijkl} = 0 in 3d)

UV action with z=3

- Kinetic terms (2nd time derivative)

$$\int N dt \sqrt{g}\, d^3x \left(K_{ij}K^{ij} - \lambda K^2 \right)$$

c.f. $\lambda = 1$ for GR

- z=3 potential terms (6th spatial derivative)

$$\int N dt \sqrt{g}\, d^3x \Big[\quad D_i R_{jk} D^i R^{jk} \qquad D_i R D^i R$$

$$R_i^{\,j} R_j^{\,k} R_k^{\,i} \qquad R R_i^{\,j} R_j^{\,i} \qquad R^3 \qquad \Big]$$

c.f. $D_i R_{jk} D^j R^{ki}$ is written in terms of other terms

Relevant deformations (with parity)

- z=2 potential terms (4th spatial derivative)

$$\int N dt \sqrt{g} d^3x \left[\quad R_i^j R_j^i \quad\quad R^2 \quad \right]$$

- z=1 potential term (2nd spatial derivative)

$$\int N dt \sqrt{g} d^3x \left[\quad R \quad \right]$$

- z=0 potential term (no derivative)

$$\int N dt \sqrt{g} d^3x \left[\quad 1 \quad \right]$$

IR action with z=1

- **UV: z=3** , power-counting renormalizability

  ⬇ RG flow

- **IR: z=1** , seems to recover GR iff λ → 1

$$\frac{1}{16\pi G_N}\int N dt \sqrt{g}\, d^3x \left(\overbrace{K_{ij}K^{ij} - \lambda K^2}^{\text{kinetic term}} + \underbrace{c_g^2 R - 2\Lambda}_{\text{IR potential}} \right)$$

note:

Renormalizability has not been proved.

RG flow has not yet been investigated.

Different versions of HL gravity

- There are at least four versions of the theory: w/wo detailed balance & w/wo projectability.
- Horava's original proposal was with the projectability condition, $N=N(t)$, and with/without the detailed balance condition.
- **Non-projectable extension requires inclusion of $a_i = (\ln N)_{,i}$ in the action [Blas, Pujolas and Sibiryakov 2009].** Other wise, non-projectable theory is inconsistent [c.f. Henneaux, et.al. (2009)].
- In this talk I will consider the projectable case.

Projectability condition

- Infinitesimal tr. $\delta t = f(t)$, $\delta x^i = \zeta^i(t,x^j)$

$$\delta g_{ij} = \partial_i \zeta^k g_{jk} + \partial_j \zeta^k g_{ik} + \zeta^k \partial_k g_{ij} + f\dot{g}_{ij}$$

$$\delta N_i = \partial_i \zeta^j N_j + \zeta^j \partial_j N_i + \dot{\zeta}^j g_{ij} + \dot{f} N_i + f\dot{N}_i$$

$$\delta N = \zeta^i \partial_i N + \dot{f} N + f\dot{N}$$

- Space-independent N cannot be transformed to space-dependent N.
- N is gauge d.o.f. associated with the space-independent time reparametrization.
- It is natural to restrict N to be space-independent.
- Consequently, Hamiltonian constraint is an equation integrated over a whole space.

“Black holes” with N=N(t)?

- Schwarzschild BH in PG coordinate

$$ds^2 = -dt_P^2 + \left(dr \pm \sqrt{\frac{2m}{r}} dt_P \right)^2 + r^2 d\Omega^2$$

exact sol
for λ = 1

- Gaussian normal coordinate

$$ds^2 = -dt_G^2 + \cdots$$

approx sol
for λ = 1

Lemaitre reference frame
Doran coordinate

Dark matter as integration constant in Horava-Lifshitz gravity

arXiv:0905.3563 [hep-th]

See also arXiv:0906.5069 [hep-th]
Caustic avoidance in Horava-Lifshitz gravity

Structure of HL gravity

- Foliation-preserving diffeomorphism
  = 3D spatial diffeomorphism
  + space-independent time reparametrization
- 3 local constraints + 1 global constraint
  = 3 momentum @ each time @ each point
  + 1 Hamiltonian @ each time integrated
- Constraints are preserved by dynamical equations.
- We can solve dynamical equations, provided that constraints are satisfied at initial time.

FRW spacetime in HL gravity

- Approximates overall behavior of our patch of the universe inside the Hubble horizon.
- No “local” Hamiltonian constraint

  E.o.m. of matter

  → conservation eq.

$$\dot{\rho}_i + 3\frac{\dot{a}}{a}(\rho_i + P_i) = 0$$

- Dynamical eq

  can be integrated to give

  Friedmann eq with “dark matter as integration constant”

$$-2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} = 8\pi G_N \sum_{i=1}^{n} P_i$$

$$3\frac{\dot{a}^2}{a^2} = 8\pi G_N \left(\sum_{i=1}^{n} \rho_i + \frac{C}{a^3} \right)$$

IR limit of HL gravity

$$\frac{1}{16\pi G_N}\int Ndt\sqrt{g}d^3x\left(K_{ij}K^{ij}-\lambda K^2+R-2\Lambda\right)$$

- Looks like GR iff λ = 1. So, we assume that λ = 1 is an IR fixed point of RG flow.
- Global Hamiltonian constraint

$$\int d^3x\sqrt{g}(G^{(4)}_{\mu\nu}+\Lambda g^{(4)}_{\mu\nu}-8\pi G_N T_{\mu\nu})n^\mu n^\nu=0$$

$$n_\mu dx^\mu=-Ndt,\quad n^\mu\partial_\mu=\frac{1}{N}(\partial_t-N^i\partial_i)$$

- Momentum constraint & dynamical eq

$$(G^{(4)}_{i\mu}+\Lambda g^{(4)}_{i\mu}-8\pi G_N T_{i\mu})n^\mu=0$$

$$G^{(4)}_{ij}+\Lambda g^{(4)}_{ij}-8\pi G_N T_{ij}=0$$

Dark matter as integration constant

- Def. $T^{HL}{}_{\mu\nu}$ $$G^{(4)}_{\mu\nu} + \Lambda g^{(4)}_{\mu\nu} = 8\pi G_N \left(T_{\mu\nu} + T^{HL}_{\mu\nu} \right)$$
- General solution to the momentum constraint and dynamical eq.

$$T^{HL}_{\mu\nu} = \rho^{HL} n_\mu n_\nu \qquad n^\mu \nabla_\mu n_\nu = 0$$

- Global Hamiltonian constraint

$$\int d^3x \sqrt{g} \rho^{HL} = 0$$

ρ^{HL} can be positive everywhere in our patch of the universe inside the horizon.

- Bianchi identity → (non-)conservation eq

$$\partial_\perp \rho^{HL} + K \rho^{HL} = n^\nu \nabla^\mu T_{\mu\nu}$$

Micro to Macro

- Overall behavior of smooth $T^{HL}_{\mu\nu} = \rho^{HL} n_\mu n_\nu$ is like pressureless dust.
- Microscopic lumps (sequences of caustics & bounces) of ρ^{HL} can collide and bounce. (cf. early universe bounce [Calcagni 2009, Brandenberger 2009]) If asymptotically free, would-be caustics does not gravitate too much.
- Group of microscopic lumps with collisions and bounces → When coarse-grained, can it mimic a cluster of particles with velocity dispersion?
- Dispersion relation of matter fields defined in the rest frame of "dark matter"
  → Any astrophysical implications?

Summary so far

- Horava-Lifshitz gravity is power-counting renormalizable and can be a candidate theory of quantum gravity.
- While there are many fundamental issues to be addressed, it is interesting to investigate cosmological implications.
- The z=3 scaling solves horizon problem and leads to scale-invariant cosmological perturbations for $a \sim t^p$ with $p>1/3$.
- HL gravity does NOT recover GR at low-E but can instead mimic GR+CDM: "dark matter as an integral constant". Constraint algebra is smaller than GR since the time slicing and the "dark matter" rest frame are synchronized.

Strong coupling vs loss of predictability

- In low-E EFT (e.g. massive gravity and DGP gravity), strong coupling usually implies loss of predictability. Prediction requires knowledge of infinite number of terms, which we do not know.
- In HL gravity, if the theory is really renormalizable then all coefficients of infinite terms in perturbative expansion are written in terms of the 11 parameters of the theory. Therefore, the strong coupling itself does not imply loss of predictability.
- However, we need to see if the strongly coupled sector decouples from the other sector. This would be an **analogue of Vainshtein effect.**

Vainshtein effect in massive gravity

- Linearized analysis results in vDVZ discontinuity of the massless limit.
- However, perturbative expansion breaks down in this limit and cannot be trusted.
- Non-perturbative analysis shows continuity and GR is recovered in the massless limit.
- Continuity is not uniform as a function of distance. (e.g. 1/r expansion does not work.) However, Vainshtein radius can be pushed to infinity in the massless limit.

Linear instability of scalar graviton

- Sign of (time) kinetic term $(\lambda-1)/(3\lambda-1) > 0$.
- The dispersion relation in flat background
  $\omega^2 = c_s^2 k^2 \times [1+ O(k^2/M^2)]$ with $c_s^2 = -(\lambda-1)/(3\lambda-1) < 0$
  → IR instability in linear level
  (Wang&Maartens; Blas,et.al.; Koyama&Arroja 2009)
- Slower than Jeans instability of "DM as integration const" if
  $t_J \sim (G_N \rho)^{-1/2} < t_L \sim L/|c_s|$.
- Tamed by Hubble friction or/and $O(k^2/M^2)$ terms if
  $H^{-1} < t_L$ or/and $L < 1/M$.
- Thus, the linear instability does not show up if
  $|c_s| = |(\lambda-1)/(3\lambda-1)|^{1/2} < \mathrm{Max}\,[|\Phi|^{1/2}, HL]$. $(\Phi \sim -G_N \rho L^2)$
  for $L > \mathrm{Max}[0.01\mathrm{mm}, 1/M]$
  (Shorter scales → similar to spacetime foam)
- Phenomenological constraint on properties of RG flow.

Analogue of Vainshtein effect

- Breakdown of perturbation in the limit λ → 1

$$N = 1, \quad N_i = \partial_i B + n_i, \quad g_{ij} = e^{2\zeta}\left[e^h\right]_{ij}$$

$$B = \frac{3\lambda - 1}{\lambda - 1}\frac{\dot{\zeta}}{\partial^2}, \quad n_i = 0$$

momentum constraint

$$\begin{aligned} I_{kin} &= M_{Pl}^2 \int dt d^3\vec{x} \left\{ (1+3\zeta)\left[\frac{3\lambda-1}{\lambda-1}\dot{\zeta}^2 + \frac{1}{8}\dot{h}^{ij}\dot{h}_{ij}\right] \right. \\ & + \frac{1}{2}\zeta\partial^i(\partial_i B\partial^2 B + 3\partial^j B\partial_i\partial_j B) + \frac{1}{2}(\partial^k h_{ij}\partial_k B - 3\dot{h}_{ij}\zeta)\partial^i\partial^j B \\ & \left. - \frac{1}{4}(\dot{h}^{ij}\partial_k h_{ij})\partial^k B \right\} + O(\epsilon^4), \end{aligned}$$

- No negative power of (λ-1) in potential part
- Non-perturbative analysis is needed for scalar graviton sector!

Analogue of Vainshtein effect

- Spherically symmetric, static ansatz

$$N=1,\quad N_i dx^i = \beta(x)dx,\quad g_{ij}dx^i dx^j = dx^2 + r(x)^2 d\Omega_2^2$$

$$\Downarrow\quad R \equiv \beta^{(\lambda-1)/(2\lambda)} r$$ without HD terms

$$R'' + \frac{\lambda-1}{\lambda}\left[\frac{(3\lambda-1)(\beta')^2 R}{4\lambda^2\beta^2} + \frac{(\lambda-1)\beta' R'}{\lambda\beta} - \frac{(R')^2}{R}\right] = 0$$

$$\frac{\beta'}{\beta} - \frac{(\lambda-1)R}{4\lambda R'}\left(\frac{\beta'}{\beta}\right)^2 + \frac{\lambda}{RR'}\frac{\beta^{(\lambda-1)/\lambda} + [(2\lambda-1)\beta^2 - 1](R')^2}{(3\lambda-1)\beta^2 + (\lambda-1)} = 0$$

- Two branches

$$\frac{\beta'}{\beta} = \frac{1 \pm \sqrt{1+4AB}}{2A},$$

$$A \equiv \frac{(\lambda-1)R}{4\lambda R'},\quad B \equiv \frac{\lambda}{RR'}\frac{\beta^{(\lambda-1)/\lambda} + [(2\lambda-1)\beta^2 - 1](R')^2}{(3\lambda-1)\beta^2 + (\lambda-1)}$$

- "-" branch recovers GR in the $\lambda \rightarrow 1$ limit

Analogue of Vainshtein effect

$$\frac{\beta'}{\beta} = \frac{1 \pm \sqrt{1+4AB}}{2A}, \quad \Rightarrow \text{choose the “-” branch}$$

$$A \equiv \frac{(\lambda-1)R}{4\lambda R'}, \quad B \equiv \frac{\lambda}{RR'} \frac{\boxed{\beta^{(\lambda-1)/\lambda}} + [(2\lambda-1)\beta^2 - 1](R')^2}{\boxed{(3\lambda-1)\beta^2 + (\lambda-1)}}$$

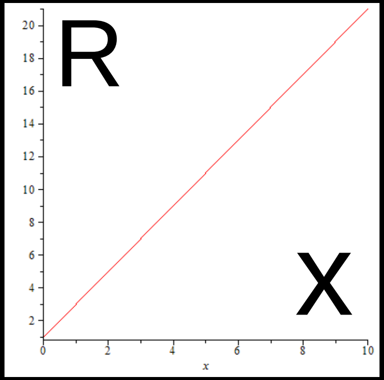
- $(3\lambda-1)\beta^2 \ll (\lambda-1)$
  perturbative regime, 1/r expansion
- $(3\lambda-1)\beta^2 \gg (\lambda-1)$
  non-perturvative regime, recovery of GR
- $(3\lambda-1)\beta^2 \sim (\lambda-1)$ with $\beta^2 \sim r_g/r$ → $r \sim r_g/(\lambda-1)$
  analogue of Vainshtein radius???

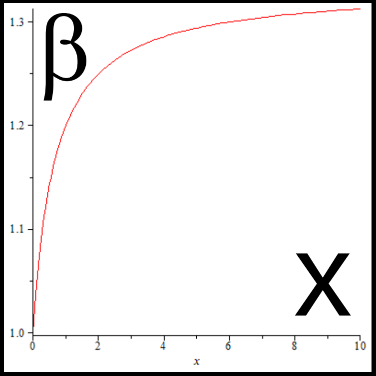
Izumi & Mukohyama 2009
“Steller center is dynamical”

Analogue of Vainshtein effect

- Numerical integration in the “-” branch with $\beta(x=0)=1$, $r(x=0)=1$, $r'(x=0)$ given

for
$\lambda-1=10^{-6}$
$r'(x=0)=2$



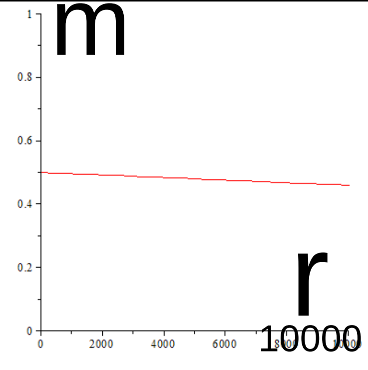


- Misner-Sharp energy

$$m \equiv \frac{r}{2}\left[1-\left(1-\beta^2\right)\left(r'\right)^2\right]$$

almost constant

GR is recovered!



Nonlinear cosmological perturbation and $\lambda \to 1$

arXiv: 1105.0246 [hep-th] with K.Izumi

- HL gravity @ IR → GR + DM [Mukohyama 2009]
- $^{\exists}$Subtleties with $\lambda \to 1$
- Nonlinear cosmological perturbation in vacuum HL gravity
- Gradient expansion up to any order
- No problem with $\lambda \to 1$
- Recovers GR+CDM with $\lambda \to 1$ @ low E

Caustic avoidance

JCAP 0909:005,2009

- In GR, congruence of geodesics forms caustics because gravity is attractive.
- HL gravity is repulsive at short distances, due to higher curvature (HC) terms. (c.f. bouncing FRW universe)
- With codimension 2 and 3, HC terms can bounce would-be caustics.
- With codimension 1, deviation of λ from 1 is also needed to bounce would-be caustics.

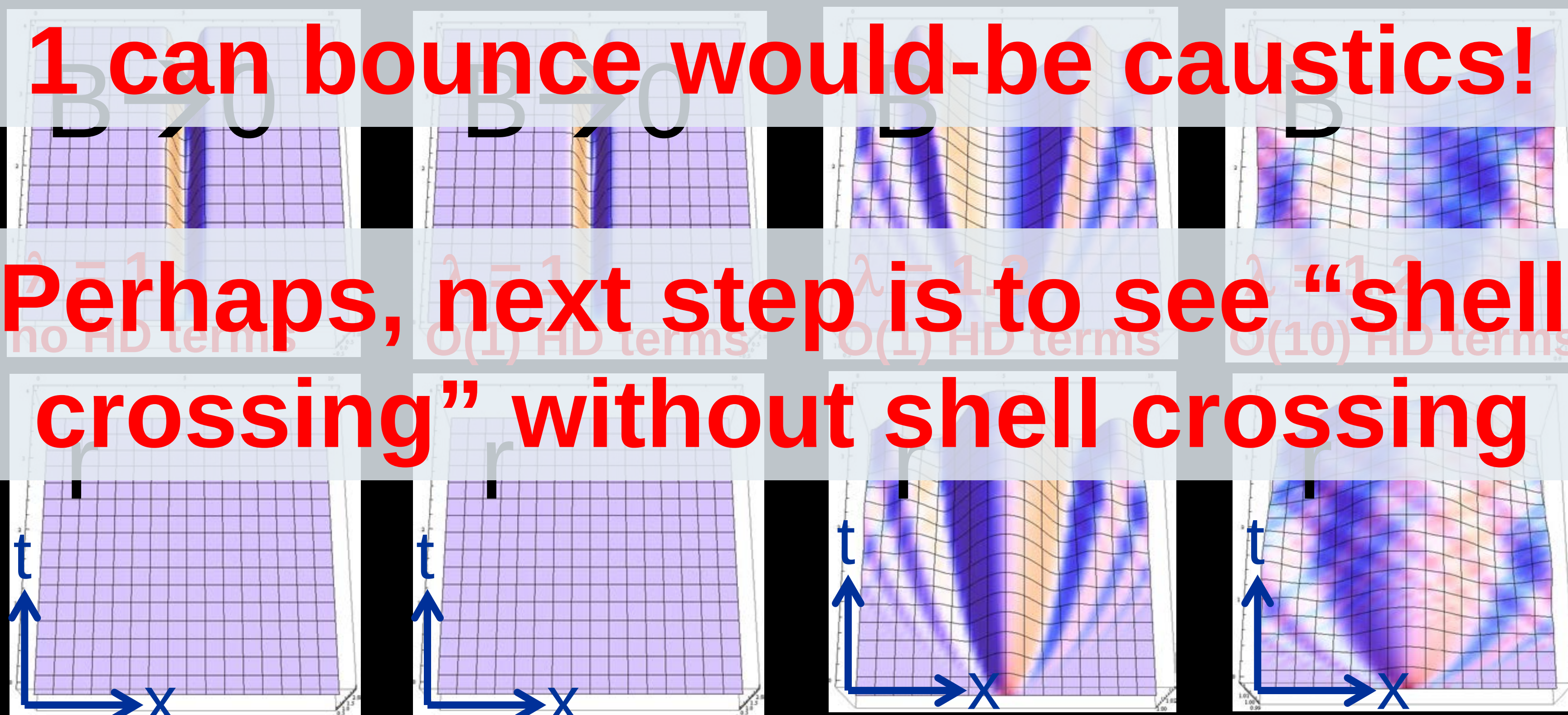
Caustic avoidance

(preliminary)

$$N = 1 \qquad N_i = 0$$

$$ds^2 = B(t,x)dx^2 + r^2(t,x)(dy^2 + dz^2)$$

HC terms and deviation of λ from 1 can bounce would-be caustics!



Perhaps, next step is to see "shell crossing" without shell crossing

What happens in the UV?

arXiv: 1104.2087 [hep-th] with E.Gumrukcuoglu

- $1/3 < \lambda < 1$ is forbidden because of ghost
- Recovery of GR requires $\lambda \rightarrow 1+0$ in the IR
- A natural candidate for the UV fixed point would be $\lambda \rightarrow \infty$
- Regular and simpler dynamics with $\lambda \rightarrow \infty$
- Weakly coupled if $M_{z=3} \ll M_{Pl}$.

Summary

- Horava-Lifshitz gravity is power-counting renormalizable and can be a candidate theory of quantum gravity.
- While there are many fundamental issues to be addressed, it is interesting to investigate cosmological implications.
- The z=3 scaling solves horizon problem and leads to scale-invariant cosmological perturbations for $a \sim t^p$ with $p>1/3$.
- HL gravity does NOT recover GR at low-E but can instead mimic GR+CDM: "dark matter as an integral constant". Constraint algebra is smaller than GR since the time slicing and the "dark matter" rest frame are synchronized.
- For spherically-symmetric, static, vacuum configurations, GR is recovered in the limit $\lambda \to 1$ non-perturbatively.
  → analogue of Vainshtein effect
- For superhorizon cosmological perturbations, GR + DM is recovered in the limit l → 1 non-perturbatively.
- Caustics avoidance requires higher curvature terms and deviation of λ from 1 in the UV.
- A natural candidate for the UV fixed point would be $\lambda \to \infty$.

Future works

- Renormalizability beyond power-counting
- RG flow: is λ = 1 an IR fixed point ? Does it satisfy the stability condition for the scalar graviton?
  ($|c_s| < \mathrm{Max}\,[|\Phi|^{1/2}, HL]$ for $L > \mathrm{Max}[M^{-1}, 0.01\mathrm{mm}]$)
- Can we get a common "limit of speed" ?
  (i) $M_{z=3} << M_{pl}$, (ii) supersymmetry, (iii) other ideas?
- How generic is 'Vainshtein effect'?
- How generic is caustic avoidance, (perhaps with $\lambda \to \infty$ & $M_{Pl}/M_{z=3} \to \infty$) ?
- Micro & macro behavior of "CDM"
- Adiabatic initial condition for "CDM" from the z=3 scaling
- Spectral tilt from anomalous dimension