Introduction to Riemannian geometry, curvature and Ricci flow, with applications to the topology of 3-dimensional manifolds. Part 2.

The second lecture of the mini-course concentrates on Ricci flow and geometrization. Geometrization is a topological classification of 3-dimensional manifolds based on homogeneous spaces. In dimension 2 every closed manifold carries a metric of constant curvature but in higher dimensions the situation is more complicated. Thurston conjectured that every 3-dimensional manifold can be decomposed in a certain way into pieces each of which has a geometric structure of one of the 8 types. Perelman proved this conjecture using the Ricci flow technique introduced by Hamilton. The Ricci flow equation features short-time existence of solutions but they can develop singularities over time. The key ingredient of Perelman's proof is understanding the metric structure near these singularities.