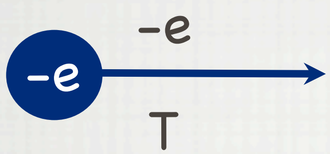
מכון ויצמן למדע
WEIZMANN INSTITUTE OF SCIENCE

Probing interactions with thermal transport

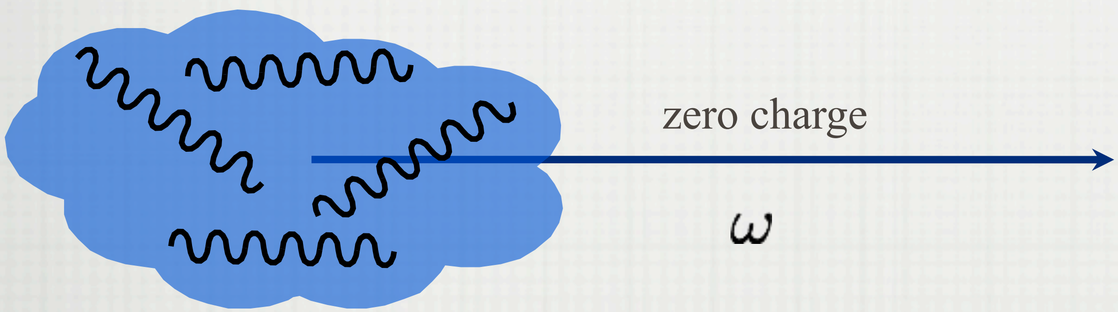
Karen Michaeli

Quasiparticles:



$$\sigma = \frac{e^2 n\tau}{m} \qquad \kappa = \frac{\pi^2 k_B^2 n\tau}{3m} T$$

Interactions:



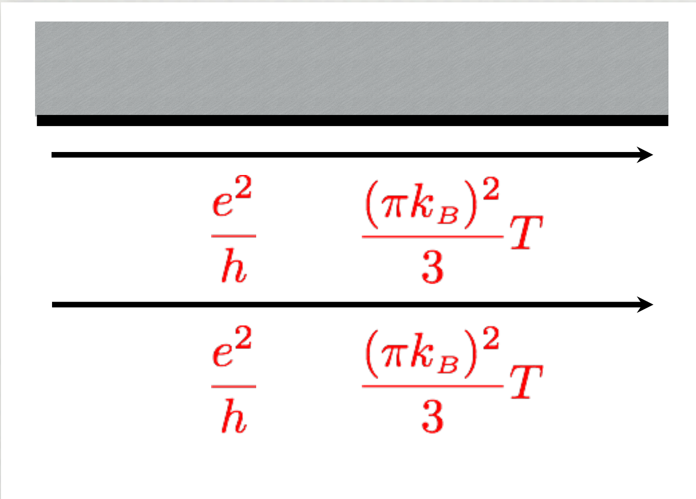
$$\sigma = 0$$
$$\kappa \neq 0$$

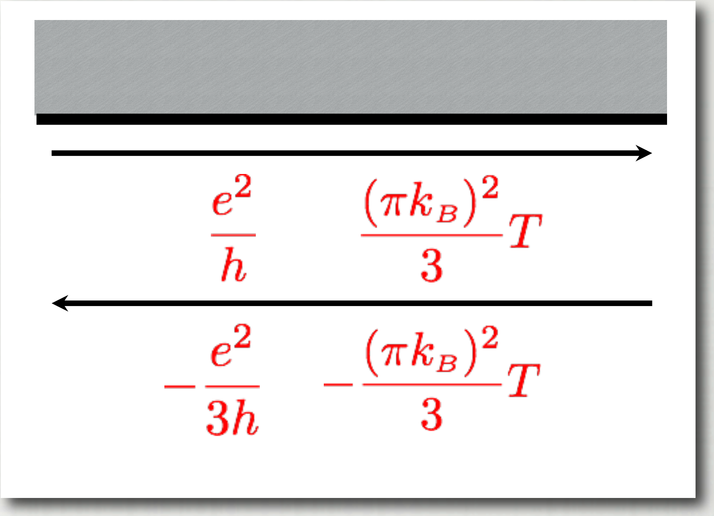
In conventional metals (Fermi-liquids), the contribution of interactions can be neglected

$$L = \frac{\pi^2 k_B^2}{3e^2} = \frac{\kappa}{\sigma T}$$

quantum Hall effect

Kane and Fisher, 1997





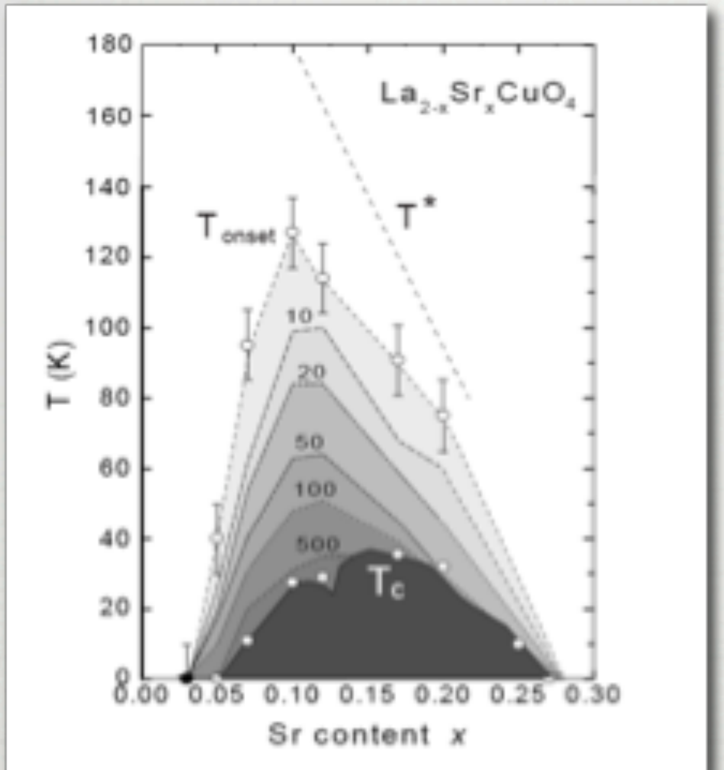
$$\nu = 2$$

$$\nu = 2/3$$

$$\sigma_H = 2\frac{e^2}{h} \qquad \kappa_H = 2\frac{(\pi k_B)^2}{3}T$$

$$\sigma_H = \frac{2e^2}{3h} \qquad \kappa_H = 0$$

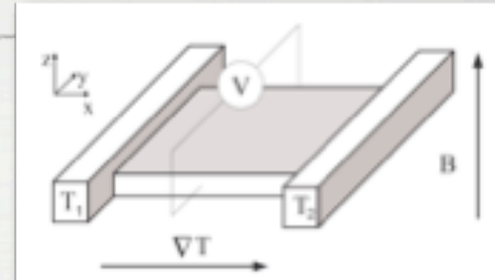
Superconductors

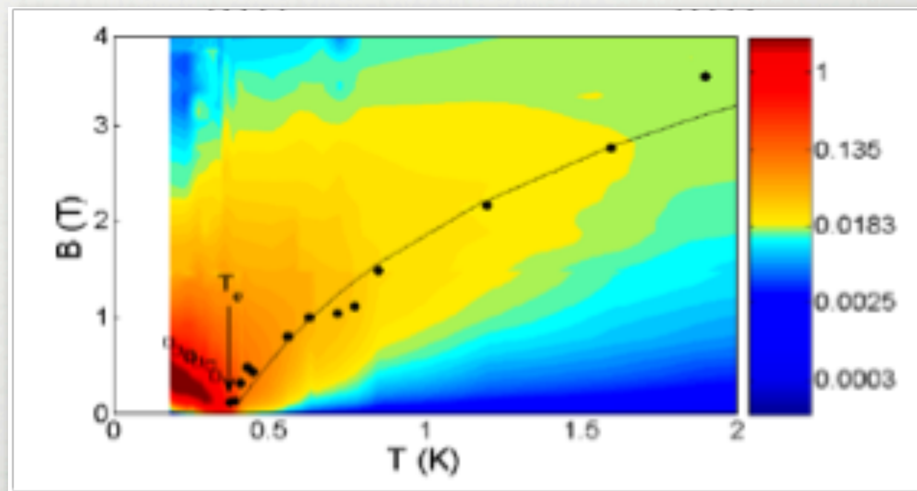


Y. Wang, et al, 2005

The Nernst signal

$$\nu = \frac{E_y}{-\nabla_x T \cdot B}$$



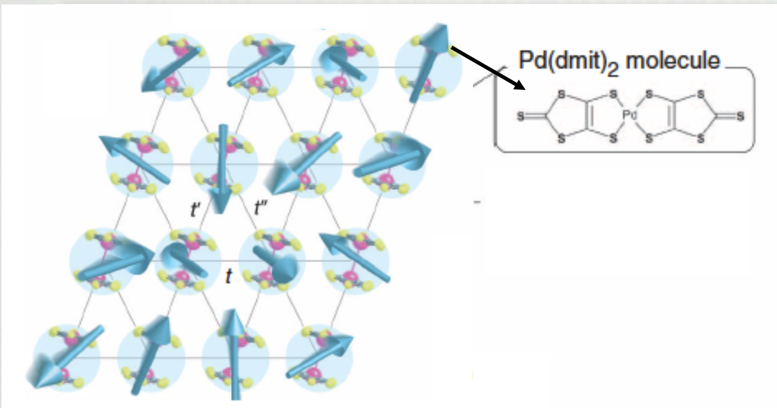


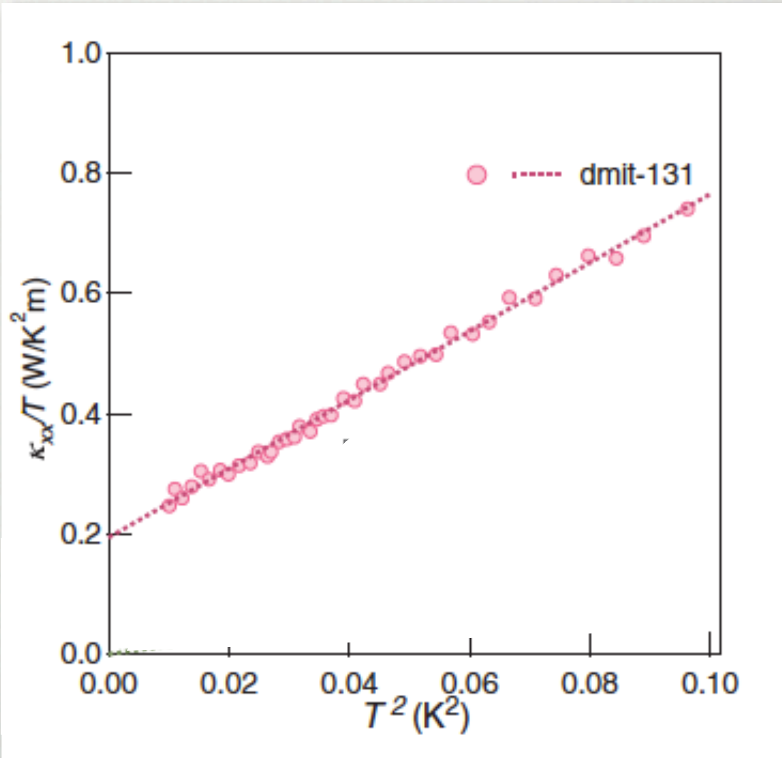
Nb$_{0.15}$Si$_{0.85}$

Pourret, et al, 2007

FRUSTRATED MAGNETS

$Pd(dmit)_2(EtMe_3Sb)$





$$\kappa = \lambda T + \beta T^3$$

M. Yamashita, et al, 2010

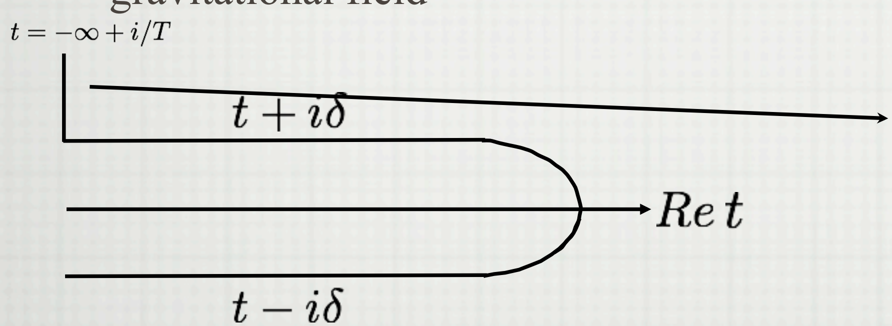
Quantum Kinetic approach

KM and A. M. Finkel’stein, PRB 80, 115111 (2009)

The derivation consists of two main stages:

1. Derive the quantum kinetic equation for the Green functions in the presence of temperature gradient -

use Einstein-like construction to relate between a temperature gradient and a gravitational field



$$\hat{\rho}=\exp\left\{-i\int d\mathbf{r}\int_{-\infty+i\beta(\mathbf{r})}^{-\infty+i\delta}dt\left[h(\mathbf{r})-\mu n(\mathbf{r})\right]\right\}$$

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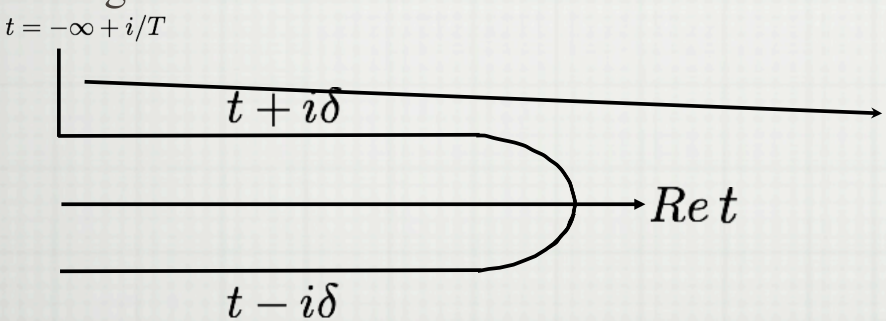

$$\hat{\rho} = \exp\left\{-i\int d\mathbf{r} \int_{-\infty+i\beta(\mathbf{r})}^{-\infty+i\delta} dt'\,\gamma(\mathbf{r})\left[h(\mathbf{r}) - \mu n(\mathbf{r})\right]\right\}$$

Quantum Kinetic approach

KM and A. M. Finkel'stein, PRB 80, 115111 (2009)

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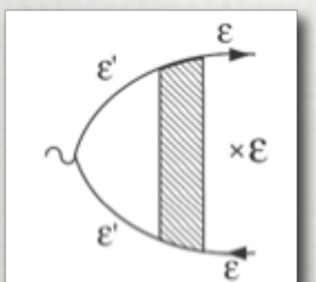
- Extract the expression for the currents from the continuity equation:



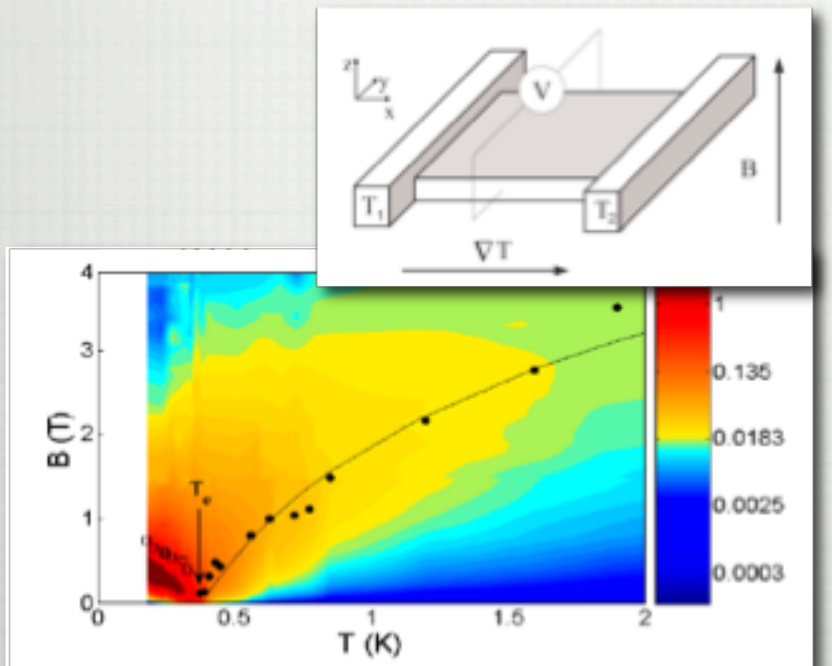
$$\vec{j}_{e,h} = -i \int \frac{d\epsilon}{2\pi} \zeta_{e,h}(\epsilon) \left[\hat{\vec{v}} \hat{G}_{e,h}(\epsilon) \right]^{<}$$

$$\zeta_e = -e \qquad \zeta_h = \epsilon$$

The renormalized velocity and Green's function

this talk

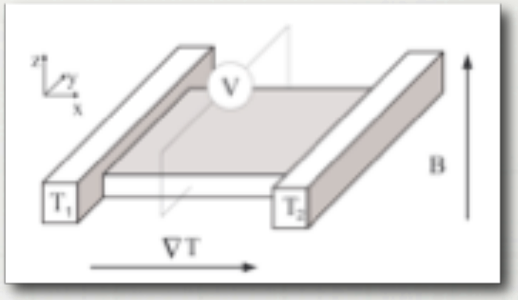
Nernst effect near the superconducting phase transition



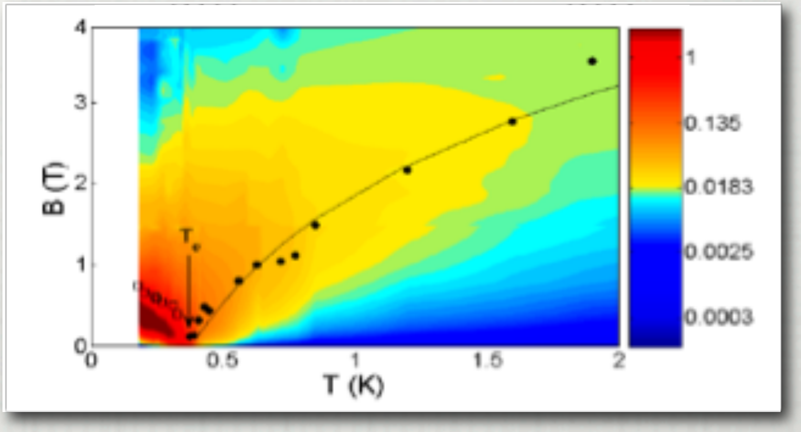
Nernst effect - superconductors

$$\begin{pmatrix} j_e \\ j_h \end{pmatrix} = \begin{pmatrix} \sigma & \alpha \\ \tilde{\alpha} & \kappa \end{pmatrix} \begin{pmatrix} E \\ -\nabla T \end{pmatrix}$$

$$e_N = \frac{E_y}{-\nabla_x T} = \alpha_{xy}\rho_{xx} + \cancel{\alpha_{xx}\rho_{xy}}$$



* ~~Quasiparticles~~
* ~~Neutral modes~~
* Superconducting fluctuations



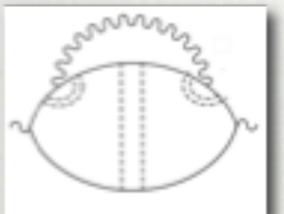
TRANSVERSE PELTIER COEFFICIENT

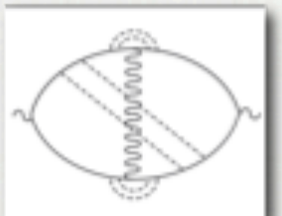
KM and A. M. Finkel'stein, EPL 86, 27007 (2009)
KM and A. M. Finkel'stein, PRB 80, 214516 (2009)

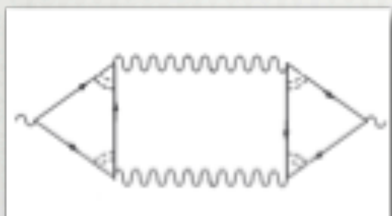
$$\vec{J}_e = \vec{J}_e^{\,\mathrm{con}} + \vec{\nabla} \times \vec{M}$$

$$\alpha_{xy}^{mag} = -\frac{\partial}{\partial B}\frac{eB}{\pi}\sum_{N=0}^{\infty}\sum_{\omega_m}\ln\left[\ln\frac{T}{T_c}+\psi\left(\frac{1}{2}+\frac{|\omega_m|+\Omega_c(N+1/2)}{4\pi T}\right)-\psi\left(\frac{1}{2}\right)\right]$$

$$\Omega_c = \frac{4eDB}{c}$$







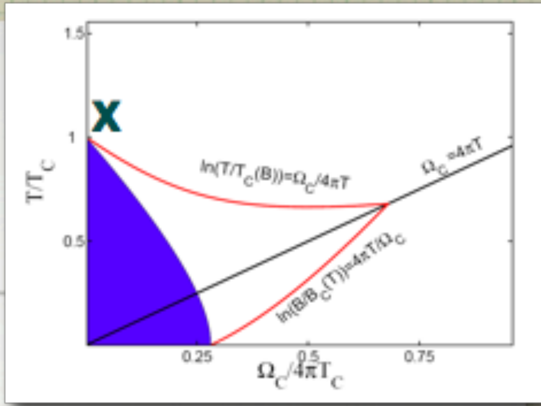
The quantum kinetic equation ≠ diagrammatic method. The contributions to α_{xy} can be interpreted in terms of diagrams

The Peltier coefficient is related to the flow of entropy

According to the third law of thermodynamics

$$\alpha \xrightarrow[T\to 0]{} 0$$

NERNST EFFECT - T ≈T_c



$$\Omega_C << T \qquad \ln\frac{T}{T_C} << 1$$

$$\Omega_c = \frac{4eDB}{c}$$

$$\alpha_{xy} \approx \frac{e\Omega_C}{192T\ln\left(T/T_C(H)\right)}$$

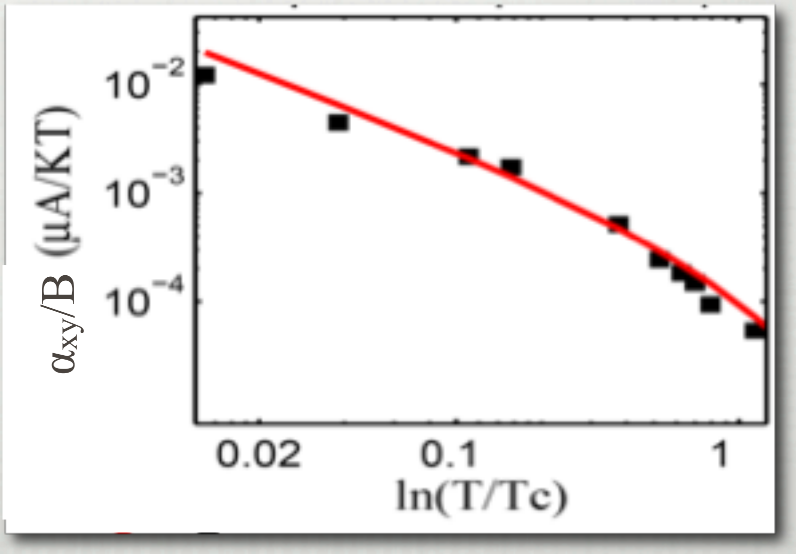
Classical fluctuations – coincide with the phenomenological result of Ussishkin, Sondhi and Huse, 2002

Experimental data from Pourret, et al, 2007

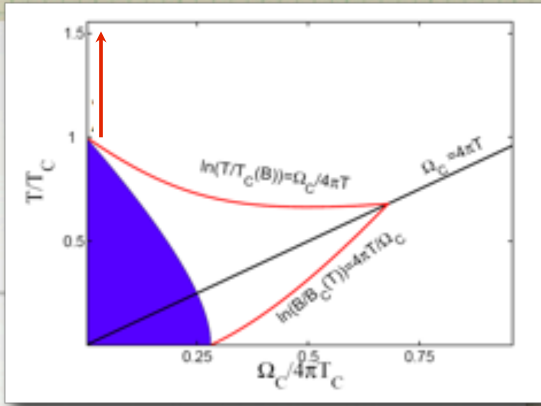
film thickness $35nm$

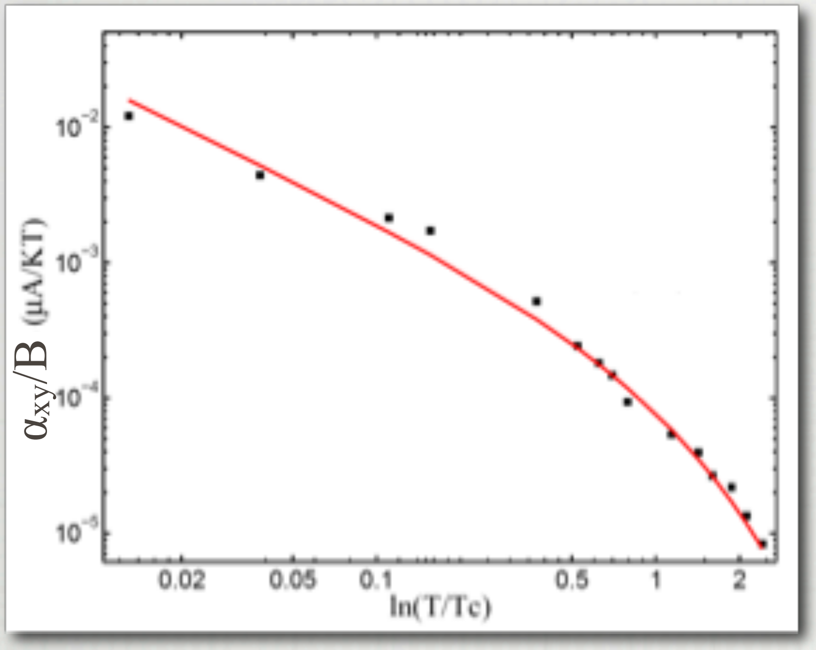
$T_C = 380mK$

$D = 0.187\, cm^2/\sec$



NERNST EFFECT - T ≫ T_c





$$\alpha_{xy}^{con}\ ,\ \alpha_{xy}^{mag}\ \sim\ \ln\left(\ln\frac{1}{T\tau}\right)-\ln\left(\ln\frac{T}{T_C}\right)$$

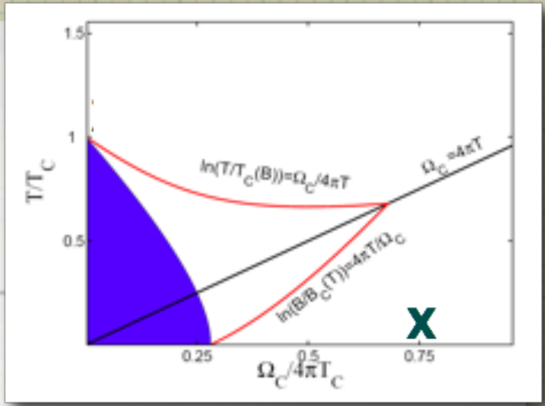
The logarithmically divergent terms are canceled out

$$\Omega_C << T \qquad \ln\frac{T}{T_C} >> 1$$

Trace of the third law of thermodynamics

$$\alpha_{xy} \approx \frac{e\Omega_C}{24\pi^2 T\ln(T/T_C)}$$

NERNST EFFECT - T → 0



$$\Omega_C >> T$$

$$\alpha_{xy}^{con}\ ,\ \alpha_{xy}^{mag}\ \sim \frac{\Omega_c}{T}\left[\ln\left(\ln\frac{1}{T\tau}\right)-\ln\left(\ln\frac{T}{T_c}\right)\right]$$

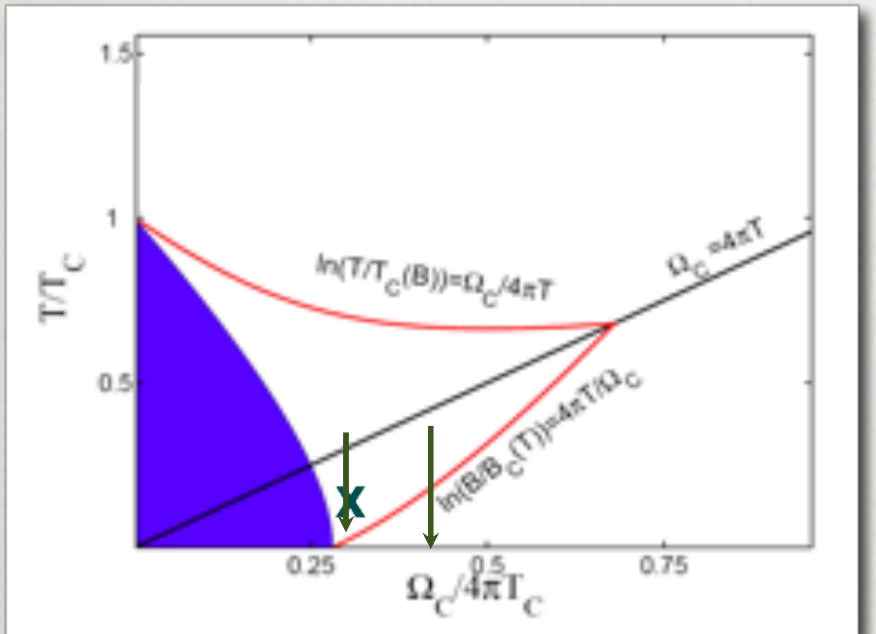
These diverging terms cancel

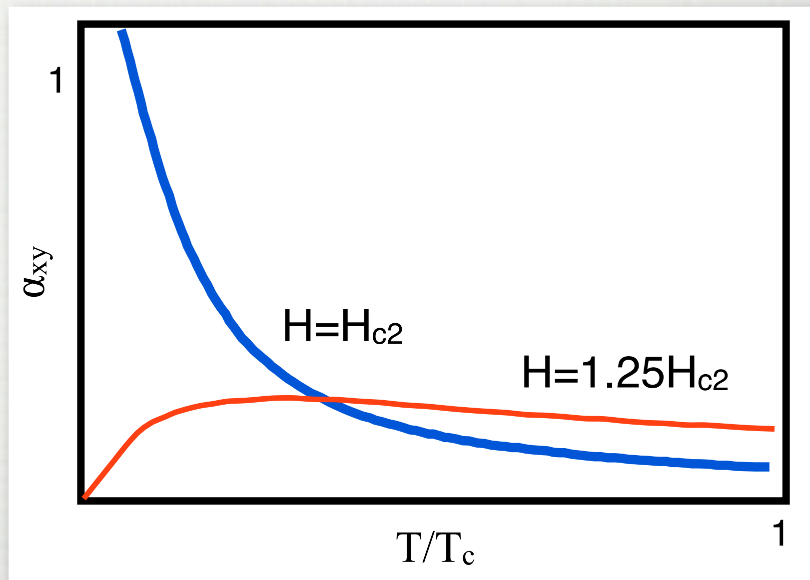
The remaining terms goes to zero at T→0

$$\alpha_{xy} \approx -\frac{eT\ln 3}{3\Omega_C \ln^2\left(H / H_C(T)\right)} \qquad \ln\frac{H}{H_{C2}} << 1$$

QUANTUM CRITICAL BEHAVIOR

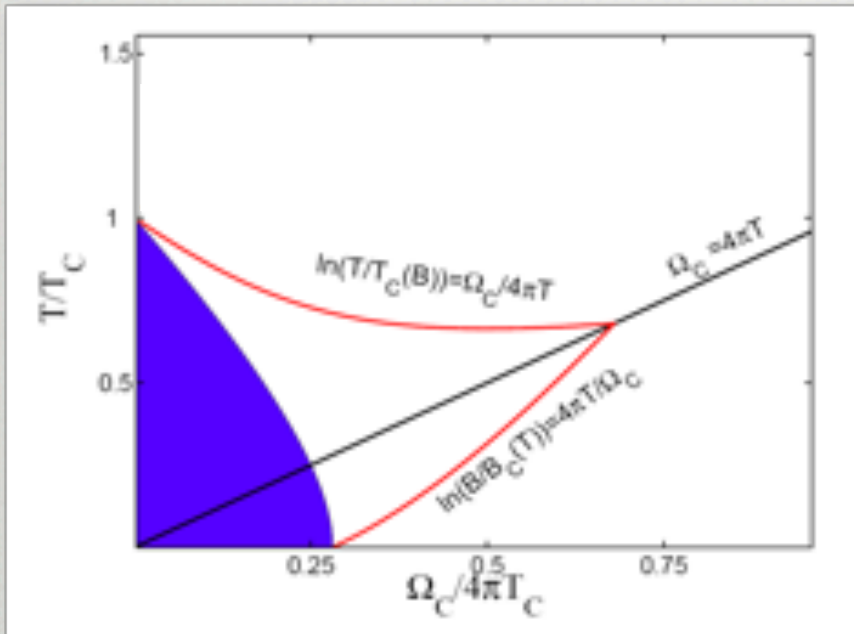
KM and A. M. Finkel'stein, in preparation





$$\ln\left(\frac{H}{H_c(T)}\right) < \frac{T}{\Omega_c} \qquad \alpha_{xy} \approx -\frac{e\ln 3}{2\pi\ln\left(H / H_c(T)\right)}$$

QUANTUM CRITICAL BEHAVIOR



$$\sigma_{xx} \sim \frac{T}{ln(H/H_c(T))}$$

V. M. Galitski, and A. I. Larkin, PRB, 2001.

Similar critical behavior is observed in occurs in the Hall conductivity

$$\ln\left(\frac{H}{H_C(T)}\right) < \frac{T}{\Omega_C} \qquad \alpha_{xy} \approx -\frac{e\ln 3}{2\pi \ln\left(H/H_C(T)\right)}$$

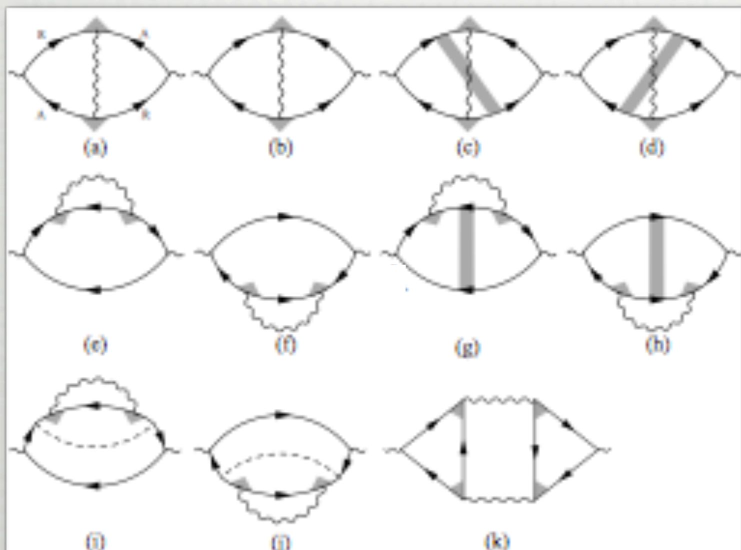
QUANTUM CRITICAL BEHAVIOR

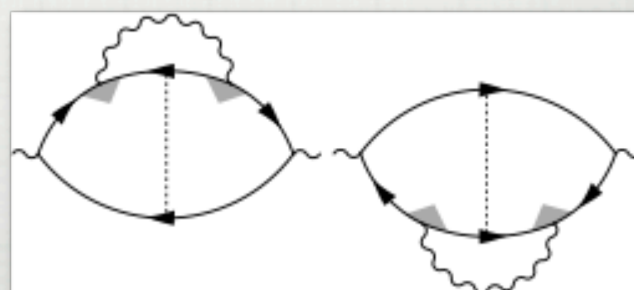
For the Hall effect, the curvature of the electronic spectrum needs to be considered

$$L_{R,A}(\mathbf{q},\omega) = -\frac{1}{\nu}\left[\ln\left(\frac{T}{T_c}\right) + \psi\left(\frac{1}{2} + \frac{\mp i\omega + Dq^2}{4\pi T}\right) - \psi\left(\frac{1}{2}\right) + \alpha\omega\right]^{-1}$$

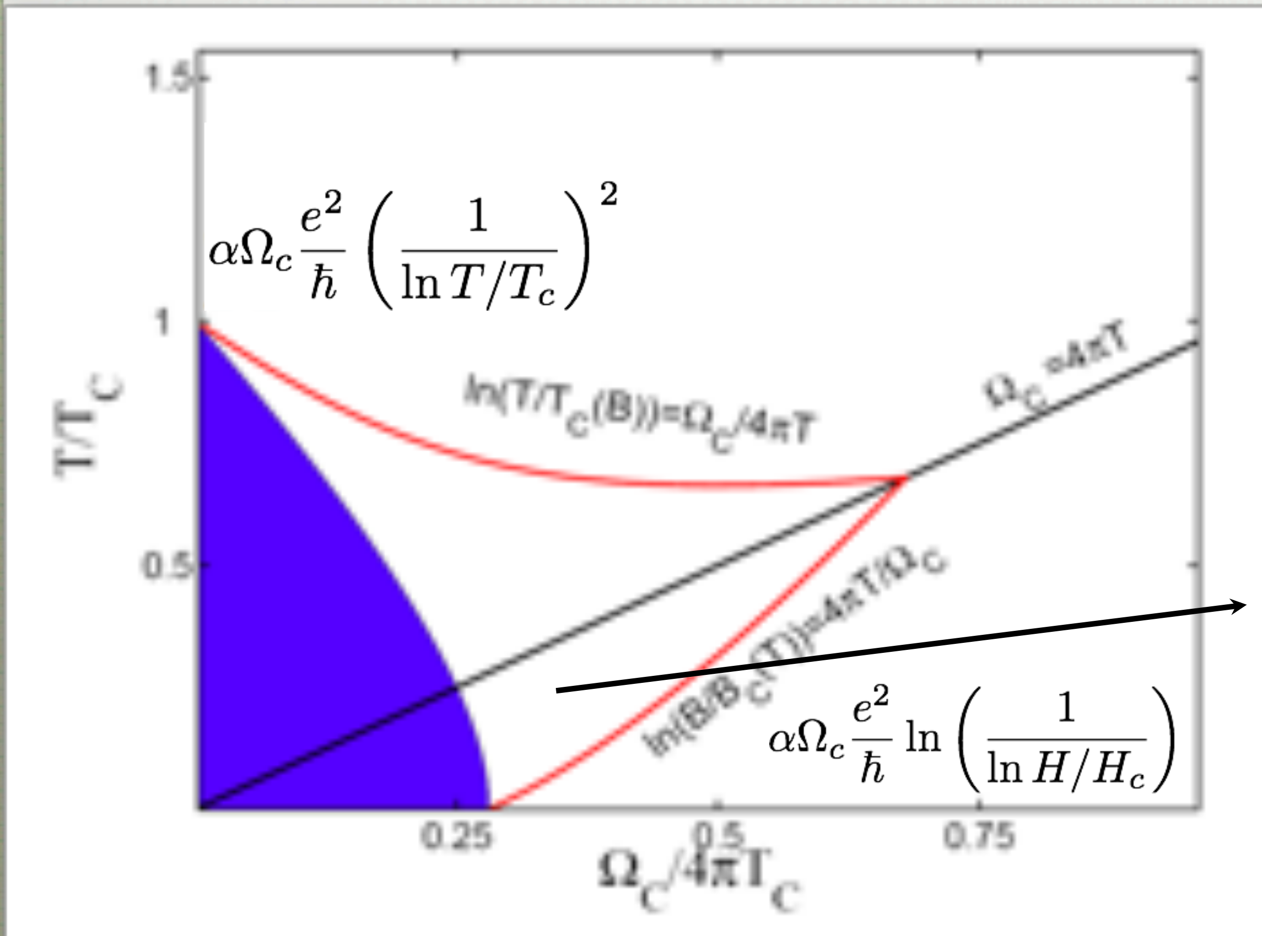
the particle-hole asymmetry parameter
(pair-breaking term)

$\alpha \sim 1/\varepsilon_F$





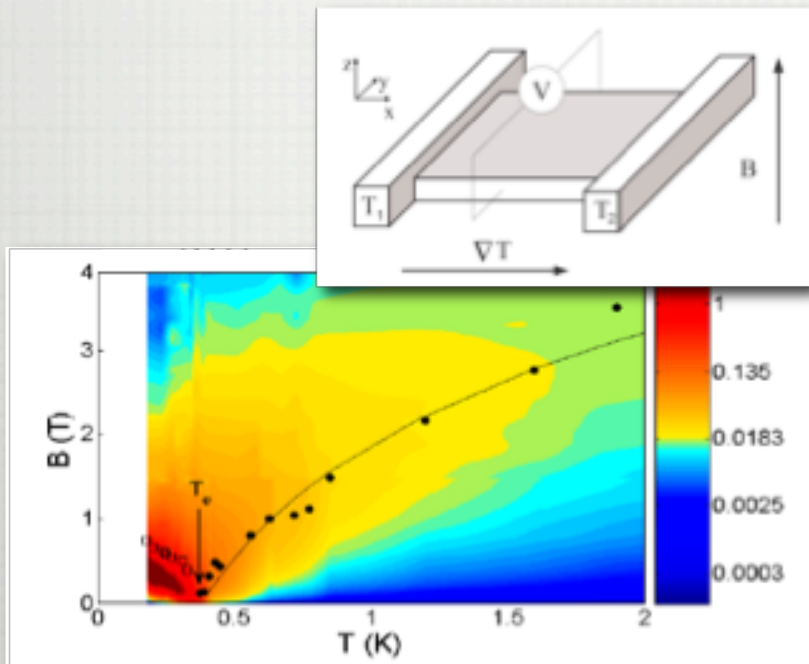
QUANTUM CRITICAL BEHAVIOR



$$\alpha T \frac{e^2}{\hbar} \frac{1}{\ln H/H_c(T)}$$

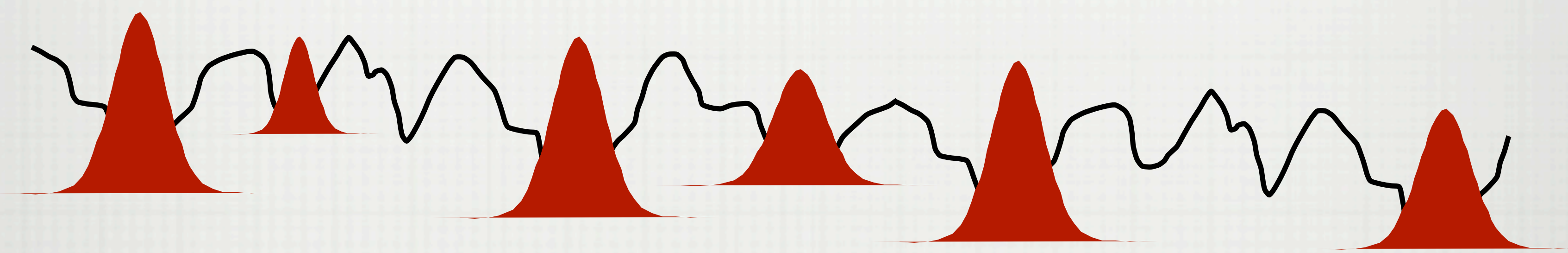
this talk

Nernst effect near the superconducting phase transition



THERMAL TRANSPORT IN INSULATORS

Anderson localization with interactions

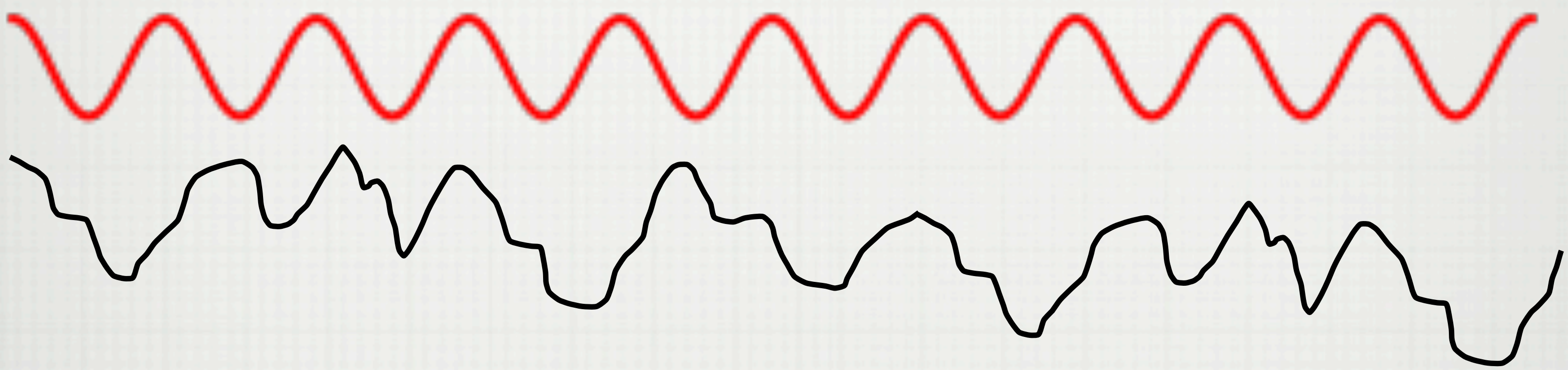


$$\sigma = 0$$

$$\kappa = \ ?$$

THERMAL TRANSPORT IN INSULATORS

Metallic side:



$$\kappa = L_0 T \sigma +$$ Terms violating the Wiedemann-Franz law?

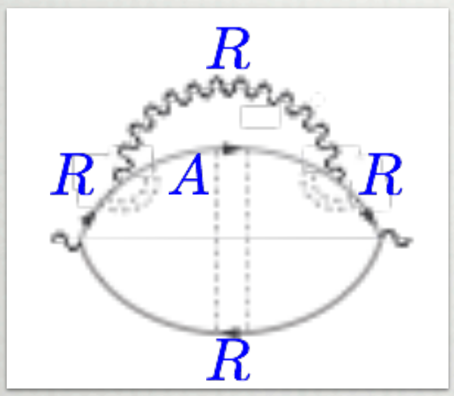
THE FATE OF THE WIEDEMANN-FRANZ LAW

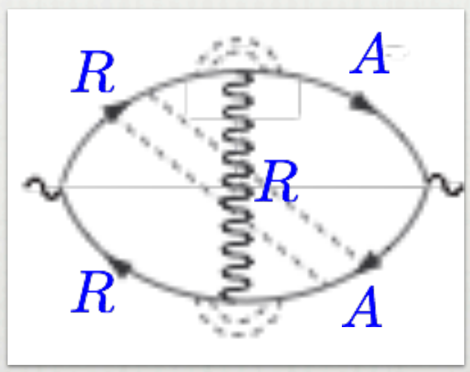
$$G^{<}_{\mathbf{E}}(\epsilon) = -\frac{ie\mathbf{E}}{2}\left[\frac{\partial \hat{g}_{eq}(\epsilon)}{\partial \epsilon} v_{eq}(\epsilon)\hat{g}_{eq}(\epsilon) - \hat{g}_{eq}(\epsilon) v_{eq}(\epsilon) \partial \hat{g}_{eq}(\epsilon) \partial \epsilon\right]^{<}$$

$$+\left[\hat{g}_{eq}(\epsilon)\Sigma(\hat{G}_{\mathbf{E}})\hat{g}_{eq}(\epsilon)\right]^{<}$$

Renormalization of the self-energy and velocity

Renormalization of the transport scattering rate $1/\tau_{tr}$ - the collision integral





THE FATE OF THE WIEDEMANN-FRANZ LAW

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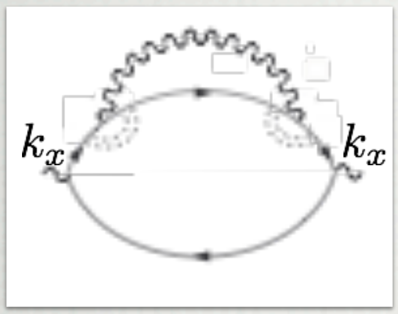
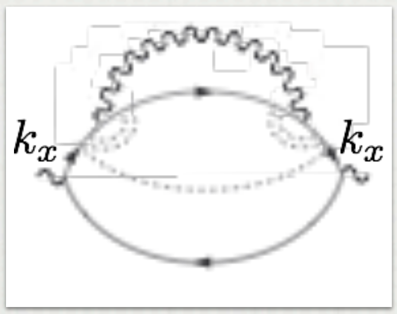
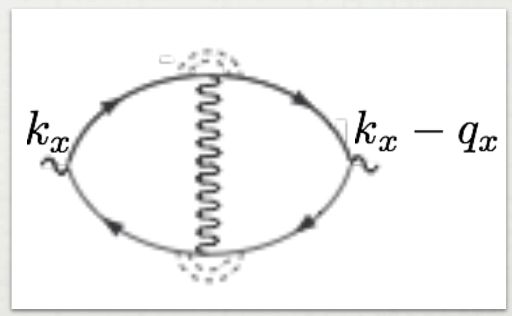
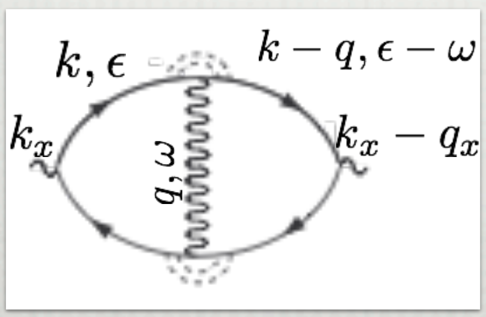
$$+\left[\hat{g}_{eq}(\epsilon)\Sigma(\hat{G}_{\mathbf{E}})\hat{g}_{eq}(\epsilon)\right]^{<}$$





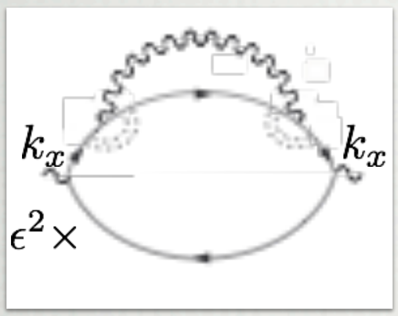


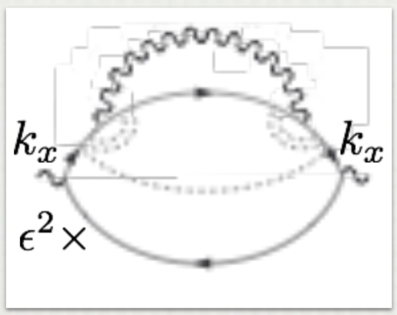


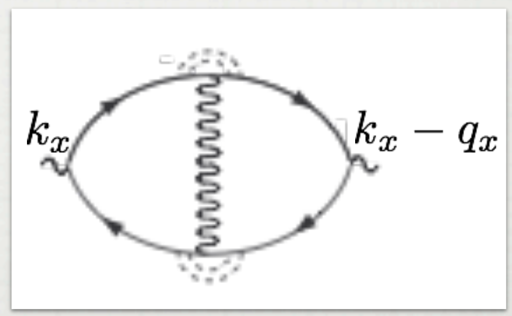
THE FATE OF THE WIEDEMANN-FRANZ LAW

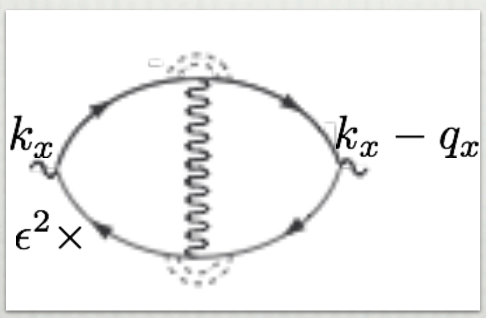
$$G^{<}_{\nabla T}(\epsilon) = -\frac{i\epsilon\nabla T}{2T}\left[\frac{\partial \hat{g}_{eq}(\epsilon)}{\partial \epsilon} v_{eq}(\epsilon)\hat{g}_{eq}(\epsilon) - \hat{g}_{eq}(\epsilon) v_{eq}(\epsilon)\partial\hat{g}_{eq}(\epsilon)\partial\epsilon\right]^{<}$$

$$+\left[\hat{g}_{eq}(\epsilon)\Sigma(\hat{G}_{\mathbf{E}})\hat{g}_{eq}(\epsilon)\right]^{<}$$





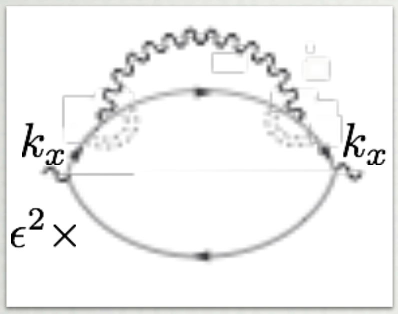


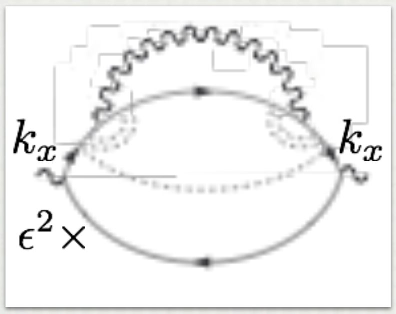


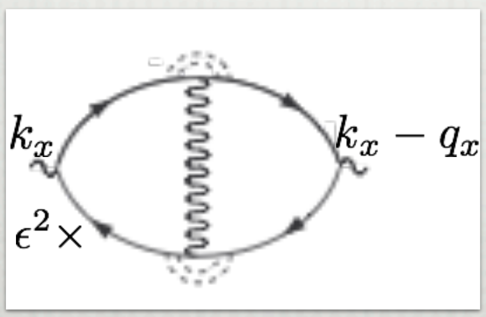
THE FATE OF THE WIEDEMANN-FRANZ LAW

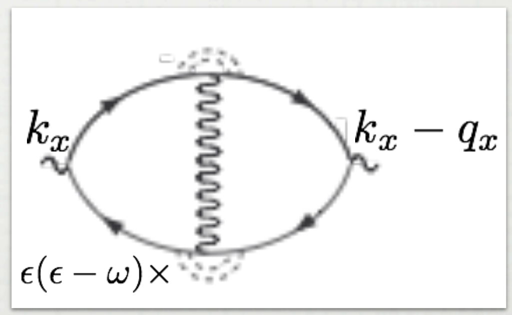
$$G^{<}_{\nabla T}(\epsilon) = -\frac{i\epsilon\nabla T}{2T}\left[\frac{\partial \hat{g}_{eq}(\epsilon)}{\partial \epsilon} v_{eq}(\epsilon)\hat{g}_{eq}(\epsilon) - \hat{g}_{eq}(\epsilon) v_{eq}(\epsilon) \partial \hat{g}_{eq}(\epsilon) \partial \epsilon\right]^{<}$$

$$+ \left[\hat{g}_{eq}(\epsilon)\Sigma(\hat{G}_{\nabla T})\hat{g}_{eq}(\epsilon)\right]^{<}$$

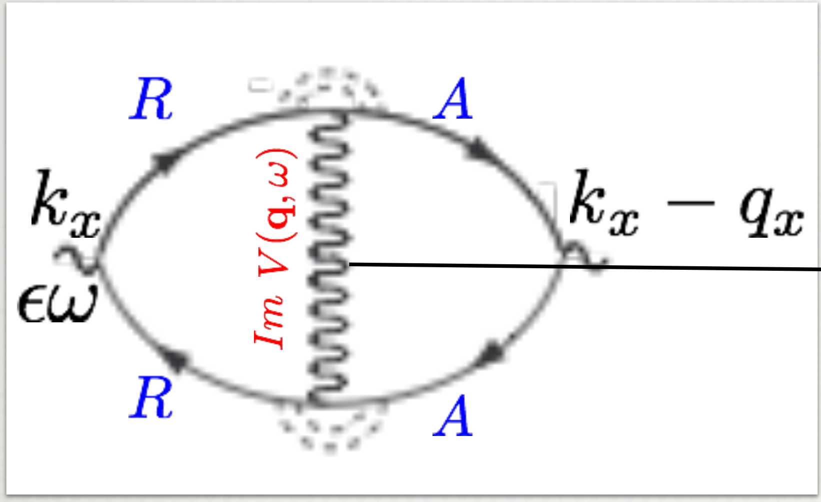








THE COLLISION INTEGRAL



The interaction propagator is taken on the mass shell

The integration over ω is from 0 to T

$$\omega/D\ell_{screen} < q < \sqrt{\omega/D}$$

$$\ell_{screen} = 2\pi e^2 \nu$$

$$\boxed{\delta\kappa_{non-WF} = \frac{T}{12} \ln\left(\frac{D\kappa^2_{screen}}{T}\right).}$$

This term is not renormalized in the usual RG scheme of the metal insulator transition

GAUGE TRANSFORMATION

$$S = \int d\mathbf{r}dt\gamma(\mathbf{r})\left\{\psi^\dagger(\mathbf{r},t)\frac{i\partial_t}{\gamma(\mathbf{r})}\psi(\mathbf{r},t) - \frac{(\nabla\psi^\dagger(\mathbf{r},t))(\nabla\psi(\mathbf{r},t))}{2m} - \phi(t)\psi^\dagger(\mathbf{r},t)\psi(\mathbf{r},t) + \frac{\phi^2(t)}{2U}\right\}$$

$$\gamma(\mathbf{r}) = 1 + \frac{\mathbf{r}\cdot\nabla T}{T}$$

$$\psi(\mathbf{r},t) \to e^{-i\int^t dt'\phi(t')}\psi(\mathbf{r},t)$$

$$S = \int d\mathbf{r}dt\gamma(\mathbf{r})\left\{\psi^\dagger(\mathbf{r},t)\frac{i\partial_t + \phi(t)}{\gamma(\mathbf{r})}\psi(\mathbf{r},t) - \frac{(\nabla\psi^\dagger(\mathbf{r},t))(\nabla\psi(\mathbf{r},t))}{2m} - \phi(t)\psi^\dagger(\mathbf{r},t)\psi(\mathbf{r},t) + \frac{\phi^2(t)}{2U}\right\}$$

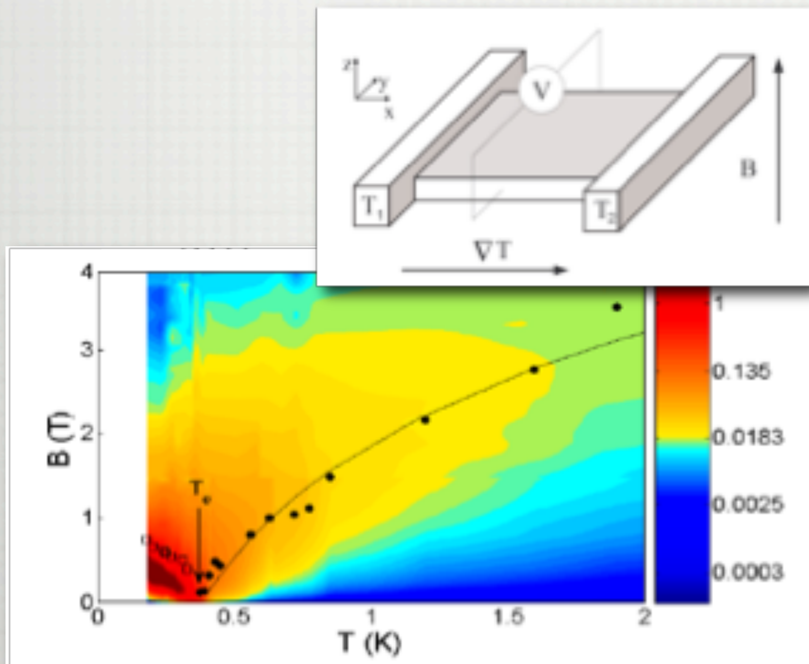
$$S = \int d\mathbf{r}dt\gamma(\mathbf{r})\left\{\psi^\dagger(\mathbf{r},t)\frac{i\partial_t}{\gamma(\mathbf{r})}\psi(\mathbf{r},t) - \frac{(\nabla\psi^\dagger(\mathbf{r},t))(\nabla\psi(\mathbf{r},t))}{2m} - \phi(t)(\mathbf{r}\nabla\gamma)\psi^\dagger(\mathbf{r},t)\psi(\mathbf{r},t) + \frac{\phi^2(t)}{2U}\right\}$$

Response of non-interacting electrons to the temperature gradient

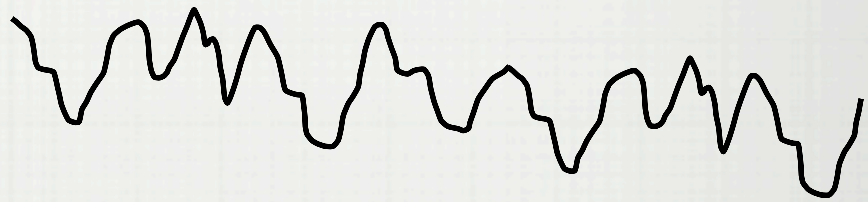
Response to a fluctuating electric field created by the interaction

this talk

Nernst effect near the superconducting phase transition

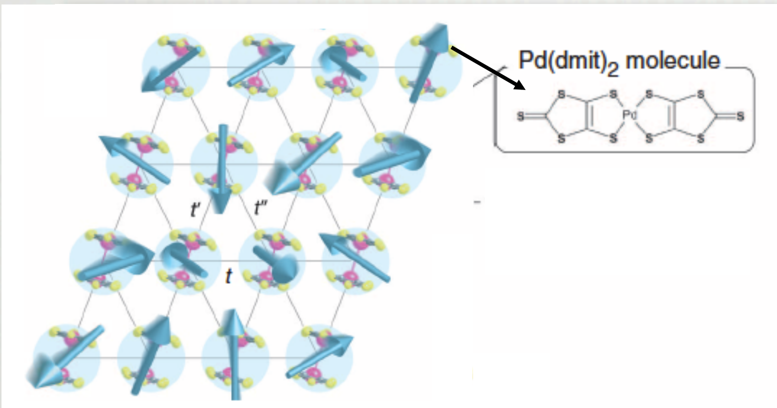


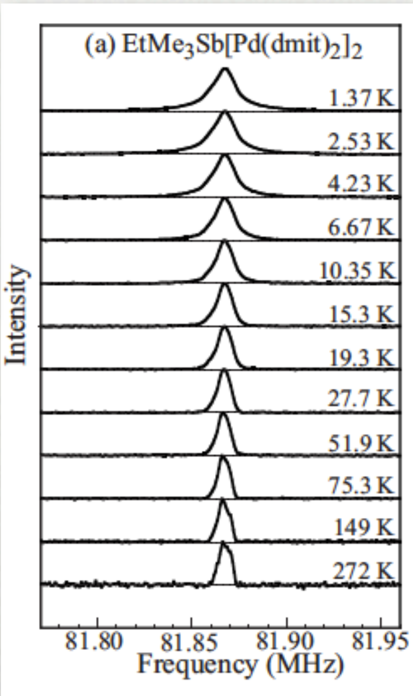
Thermal conductivity in disordered metal

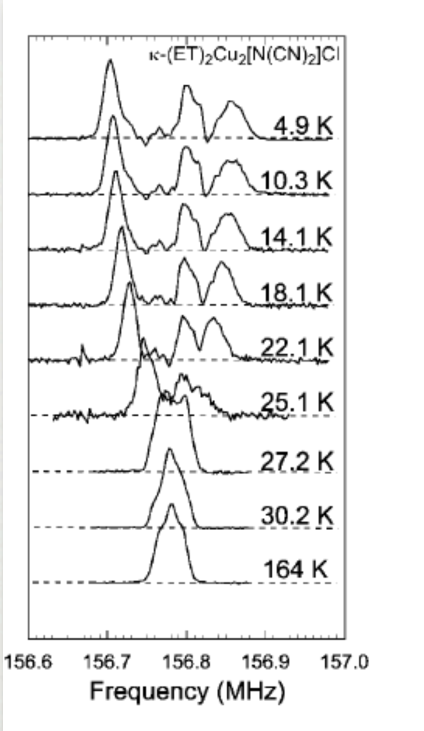


FRUSTRATED MAGNETS

$Pd(dmit)_2(EtMe_3Sb)$





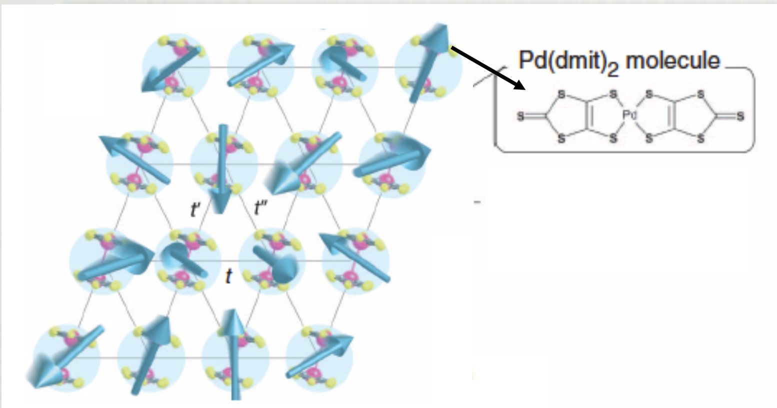


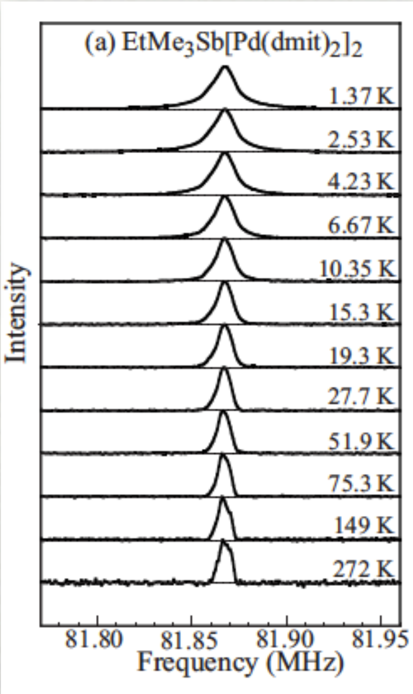
Miyagawa, et al, 1995

Itou, et al, 2008

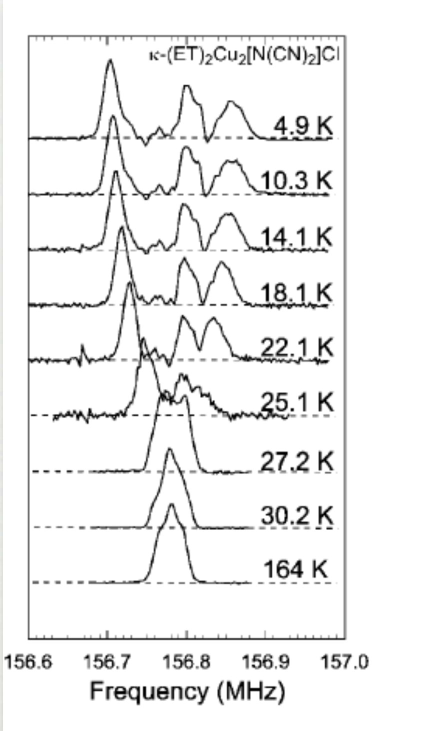
FRUSTRATED MAGNETS

$Pd(dmit)_2(EtMe_3Sb)$





Itou, et al, 2008



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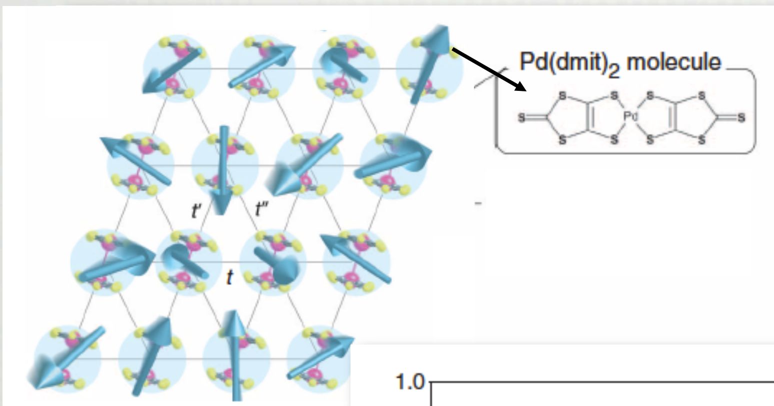
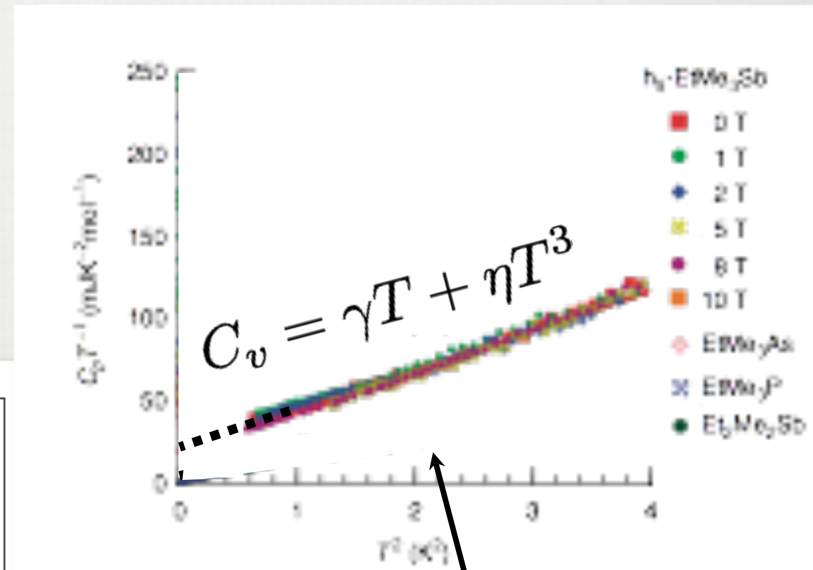
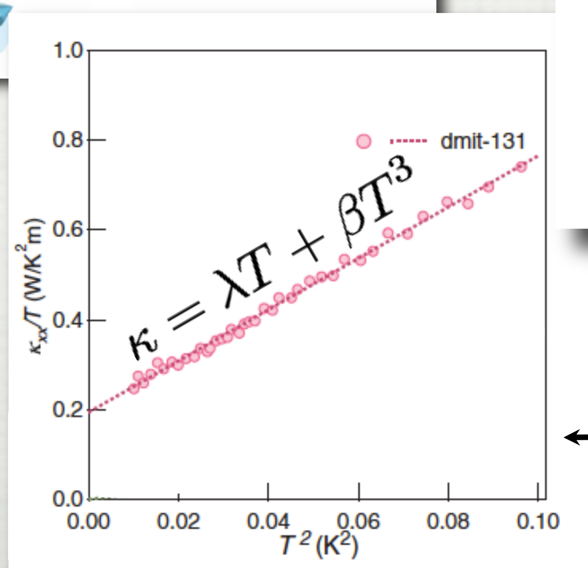




S. Yamashita, et al, 2010



$\kappa = \beta T^3$

$C_v = \eta T^3$

M. Yamashita, et al, 2010

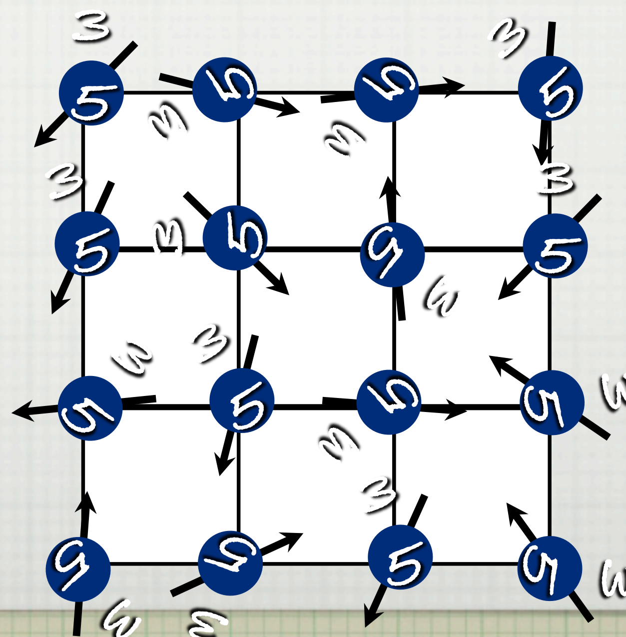
MOTT INSULATORS

$$\mathcal{H} = -t \sum_{<i,j>,\sigma} \left[c^{\dagger}_{i,\sigma} c_{j,\sigma} + c^{\dagger}_{j,\sigma} c_{i,\sigma} \right] + U \sum_{i} c^{\dagger}_{i,\uparrow} c_{i,\uparrow} c^{\dagger}_{i,\downarrow} c_{i,\downarrow}$$

Kinetic energy

Potential energy
(electron- electron interaction)

$$t \ll U$$



MOTT INSULATORS

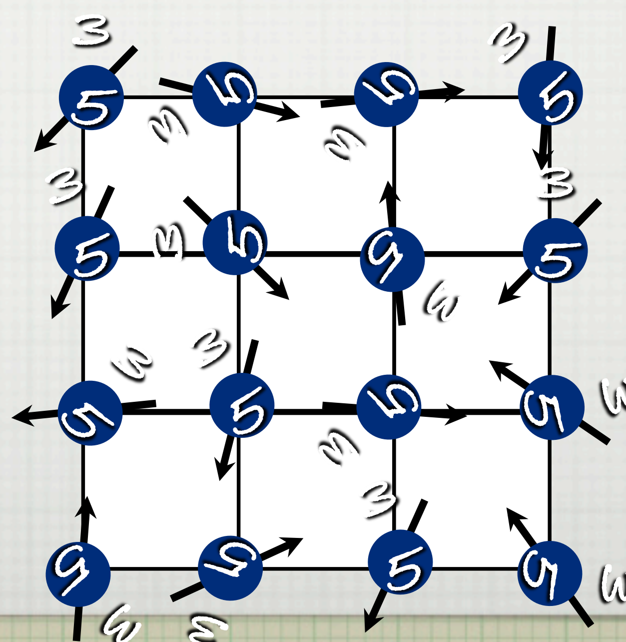
$$\mathcal{H} = -t \sum_{<i,j>,\sigma} \cancel{\left[c^{\dagger}_{i,\sigma} c_{j,\sigma} + c^{\dagger}_{j,\sigma} c_{i,\sigma}\right]} + U \sum_{i} c^{\dagger}_{i,\uparrow} c_{i,\uparrow} c^{\dagger}_{i,\downarrow} c_{i,\downarrow}$$

$$+J \sum_{<i,j>} \vec{S}_i \cdot \vec{S}_j \; + \; ...$$

$J \sim t^2/U$

Spin exchange

$$t \ll U$$



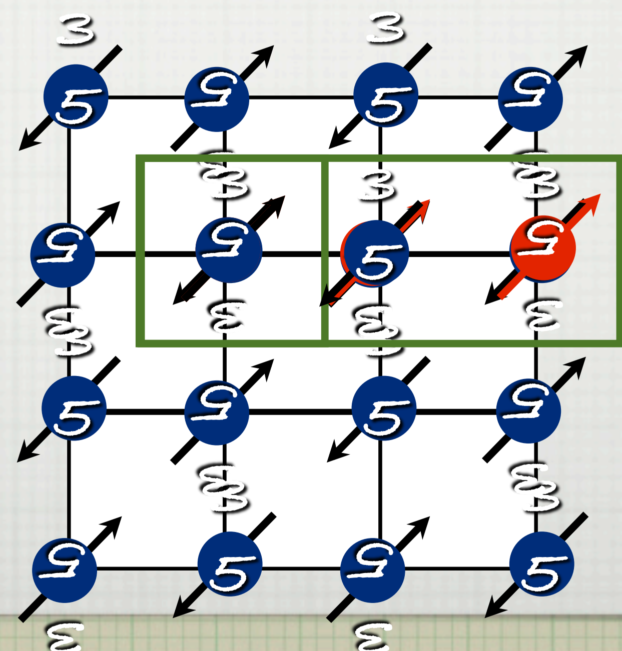
NÉEL STATE

$$\mathcal{H} = -t \sum_{<i,j>,\sigma} \left[c^\dagger_{i,\sigma} c_{j,\sigma} + c^\dagger_{j,\sigma} c_{i,\sigma} \right] + U \sum_i \cancel{c^\dagger_{i,\uparrow} c_{i,\uparrow} c^\dagger_{i,\downarrow}} c_{i,\downarrow}$$

$$+J \sum_{<i,j>} \vec{S}_i \cdot \vec{S}_j + \ldots \qquad J \sim t^2/U$$

Spin exchange

$$S_{stag} = \sum_{i \in A} S_i^Z - \sum_{i \in B} S_i^Z$$



low energy excitations:

Magnons = spin 1 excitations corresponding to flipping one spin

FROM NÉEL STATE TO SPIN LIQUID

$$\mathcal{H}_{\text{exchange}} = J \sum_{<i,j>} \vec{S}_i \cdot \vec{S}_j$$

$$S_{stag} = \sum_{i \in A} S_i^Z - \sum_{i \in B} S_i^Z$$

The Néel state is classical and can only occur in the thermodynamic limit

Quantum fluctuations can destroy the spin order (e.g. 1D spin chains).

FROM NÉEL STATE TO SPIN LIQUID

$$\mathcal{H}_{\text{exchange}} = J \sum_{<i,j>} \vec{S}_i \cdot \vec{S}_j \qquad S_{stag} = \sum_{i \in A} S_i^Z - \sum_{i \in B} S_i^Z$$

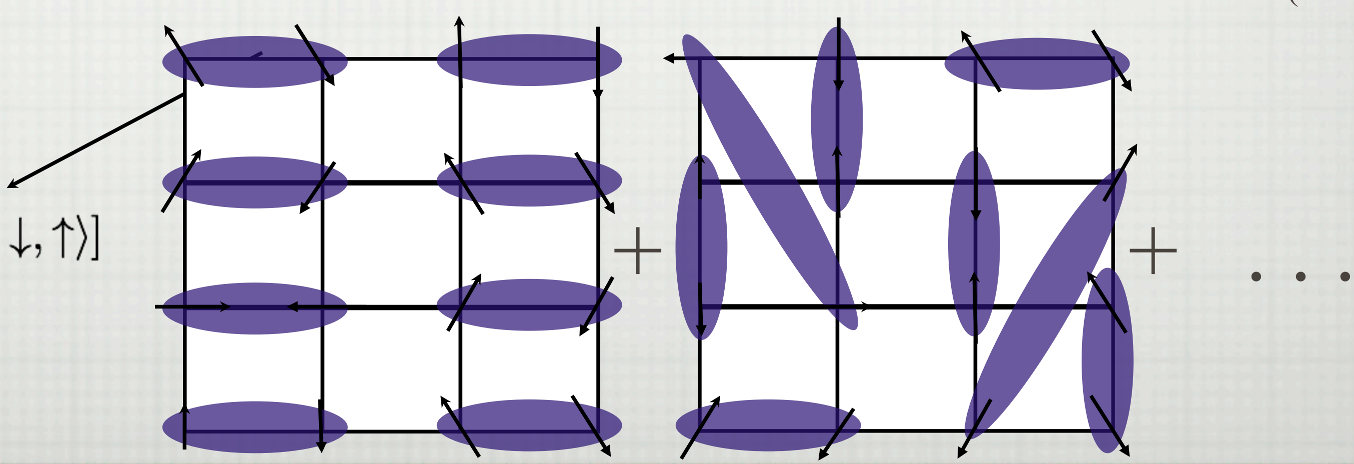
The Néel state is classical and can only occur in the thermodynamic limit

Quantum fluctuations can destroy the spin order (e.g. 1D spin chains).

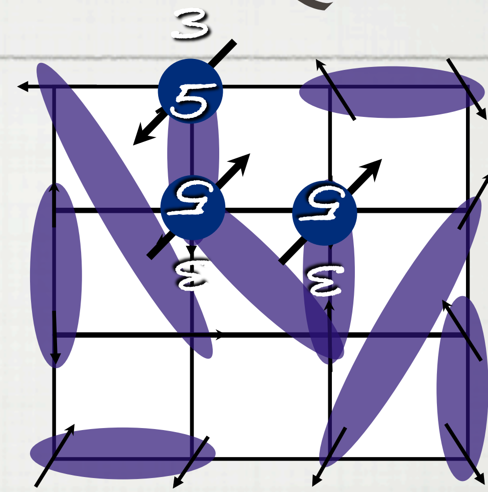
The Néel state (spin solid) can melt into a resonating valence bond state (spin liquid)

P. W. Anderson (1973)

$$\frac{1}{\sqrt{2}}\left[|\uparrow,\downarrow\rangle - |\downarrow,\uparrow\rangle\right]$$



FROM NÉEL STATE TO SPIN LIQUID



Low energy excitations:

breaking a bond

In attempt to capture the low energy physics of spin liquids we represents the spin excitations using spinons - spin 1/2 neutral fermions:

$$\vec{S}_i = \frac{1}{2} f^{\dagger}_{i,\alpha} \vec{\sigma}_{\alpha,\beta} f_{i,\beta}$$

The spinons still interact with each other

$$J \sum_{<i,j>} \vec{S}_i \cdot \vec{S}_j \longrightarrow \frac{J}{2} \sum_{\substack{<i,j> \\ \alpha,\beta=\uparrow,\downarrow}} f^{\dagger}_{i,\alpha} \overbrace{f_{j,\alpha} f^{\dagger}_{j,\beta}}^{\text{current-current interaction}} f_{i,\beta} + \frac{J}{4} \sum_{\substack{<i,j> \\ \alpha,\beta=\uparrow,\downarrow}} f^{\dagger}_{i,\alpha} \underbrace{f_{i\alpha} f^{\dagger}_{j,\beta}}_{\text{density-density interaction}} f_{j,\beta}$$

The interaction can be described by coupling of the spinons to an effective electromagnetic field

STRONG CORRELATIONS

The spinons interact through a 2D electromagnetic field

$$S = \int d^2r dt \sum_{\alpha} f^{\dagger}_{\alpha}(\vec{r},t) \left[i\partial_t + \mu - \varphi(\vec{r},t) + \frac{(\vec{\nabla} - i\vec{a}(\vec{r}))^2}{2m} \right] f_{\alpha}(\vec{r},t)$$

The transverse component of the electromagnetic field generates the strong correlations of the spin liquid

STRONG CORRELATIONS

The spinons interact through a 2D electromagnetic field

$$S = \int d^2r dt \sum_\alpha f_\alpha^\dagger(\vec{r},t) \left[i\partial_t + \mu - \varphi(\vec{r},t) + \frac{(\vec{\nabla} - i\vec{a}(\vec{r}))^2}{2m} \right] f_\alpha(\vec{r},t)$$

$$D_{\text{gauge}}(\omega, \vec{q}) = [\gamma|\omega|/q + \chi q^2]^{-1}$$

$$G_{\text{spinon}}(\varepsilon, \vec{k}) = [i\varepsilon - \epsilon(\vec{k}) + i|\varepsilon|^{2/3}\text{sign}(\varepsilon)]^{-1}$$

The strong interactions lead to the breakdown of the quasiparticle picture

$$m^* \xrightarrow[\epsilon=\epsilon_F]{} \infty$$

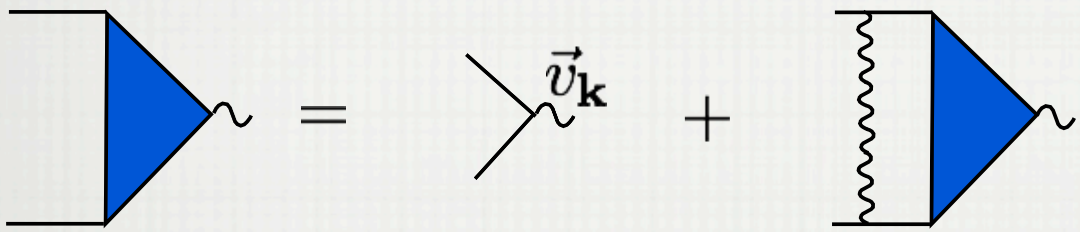
$$\tau_{qp} \sim |\varepsilon - \varepsilon_F|^{-2/3}$$

The excess entropy is expected to be large

$$c \sim T^{2/3}$$

P. A. Lee and N. Nagaosa, 1992
O. I. Motrunich, 2005

TRANSPORT SCATTERING RATE

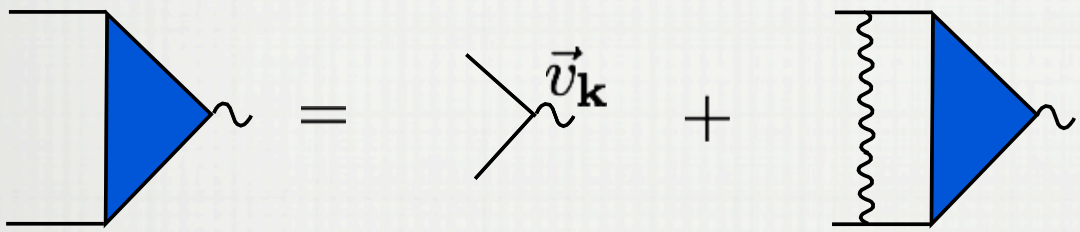


$$\frac{1}{\tau^s_{tr}} = \frac{1/\tau}{1 - \tau \int d\mathbf{q} d\omega Im D(\mathbf{q}, \omega) Im G(\mathbf{k} - \mathbf{q}, \epsilon - \omega)[1 - q^2/k_F^2]}$$

$$D_{\text{gauge}}(\omega, \vec{q}) = [\gamma|\omega|/q + \chi q^2]^{-1} \qquad q \sim \omega^{1/3} \qquad \tau_{qp} \sim |\varepsilon - \varepsilon_F|^{-2/3}$$

$$\rho_s \sim T^{4/3}$$

TRANSPORT SCATTERING RATE

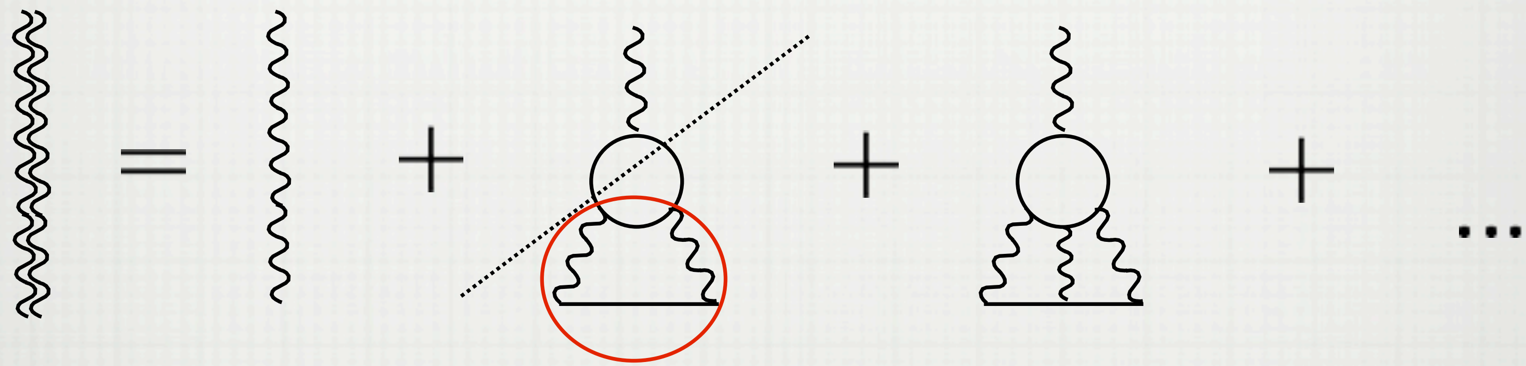


$$\frac{1}{\tau_{tr}^{h}} = \frac{1/\tau}{1 - \tau/T^2 \int d\mathbf{q} d\omega Im D(\mathbf{q},\omega) Im G(\mathbf{k}-\mathbf{q}, \epsilon-\omega)[1-\epsilon\omega-\epsilon^2 q^2/k_F^2]}$$

$$D_{\text{gauge}}(\omega,\vec{q}) = [\gamma|\omega|/q + \chi q^2]^{-1} \qquad q \sim \omega^{1/3} \qquad \tau_{qp} \sim |\varepsilon - \varepsilon_F|^{-2/3}$$

$$\frac{\kappa}{T} \sim T^{-2/3}$$

TRANSPORT SCATTERING RATE

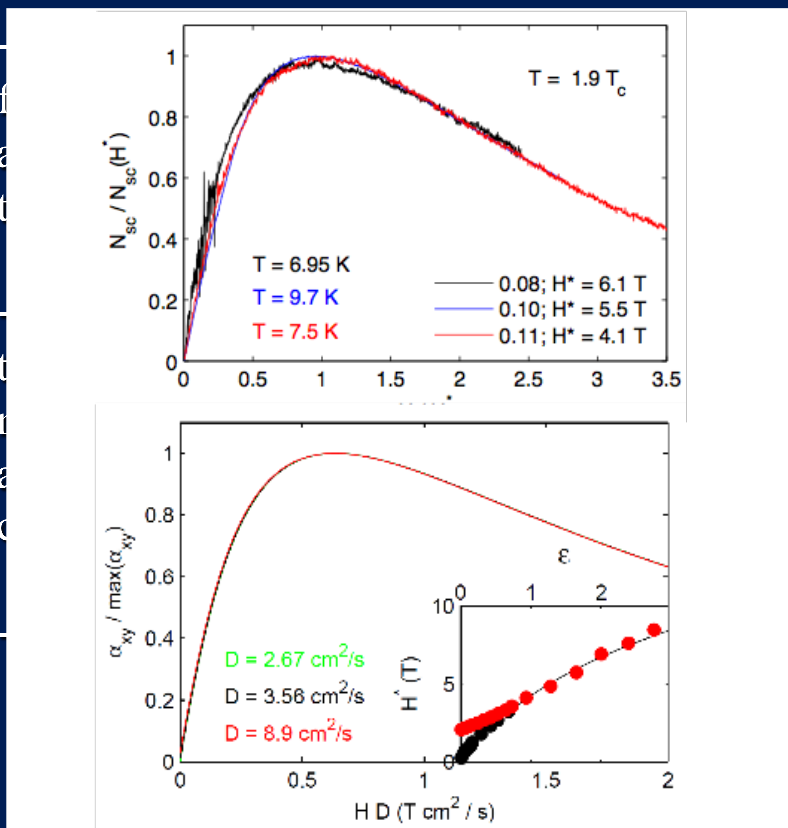


$$\frac{\kappa}{T} \sim 1$$

KM, in preparation

this talk

Summary



Chang, et al, 2012

Summary

- For the thermal conductivity there is a contribution from the collision integral that is not restricted to small angle scattering. This gives a dominant contribution with and without disorder

- Strong enough correlations, although cannot relax momentum, can result in finite thermal transport coefficients