*CLOSED GEODESICS IN RIEMANNIAN AND FINSLER MANIFOLDS*

This series of lectures will be devoted to the problem of the existence and the multiplicity of closed geodesics in Riemannian and Finsler manifolds. The problem originated from the seminal work of Hadamard and PoincarЋ in the late 19th century, and inspired the development of critical point theory in the 20th century. The celebrated closed geodesics conjecture, which in its maximal generality is still open, claims that every closed Riemannian manifold of dimension at least 2 possesses infinitely many closed geodesics. In the course of the lectures, I will provide the background and the proofs of some among the most celebrated results that confirm this conjecture for several classes of manifolds. In the setting of Finsler manifolds, the analogous conjecture fails, as it was shown by Katok in his celebrated example on the 2-sphere. A more general setting, which includes the Finsler one as a special case, is the one of magnetic geodesic flows. The last part of this series of lectures will be devoted to present a few recent results on the multiplicity of closed magnetic geodesics, as well as related open problems from symplectic dynamics.