



РОССИЙСКАЯ АКАДЕМИЯ НАУК

МУЗЕЙ АНТРОПОЛОГИИ И ЭТНОГРАФИИ  
ИМЕНИ ПЕТРА ВЕЛИКОГО (КУНСТКАМЕРА)

Санкт-Петербург, 199034, Университетская наб., 3, тел.: (812) 328 0812, (812) 328 0712  
факс: (812) 328 0811, e-mail: museum@kunstcamera.ru, http://www.kunstcamera.ru

при поддержке  
фонда



Династия

**30 АПРЕЛЯ – 5 МАЯ 2012**  
**САНКТ-ПЕТЕРБУРГСКИЙ СЕМИНАР ПО ГЕОМЕТРИЧЕСКОЙ**  
**МОРФОМЕТРИИ**  
**ДЛЯ АНТРОПОЛОГОВ, БИОЛОГОВ И АРХЕОЛОГОВ**



DR. GERMAN MANRIQUEZ. Lecture  
"Basics of GMM (Shape and size, Morphospaces and  
Procrustes analysis, TPS function).



THE ROYAL SOCIETY



Geometric morphometrics: Internet resources

**Morphometrics**  
at SUNY Stony Brook

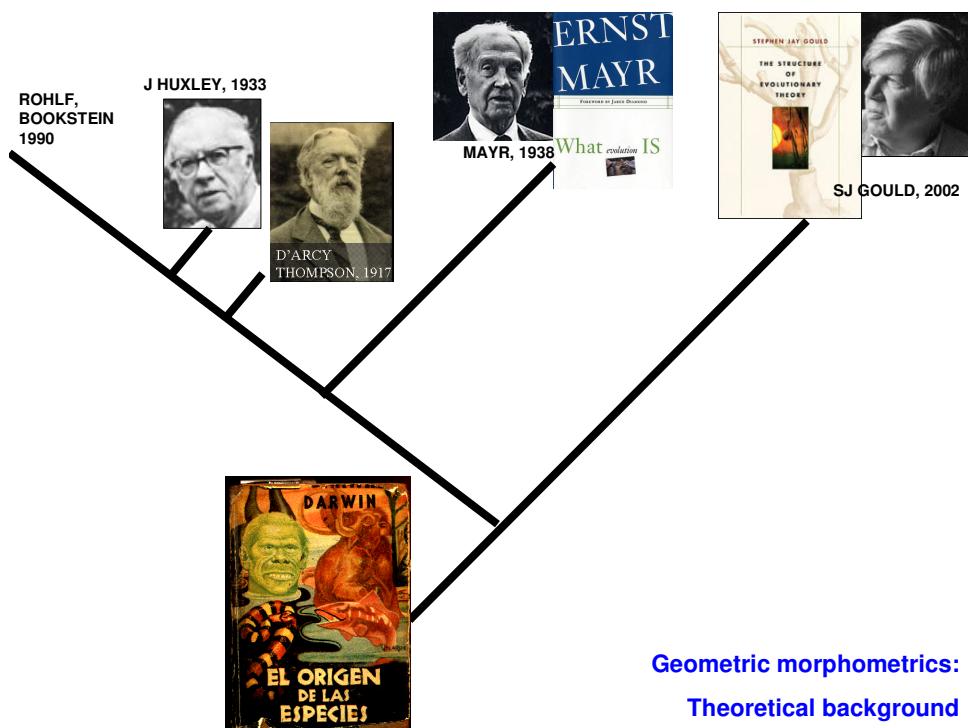
<http://life.bio.sunysb.edu/morph/>

- [Meetings](#)
- [Softwares, Data adquisition](#)
- [Books](#)
- [Glossaries](#)

Functional Morphology and Evolution, Hull York University, UK

<http://www.york.ac.uk/res/fme/index.htm>

- [Paul O'Higgins Lab](#)
- [Research](#)
- [Morphologika download](#)

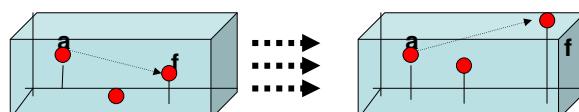


### MAP FUNCTION IN MORPHOMETRY

Distance & homology



*"Morphometric distances express the patterns of relative location among the parts of one organism in comparison with those of another"* ("Distances" and distances, after Bookstein 1991 (Ch. 3))



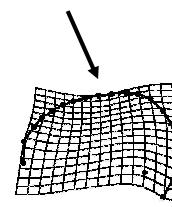
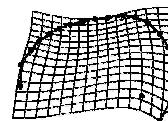
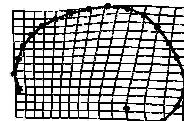
*"What we are measuring is ultimately a matter not of shape but of homology. To pass from the biological to the biometrical context, homology must be considered as a mapping function, a correspondance relating points to points rather than parts to parts" (idem)*

## LANDMARK'S DEFINITION

**LM= Discrete anatomical loci that can be recognized as the same loci in all specimens in the study”**(Zelditch, 2004, Ch. 2)

**LM= A specific point on a biological form or image of a form located according to some rule. Landmarks with the same name, homologues in the purely semantic sense, are presumed to correspond in some sensible way over the forms of a data set ”**(Slice, Rohlf, Bookstein. Glossary...)

**LM= The points at which one’s explanations of biological processes are grounded (...) the boundary points between organ and organ or between organism and organism, at which our epigenetic explanations adhere”**  
(Bookstein, 1991)



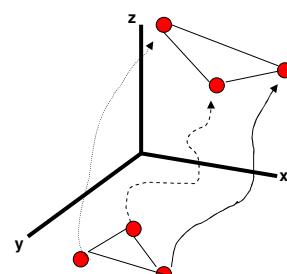
## PROPERTIES OF LANDMARKS OF COORDINATES

Landmarks are anatomically /evolutionary, functionally, structurally, ontogenetically/ **homologous** (O'Higgins 2000)

For Mathematics biological change is interpreted as a deformation= a smooth mapping of one set of points to **corresponding** points in another form.

In Maths, Homology means correspondence point to point, **in Biology** instead, it means a correspondence among two or more structures (the region in between the points, but not the points themselves)

They are explaining the change hierarchically : **Type I, Type II, Type III**, sensu Bookstein. After Zelditch et al interpretation, they can be assumed as anchoring different amounts of biological spaces rather than logical explanations (implies forces acting onto them)



3.3.1. Juxtapositions

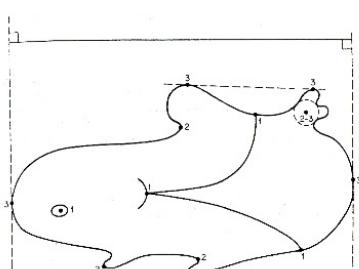


Figure 3.3.1. The three types of landmarks. Type 1: Juxtapositions of tissues. Type 2: Maxima of curvature. Type 3: Extremal points.

(Zelditch et al, 2005)

## SUPERPOSITION METHODS

### Bookstein shape coordinates

52

GEOMETRIC MORPHOMETRICS FOR BIOLOGISTS

SIMPLE SIZE AND SHAPE VARIABLES: BOOKSTEIN SHAPE COORDINATES 53

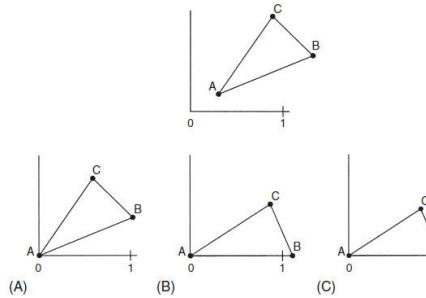


Figure 3.1 Three operations that do not alter shape, applied to a triangle: (A) translation; (B) rotation; (C) rescaling.

$$SC_x = \frac{(B_x - A_x)(C_x - A_x) + (B_y - A_y)(C_y - A_y)}{(B_x - A_x)^2 + (B_y - A_y)^2}$$

$$SC_y = \frac{(B_x - A_x)(C_y - A_y) - (B_y - A_y)(C_x - A_x)}{(B_x - A_x)^2 + (B_y - A_y)^2}$$

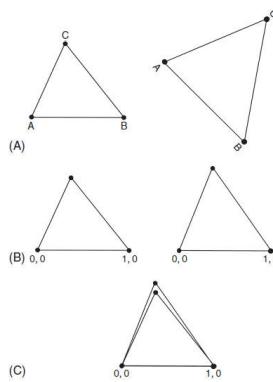


Figure 3.2 Two triangles whose shape difference is the subject of investigation: (A) the two triangles as initially recorded; (B) the same two triangles after being translated, rotated and rescaled by the two-point registration; (C) the same two triangles, superimposed.

Do these triangles differ in shape?

(Zelditch et al, 2005)

## SUPERPOSITION METHODS

### Bookstein shape coordinates

54

GEOMETRIC MORPHOMETRICS FOR BIOLOGISTS

any two points as the endpoints of the baseline (A and B); we will discuss how to choose them later.

1.	54.00000	306.00000
2.	223.00000	447.00000
3.	632.00000	300.00000

Now take the next three coordinate pairs, and draw that triangle:

1.	11	342
2.	251	520
3.	769	318

Now draw both triangles (with point A and B superimposed), and draw the vector extending between the one free ( $C_{xy}$ ) landmark on both of the triangles (A). That vector is the shape variable describing the difference between the triangles.

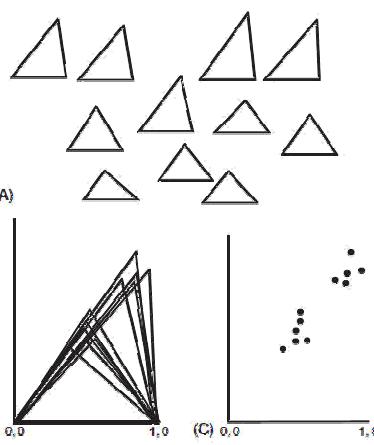
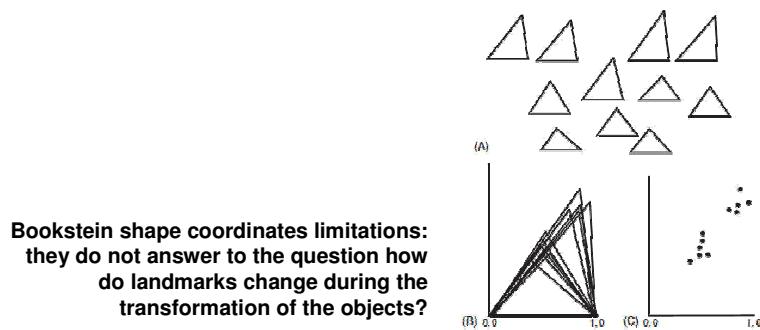


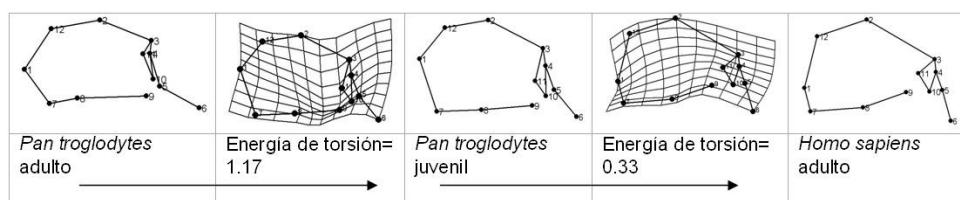
Figure 3.3 Comparing shapes of triangles: (A) the collection of triangles whose shape differences are the subject of investigation; (B) the same collection of triangles, put in a common coordinate system by the two-point registration; (C) scatter plot depicting the location of the free landmarks.

(Zelditch et al, 2005)



**Thin-plate spline function allows to**  
**describe visually the changes occurring**  
**during the transformation**

(Manríquez, 2009)



## SHAPE AND SIZE

### Procrustes fitting

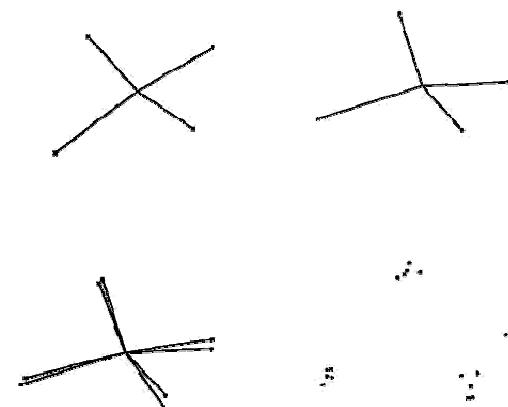


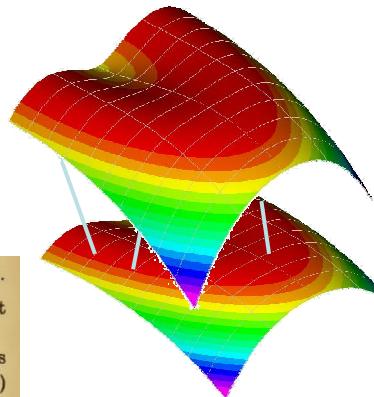
Figure 1. Geometry of Procrustes fits. Upper row: Two forms with four landmarks. Center row: Each form is scaled to unit Centroid Size (sum of squared distances from the centroid to the landmarks). Lower left: The centroids are superposed and then one form is rotated over the other to a position of least squared differences between the homologues. Lower right: If the fixed form is a sample average shape (see text), vectors of displacement in Procrustes space, shown here as four scatters around the average (dots), serve as one set of shape coordinates for the sample of forms. These coordinates total eight (four sets of two), which is four in excess of the actual dimension of the shape space of a quadrilateral.

(Zelditch et al, 2005)

### The model: the thin-plate spline

### The parameter: Shape changes

**The variable: Bending energy.** It's the mean value of the local vectors's change in magnitude and direction when they are superposed to the homologous landmarks located on a new object



### 1062 THE THEORY OF TRANSFORMATIONS [CH.]

which fossils are subject (as we have seen on p. 811) as the result of shearing-stresses in the solid rock.

Fig. 519 is an outline diagram of a typical Scaroid fish. Let us deform its rectilinear coordinates into a system of (approximately) coaxial circles, as in Fig. 520, and then filling into the new system,

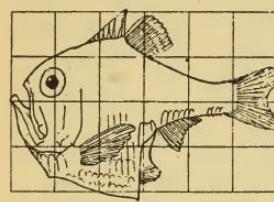


Fig. 517. *Argyropelecus Olfersi*.

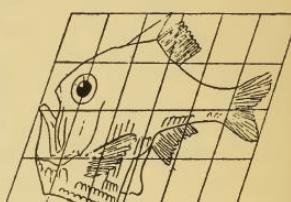


Fig. 518. *Sternopyx diaphana*.

### ¿Qué es una deformación?

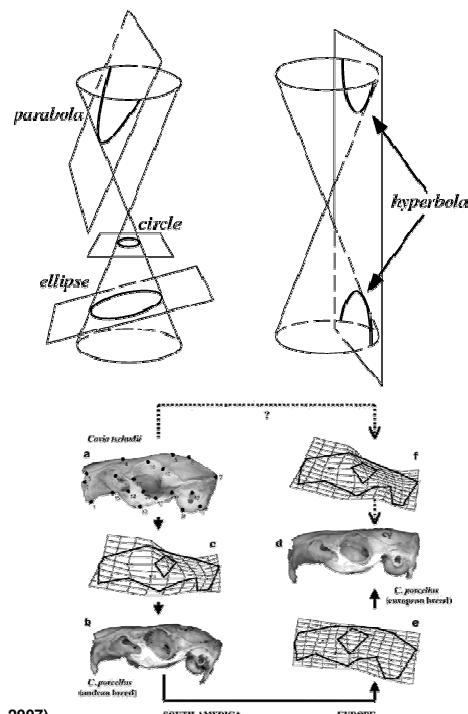
Es una función que mapea hitos en una forma que se corresponden con los hitos ubicados en otra forma, es decir, una función tal que posee continuidad geométrica

Las cinco secciones de un cono como cinco versiones diferentes de una misma forma

La línea como círculo de radio infinito:  $x = \infty$  sea un punto en el círculo,  $x = +\infty$  and  $x = -\infty$  sean idénticas

El cambio de la forma no es una deformación cuando dicho cambio se concentra en un sólo hito...

La grilla es un gráfico que describe el desplazamiento de los hitos discretos en deformación



(Spotorno et al, 2007)

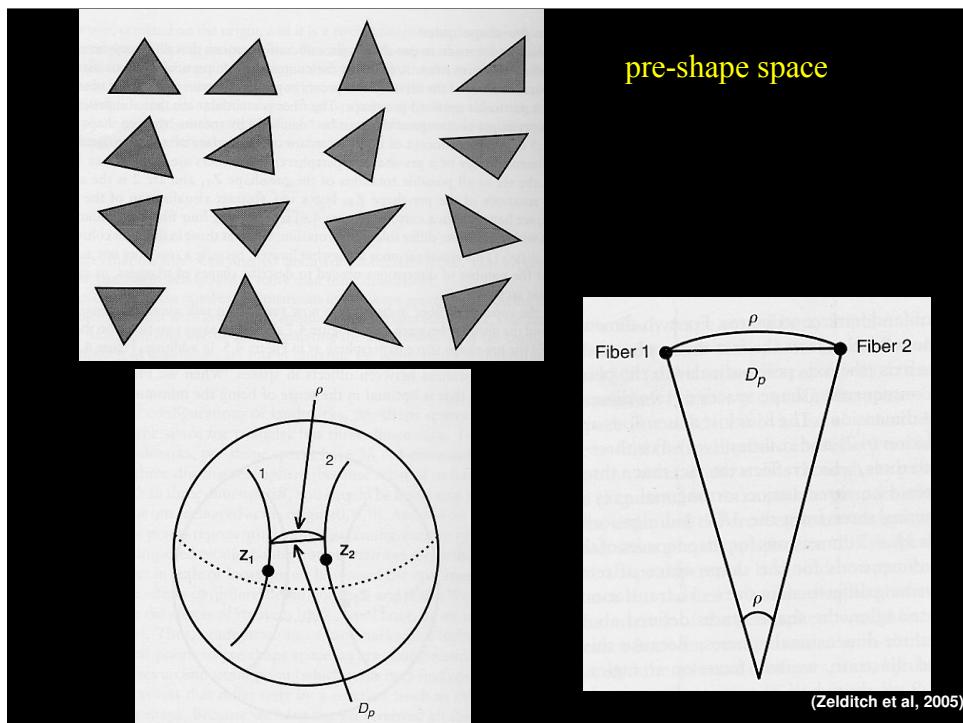
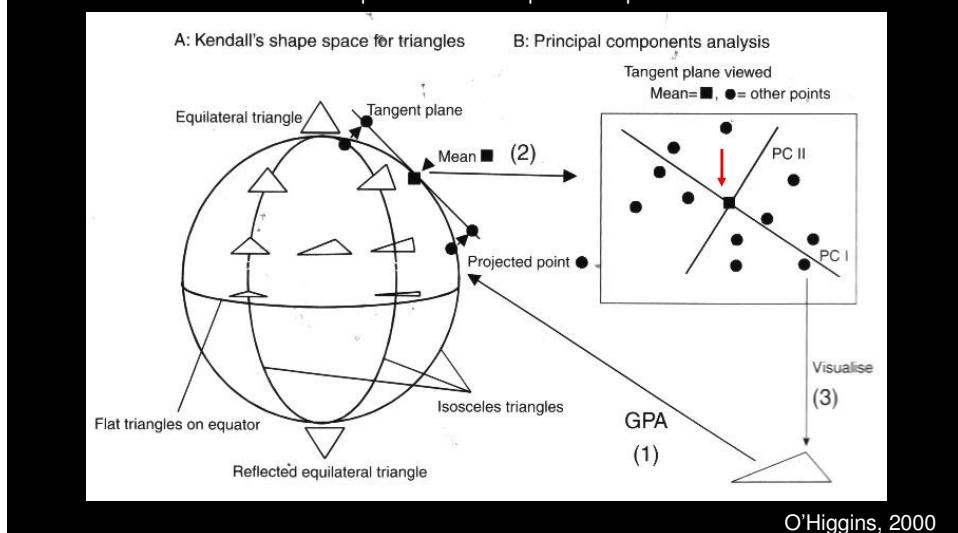
Remoción de diferencias de escala, rotación y traslación (**Análisis de Procrustes generalizado**)

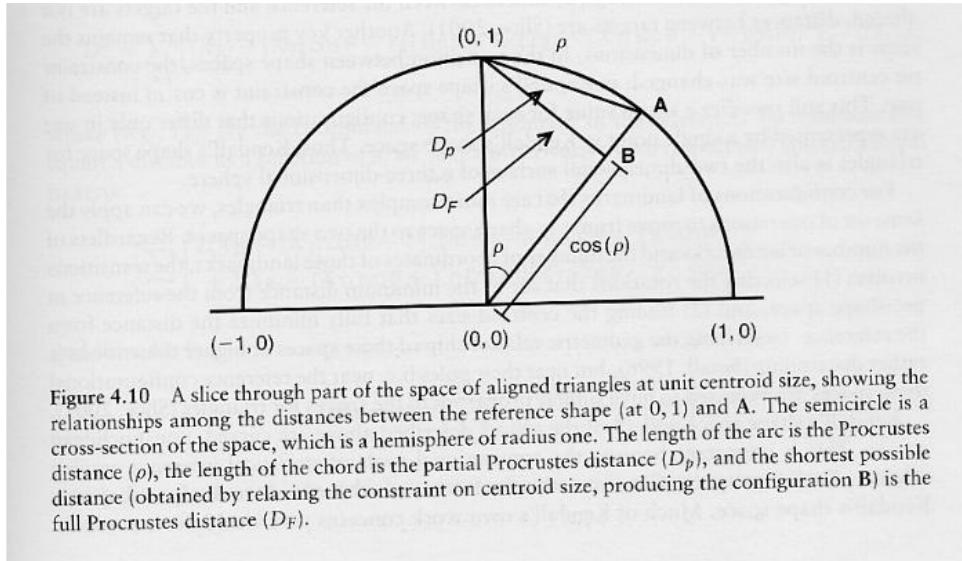
Obtención de configuración de consenso

Proyección ortogonal de matrices alineadas (**Relative warp analysis**)

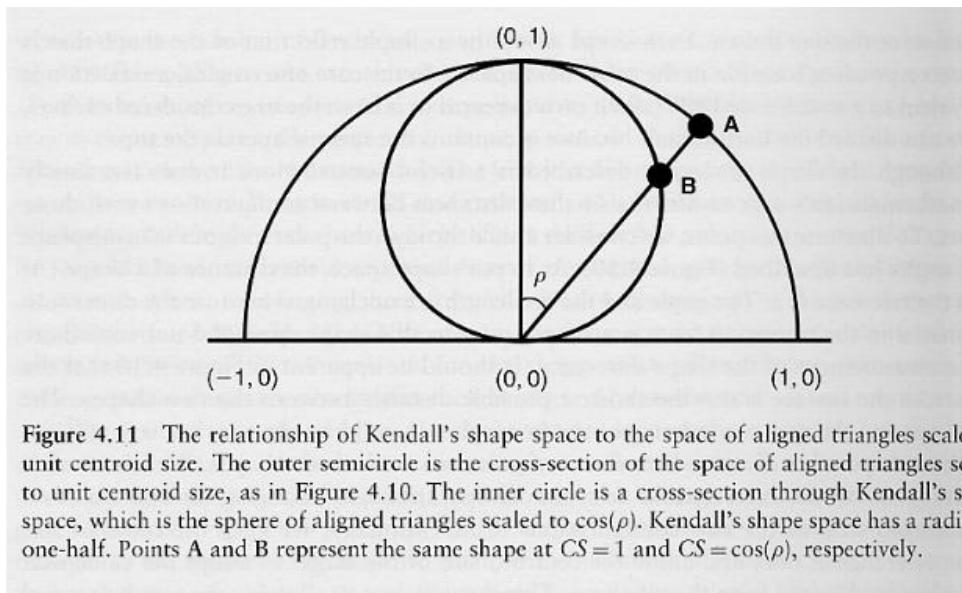
Función de placa delgada para interpolar los datos de la variación no uniforme.

Sus componentes son los partial warps





(Zelditch et al, 2005)



(Zelditch et al, 2005)

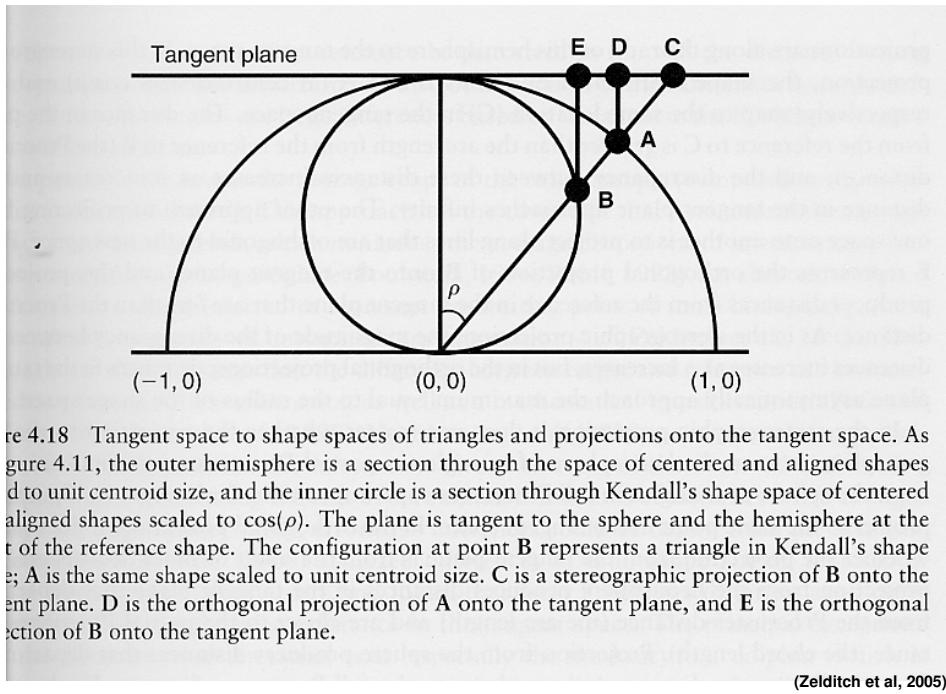
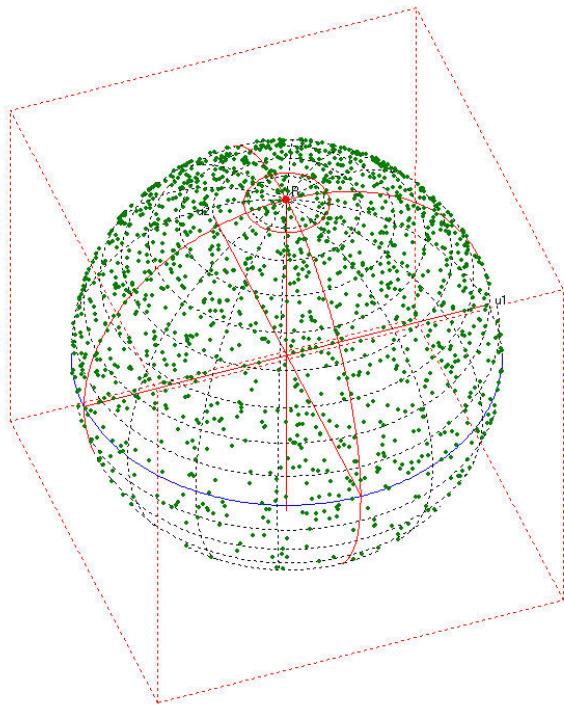
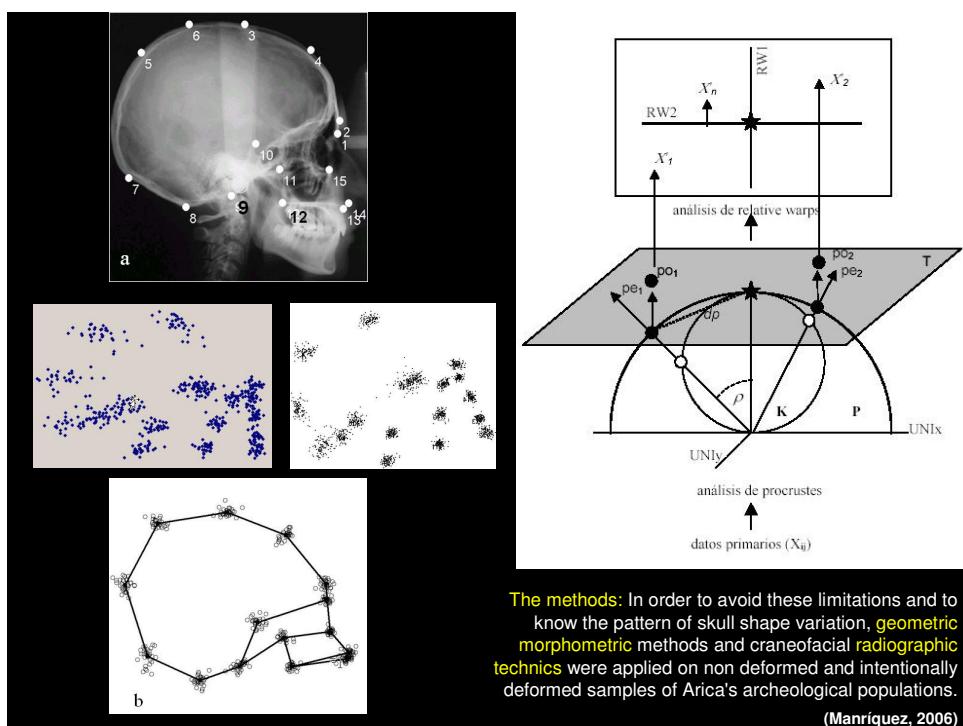


Figure 4.18 Tangent space to shape spaces of triangles and projections onto the tangent space. As in Figure 4.11, the outer hemisphere is a section through the space of centered and aligned shapes scaled to unit centroid size, and the inner circle is a section through Kendall's shape space of centered and aligned shapes scaled to  $\cos(\rho)$ . The plane is tangent to the sphere and the hemisphere at the point of the reference shape. The configuration at point B represents a triangle in Kendall's shape space; A is the same shape scaled to unit centroid size. C is a stereographic projection of B onto the tangent plane. D is the orthogonal projection of A onto the tangent plane, and E is the orthogonal projection of B onto the tangent plane.

(Zelditch et al, 2005)

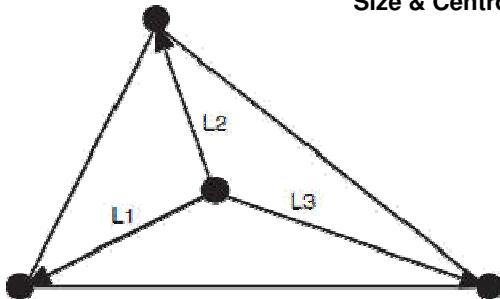
#### Population character of GM data





### FORMA Y TAMAÑO EN MORFGEOM

#### Size & Centroid size



##### Cálculo del tamaño del centroide (**centroid size**):

-calcular el centroide, ubicado en el punto medio de las coordenadas. Corresponde al promedio de las coordenadas en X, y al promedio de las coordenadas en Y (X, Y)

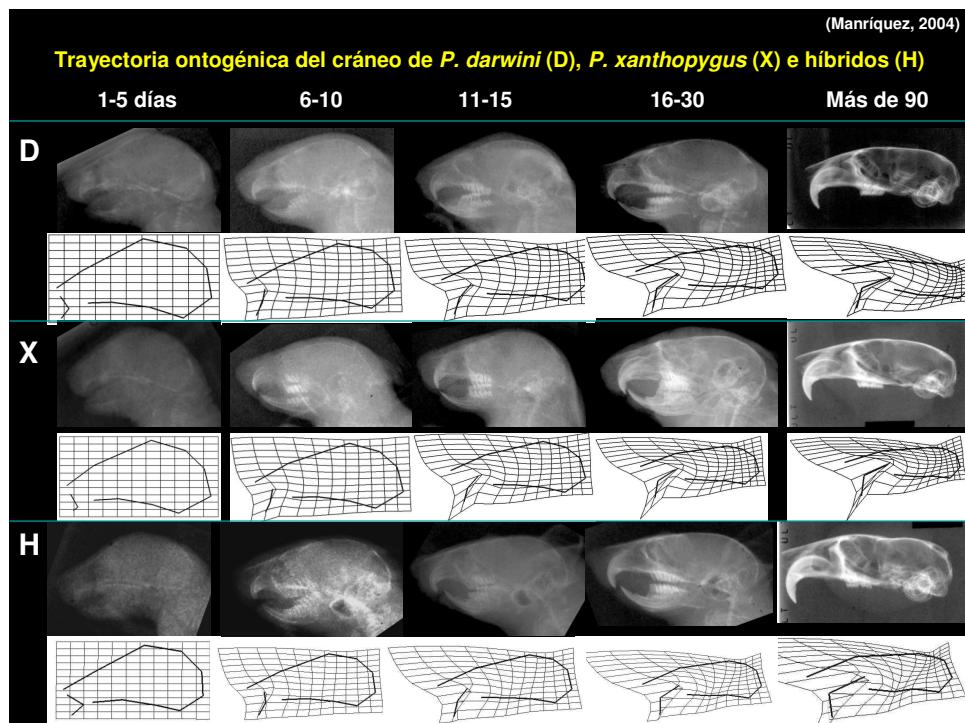
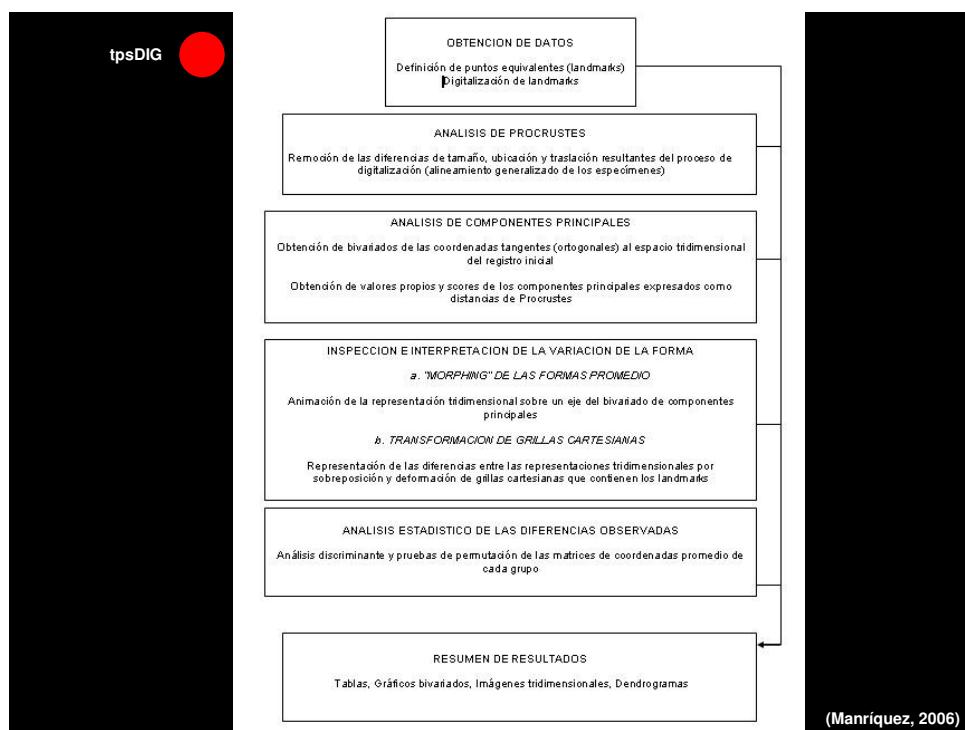
-Para

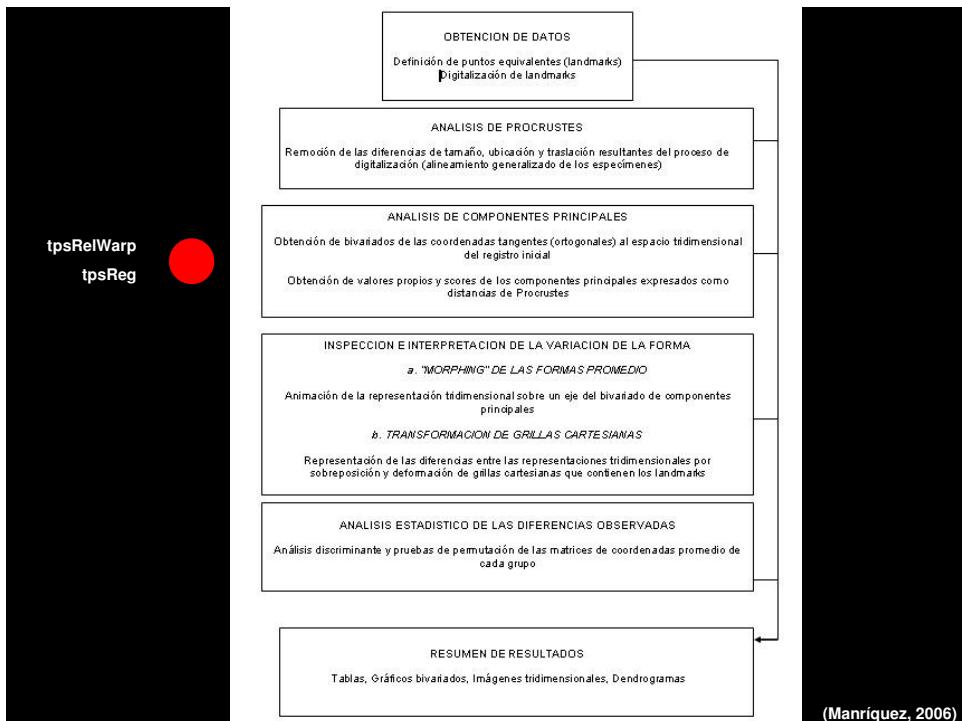
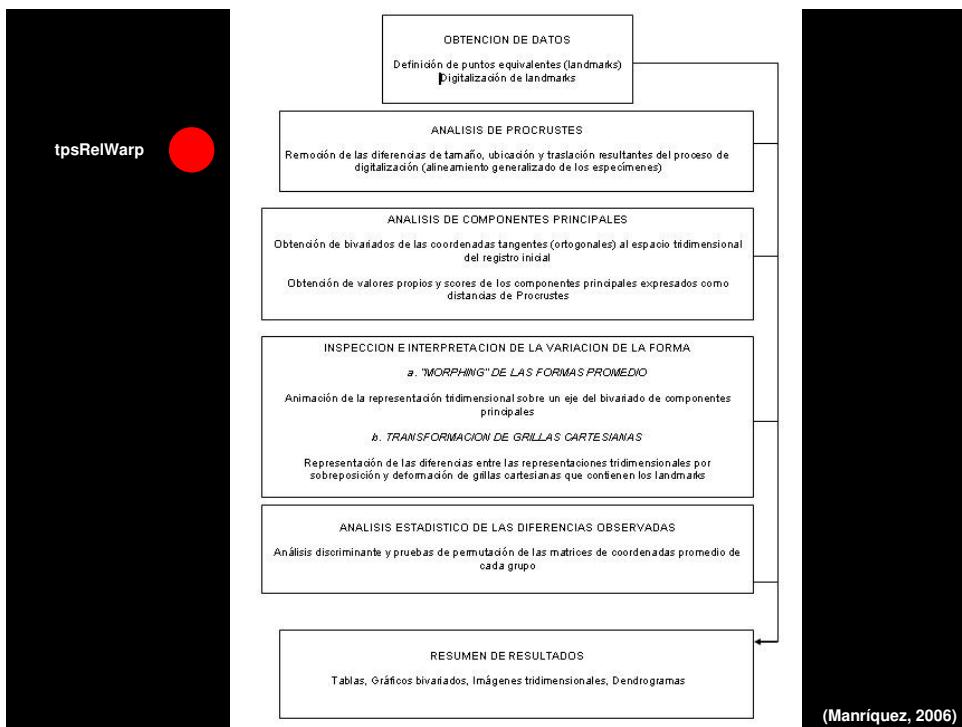
$$(0, 0), (1, 0), (0.3, 0.8)$$

$$(X, Y) = (0.433, 0.267)$$

-Luego, se calcula la distancia cuadrática de cada coordenada al centroide:  $(Xi - X)^2 + (Yi - Y)^2$   
 $(0 - 0.433)^2 + (1 - 0.433)^2 + \dots$

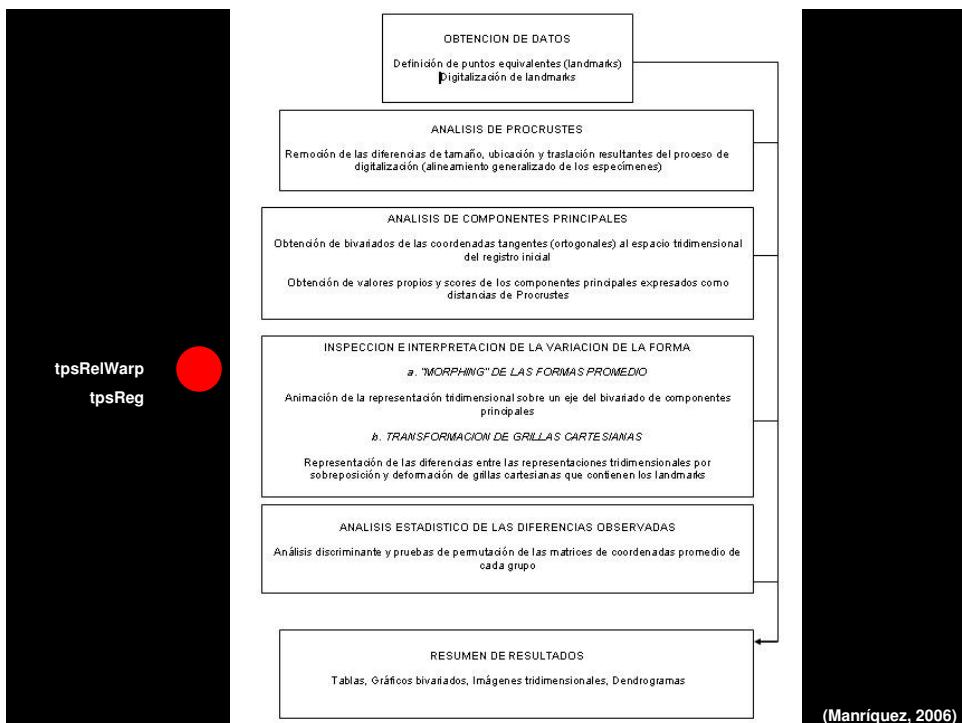
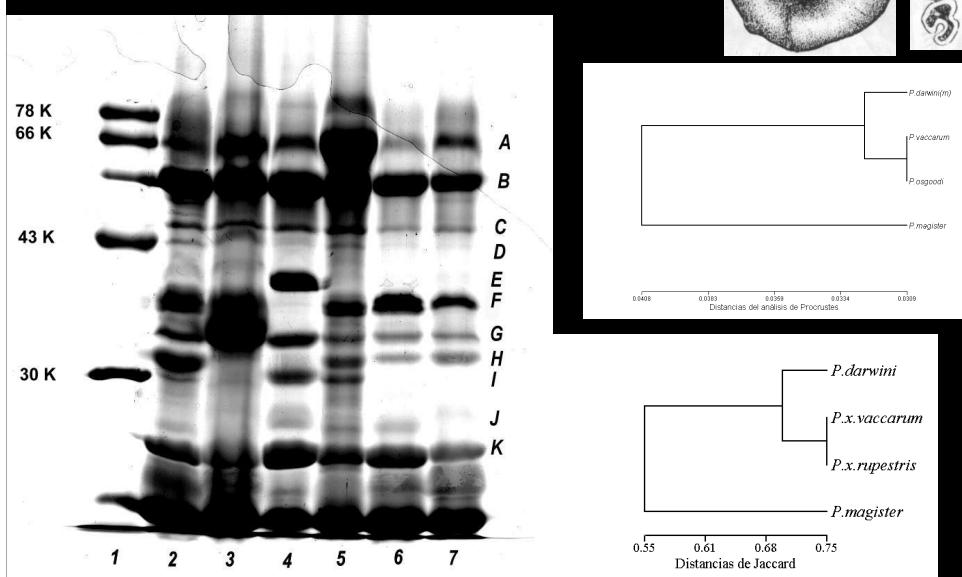
-Se obtiene una medida del área. Se lineariza obteniendo la raíz cuadrada de ese valor cuadrático, y tenemos el valor del tamaño del centroide (raíz cuadrada de la suma cuadrática de las coordenadas de cada hito respecto de las coordenadas del centroide)



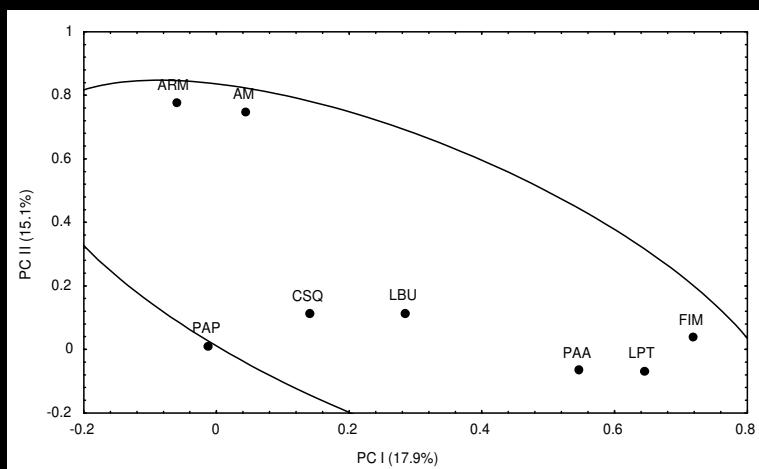


Hipsodontía y herbivoría progresiva debieran tener un correlato funcional, y no sólo anatómico (hipótesis *ad-hoc*, análisis *a posteriori*)

(Manríquez, 2004)

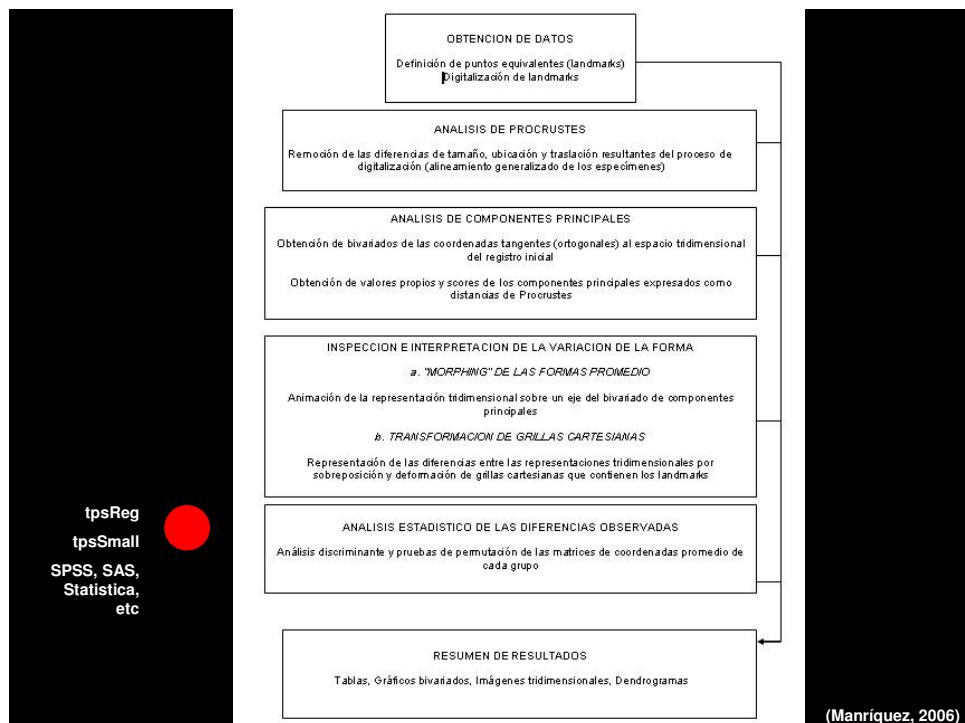


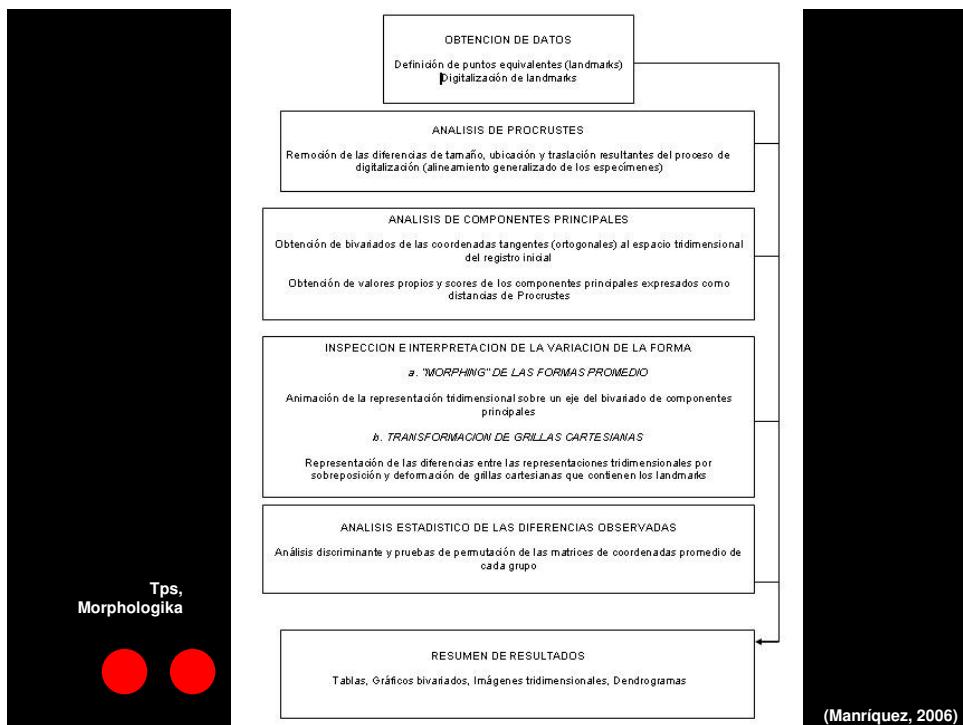
**Eigenloads for variables explaining main variation of masticatory apparatus (FIM, ARM, AM1). They are also the less asymmetric.**



### CANALIZATION OF THE STRUCTURES OF THE MASTICATORY APPARAT?

(Manríquez, 2004)



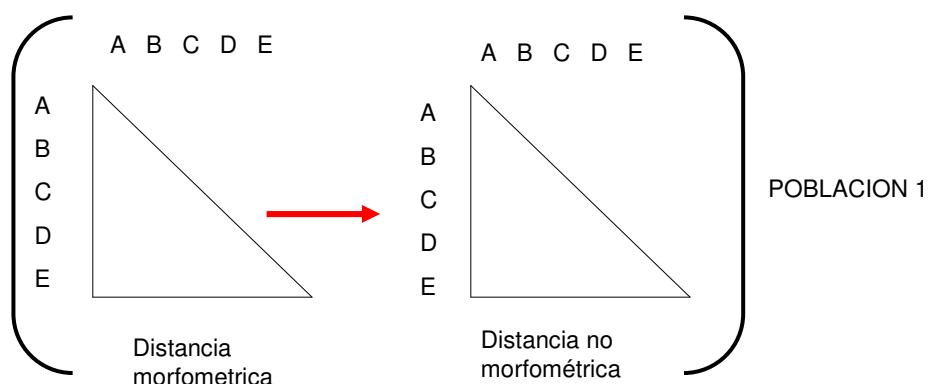


### La concordancia taxonómica como función de mapa

Concepto de matriz de distancias (matrices rectangulares)

Uso de matrices de distancias de Procrustes y configuraciones de consenso en análisis de concordancia ([tpsRelwarp](#) [tpsSmall](#))

Puesta a prueba de hipótesis con Mantel-test



+ Mantel-test (pruebas de permutación)

(Manríquez, 2009)