Flavor symmetry and competing orders in graphene bilayer

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Outline

- Background (excitonic instability in graphene)
- Recent work on bilayer graphene
- SU(4) flavor symmetry and relation between different proposals
- Phase diagram in E and B fields
- Gap enhancement due to dynamical screening

Excitonic instability in graphene bilayer

Motivation:

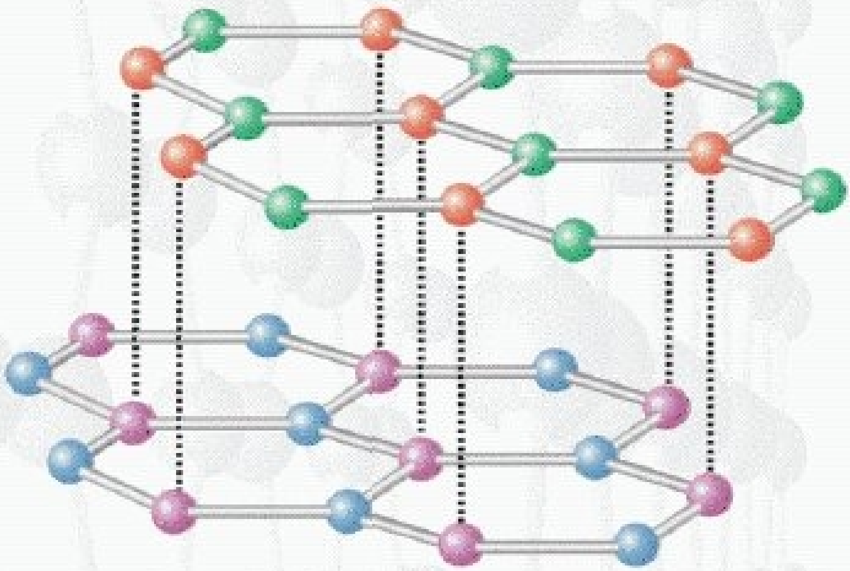
- Fundamental many-body phenomenon: long anticipated (by Mott, Keldysh & others), not yet observed
- Analogy with high energy physics, chiral symmetry breaking
- A knob to control electronic properties

Rahul Nandkishore, LL arXiv:1002.1966, arXiv:0907:5395, PRL to appear

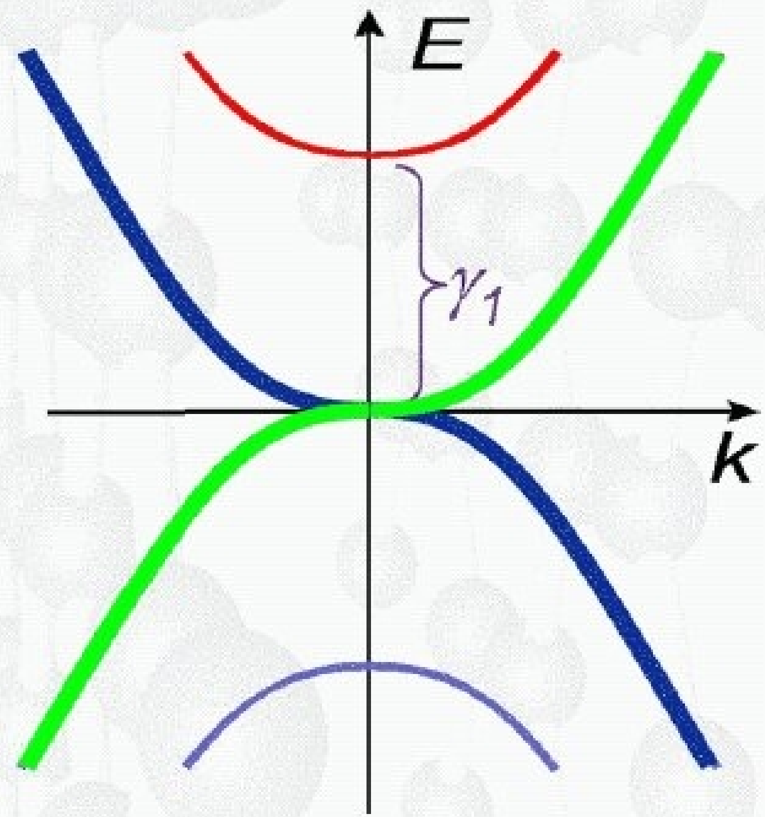
Gap opening at the Dirac point

- Gapless state robust in a single layer (Khveshchenko 2001) but can be induced by B field (Gusynin et al; Checkelsky et al 2008)
- Bilayer in external E field (McCann, Falko, 2006, Oostinga et al 2007)
- Excitonic instability for a pair of single layers (Min, Su, MacDonald; Zhang, Joglekar; Kharitonov, Efetov 2008): BCS-like pairing (*a la* Keldysh-Kopaev, Kozlov-Maksimov), exponentially small energy scales Δ, T_c due to small DOS near the Dirac point
- Suspended bilayer in magnetic field (Yacoby's group 2009) gap at $\nu=0$ for B>0.1 Tesla

Graphene bilayer: electronic structure and QHE



$$E(p) = \pm \tfrac{1}{2}\gamma_1 \pm \sqrt{\tfrac{1}{4}\gamma_1^2 + v_F^2 p^2}$$



$$\hat{F} = -\frac{1}{2m}\begin{pmatrix} 0 & (\hat{p}_x + i\hat{p}_y)^2 \\ (\hat{p}_x - i\hat{p}_y)^2 & 0 \end{pmatrix}$$

$$E_N = \pm\hbar\omega_c\sqrt{N(N-1)}$$

McCann & Falko 2006

Excitonic ordering in a bilayer

Dynamically generated UV cutoff (characteristic "Rydberg energy" and "Bohr radius")

$$E_0 = \frac{me^4}{\kappa^2} \approx \frac{1.47}{\kappa^2}\,\mathrm{eV}, \qquad a_0 = \frac{\kappa}{me^2} \approx \kappa \times 1.1\,\mathrm{nm}$$

- Real-valued order parameter; phase locking, no exciton superfluidity
- 'Which-layer' symmetry breaking;
- Spontaneous ferroelectric-type layer polarization
- Gap scales as a power law with interaction
- Δ may reach 10-20 K in a clean system

$$\Delta \approx 10^{-3} E_0$$

Nandkishore, LL
arXiv:0907:5395

Summary of proposed states

$$H_K = \begin{pmatrix} \Delta_K & p_-^2/2m \\ p_+^2/2m & -\Delta_K \end{pmatrix} \qquad H_{K'} = \begin{pmatrix} \Delta_{K'} & p_+^2/2m \\ p_-^2/2m & -\Delta_{K'} \end{pmatrix}$$

$$\Delta_{K,\sigma} = \pm\Delta_{K',\sigma} = \pm\Delta_{K,-\sigma} = \pm\Delta_{K',-\sigma} \qquad p_\pm = p_1 \pm i p_2$$

- Four-fold spin/valley degeneracy
- Many gapped states: valley “antiferromagnet”, ferromagnetic, ferrimagnetic, ferroelectric, etc
- Degeneracy on a mean field level: instability threshold the same for all states: short-range interaction model (MacDonald et al), screened long-range interaction (Nandkishore & LL)
- SU(4) symmetry?

SU(4) symmetry

Pauli matrices for layer, valley and spin: $\tau_i\,,\eta_i\,,\sigma_i(i=1,2,3)$

8x8 single-particle Hamiltonian:

$$H_0=\frac{(p_1+i\,p_2\eta_3)^2}{2m}\tau_+ + \frac{(p_1-i\,p_2\eta_3)^2}{2m}\tau_- + \epsilon\tau_3$$

Interchage layer indices in one valley to obtain an SU(4) invariant Hamiltonian

$$U=\frac{1+\eta_3}{2}+\frac{1-\eta_3}{2}\tau_1$$

transverse E field breaks SU(4)

$$\tilde{H}_0=U\,H_0U^{-1}=\frac{p_+^2}{2m}\tilde{\tau}_+ + \frac{p_-^2}{2m}\tilde{\tau}_- + \epsilon\tilde{\tau}_3\tilde{\eta}_3$$

Symmetrized interlayer and intralayer interactions

$$H=\sum_p \psi_p^+ \tilde{H}_0 \psi_p + \frac{1}{2}\sum_q V_+(q)\rho_q\rho_{-q} + V_-(q)\lambda_q\lambda_{-q}$$

$$V_+(q)=2\pi e^2/\kappa q$$

$$V_-(q)=\pi e^2 d/\kappa$$

$$\lambda(x)=(\lambda_1(x)-\lambda_2(x))/2$$

$$\rho(x)=(\rho_1(x)+\rho_2(x))/2$$

d=2.5 A, layer separation

$$V_-(q)\ll V_+(q)$$

Full Hamiltonian SU(4) invariant up to a weak λ–λ term

Ordering types

Order parameter a 4x4 hermitian matrix

$$\hat{\Delta} = \Delta \hat{Q} \qquad \hat{Q}^2 = 1$$

sign plus or minus

eigenvalues +1 or -1

Classify according to the number of positive and negative eigenvalues: (4,0), (3,1), (2,2)

(4,0)	(3,1), (2,2)
discrete Z_2 symmetry	continuous symmetry
SU(4) singlet	SU(3) or SU(2)xSU(2)
broken time reversal symmetry	Goldstone modes suppress ordering

Anomalous Hall insulator state

Dirac Hamiltonian with a T-noninvariant mass term: realization of parity anomaly (Jackiw '84, Haldane '88), nonzero Hall conductance even at B=0

$$\Delta_K = \Delta_{K'}$$

Contrast with the gap opened by transverse E field: zero Hall conductance

$$\Delta_K = -\Delta_{K'} = \epsilon$$

Phase diagram

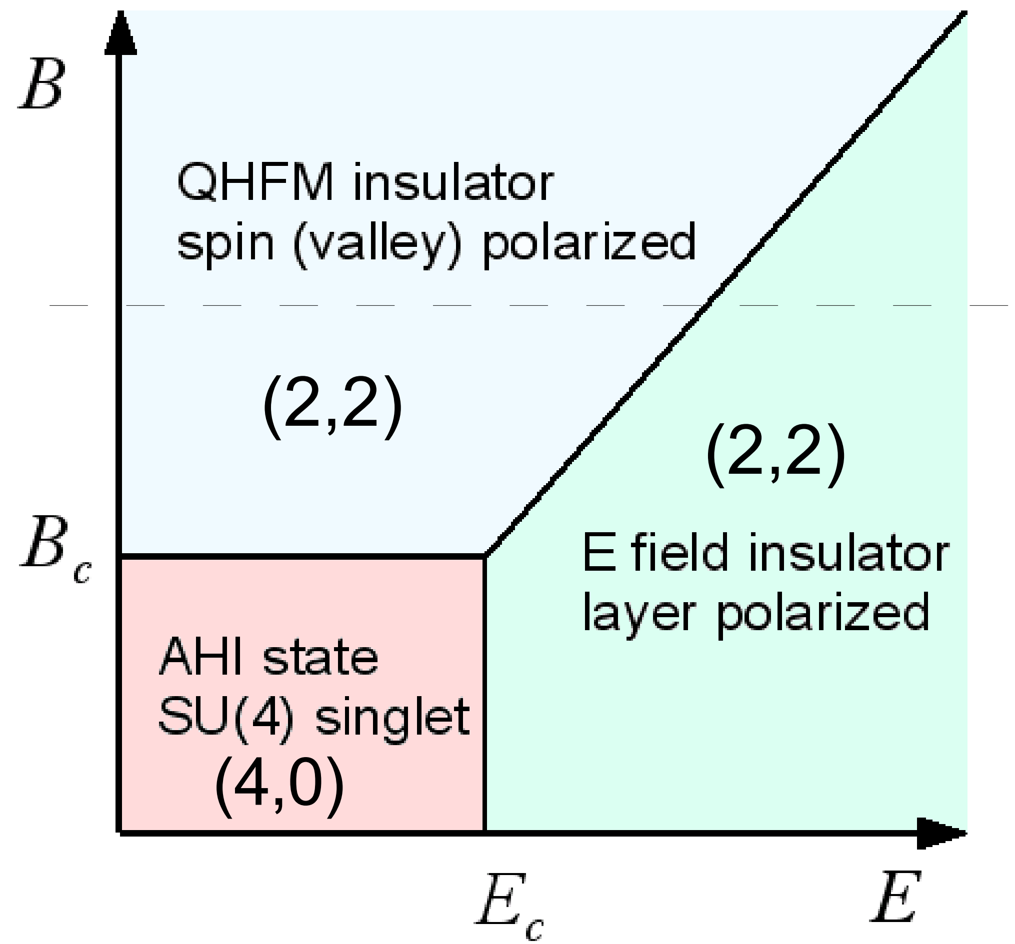
Magnetic field orbital coupling SU(4) invariant;
Zeeman coupling (non-invariant but weak) tends to polarize zeroth Landau level: Quantum Hall Ferromagnet state

Sigma model

$$F(Q) = \int J(\nabla Q)^2 + \frac{\epsilon}{4} tr\, Q \lambda_E$$

$$+ v_-(tr\, Q \lambda_E)^2 + \frac{E_z}{4} tr\, Q \lambda_z$$

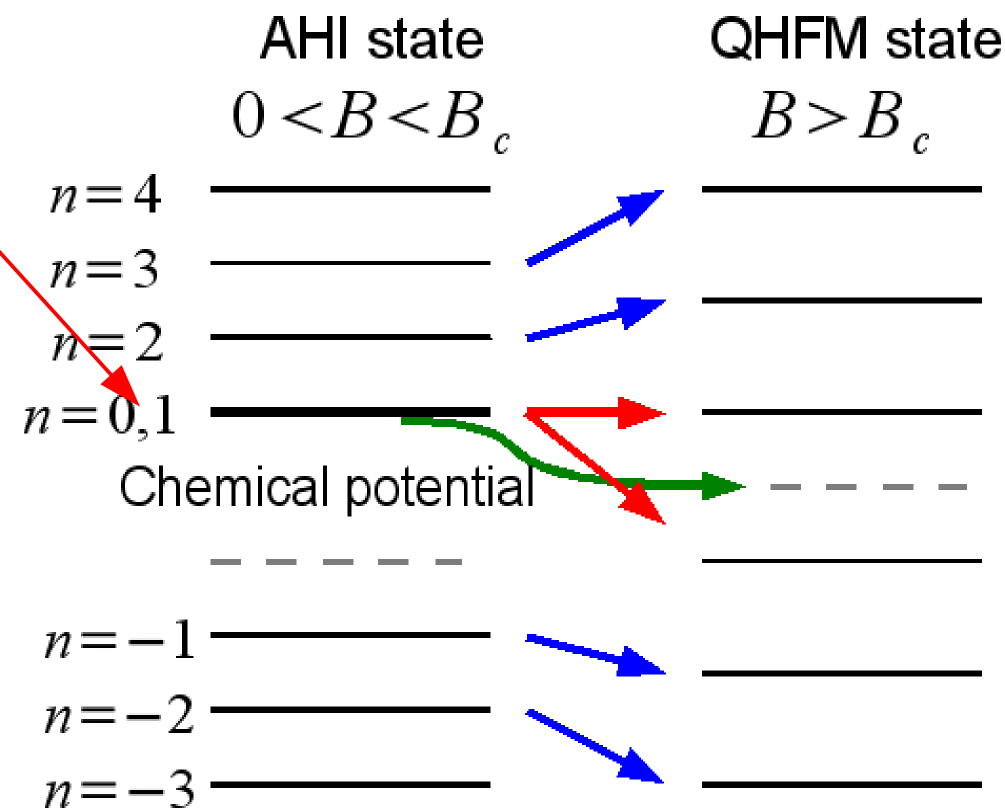
$$\lambda_E = \tilde{\eta}_{3,} \lambda_z = \tilde{\sigma}_{3,} v_- = \frac{NB}{32 \Phi_0} V_-$$



Transition from AHI to QHFM

anomalous Landau level

In AHI state B>0 the Fermi level is pinned to the anomalous Landau level: band bending at the domain boundaries



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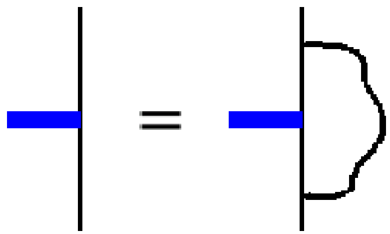
Excitonic gap without accounting for Z renormalization

$$\Delta = \int \frac{\Delta}{(iE + H_0)(iE - H_0)} U \qquad \tilde{U}_{q,\omega} = \frac{2\pi e^2}{q - 2\pi e^2 \Pi_q}$$



$$\Pi_{q,\omega} = \int G(p_+, \epsilon_+) G(p_-, \epsilon_-) = -\frac{m}{2\pi} f(2\mathrm{m}\omega/q^2)$$

$$f(w) = \frac{2\arctan w - \arctan 2\mathrm{w}}{w} + \ln\frac{w^2+1}{w^2+1/4} \approx \frac{\ln 4}{\sqrt{1+u\,w^2}}$$

$$u = (2\ln 4\pi)^2$$

Dynamical screening crucial:

1) Log^2 divergence at IR

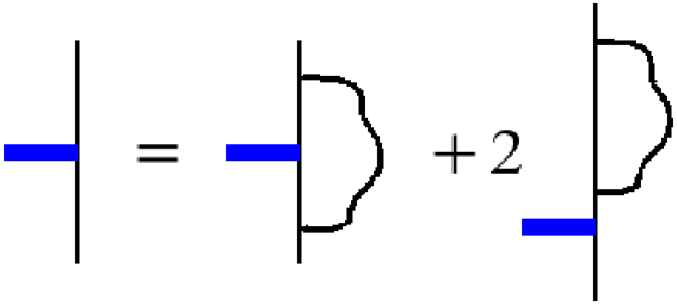
2) No UV divergence: dynamically generated cutoff

$$1 = \frac{1}{\pi^2 N}\ln^2\left(aN^2E_0/\Delta\right) \qquad N = 4$$

$$\Delta = aN^2 e^{-\pi\sqrt{N}} E_0 \qquad \Delta = 0.09E_0$$

Excitonic gap with Z renormalization

1) cancellation of Log^2 contributions
2) subleading Log term positive
(drives instability)
3) the gap Δ power-law in coupling strength

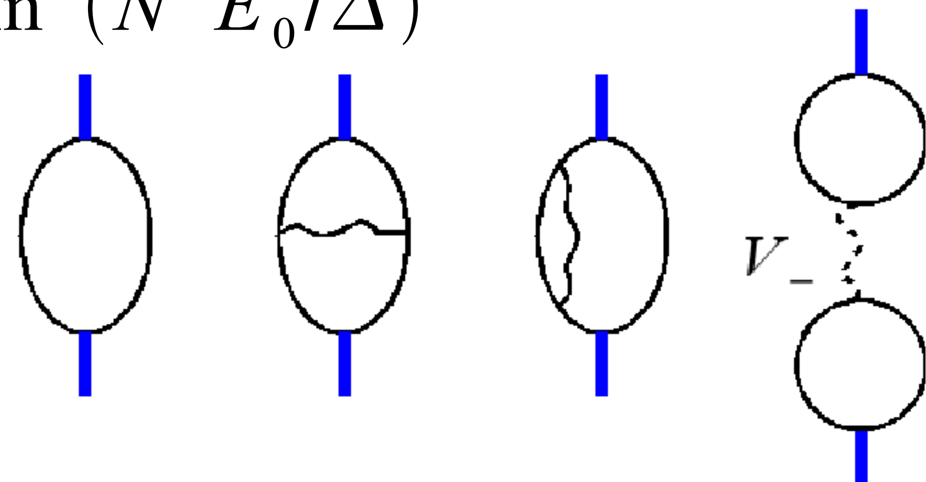


$$\delta E = \int \frac{d\omega\, d^2 p}{(2\pi)^3} \ln\left(1 - \tilde{U}_{\omega,q}\left(\Pi_{\omega,q,\Delta} - \Pi_{\omega,q,0}\right)\right)$$

$$F(\Delta) = \frac{m}{2\pi}\Delta^2 \ln \Lambda/\Delta - \frac{13\, m\, \Delta^2}{6\pi^3} \ln^2\left(N^2 E_0/\Delta\right)$$

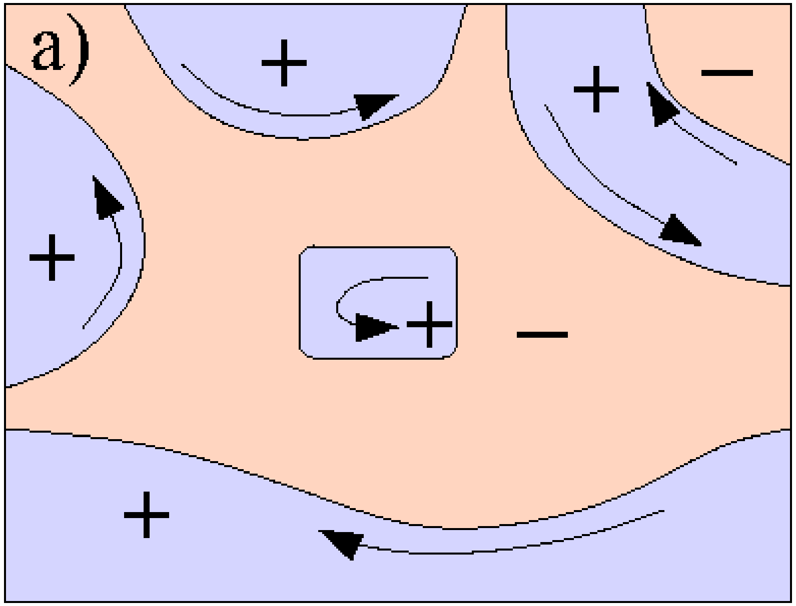
$$\Delta = N^2 E_0 \exp\left(-3\pi^2 N/13\right)$$

$$\Delta \approx 10^{-3} E_0$$



Excitonic instability features

- 'Which-layer' symmetry breaking
- Domains of + and – polarization in a uniform system, domain size controled by long-range dipole interactions
- Valley or charge polarized current along domain boundaries (Blanter, Martin, Morpurgo 2007)
- In a spatially nonuniform system (e/h droplets *a la* Yacoby): instability near p-n boundaries



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