Open books and twisted open books in 3-dimensional topology

A mini-course by F. Laudenbach (Lab. Jean Leray, Nantes)

Table of contents

The open book approach sets a deep relationship between 2-dimensional topology and geometric structures in dimension 3 and 4. Here is a striking example, which is a consequence of difficult results in contact geometry. Consider a compact oriented surface S with non-empty boundary and the relative *mapping class group* $MCG(S, \partial S)$ of the isotopy class of diffeomorphisms of S which are the Identity on the boundary.

> *There exists a diffeomorphism f of some S which is which is the identity on ∂S and right-veering, but which is not composed of positive Dehn twists.*

Here are the necessary definitions. A diffeomorphism $f : S \to S$ which is the identity on the boundary is said to be *right-veering* if for every proper simple α (proper means that the ends points are in ∂S) the image $f(\alpha)$ *lies on the right* of α, up to isotopy, in the following sense. If $\tilde{\alpha}$ is a lift of α to the universal cover $\widetilde{S}$ and if $\tilde{\beta}$ is a lift of the image $f(\alpha)$ from the same origin, $\tilde{\beta}$ lies on the right of $\tilde{\alpha}$ (except its origin), which makes sense since $\tilde{\alpha}$ separates $\widetilde{S}$ into two parts, a right one and a left one (with respect to the orientation of Of course, a positive Dehn twist along any simple closed curve which is not null-homotopic in S has this property.

The above-mentioned statement is not reachable in this course which intends to be only an introduction to open books. Just for exciting curiosity, this statement is a consequence of the fact that there are tight contact structures in dimension 3 which are not Stein fillable in dimension 4 (Giroux, Honda-Kazez-Matić, Etnyre-Honda).