

# OPEN BOOKS AND TWISTED OPEN BOOKS IN 3-DIMENSIONAL TOPOLOGY

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The open book approach sets a deep relationship between 2-dimensional topology and geometric structures in dimension 3 and 4. Here is a striking example, which is a consequence of difficult results in contact geometry. Consider a compact oriented surface  $S$  with non-empty boundary and the relative *mapping class group*  $MCG(S, \partial S)$  of the isotopy class of diffeomorphisms of  $S$  which are the Identity on the boundary.

*There exists a diffeomorphism  $f$  of some  $S$  which is which is the identity on  $\partial S$  and right-veering, but which is not composed of positive Dehn twists.*

Here are the necessary definitions. A diffeomorphism  $f : S \rightarrow S$  which is the identity on the boundary is said to be *right-veering* if for every proper simple  $\alpha$  (proper means that the ends points are in  $\partial S$ ) the image  $f(\alpha)$  lies on the right of  $\alpha$ , up to isotopy, in the following sense. If  $\tilde{\alpha}$  is a lift of  $\alpha$  to the universal cover  $\tilde{S}$  and if  $\tilde{\beta}$  is a lift of the image  $f(\alpha)$  from the same origin,  $\tilde{\beta}$  lies on the right of  $\tilde{\alpha}$  (except its origin), which makes sense since  $\tilde{\alpha}$  separates  $\tilde{S}$  into two parts, a right one and a left one (with respect to the orientation of  $\tilde{S}$ ). Of course, a positive Dehn twist along any simple closed curve which is not null-homotopic in  $S$  has this property.

The above-mentioned statement is not reachable in this course which intends to be only an introduction to open books. Just for exciting curiosity, this statement is a consequence of the fact that there are tight contact structures in dimension 3 which are not Stein fillable in dimension 4 (Giroux, Honda-Kazez-Matić, Etnyre-Honda).