Heat transport (measurements) in nanostructures

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Outline:

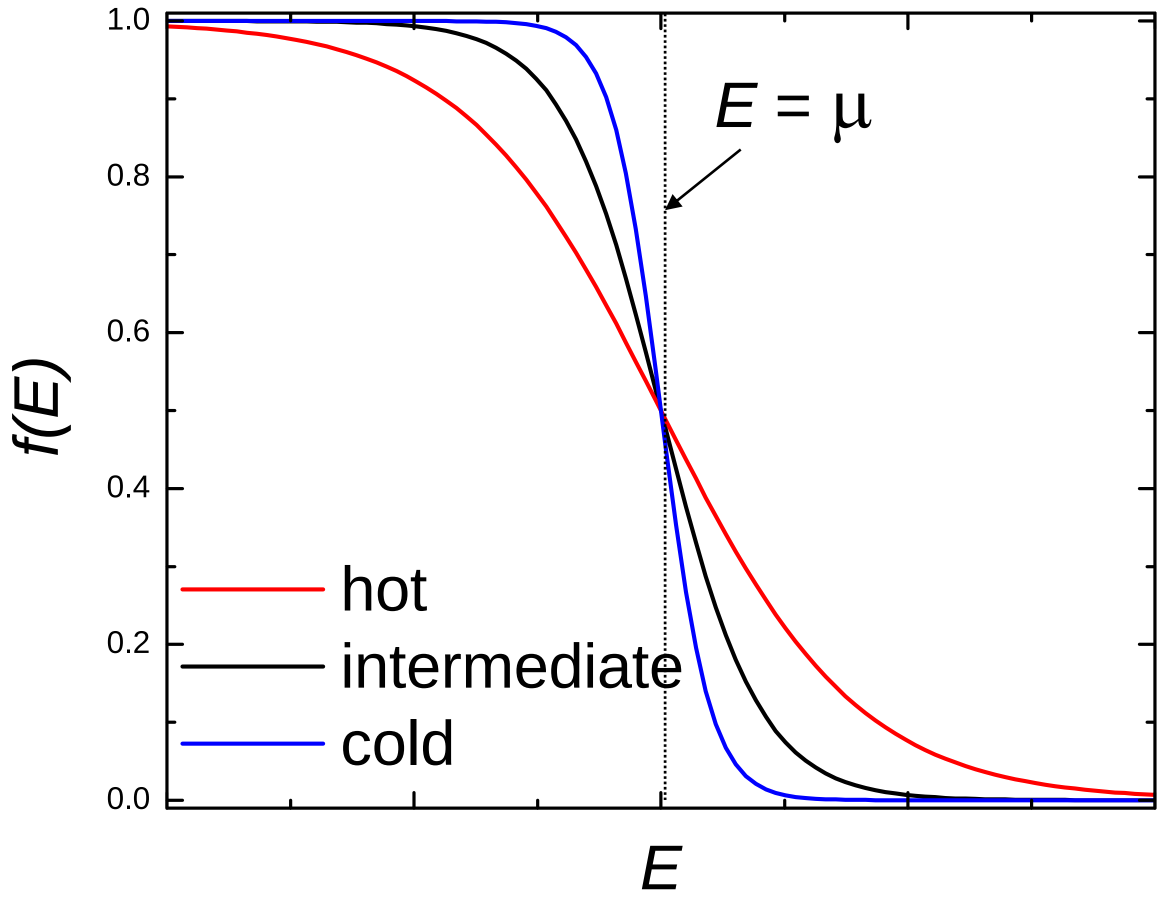
1. General considerations, thermometry, cooling
2. Electron-electron and electron-phonon relaxation in metals
3. Photonic heat transport in nanostructures
4. Heat conduction in metals and superconductors
5. Relaxation by recombination in superconductors

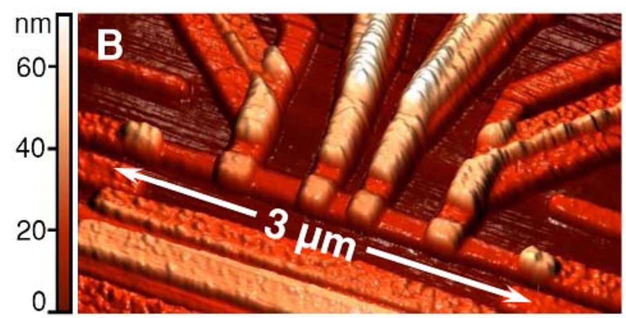
Low temperature laboratory

Temperature and energy relaxation

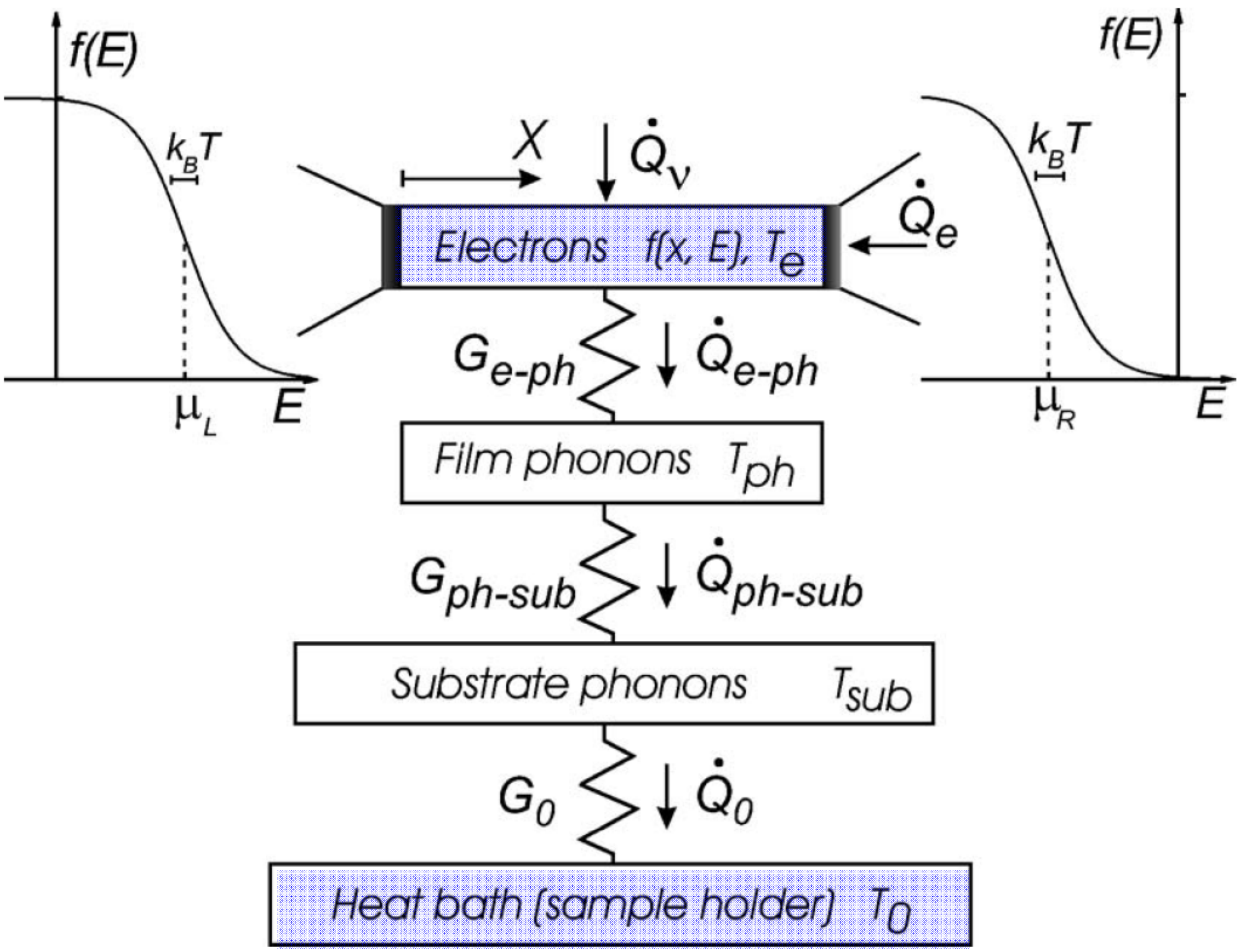
Temperature in an electronic device

$$f(E) = \frac{1}{1 + e^{(E-\mu)/k_B T}}$$

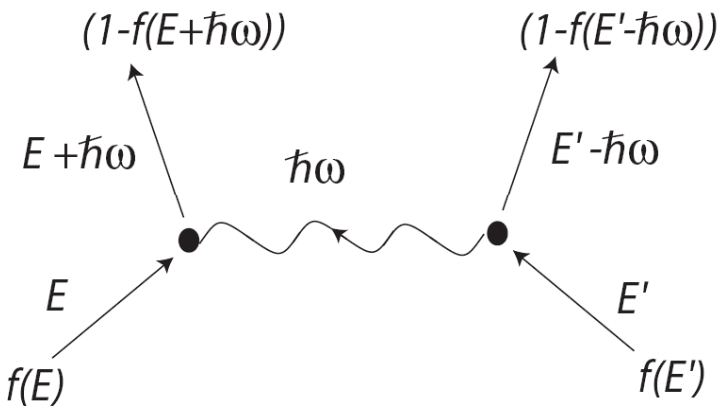




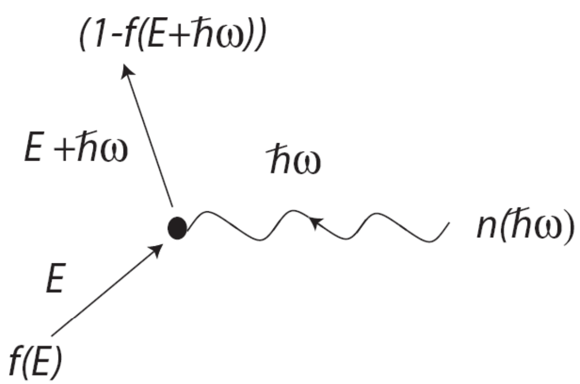
Generic thermal model for an electronic "thermometer"

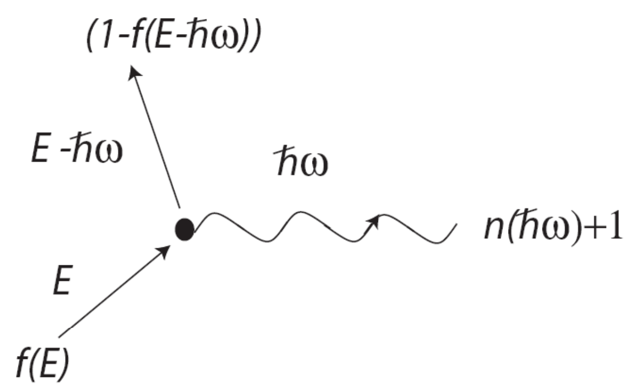


Electron-electron and electron-phonon relaxation



e-e relaxation drives the system towards *quasi-equilibrium*



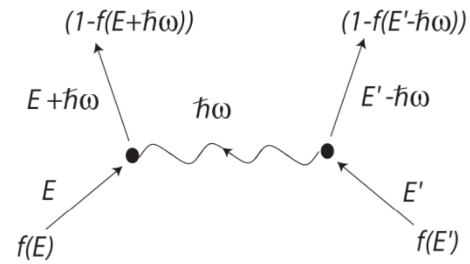


e-p relaxation drives the system towards *equilibrium*

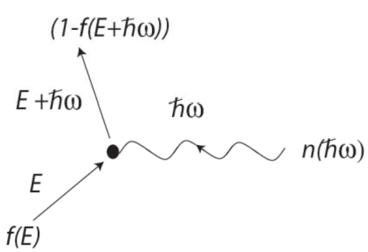
Kinetic equation

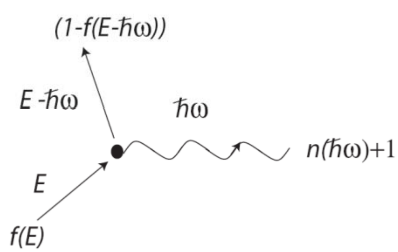
$$\frac{\partial f(E,t)}{\partial t} = I_{\rm inel}[f] = I_{\rm e-e}[f] + I_{\rm e-ph}[f] + ...$$

(in a spatially homogeneous case)



$$I_{\rm e-e}[f] = -\int d\omega dE' K_{ee}(E,E',\omega)[f(E)f(E')(1-f(E+\hbar\omega))(1-f(E'-\hbar\omega)) \\ -(1-f(E))(1-f(E'))f(E+\hbar\omega)f(E'-\hbar\omega)]$$





$$I_{\rm e-ph}[f] = -\int_0^\infty d\omega K_{eph}(E,\omega)\{[f(E)(1-f(E-\hbar\omega)) - f(E+\hbar\omega)(1-f(E))] \\ \times [n(\hbar\omega)+1] - [f(E-\hbar\omega)(1-f(E)) - f(E)(1-f(E+\hbar\omega))]\, n(\hbar\omega)\}.$$

The energy distribution of electrons in a small metal conductor

The distribution is determined by energy relaxation:

Equilibrium –**Thermometer measures the temperature of the ”bath”**
Quasi-equilibrium –**Thermometer measures the temperature of the electron system which can be different from that of the ”bath”**
Non-equilibrium –**There is no well defined temperature measured by the ”thermometer”**

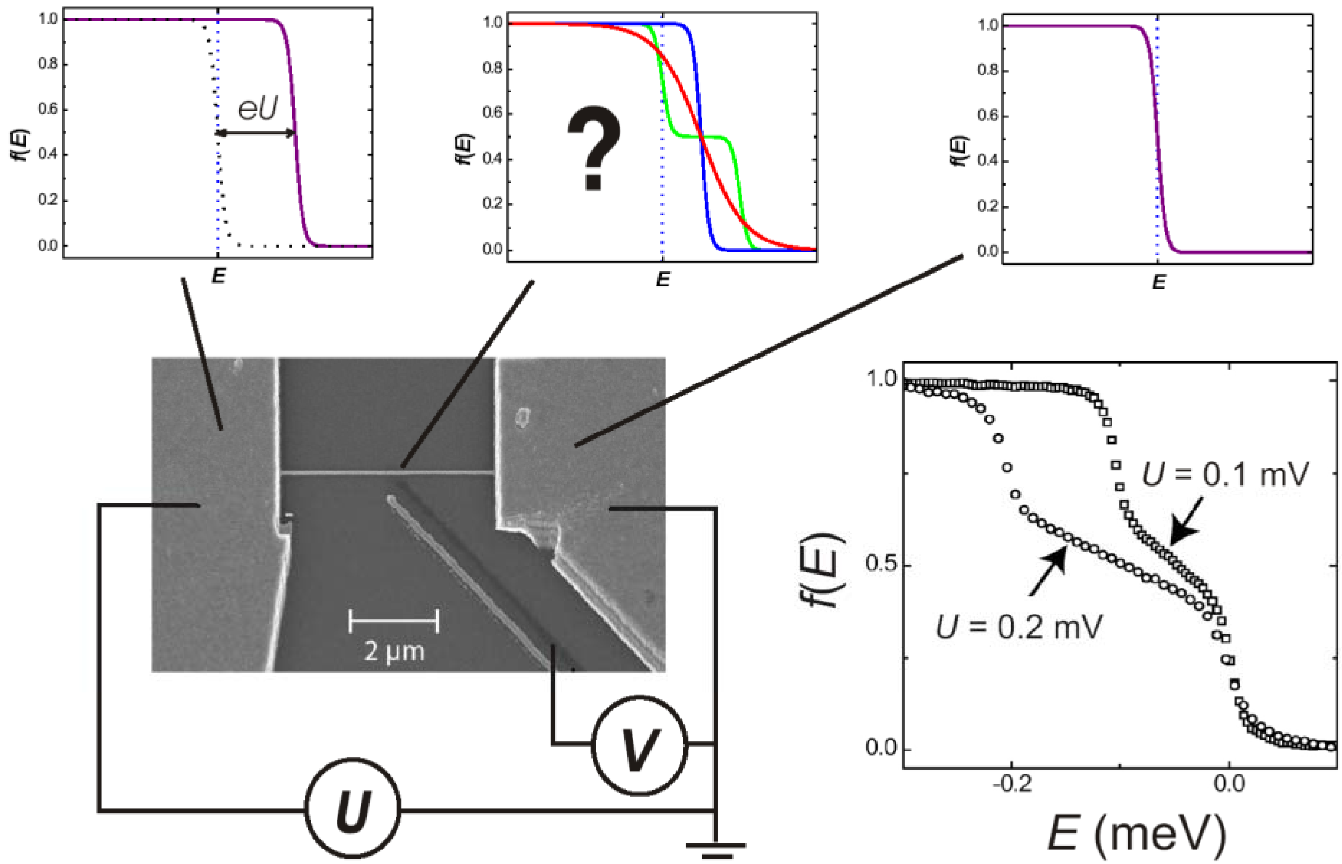
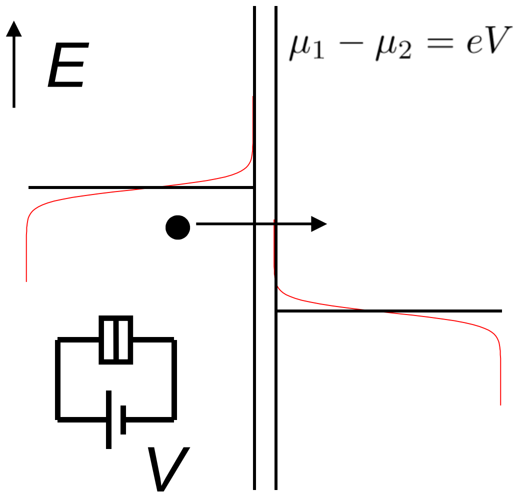


Illustration: diffusive normal metal wire
H. Pothier et al. 1997

THERMOMETRY

Metal – Insulator – Metal (MIM or NIN) tunnel junction



$$I(V) = \mathcal{T}^2 \int eN_1(E - eV)N_2(E)[f_1(E - eV) - f_2(E)]dE$$

Now, density of states (DOS) is almost constant over the small energy interval:

$$I(V) = e\mathcal{T}^2 N_1(0)N_2(0) \int [f_1(E - eV) - f_2(E)]dE$$

Quite generally:

$$\int [f_1(E - eV) - f_2(E)]dE = eV$$

$$\boxed{I = V/R_{\mathrm{T}}}$$

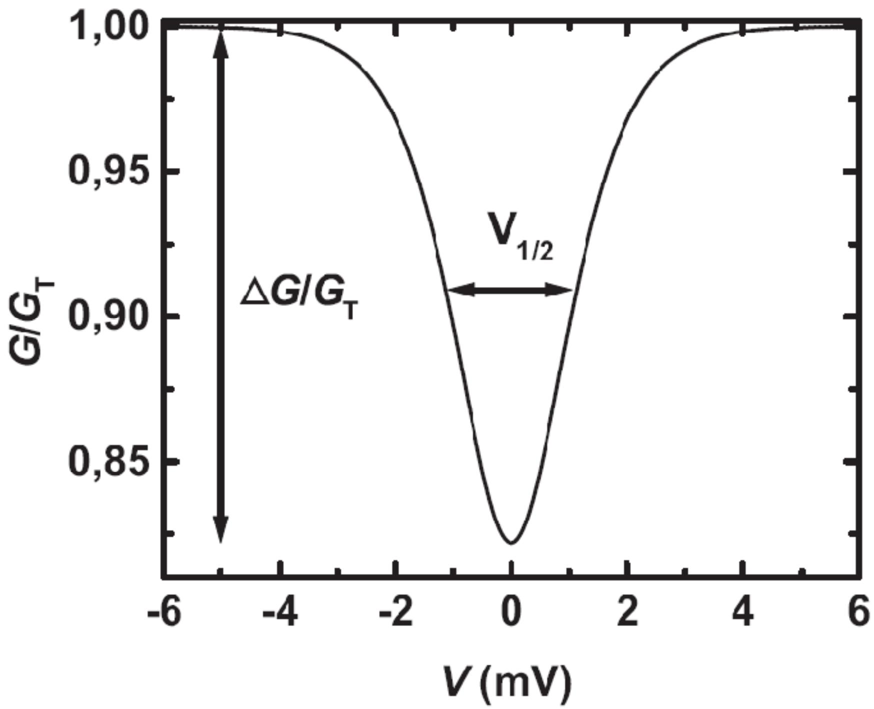
Ohmic, no temperature dependence

$$R_{\mathrm{T}} = [e^2\mathcal{T}^2 N_1(0)N_2(0)]^{-1}$$

NOT A THERMOMETER!

Basic properties of a Coulomb blockade thermometer (CBT)

A series array of *N* tunnel junctions in weak Coulomb blockade, $E_C << k_B T$:



$$V_{1/2} \simeq 5.439 N k_B T / e$$

primary thermometer

$$\Delta G / G_T = \frac{1}{6} \frac{E_C}{k_B T}$$

secondary thermometer

$$E_C \equiv [(N-1)/N] \, e^2 / C$$

J.P. et al., PRL 73, 2903 (1994)

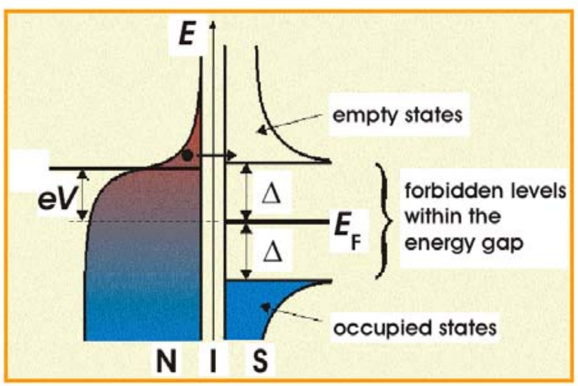
NIS-tunneling

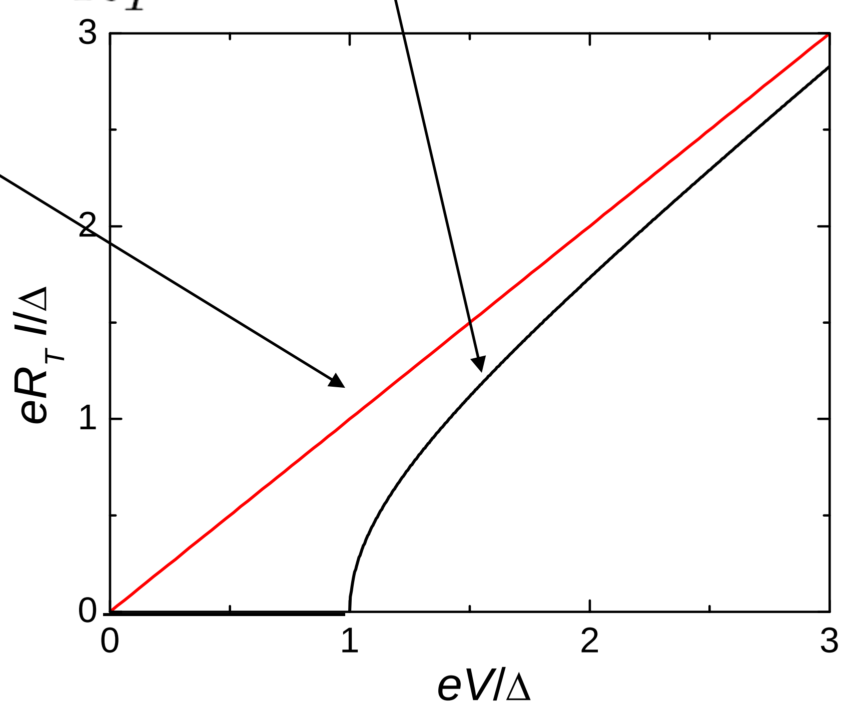
$$I = \frac{1}{eR_T}\int N_S(E)[f_N(E-eV) - f_S(E)]dE = \frac{1}{2eR_T}\int N_S(E)[f_N(E-eV) - f_N(E+eV)]dE$$

$$N_S(E) = \frac{|E|}{\sqrt{E^2-\Delta^2}}$$

$$I = \frac{1}{eR_T}\int_0^{eV} \frac{E}{\sqrt{E^2-\Delta^2}}dE = \frac{1}{R_T}\sqrt{V^2-(\Delta/e)^2},\ T=0$$

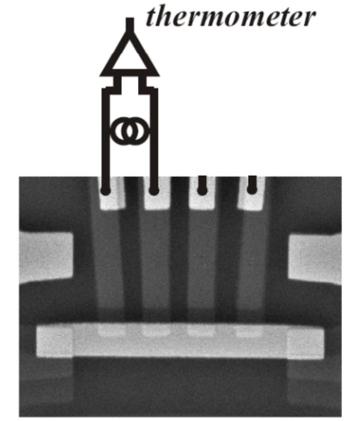
$$I = \frac{V}{R_T},\ T \geq T_C$$



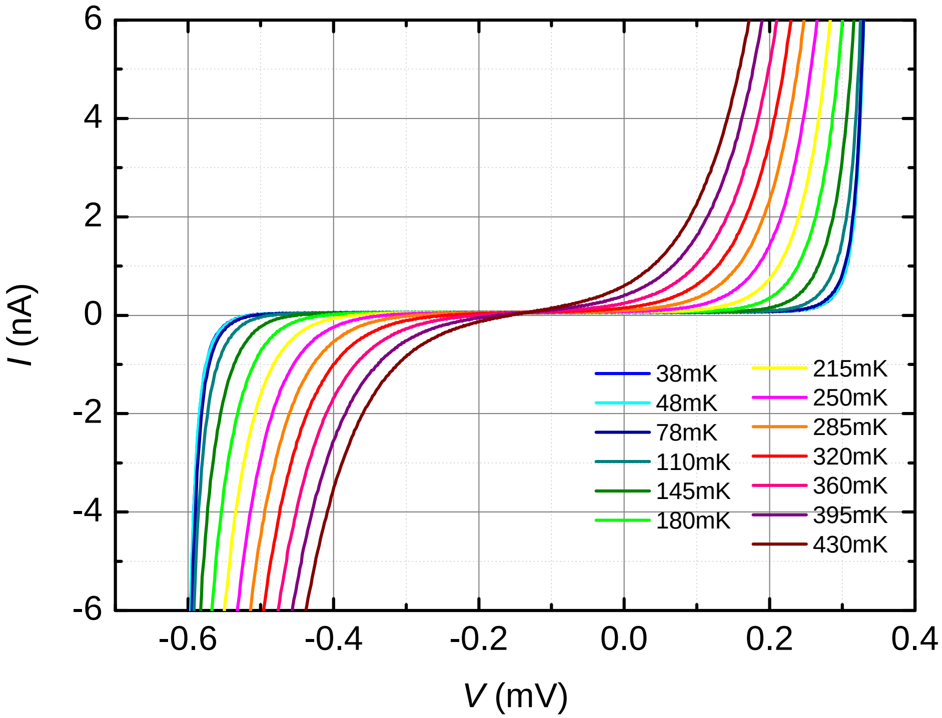


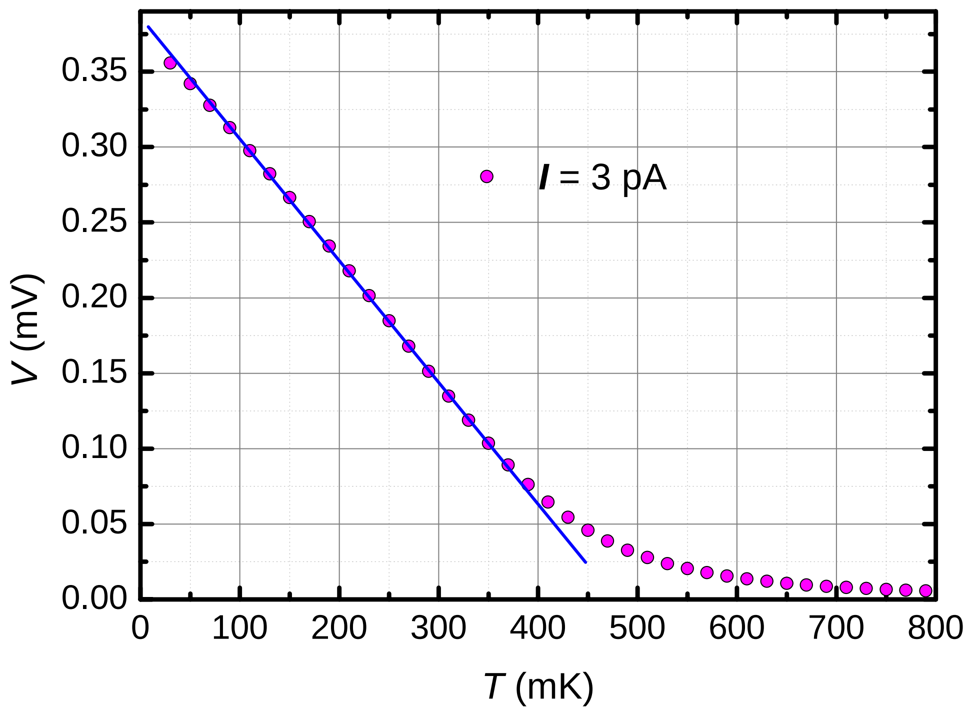
NIS-thermometry

$$I(V) = \frac{1}{2eR_{\mathrm{T}}} \int_{-\infty}^{\infty} N_S(E)[f_{\mathrm{N}}(E-eV) - f_{\mathrm{N}}(E+eV)]dE$$

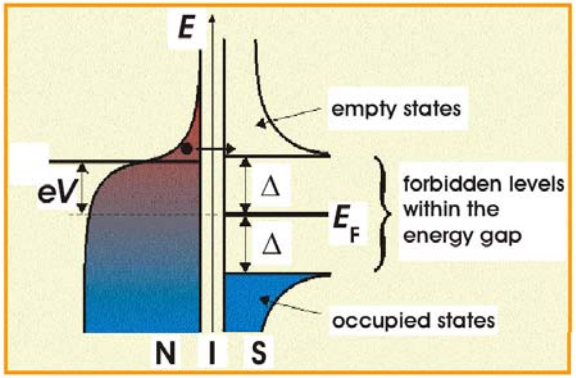


Probes electron temperature of N island (and not of S!)

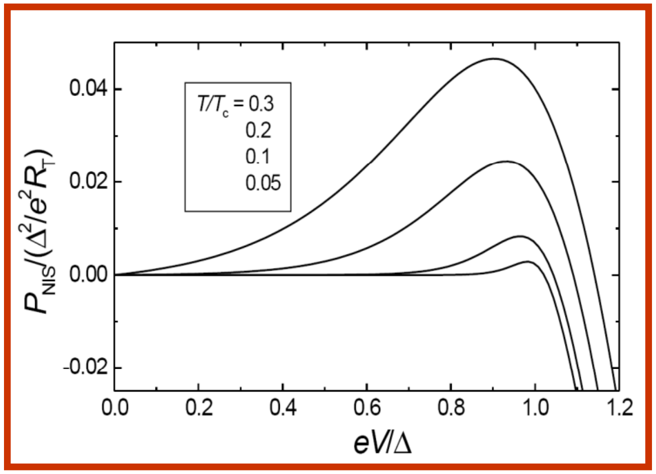




NIS junction as a refrigerator



Cooling power of a NIS junction:



$$P(V) = \frac{1}{eR_T}\int (E - eV)N_S(E)[f_N(E - eV) - f_S(E)]dE$$

Optimum cooling power is reached at $V \cong \Delta/e$:

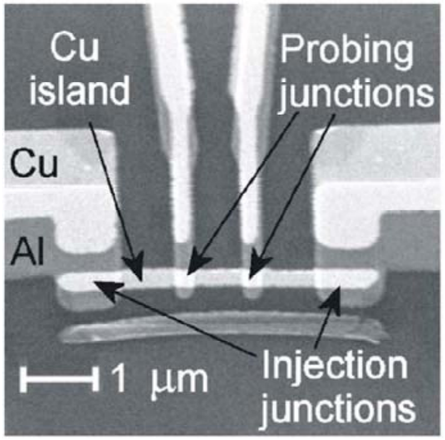
$$P_{\mathrm{NIS,max}} \simeq 0.59\frac{\Delta^2}{e^2R_T}(\frac{k_BT_N}{\Delta})^{3/2} - \frac{\Delta^2}{e^2R_T}\sqrt{\frac{2\pi k_BT_S}{\Delta}}\exp(-\frac{\Delta}{k_BT_S})$$

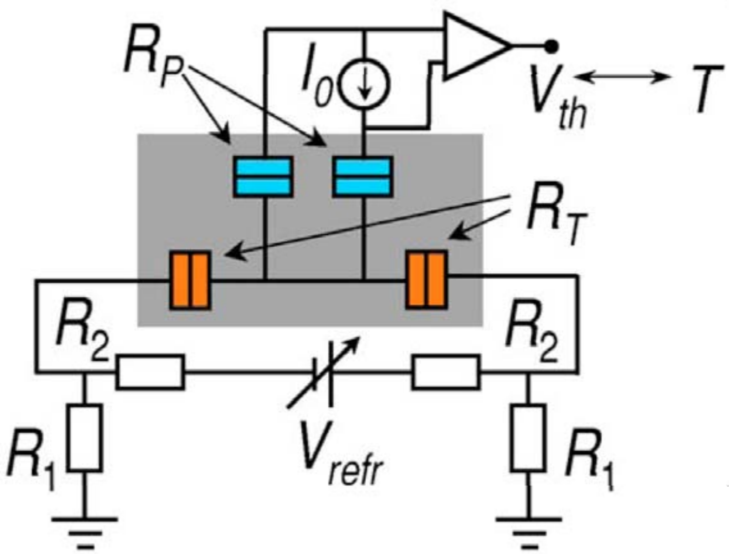
Optimum cooling power of a NIS junction at T_S ,T_N<< T_C

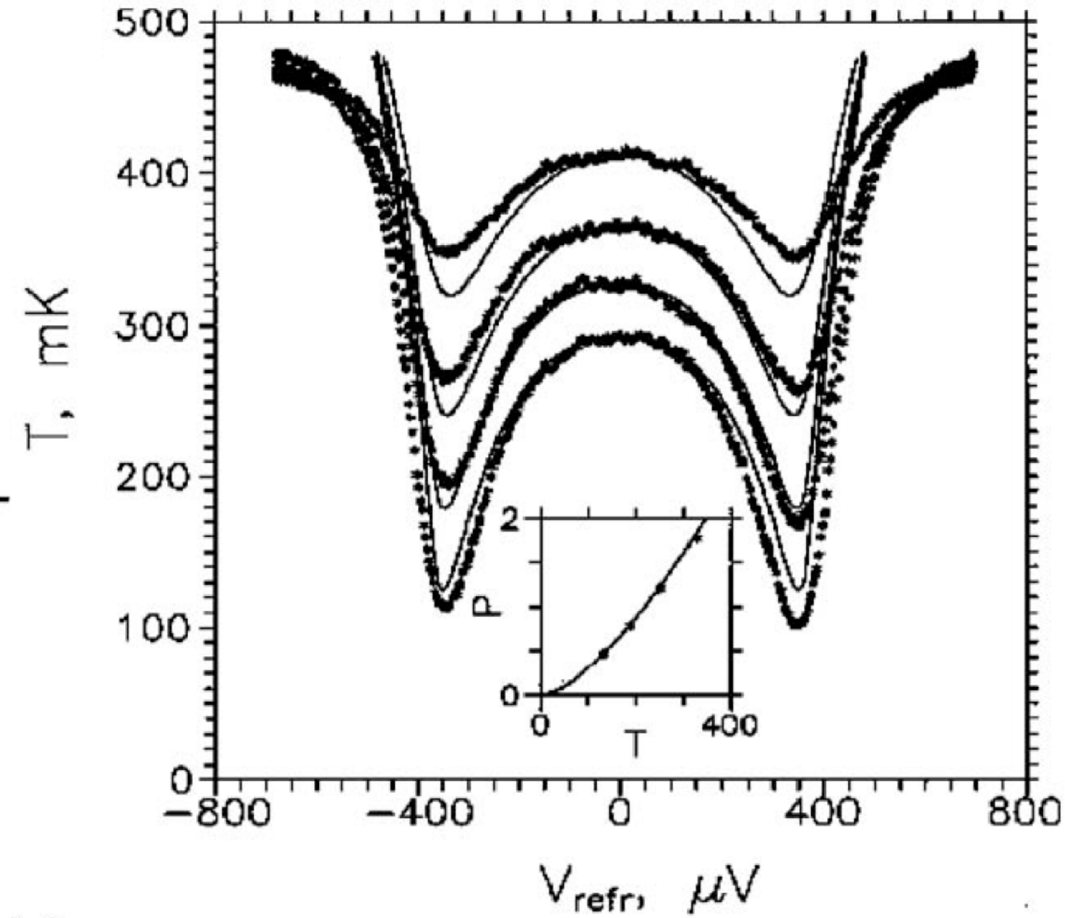
Efficiency (coefficient of performance) of a NIS junction refrigerator:

$$\eta \simeq k_BT/\Delta$$

Early experiments

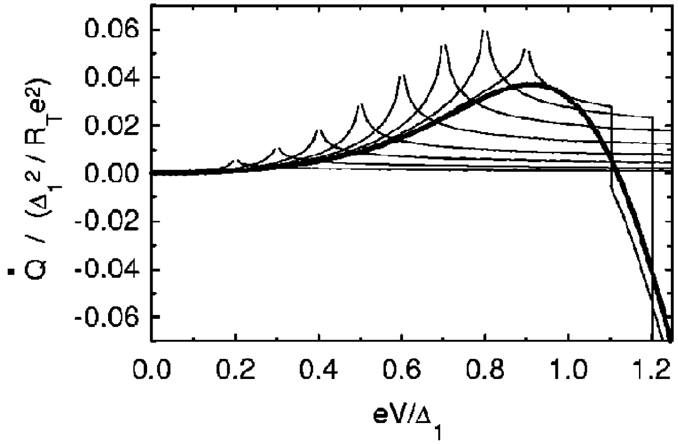






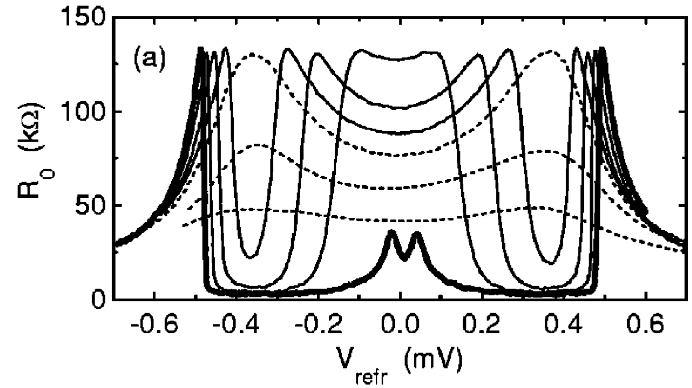
M. Leivo et al., 1996

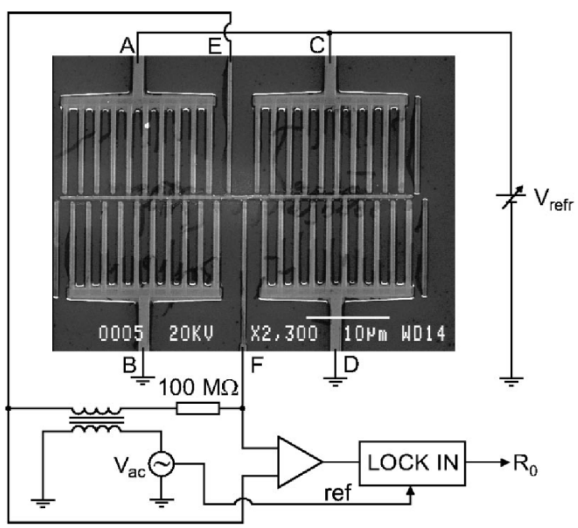
Cooling of a superconductor (SIS'IS cooler)



$$\dot{Q} = \frac{1}{e^2 R_T} \int_{-\infty}^{\infty} [f(\epsilon, T_{e2}) - f(\epsilon - eV, T_{e1})]$$

$$\times N_2(\epsilon) N_1(\epsilon - eV)\epsilon\, d\epsilon$$



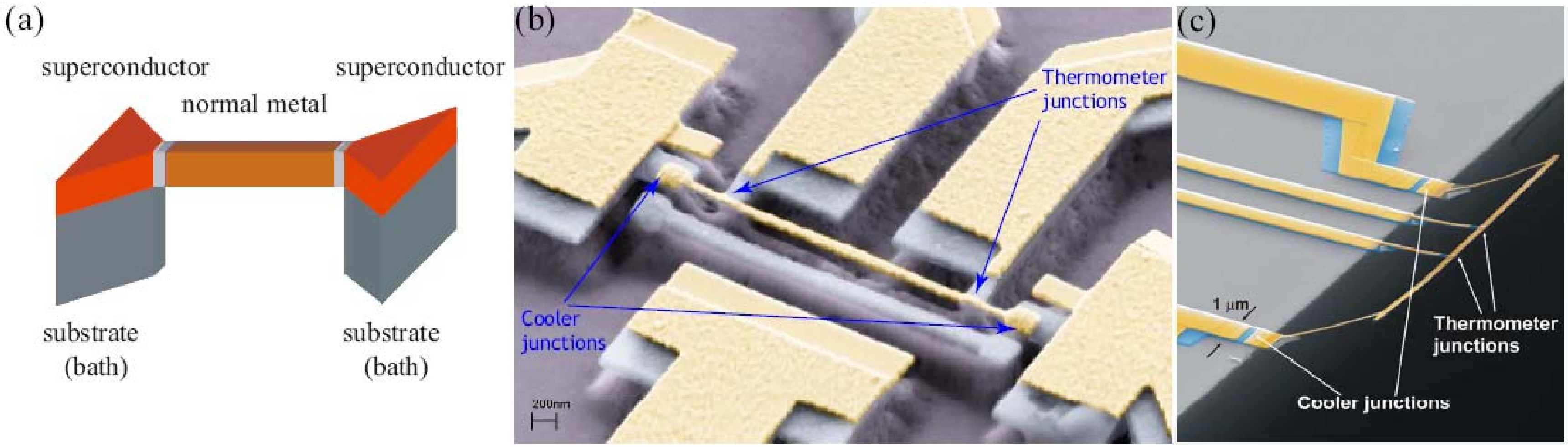


Ti – Al sample [T_C(Ti) = 0.5 K, T_C(Al) = 1.3 K]

COOLING FROM NORMAL TO SUPERCONDUCTING STATE

A. J. Manninen et al., Appl. Phys. Lett. **74**, 3020 (1999).

Cooling nanomechanical beams



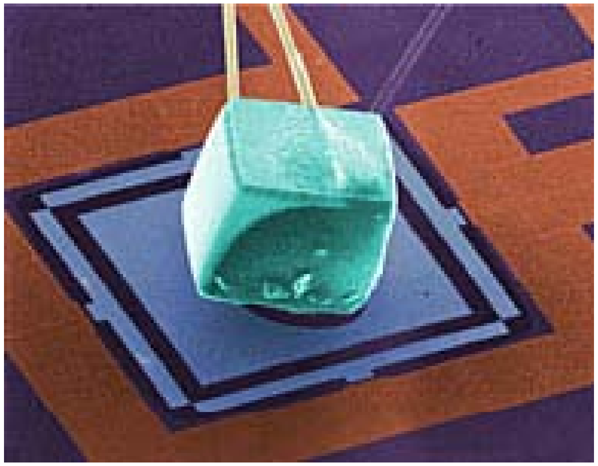
Cooling phonons as well
Koppinen et al PRL 2009
Is e-ph coupling as in bulk? T^3 instead of T^5?
Hekking et al, PRB 2008, Muhonen et al, APL 2009

Experimental status

Nahum, Eiles, Martinis 1994 *Demonstration of NIS cooling*
Leivo, Pekola, Averin 1996, Kuzmin 2003, Rajauria et al. 2007 *Cooling electrons 300 mK -> 100 mK by SINIS*
Manninen et al. 1999 *Cooling by SIS'IS* see also Chi and Clarke 1979 and Blamire et al. 1991, Tirelli et al. 2008
Manninen et al. 1997, Luukanen et al. 2000 *Lattice refrigeration by SINIS*
Savin et al. 2001 *S – Schottky – Semic – Schottky – S cooling*
Clark et al. 2005, Miller et al. 2008 *x-ray detector refrigerated by SINIS*
Prance et al. 2009 *Electronic refrigeration of a 2DEG*
Kafanov et al. 2009 *RF-refrigeration*
Koppinen et al. And Muhonen et al. 2009 *Cooling nanomechanical beams*
Quaranta et al. 2011 *Cooling from 1 K to 0.4 K*

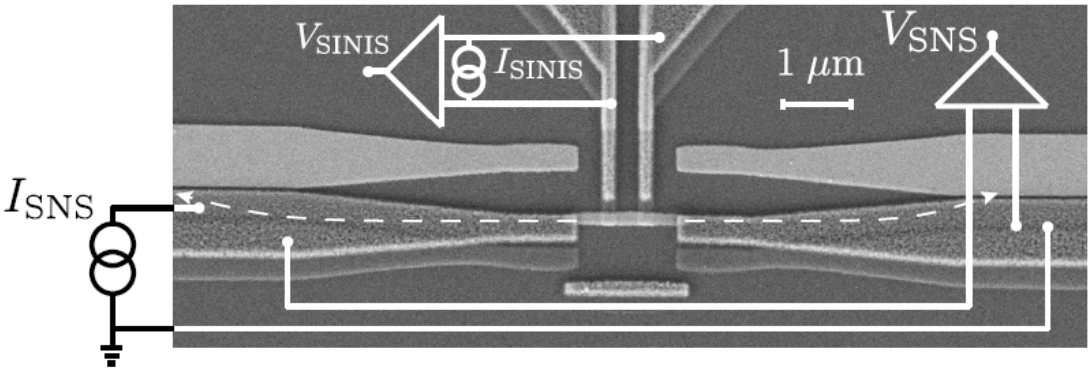
For a review, see Rev. Mod. Phys. 78, 217 (2006).

Refrigeration of a "bulk" object



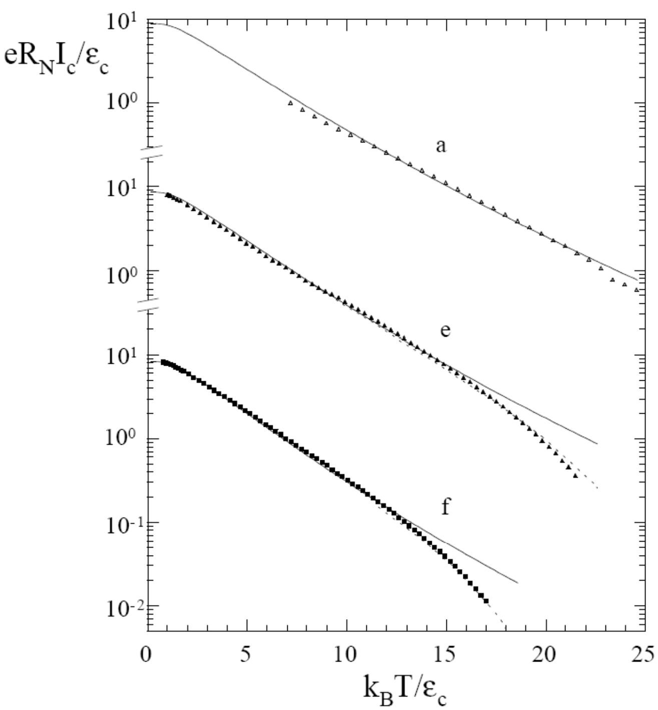
A. Clark et al., Appl. Phys. Lett. **86**, 173508 (2005).

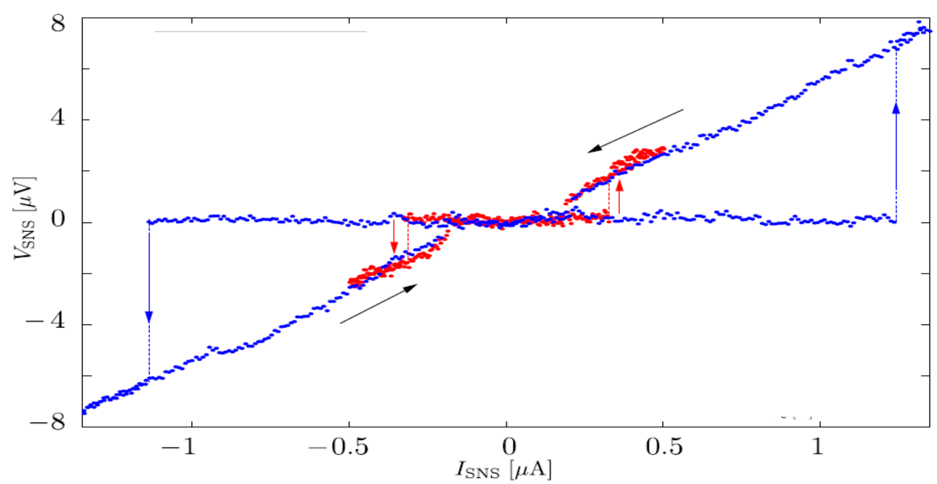
SNS Josephson junction as a thermometer



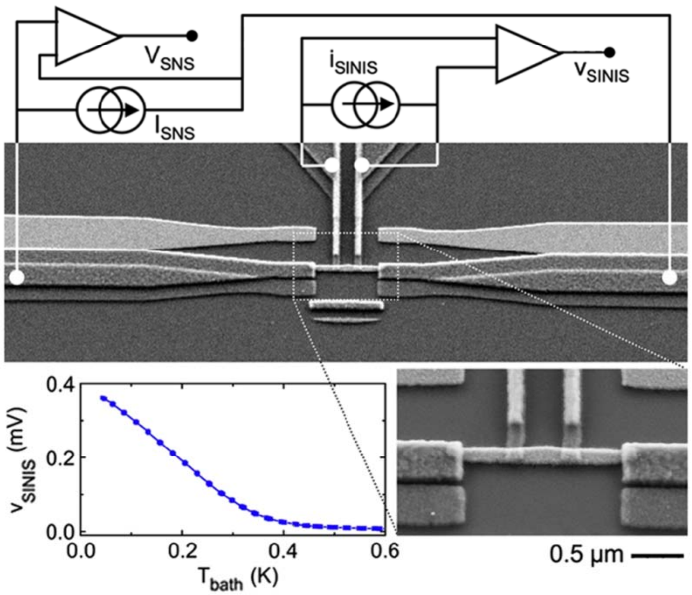
P. Dubos et al, 2001
H. Courtois et al, 2008
J. Peltonen et al, 2011

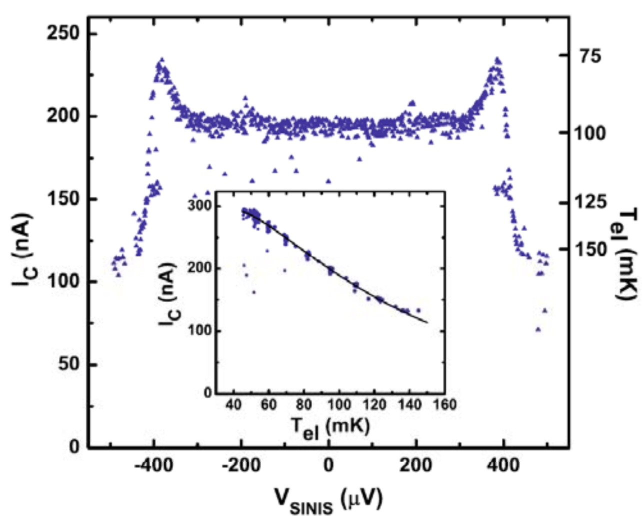
$$I_C = c\frac{(k_B T)^{3/2}}{eR_N\sqrt{E_T}}\exp\left(-\sqrt{\frac{2\pi k_B T}{E_T}}\right)$$

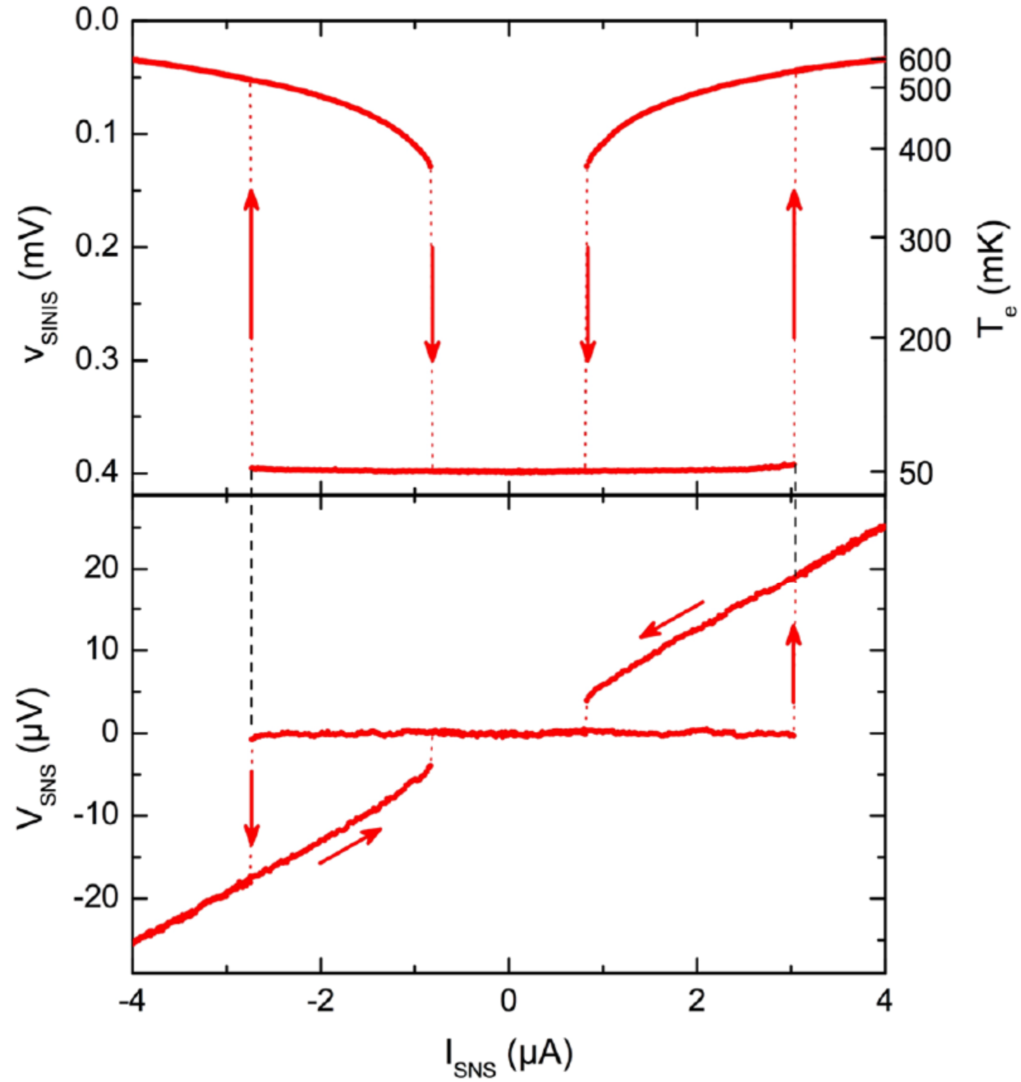




Thermal origin of hysteresis in SNS junctions



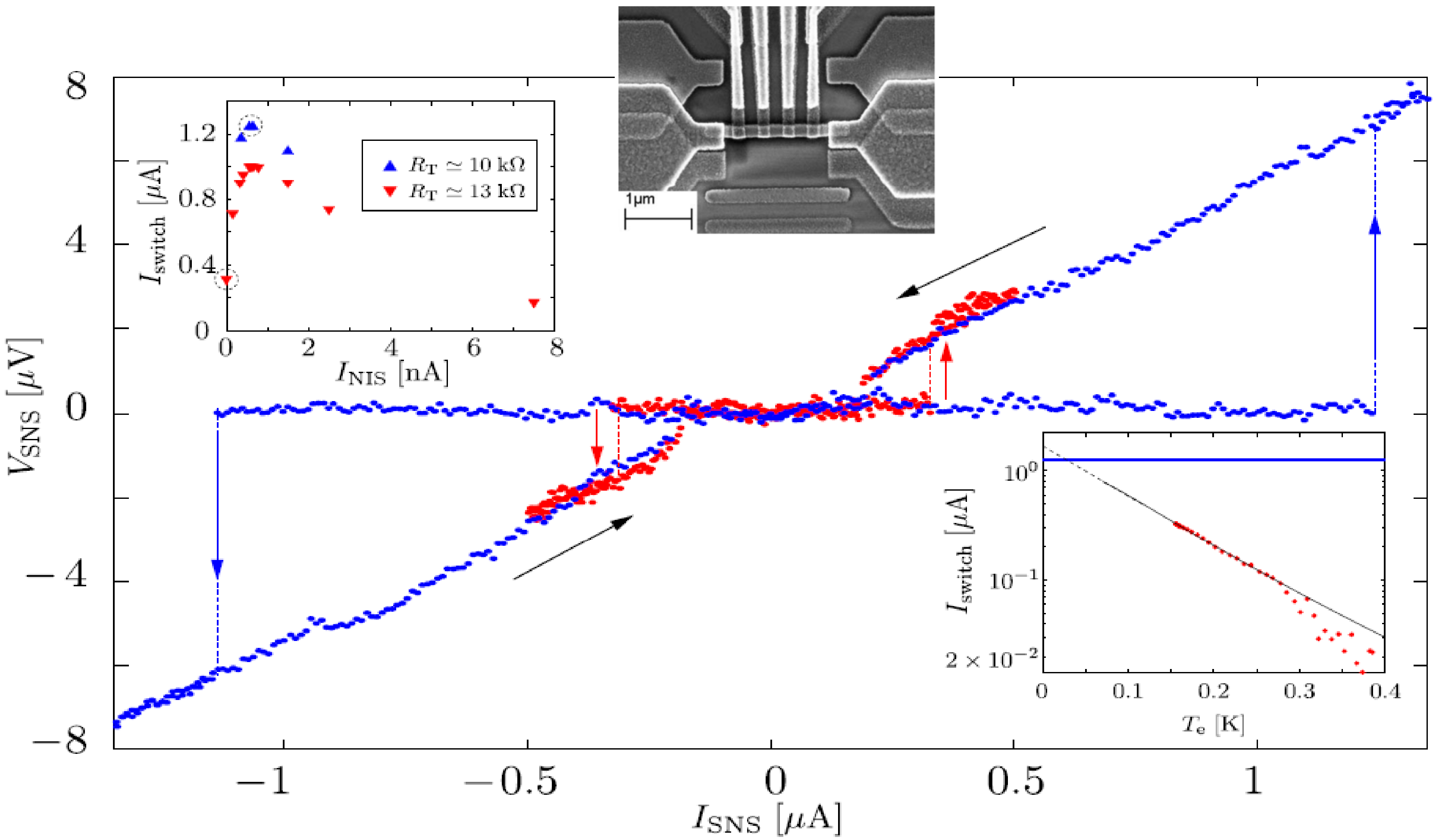




J. Peltonen PhD thesis, Courtois et al., PRL 2008

Low temperature limit

SNS proximity Josephson junction is a low-dissipative, unsaturating thermometer at low *T*: lowest *T* = 20 mK (+/- 10 mK). J. Peltonen et al, unpublished.

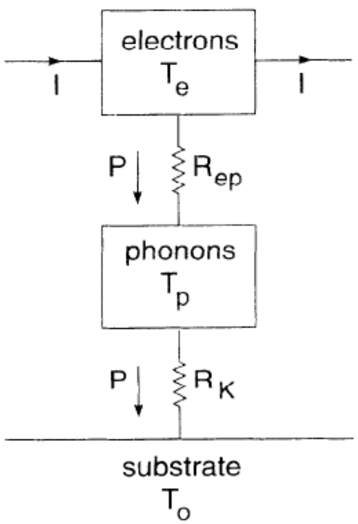


Electron-phonon relaxation in metals at low *T*

PHYSICAL REVIEW B VOLUME 49, NUMBER 9 1 MARCH 1994-I

Hot-electron effects in metals

F. C. Wellstood,* C. Urbina,† and John Clarke
Department of Physics, University of California, Berkeley, California 94720
and Materials Sciences Division, Lawrence Berkeley Laboratory, Berkeley, California 94720
(Received 21 July 1993)



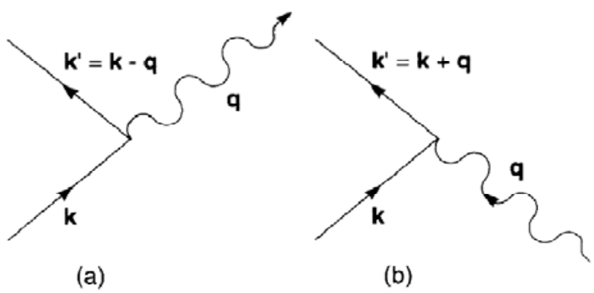
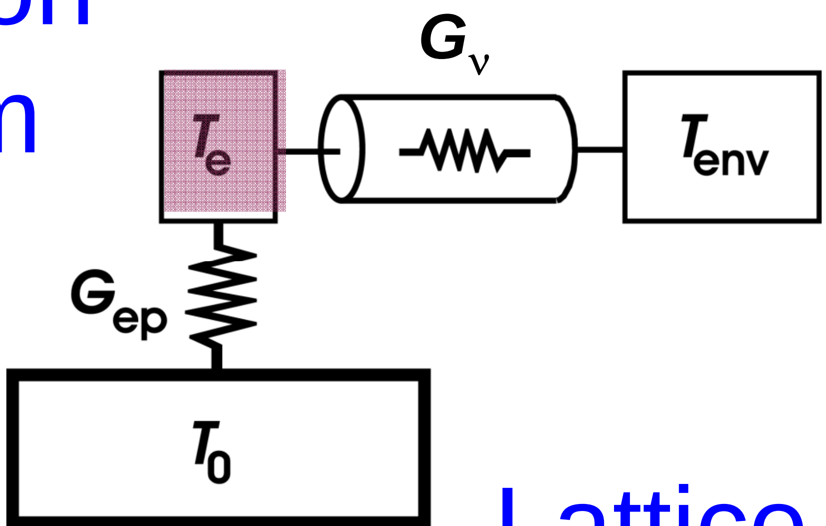


FIG. 2. Emission and absorption of phonons of wave vector **q** by an electron of wave vector **k**.

$$\boxed{\dot{Q}_{\mathrm{ep}} = \Sigma\Omega(T_{\mathrm{e}}^{5} - T_{\mathrm{p}}^{5})}$$

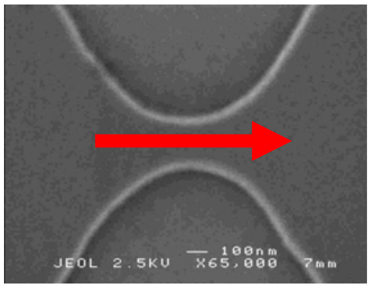
Electromagnetic transfer of heat (photons)



Schmidt et al., PRL 93, 045901 (2004)
Meschke et al., Nature 444, 187 (2006)
Ojanen et al., PRB 76, 073414 (2007), PRL 100, 155902 (2008)
D. Segal, PRL 100, 105901 (2008)

Quantized conductance

Electrons:



Electrical conductance in a ballistic contact:

$$\sigma_{\mathrm{Q}} = 2e^2/h$$

Thermal conductance:

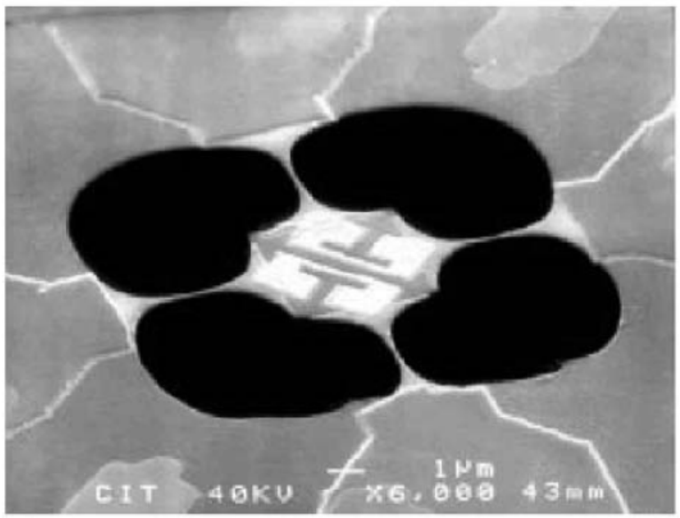
$$G_{\mathrm{Q}} = \frac{\pi k_{\mathrm{B}}^2}{6\hbar} T$$

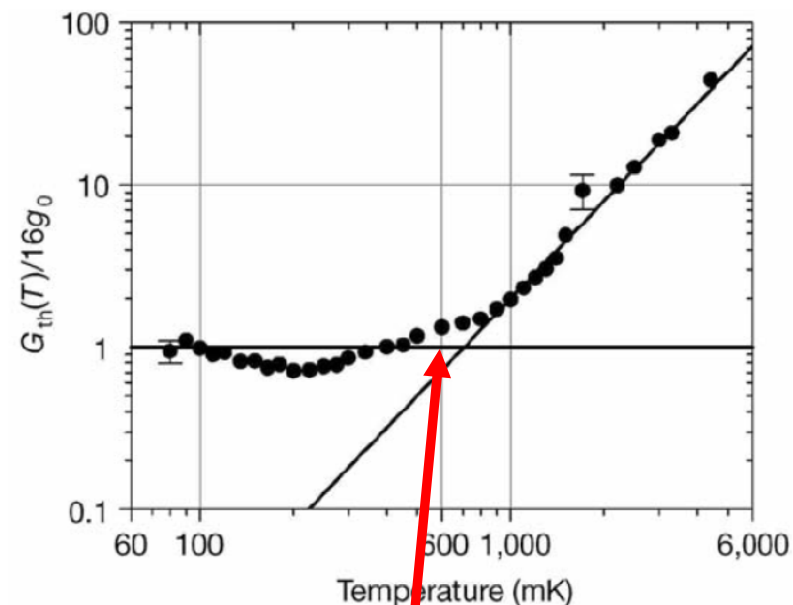
G_Q and σ_Q related by Wiedemann-Franz law

More generally:

Expression of G_Q is expected to hold for carriers obeying arbitrary statistics, in particular for electrons, phonons, photons (Pendry 1983, Greiner et al. 1997, Rego and Kirczenow 1999, Blencowe and Vitelli 1999).

Example of quantized thermal conductance: phonons in a nanobridge

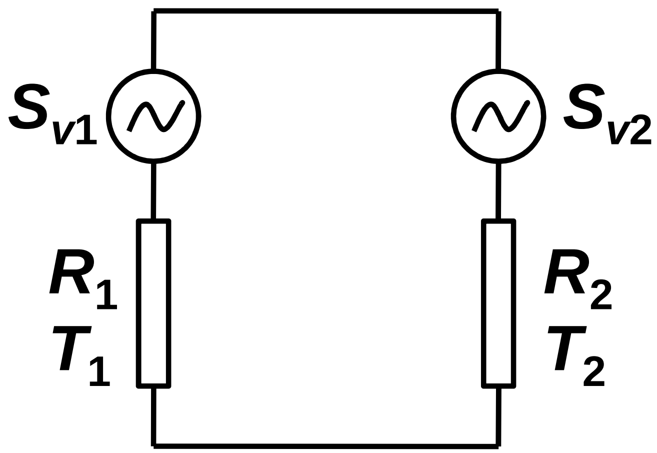




K. Schwab et al., Nature 404, 974 (2000)
C. Yung, D. Schmidt and A. Cleland, Appl. Phys. Lett. 81 31 (2002)

$$G = 4 \times 4 \times G_{\mathrm{Q}}$$

Radiative heat transport in an electrical circuit



Voltage noise of a resistor:

$$S_{Vi}(\omega) \simeq 4\hbar\omega R_i n_i(\omega)$$

Bose distribution:

$$n_i(\omega) = \frac{1}{e^{\hbar\omega/k_B T_i} - 1}$$

Current noise created by resistor 1:

$$S_{I1}(\omega) = S_{V1}(\omega)/|Z_{\text{tot}}|^2$$

$$Z_{\text{tot}} = R_1 + R_2$$

Spectrum of dissipation of energy created by resistor 1 and absorbed by resistor 2:

$$S_{P12}(\omega) = R_2 S_{I1}(\omega)$$

Heat transported between two resistors

Radiative contribution to net heat flow between electrons of 1 and 2:

$$P_\nu = \int_0^\infty \frac{d\omega}{2\pi}\left[S_{P12}(\omega) - S_{P21}(\omega)\right] = r\frac{\pi k_B^2}{12\hbar}(T_1^2 - T_2^2)$$

Coupling constant:

$$r \equiv \frac{4R_1R_2}{(R_1+R_2)^2}$$

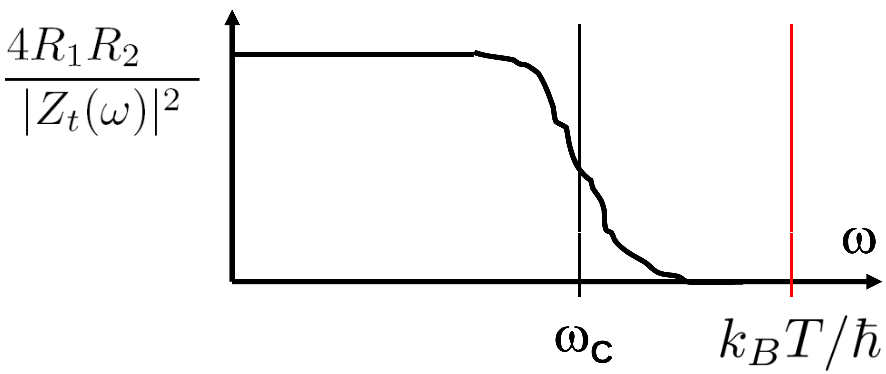
Linearized expression for small temperature difference $\Delta T = T_1 - T_2$:

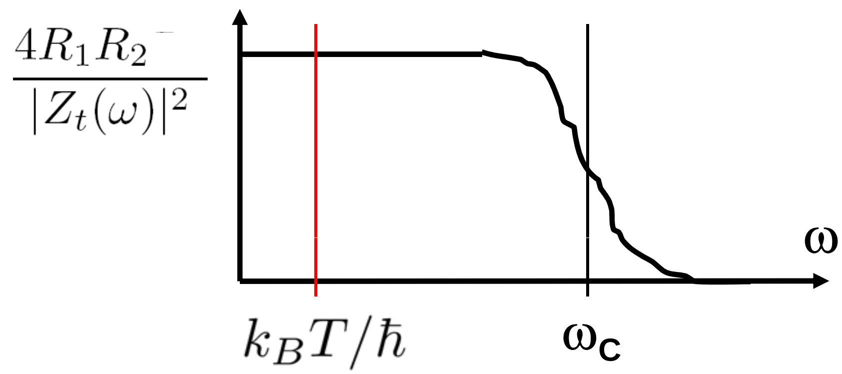
$$P_\nu = rG_\mathrm{Q}\Delta T \qquad \boxed{G_\mathrm{Q} = \frac{\pi k_\mathrm{B}^2}{6\hbar}T}$$

$$G_\nu = rG_\mathrm{Q}$$

Classical or quantum heat transport?

$$P_\nu = \int_0^\infty \frac{d\omega}{2\pi} \frac{4R_1 R_2 \hbar\omega}{|Z_t(\omega)|^2} \left(\frac{1}{e^{\hbar\omega/k_B T_1} - 1} - \frac{1}{e^{\hbar\omega/k_B T_2} - 1} \right)$$





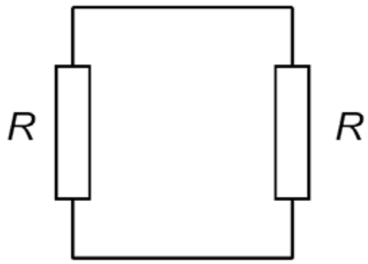
”Classical”

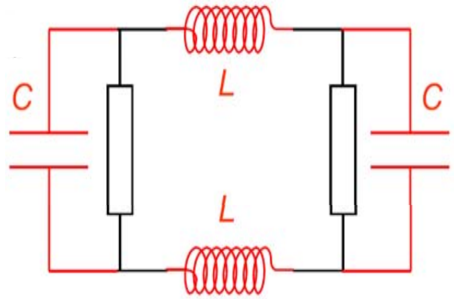
$$G_\nu \sim r k_B \omega_C$$

”Quantum”

$$G_\nu = r G_Q$$

Classical or quantum heat transport?





$$C \sim \epsilon \ell$$

$$L \sim \mu \ell$$

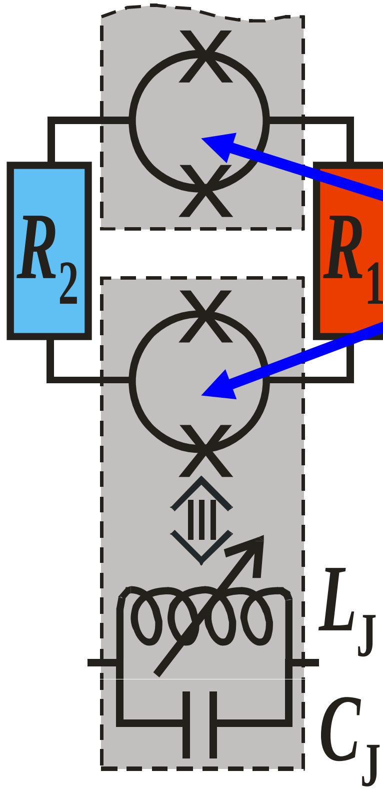
Classical: $\frac{\hbar}{k_B T} \frac{1}{RC} \ll 1$ $\quad \frac{\hbar}{k_B T} \frac{R}{L} \ll 1$ Johnson, Nyquist 1928

Quantum limited: $\frac{\hbar}{k_B T} \frac{1}{RC} \gg 1$ $\quad \frac{\hbar}{k_B T} \frac{R}{L} \gg 1$

$$T = 300\ \mathrm{K},\ \ell = 1\ \mathrm{cm}:\ \frac{\hbar}{k_B T} \frac{1}{RC} \sim \frac{\hbar}{k_B T} \frac{R}{L} < 10^{-3} \ll 1$$

$$T = 100\ \mathrm{mK},\ \ell = 100\ \mu\mathrm{m}:\ \frac{\hbar}{k_B T} \frac{1}{RC} \sim \frac{\hbar}{k_B T} \frac{R}{L} \sim 10^{2} \gg 1$$

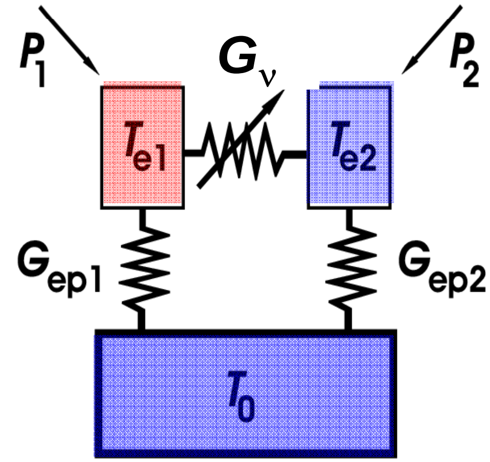
Demonstration of photonic heat conduction

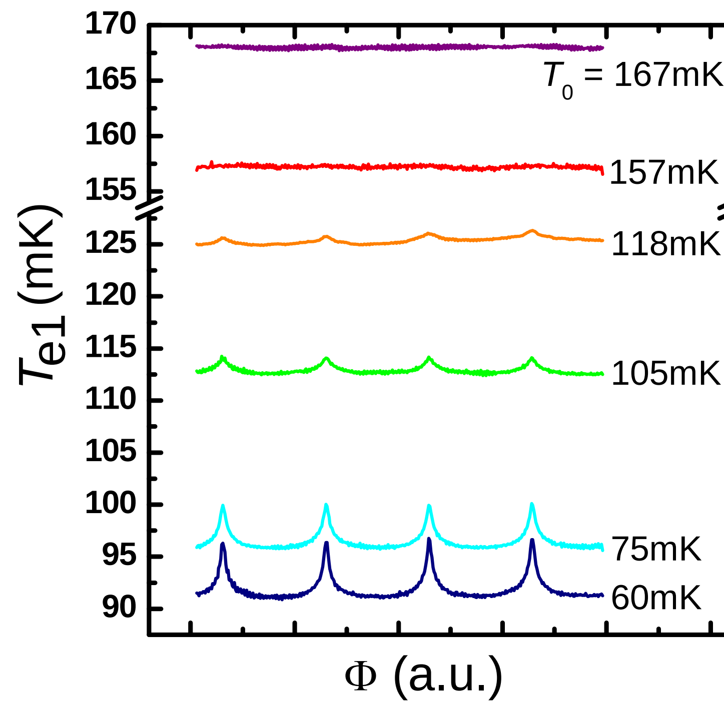


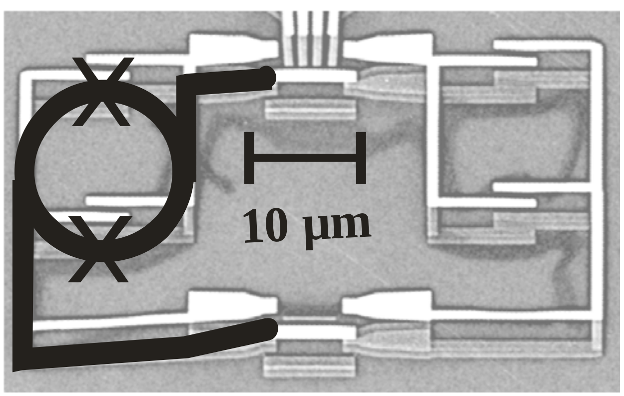
Tunable impedance matching using DC-SQUIDs

$$L_{\mathrm{J}} = \frac{\hbar}{2eI_{\mathrm{C},0}|\cos(\pi\Phi/\Phi_0)|}$$

Thermal model

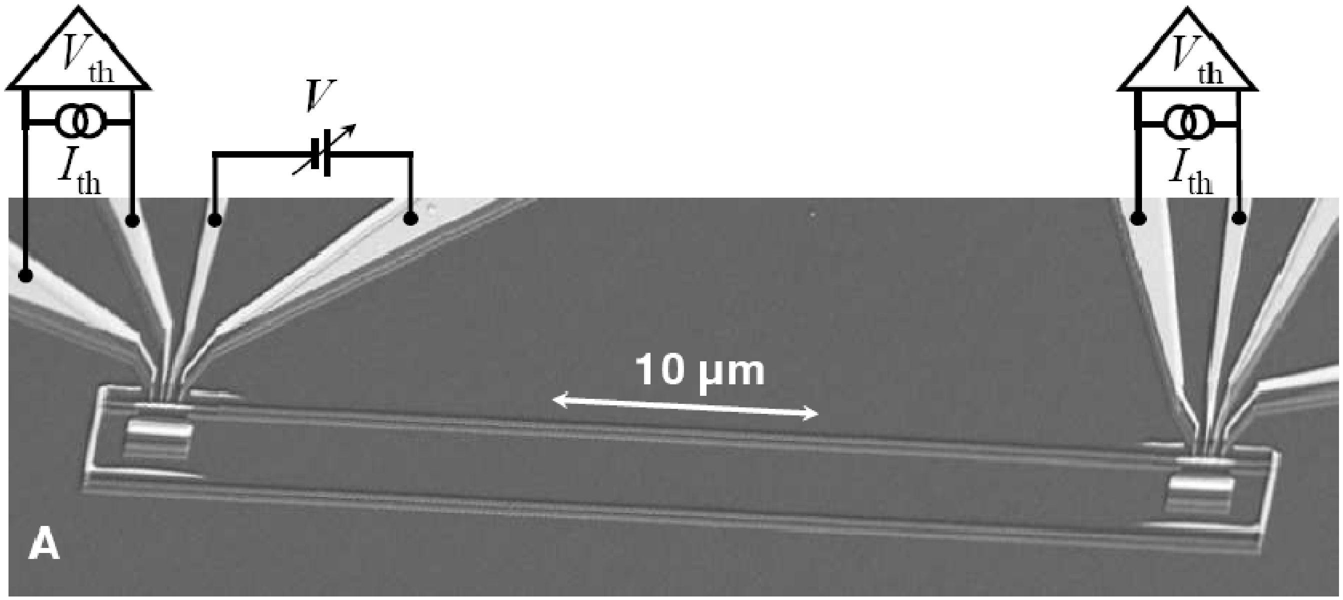


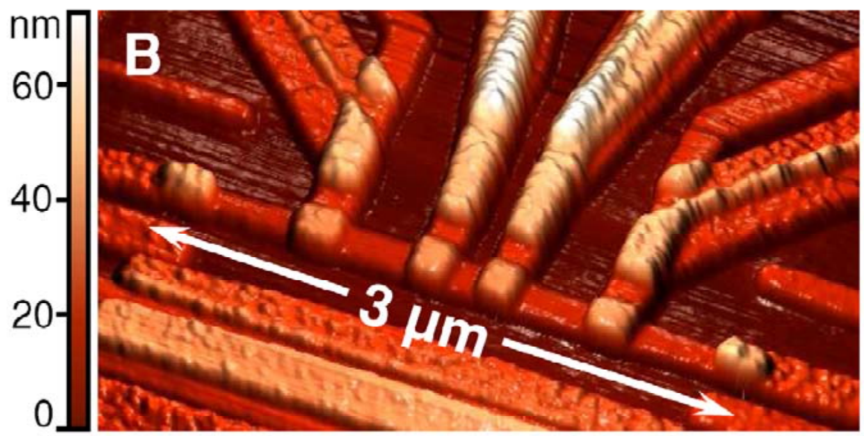




M. Meschke, W. Guichard and J.P., Nature 444, 187 (2006)

2nd experiment

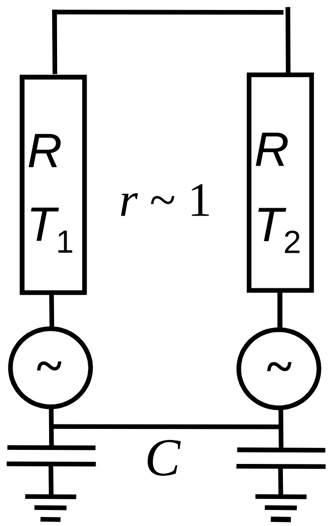




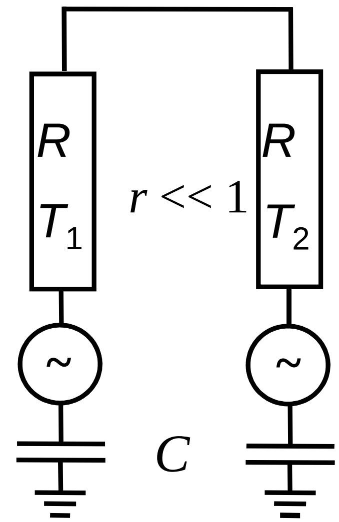
SAMPLE A in a loop (”matched”) [SAMPLE B without loop (”not matched”)]

Heat transport in different set-ups

Loop geometry (Sample A)



Linear geometry (Sample B)



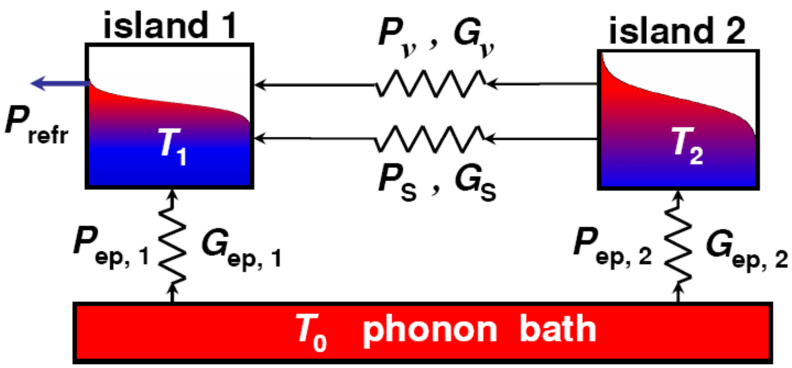
$$P_\nu^{\mathrm{A}} = G_{\mathrm{Q}} \Delta T$$

for small temperature difference

$$P_\nu^{\mathrm{B}} / P_\nu^{\mathrm{A}} = \frac{2}{5} (k_{\mathrm{B}} T R C / \hbar)^2$$
$$\simeq 10^{-3}$$

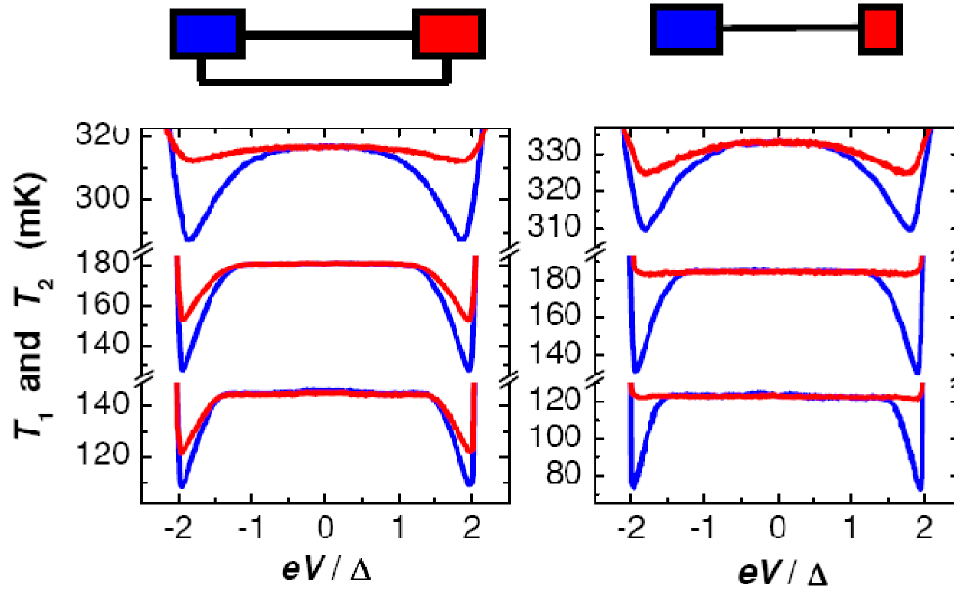
in the present experiment

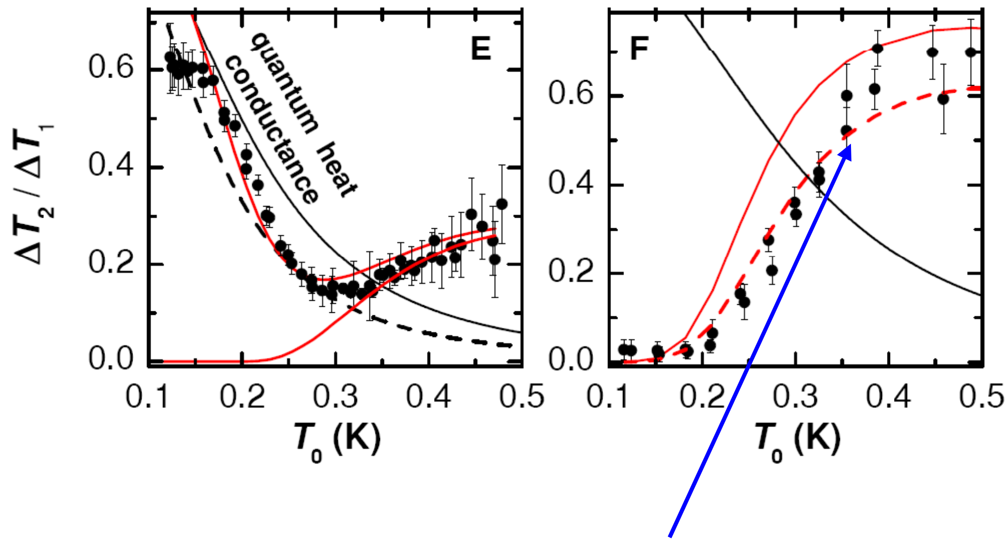
Results in the two sample geometries



$$\frac{\Delta T_2}{\Delta T_1} = \frac{G_\nu + G_s}{G_\nu + G_s + G_{ep,2}}$$

Heat transported by residual quasiparticles at T > 0.3 K and by photons (in the loop sample) at T < 0.3 K





Quasiparticles

Electronic heat conduction

1D heat diffusion along x-axis of a uniform wire with cross-sectional area *A*

$$\dot{Q} = -G_{\mathrm{th}} A \frac{dT}{dx}$$

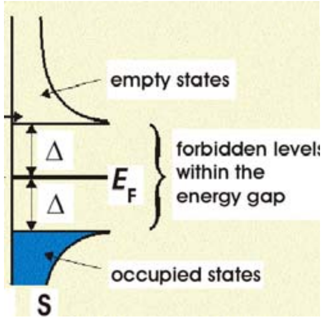
In a metal, diffusive heat transport is governed by the Wiedemann-Franz law:

$$G_{\mathrm{th}}^{\mathrm{N}} = \mathcal{L}_0 G_{\mathrm{N}} T$$

$\mathcal{L}_0 = \pi^2 (k_{\mathrm{B}}/e)^2/3$ is the Lorenz number and

G_{N} is the electrical conductivity.

Quasiparticle heat conduction in a superconductor



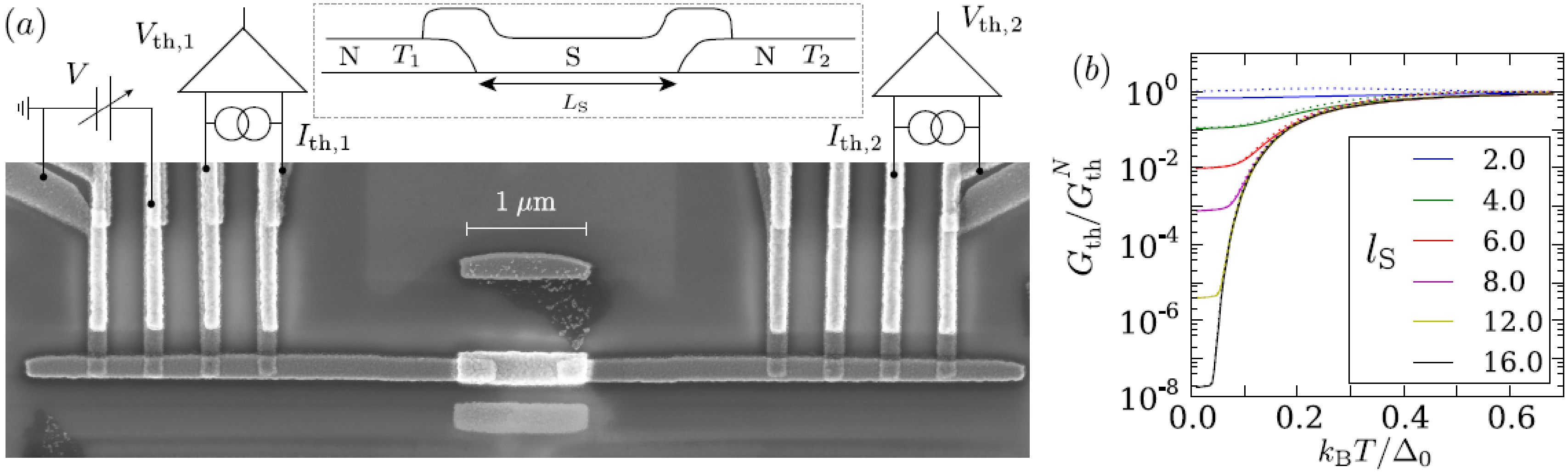
Bardeen et al. 1958

$$\gamma(T) = \frac{G_{\text{th}}}{G_{\text{th}}^{\text{N}}} = \frac{3}{2\pi^2} \int_{\Delta/k_{\text{B}}T}^{\infty} dx \frac{x^2}{\text{sech}^2(x/2)} \simeq \frac{3}{2\pi^2} \left(8 + 8a + 4a^2\right) e^{-a}$$

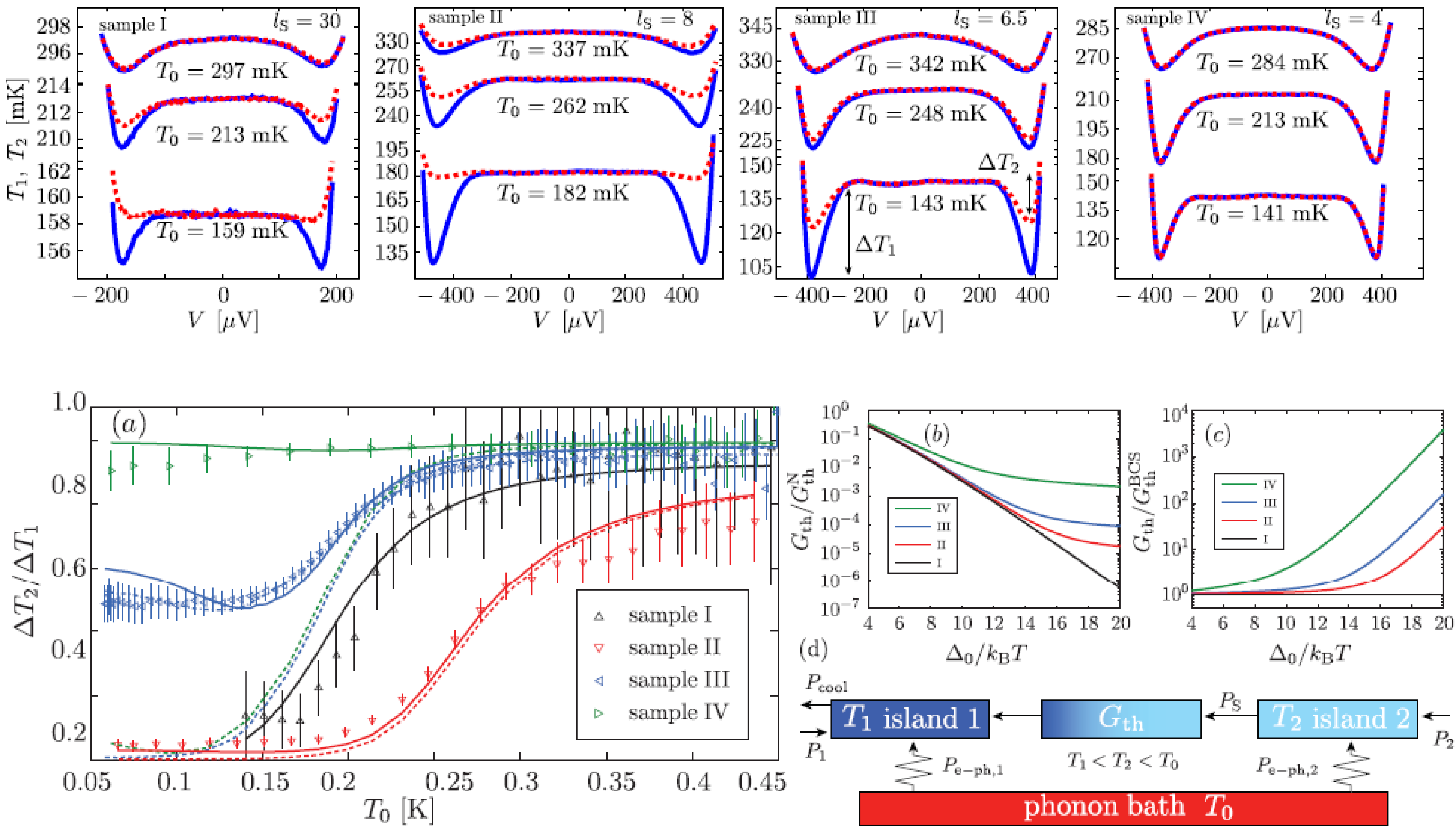
$$a = \Delta/k_{\text{B}}T$$

Heat transport is exponentially suppressed at low temperatures in a superconductor!

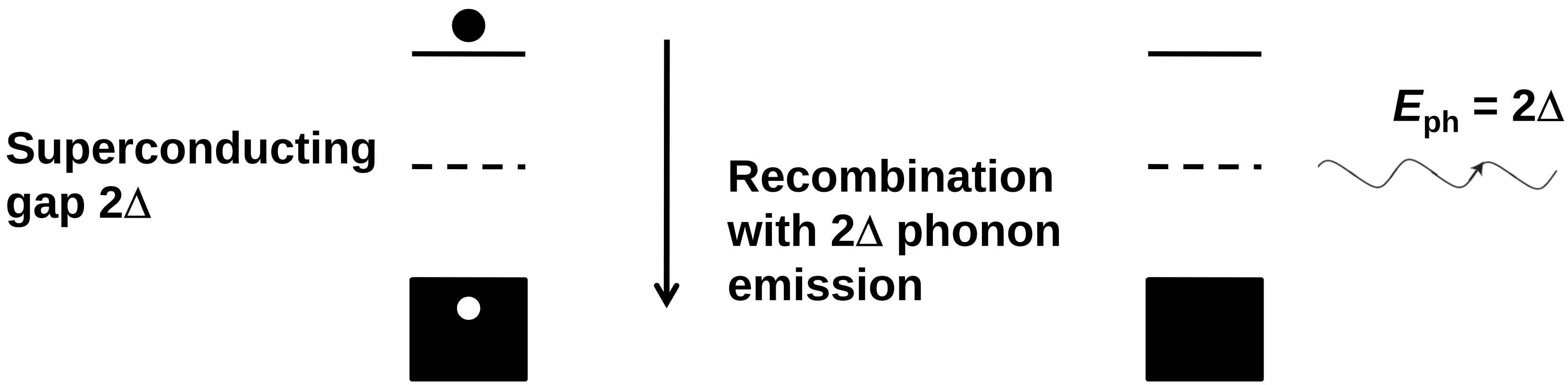
Measurement inc. inverse proximity effect, Peltonen et al. 2010.



Experiment on quasiparticle heat conduction in a superconductor



Quasiparticle recombination



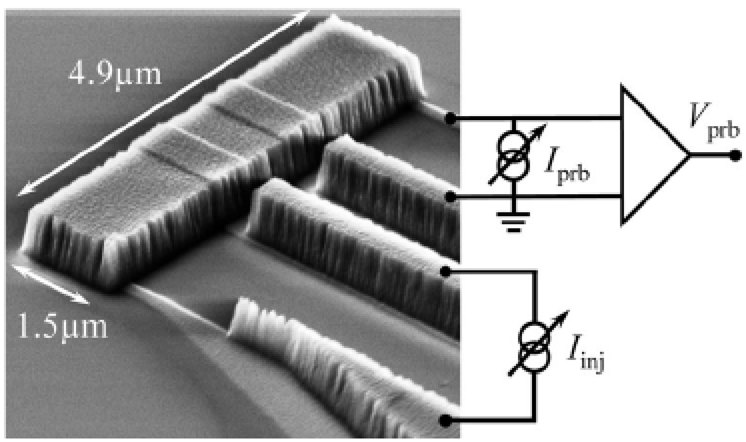
$$\frac{1}{\tau_{rec}} = \frac{1}{\tau_0}\sqrt{\pi}\left(\frac{2\Delta}{kT_c}\right)^{5/2}\sqrt{\frac{T}{T_c}}\,e^{-\frac{\Delta}{kT}}$$

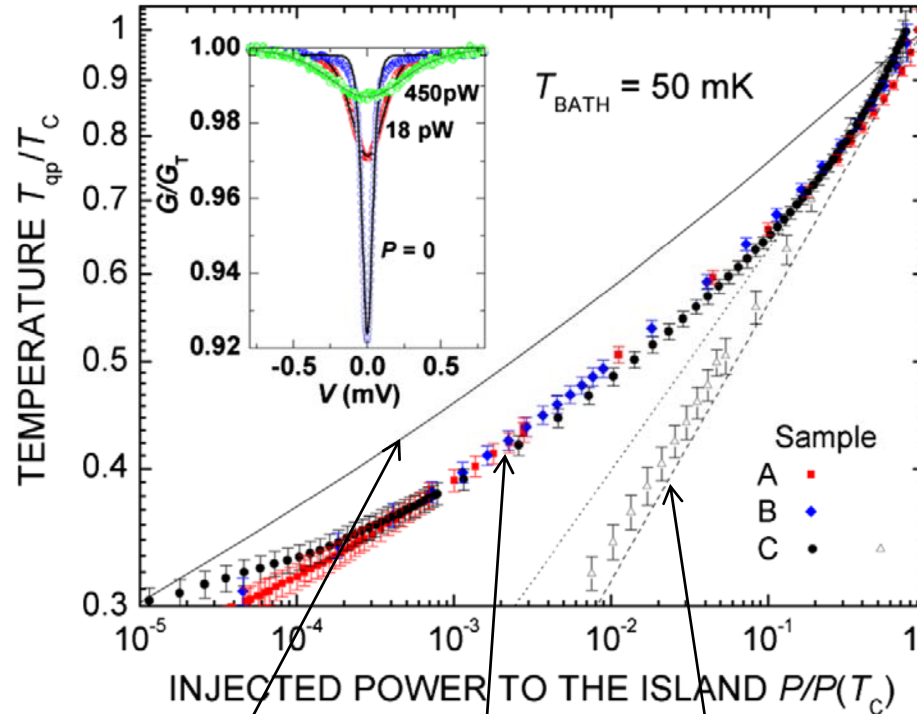
Kaplan et al, 1976
Barends et al., 2008

This process represents electron-phonon relaxation in a superconductor at low *T*.
The corresponding heat current is suppressed exponentially.

$$P_{\text{qp-ph}} \simeq \frac{64}{63\zeta(5)}\Sigma \mathcal{V} T^5 \; e^{-\Delta/k_B T}$$

Measurement of weak recombination in a superconductor





Measurement of relaxation in an aluminium bar, A. Timofeev et al, PRL 2009

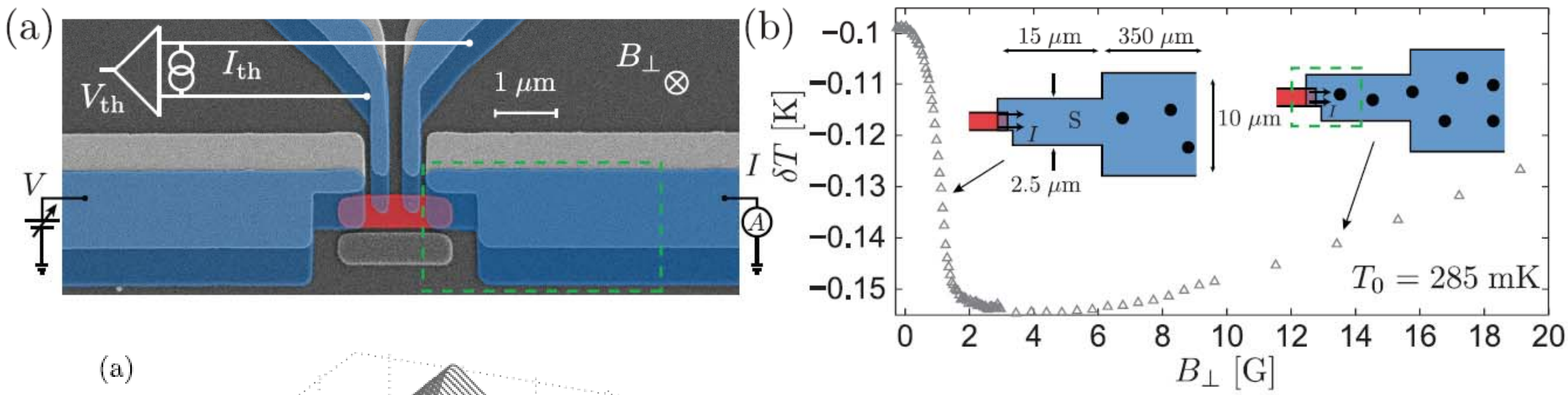
Normal state

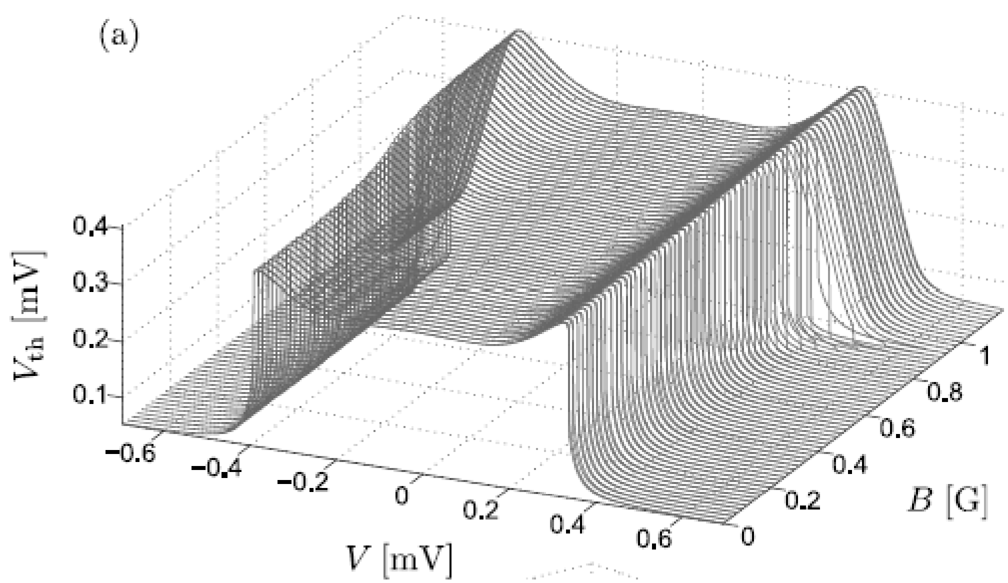
Superconducting (theory)

Superconducting (exp)

Influence of magnetic field on coolers

Magnetic field enhanced cooling





Quasiparticle relaxation is improved in the superconductor due to the presence of vortices, J. Peltonen et al, PRB 84, 220502 (2011)