Composite fermion state of spin-orbit bosons and bosons in Moat Lattices

Alex Kamenev

Fine Theoretical Physics Institute, University of Minnesota

Tigran Sedrakyan

Leonid Glazman

UNIVERSITY OF MINNESOTA

YALE

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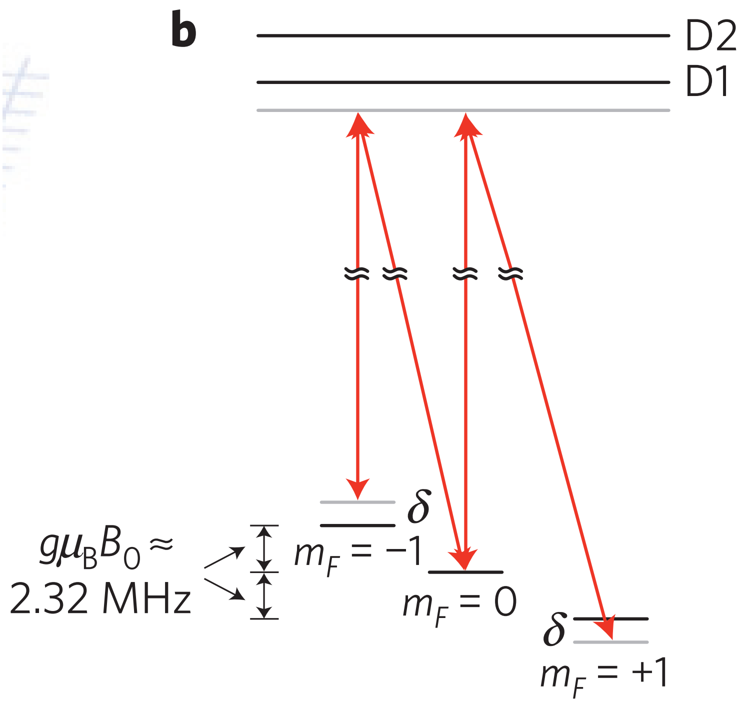
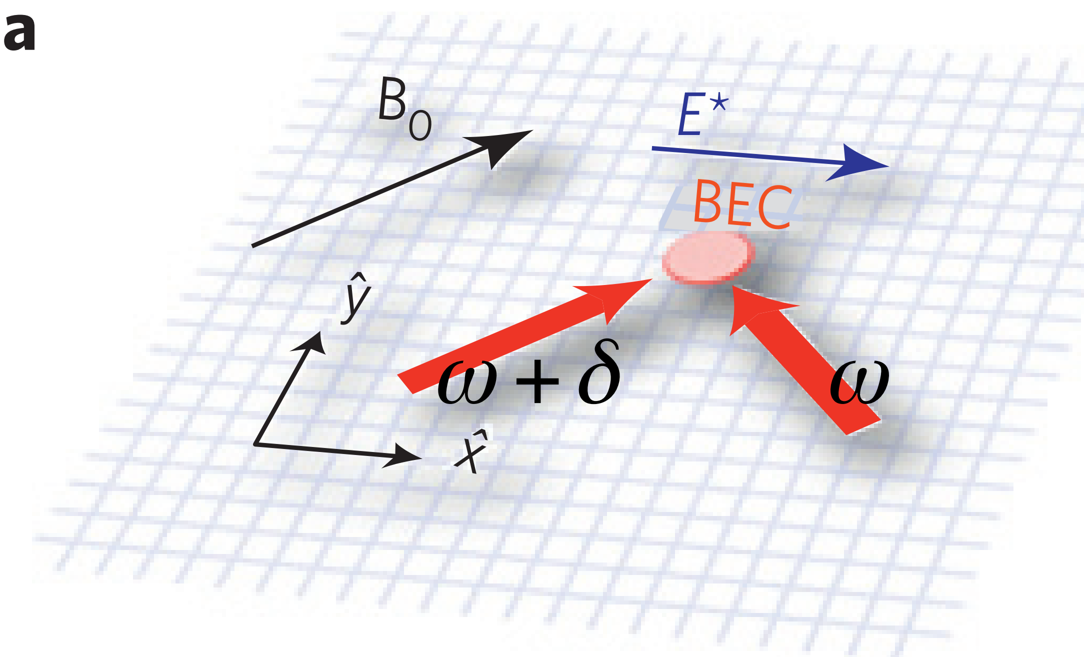
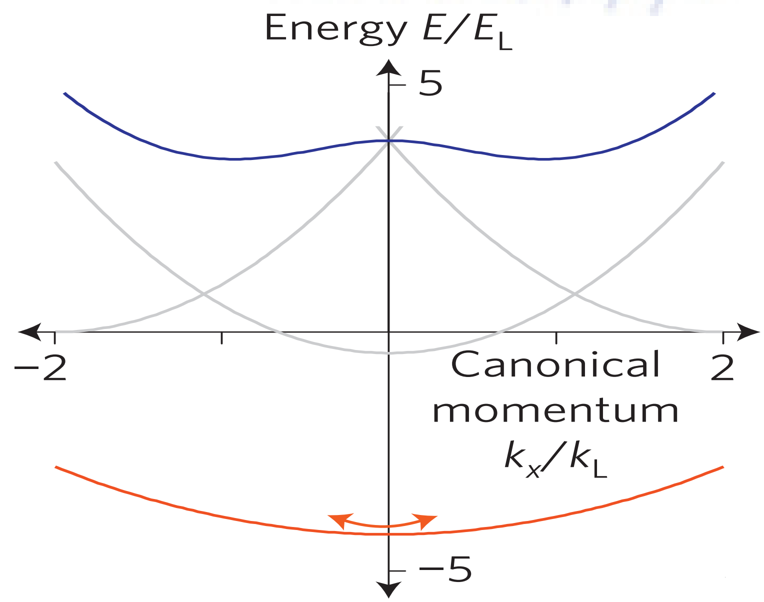
U.S. DEPARTMENT OF ENERGY

Dispersion Relation Engineering



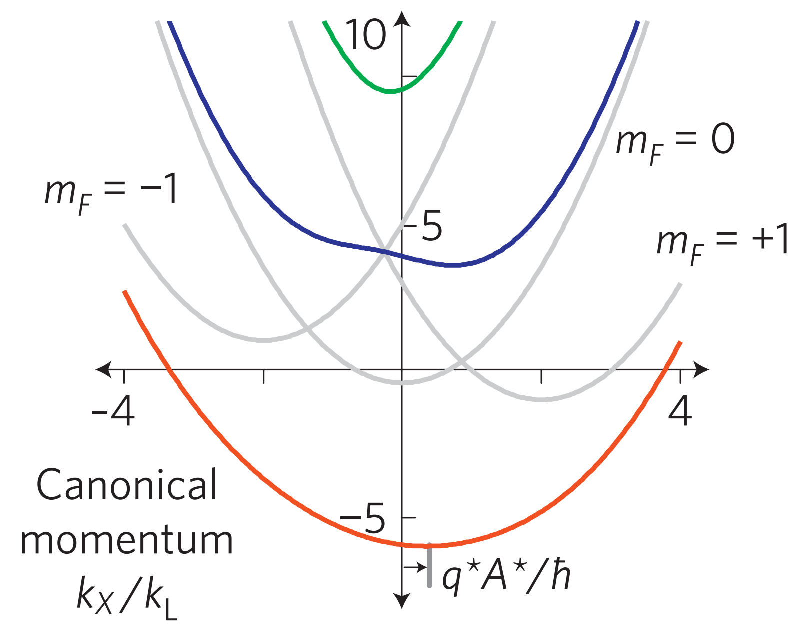


$$H(\tilde{k}_x) = \begin{pmatrix} (\tilde{k}_x - 2)^2 + \delta & \Omega/2 & 0 \\ \Omega/2 & \tilde{k}_x^2 + \epsilon & \Omega/2 \\ 0 & \Omega/2 & (\tilde{k}_x + 2)^2 - \delta \end{pmatrix}.$$



Synthetic Magnetic and Electric Fields

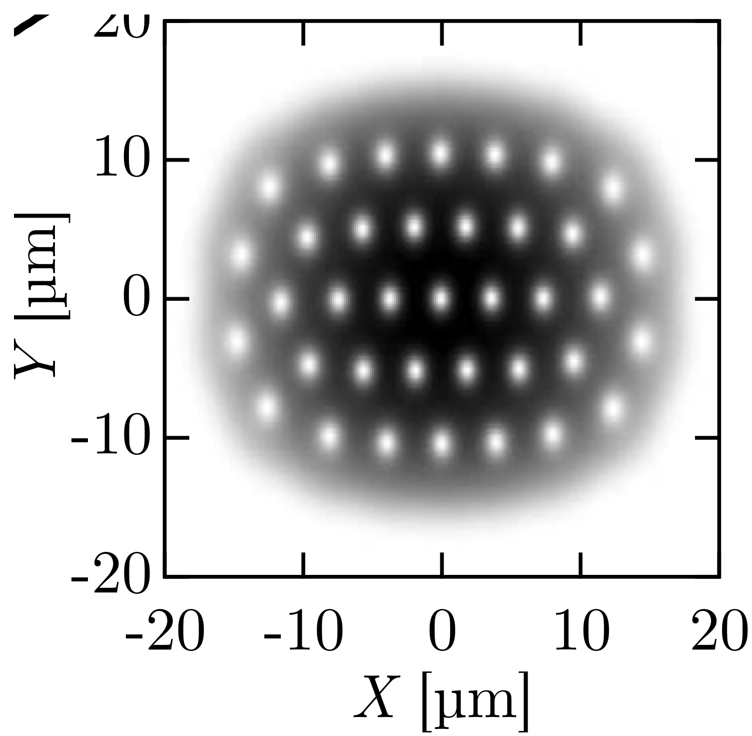
c



d

$$H = (\mathbf{p}_{\text{can}} - q\mathbf{A})^2/2m,$$

$$\mathbf{B} = \nabla \times \mathbf{A} \qquad E(t)\hat{x} = -\partial\mathbf{A}/\partial t$$

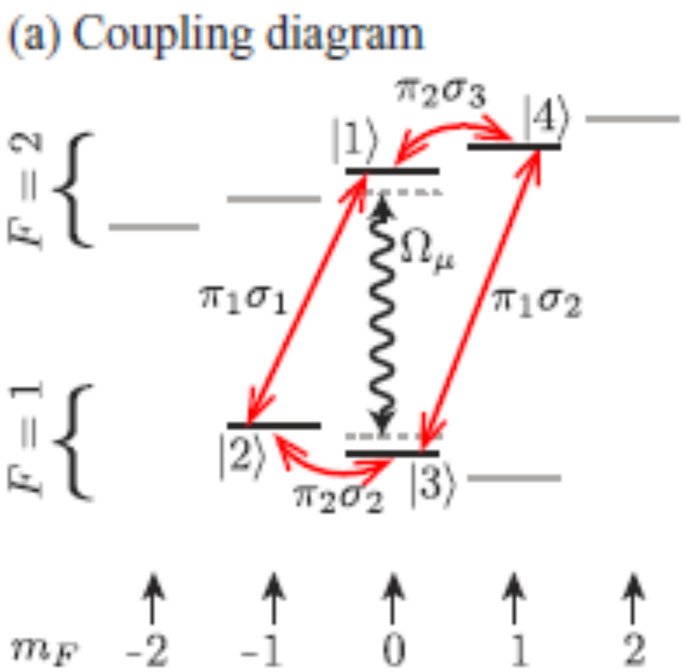


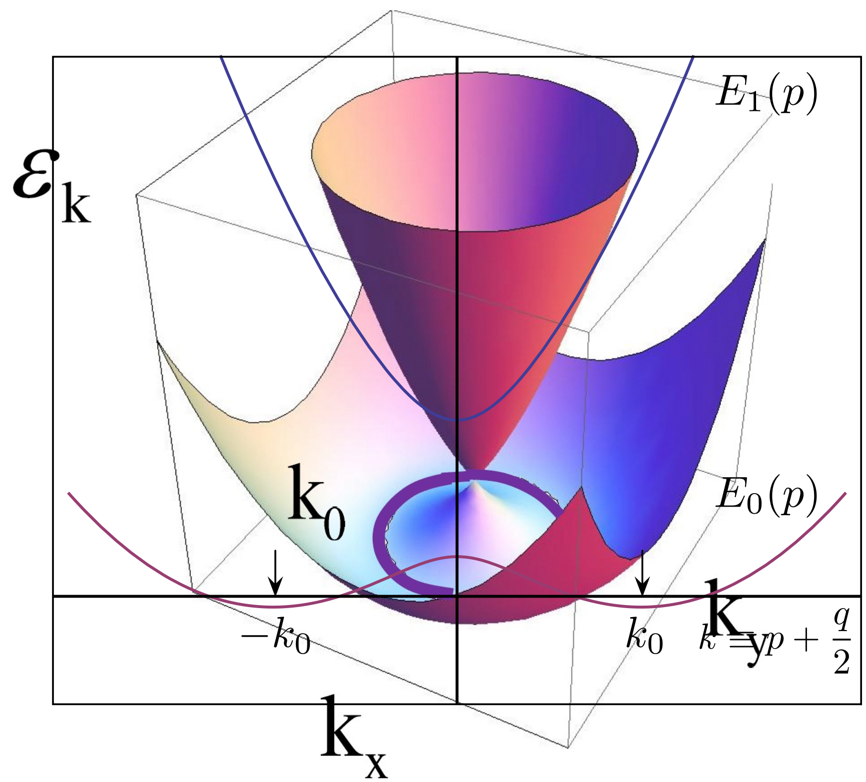
Y-J Lin *et al.* *Nature* **462**, 628-632 (2009) doi:10.1038/nature08609

Spin-Orbit Coupling

Campbell, Juzeliunas, Spielman 2011

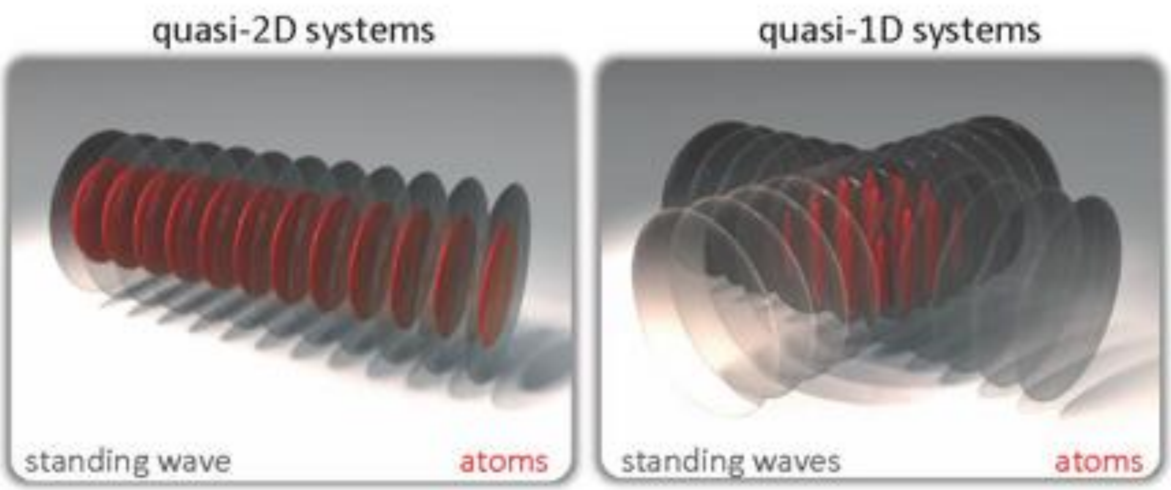
$$\varepsilon_{\mathbf{k}} = \frac{k^2}{2m} \pm vk$$

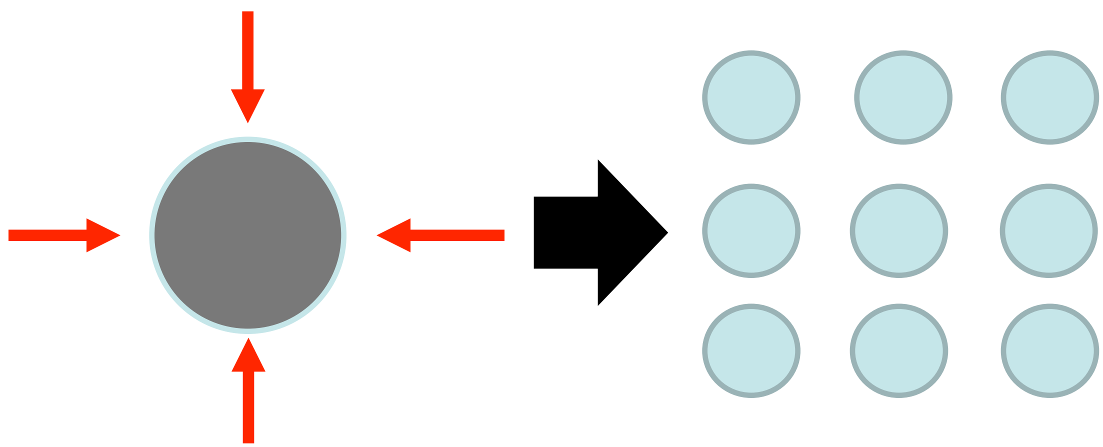


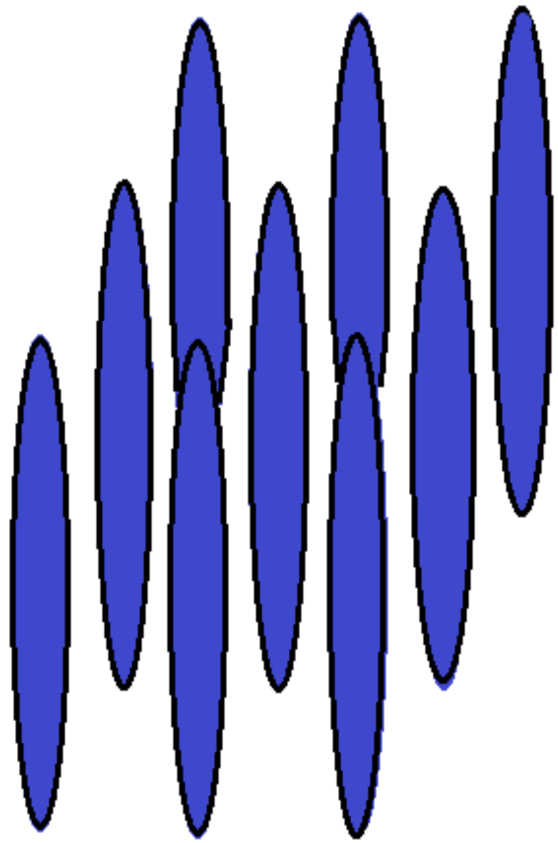


$$H_0 = -\frac{\nabla_{\mathbf{r}}^2}{2m} + iv\hat{\mathbf{z}} \cdot [\boldsymbol{\sigma} \times \nabla_{\mathbf{r}}]$$

Optical Lattices





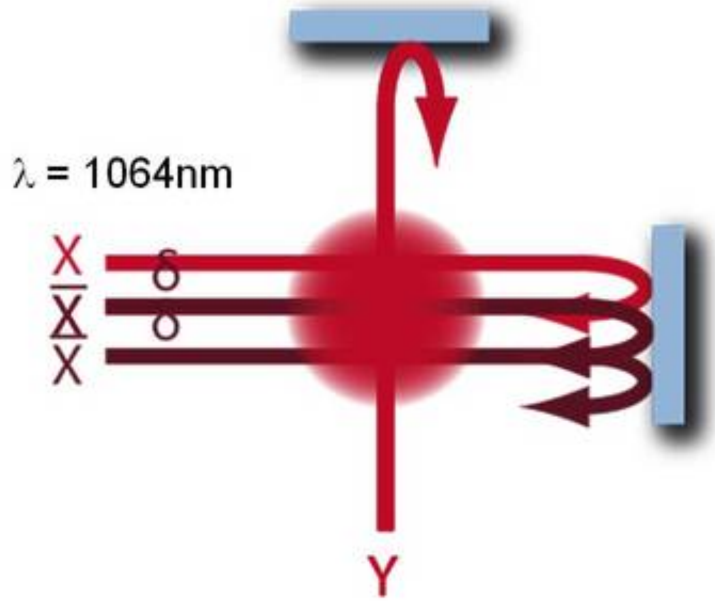


1D: $T, \mu \ll \omega_\perp$

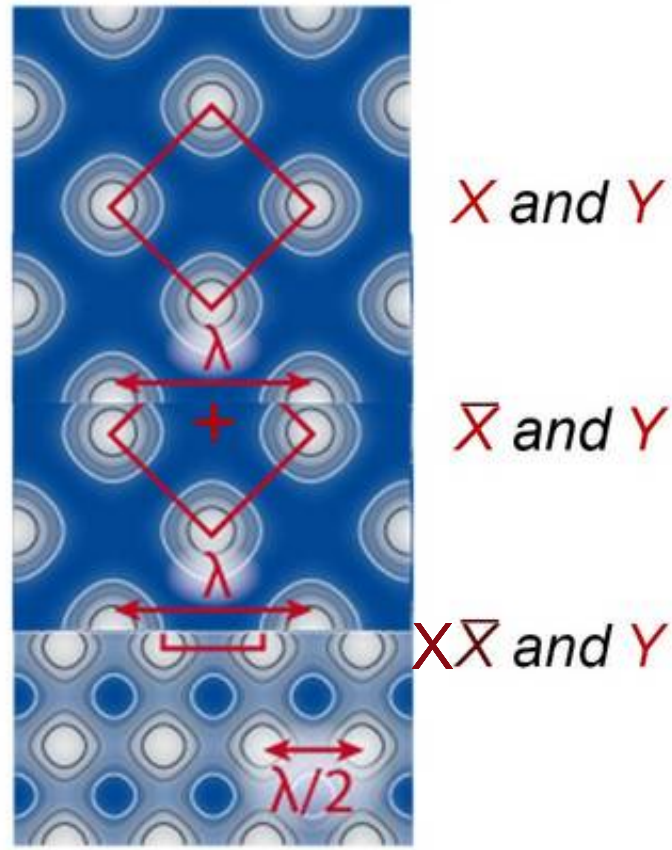
An optical lattice of tunable geometry

ETH

Setup



Optical potential

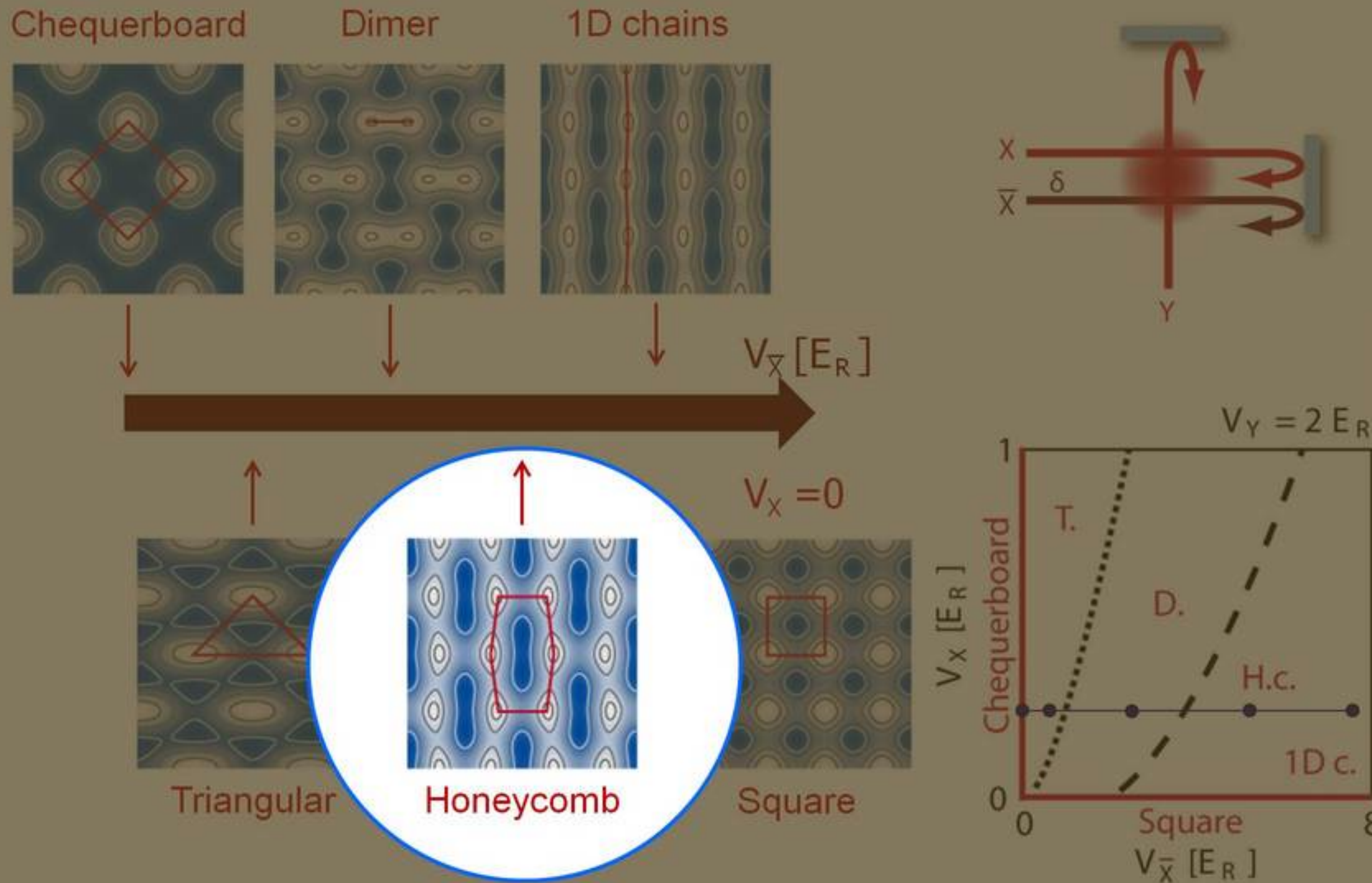


L. Tarruell, T. Esslinger, et al. ETH Zurich

$$V(x,y) = \boxed{V_{\bar{X}} \cos^2(kx + \theta/2)} + \boxed{V_X \cos^2(kx) + V_Y \cos^2(ky) + 2\alpha\sqrt{V_X V_Y} \cos(kx)\cos(ky)}$$

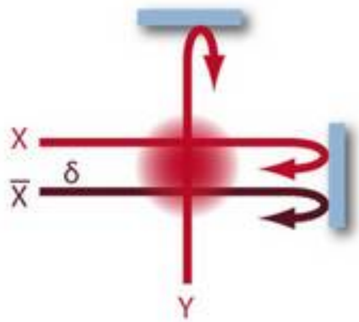
An optical lattice of tunable geometry

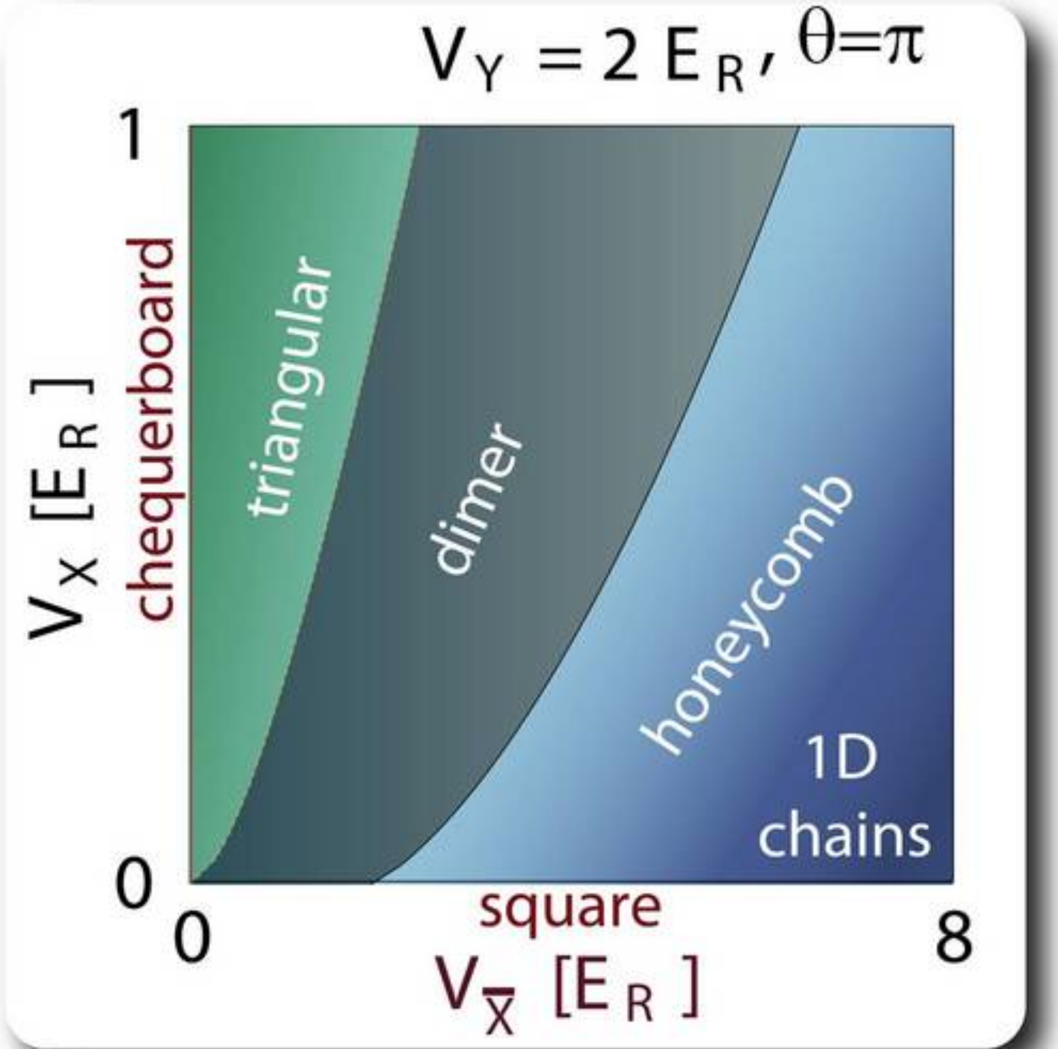
ETH



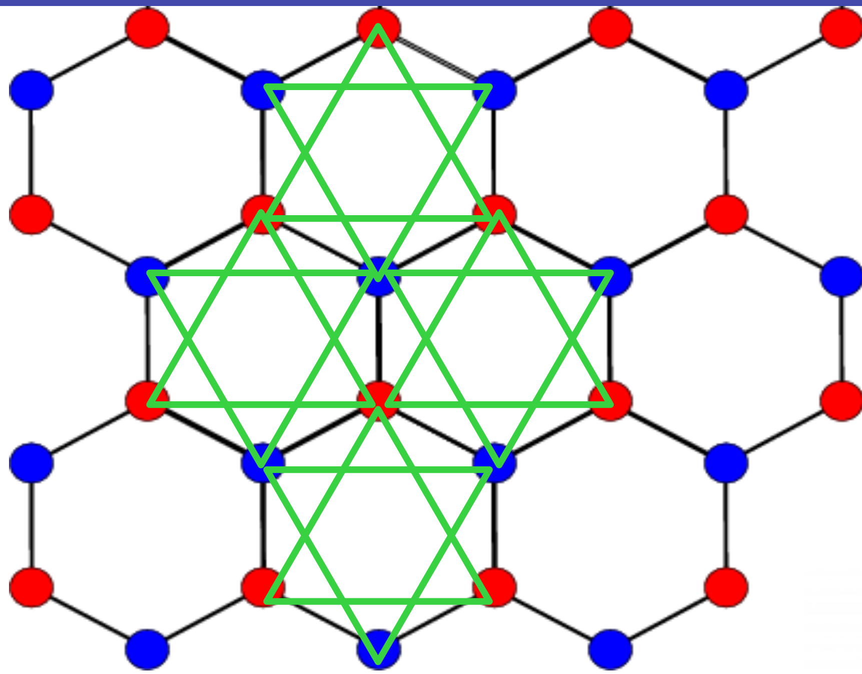
An optical lattice of tunable geometry

ETH

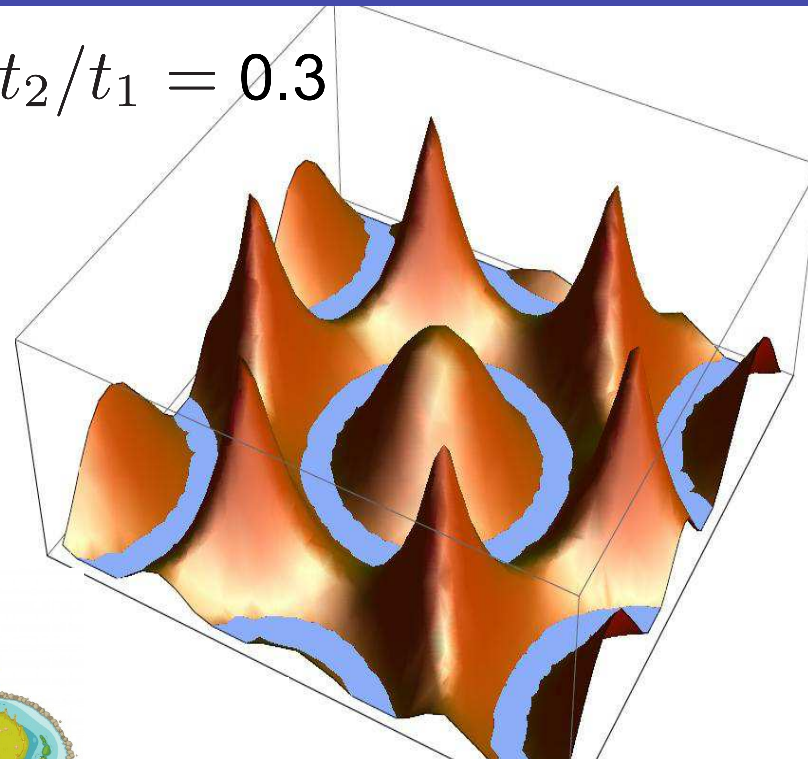




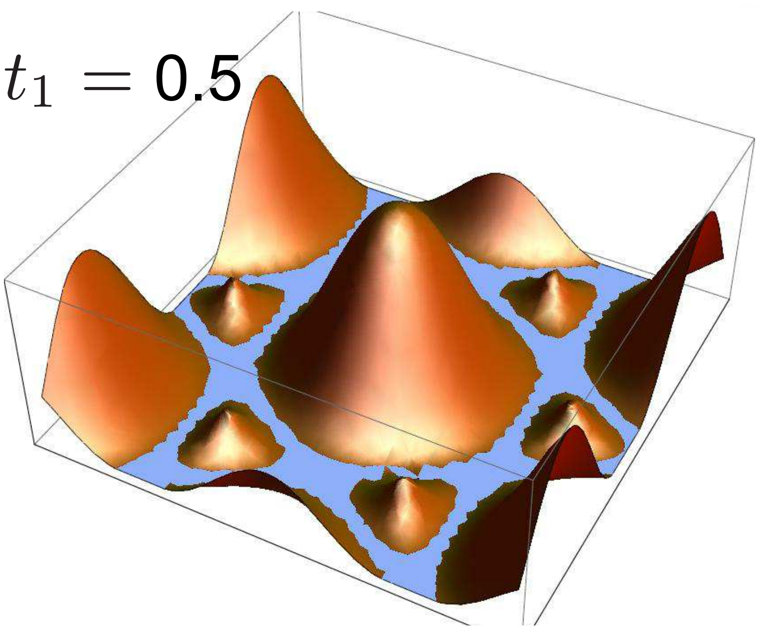
Moat Lattices



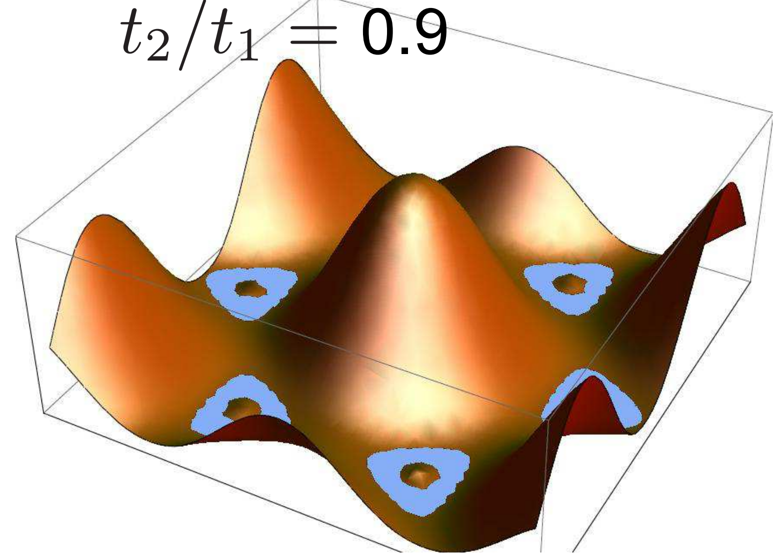
$t_2/t_1 = 0.3$



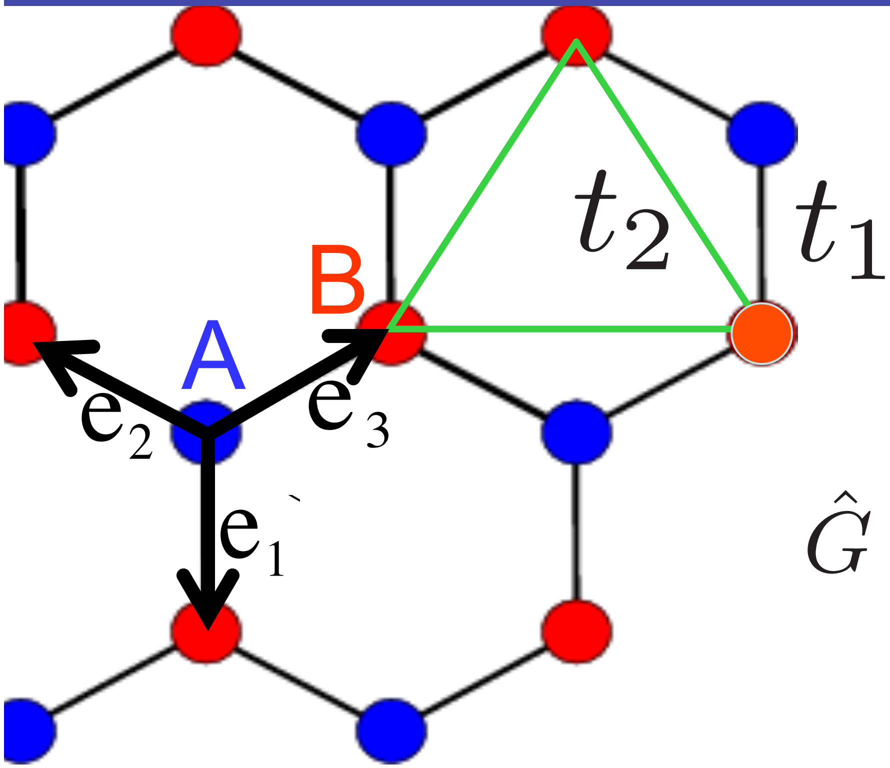
$t_2/t_1 = 0.5$



$t_2/t_1 = 0.9$



Moat Lattices



$$\begin{array}{cc} & \begin{array}{cc} \text{A} & \text{B} \end{array} \\ \hat{T} = & \begin{pmatrix} 0 & \hat{G} \\ \hat{G}^{\dagger} & 0 \end{pmatrix} \begin{array}{c} \text{A} \\ \text{B} \end{array} \end{array}$$

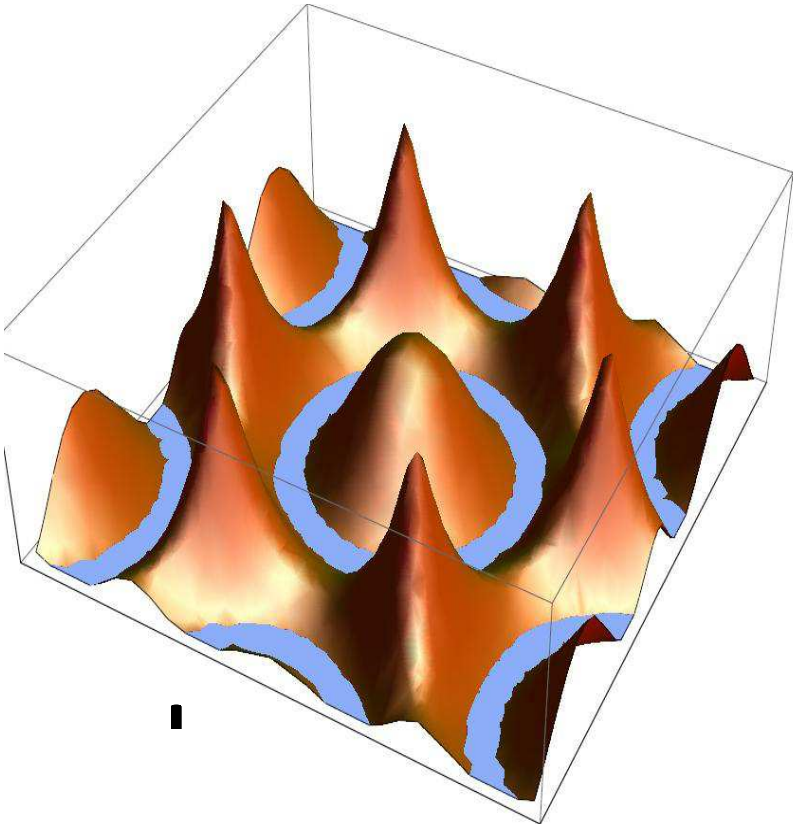
$$\hat{G} \; = \; G_{\mathbf{k}} \; = \; \sum_{j=1,2,3} e^{i\mathbf{k}\cdot\mathbf{e}_j}$$

$$\hat{H} = t_1 \hat{T} + t_2 \hat{T}^2$$

Nearest-neighbor
graphene

Next-nearest-neighbor
graphene square

Moat Lattices



$$\hat{H} = t_1 \hat{T} + t_2 \hat{T}^2$$

$$E_{\mathbf{k}}^{(\mp)} = \mp |t_1||G_{\mathbf{k}}| + t_2 |G_{\mathbf{k}}|^2$$

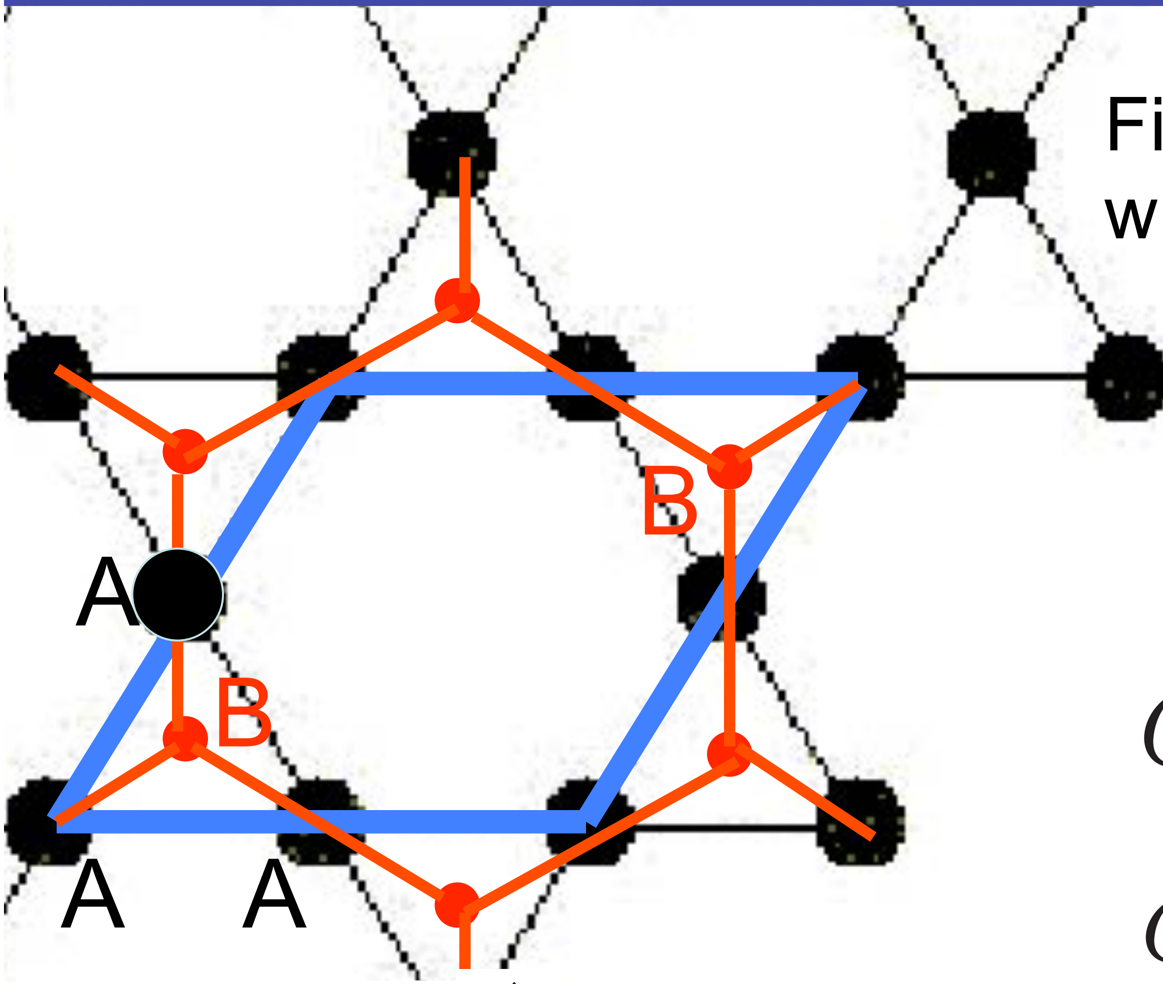
graphene

graphene square

The Moat: $|G_{\mathbf{k}}| = |t_1|/2t_2.$

Non-trivial if t_2/t_1 > 1/6

Kagome



Fictitious bipartite lattice with 5 sites per unit cell

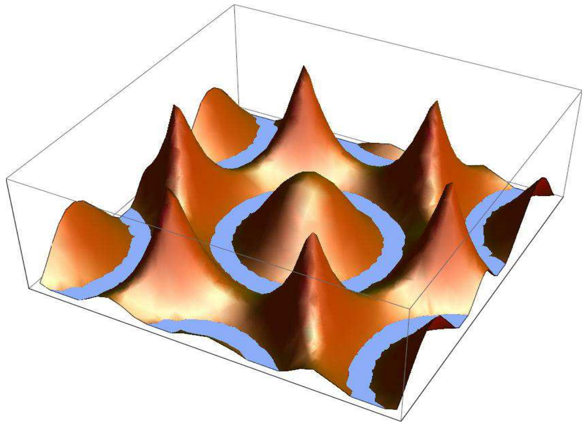
$$\hat{T} = \begin{pmatrix} 0 & \hat{G} \\ \hat{G}^\dagger & 0 \end{pmatrix}$$

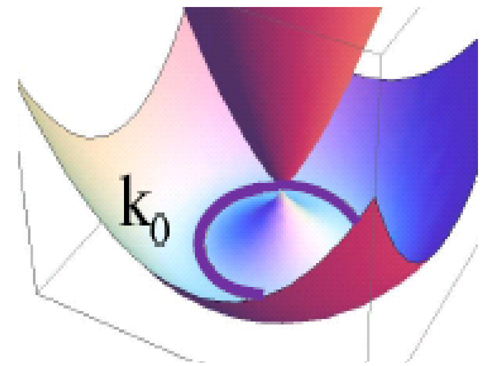
$\hat{G}$ 2 x 3 matrix

$\hat{G}^\dagger$ 3 x 2 matrix

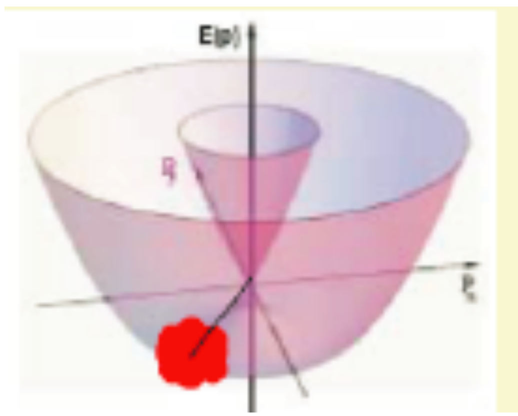
Kagome: $\hat{G}^\dagger \hat{G}$ = (3 x 2)*(2 x 3), thus zero determinant, i.e. flat band.

Bosons in a Moat: do they condense?



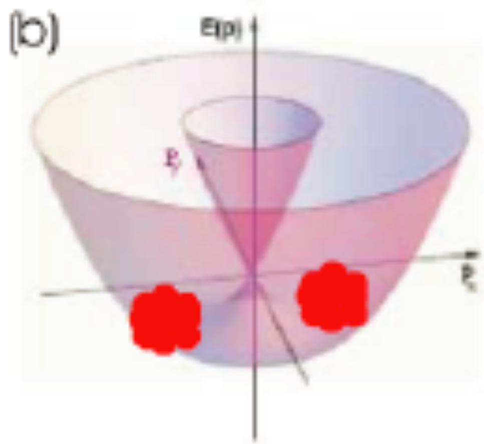


Time-Reversal Symmetry Breaking



$$\Psi_k \sim \frac{1}{\sqrt{2}} e^{ikr} \begin{pmatrix} 1 \\ -e^{i\arg(k)} \end{pmatrix}$$

(Spin-)Density Wave



$$\Psi_k \sim \frac{1}{\sqrt{2}} \begin{pmatrix} \cos(kr) \\ i\sin(kr) \end{pmatrix}$$

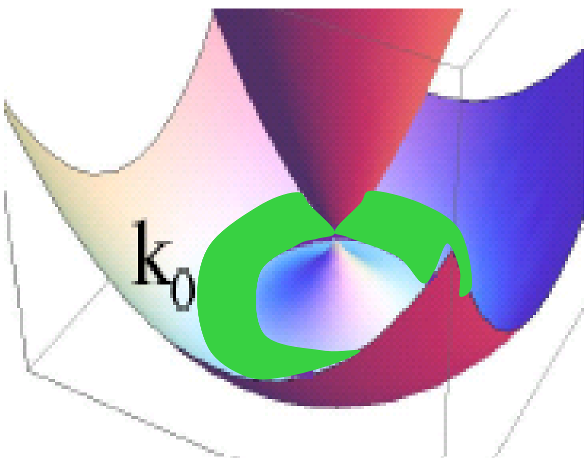
Chunji Wang, Chao Gao, Chao-Ming Jian, and Hui Zhai, PRL 105, 160403 (2010)

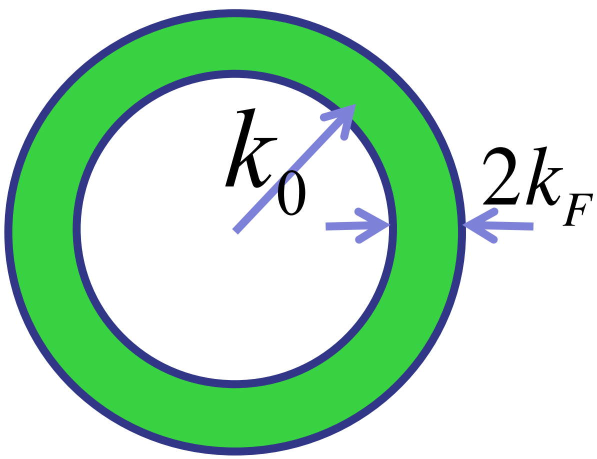
However…

DOS diverge as $\rho(E) \sim 1/\sqrt{E}$ at the Moat bottom, as in 1D:

No condensation at finite temperature. How about $T = 0$?

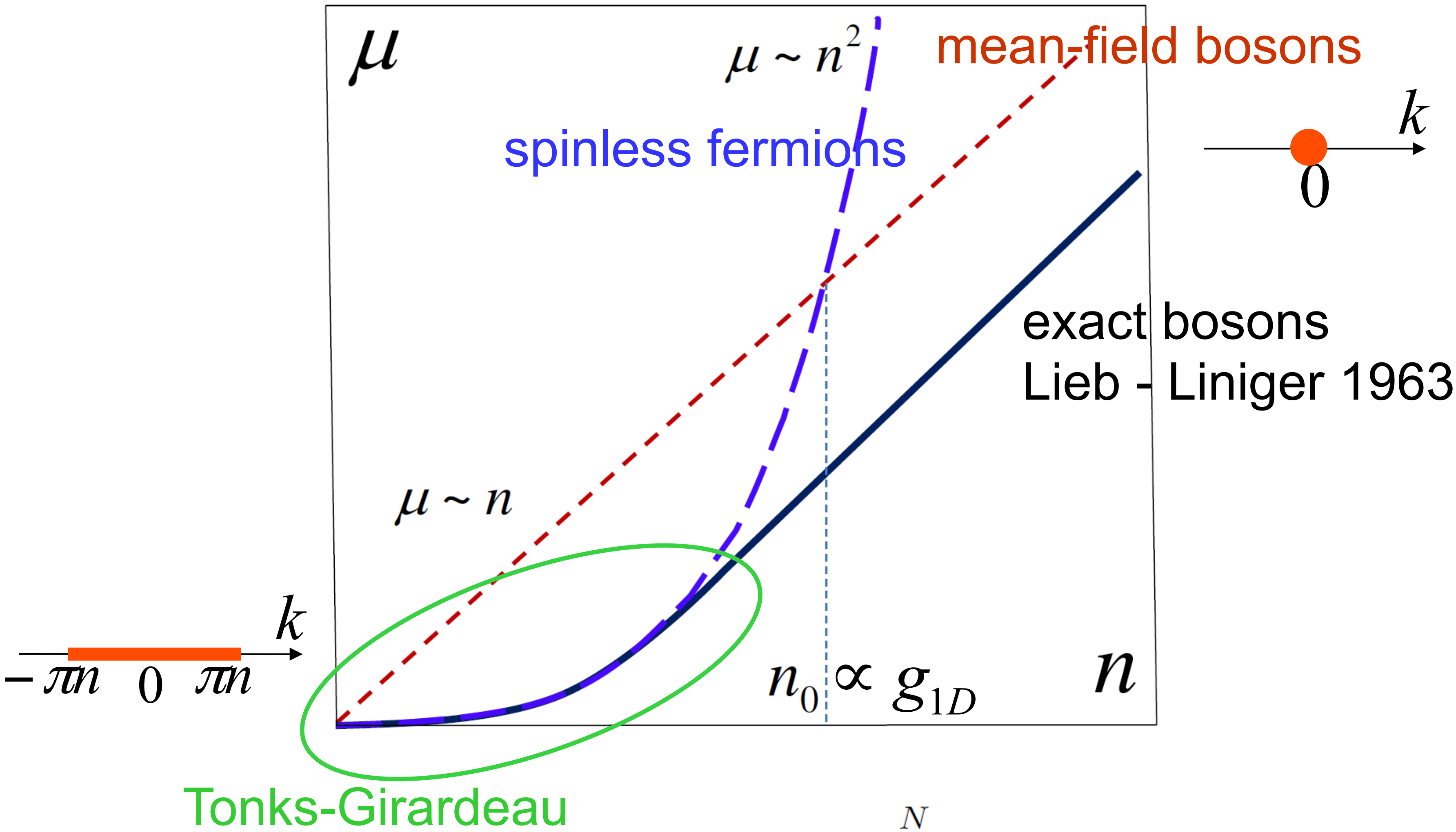
Consider for a moment fermions instead of bosons:





Fermi-chemical potential: $(2\pi k_0)(2k_F) \sim 4\pi^2 n \Rightarrow$ $\boxed{E_F = \frac{k_F^2}{2m} \sim n^2}$

Reminder: spinless 1D model



$$\Psi_B(x_1,\ldots,x_N) = \prod_{i,j}^{N} \mathrm{sign}(x_i - x_j)\Psi_F(x_1,\ldots,x_N)$$

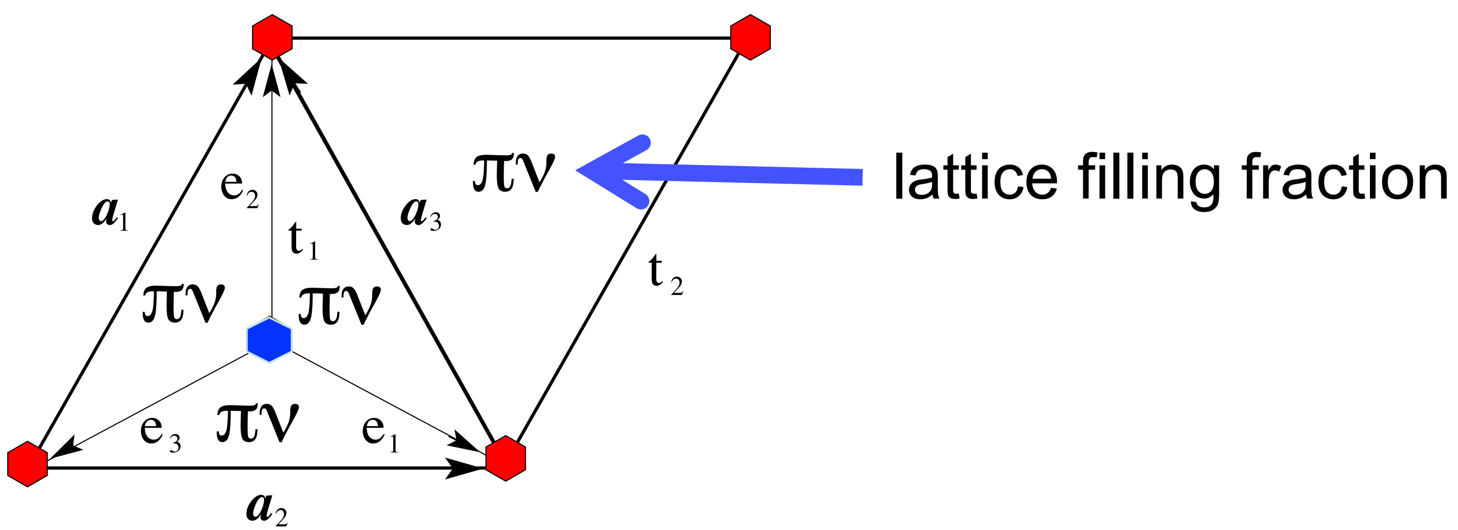
Can one Fermionize Bosons on 2D lattice?

Chern-Simons transformation:

$$b_{\mathbf{r}}^{(\dagger)} = c_{\mathbf{r}}^{(\dagger)} \, e^{\pm i \sum_{\mathbf{r}' \neq \mathbf{r}} \arg[\mathbf{r}-\mathbf{r}'] n_{\mathbf{r}'}}$$

boson fermion

One obtains non-interacting fermions on the lattice in an effective magnetic field (constant + staggered a-la Haldane).



Effective Fermion Hamiltonian

Hamiltonian preserves the algebraic structure!:

$$\hat{H} = t_1 \hat{T} + t_2 \hat{T}^2$$

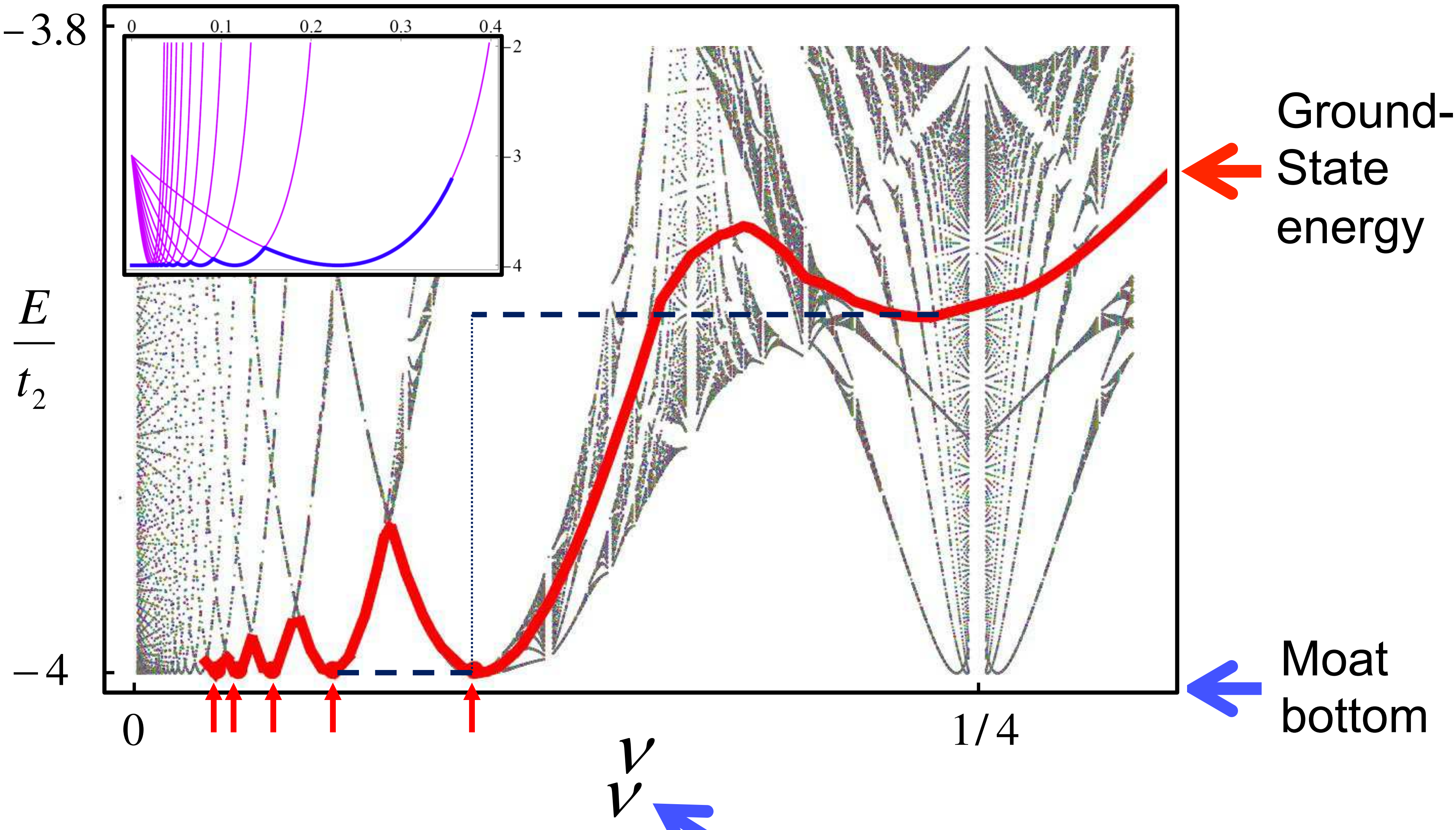
$$\hat{T} = \begin{pmatrix} 0 & \hat{G} \\ \hat{G}^\dagger & 0 \end{pmatrix}$$

Hofstadter problem on graphene

$$\hat{G} = \sum_j e^{i\mathbf{e}_j \cdot (\mathbf{k}+\mathbf{A}_\mathbf{r})}$$

$\hat{G}^\dagger$ and $\hat{G}$ do not commute

Butterflies in the Moat

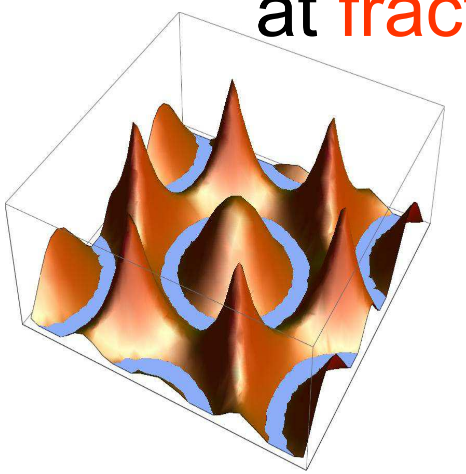


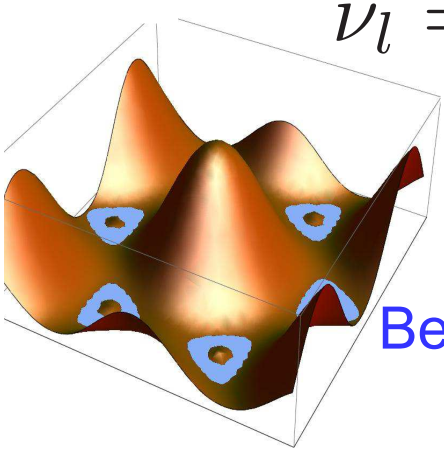
Boson filling fraction = average magnetic field

Fractional density quantization

Chemical potential exhibits discontinuities

at fractionally quantized filling factors





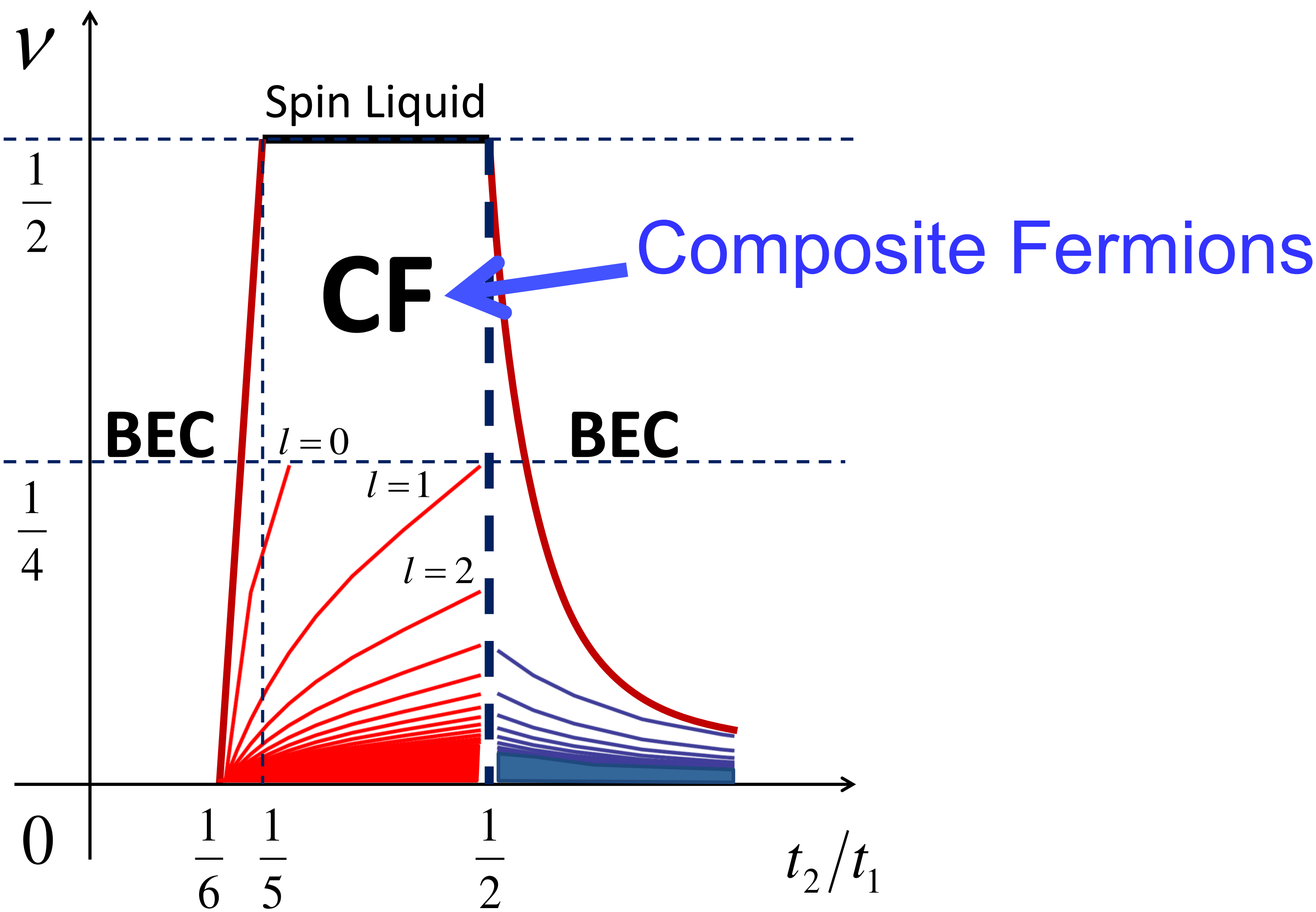
Area of the Brillouin zone encircled by the Moat

$$\nu_l = \frac{\mathcal{A}_{\mathcal{C}}}{2l+1+\gamma/\pi}, \qquad l = 0, 1, \ldots ,$$

Berry phase: $\gamma = 0$ if Moat encircles Γ point

$\gamma = \pi$ if Moat encircles K and K’

Phase Diagram of Hard-core bosons

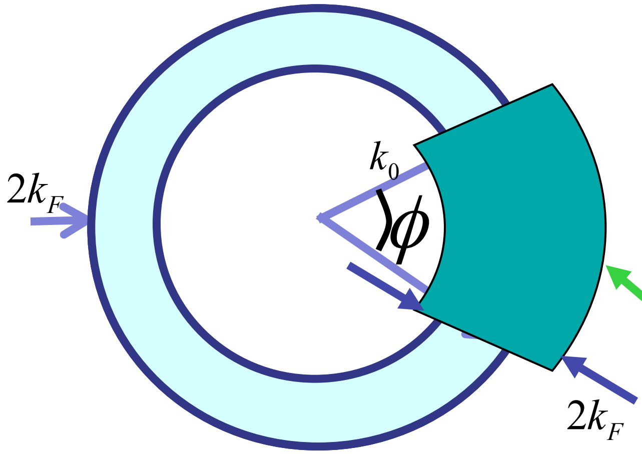


Incompressible domains with fractional fillings and edge states

Conclusions

Life is good with a Moat

Fermions with spin do interact



$$E_{\text{int}} \propto g(1-\cos\phi) \propto g\phi^2$$

Hartree   Fock

Variational Fermi surface

$$E_{\text{kin}} \propto k_F^2 \propto \left(\frac{n}{k_0\phi}\right)^2$$

$$E_{\text{int}} \propto gn\phi^2$$

minimizing w.r.t. ϕ

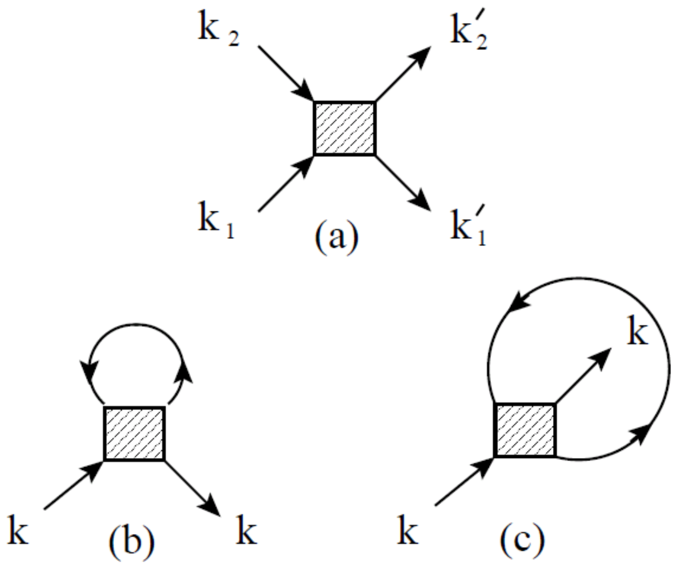
$$\phi = \left(\frac{n}{gk_0^2}\right)^{1/4}$$

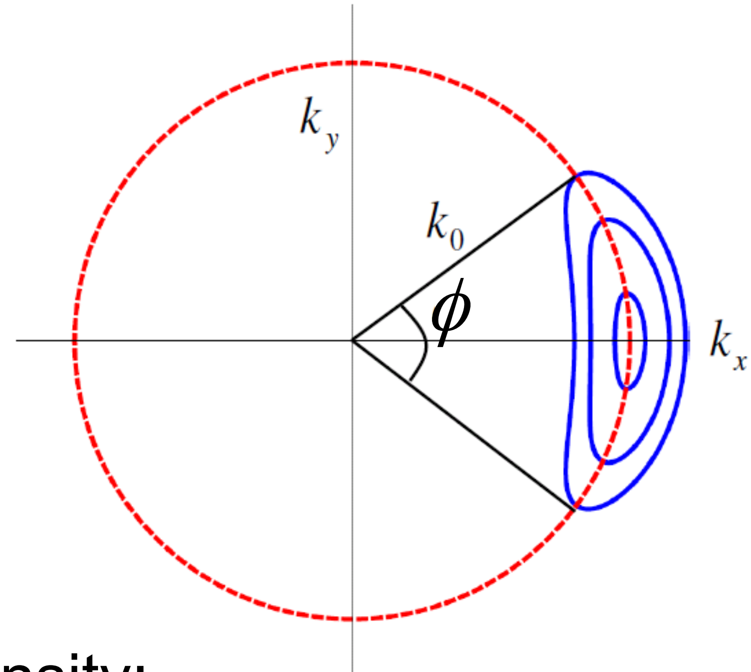
$$E_{\text{kin}} \propto E_{\text{int}} \propto \boxed{\sqrt{\frac{g}{k_0}}n^{3/2} << n}$$

Berg, Rudner, and Kivelson, (2012)

Self-consistent Hartree-Fock for Rashba fermions

Berg, Rudner, and Kivelson, (2012) *nematic state*





Elliptic Fermi surface at small density:

$$H_{HF} = \frac{(k_x - k_0)^2}{2m} + \frac{k_y^2}{2m_y} \qquad m_y = m\frac{k_0^2}{gn} >> m$$

$$\mu \propto \sqrt{\frac{g}{k_0}}\,n^{3/2} << gn$$

nematic fermions

mean-field bosons

Fermionization in 2D? Chern-Simons!

$$\Psi_{\downarrow\ldots\downarrow}(\mathbf{r}_1,\ldots,\mathbf{r}_N) = \prod_{i<j} e^{\pm i\,\mathrm{arg}(\mathbf{r}_i-\mathbf{r}_j)}\Psi_F(\mathbf{r}_1,\ldots,\mathbf{r}_N)$$

bosonic wave function

Chern-Simons phase

fermionic wave function

- (plus/minus) One flux quantum per particle
- Broken parity P
- Higher spin components are uniquely determined by the ***projection*** on the lower Rashba brunch
- Fermionic wave function is Slater determinant, minimizing kinetic and interaction energy

Chern-Simons magnetic Field

$$\Psi_{\downarrow\ldots\downarrow}(\mathbf{r}_1,\ldots,\mathbf{r}_N) = \prod_{i<j} e^{\pm i\,\mathrm{arg}(\mathbf{r}_i-\mathbf{r}_j)}\Psi_F(\mathbf{r}_1,\ldots,\mathbf{r}_N)$$

$$\hat{\mathbf{k}} \to \hat{\mathbf{k}} \pm \mathbf{A} \qquad\qquad \mathbf{A}_\alpha(\mathbf{r}_j) = \sum_{i\neq j} \epsilon_{\alpha\beta}\frac{(\mathbf{r}_j-\mathbf{r}_i)_\beta}{|\mathbf{r}_j-\mathbf{r}_i|^2}$$

$$B_{\mathrm{CS}}(\mathbf{r}_j) = \mathrm{rot}\mathbf{A}(\mathbf{r}_j) = 2\pi\sum_{i\neq j}\delta(\mathbf{r}_j-\mathbf{r}_i) \to 2\pi n$$

mean-field approximation

Particles with the cyclotron mass: $m_c = \sqrt{m_x m_y} = m\sqrt{\frac{k_0^2}{gn}}$

in a uniform magnetic field: $B_{\mathrm{CS}} = 2\pi n$

Integer Quantum Hall State

Particles with the cyclotron mass: $m_c = \sqrt{m_x m_y} = m\sqrt{\frac{k_0^2}{gn}}$

in a uniform magnetic field: $B_{CS} = 2\pi n$

Landau levels: $$\varepsilon_l = \frac{B_{CS}}{m_c}\left(l + \frac{1}{2}\right)$$

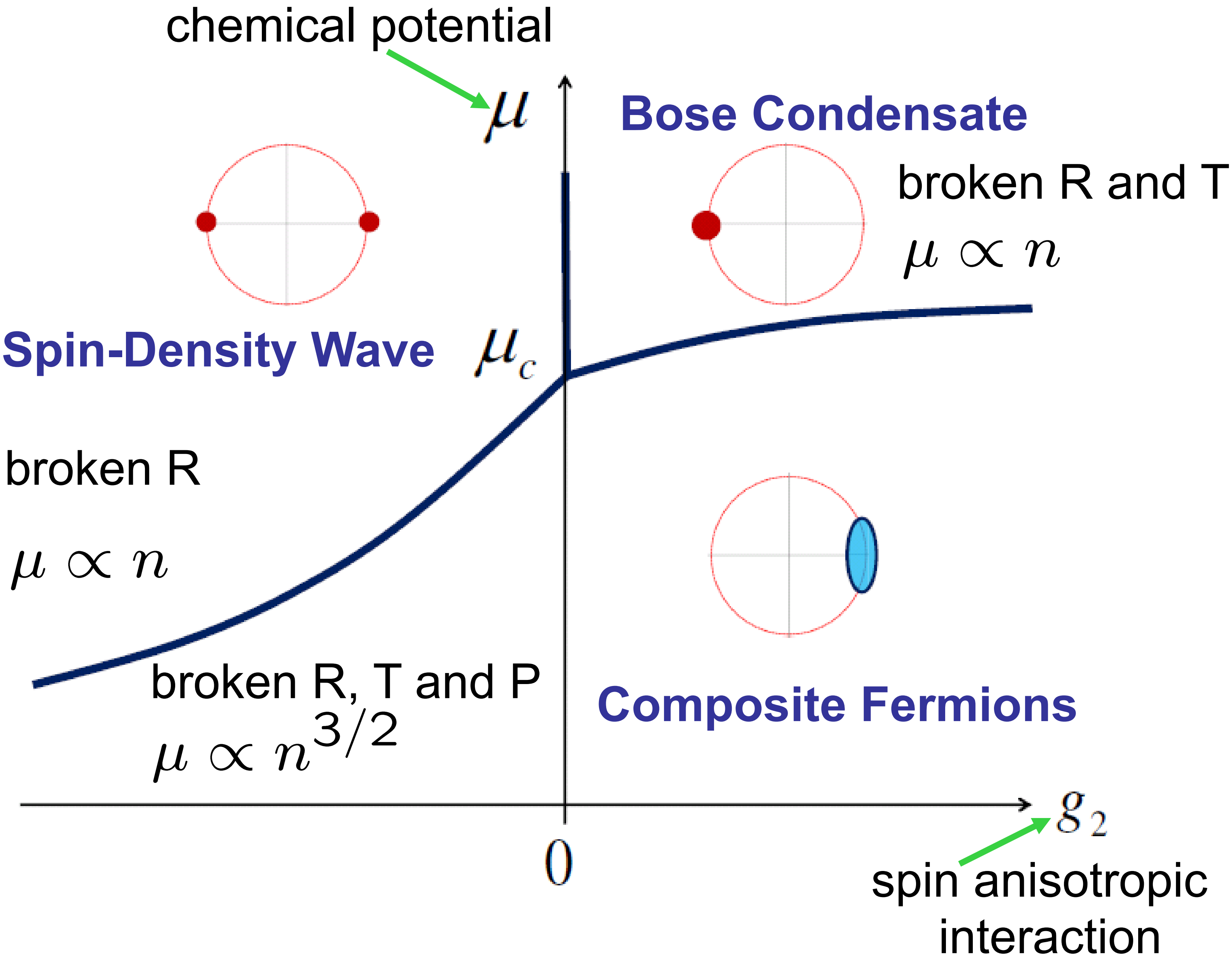
One flux quanta per particle, thus ν=1 filling factor: IQHE

$$\boxed{\mu = \varepsilon_0 = \frac{\pi\sqrt{g}}{mk_0} n^{3/2}}$$

✓ Gapped bulk and chiral edge mode:

interacting topological insulator

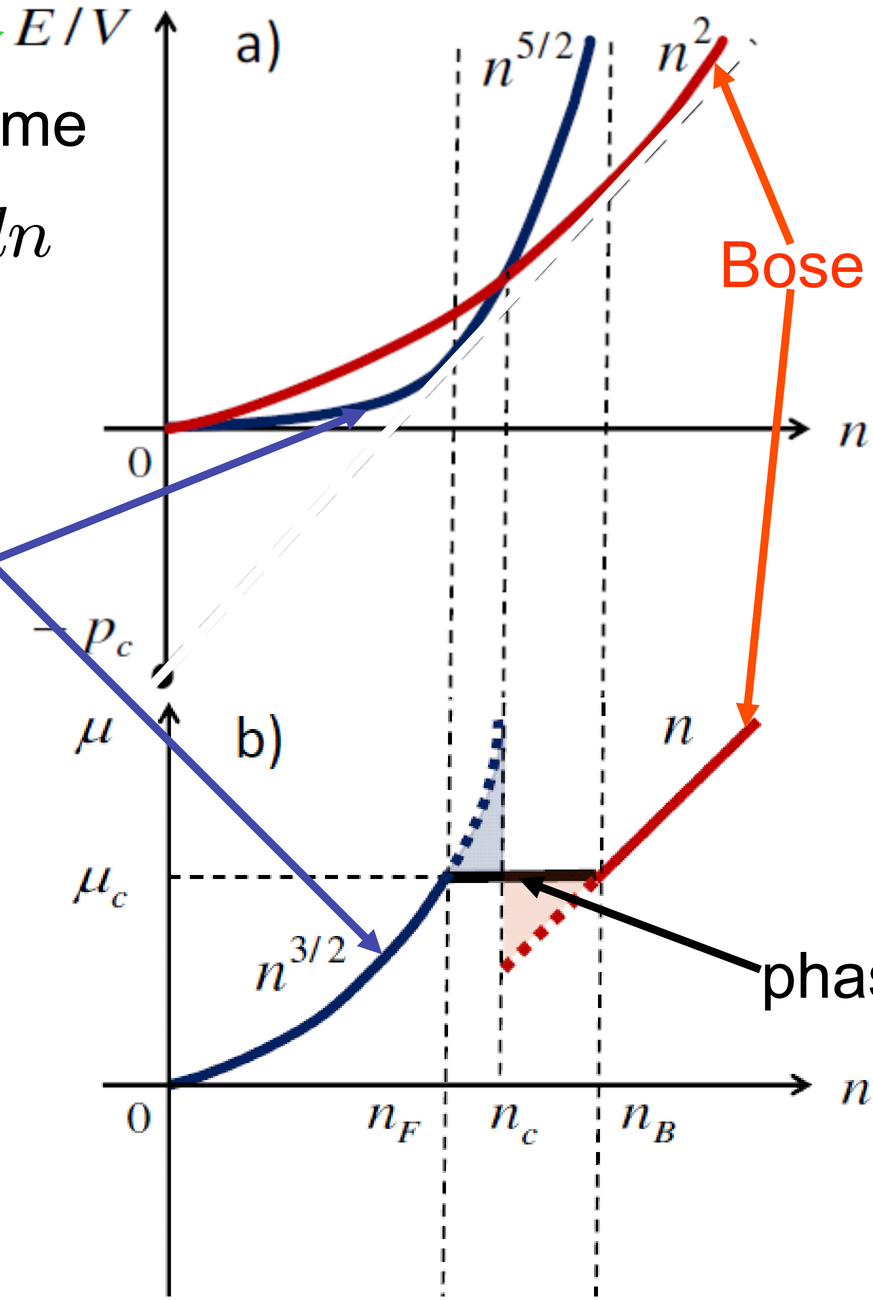
Phase Diagram



Phase Separation

total energy per volume

$$E(n) = \int^{n} \mu(n)dn$$



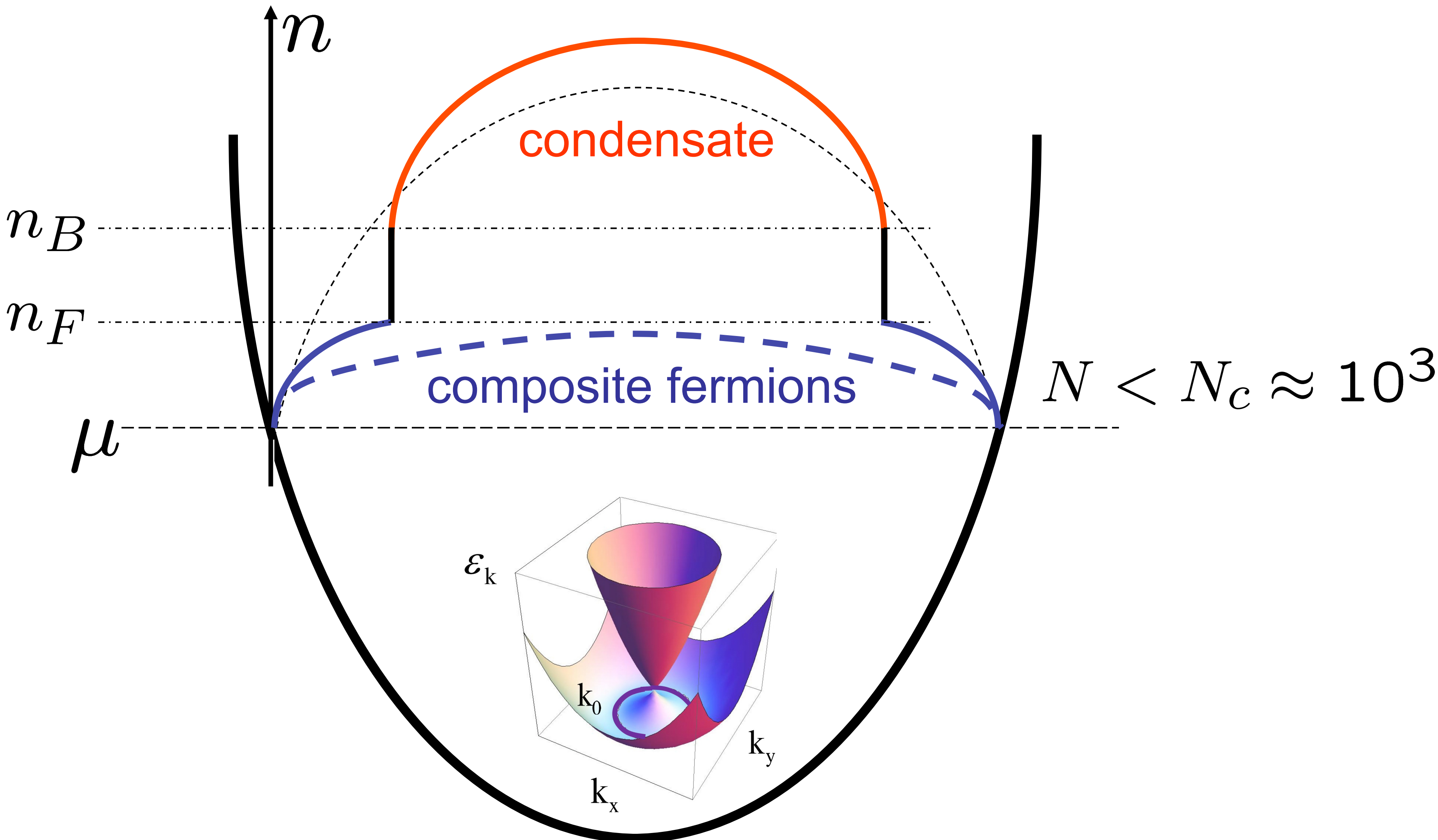
composite fermions

Bose condensate

phase separation

$$\boxed{n_{B,F} \propto g k_0^2}$$

Rashba Bosons in a Harmonic Trap



✓ high density Bose condensate in the middle and low density composite fermions in the periphery

Conclusions

- At low density Rashba bosons exhibit Composite Fermion groundstate
- CF state breaks **R, T** and **P** symmetries
- CF state is gaped in the bulk, but supports gapless edge mode, realizing interacting **topological insulator**
- CF equation of state: $\mu(n) \propto n^{3/2}$
- There is an interval of densities where CF **coexists** with the Bose condensate
- In a trap the low-density CF fraction is pushed to the **edges** of the trap

M