SUPERINDUCTORS: NEW TYPE OF QUBITS AND THEIR DECOHERENCE

Theoretical effort (I. Sadovski, B. Doucot, A. Kitaev & LI)
in collaboration with experimental group of M. Gershenson (Rutgers): M. Bell and M.G.

PLAN

1. What are superinductors?
2. Why do we need them?
3. Main idea – use quantum phase transition
4. What is really measured in experiment?
5. Results and theoretical expectations.
6. Ordered state and its excitations
7. Conclusions.

WHAT ARE SUPERINDUCTORS?

Superinductors:

Dissipationless inductors with $Z \gg R_Q = \frac{h}{(2e)^2} \approx 6.5\ \mathrm{k}\Omega$

and no extra dephasing

Potential applications:

- reduction of the sensitivity of Josephson qubits to the charge noise,
- Implementation of fault tolerant computation based on pairs of Cooper pairs and pairs of flux quanta
- *ac* isolation of the Josephson junctions in the electrical current standards based on Bloch oscillations.

Specific to our design (tunable non-linearity)

- Simple non-linear qubits that can be tuned from linear to non-linear regime
- Formation of protected low energy modes expected in quantum Ising models (Majorana fermions) and their experimental study.

GEOMETRICAL VS. KINETIC INDUCTANCE

Geometrical inductance of a wire: ~ 1 pH/μm.
Hence, it is difficult to make a large (1 μH ⟶ 6 kΩ @ 1 GHz) *L* in a planar geometry.

Moreover, a wire loop possesses not only geometrical inductance, but also a parasitic capacitance, and its microwave impedance is limited:

$$Z = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 8\alpha R_Q \approx 0.4\ \mathrm{k\Omega} \quad \alpha = \frac{e^2}{\hbar c} \approx \frac{1}{137}$$ - fine structure constant

Kinetic inductance of a superconducting wire

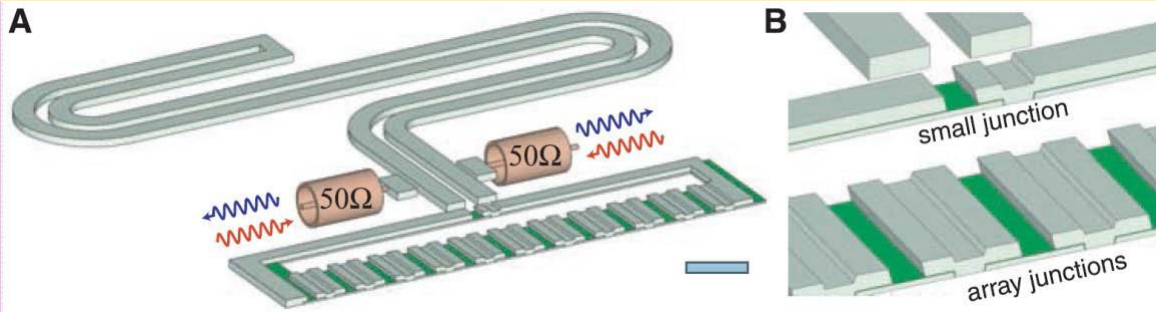
can be much larger for highly resistive films: $L = \frac{\hbar R}{\pi \Delta}$

For d=5 nm NbN films ($R_\square$=1 kΩ) $L_\square = 0.5$ nH,

for d=30 nm InO films ($R_\square$=3 kΩ) $L_\square = 2-4$ nH,

Manucharian, Science 2009

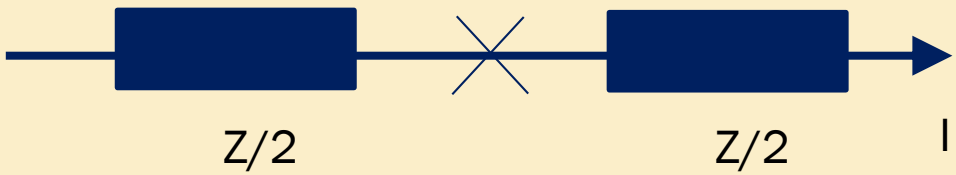
But it is still difficult to achieve inductances in μH range!
Alternative solution – chain of large Josephson
junctions (Yale) gave 0.3 μH



WHY DO WE NEED THEM?

WHY DOES ONE CARE? - BLOCH OSCILLATIONS

1. Bloch oscillations.



Semiclassical equations:

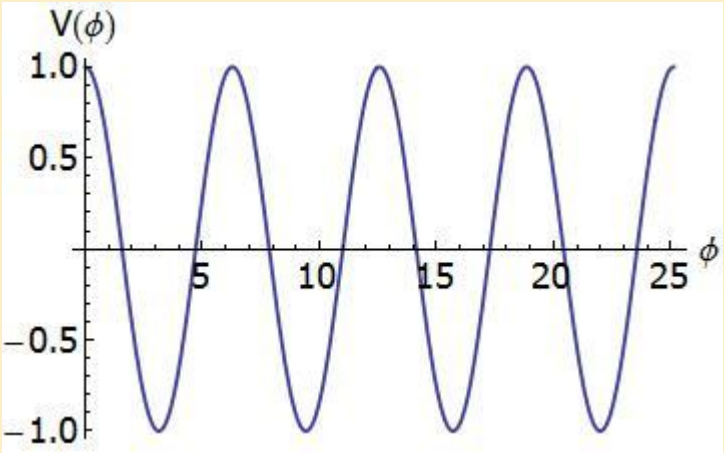
$$\frac{d\phi}{dt} = \frac{1}{\hbar}\frac{dE_n(q)}{dq}$$

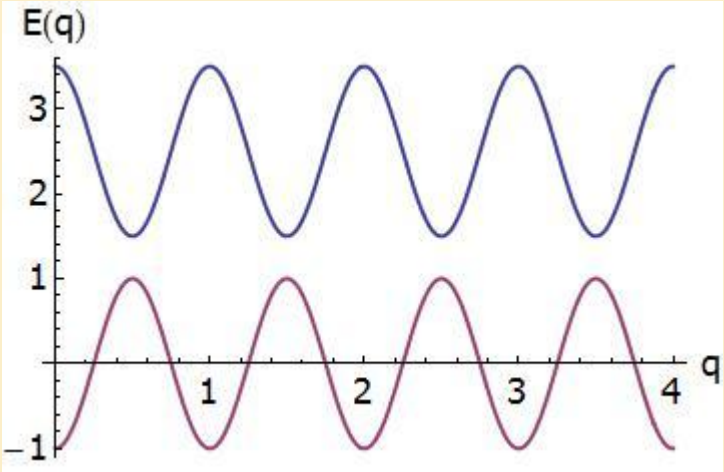
$$\frac{dq}{dt} = \frac{I}{2e} - \frac{Z_Q}{Z}\frac{d\phi}{dt}$$

Energy band $E_n(q)$ is obtained by diagonalization of

$$H = V(\phi) + 4E_c q^2$$

External current leads to $-I\phi/2e$ term that violates periodicity.





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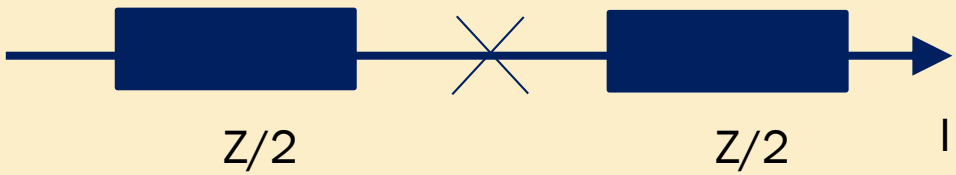

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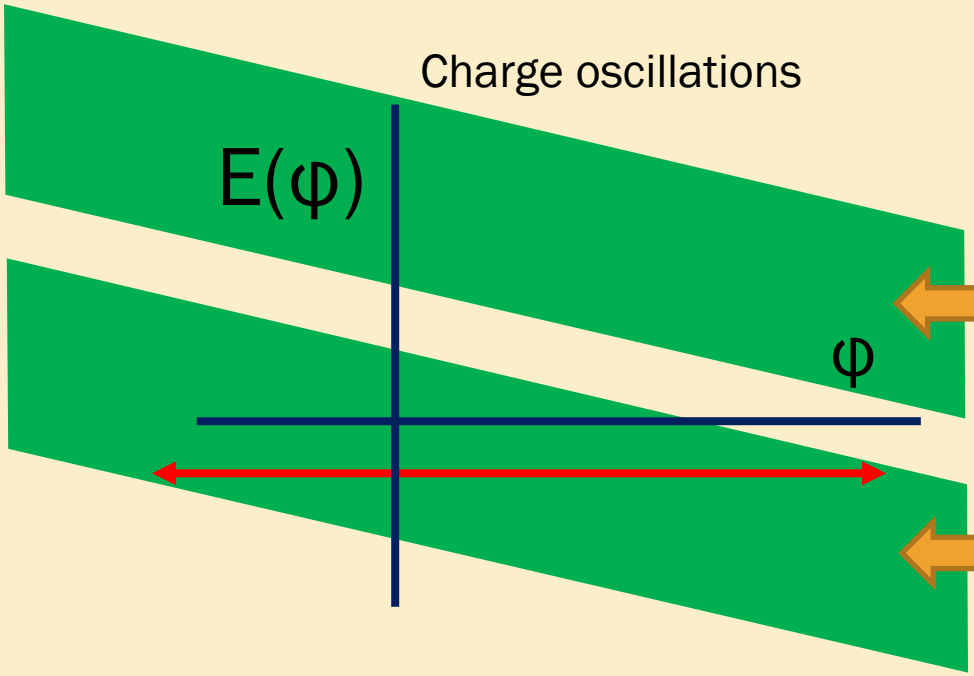
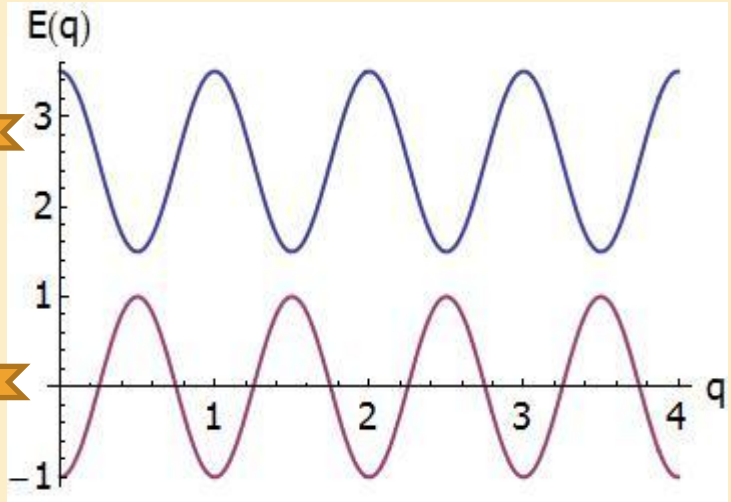
Charge oscillations





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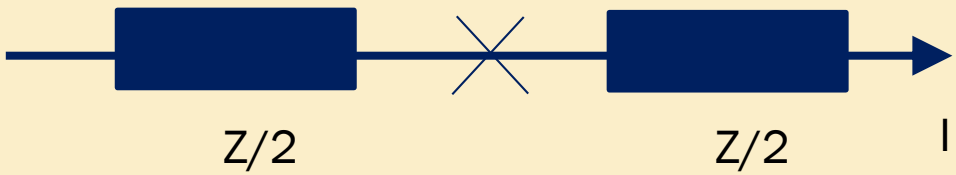

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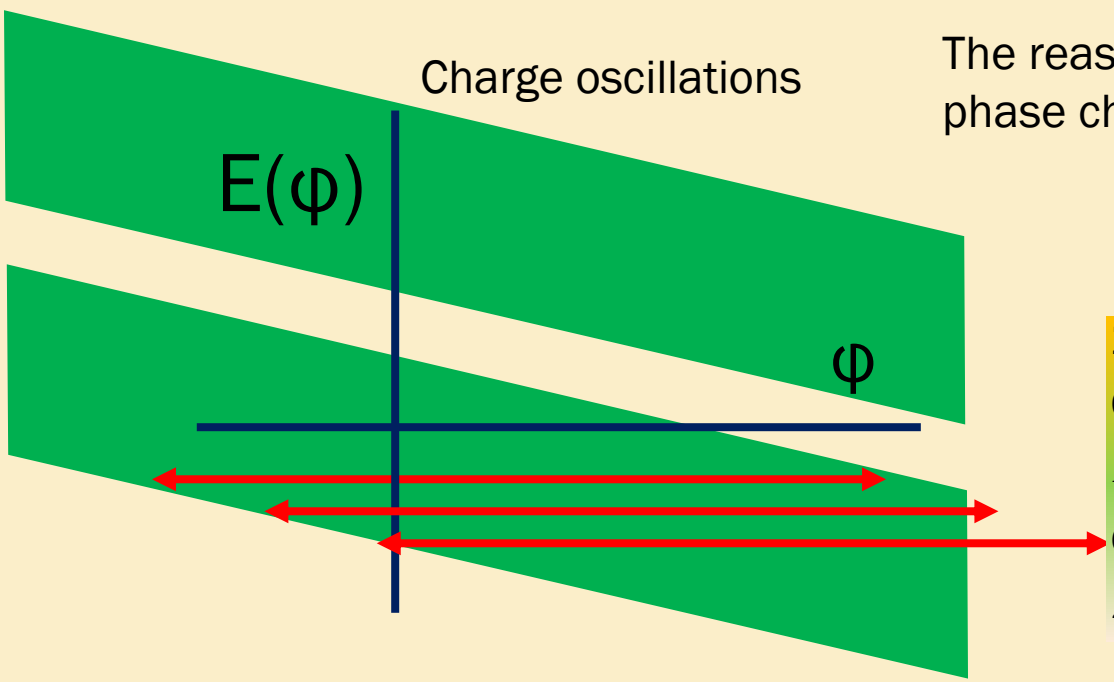
Oscillations with frequency f=I/2e
Plus slow drift down due to dissipation.

The reasoning is correct provided that phase changes by many periods:



$2eW / I \gg 2\pi$

Oscillatory solution appears only if

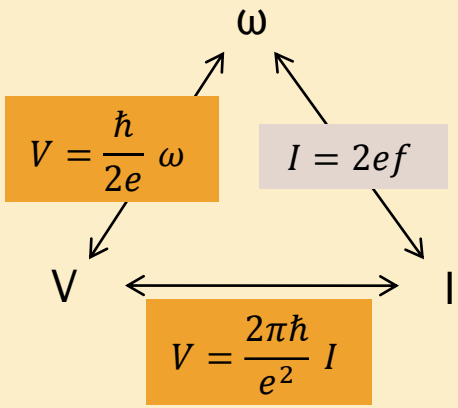
$I / 2e > W(Z_Q / Z)$

Conditions are compatible only if

$Z \gg 2\pi Z_Q$

BLOCH OSCILLATIONS

Allow to convert current into frequency I=2e f
provided that $Z \gg 2\pi Z_Q$

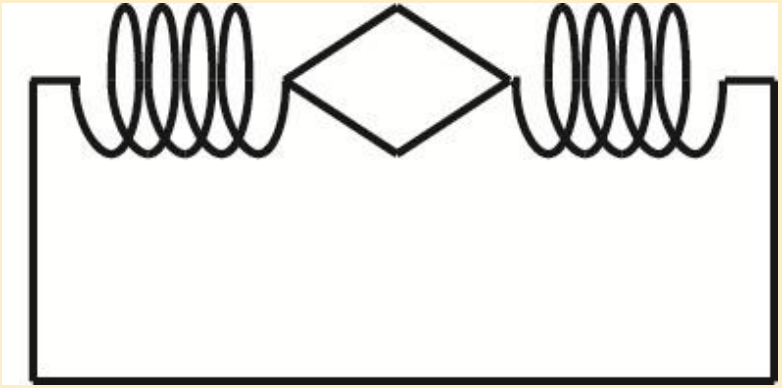


Application:

Josephson relation $V = \frac{\hbar}{2e}\,\omega$ is the basis of the voltage standard

Dual to it : $I = 2e\omega$ *would be* the basis of current standard.

ENCODING QUBIT IN THE OSCILLATOR



$$H_{dosc} = -\frac{1}{2}E_p(X^n + X^{-n}) + \frac{1}{2}E_L\left(2\pi k/n\right)^2$$

$$X\left|k\right\rangle = \left|k+1\right\rangle \qquad \text{n=2}$$

Discrete oscillator

$$\delta E_k = \cos\left(\frac{2\pi k}{n}\right)\left[\frac{4\sqrt{2}}{\pi}\left(\frac{E_p}{V_0}\right)^{1/4} e^{-\frac{4}{\pi}\sqrt{\frac{E_p}{E_L}}}\right]\omega_0$$

Forms two, almost degenerate states for $E_p \gg E_L$

For time dependent $E_L(t)$

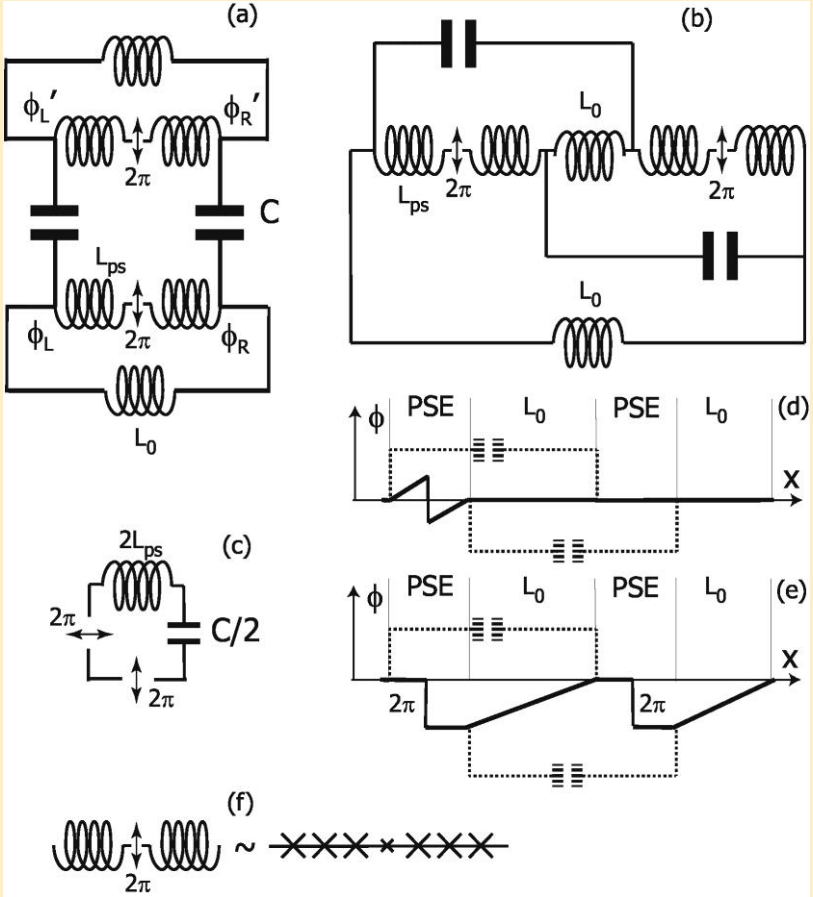
$$\psi_{k+pn} \to \psi_{k+pn} e^{2\pi^2 i \int dt E_L(t)(p+k/n)^2}$$

If $2\pi\int dt E_L(t) = nr$ the phases with different p are equal but

different k are different: $e^{\pi i k^2 r/2}$ - non trivial rotations

Need $\pi^2 E_L \ll E_p$ which is equivalent to Z>>$\pi^2 Z_Q$

FLUX PAIRING AND OTHER DEVICES



Simplest protected qubits - phase slip elements

For phase slip elements we need inductors with E_L that is of the order of $E_J \sim E_c$ of small junctions.

We also need even bigger inductors to form a large loop in which the flux would change by a multiple of 4π

$$H_{loop} = \frac{1}{2}E_{ps}(X + X^{-1}) + \frac{1}{2}E_{2ps}(X^2 + X^{-2}) + \frac{1}{2}E_L\left(2\pi k / n\right)^2$$

$$H_{pse} = \frac{1}{2}E_{Le}(\phi - 2\pi n)^2 + \frac{1}{2}E_0(X + X^{-1})$$

$E_{Le} \gg E_L \rightarrow$ so the state after the phase slip is purely virtual, its energy remains high for time $\tau = 1/\sqrt{8E_C E_{Le}}$

$$\rightarrow E_{ps} = \exp(-\pi\sqrt{E_{Le}/8E_C})E_0$$

$$E_{2ps} = E_0^2/(\pi^2 E_{Le})$$

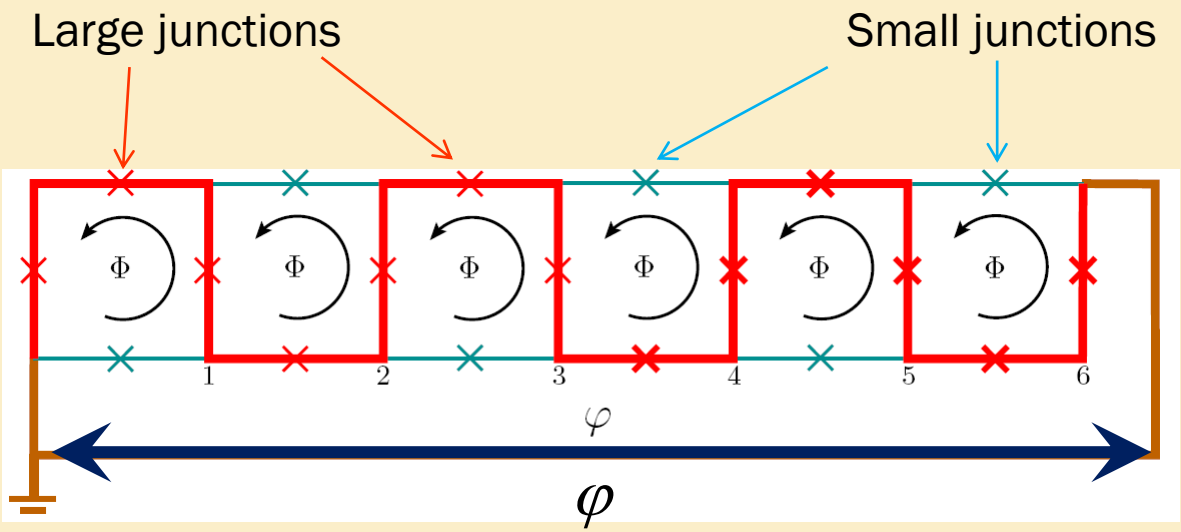
Need $E_L \ll \pi^2 E_0$

$L \geq 0.1$ mH

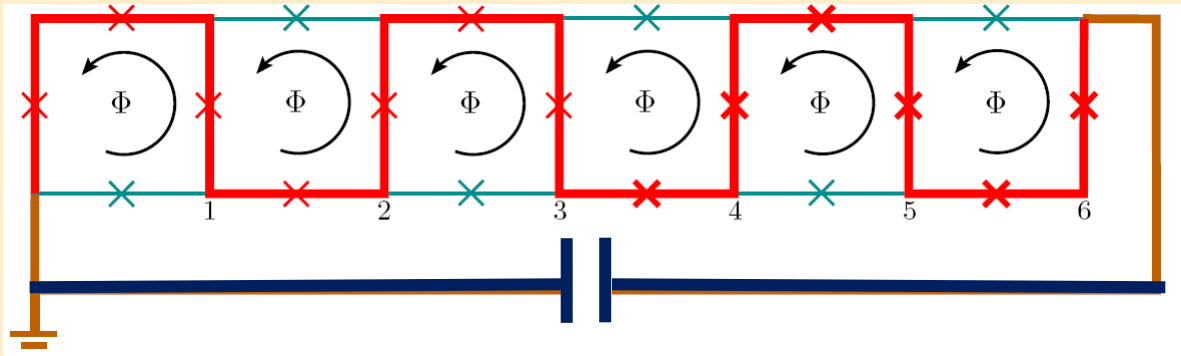
MAIN IDEA : USE QUANTUM PHASE TRANSITION

OUR SUPERINDUCTOR

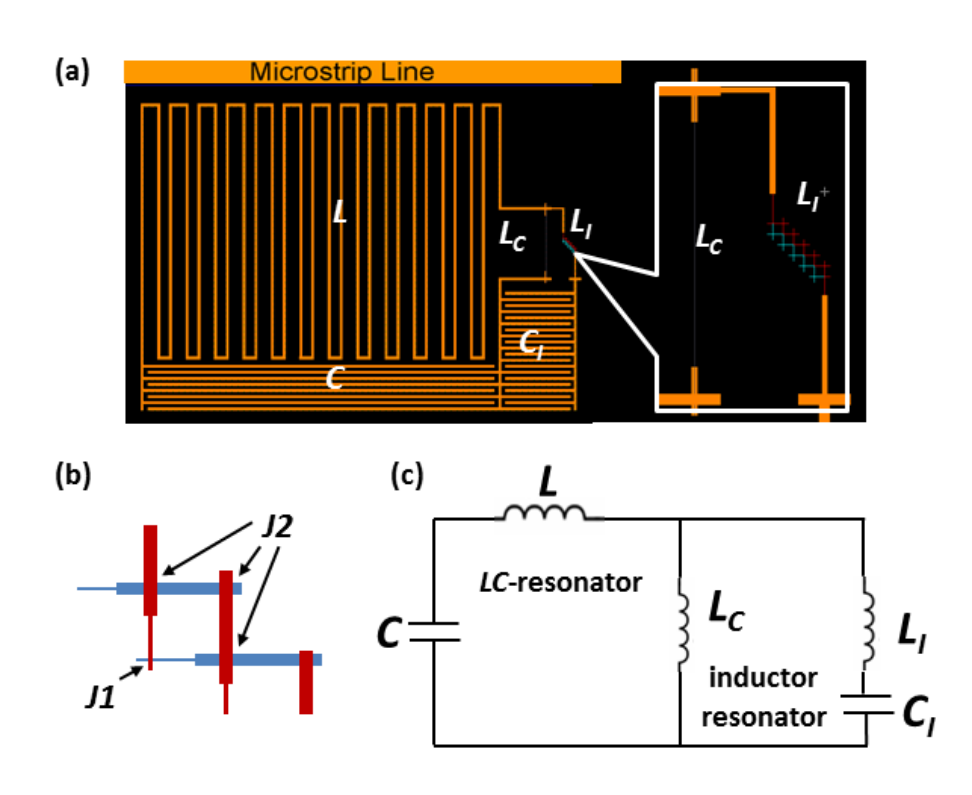
Frustrated **ladder** of Josephson junctions:



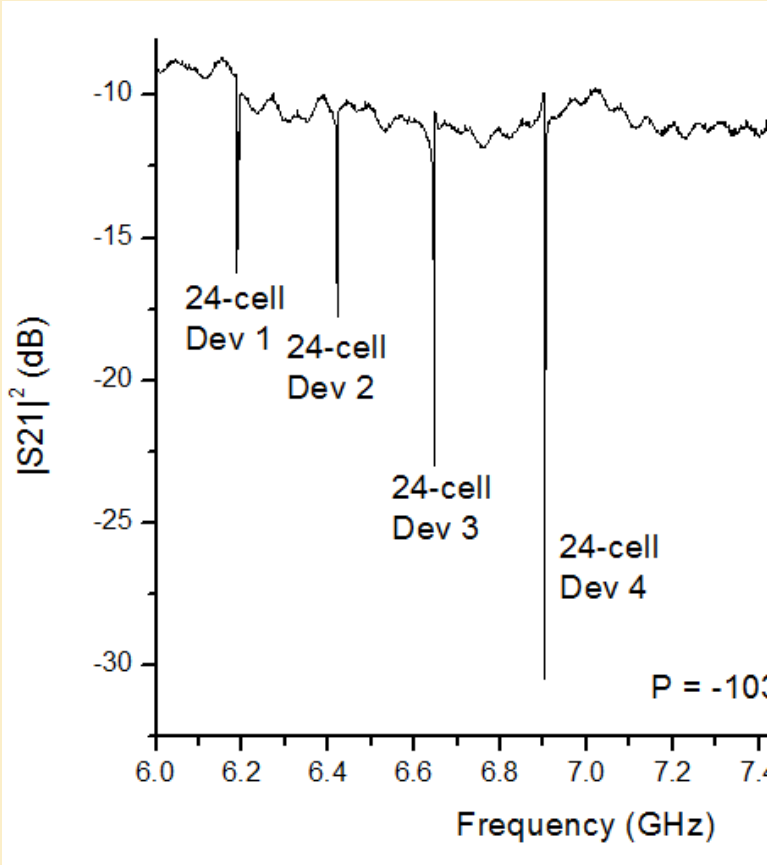
Simplest qubit based on superinductor:



EXPERIMENT SCHEMATICS



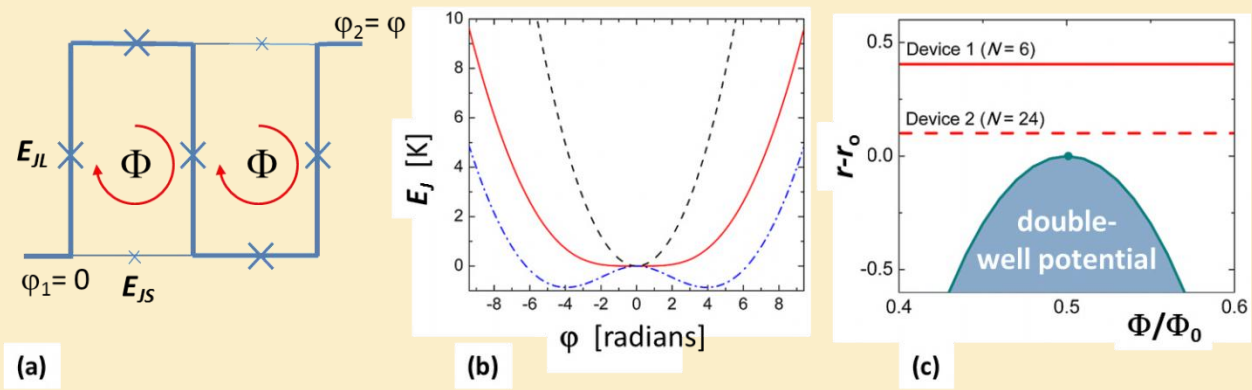
Multiplexing – many devices with different E_J of large and small contacts in one microwave line



MAIN IDEA OF SUPERINDUCTORS

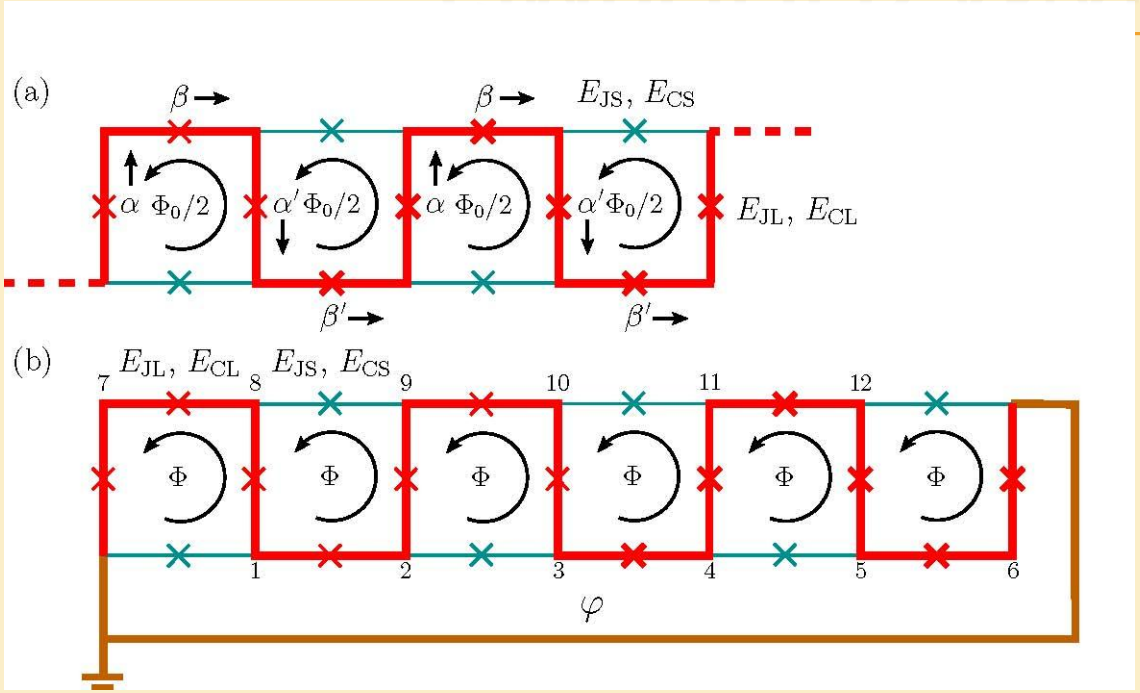
Form effective potential that is soft at the bottom but does not allow phase slips
Advantages:

- No offset charge sensitivity.
- Large tunable non-linearity.
- Protection from the flux noise (no linear coupling).



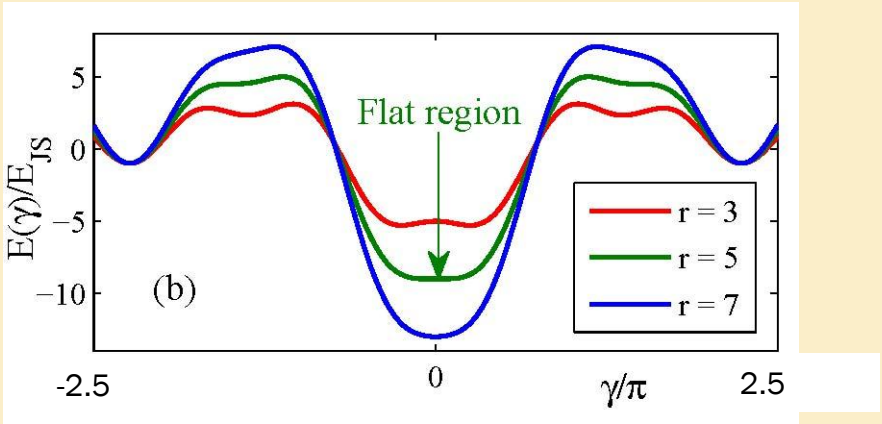
Potential can be varied from a single well to double well either by changing the ratio of small to large junction and by changing the flux.

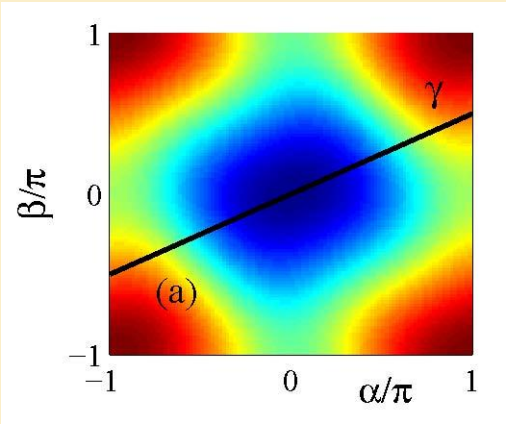
SIMPLIFIED ANALYSIS



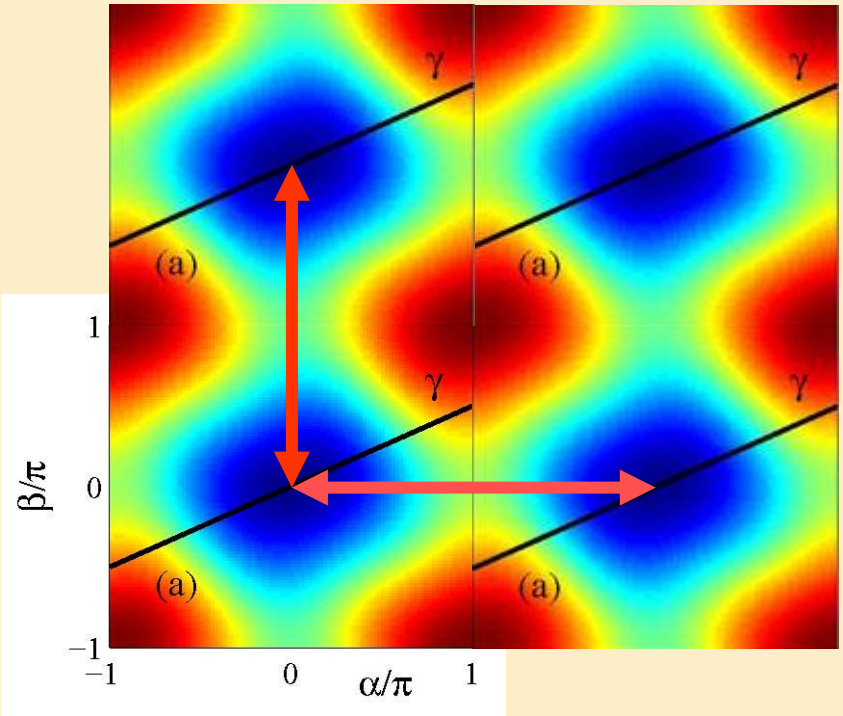
Exactly at half flux per plaquette, assuming uniform mode:

$$E(\alpha,\beta) = -E_{JL}(\cos\alpha + \cos\beta) - E_{JS}\cos(\pi - 2\alpha - \beta).$$





ESTIMATES OF PHASE SLIP AMPLITUDE



Transition paths that change the phase by 2π

$$t \approx E_{JL}^{1/4} E_{CS}^{3/4} Exp(-c\sqrt{E_{JL} / E_{CS}})$$

c depends on path, $c = 2.5 - 2.8$

For a typical device $E_{JL} / E_{CS} \approx 100$

So phase slip amplitude is negligible:

$$t \sim E_J 10^{-10}$$

FULL QUANTUM PROBLEM

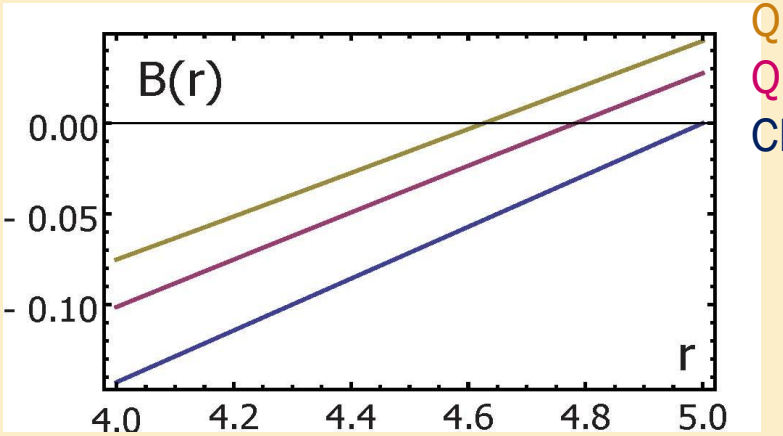
For physical parameters (charging and Josephson energies) quantum fluctuations result in modest renormalization of the effective energy due to short scale fluctuations:

Classical $E(\alpha,\beta) = -E_{\mathrm{JL}}(\cos\alpha + \cos\beta) - E_{\mathrm{JS}}\cos(\pi - 2\alpha - \beta).$

Quantum corrections near minimum $E(\varphi) = \dfrac{B_{\mathrm{q}}(r)}{N} E_{\mathrm{JS}}\varphi^2 + \dfrac{C_{\mathrm{q}}(r)}{N^3} E_{\mathrm{JS}}\varphi^4.$

$$B_{\mathrm{q}}(r) = B_{\mathrm{cl}}(r + \delta r) \qquad B_{\mathrm{cl}}(r) = \frac{5}{18} E_{\mathrm{JS}}(r - r_o)$$

$$C_{\mathrm{q}}(r) = 0.7 C_{\mathrm{cl}}(r = r_{\mathrm{o}})$$



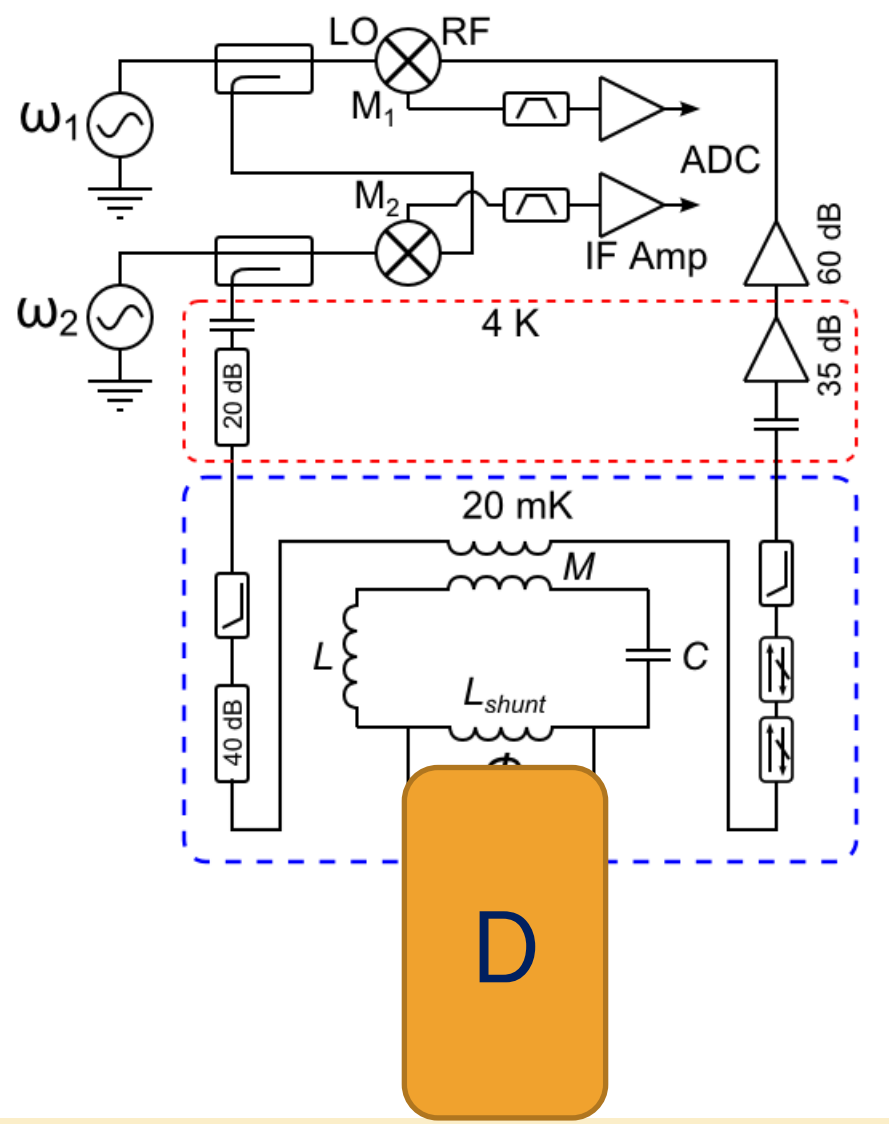
Quantum 4 rung
Quantum 3 rung
Classical

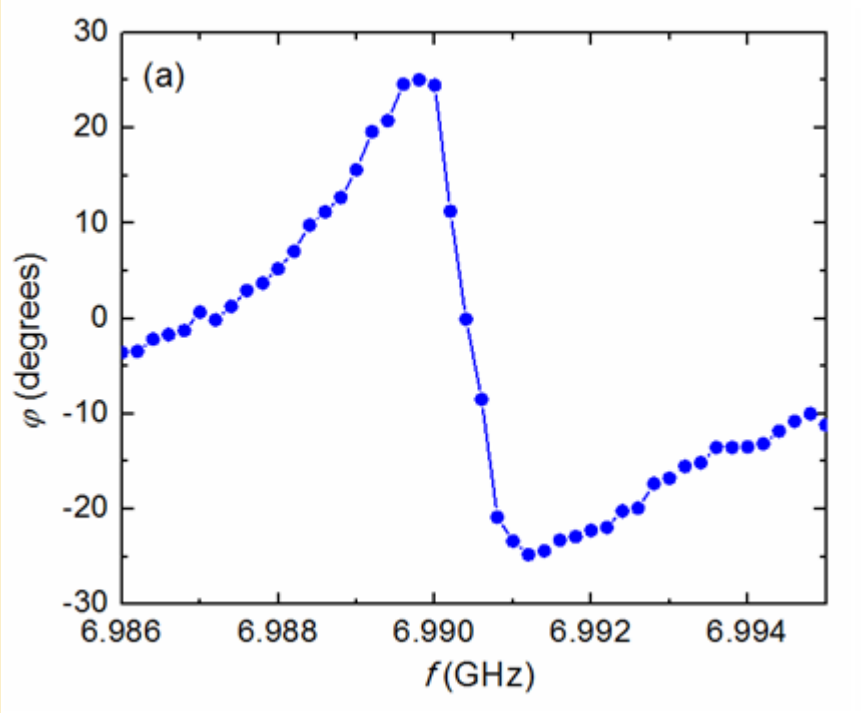
$$H = V(\varphi) + 4E_C q^2$$

Gives low energy levels of the device

UNDERSTANDING THE DATA

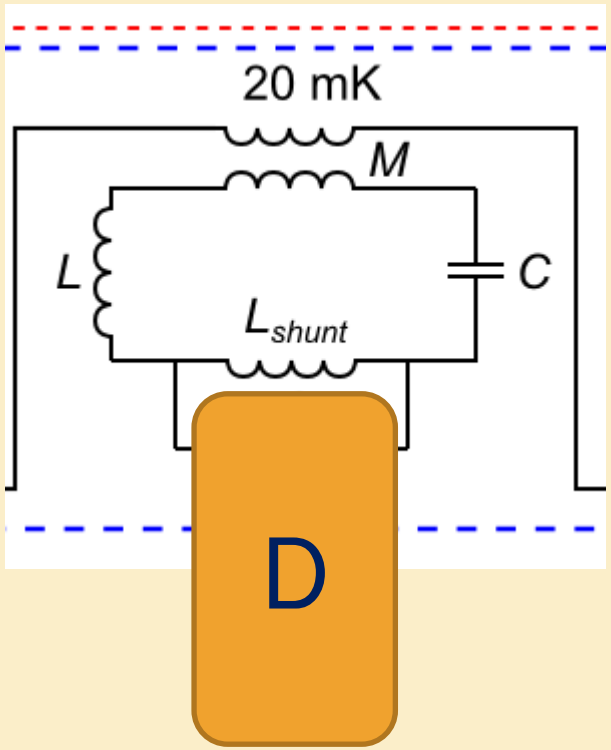
WHAT IS REALLY MEASURED?





Small (tiny) frequency shifts of the LC – resonator when “device” is excited by the second tone.

WHAT IS REALLY MEASURED?



$$\mathcal{L} = T_{sh}(V_{sh}) + T_m(V_m) + \frac{C}{2}(V_{sh} + V_m)^2 - \frac{1}{2}E_{sh}\phi_{sh}^2 - \frac{1}{2}E_m\phi_m{}^2$$

$$+\mathcal{L}_D(\phi_0, V_0).$$

$$\mathcal{L}_{eff} = \frac{C_L}{2}V_0^2 - \frac{1}{2}E_L(\phi_0 - \phi_B)^2 + \mathcal{L}_D(\phi_0, V_0)$$

$$C_L = C(1 + E_{sh} / E_m)^2$$

$$E_L = E_{sh}(1 + E_{sh} / E_m)$$

$$H_R = \frac{\omega_0^2}{2E_L}q_0^2 + \frac{1}{2}E_L\phi_0{}^2$$

$$H_{int} = C_L^{-1}\sum_{i,j>0} q_0 C_{0j} C_{ji}^{-1}(\mathbf{q}_{ji} - n_i) - \sum_i J_{i0}\cos(\phi_0 - \phi_j - \Phi_{0i})$$

$$H_D = \frac{1}{2}\sum_{ij>0}(\mathbf{q_i} - n_i)C_{ij}^{-1}(\mathbf{q}_j - n_j) - \frac{1}{2}\sum_{ij>0} J_{ij}\cos(\phi_i - \phi_j - \Phi_{ij})$$

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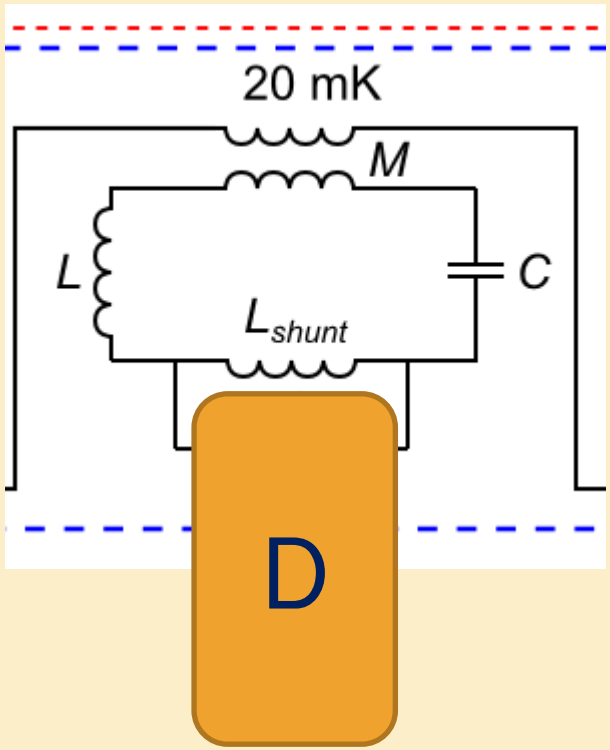


$$H_{eff} = (\omega_0 + 2A^2\Xi)(a^\dagger a + 1/2) + (A\mathbf{J} + \frac{i}{2A}\mathbf{Q})a + h.c.$$

$$\mathbf{J} = \left.\frac{dL}{d\phi_0}\right|_{\phi_0=0} = -\sum_i J_{i0}\sin(\phi_j + \Phi_{0i})$$

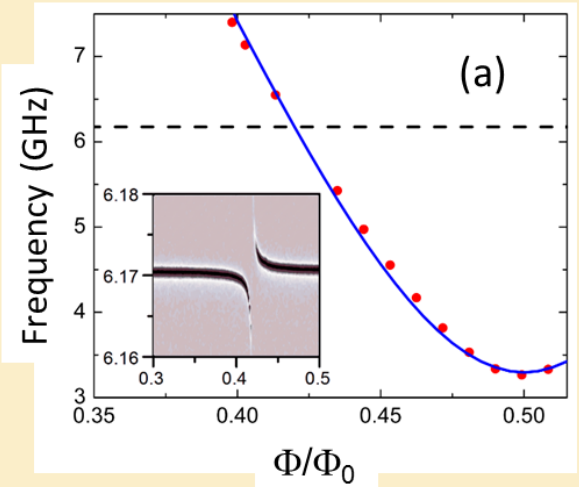
$$\mathbf{Q} = C_L^{-1}\sum_{i,j>0} C_{0j}C_{ji}^{-1}(\mathbf{q}_{ji} - n_i)$$

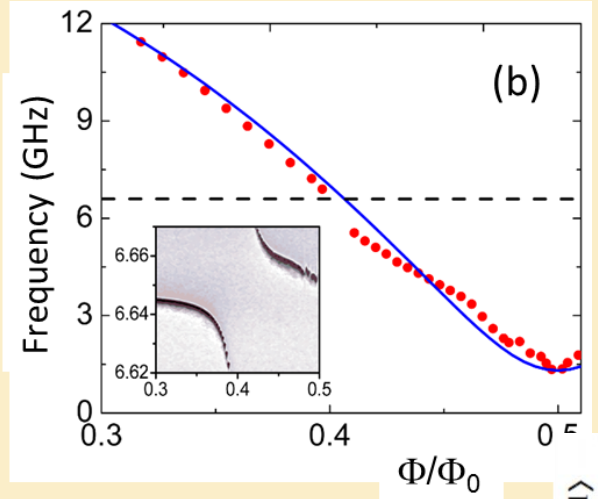
$$\Xi = \frac{1}{2}\left.\frac{d^2L}{d\phi_0^2}\right|_{\phi_0=0} = \frac{1}{2}\sum_i J_{i0}\cos(\phi_j + \Phi_{0i})$$

Conclusion: measurement tests for the current operator and charge operators of the devices in the ground and excited states.

RESULTS

THEORETICAL PREDICTIONS AND COMPARISON WITH DATA FOR MEDIUM AND SHORT LADDERS



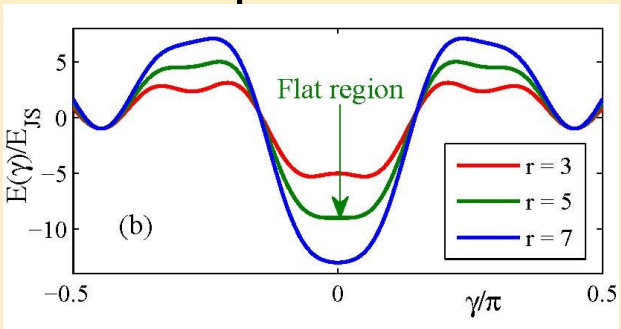


N=6

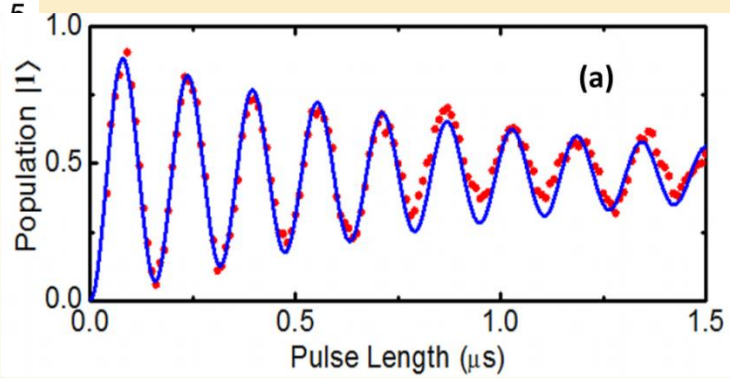
$$H = V_{eff}(\varphi) + 4E_C q^2$$

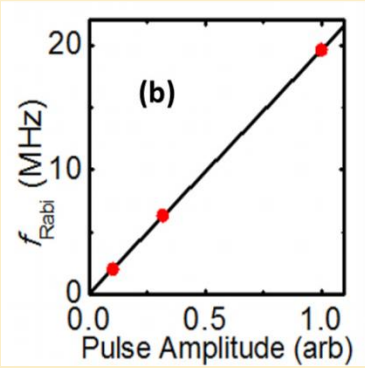
N=24

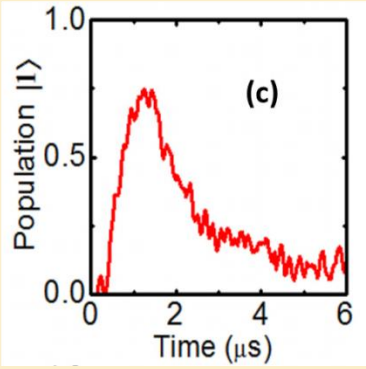
Gives low energy levels of the device in the effective potential $V_{eff}(\varphi)$ (classical + quantum corrections)



Rabi oscillations with time constant 1.4 μs







LONG CHAINS (THEORETICAL EXPECTATIONS)

Need to take into account spatially non-uniform fluctuations

$$\mathcal{L} = \int dz \left[\frac{1}{16 E_{\mathrm{C}}^{(\mathrm{eff})}} \left(\frac{\partial \gamma}{\partial t} \right)^2 - V(\gamma) \right],$$

$$V(\gamma) = E_{\mathrm{JS}} \left[\frac{3}{5} \left(\frac{\partial \gamma}{\partial z} \right)^2 + \frac{1}{2}(r - r^*)\gamma^2 + \left(\frac{25}{24} - \frac{17r}{600} \right) \gamma^4 \right],$$

Close to critical point $r = r^*$ JJ ladder is equivalent to

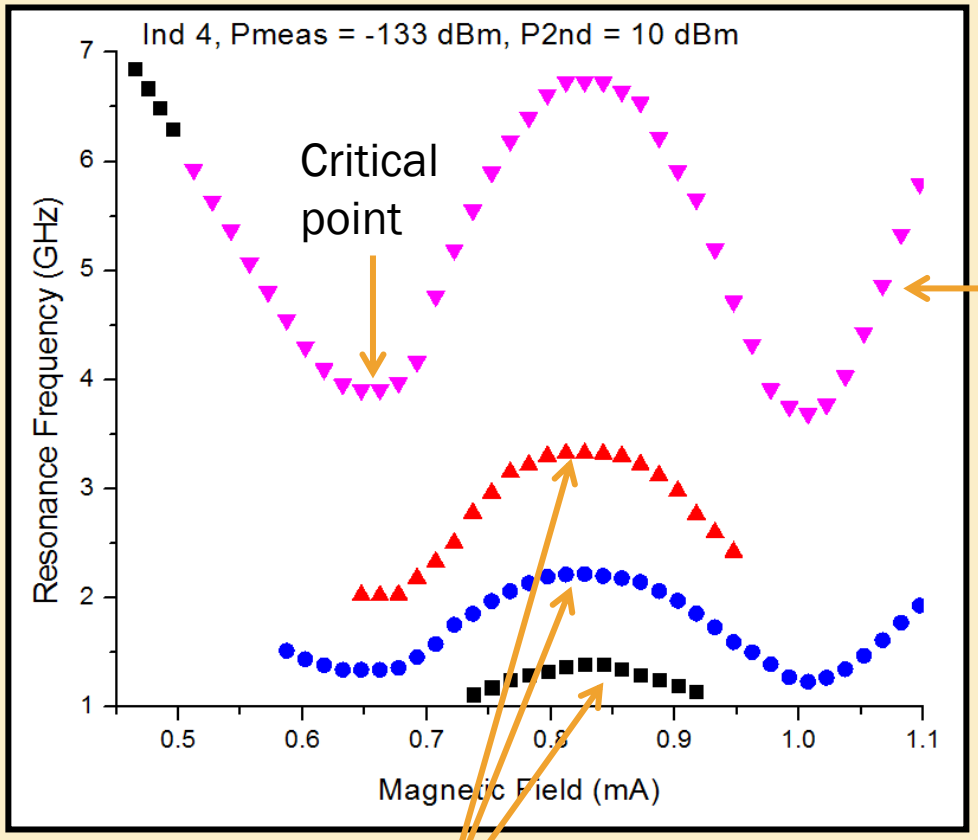
Quantum Ising model (critical point at $J = h$):

$$H = -J \sum_{i=0}^{N} S_i^z S_{i+1}^z + h \sum_{i=0}^{N} S_i^x$$

Low energy modes - Majorana fermions with pseudo-linear spectrum

$$\omega = \sqrt{E_C \left(Jk^2 + |J - h| \right)}$$

CRITICAL BEHAVIOR OF LONG CHAINS - DATA

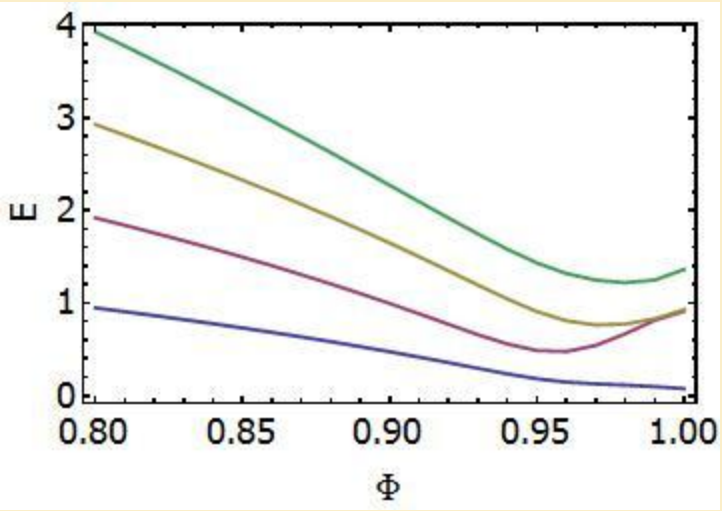


Internal low energy (Majorana) mode.

2-photon 3-photon and 4 photon transitions

LOW LEVELS OF LADDER IN THE ORDERED STATES

Numerical similations on small ladder



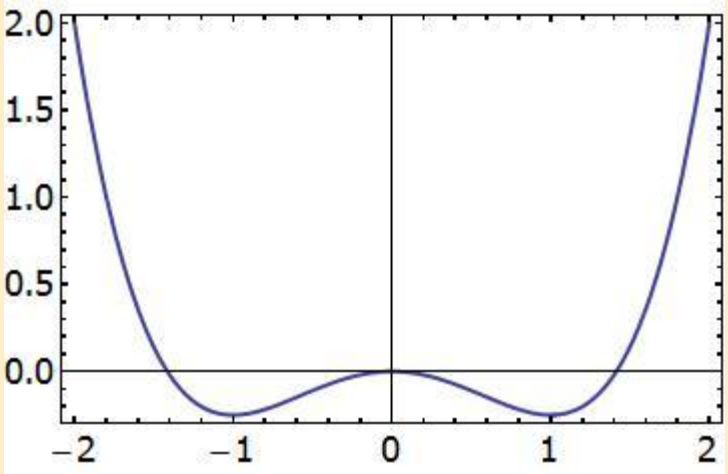
Lowest level – two degenerate (in the thermodynamic limit) states.

Not observable because charge operators in two lowest states of the global potential are equal.

$$|0\rangle = \frac{1}{\sqrt{2}}(|L\rangle + |R\rangle)$$

$$|1\rangle = \frac{1}{\sqrt{2}}(|L\rangle - |R\rangle)$$

$$\langle L,R|\hat{Q}|L,R\rangle \approx 0$$

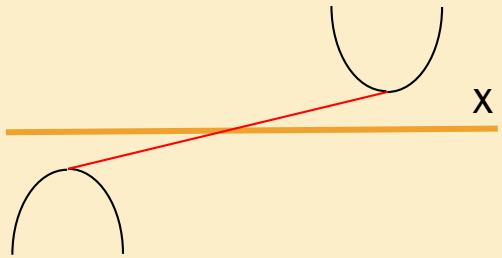


CURRENT RESEARCH

1. Prove that low energy mode observed experimentally is indeed Majorana fermion.
2. What is its coherence time (experiment)?
3. What is the dominant mechanism of decoherence (theory)?
4. Can we create very low energy Majorana modes by constructing smooth boundaries between disordered and ordered phases?

$$V(\gamma) = E_{\mathrm{JS}}\left[\frac{3}{5}\left(\frac{\partial\gamma}{\partial z}\right)^2 + \frac{1}{2}(r(x) - r^*)\gamma^2 + C\gamma^4\right],$$

$$r(0) > r^*,\ r(L) < r^*$$



5. What is the dominant mechanism of decoherence in these low energy modes?
6. Non-Abelian excitations formed when three chains interact?

CONCLUSIONS

1. Superinductors realized in the frustrated Josephson junction ladders show L up to 3μH in the fully frustrated regime.
2. Expect no phase slip amplitude.
3. Simplest qubit realized by superinductance has a reasonbly long decoherence time > 1 μs
4. Tunable non-linearity
5. Can be used to implement critical quantum Ising model and to realize qubits built on non-local Majorana modes.
6. What are the main mechanisms of decoherence and what is intrinsic decoherence time of the ladders remains to be studied – the current decoherence time is dominated by the coupling to the microwave line?
7. What is the coherence of internal modes?