Exact solutions to 4D higher-spin gravity

Carlo IAZEOLLA

Lebedev Physical Institute, Moscow
&
Institute of Physics of the Czech Academy of Sciences, Prague

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Summary

- The 4D Vasiliev equations
  - Oscillator algebras
  - Full equations (bosonic)

- Solving the equations
  - Gauge function method
  - Moduli space
  - Gauge-invariant characterization via zero-form charges

- Exact solutions
  - Zoology of known exact solutions
  - HS black-hole-like solutions

- Conclusions and Outlook

Oscillator algebra

- Commuting variables $Y_{\underline{\alpha}} = (y_\alpha, \bar{y}_{\dot{\alpha}})\ , \quad Z_{\underline{\alpha}} = (z_\alpha, -\bar{z}_{\dot{\alpha}}) \ \rightarrow\ \mathfrak{sp}(4,\mathbb{R})$ quartets

$$[Y_{\underline{\alpha}}, Y_{\underline{\beta}}]_\star = 2iC_{\underline{\alpha\beta}} = 2i \begin{pmatrix} \varepsilon_{\alpha\beta} & 0 \\ 0 & \varepsilon_{\dot{\alpha}\dot{\beta}} \end{pmatrix}\ , \qquad [Z_{\underline{\alpha}}, Z_{\underline{\beta}}]_\star = -2iC_{\underline{\alpha\beta}}\ , \quad [Y_{\underline{\alpha}}, Z_{\underline{\beta}}]_\star = 0$$

- Star-product [normal-ordering wrt $A^+ = (Y-Z)/2i$, $A^- = (Y+Z)/2$]

$$\widehat{F}(Y,Z) \star \widehat{G}(Y,Z) \ = \ \int_{\mathcal{R}} \frac{d^4U d^4V}{(2\pi)^4}\, e^{iV^{\underline{\alpha}}U_{\underline{\alpha}}}\, \widehat{F}(Y+U, Z+U)\, \widehat{G}(Y+V, Z-V)$$

- π automorphism generated by the inner kleinian operator κ:

$$\pi(\widehat{f}(y,\bar{y};z,\bar{z})) \ = \ \widehat{f}(-y,\bar{y};-z,\bar{z})\ , \quad \bar{\pi}(\widehat{f}(y,\bar{y};z,\bar{z})) \ = \ \widehat{f}(y,-\bar{y};z,-\bar{z})$$

$$\begin{aligned} \pi(\widehat{f}) \ &= \ \kappa \star \widehat{f} \star \kappa\ , \qquad \kappa \ = \ e^{iy^\alpha z_\alpha}\ , \qquad \kappa\star\kappa \ = \ 1 \\ \kappa \ &= \ \kappa_y \star \kappa_z\ , \quad \kappa_y \star \kappa_y \ = \ 1 \quad \textit{idem}\ \kappa_z,\ \bar{\kappa}_{\bar{y}} \text{ and } \bar{\kappa}_{\bar{z}} \\ \kappa_y \ &= \ 2\pi\delta^2(y) \ = \ 2\pi\delta(y_1)\delta(y_2) \end{aligned}$$

- Fields live on correspondence space, locally $\mathcal{X}$ x $\mathcal{Y}$ x $\mathcal{Z}$:

$$d \rightarrow \hat{d} = d + d_Z \ = \ dx^\mu \frac{\partial}{\partial x^\mu} + dz^\alpha \frac{\partial}{\partial z^\alpha} + d\bar{z}^{\dot{\alpha}} \frac{\partial}{\partial \bar{z}^{\dot{\alpha}}}$$

$$A(x|Y) \rightarrow \hat{A}(x|Z,Y) \equiv (dx^\mu \hat{A}_\mu + dz^\alpha \hat{A}_\alpha + d\bar{z}^{\dot{\alpha}} \hat{A}_{\dot{\alpha}})(x|Z,Y)\ , \quad A_\mu(x|Y) = \hat{A}_\mu\big|_{Z=0}$$

$$\Phi(x|Y) \rightarrow \hat{\Phi}(x|Z,Y)\ , \quad \Phi(x|Y) = \hat{\Phi}(x|Z,Y)\big|_{Z=0}$$

The Vasiliev Equations

- Gauge field $\in$ Adj($\mathfrak{hs}$(3,2)) (*master 1-form connection*):

$$A_\mu(x|y,\bar{y}) = \sum_{n+m=2\text{mod}4}^{\infty} \frac{i}{2n!m!} dx^\mu A_\mu{}^{\alpha_1\ldots\alpha_n\dot{\alpha}_1\ldots\dot{\alpha}_m}(x) y_{\alpha_1}\ldots y_{\alpha_n}\bar{y}_{\dot{\alpha}_1}\ldots\bar{y}_{\dot{\alpha}_m}$$

(every spin-s sector contains all one-form connections that are necessary for a frame-like formulation of HS dynamics (finitely many))

Generators of $\mathfrak{hs}$(3,2): $T_s \sim y_{\alpha_1}\ldots y_{\alpha_n}\bar{y}_{\dot{\alpha}_1}\ldots\bar{y}_{\dot{\alpha}_m}\ ,\ \frac{n+m}{2}+1=s$

Bilinears in osc. → $\mathfrak{so}$(3,2): $M_{AB} = -\frac{1}{8} Y^{\underline{\alpha}}(\Gamma_{AB})_{\underline{\alpha\beta}} Y^{\underline{\beta}} = \{M_{ab}, P_a\}$

- **Massless UIRs** with all spins in AdS include **a scalar**!

→ "twisted adjoint" *master 0-form* (contains scalar, Weyl, HS Weyl and derivatives)

$$T(X)(\Phi) = [X,\Phi]_{\star,\pi} \equiv X\star\Phi - \Phi\star\pi(X)$$

- *Weyl 0-form* : $\Phi(x|y,\bar{y}) = \sum_{|n-m|=0\text{mod}4}^{\infty} \frac{1}{n!m!} \Phi^{\alpha_1\ldots\alpha_n\dot{\alpha}_1\ldots\dot{\alpha}_m}(x)\, y_{\alpha_1}\ldots y_{\alpha_n}\bar{y}_{\dot{\alpha}_1}\ldots\bar{y}_{\dot{\alpha}_m}$

N.B.: spin-s sector → infinite-dimensional

(upon constraints, all on-shell-nontrivial covariant derivatives of the physical fields, *i.e.*, all the local dof encoded in the 0-form at a point)

The Vasiliev Equations

- Full eqs: $$\hat{F} \equiv \hat{d}\hat{A} + \hat{A} \star \hat{A} = \frac{i}{4}(dz^{\alpha} \wedge dz_{\alpha}\,\hat{\mathcal{B}} \star \hat{\Phi} \star \kappa + \; d\bar{z}^{\dot{\alpha}} \wedge d\bar{z}_{\dot{\alpha}}\hat{\bar{\mathcal{B}}} \star \hat{\Phi} \star \bar{\kappa})$$

  (Vasiliev '90) $$\hat{\mathcal{D}}\hat{\Phi} \equiv \hat{d}\hat{\Phi} + \hat{A} \star \hat{\Phi} - \hat{\Phi} \star \bar{\pi}(\hat{A}) = 0$$

Local sym: $$\delta\hat{A} = \hat{D}\hat{\epsilon} \;, \quad \delta\hat{\Phi} = -[\hat{\epsilon}, \hat{\Phi}]_{\pi}$$

- In components:

$$\begin{aligned}
\widehat{F}_{\mu\nu} \;&=\; \widehat{F}_{\mu\alpha} \;=\; \widehat{F}_{\mu\dot{\alpha}} \;=\; 0 \;, \qquad \widehat{D}_{\mu}\widehat{\Phi} \;=\; 0 \;, \\
\left[\widehat{S}_{\alpha}, \widehat{S}_{\beta}\right]_{\star} \;&=\; -2i\epsilon_{\alpha\beta}(1 - \mathcal{B} \star \widehat{\Phi} \star \kappa) \;, \\
\left[\widehat{S}_{\dot{\alpha}}, \widehat{S}_{\dot{\beta}}\right]_{\star} \;&=\; -2i\epsilon_{\dot{\alpha}\dot{\beta}}(1 - \bar{\mathcal{B}} \star \widehat{\Phi} \star \bar{\kappa}) \\
\left[\widehat{S}_{\alpha}, \widehat{S}_{\dot{\beta}}\right]_{\star} \;&=\; 0 \;, \\
\widehat{S}_{\alpha} \star \widehat{\Phi} \;&+\; \widehat{\Phi} \star \pi(\widehat{S}_{\alpha}) \;=\; 0 \;, \\
\widehat{S}_{\dot{\alpha}} \star \widehat{\Phi} \;&+\; \widehat{\Phi} \star \bar{\pi}(\widehat{S}_{\dot{\alpha}}) \;=\; 0
\end{aligned}$$

$$\hat{S}_{\alpha} \;=\; z_{\alpha} - 2i\hat{A}_{\alpha}$$

- Z-evolution determines Z-contractions in terms of original dof. Solution of Z-eqs. yields consistent nonlinear corrections as an expansion in Φ.

Exact solutions in HSGRA

- Crucial to look into the non-perturbative sector of the theory, may shed some light on peculiarities of HS physics and prompts to study global issues in HS gravity (boundary conditions, asymptotic charges, global dof in $\mathcal{Z}$…).

- The Z-extended unfolded system encodes physical field equations that are highly non-local: all higher-derivative terms are induced by HS symmetries, come with the non-linear corrections, and are normalized by the inverse cosmological constant .
  For any order in the coupling constant, infinite derivative expansion.
  → radical departure from the familiar setups of lower-spin field theories, quantum effective theories or even SUGRA with stringy higher-derivative corrections.

- Very likely new tools, and HS geometry adapted to HS symmetries, have to be developed for a proper physical interpretation of the theory.

Black Holes and Higher Spins

- HS Gravity contains a spin-2 field. Natural to look for black-hole-like solutions, among the simplest possible solutions in Gravity.

- HS Gravity does not admit a consistent truncation to spin 2. No obvious embedding of gravitational bhs.

- Characterization of bhs rests on geodesic motion, but relativistic interval $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$ is NOT HS-invariant . What is to be called a "higher-spin black hole"?

- Do non-local interactions & HS gauge symmetries smooth out singularities? (already from String Theory we are used to higher-derivative stringy correction affecting the nature of singularities)

Exact solutions: gauge function method

- $\mathcal{X}$ x $\mathcal{Y}$ x $\mathcal{Z}$-space eqns:
- $\mathcal{Y}$ x $\mathcal{Z}$-space eqns:

$$
\begin{array}{rcl}
\widehat{F}_{\mu\nu} & = & \widehat{F}_{\mu\alpha} \;=\; \widehat{F}_{\mu\dot{\alpha}} \;=\; 0 \;, \qquad \widehat{D}_{\mu}\widehat{\Phi} \;=\; 0 \;, \\
\left[\widehat{S}'_{\alpha}, \widehat{S}'_{\beta}\right]_{\star} & = & -2i\epsilon_{\alpha\beta}(1 - \mathcal{B} \star \widehat{\Phi}' \star \kappa) \;, \\
\left[\widehat{S}'_{\dot{\alpha}}, \widehat{S}'_{\dot{\beta}}\right]_{\star} & = & -2i\epsilon_{\dot{\alpha}\dot{\beta}}(1 - \bar{\mathcal{B}} \star \widehat{\Phi}' \star \bar{\kappa}) \\
\left[\widehat{S}'_{\alpha}, \widehat{S}'_{\dot{\beta}}\right]_{\star} & = & 0 \;, \\
\widehat{S}'_{\alpha} \star \widehat{\Phi}' & + & \widehat{\Phi}' \star \pi(\widehat{S}'_{\alpha}) \;=\; 0 \;, \\
\widehat{S}'_{\dot{\alpha}} \star \widehat{\Phi}' & + & \widehat{\Phi}' \star \bar{\pi}(\widehat{S}'_{\dot{\alpha}}) \;=\; 0
\end{array}
$$

- Project on $\mathcal{Z}$! (base ↔ fiber evolution)
  Locally give x-dep. via gauge functions (spacetime ~ pure gauge!)

$$\hat{A}_{\mu} \;=\; \hat{L}^{-1} \star \partial_{\mu}\hat{L} \;, \quad \hat{S}_{\alpha} \;=\; \hat{L}^{-1} \star (\hat{S}'_{\alpha}) \star \hat{L} \;, \quad \hat{\Phi} \;=\; \hat{L}^{-1} \star \hat{\Phi}' \star \pi(\hat{L})$$

$$\hat{L} = \hat{L}(x|Z,Y) \;, \; \hat{L}(0|Z,Y) = 1 \qquad \hat{S}'_{\alpha} = \hat{S}_{\alpha}(0|Z,Y) \;, \quad \hat{\Phi}' = \hat{\Phi}(0|Z,Y)$$

- Z-eq.^{ns} can be solved exactly: 1) imposing symmetries on primed fields
  2) via projectors

- "Dress" with x-dependence by performing star-products with gauge function.

Gauge fields sector

- We want to interpret the coefficients of the master fields as space-time tensors → it should be possible to extract Lorentz tensors (and a Lorentz connection) out of the gauge fields generating function.

- But, in general, expansion coefficients in $\hat{A}_\mu$ are not Lorentz tensors!

- The proper Lorentz generator, at the full level, is

$$\widehat{M}_{\alpha\beta} = y_\alpha y_\beta - z_\alpha z_\beta + \frac{1}{2}\{\widehat{S}_\alpha, \widehat{S}_\beta\}_\star =: \widehat{M}^{(0)}_{\alpha\beta} + \widehat{M}^{(S)}_{\alpha\beta}$$

under which

$$\begin{aligned}
\delta_L \hat{\Phi} &\equiv -[\widehat{\epsilon}_L, \widehat{\Phi}]_\pi = -[\widehat{\epsilon}_0, \widehat{\Phi}]_\star \ , \\
\delta_L \widehat{A}_\alpha &\equiv \widehat{D}_\alpha \widehat{\epsilon}_L = -[\widehat{\epsilon}_0, \widehat{A}_\alpha]_* + \Lambda_\alpha{}^\beta \widehat{A}_\beta \ , \\
\delta_L \widehat{A}_\mu &\equiv \widehat{D}_\mu \widehat{\epsilon}_L = -[\widehat{\epsilon}_0, \widehat{A}_\mu]_\star + \left(\frac{1}{4i}\partial_\mu \Lambda^{\alpha\beta} \widehat{M}_{\alpha\beta} - \text{h.c.}\right) .
\end{aligned}$$

- Complicated, field-dependent transformation, but the field-redefinition

$$\widehat{A}_\mu - \frac{1}{4i}\omega_\mu^{\alpha\beta}\widehat{M}_{\alpha\beta} - \text{h.c.} = \widehat{W}_\mu$$

only contain Lorentz tensors!

Moduli space

- What are the quantities that build the space of solutions?

1. Local dof in $\Phi'(Y) := \hat{\Phi}'(Y, Z)|_{Z=0}$. Initial condition of Z-evolution.
   All on-shell nontrivial derivatives of physical fields at a spacetime point. A given gauge function spreads this datum over space-time (more precisely, over a chart)

The functional form of C'(Y) matters, since, for any chosen gauge function, it contributes to the space-time behaviour of the fields, different asymptotics, etc. .
Moreover (and not disconnectedly) specific functions C'(Y) may have a peculiar behaviour under ★-product (span some subalgebra, diverge, etc.) → different SECTORS of HSGRA.

2. Monodromies and projectors in $\hat{A}'_\alpha{}^{(0)} = \hat{A}'_\alpha|_{C'=0}$.
   Z-space connection can be flat but nontrivial. New vacua?
   Global dof in Z?

3. Choice of gauge function $\hat{L}$ (boundary values in (x,Y,Z) may affect observables)

4. Winding numbers in transition functions…

HS Invariants

- Define classical observables, gauge invariant off-shell. Weyl-curvature invariants:

$$\mathcal{C}^{\pm}_{2p} = \mathcal{N}_{\pm}\widehat{Tr}_{\pm}[\mathcal{C}_{2p}] \, , \qquad \mathcal{C}_{2p} = [\widehat{\Phi}\star\pi(\widehat{\Phi})]^{\star p}$$

$$\widehat{Tr}_{+}[f(Y,Z)] = \int \frac{d^4Y\, d^4Z}{(2\pi)^4} f(Y,Z) \, , \qquad \widehat{Tr}_{-}[f(Y,Z)] = \widehat{Tr}_{+}[f(Y,Z)\star\kappa\bar{\kappa}]$$

- Ciclicity: $\widehat{Tr}_{\pm}[f(Y,Z)\star g(Y,Z)] = \widehat{Tr}_{\pm}[g(\pm Y,\pm Z)\star f(Y,Z)]$

- Conserved on the field equations:

$$\widehat{D}_{\mu}\widehat{\Phi} = 0 \;\Rightarrow\; \partial_{\mu}(\widehat{\Phi}\star\kappa)^{\star q} = -[\widehat{A}_{\mu},(\widehat{\Phi}\star\kappa)^{\star q}]_{\star}$$

Ciclicity + A_{μ} even function of oscilllators $\Rightarrow$

$$d\,\widehat{Tr}_{\pm}[\mathcal{C}^{\pm}_{2p}] = 0$$

$$\mathcal{C}^{[0]}_{k} = \widehat{\mathrm{Tr}}_{+}\left[(\widehat{\Phi}\star\pi(\widehat{\Phi}))^{\star k}\star\kappa\bar{\kappa}\right]$$

$$\mathcal{I}(\sigma,k,\bar{k};\lambda,\bar{\lambda}) = \widehat{\mathrm{Tr}}_{+}\left[(\widehat{\kappa}\widehat{\bar{\kappa}})^{\star\sigma}\star\exp_{\star}(\lambda^{\alpha}\widehat{S}_{\alpha}+\bar{\lambda}^{\dot{\alpha}}\widehat{\bar{S}}_{\dot{\alpha}})\star(\widehat{\Phi}\star\widehat{\kappa})^{\star k}\star(\widehat{\Phi}\star\widehat{\bar{\kappa}})^{\star\bar{k}}\right]$$

Summary of known exact solutions

- *AdS_4 vacuum solution.*

$$\widehat{\Phi} = 0\,,\quad \widehat{S}_\alpha = \widehat{S}^{(0)}_\alpha = z_\alpha\,,\quad \widehat{S}_{\dot\alpha} = \widehat{S}^{(0)}_{\dot\alpha} = \bar z_{\dot\alpha}\,,\quad \widehat{A}_\mu = \Omega^{(0)}_\mu = L^{-1}\star\partial_\mu L$$

$$L(x;y,\bar y) = e_\star^{i\lambda\tilde x^\mu(x)\delta^a_\mu P_a} \;:\; \mathcal{R}^{3,1} \longrightarrow \frac{SO(3,2)}{SO(3,1)} \qquad \longrightarrow \qquad ds^2_{(0)} = \frac{4dx^2}{(1-\lambda^2x^2)^2}$$

- *SO(3,1)-invariant solution*: simplest deformation of AdS vacuum

$$\widehat{\Phi}' = \nu\,,\qquad \widehat{S}'_\alpha = z_\alpha S(y^\alpha z_\alpha)\,,\qquad \widehat{S}_{\dot\alpha} = \bar z_{\dot\alpha}\bar S(\bar y^{\dot\alpha}\bar z_{\dot\alpha})$$

→ gives rise to a scalar profile over a conformally rescaled AdS-metric,

$$\phi(x) = \nu(1-x^2)\,,\qquad ds^2 = \frac{4\Omega^2 d\tilde x^2}{(1-\tilde x^2)^2}$$ *(Sezgin-Sundell '05)*

- *Projectors in Z-space connection*: more complicated vacua? *(C.I.-Sezgin-Sundell '07)*

$$\widehat{\Phi}' = 0\,,\quad \widehat{S}'_\alpha = z_\alpha\left(1-2\sum_{n=0}^{\infty}\theta_n P_n(Y,Z)\right)\,,\qquad P_n\star P_m = \delta_{nm}P_n\,,\qquad \theta_n = \{0,1\}$$

- ➢ Can also dress up non-vacuum solutions (like windings in String Theory…)
- ➢ Different choices for the projectors lead to different global symmetries

Summary of known exact solutions

- What if we turn on projectors also in the Weyl zero-form?

The special Ansatz: *(Didenko-Vasiliev '09 , C.I.-Sundell '11)*

$$\Phi'(Y) = \sum_{\mathbf{n}} \nu_{\mathbf{n}} P_{\mathbf{n}}(Y) \star \kappa_y , \qquad \widehat{S}'_\alpha = z_\alpha - 2i \sum_{\mathbf{n}}^{\infty} P_{\mathbf{n}}(Y) \star A^{\mathbf{n}}_\alpha(z) ,$$

generates a subalgebra of the ★-product algebra → various families of exact solutions with fields of all spins turned on, and different isometries (induced by the symmetries of the projectors), among which:

- Spherically-symm. → candidate HS black-hole solutions!, generalize AdS Schwarzschild black hole.
- Cylindrically-symm. → reminiscent of Melvin universe in GR.

- In Euclidean signature, solutions that turn on all spins, are half-flat and generalize gravitational type-D instantons. *(C.I.-Sezgin-Sundell '07)*

N.B.:
- continuous parameters ν_n → Φ-moduli;
- discrete parameters θ_k → S-moduli, a "landscape" of vacua.

- In progress: bhs + massless particles , topological bhs, black branes, …

AdS$_4$ Vacuum Solution

- AdS$_4$ vacuum sol.:

$$\boxed{\widehat{\Phi} = 0\,, \quad \widehat{S}_\alpha = \widehat{S}^{(0)}_\alpha = z_\alpha\,, \quad \widehat{S}_{\dot\alpha} = \widehat{S}^{(0)}_{\dot\alpha} = \bar{z}_{\dot\alpha}\,, \quad \widehat{A}_\mu = \Omega^{(0)}_\mu = L^{-1}\star\partial_\mu L}$$

The gauge function $(h = \sqrt{1-\lambda^2x^2})$

$$L(x;y,\bar{y}) = e_\star^{i\lambda\tilde{x}^\mu(x)\delta^a_\mu P_a} = \frac{2h}{1+h}\exp\left[\frac{i\lambda x^{\alpha\dot\alpha}y_\alpha\bar{y}_{\dot\alpha}}{1+h}\right]$$

gives AdS$_4$ connection

$$\Omega^{(0)}_\mu = -i\left(\frac{1}{2}\omega^{ab}_{(0)}M_{ab} + e^a_{(0)}P_a\right) = \frac{1}{4i}\left(\omega^{\alpha\beta}_{(0)}y_\alpha y_\beta + \bar{\omega}^{\dot\alpha\dot\beta}_{(0)}\bar{y}_{\dot\alpha}\bar{y}_{\dot\beta} + 2e^{\alpha\dot\beta}_{(0)}y_\alpha\bar{y}_{\dot\beta}\right)$$

leading to AdS$_4$ metric in stereographic coords.: $$ds^2_{(0)} = \frac{4dx^2}{(1-\lambda^2x^2)^2}$$

- Global symmetries:

$$\delta S^{(0)}_\alpha = [z_\alpha, \widehat{\epsilon}]_\star = 0 \quad\Rightarrow\quad \widehat{\epsilon} = \epsilon^{(0)}(x|Y)$$
$$\delta\Omega^{(0)}_\mu = D^{(0)}_\mu\epsilon^{(0)}(x|Y) = 0$$

Y^2-sector: $\epsilon^{(0)} = -i\left(\frac{1}{2}\kappa^{ab}M_{ab} + v^aP_a\right)$ $\longrightarrow$

$$\delta e^a_{(0)} = 0 \quad\Rightarrow\quad \nabla^{(0)}_a v_b = \kappa_{ab}$$
$$\delta\omega^{ab}_{(0)} = 0 \quad\Rightarrow\quad \nabla^{(0)}_a\kappa_{bc} = g^{(0)}_{ac}v_b - g^{(0)}_{ab}v_c$$

Local properties of 4D black holes

- Bh Weyl tensor is of Petrov-type D, ((anti-)selfdual part) has 2 principal spinors :

$$\Phi_{\alpha\beta\gamma\delta} = \nu(x)\, u^+_{(\alpha} u^-_\beta u^+_\gamma u^-_{\delta)} \ , \qquad u^{+\alpha} u^-_\alpha = 1$$

- Local characterization of 4D bhs: sol.ns of Einstein's eqs. in vacuum (flat or AdS) such that their Weyl tensor's principal spinors are collinear with those of the Killing 2-form of an asymptotically *timelike* KVF, $\kappa_{\mu\nu} = \nabla_\mu v_\nu$ *(Mars, '99; Didenko-Matveev-Vasiliev, '08-'09)* ,

$$\Phi_{\alpha\beta\gamma\delta} \sim \frac{M}{(\varkappa^2)^{5/2}} \varkappa_{(\alpha\beta} \varkappa_{\gamma\delta)} \ , \qquad \varkappa^2 := \frac{1}{2} \varkappa^{\alpha\beta} \varkappa_{\alpha\beta}$$

- A generic bh is completely determined by a chosen background global symmetry parameter $Y^{\underline{\alpha}} K_{\underline{\alpha\beta}} Y^{\underline{\beta}}$ *(Didenko-Matveev-Vasiliev, '09)*

$$K_{\underline{\alpha\beta}} = \begin{pmatrix} \varkappa_{\alpha\beta} & v_{\alpha\dot\beta} \\ \bar v_{\dot\alpha\beta} & \bar\varkappa_{\dot\alpha\dot\beta} \end{pmatrix} \ , \qquad D_0 K_{\underline{\alpha\beta}} = 0$$

Properties of bh encoded in algebraic conditions: $K^2 = -1 \rightarrow$ static:

$$K_{\underline{\alpha}}{}^{\underline{\beta}} K_{\underline{\beta}}{}^{\underline{\gamma}} = -\delta_{\underline{\alpha}}{}^{\underline{\gamma}} \quad \Leftrightarrow \quad \begin{cases} \varkappa^2 + v^2 = 1 \\ \varkappa^2 = \bar\varkappa^2 \\ \varkappa_\alpha{}^\beta v_\beta{}^{\dot\gamma} + v_\alpha{}^{\dot\beta} \bar\varkappa_{\dot\beta}{}^{\dot\gamma} = 0 \end{cases} \longrightarrow \quad v_{[\mu} \nabla_\nu v_{\rho]} = 0$$

HS black-hole-like Ansatz

- Weyl zero-form $\widehat{\Phi} = \widehat{L}^{-1} \star \widehat{\Phi}' \star \pi(\widehat{L})$ with $\Phi'(Y) = \sum_{\mathbf{n}} \nu_{\mathbf{n}} P_{\mathbf{n}}(Y) \star \kappa_y$

➢ Linearizes its field eqs.: $\partial_\mu \widehat{\Phi} + [\widehat{A}_\mu, \widehat{\Phi}]_\pi = 0 \quad \rightarrow \quad \partial_\mu \Phi + [\Omega^{(0)}_\mu, \Phi]_\pi = 0$

(this projector Ansatz projects eqs. onto a subsector in which all of them linearize)

➢ Relation with global sym: to any HS global sym parameter $\varepsilon^{(0)}(x|Y)$ ($D^{(0)}\epsilon^{(0)} = 0$) is associated a solution $\epsilon^{(0)} \star \kappa_y$ of the linearized Weyl 0-form eqn.

$$\partial_\mu(\epsilon^{(0)} \star \kappa_y) + [\Omega^{(0)}_\mu, (\epsilon^{(0)} \star \kappa_y)]_\pi = (D^{(0)}\epsilon^{(0)}) \star \kappa_y = 0$$

$$\Phi(x|Y) = \epsilon^{(0)}(x|Y) \star \kappa_y = L^{-1} \star \epsilon'_{(0)}(Y) \star L \star \kappa_y \quad \Rightarrow \quad \Phi'(Y) = \epsilon'_{(0)}(Y) \star \kappa_y$$

- Build projectors via AdS KVF $K_{\underline{\alpha\beta}}(x)$ → on rigid $K'_{\underline{\alpha\beta}} \in \mathfrak{sp}(4,\mathbb{C})$.

  Generalize to a HS global sym parameter *(Didenko-Vasiliev '09)*

$$P_{\mathbf{n}}(Y) = P_{\mathbf{n}}(Y^{\underline{\alpha}} K'_{\underline{\alpha\beta}} Y^{\underline{\beta}})$$

- Assume two commuting $K^{(+)}_{\underline{\alpha\beta}}(x)$ and $K^{(-)}_{\underline{\alpha\beta}}(x)$, $[K^{(+)}, K^{(-)}]_{\underline{\alpha\beta}} = 0$.

  Rigid elements $K'^{(\pm)} := Y^{\underline{\alpha}} K'^{(\pm)}_{\underline{\alpha\beta}} Y^{\underline{\beta}}$ generate $\mathfrak{so}(2)_{(+)} \oplus \mathfrak{so}(2)_{(-)}$.

HS black-hole-like Ansatz

- Projectors depending on generators of $\mathfrak{sp}(4,\mathbb{C})$ can be built as

$$P_{n_1n_2}(K'_{(+)},K'_{(-)}) = \mathcal{N}_{n_1n_2}\, e^{-4K'_{(+)}}\, L_{n_1-\frac{1}{2}}\left(K'_{(+)}-K'_{(-)}\right) L_{n_2-\frac{1}{2}}\left(K'_{(+)}+K'_{(-)}\right)$$

$$P_{n_1,n_2}\star P_{n'_1,n'_2} = \delta_{n_1n'_1}\delta_{n_2n'_2}P_{n_1,n_2}\ , \qquad (n_1,n_2)\in(\mathbb{Z}+\frac{1}{2})\times(\mathbb{Z}+\frac{1}{2})$$

axisymmetric excitations of vacuum with enhanced sym $\mathfrak{c}_{\mathfrak{sp}(4,\mathbb{R})}(K'^{(q)})$

$$P_{1/2,1/2}(K'_{(q)}) := 4e^{-\frac{1}{2}Y^{\underline{\alpha}}K'^{(q)}_{\underline{\alpha\beta}}Y^{\underline{\beta}}}\ , \qquad P_{1/2,1/2}\star P_{1/2,1/2} = P_{1/2,1/2}\ , \qquad \left(K'^{(q)}_{\underline{\alpha}}{}^{\underline{\beta}}K'^{(q)}_{\underline{\beta}}{}^{\underline{\gamma}} = -\delta_{\underline{\alpha}}{}^{\underline{\gamma}}\right)$$

$$K'_{\underline{\alpha}}{}^{\underline{\beta}}K'_{\underline{\beta}}{}^{\underline{\gamma}} = -\delta_{\underline{\alpha}}{}^{\underline{\gamma}} \quad\Rightarrow\quad K'_{\underline{\alpha\beta}} \sim (\Gamma_{AB})_{\underline{\alpha\beta}}\ , \quad M_{AB} = -\frac{1}{8}Y^{\underline{\alpha}}(\Gamma_{AB})_{\underline{\alpha\beta}}Y^{\underline{\beta}}$$

- 3 inequivalent embeddings of $\mathfrak{so}(2)_{(+)}\oplus\mathfrak{so}(2)_{(-)}$ in $\mathfrak{sp}(4,\mathbb{C})$:
  (E , J) ; (J , iB) ; (iB , iP) . [E:= $M_{0'0}$=P_0 ; J: = M_{12} ; B:=M_{03} ; P:=P_1]

- Each gives rise to two families ($|K'_{(+)}| > |K'_{(-)}|$ or $|K'_{(+)}| < |K'_{(-)}|$) based on choice of "principal" $K'_{(q)}$, determining symmetries of vacuum (and behaviour of Weyl tensors) → six infinite families of solutions.

HS black-hole-like Ansatz

- Specific combinations of P_{n1n2} ($K'_{(+)}$, $K'_{(-)}$) give rank-|n| projectors depending on $K'_{(+)}$ only → enhanced sym under $\mathfrak{c}_{\mathfrak{sp}(4,\mathbb{R})}(K^{(q)})$

$$\mathcal{P}_n(K'_{(q)}) = \sum_{\substack{n_2 + q n_1 = n \\ \epsilon_1 \epsilon_2 = q}} P_{n_1,n_2} = 4(-)^{n-\frac{1+\varepsilon}{2}} e^{-4K'_{(q)}} L^{(1)}_{n-1}(8K'_{(q)})$$

$$= 2(-)^{n-\frac{1+\varepsilon}{2}} \oint_{C(\varepsilon)} \frac{d\eta}{2\pi i} \left(\frac{\eta+1}{\eta-1}\right)^n e^{-4\eta K'_{(q)}} \, , \qquad n \in \mathbb{Z}$$

- ⇒ Φ'(Y) = any f(Y) diagonalizable on such bases of projectors $*\,\kappa_y$:

$$\Phi'(Y) = \sum_{\mathbf{n}} \nu_{\mathbf{n}} P_{\mathbf{n}}(Y) \star \kappa_y$$

- Weyl 0-form: $$\Phi(x|Y) = \sum_n \nu_n \mathcal{N}_n \oint_{C(\varepsilon)} \frac{d\eta}{2\pi i} \left(\frac{\eta+1}{\eta-1}\right)^n \underbrace{L^{-1}(x) \star e^{-4\eta K'_{(q)}} \star L(x) \star \kappa_y}$$

Type-D
Weyl 0-form
generating f :

$$\bar{y} = 0 : \quad \frac{1}{\eta\sqrt{\varkappa^2(x)}} \exp\left(\frac{1}{2\eta} y^\alpha \varkappa^{-1}_{\alpha\beta}(x) y^\beta\right) \, , \quad \varkappa^{-1}_{\alpha\beta} = -\frac{\varkappa_{\alpha\beta}}{\varkappa^2} \, , \quad \varkappa^2 = \frac{1}{2}\varkappa^{\alpha\beta}\varkappa_{\alpha\beta}$$

Tower of spin-s bh-like Weyl tensors:
(s = 0, 1, 2, …)

$$\Phi^{(n)}_{\alpha(2s)} \sim \frac{\nu_n}{(\varkappa^2)^{s+1/2}} \varkappa_{(\alpha_1\alpha_2} \cdots \varkappa_{\alpha_{2s-1}\alpha_{2s})}$$

Spherically symmetric type-D solutions

- Based on enhanced E-dependent projectors, s.t. residual symmetry → centralizer of E $\Rightarrow$ $\mathfrak{c} = \mathfrak{so}(2)_E \oplus \mathfrak{so}(3)_{M_{ij}}$.

$$\delta\Phi(x|Y) = -[\epsilon(x|Y), \Phi(x|Y)]_{\star,\pi} = 0 \Leftrightarrow [\epsilon'(Y), e^{-4sK_{(q)}}]_\star = 0 \Rightarrow K_{(q)} = E$$

- Spherical symm. solutions based on scalar singleton ground-state projector!,

$$\begin{aligned} E \star e^{-4E} &= e^{-4E} \star E = \frac{1}{2} e^{-4E} , \\ L_r^- \star e^{-4E} &= 0 = e^{-4E} \star L_r^+ , \\ M_{rs} \star e^{-4E} &= 0 \end{aligned} \quad \Longrightarrow \quad 4e^{-4E} \simeq |1/2;0\rangle\langle 1/2;0| \in \mathcal{D}_0 \otimes \mathcal{D}_0^*$$

(C.I., P. Sundell '08)

- General spherically symm. type-D sol.ns include *all* projectors on scalar (super)singleton modes (all $\mathfrak{so}(3)$-invariant excitations of 4 exp(-4E)) and their negative-energy counterparts.

$$\mathcal{P}_n(E) \sim \begin{cases} a^{\dagger\, i_1} \dots a^{\dagger\, i_n} \star |1/2;0\rangle\langle 1/2;0| \star a_{i_1} \dots a_{i_n} , & n > 0 \\ a_{i_1} \dots a_{i_{|n|}} \star |-1/2;0\rangle\langle -1/2;0| \star a^{\dagger\, i_1} \dots a^{\dagger\, i_{|n|}} , & n < 0 \end{cases}$$

$$\begin{aligned} a_1 &= \frac{1}{2}(y_1 + i\bar{y}_{\dot 2}) , \quad a^{\dagger\, 1} = \frac{1}{2}(\bar{y}_{\dot 1} - iy_2) , \\ a_2 &= \frac{1}{2}(-y_2 + i\bar{y}^{\dot 1}) , \quad a^{\dagger\, 2} = \frac{1}{2}(-\bar{y}_{\dot 2} - iy_1) \end{aligned}$$

$$[a_i, a^{\dagger\, j}]_\star = \delta_i^j$$

Spherically symmetric type-D solutions

- Using the gauge function:

$$\mathcal{P}_1(Y) = 4e^{-\frac{1}{2}Y^{\underline{\alpha}}K'_{\underline{\alpha\beta}}Y^{\underline{\beta}}} = 4e^{-y^\alpha \sigma^0_{\alpha\dot\alpha}\bar y^{\dot\alpha}} = 4e^{-4E} \quad \to \quad L^{-1}\star\mathcal{P}_1(Y)\star L = 4e^{-\frac{1}{2}Y^{\underline{\alpha}}K_{\underline{\alpha\beta}}(x)Y^{\underline{\beta}}}$$

In AdS_4 spherical coords. (t,r,θ,φ) $[ds^2 = (1+r^2)\,dt^2 + (1+r^2)^{-1}\,dr^2 + r^2\,d\Omega^2]$

$$K'_{\underline{\alpha\beta}} = (\Gamma_0)_{\underline{\alpha\beta}} = \begin{pmatrix} 0 & u^+_\alpha \bar u^+_{\dot\beta} + u^-_\alpha \bar u^-_{\dot\beta} \\ \bar u^+_{\dot\alpha} u^+_\beta + \bar u^-_{\dot\alpha} u^-_\beta & 0 \end{pmatrix} \longrightarrow K_{\underline{\alpha\beta}} = \begin{pmatrix} 2r\tilde u^+_{(\alpha}\tilde u^-_{\beta)} & \sqrt{1+r^2}(\tilde u^+_\alpha \tilde{\bar u}^+_{\dot\beta} + \tilde u^-_\alpha \tilde{\bar u}^-_{\dot\beta}) \\ \sqrt{1+r^2}(\tilde{\bar u}^+_{\dot\alpha}\tilde u^+_\beta + \tilde{\bar u}^-_{\dot\alpha}\tilde u^-_\beta) & 2r\tilde{\bar u}^+_{(\dot\alpha}\tilde{\bar u}^-_{\dot\beta)} \end{pmatrix}$$

$$u^{+\alpha}u^-_\alpha = 1 = \tilde u^{+\alpha}(x)\tilde u^-_\alpha(x)\,, \qquad \varkappa^2(x) = -r^2$$

$$\Phi(x|Y) = \sum_n \nu_n \mathcal{N}_n \oint_{C(\varepsilon)} \frac{d\eta}{2\pi i}\left(\frac{\eta+1}{\eta-1}\right)^n \underbrace{L^{-1}(x)\star e^{-4\eta E}\star L(x)\star \kappa_y}$$

$$\bar y = 0: \quad \frac{1}{\eta r}\exp\left(\frac{1}{2\eta r}\, y^\alpha \tilde u^+_\alpha \tilde u^-_\beta y^\beta\right) \qquad \Longrightarrow \qquad \Phi^{(n)}_{\alpha(2s)} \sim \frac{i^{n-1}\mu_n}{r^{s+1}}\,(\tilde u^+\tilde u^-)^s_{\alpha(2s)}$$

- Deformation parameter is real for scalar singleton, imaginary for spinor singl.
  → generalized electric/magnetic charge (or mass/NUT).
  E/m duality connects Type A/B models?
- Spacetime coords. enter as parameter of a limit representation of a delta function .

$$\widehat\Phi_1 \xrightarrow{r\to 0} \widehat\Phi'_1 = \nu_1 \kappa_{y-i\sigma_0\bar y} = 2\pi\nu_1\left[\delta^2(y-i\sigma_0\bar y)\right]$$

Cylindrically-symmetric type-D solutions

- Condition $K'_{\underline{\alpha}}{}^{\underline{\beta}} K'_{\underline{\beta}}{}^{\underline{\gamma}} = -\delta_{\underline{\alpha}}{}^{\underline{\gamma}}$ solved by any $Y^{\underline{\alpha}} K'_{\underline{\alpha\beta}} Y^{\underline{\beta}} \sim E, J, iB, iP$

➔ Solutions with $\mathfrak{so}(2,1)_{\mathfrak{h}} \oplus \mathfrak{so}(2)_{YK'Y}$ symmetry (centralizer of YK'Y).

- In particular, for $K' = \Gamma_{12}$, $\mathcal{P}_1(Y) := 4e^{-\frac{1}{2}Y^{\underline{\alpha}}K'_{\underline{\alpha\beta}}Y^{\underline{\beta}}} = 4e^{-4J}$

  Again a ground state of a 2D Fock-space (a non-compact ultra-short irrep, singleton-like but with roles of E and J exchanged, |E| < |J| instead of |E| > |J|).

  [Systematic procedure to extract creation/annihilation operators]

- Same steps yield $\Phi(x|Y) = \sum_n \nu_n \mathcal{N}_n \oint_{C(\varepsilon)} \frac{d\eta}{2\pi i} \left(\frac{\eta+1}{\eta-1}\right)^n \underbrace{L^{-1}(x) \star e^{-4\eta J} \star L(x) \star \kappa_y}$

$$\bar{y} = 0 : \quad \frac{1}{\eta\sqrt{\varkappa^2}} \exp\left(\frac{1}{2\eta}\, y^\alpha \varkappa^{-1}_{\alpha\beta} y^\beta\right) , \quad \varkappa^{-1}_{\alpha\beta} = -\frac{\varkappa_{\alpha\beta}}{\varkappa^2} , \quad \varkappa^2 = 1 + r^2 \sin^2\theta$$

$$\Phi^{(n)}_{\alpha(2s)} \sim \frac{i^{n+s+1}\mu_n}{(1+r^2\sin^2\theta)^{(s+1)/2}} \left(\tilde{u}^+\tilde{u}^-\right)^s_{\alpha(2s)}$$

Deformation parameters and asymptotic charges

- Building solutions on more than one projector opens up interesting possibilities.

➢ Every singleton-state projector contains a tower of fields of all spins → can change basis and diagonalize on spin (and not occupation number)

$$\mathcal{C}(x|y) \;=\; \sum_n \nu_n \mathcal{N}_n \oint_{C(\varepsilon)} \frac{d\eta}{2\pi i} \left(\frac{\eta+1}{\eta-1}\right)^n \frac{1}{\eta\sqrt{\varkappa^2}} \exp\left(\frac{1}{2\eta}\, y^\alpha \varkappa^{-1}_{\alpha\beta} y^\beta\right)$$

$$\mathcal{C}(x|y) \;=\; \sum_{s=0}^{\infty} \frac{1}{(2s)!} C_{\alpha(2s)}(x)\, y^{\alpha(2s)} \;, \qquad C_{\alpha(2s)} \;\sim\; \frac{\mathcal{M}_s}{r^{s+1}} \left(\tilde{u}^+ \tilde{u}^-\right)^s_{\alpha(2s)}$$

"HS asymptotic charge", f(ν_n): $\mathcal{M}_s \sim \sum_n \nu_n \tilde{\mathcal{N}}_n \oint_{C(\epsilon)} \frac{d\eta}{2\pi i \eta^{s+1}} \left(\frac{\eta+1}{\eta-1}\right)^n$

(Can we choose ν_n such that $\mathcal{M}_s \sim \delta_{s,k}$, switching off all spins except one?)

➢ Possible to turn on an angular dependence in the Weyl tensor singularity via specific choices of deformation parameters (e.g. $\nu_n = q^n$, exchanging sum and integral) → Kerr-like HS black-hole?

Reading off asymptotic charges

- Having found the gauge-fields generating functions, one may try to read off asymptotic charges from the sources of field strengths for r → ∞, i.e. analyzing the asymptotics of the gauge field eq.

$$\nabla\widehat{W} + \widehat{W}\star\widehat{W} + \tfrac{1}{4i}\left(r^{\alpha\beta}\widehat{M}_{\alpha\beta} + \bar{r}^{\dot\alpha\dot\beta}\widehat{\overline{M}}_{\dot\alpha\dot\beta}\right) \;=\; 0$$

$$r^{\alpha\beta} \;:=\; d\omega^{\alpha\beta} + \omega^{\alpha\gamma}\omega^{\beta}{}_{\gamma}\;, \qquad \nabla\widehat{W} \;=\; d\widehat{W} + \tfrac{1}{4i}\left[\omega^{\alpha\beta}\widehat{M}^{(0)}_{\alpha\beta} + \bar{\omega}^{\dot\alpha\dot\beta}\widehat{\overline{M}}^{(0)}_{\dot\alpha\dot\beta}\;,\;\widehat{W}\right]_{\star}$$

after moving to the standard gauge of perturbation theory and reducing to spacetime submanifold {Z=0}.

- Possible mixing between different orders in $\mathcal{M}_s$ due to s-dependent r-behaviour of spin-s component fields

$$(\nabla_{(0)}W + \{e_{(0)},W\}_{\star})_{\alpha(2s)} \;\sim\; e_{(0)}\wedge e_{(0)}\,\partial^{(y)}_{\alpha(2s)}\Phi + \text{h.o.t.} \;=\; \frac{\widehat{\mathcal{M}}_s}{r^{s+1}}(u^+u^-)^s_{\alpha(2s)}$$

leads to possible asymptotic charge redefinition

$$\widehat{\mathcal{M}}_s \;=\; \mathcal{M}_s + O(\mathcal{M}_s^2)$$

Twistor gauge and asymptotic charges

- To compare solutions in x-space, need to bring them in "universal" twistor gauge via some extended HS gauge transformation $G_{(v)}^{(K)}(x|Y,Z)$.

$$\widehat{v}^{\alpha}(x|Y,Z)\widehat{A}_{\alpha}(x|Y,Z) = \widehat{f}(x|Y,Z)\,, \qquad \frac{\partial}{\partial\nu_n}\widehat{v}^{\alpha} = \frac{\partial}{\partial K_{\underline{\alpha\beta}}}\widehat{v}^{\alpha} = 0 = \frac{\partial}{\partial\nu_n}\widehat{f} = \frac{\partial}{\partial K_{\underline{\alpha\beta}}}\widehat{f}$$

with residual gauge symmetries → $\mathfrak{ho}(3,2)$, e.g., standard choice $v^{\alpha} = z^{\alpha}$.

- Our solutions are in some twistor gauge but NOT in universal twistor gauge (v^{α} depends explicitly on K). Can be brought to twistor gauge, e.g., the standard gauge of perturbative analysis, order by order in ν_n.

- The action of G^{v}_{K} on solutions will redefine the HS asymptotic charges, too!

$$\begin{aligned} \widehat{\Phi}_{(v)} &= (\widehat{G}^{(K)}_{(v)})^{-1} \star \widehat{\Phi}_{(K)} \star \pi(\widehat{G}^{(K)}_{(v)}) \\ \longrightarrow \quad \mathcal{M}_s\Big|_{(v)} &= \mathcal{M}_s\Big|_{(K)} + \sum_{s,'s''} \mathcal{M}_{s'}\Big|_{(K)} \mathcal{M}_{s''}\Big|_{(K)} f_s^{s's''} + \ldots \end{aligned}$$

- Finally, $\mathfrak{ho}(3,2)$ asymptotic symmetries (possibly enhanced to current algebra of free fields) will act on $\mathcal{M}_s$. Invariants?

Singularity?

- Radial dependence of individual spin-s Weyl tensor $\sim r^{-s-1}$. However, HS-invariants for finitely many projectors are finite!

$$Tr_+\left[(\widehat{\Phi}\star\pi(\widehat{\Phi}))^N\star\kappa\bar{\kappa}\right] \;=\; -4\sum_{n=\pm1,\pm2,\ldots}|n|(-1)^{(N+1)n}\mu_n^{2N}$$

Note: invariants are also (formally) insensitive to changes of ordering!
Can the singularity be only an artefact of basis choice for function of operators?
(crucial with non-polynomial f(operators))

- Examine master-fields in r = 0:

$$\Phi(r=0)\;=\;L^{-1}\,|_{r=0}\,\star\,P_1(E)\,\star\,L\,|_{r=0}\,\star\,\kappa_y\;=\;P_1(E)\star\kappa_y\;\sim\;\delta^2(y-i\sigma^0\bar{y})$$

$\downarrow$

[L (r = 0) = f(E)]

$\Rightarrow$ Weyl tensors generating function ~ $\delta^2(y)$
$\rightarrow$ a regular function ($\exp(-2N_y)$) in normal ordering!

Internal Z-Space Solution

- Ansatz for internal eqs., separation of Y and Z variables, absorb Y-dep. in $P_n(Y)$:

$$\widehat{S}'_\alpha = z_\alpha - 2i \sum_{n=0}^{\infty} P_n(Y) \star A^n_\alpha(z) \ , \quad \widehat{\bar{S}}'_{\dot\alpha} = \bar{z}_{\dot\alpha} - 2i \sum_{n=0}^{\infty} P_n(Y) \star \bar{A}^n_{\dot\alpha}(\bar{z})$$

Reduced deformed oscillators: $\Sigma^n_\alpha = z_\alpha - 2iA^n_\alpha \ , \quad \bar{\Sigma}^n_{\dot\alpha} = \bar{z}_{\dot\alpha} - 2i\bar{A}^n_{\dot\alpha}$

- ✓ Orthogonality of projectors ⇒ eqs. for different n split;
- ✓ Projectors only Y-dep. ⇒ spectators, out of commutators;
- ✓ ν_n = cost and $\pi(\Sigma) = -\Sigma$ solve $\{S',\Phi'\}_\pi = 0$;
- ✓ Holomorphicity in z of S' solves $[S', \overline{S}'] = 0$

- Left with the deformed oscillator problem :

$$\left[\Sigma^n_\alpha, \Sigma^n_\beta\right]_\star = -2i\epsilon_{\alpha\beta}(1 - \mathcal{B}_n\nu_n\kappa_z) \ ,$$
$$\left[\bar{\Sigma}^n_{\dot\alpha}, \bar{\Sigma}^n_{\dot\beta}\right]_\star = -2i\epsilon_{\dot\alpha\dot\beta}(1 - \bar{\mathcal{B}}_n\bar{\nu}_n\bar{\kappa}_{\bar{z}})$$

Can solve by a general method *(Prokushkin-Vasiliev '98, Sezgin-Sundell '05)* for regular deformation terms. Use a limit representation of $\kappa_z \sim \delta^2(z)$ or first go to normal-ordering where κ_z = gaussian .

Solution for Z-space deformed oscillators

- Introduce basis spinors $u^{\pm}_{\alpha}$ (a priori non-collinear with $\tilde{u}^{\pm}_{\alpha}(x)$) :

$$z^{\pm} := u^{\pm\alpha} z_{\alpha} \ , \quad w_z := z^{+} z^{-} \ , \quad [z^{-}, z^{+}]_{\star} = -2i$$

- Solve $[\Sigma^{+}_{n}, \Sigma^{-}_{n}]_{\star} \ = \ -1 + \mathcal{B}_n \nu_n \kappa_z$ w/ Laplace-like transform:

$$\Sigma^{\pm}_{n} \ = \ 4z^{\pm} \int_{-1}^{1} \frac{dt}{(t+1)^2} \, f^{n\pm}_{\sigma_n}(t) \, e^{i\sigma_n \frac{t-1}{t+1} w_z}$$

and using the limit representation $\lim_{\varepsilon \to 0} \frac{1}{\varepsilon} e^{-i\frac{\sigma}{\varepsilon} z^{+} z^{-}} \ = \ \sigma \, [\kappa_z]^{\mathrm{Weyl}}$.

Leads to manageable algebraic eqns for $f^{n}_{\pm}(t)$. Can either solve symmetrically, $f^{n}_{+} = f^{n}_{-}$, or asymmetrically (gauge freedom on S).
Study sym case: particular, ν-dependent solution

$$f^{n\pm}_{\sigma_n}(t) \ = \ \delta(t-1) \ - \ \frac{\sigma_n \mathcal{B}_n \nu_n}{4} \, {}_1F_1 \left[\frac{1}{2}; 2; \frac{\sigma_n \mathcal{B}_n \nu_n}{2} \log \frac{1}{t^2} \right]$$

- Also: a general way of solving the homogeneous ($\nu_n = 0$) eq. is the projector solution: $X^2 = 1 \ \rightarrow \ X = 1 - 2P \ , \quad P^2 = P$

Solution for Z-space deformed oscillators

- Internal Z-space connection:

$$
\begin{aligned}
A^n_\pm &= A^{n\,(reg)}_\pm + A^{n\,(proj)}_\pm \\
A^{n\,(reg)}_\pm &= \frac{i\sigma_n \mathcal{B}_n \nu_n}{2} z^\pm \int_{-1}^{1} \frac{dt}{(t+1)^2}\, e^{i\sigma_n \frac{t-1}{t+1} w_z} \left[{}_1F_1\left(1/2; 2; \frac{\nu_n}{2} \log \frac{1}{t^2}\right)\right] \\
A^{n\,(proj)}_\pm &= -iz^\pm \sum_{k=0}^{\infty} (-1)^k \theta_k L_k[\nu_n] P_k(z)\ , \quad P_k(z) = \frac{(z^+ z^-)^k}{k!}\, e^{-z^+ z^-}\ , \\
L_k[\nu] &= \int_{-1}^{1} dt\, t^k f^n_\pm(t) \longrightarrow 1 \ \text{ as } \ \nu_n \longrightarrow 0\ , \quad \theta_k = 0, 1
\end{aligned}
$$

Sol.ns depend on two infinite sets of parameters:

- continuous parameters ν_n → Φ-moduli;
- discrete parameters θ_k → S-moduli, a “landscape” of vacua.

- Divergent deformed oscillators ($t = -1$) but $S(x|Y,Z)$ only singular in $r = 0$! Pushed out of integration domain by star-product with $\mathcal{P}_n(x|Y)$. For n=1:

$$
\widehat{S}^\pm = \tilde{z}^\pm + 8\,\mathcal{P}_1(x|Y)\,\tilde{a}^\pm \int_{-1}^{1} \frac{dt}{(t+1+i\sigma_n r(t-1))^2}\, j_1^\pm(t)\, e^{\frac{i\sigma_n(t-1)}{t+1+i\sigma r(t-1)}\,\tilde{a}^+\tilde{a}^-}
$$

$$
\tilde{a}^\pm := \tilde{u}^{\alpha\pm} a_\alpha\ , \qquad a_\alpha = z_\alpha + i(\varkappa_\alpha{}^\beta y_\beta + v_\alpha{}^{\dot\beta}\bar{y}_{\dot\beta})\ , \quad z_\alpha \star \mathcal{P}_1 = a_\alpha \mathcal{P}_1\ , \quad [a_\alpha, a_\beta]_\star = -2i\epsilon_{\alpha\beta}
$$

Conclusions & Outlook

- Found a general class of (almost) type-D solutions, with various symmetries:
  - spherical, HS generalization of Schwarzschild bh
  - cylindrical, HS counterpart of GR Melvin solution (regular everywhere)
  - biaxial (building blocks of the previous two, "almost type-D")

and other ones whose physical interpretation and GR analogues are yet to be studied.

- Singular? Not obvious, not at the level of invariants nor master-fields.
  1) A closed 2-form charge could detect singularities
  2) Divergent curvature invariants with infinitely many excitations

  A HS-invariant characterization of bhs is yet to be found.

- Must gain a better understanding of HS invariants and evaluate more of them.
  [HS "metrics" $G_{\mu_1\dots\mu_s} = \widehat{Tr}_+\left[\widehat{\kappa}\widehat{\bar{\kappa}} \star \widehat{E}_{(\mu_1} \star \cdots \star \widehat{E}_{\mu_s)}\right] \,, \quad \widehat{E}_\mu \;=\; \frac{1}{2}(1-\pi)\widehat{W}_\mu$]

- Multi-body solutions? [Preliminary analysis of consistency of a 2-body problem by evaluating 0-form invariants for $\Phi(x) + \Phi(y)$. Cross terms fall off as $V\left((1+r^2)^{-1/2}; n\right)$. Hierarchy of excitations ?]

- Thermodynamics in invariants? Horizon? Trapped surfaces?...

Conclusions & Outlook

- Study the boundary duals of such solutions. Many interesting questions:
  - ➢ What are the dual configurations in U(N)/O(N) vector models?
  - ➢ Hawking-Page phase transitions? *(Shenker-Yin '11 → No uncharged bhs in Type A minimal model)*
  - ➢ Are spacetime boundary conditions (partly) encoded in (Y,Z)-space behaviour? [Distiction small/large gauge transformation and superselection sectors]

- Role of Z-space in non-perturbative sector of the theory . In particular, "Z-space vacua", topologically non-trivial flat Z-connections.

- Solutions mixing AdS massless particle state + soliton-like state. [Particles alone are inconsistent as solutions of the full eqs., backreaction forces addition of non-perturbative states]