Coulomb drag in graphene

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Phys. Rev. B 85, 195421 (2012),
Phys. Rev. Lett. 110, 026601 (2013),

Moscow, September 5, 2013
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Collaboration

Theory:
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P. M. Ostrovsky (MPI Stuttgart + Landau Institute)
M. Titov, M. I. Katsnelson, T. Tudorovskiy (Nijmegen)

Experiment:
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L. A. Ponomarenko (Manchester)
What is Coulomb drag?

Coulomb drag = response of the passive layer to a current in the active layer mediated by Coulomb interaction
Coulomb drag: why interesting?

• no drag without interaction: probe of inter-electron correlations

• provides information about inelastic processes, phase-coherent phenomena

• drag is related to particle-hole asymmetry

Drag in graphene near the Dirac point?
FIG. 2. Temperature dependence of observed frictional drag between two 2D electron systems separated by 175-Å barrier. Data are plotted as an equivalent resistance and a momentum-transfer rate. Inset: An idealized conduction-band diagram for a DQW structure indicating the ground subband energy $E_0$ and the Fermi energy $E_F$. 
Drag from Drude

\[ \mathbf{j} = e n \mathbf{v} \]

Equations of motion \( t \gg \tau, \tau_D \)

\[
\frac{d \mathbf{v}_1}{dt} = \frac{e}{m} \mathbf{E}_1 + \frac{\mathbf{v}_2 - \mathbf{v}_1}{\tau_D} - \frac{\mathbf{v}_1}{\tau} = 0
\]

\[
\frac{d \mathbf{v}_2}{dt} = \frac{e}{m} \mathbf{E}_2 + \frac{\mathbf{v}_1 - \mathbf{v}_2}{\tau_D} - \frac{\mathbf{v}_2}{\tau} = 0
\]

\[ \frac{1}{\tau} \text{ - impurity scattering rate} \]
\[ \frac{1}{\tau_D} \text{ - interlayer scattering rate} \]

\[
\begin{pmatrix} \mathbf{E}_1 \\ \mathbf{E}_2 \end{pmatrix} = \frac{m}{e^2 n} \begin{pmatrix} \frac{1}{\tau} + \frac{1}{\tau_D} & -\frac{1}{\tau_D} \\ -\frac{1}{\tau_D} & \frac{1}{\tau} + \frac{1}{\tau_D} \end{pmatrix} \begin{pmatrix} \mathbf{j}_1 \\ \mathbf{j}_2 \end{pmatrix} = \begin{pmatrix} \rho_1 & -\rho_D \\ -\rho_D & \rho_2 \end{pmatrix} \begin{pmatrix} \mathbf{j}_1 \\ \mathbf{j}_2 \end{pmatrix}
\]

Drude-like formula

\[ \rho_D = \left( \frac{e^2 n \tau_D}{m} \right)^{-1} \]
Fermi-liquid theory

Additional natural assumptions:
- the spacer is much wider than the screening length $q_{TF} d \gg 1$
- the spacer is much wider than the Fermi wave length $k_F d \gg 1$

Interlayer scattering time from the **Fermi golden rule**

$$
\tau_D^{-1} = \frac{\pi \zeta(3)}{32} \frac{(k_B T)^2}{(q_{TF} d)^2 (k_F d)^2} \quad \rho_D = \left( \frac{e^2 n \tau_D}{m} \right)^{-1}
$$

Transresistivity is essentially expressed via electron concentration $n$, temperature $T$, and spacing $d$

$$
\rho_D \sim \frac{T^2}{n^3 d^4} \quad n \propto k_F^2 \quad \text{in 2D}
$$
Particle-hole asymmetry

Example: strong magnetic field

(i) Curvature $\rightarrow$ normal positive drag
(ii) Landau levels DoS $\rightarrow$ anomalous oscillatory drag

IG, Mirlin, von Oppen (2004)
Drag in graphene

\[ j \]
News in graphene

Dirac spectrum at low energies

\[ E_\nu(k) = \nu v k, \quad \nu = \pm \]

electron-hole symmetry at the Dirac point

linear spectrum – no Galilean invariance

- non-trivial single-layer conductivity

small interlayer distance \( d \)

- electron density can be positive, negative, large and low

- Fermi wave length at low density is much larger than \( d \)

- screening length at low density is much larger than \( d \)
Drag in graphene: experiment

Austin group: single-gate device
Drag in graphene: experiment

single-gate device

Drag in graphene: experiment

Austin group: double-gate device
Drag in graphene: experiment

double-gate device

Drag in graphene: experiment

Manchester group:

- “clean” substrate and spacer – BN

- smaller inter-layer spacing
  \[ d = 1-10 \text{ nm} \]

- Double-gate setup:
Drag in graphene: experiment

Manchester group:
Drag in graphene: experiment

double-gate device

Drag in graphene: experiment

Our theory (zero magnetic field)

\[ \rho_D \sim 1 \]

\[ \rho_D \sim \alpha^2 \mu^2 T \tau^3 + \alpha^3 (T \tau)^{-3/2} \]

\[ \rho_D \sim \alpha^2 \frac{T^2}{\mu^2} \ln \frac{T}{\mu} \]

\[ \rho_D \sim \alpha^4 \frac{\mu^2}{T} \tau + \rho_D^{(3)} \]

\[ \rho_D \sim \alpha^2 \frac{T^2}{\mu^2} \ln \frac{T}{\mu} \]
disordered graphene ($\tau \ll \tau_{ee}$)
Results for $\alpha \ll 1 \quad d \ll \hbar v / T$

$$\sigma_D(\mu) / \alpha^2 e^2 \tau^2$$

Tse, Hu, Das Sarma, PRB'07

Fermi-liquid asymptotic!

$$\sigma_D = \frac{\zeta(3)}{4} \frac{e^2 \tau^2 T^2}{(k_F d)^2 (\chi d)^2}$$

$$\chi = 4 \alpha k_F$$

$$\chi d \gg 1$$

USEFUL to REMEMBER

$$\mu \simeq \hbar v \sqrt{\pi n}$$
comparison with experiment using second order perturbation theory

\[
\rho_D[\Omega] \quad \text{vs} \quad n \left[10^{12} \text{ cm}^{-2}\right]
\]

- \(d = 4 \text{ nm} \quad T = 240 \text{ K} \quad \alpha = 0.3 \quad h/\tau = 51 \text{ K}

- \(d = 6 \text{ nm} \quad T = 240 \text{ K} \quad \alpha = 0.3 \quad h/\tau = 36 \text{ K}

- \(d = 9 \text{ nm} \quad T = 240 \text{ K} \quad \alpha = 0.3 \quad h/\tau = 31 \text{ K}

- \(d = 12 \text{ nm} \quad T = 240 \text{ K} \quad \alpha = 0.3 \quad h/\tau = 25 \text{ K}

\[
\tau^{-1} \gg \alpha^2 T \approx 22 \text{ K}
\]
ultra-clean graphene \((- \uparrow \downarrow -_{ee})\)

Schütt, Ostrovsky, Titov, IG, Narozhny, Mirlin, PRL (2013)

see also J. Lux and L. Fritz, PRB (2012)
\[ \rho_D \sim \alpha^2 \frac{\mu^2}{T^2} + \rho_D^{(3)} \]

\[ \rho_D \sim \alpha^4 \frac{\mu^2}{T} \tau + \rho_D^{(3)} \]

\[ \rho_D \sim \alpha^2 \frac{T^2}{\mu^2} \ln \frac{T}{\mu} \]

\[ \rho_D \sim \alpha^2 \frac{\mu^2}{T^2} \left[ \ln(\mu \tau) + (T \tau)^{-1} \right] \]
Clean vs. disordered graphene

\[ \rho_D(n,n)[\Omega] \]

- $\alpha_s = 0.1$
- $T = 250K$

\[ \tau^{-1} = 0K, \tau^{-1} = 0.05K, \tau^{-1} = 0.5K, \tau^{-1} = 50K \]
Kinetic theory of the drag

Linearized kinetic equation:

\[ n_i(\epsilon, \hat{\mathbf{v}}) = n_F^{(i)}(\epsilon) + T \frac{\partial n_F^{(i)}(\epsilon)}{\partial \epsilon} h_i(\epsilon, \hat{\mathbf{v}}) \]

\[ \frac{\partial h_1}{\partial t} + \frac{e \mathbf{E}_1 \mathbf{v}}{T} = -\frac{h_1}{\tau} + I_{11}\{h_1\} + I_{12}\{h_1, h_2\}, \]

\[ \frac{\partial h_2}{\partial t} = -\frac{h_2}{\tau} + I_{22}\{h_2\} + I_{21}\{h_2, h_1\}, \]

\[ I_{ij} = -\int d2 d3 d4 W^{ij}(h_{i,1} - h_{i,2} + h_{j,3} - h_{j,4}) \]
Inelastic scattering in graphene

Kashuba '08; Müller & Sachdev '08; Fritz, Müller, Schmalian, Sachdev '08

Linear spectrum:

Velocity is not equivalent to momentum: momentum conservation does not prevent current relaxation
- Finite transport rate due to inelastic e-e scattering

Collinear scattering singularity: momentum conservation = energy conservation
- Fast thermalization within a given direction

\[ n(\epsilon, \hat{v}) = \frac{1}{1 + \exp \left[ \frac{\epsilon - \mu(\hat{v})}{T(\hat{v})} \right]} \]
Collinear scattering singularity

collinear scattering

– equivalence of energy and momentum conservation laws:

\[ p_1 + p_3 = p_2 + p_4 \]
\[ \epsilon_i = \pm v |p_i| \]
\[ \epsilon_1 + \epsilon_3 = \epsilon_2 + \epsilon_4 \]

linearized collision integrals

“momentum mode”

\[ h_i \propto p \Rightarrow I_{ij} = 0 \]

“velocity mode”

\[ h_i \propto v \Rightarrow I_{ij} = 0 \]

if \( v_i \parallel v_j \)

\[ I_{ij} = - \int d2 \, d3 \, d4 \, W^{ij}(h_{i,1} - h_{i,2} + h_{j,3} - h_{j,4}) \]

\[ W^{ij} = \delta(p_1 - p_2 + p_3 - p_4) \delta(\epsilon_1 - \epsilon_2 + \epsilon_3 - \epsilon_4) \]

\[ \times \frac{\cosh \frac{\epsilon_1 - \mu_1}{2T}}{2 \cosh \frac{\epsilon_2 - \mu_1}{2T} \cosh \frac{\epsilon_3 - \mu_1}{2T} \cosh \frac{\epsilon_4 - \mu_1}{2T}} K_{i,2,3,4}^{ij} \]
Scattering rates: Golden Rule

\[ \frac{1}{\tau_{ee}^a} = \frac{N}{8T^2B_2} \int d\{\epsilon_i\} d\{\mathbf{v}_i\} \left[ (\mathbf{v}_1 - \mathbf{v}_2 + \mathbf{v}_3 - \mathbf{v}_4)^2 \mathcal{W}^{aa} + 2(\mathbf{v}_1 - \mathbf{v}_2)^2 \mathcal{W}^{ab} \right] \]

\[ \frac{1}{\tau_{ee}^b} = \frac{N}{8T^2B_2} \int d\{\epsilon_i\} d\{\mathbf{v}_i\} \left[ (\mathbf{v}_1 - \mathbf{v}_2 + \mathbf{v}_3 - \mathbf{v}_4)^2 \mathcal{W}^{bb} + 2(\mathbf{v}_1 - \mathbf{v}_2)^2 \mathcal{W}^{ba} \right] \]

\[ \frac{1}{\tau_D} = \frac{N}{4T^2B_2} \int d\{\epsilon_i\} d\{\mathbf{v}_i\} \left( \mathbf{v}_1 - \mathbf{v}_2 \right) \left( \mathbf{v}_4 - \mathbf{v}_3 \right) \mathcal{W}^{ba} \]

Velocity (not momentum!) relaxation / transfer rates

**close to the Dirac point**

\[ \frac{1}{\tau_D} \sim \alpha^2 N \frac{\mu_a \mu_b}{T} \]

\[ \frac{1}{\tau_{ee}^{a,b}} \sim \alpha^2 N T \]

**away from the Dirac point**

\[ \frac{1}{\tau_D} \sim \alpha^2 N \frac{T^2}{\mu} \ln \frac{\mu}{T} \]

\[ \frac{1}{\tau_{ee}} - \frac{1}{\tau_D} \sim \frac{1}{\tau_{ee}} \frac{T^2}{\mu^2} \ll \frac{1}{\tau_{ee}} \]
Drag resistivity

Equal layers:

\[ \rho_D = \frac{\hbar C_2}{e^2 \epsilon_0} \left( \frac{\tau \tau_D}{\tau D} \right)^{-1} + \frac{C_1^2}{\tau D} \left[ \tau ee - \tau D^{-2} \right] \]

\[ C_1 = \frac{\langle \epsilon \rangle_e}{T} \sim \frac{\mu}{T}, \quad C_2 = \frac{\langle \epsilon^2 \rangle_e - \langle \epsilon \rangle_e^2}{T^2} \sim \text{const}, \]

Non-equal layers near the Dirac point:

\[ \rho_D (\mu_i \ll T) \approx 2.87 \frac{\hbar}{e^2} \alpha^2 \frac{\mu_1 \mu_2}{\mu_1^2 + \mu_2^2 + 0.49T/(\alpha^2 \tau)} \]

Finite drag at the double Dirac point in the clean case:

Fast momentum (energy current) transfer followed by the intralayer velocity relaxation due to the e-e interaction
neutrality point
(additional correlations)

Schütt, Ostrovsky, Titov, IG, Narozhny, Mirlin, PRL (2013)

alternative mechanism: Song & Levitov, PRL (2012)
Nonzero drag at the Dirac point

Third-order interlayer scattering rate and correlated disorder
magnetodrag and Hall drag

Titov et al. arXiv:1303.6264

see also Song & Levitov, arXiv:1303.3529
Song, Abanin, Levitov, arXiv:1304.1450
Magnetodrag


Graphs showing the dependence of magnetodrag and magnetoresistance on various parameters such as temperature and magnetic field.
Experiment
Drude-like model in magnetic field

momentum transfer rate

passive layer

\[ v_2 = 0 \]
\[ v_1^x = \frac{j_1}{en} \]

active layer

\[ E_1^x = \frac{m}{e^2 n} \left[ \frac{1}{\tau} + \frac{1}{\tau_D} \right] j_1^x \Rightarrow \rho_{xx}^{11} \approx \frac{m}{e^2 n \tau} \]

- no magneto-resistance

- classical Hall effect

\[ E_1^y = \frac{B}{n \epsilon c} j_1^x \Rightarrow \rho_{yx}^{11} = \frac{B}{n \epsilon c} \]

equations of motion

- passive layer:

\[ \frac{dv_2}{dt} = \frac{e}{m} E_2 + \frac{e}{mc} [v_2 \times B] - \frac{v_2 - v_1}{\tau_D} - \frac{v_2}{\tau} \]

- active layer:

\[ \frac{dv_1}{dt} = \frac{e}{m} E_1 + \frac{e}{mc} [v_1 \times B] - \frac{v_1 - v_2}{\tau_D} - \frac{v_1}{\tau} \]

passive layer

- no magnetodrag

\[ E_2^x = \frac{m}{e^2 n \tau_D} j_1^x \Rightarrow \rho_{xx}^{12} = \frac{m}{e^2 n \tau_D} \]

- no Hall drag

\[ E_2^y = 0 \]

No effect in a single-band model!
Drude-like model in magnetic field

momentum transfer rate

passive layer

\[ v_2 = 0 \]

\[ v_1^x = \frac{j_1}{en} \]

active layer

No effect in a single-band model!

active layer

- no magneto-resistance

\[ E_1^x = \frac{m}{e^2 n} \left[ \frac{1}{\tau} + \frac{1}{\tau_D} \right] j_1^x \Rightarrow \rho_{xx}^1 \approx \frac{m}{e^2 n \tau} \]

- classical Hall effect

\[ E_1^y = \frac{B}{ne} j_1^x \Rightarrow \rho_{yx}^1 = \frac{B}{ne} \]

passive layer

- no magnetodrag

\[ E_2^x = \frac{m}{e^2 n \tau_D} j_1^x \Rightarrow \rho_{xx}^{12} = \frac{m}{e^2 n \tau_D} \]

- no Hall drag

\[ E_2^y = 0 \]

equations of motion

- passive layer:

\[ \frac{dv_2}{dt} = \frac{e}{m} E_2 + \frac{e}{mc} [v_2 \times B] - \frac{v_2 - v_1}{\tau_D} - \frac{v_2}{\tau} \]

- active layer:

\[ \frac{dv_1}{dt} = \frac{e}{m} E_1 + \frac{e}{mc} [v_1 \times B] - \frac{v_1 - v_2}{\tau_D} - \frac{v_1}{\tau} \]
Why is drag finite at the double Dirac point?

Four fluids (e1, h1 and e2, h2)

Opposite Lorentz forces for electrons and holes: charge and quasiparticle currents noncollinear

infinite sample
``macroscopic'' vs ``mesoscopic'' samples:
energy-escape length (phonons) important
Drude-like description (infinite sample)

\[
\begin{align*}
 eE_1 + e[v_{1e} \times B] &= F_{1e} + e v_{1e}/M_1, \\
 -eE_1 - e[v_{1h} \times B] &= F_{1h} + e v_{1h}/M_1, \\
 eE_2 + e[v_{2e} \times B] &= F_{2e} + e v_{2e}/M_2, \\
 -eE_2 - e[v_{2h} \times B] &= F_{2h} + e v_{2h}/M_2.
\end{align*}
\]

\[M = \text{mobility}\]

\[F = \text{friction force}\]

\[
\rho^D_{xx} = \frac{\hbar \gamma}{e^2} \frac{B^2 M_1 M_2}{1 + \hbar \gamma \rho_0 (M_1 + M_2)/e},
\]

\[n_i = 0\]

positive drag
Drude-like description (finite sample)

\[ j_i = e(n_{ie}v_{ie} - n_{ih}v_{ih}), \quad P_i = n_{ie}v_{ie} + n_{ih}v_{ih} \]

\[ n_i = n_{ie} - n_{ih} \quad \rho_i = n_{ie} + n_{ih} \]

\[ F_{1a} = -F_{2a} = \hbar \gamma (P_1 - P_2) \]

\[ -K_1 \nabla \rho_1 + en_1 E_1 + [j_1 \times B] = \rho_1 F_1 + eP_1/M_1, \]

\[ e\rho_1 E_1 + e[P_1 \times B] = n_1 F_1 + j_1/M_1, \]

\[ \nabla P_1 = - (\rho_1 - \rho_0)/\tau_{ph} - (\rho_1 - \rho_2)/(2\tau_Q). \]

energy loss due to phonons

quasiparticle imbalance relaxation

\[ K_i = (\pi \hbar^2 v^2 / 2)(\partial n_i / \partial \mu_i) = 2T \ln(2 \cosh \mu_i / 2T) \]
Drude-like description (finite sample)

solution at charge neutrality

\[ \rho_{xx}^D = \frac{\rho_{xx}^{(0)}}{2} \left[ F(0, 0) - F \left( \frac{\hbar \gamma}{e^2 \rho_{xx}^0}, \frac{\tau_{ph}}{\tau_Q} \right) \right], \]

\[ F(X, Y) = \frac{1 + 2X + (MB)^2}{1 + 2X + (MB)^2 \frac{\tanh \Theta(X,Y)}{\Theta(X,Y)}}, \]

\[ \Theta(X, Y) = \frac{W}{2\ell_{ph}} \sqrt{(1 + 2X + (MB)^2)(1 + Y)}, \]

- drag sign depends on energy relaxation and sample geometry
- positive drag in “wide” samples
- negative drag in “narrow” samples
Magnetodrag at the double Dirac point

$\ell_{ph} = 1.2 \, \mu m$

$M = 4 \, m^2/\text{Vs}$
$T = 240 \, K$
$\alpha = 0.2$

Graph showing $\rho_{xx}^D [k\Omega]$ as a function of $B [T]$ with different widths $W$ (50 $\mu m$, 10 $\mu m$, 5 $\mu m$, 2 $\mu m$, 0.5 $\mu m$).
Magnetodrag: experiment vs theory

\[ \rho_{xx} \left( B \right) \]

\[ \alpha = 0.2 \]

\[ T = 160 \text{ K} \]

\[ M = 4 \text{ m}^2/\text{Vs} \]
Magnetodrag: experiment vs theory

\[ \rho_{xx}^D \quad \Omega \]

\( n [10^{11} \text{ cm}^{-2}] \)

\( B [\text{T}] \)

\( 0 \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.4 \)

\( \alpha = 0.2 \)

\( T = 240 \text{ K} \)

\( M = 4 \text{ m}^2/\text{Vs} \)
Hall drag

- non-zero in two-band systems, but
  - vanishes at charge neutrality and in the case of oppositely doped layers
- non-trivial density dependence
Hall drag: experiment vs theory

\( B = 100 \text{ mT} \)
\( \hbar/\tau = 50 \text{ K} \)
\( T = 240 \text{ K} \)
\( d = 1 \text{ nm} \)
\( \alpha = 0.2 \)

\[ \rho_{xy}^D [\Omega] \]
\[ \rho_{xy}^D [\Omega] \]

\( n [10^{11} \text{ cm}^{-2}] \)
Summary

Coulomb drag in graphene:

- Perturbation theory (disordered graphene)
- Kinetic theory in clean graphene (equilibrated drag)
- Peak at the Dirac point
  3\textsuperscript{rd} order drag, drag with correlated disorder
- Giant magnetodrag at the Dirac point
  (four liquid model: positive & negative drag)