Coulomb drag in graphene

Igor Gornyi

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Phys. Rev. Lett. **110**, 026601 (2013),
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Collaboration

Theory:

M. Schütt, B. N. Narozhny, A. D. Mirlin (Karlsruhe)

P. M. Ostrovsky (MPI Stuttgart + Landau Institute)

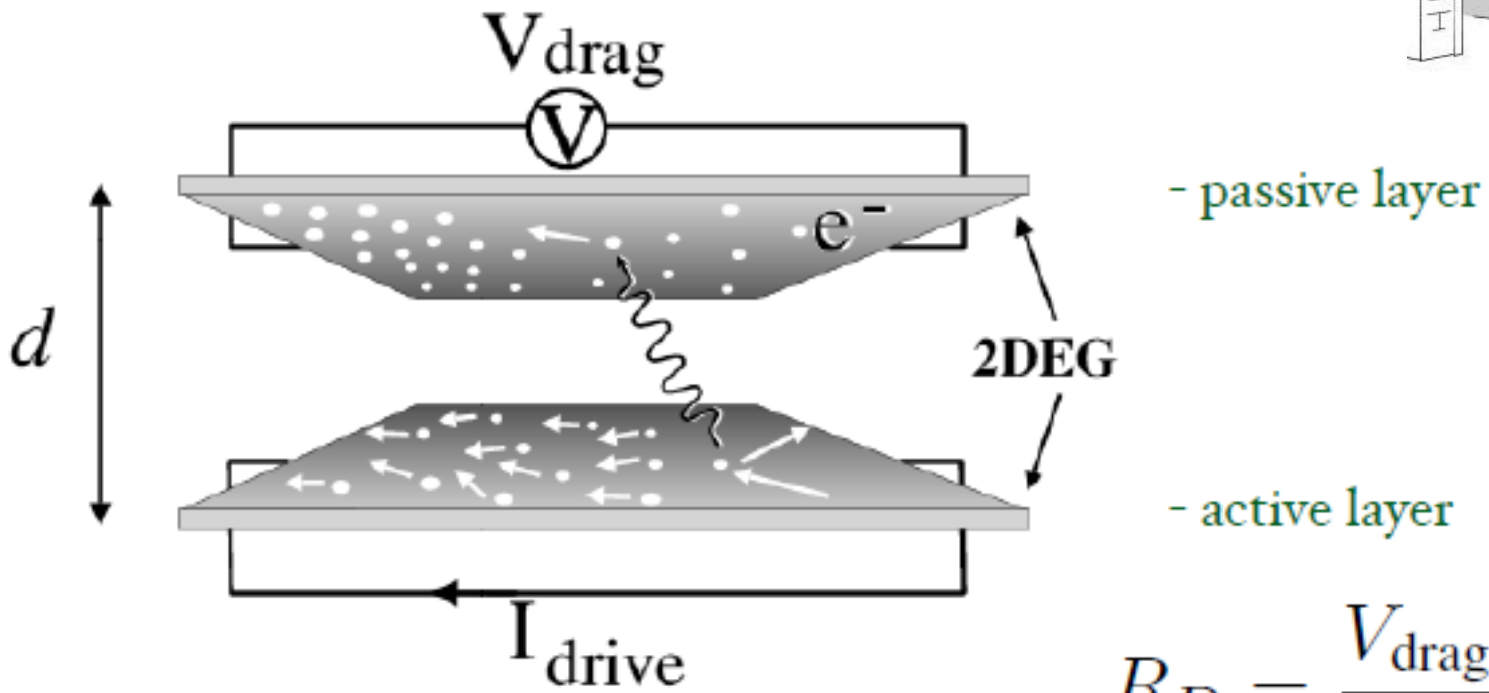
M. Titov, M. I. Katsnelson, T. Tudorovskiy (Nijmegen)

Experiment :

R. V. Gorbachev, K. S. Novoselov, A. K. Geim,

L. A. Ponomarenko (Manchester)

What is Coulomb drag?



$$R_D = \frac{V_{\text{drag}}}{I_{\text{drive}}} = \frac{L}{W}\rho_D$$

Coulomb drag = response of the passive layer
to a current in the active layer
mediated by Coulomb interaction

Coulomb drag: why interesting?

- no drag without interaction:

  probe of inter-electron correlations

- provides information about inelastic processes,

  phase-coherent phenomena

- drag is related to particle-hole asymmetry

Drag in graphene near the Dirac point ?

Coulomb drag measurements

Gramila, Eisenstein, MacDonald *et.al* ., Phys. Rev. Lett. 66, 1216 (1991)

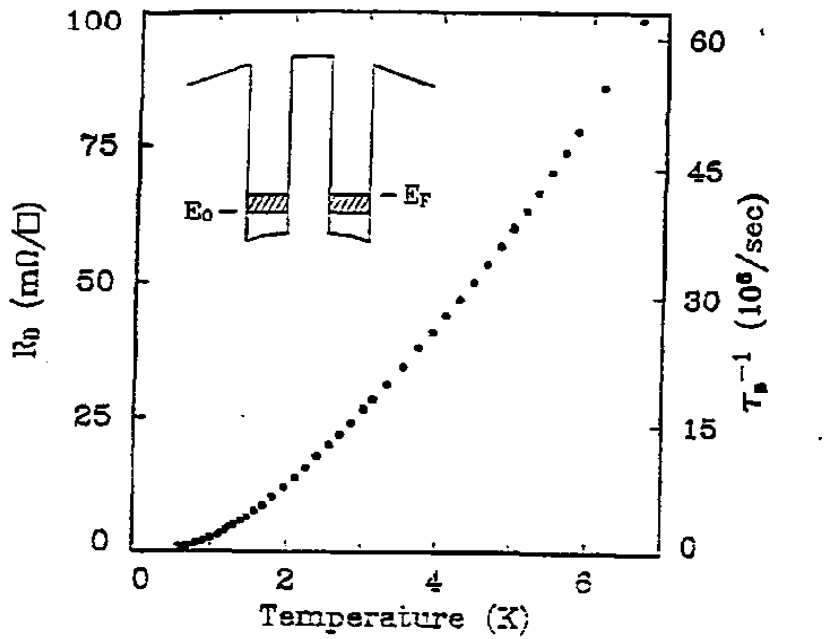


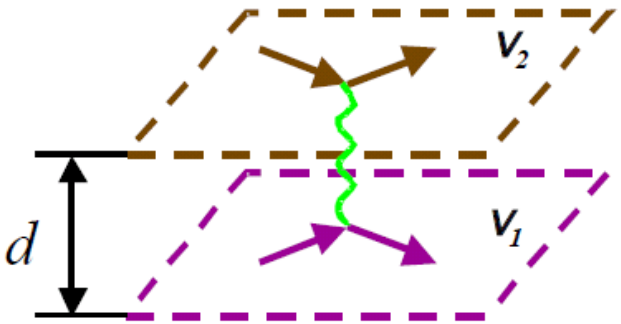
FIG. 2. Temperature dependence of observed frictional drag between two 2D electron systems separated by 175-Å barrier. Data are plotted as an equivalent resistance and a momentum-transfer rate. Inset: An idealized conduction-band diagram for a DQW structure indicating the ground subband energy E_0 and the Fermi energy E_F.

Drag from Drude

$$\boldsymbol{j} = e\,n\,\boldsymbol{v}$$

Equations of motion

$$t \gg \tau, \tau_D$$



$$\frac{d\boldsymbol{v}_1}{dt} = \frac{e}{m}\boldsymbol{E}_1 + \frac{\boldsymbol{v}_2 - \boldsymbol{v}_1}{\tau_D} - \frac{\boldsymbol{v}_1}{\tau} = 0$$

$$\frac{d\boldsymbol{v}_2}{dt} = \frac{e}{m}\boldsymbol{E}_2 + \frac{\boldsymbol{v}_1 - \boldsymbol{v}_2}{\tau_D} - \frac{\boldsymbol{v}_2}{\tau} = 0$$

$\frac{1}{\tau}$ - impurity scattering rate

$\frac{1}{\tau_D}$ - interlayer scattering rate

$$\begin{pmatrix} \boldsymbol{E}_1 \\ \boldsymbol{E}_2 \end{pmatrix} = \frac{m}{e^2 n} \begin{pmatrix} \frac{1}{\tau} + \frac{1}{\tau_D} & -\frac{1}{\tau_D} \\ -\frac{1}{\tau_D} & \frac{1}{\tau} + \frac{1}{\tau_D} \end{pmatrix} \begin{pmatrix} \boldsymbol{j}_1 \\ \boldsymbol{j}_2 \end{pmatrix} = \begin{pmatrix} \rho_1 & -\rho_D \\ -\rho_D & \rho_2 \end{pmatrix} \begin{pmatrix} \boldsymbol{j}_1 \\ \boldsymbol{j}_2 \end{pmatrix}$$

Drude-like formula

$$\rho_D = \left(\frac{e^2 n \tau_D}{m} \right)^{-1}$$

Fermi-liquid theory $T \ll E_F$

Pogrebinskii '77; Zheng, MacDonald '93; Jauho, Smith '93; Kamenev, Oreg '95; Flensberg et al. '95

additional natural assumptions:

- the spacer is much wider than the screening length $q_{TF} d \gg 1$
- the spacer is much wider than the Fermi wave length $k_F d \gg 1$

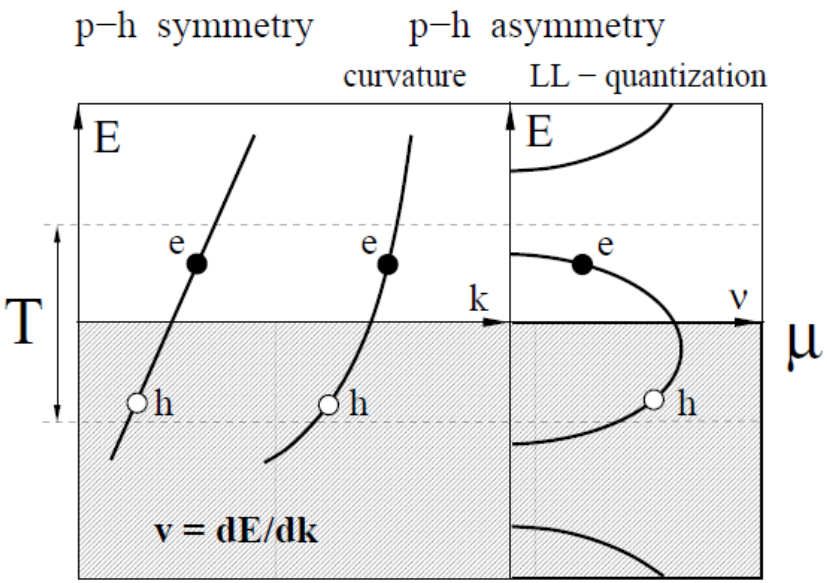
Interlayer scattering time from the **Fermi golden rule**

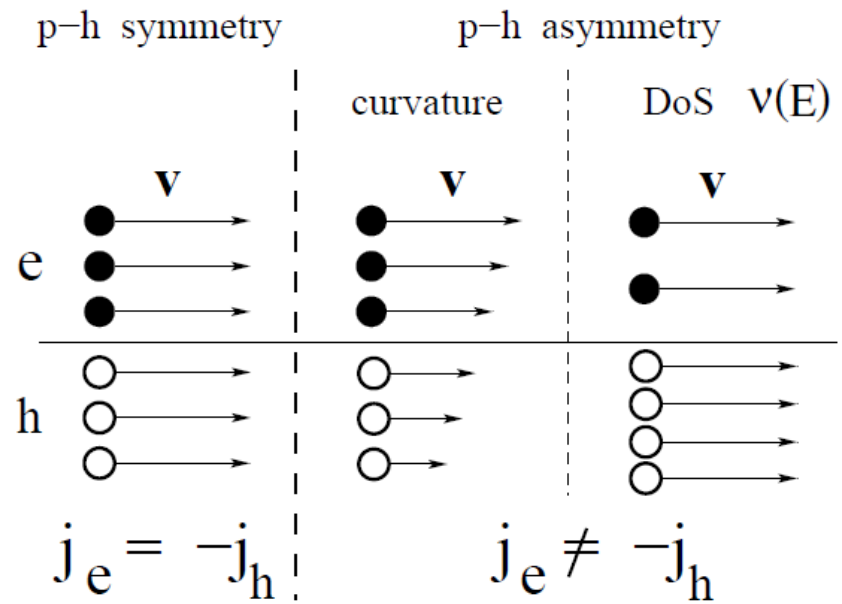
$$\tau_D^{-1} = \frac{\pi\zeta(3)}{32} \frac{(k_B T)^2}{(q_{TF} d)^2 (k_F d)^2} \qquad \rho_D = \left(\frac{e^2 n \tau_D}{m}\right)^{-1}$$

transresistivity is essentially expressed via electron concentration $\boldsymbol{n}$, temperature $\boldsymbol{T}$, and spacing $\boldsymbol{d}$

$$\rho_D \sim \frac{T^2}{n^3 d^4} \qquad n \propto k_F^2 \quad \text{in 2D}$$

Particle-hole asymmetry



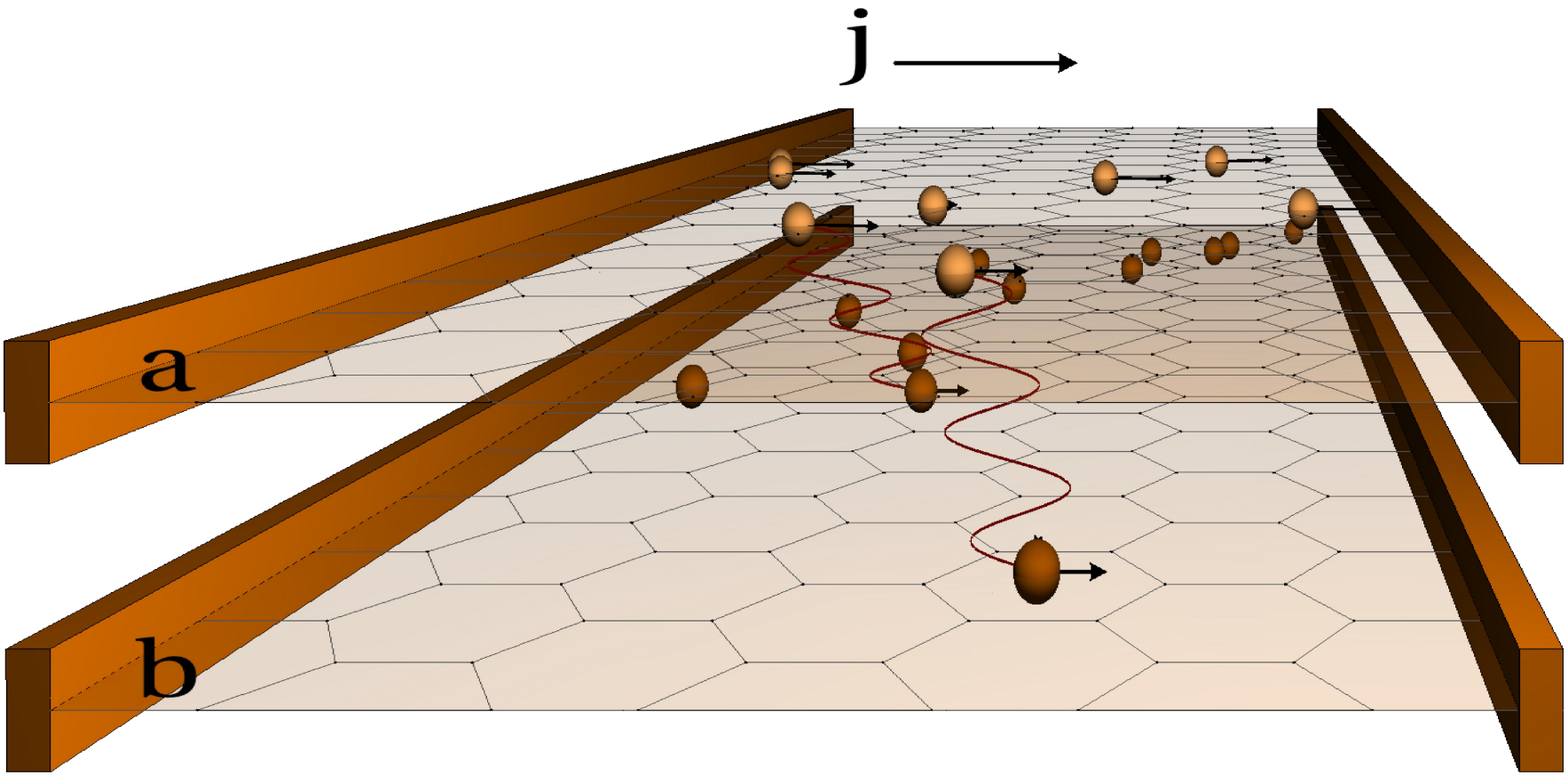


Example: strong magnetic field

(i) Curvature ➔ normal positive drag
(ii) Landau levels DoS ➔ anomalous oscillatory drag

IG, Mirlin, von Oppen (2004)

Drag in graphene



News in graphene

Dirac spectrum at low energies

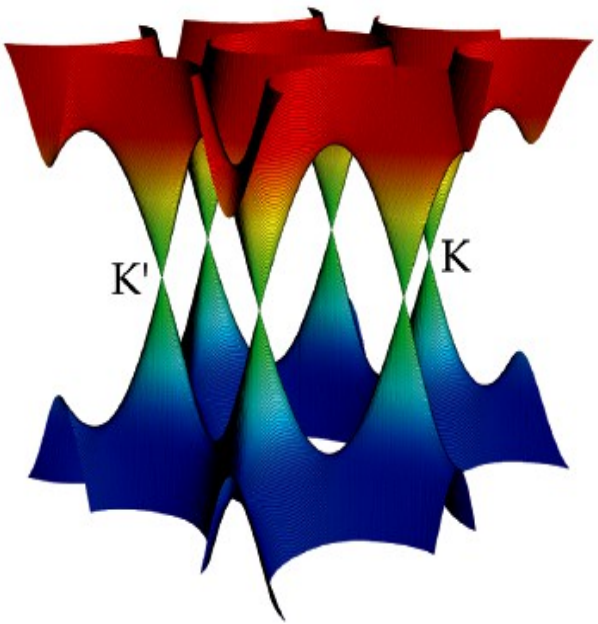
$$E_\nu(\boldsymbol{k}) = \nu v k, \quad \nu = \pm$$

electron-hole symmetry at the Dirac point

linear spectrum – no Galilean invariance

- non-trivial single-layer conductivity

small interlayer distance *d*

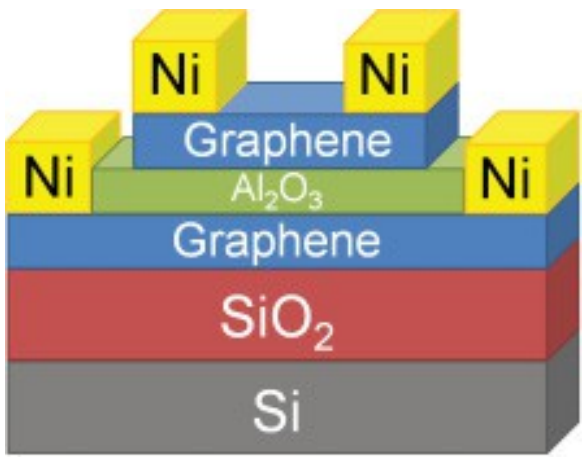


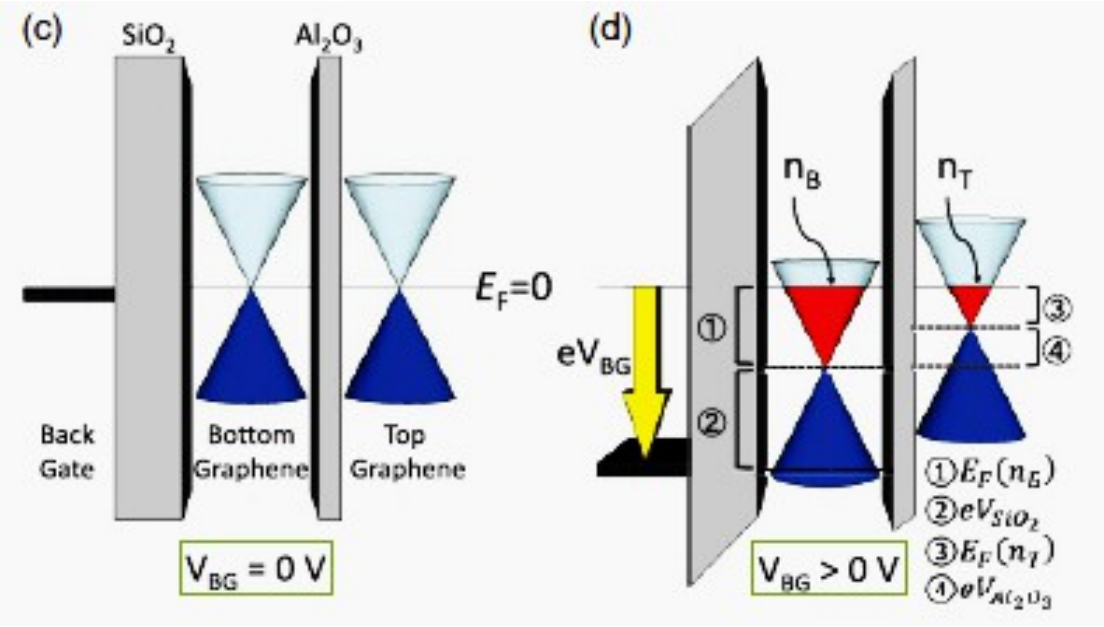
- electron density can be positive, negative, large and low
- Fermi wave length at low density is much larger than *d*
- screening length at low density is much larger than *d*

Drag in graphene: experiment

Austin group: single-gate device

Kim, Jo, Nah, Yao, Banerjee, and Tutuc, Phys. Rev. B **83**, 161401 (2011)





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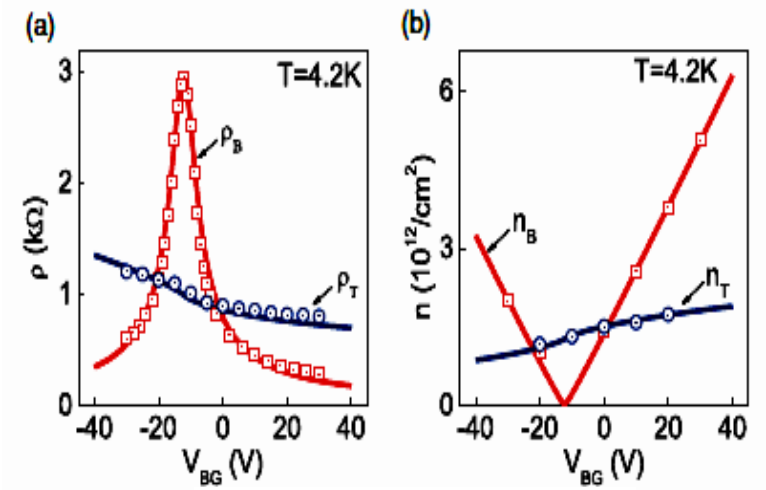
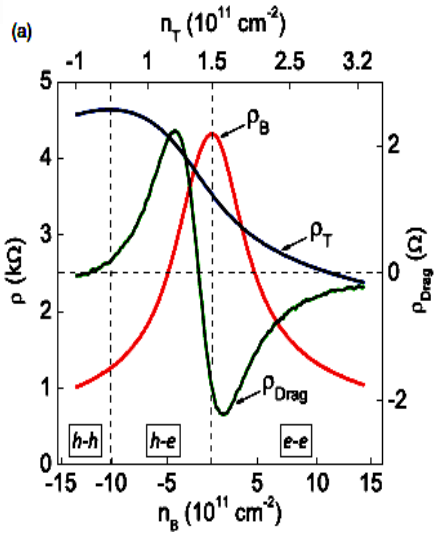
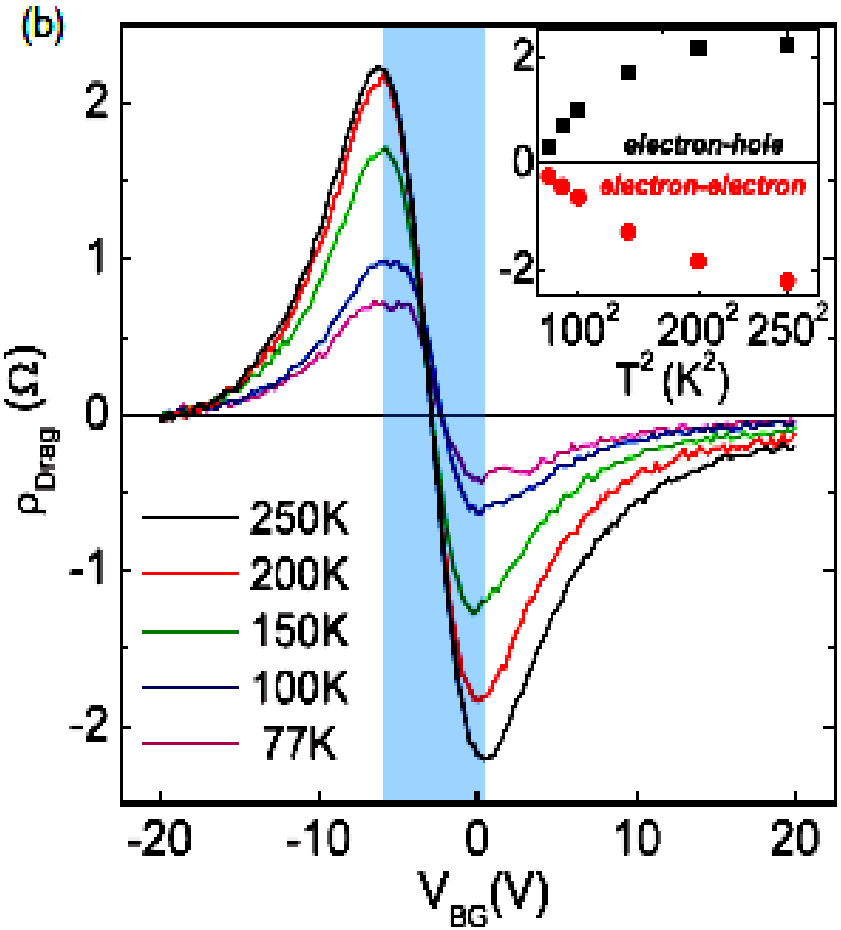


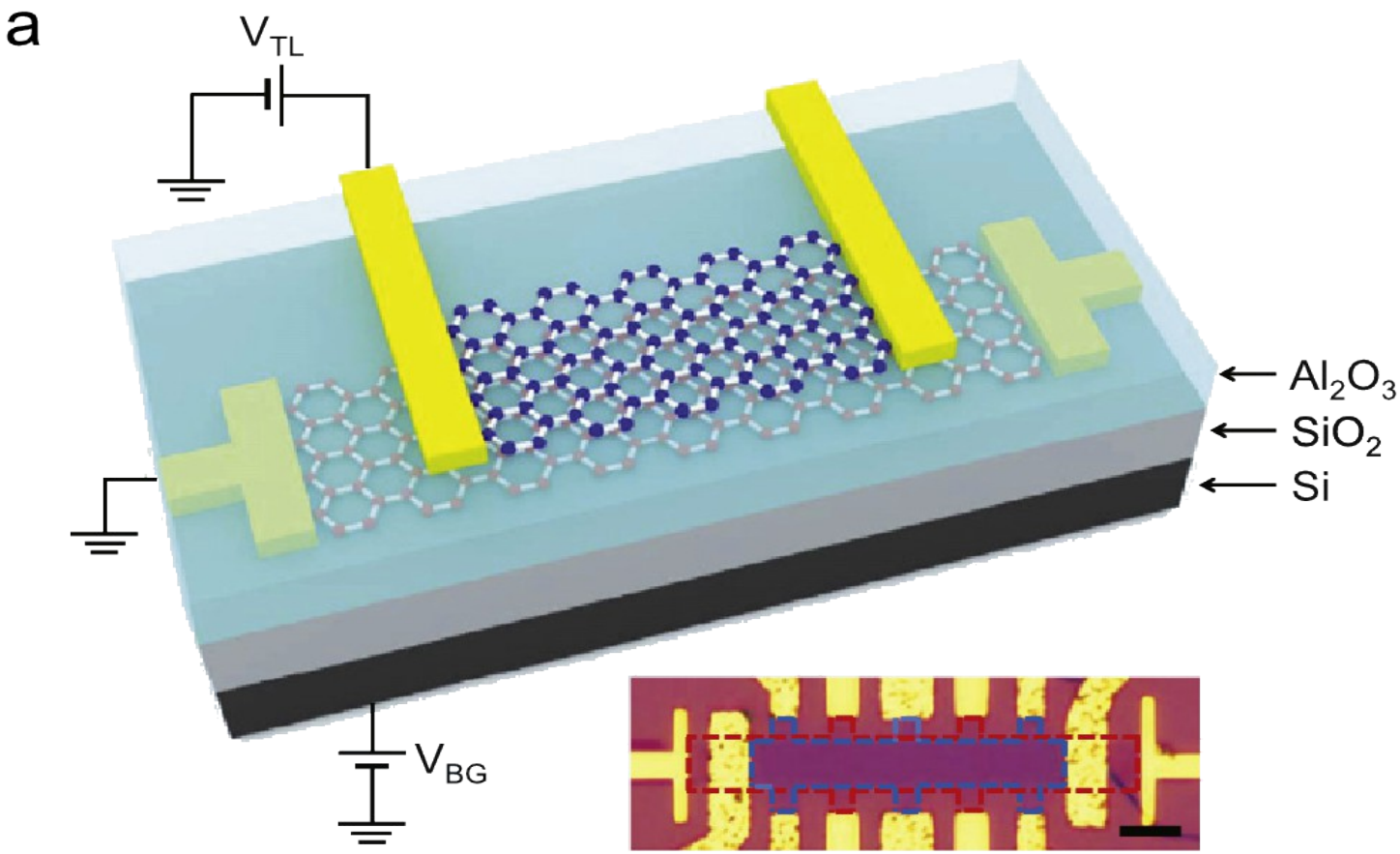




Drag in graphene: experiment

Austin group: double-gate device

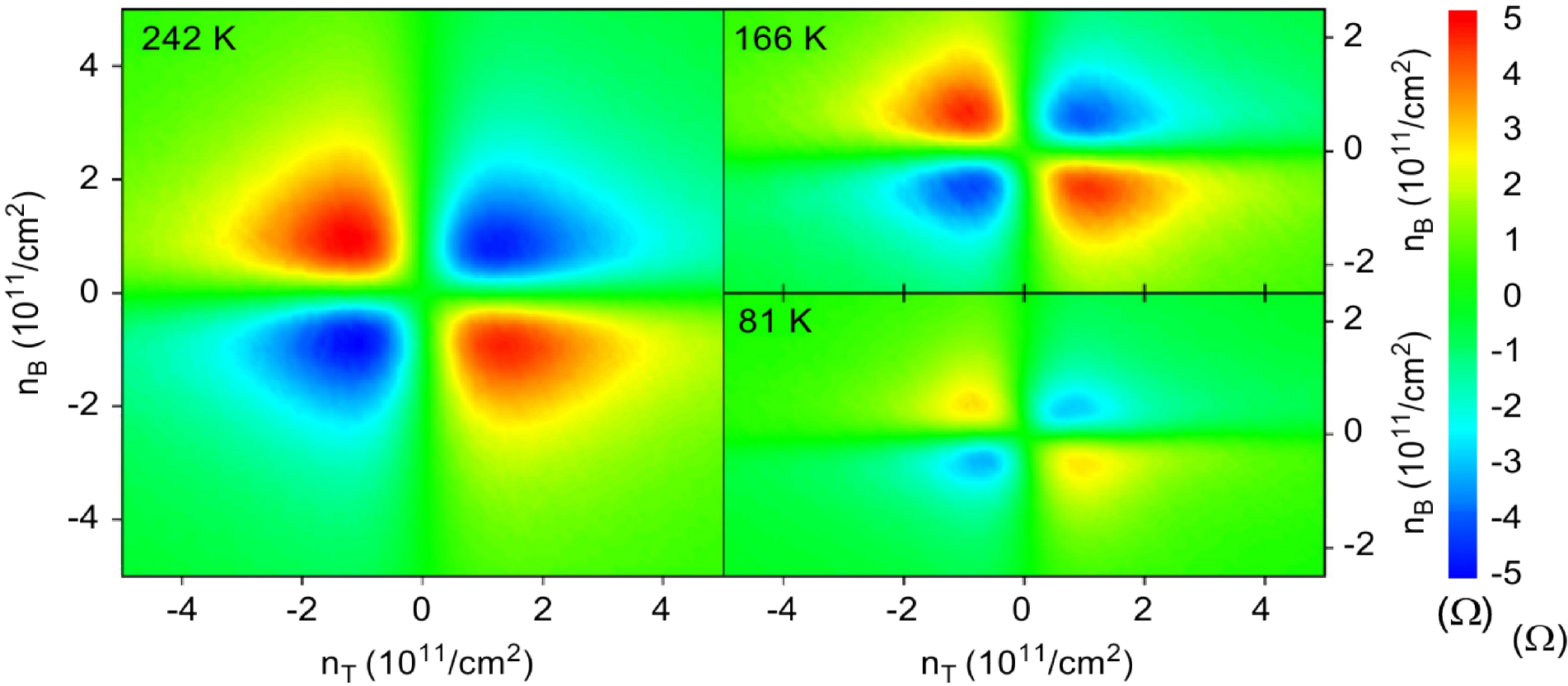
Tutuc and Kim, Solid State Comm. (2012)



Drag in graphene: experiment

double-gate device

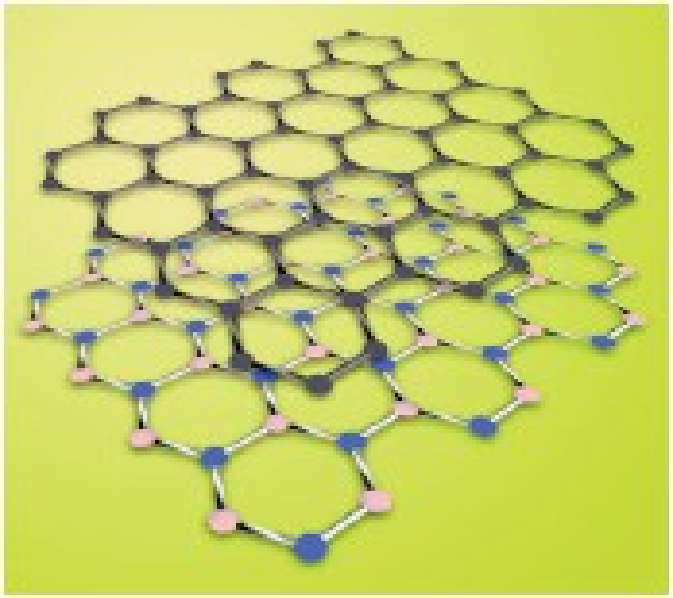
Tutuc and Kim, Solid State Comm. (2012)



Drag in graphene: experiment

Manchester group:

Gorbachev, Geim, Novoselov, Ponomarenko et al. Nature Phys. (2012)



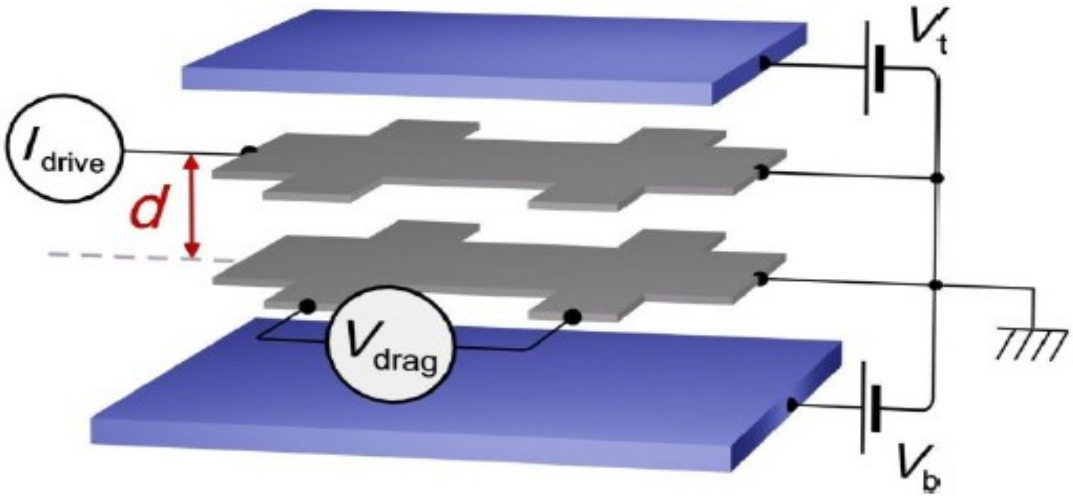
- "clean"

  substrate and spacer – BN

- smaller inter-layer spacing

  d = 1-10 nm

- Double-gate setup:



Drag in graphene: experiment

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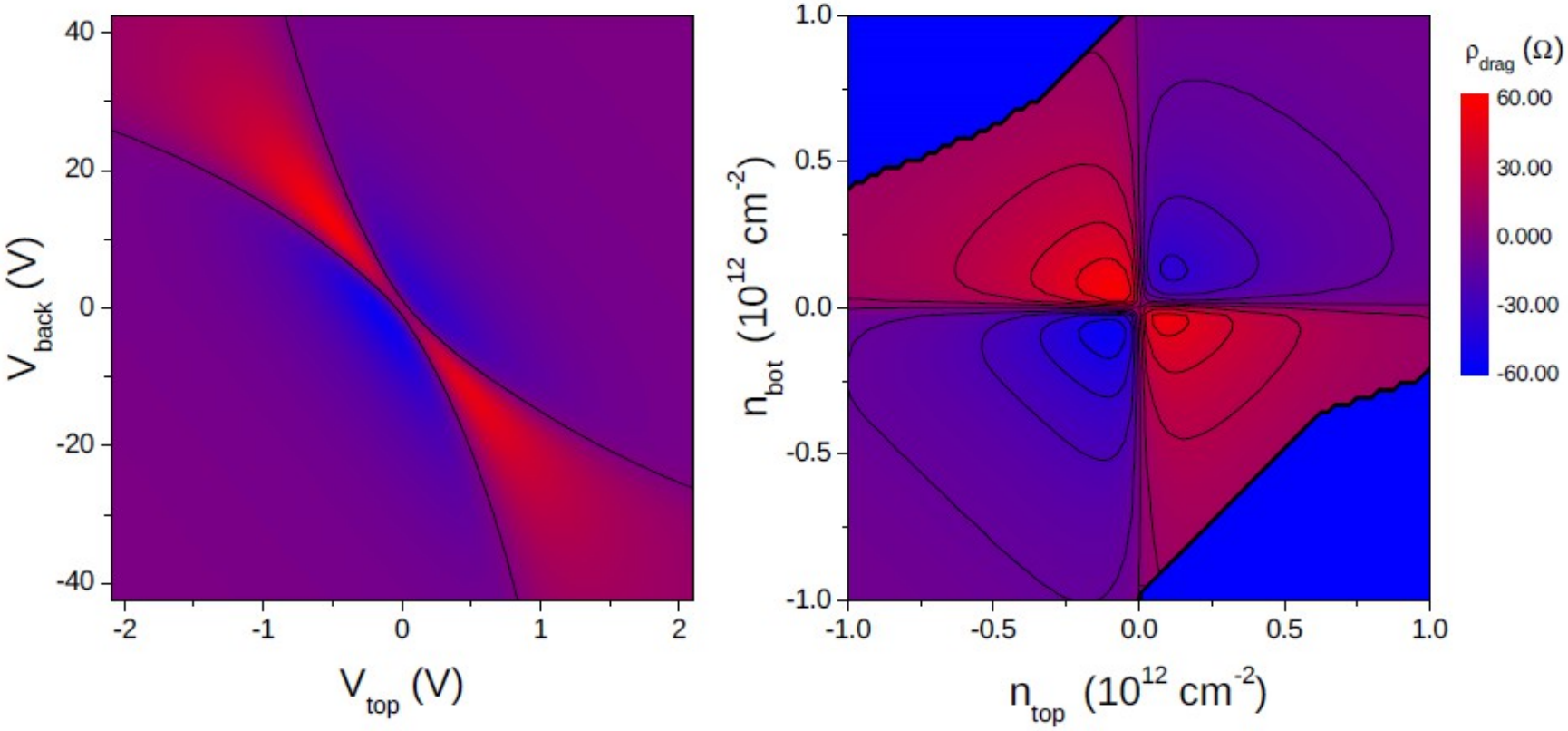
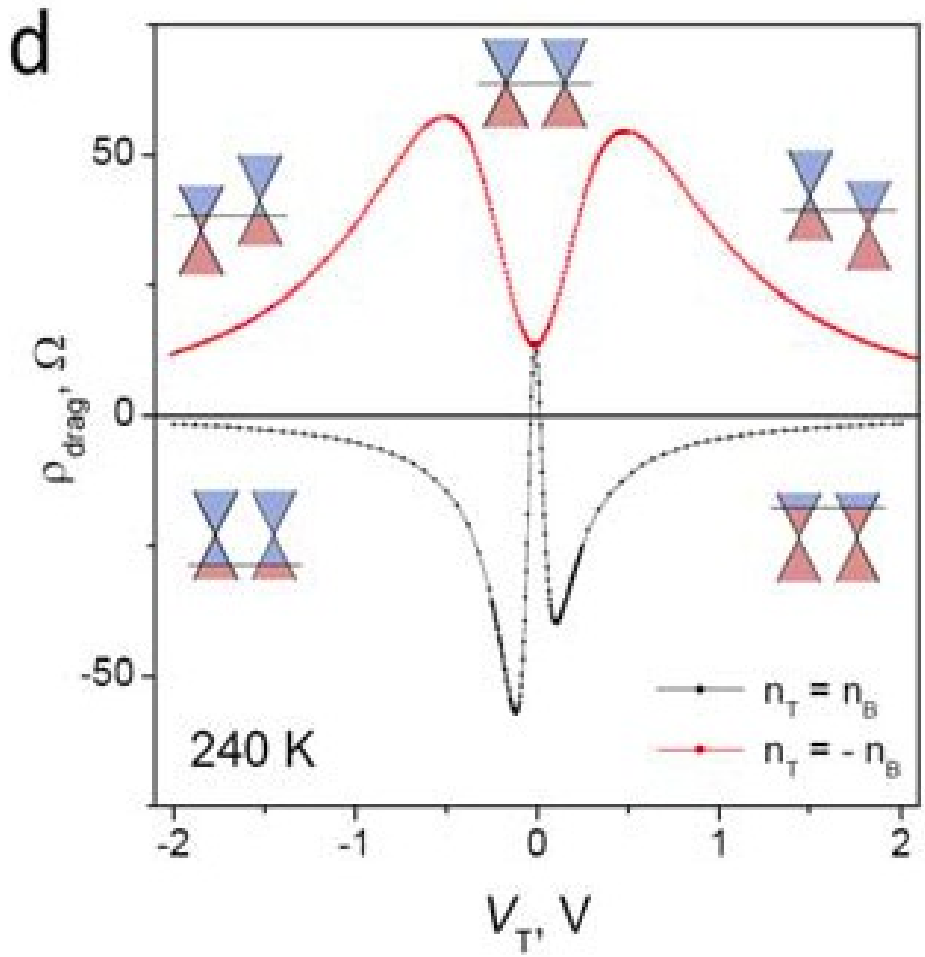
Gorbachev, Geim, Novoselov, Ponomarenko et al. Nature Phys. (2012)



Drag in graphene: experiment

double-gate device

Manchester group, Nature Phys. (2012)



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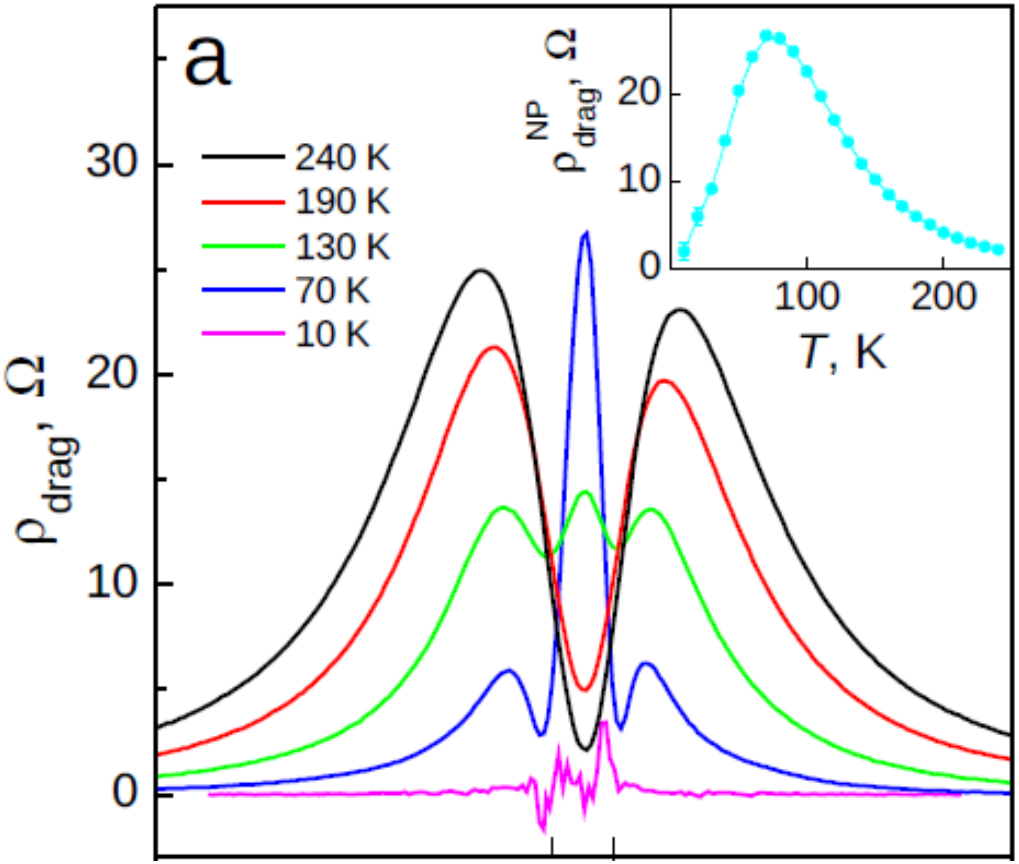
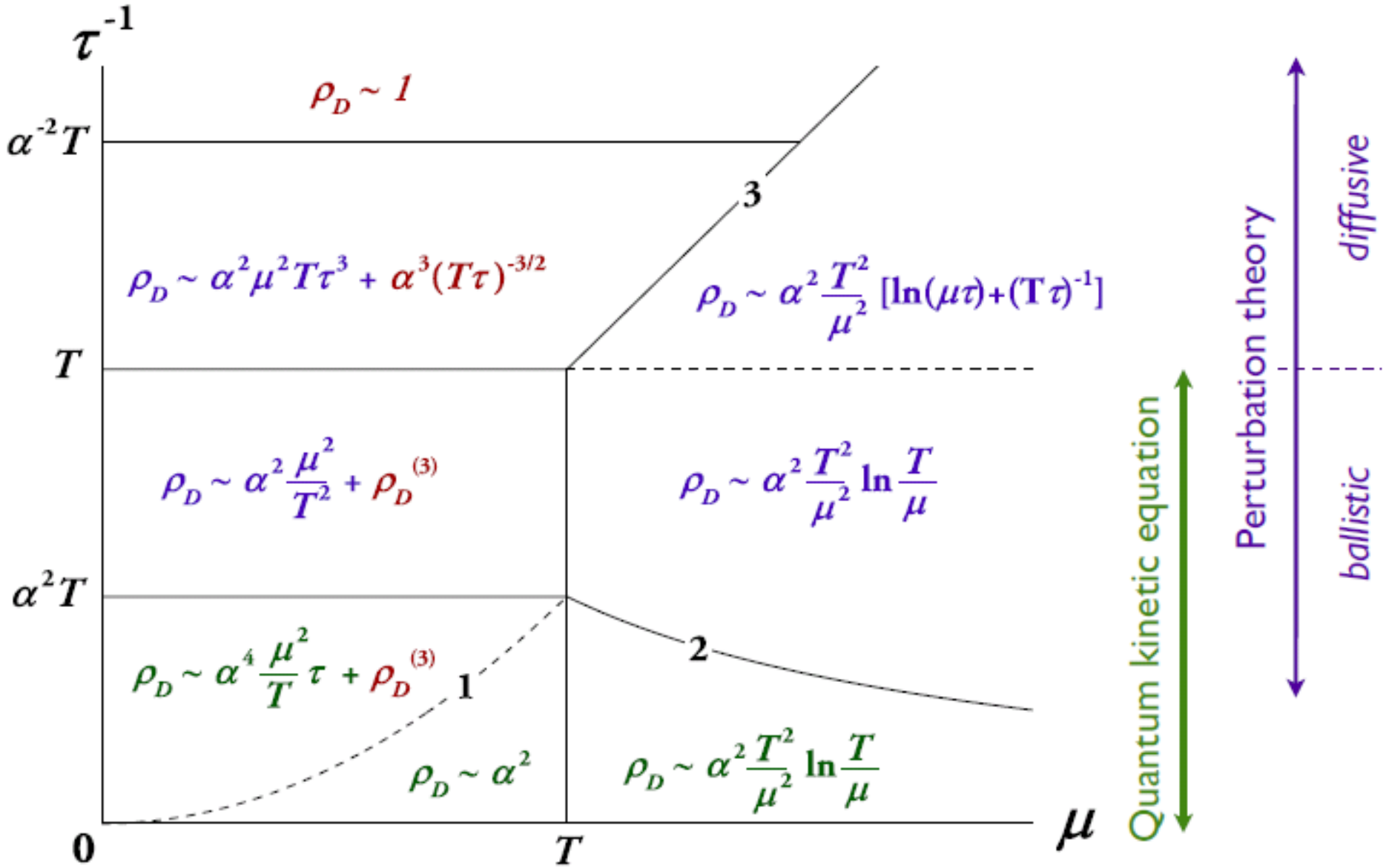
Manchester group, Nature Phys. (2012)



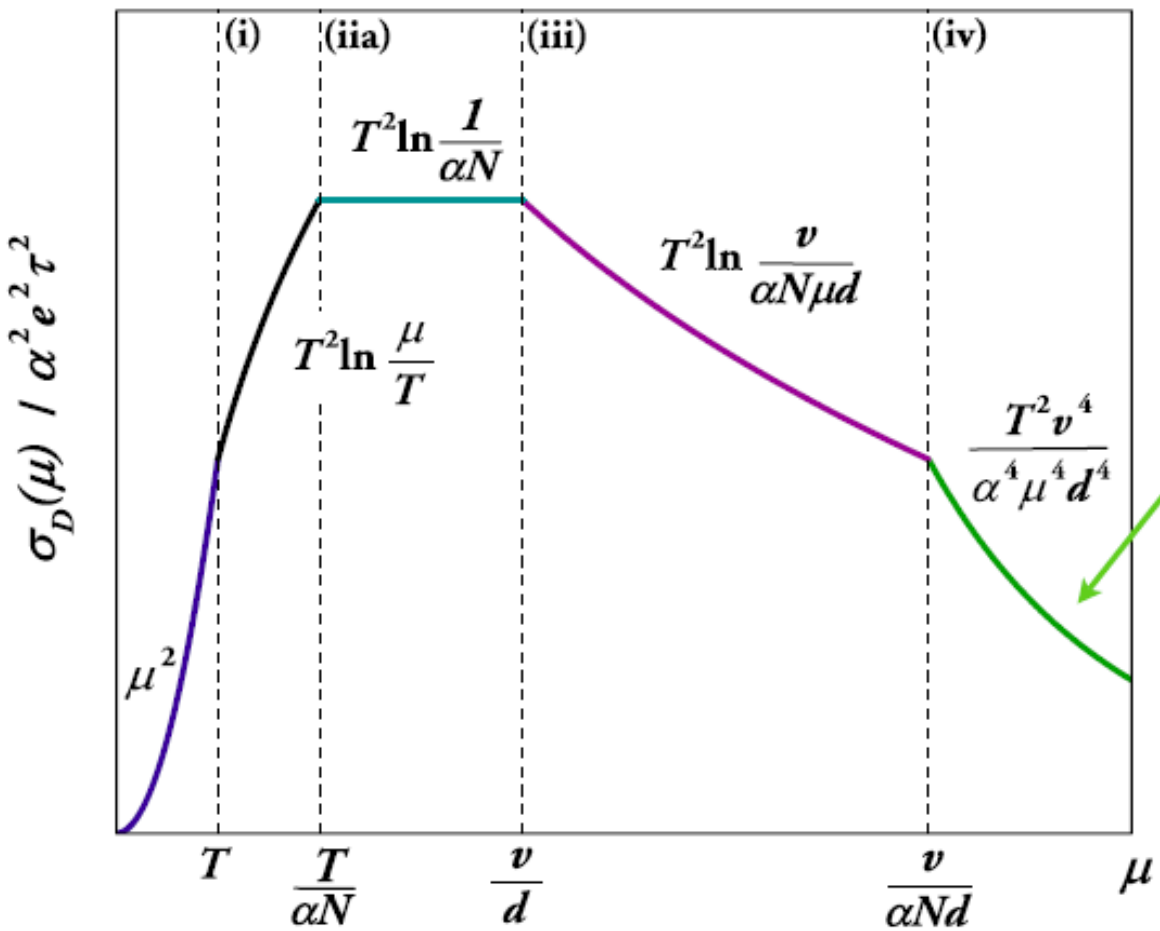
Our theory (zero magnetic field)



disordered graphene (- 11 - $_{ee}$)

Narozhny, Titov, IG, Ostrovsky, PRB (2012)

Results for $\alpha \ll 1 \qquad d \ll \hbar v/T$



Fermi-liquid asymptotic!

$$\sigma_D = \frac{\zeta(3)}{4} \frac{e^2 \tau^2 T^2}{(k_F d)^2 (\varkappa d)^2}$$

$$\varkappa = 4\alpha k_F$$

$$\varkappa d \gg 1$$

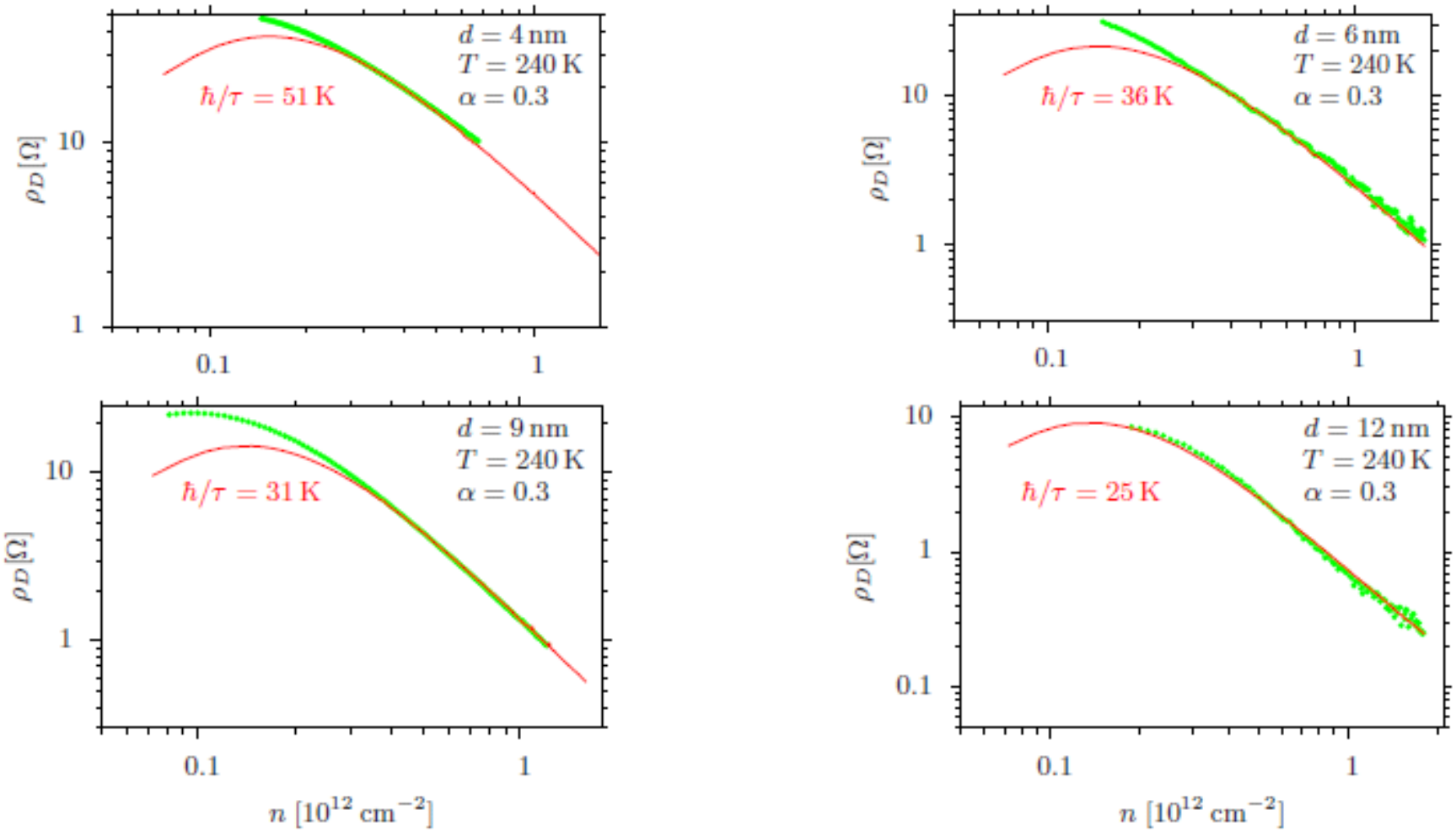
Tse, Hu, Das Sarma, PRB'07

USEFUL to REMEMBER

$$\mu \simeq \hbar v \sqrt{\pi n}$$

comparison with experiment

using second order perturbation theory

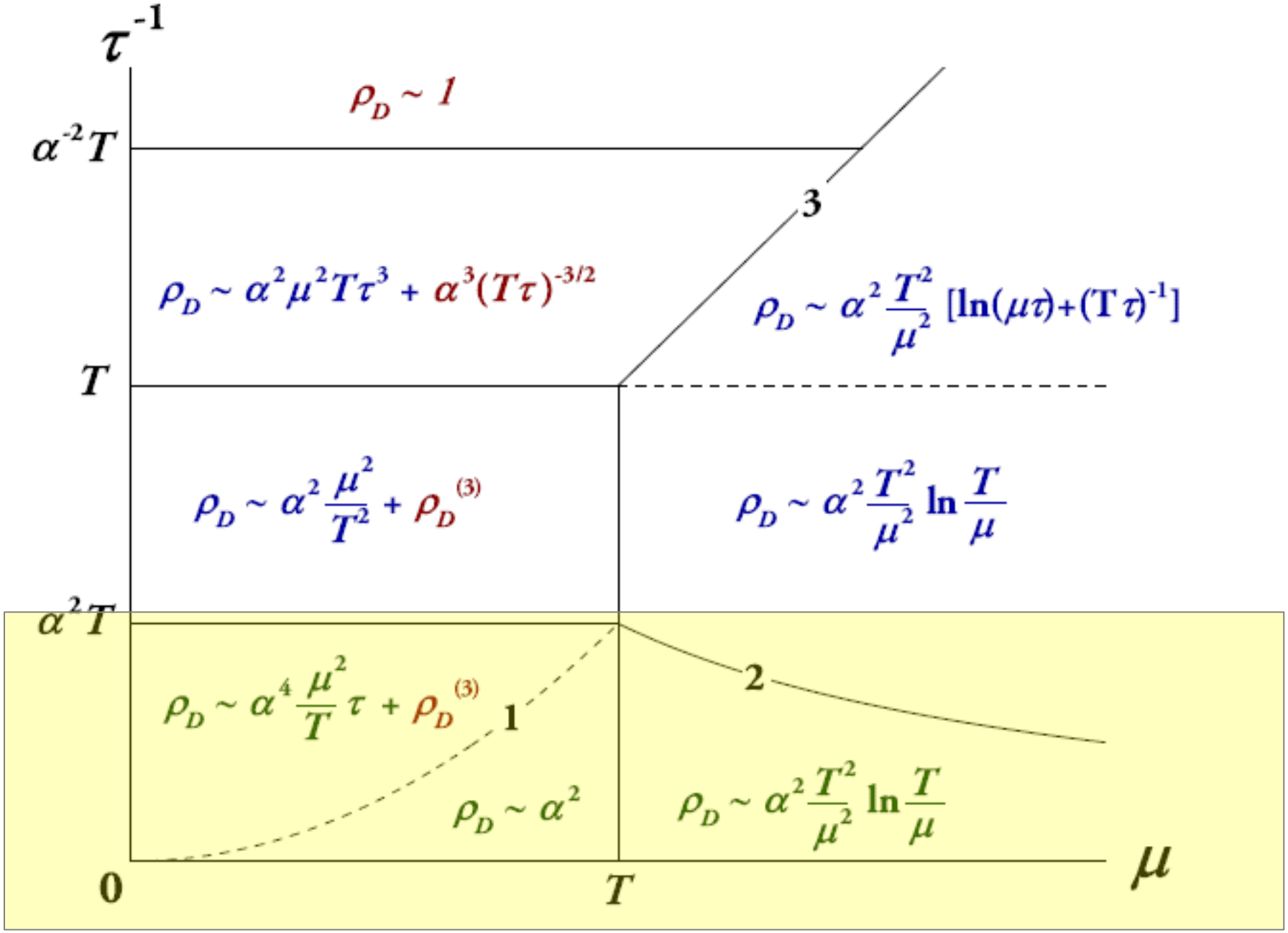


$$\tau^{-1} \gg \alpha^2 T \approx 22\ \mathrm{K}$$

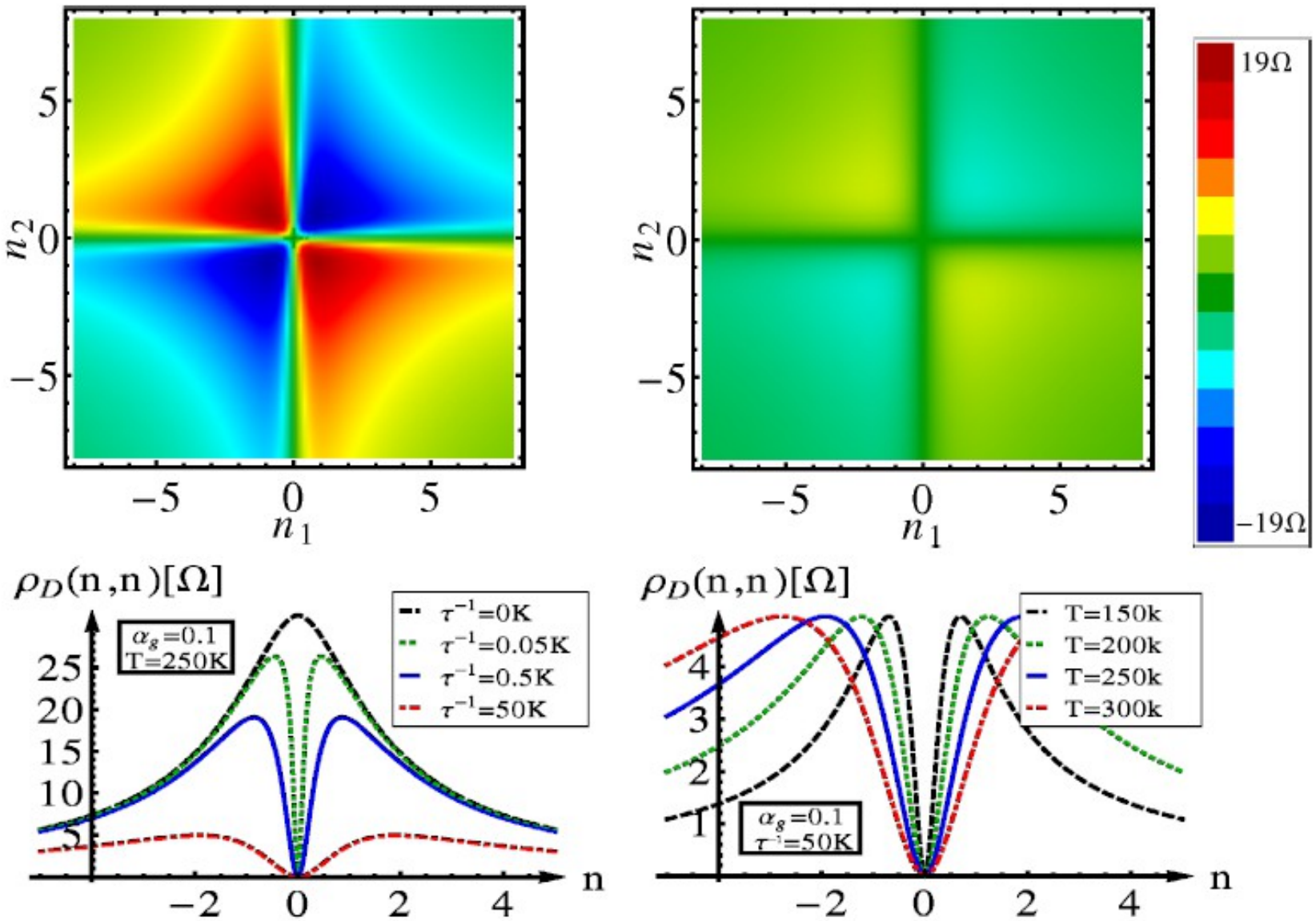
ultra-clean graphene (τ_{ee})

Schütt, Ostrovsky, Titov, IG, Narozhny, Mirlin, PRL (2013)

see also J. Lux and L. Fritz, PRB (2012)


τ^{-1}
$\rho_D \sim 1$
$\alpha^{-2}T$
3
$\rho_D \sim \alpha^2\mu^2 T\tau^3 + \alpha^3(T\tau)^{-3/2}$
$\rho_D \sim \alpha^2 \frac{T^2}{\mu^2}[\ln(\mu\tau)+(T\tau)^{-1}]$
T
$\rho_D \sim \alpha^2 \frac{\mu^2}{T^2} + \rho_D^{(3)}$
$\rho_D \sim \alpha^2 \frac{T^2}{\mu^2} \ln \frac{T}{\mu}$
$\alpha^2 T$
2
$\rho_D \sim \alpha^4 \frac{\mu^2}{T}\tau + \rho_D^{(3)}$
1
$\rho_D \sim \alpha^2$
$\rho_D \sim \alpha^2 \frac{T^2}{\mu^2} \ln \frac{T}{\mu}$
0
T
μ

Clean vs. disordered graphene



Kinetic theory of the drag

Linearized kinetic equation:

$$n_i(\epsilon, \hat{\mathbf{v}}) = n_F^{(i)}(\epsilon) + T \frac{\partial n_F^{(i)}(\epsilon)}{\partial \epsilon} h_i(\epsilon, \hat{\mathbf{v}})$$

$$\frac{\partial h_1}{\partial t} + \frac{e\boldsymbol{E}_1 \boldsymbol{v}}{T} = -\frac{h_1}{\tau} + I_{11}\{h_1\} + I_{12}\{h_1, h_2\},$$

$$\frac{\partial h_2}{\partial t} = -\frac{h_2}{\tau} + I_{22}\{h_2\} + I_{21}\{h_2, h_1\},$$

$$I_{ij} = -\int d2\ d3\ d4\ W^{ij}(h_{i,1} - h_{i,2} + h_{j,3} - h_{j,4})$$

Inelastic scattering in graphene

Kashuba '08; Müller & Sachdev '08; Fritz, Müller, Schmalian, Sachdev '08

Linear spectrum:

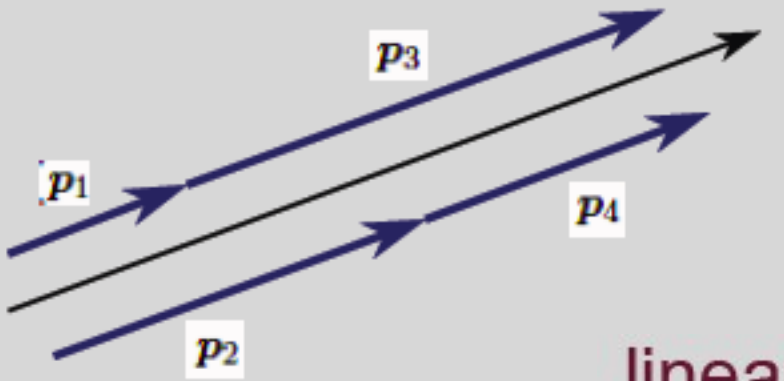
Velocity is not equivalent to momentum:
momentum conservation does not prevent current relaxation
- Finite transport rate due to inelastic e-e scattering

Collinear scattering singularity:
momentum conservation = energy conservation
- Fast thermalization within a given direction

$$n(\epsilon, \hat{\mathbf{v}}) = \frac{1}{1 + \exp\left[\frac{\epsilon - \mu(\hat{\mathbf{v}})}{T(\hat{\mathbf{v}})}\right]}$$

Collinear scattering singularity

collinear scattering



– equivalence of energy and momentum conservation laws

$$\boldsymbol{p}_1 + \boldsymbol{p}_3 = \boldsymbol{p}_2 + \boldsymbol{p}_4$$

$$\epsilon_i = \pm v |\boldsymbol{p}_i|$$

$$\epsilon_1 + \epsilon_3 = \epsilon_2 + \epsilon_4$$

linearized collision integrals

"momentum mode"

$$h_i \propto \boldsymbol{p} \;\Rightarrow\; I_{ij} = 0$$

"velocity mode"

$$h_i \propto \boldsymbol{v} \;\Rightarrow\; I_{ij} = 0$$

if $\boldsymbol{v}_i \| \boldsymbol{v}_j$

$$I_{ij} = -\int d2\; d3\; d4\; W^{ij} \underline{(h_{i,1} - h_{i,2} + h_{j,3} - h_{j,4})}$$

$$W^{ij} = \underline{\delta(\boldsymbol{p}_1 - \boldsymbol{p}_2 + \boldsymbol{p}_3 - \boldsymbol{p}_4)}\; \delta(\epsilon_1 - \epsilon_2 + \epsilon_3 - \epsilon_4)$$

$$\times \frac{\cosh \frac{\epsilon_1 - \mu_1}{2T}}{2 \cosh \frac{\epsilon_2 - \mu_1}{2T} \cosh \frac{\epsilon_3 - \mu_1}{2T} \cosh \frac{\epsilon_4 - \mu_1}{2T}} K^{ij}_{1,2;3,4},$$

Scattering rates: Golden Rule

$$\frac{1}{\tau_{ee}^{a}} = \frac{N}{8T^2 B_2} \int \mathrm{d}\{\epsilon_i\}\mathrm{d}\{\hat{\mathbf{v}}_i\} \left[(\mathbf{v}_1 - \mathbf{v}_2 + \mathbf{v}_3 - \mathbf{v}_4)^2 \ \mathcal{W}^{aa} + 2\,(\mathbf{v}_1 - \mathbf{v}_2)^2 \ \mathcal{W}^{ab} \right]$$

$$\frac{1}{\tau_{ee}^{b}} = \frac{N}{8T^2 B_2} \int \mathrm{d}\{\epsilon_i\}\mathrm{d}\{\hat{\mathbf{v}}_i\} \left[(\mathbf{v}_1 - \mathbf{v}_2 + \mathbf{v}_3 - \mathbf{v}_4)^2 \ \mathcal{W}^{bb} + 2\,(\mathbf{v}_1 - \mathbf{v}_2)^2 \mathcal{W}^{ba} \right]$$

$$\frac{1}{\tau_D} = \frac{N}{4T^2 B_2} \int \mathrm{d}\{\epsilon_i\}\mathrm{d}\{\hat{\mathbf{v}}_i\} \ (\mathbf{v}_1 - \mathbf{v}_2)(\mathbf{v}_4 - \mathbf{v}_3) \ \mathcal{W}^{ba}$$

Velocity (not momentum!) relaxation / transfer rates

close to the Dirac point

$$\frac{1}{\tau_D} \sim \alpha^2 N \frac{\mu_a \mu_b}{T}$$

$$\frac{1}{\tau_{ee}^{a,b}} \sim \alpha^2 N T$$

away from the Dirac point

$$\frac{1}{\tau_D} \sim \alpha^2 N \frac{T^2}{\mu} \ln \frac{\mu}{T}$$

$$\frac{1}{\tau_{ee}} - \frac{1}{\tau_D} \sim \frac{1}{\tau_{ee}} \frac{T^2}{\mu^2} \ll \frac{1}{\tau_{ee}}$$

Drag resistivity

Equal layers:

$$\rho_D = \frac{\hbar}{e^2}\frac{C_2}{\epsilon_0}\frac{(\tau\tau_D)^{-1} + C_1^2\left[\tau_{ee}^{-2} - \tau_D^{-2}\right]}{\tau^{-1} + C_1^2\left[\tau_{ee}^{-1} - \tau_D^{-1}\right]}$$

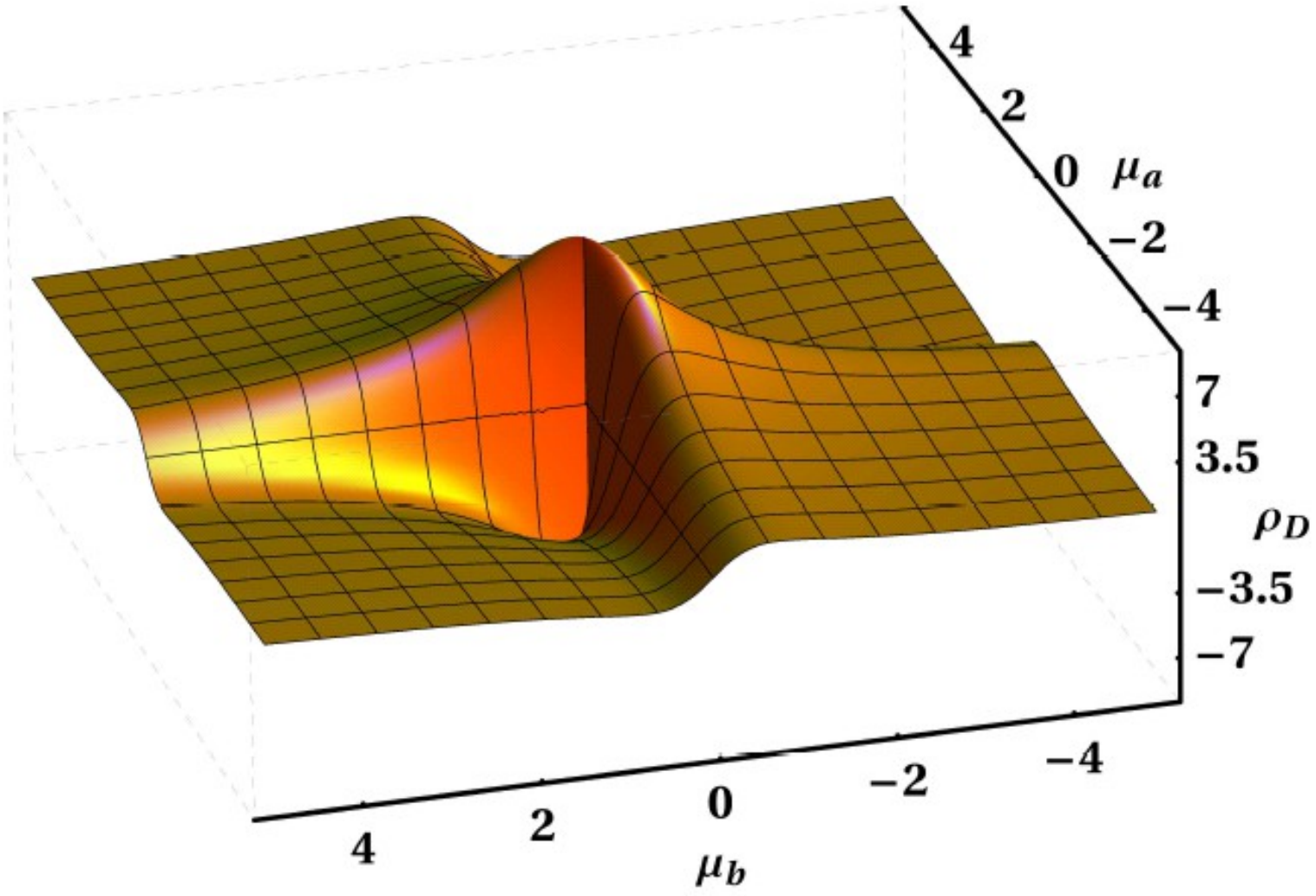
$$C_1 = \frac{\langle\epsilon\rangle_\epsilon}{T} \sim \frac{\mu}{T}, \quad C_2 = \frac{\langle\epsilon^2\rangle_\epsilon - \langle\epsilon\rangle_\epsilon^2}{T^2} \sim const,$$

Non-equal layers near the Dirac point:

$$\rho_D(\mu_i \ll T) \approx 2.87\frac{h}{e^2}\,\alpha^2\frac{\mu_1\mu_2}{\mu_1^2 + \mu_2^2 + 0.49T/(\alpha^2\tau)}$$

Finite drag at the double Dirac point in the clean case:

Fast momentum (energy current) transfer followed by the intralayer velocity relaxation due to the e-e interaction

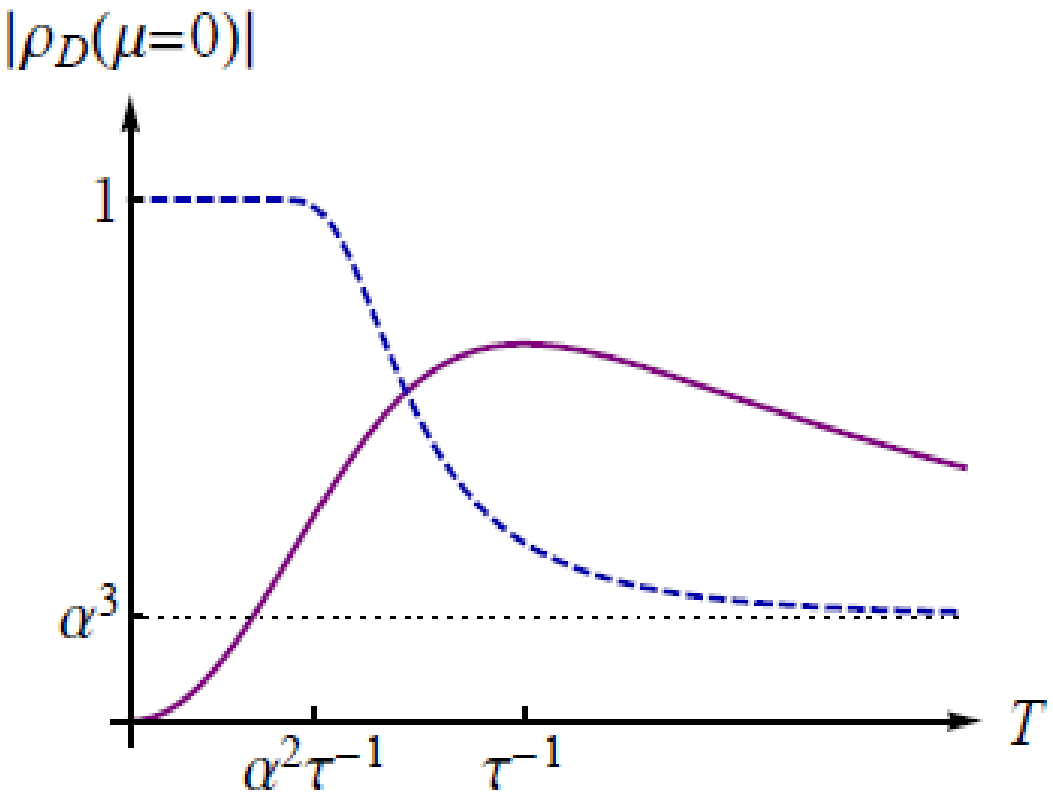

4
2
0
μ_a
−2
−4
7
3.5
ρ_D
−3.5
−7
4
2
0
−2
−4
μ_b

neutrality point
(additional correlations)

Schütt, Ostrovsky, Titov, IG, Narozhny, Mirlin, PRL (2013)

alternative mechanism: Song & Levitov, PRL (2012)

Nonzero drag at the Dirac point



Third-order interlayer scattering rate and correlated disorder

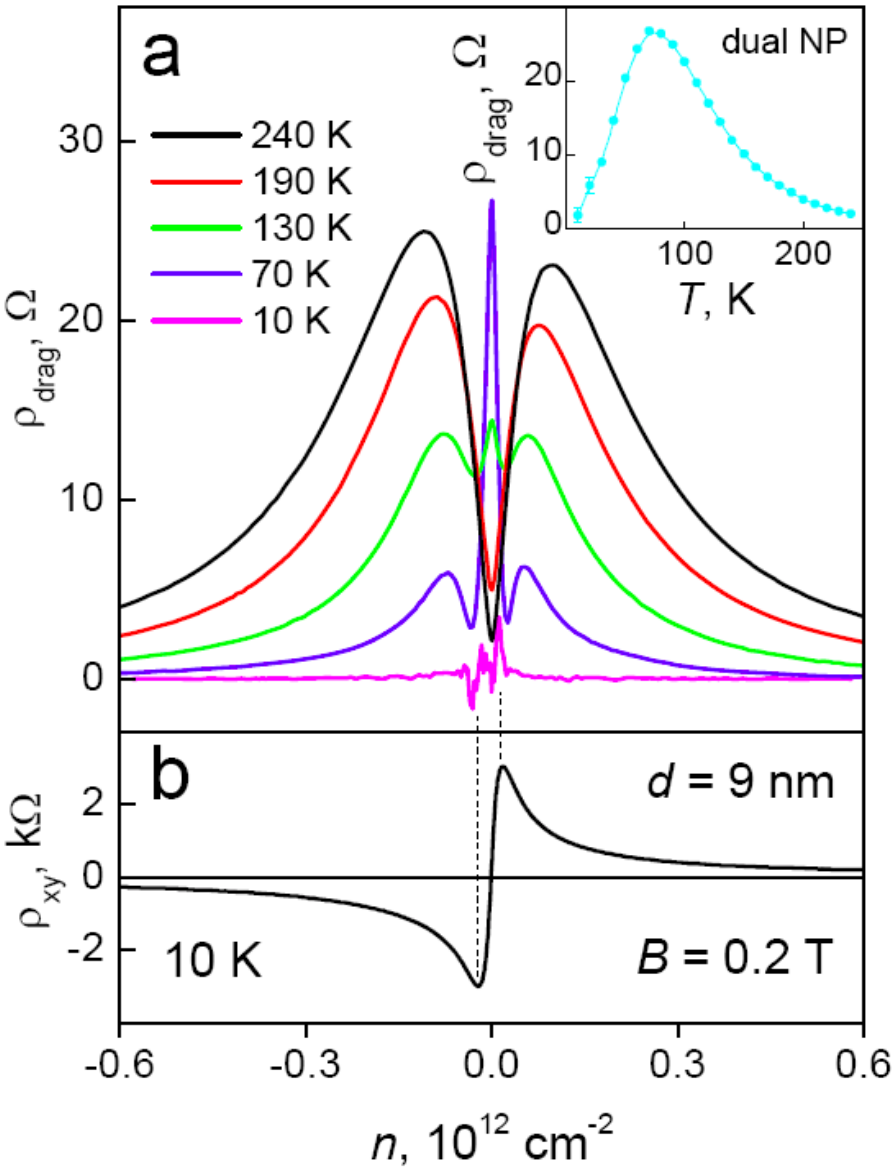
magnetodrag and Hall drag

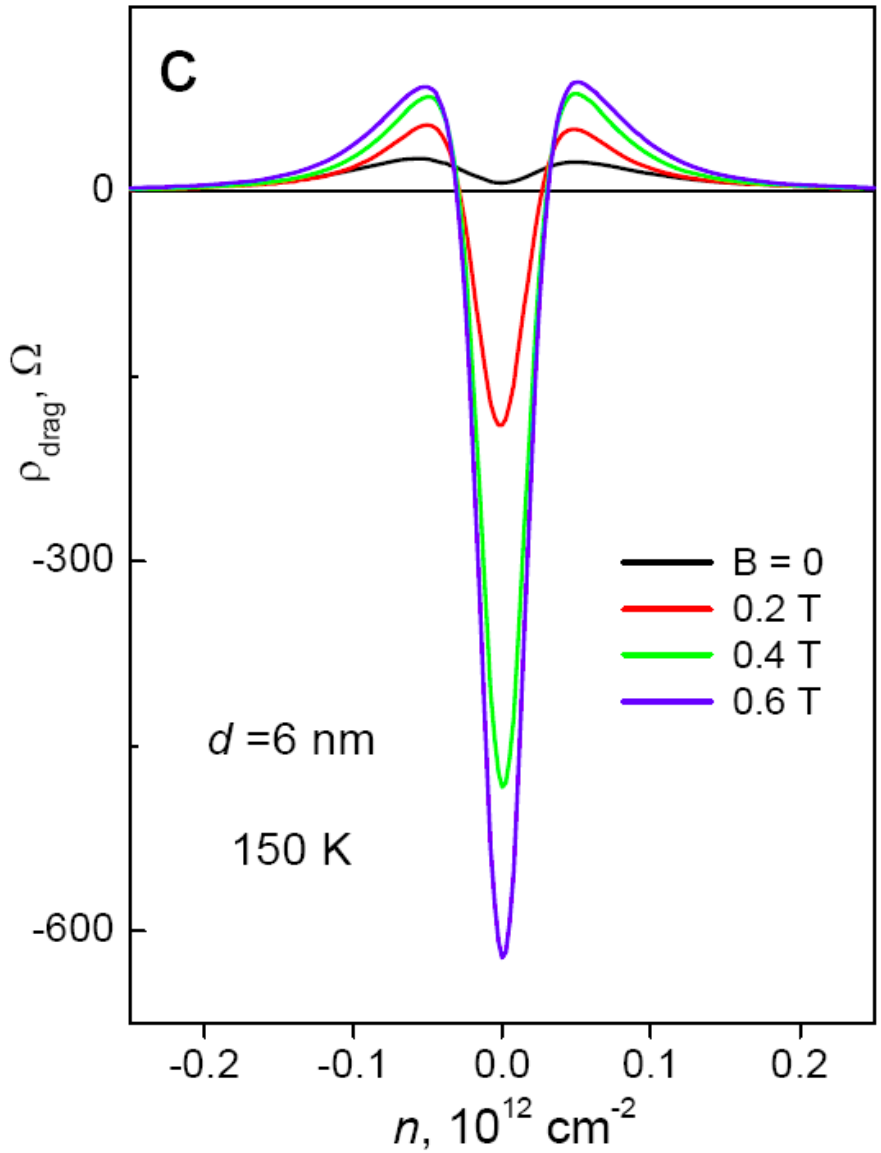
Titov et al. arXiv:1303.6264

see also Song & Levitov, arXiv:1303.3529
Song, Abanin, Levitov, arXiv:1304.1450

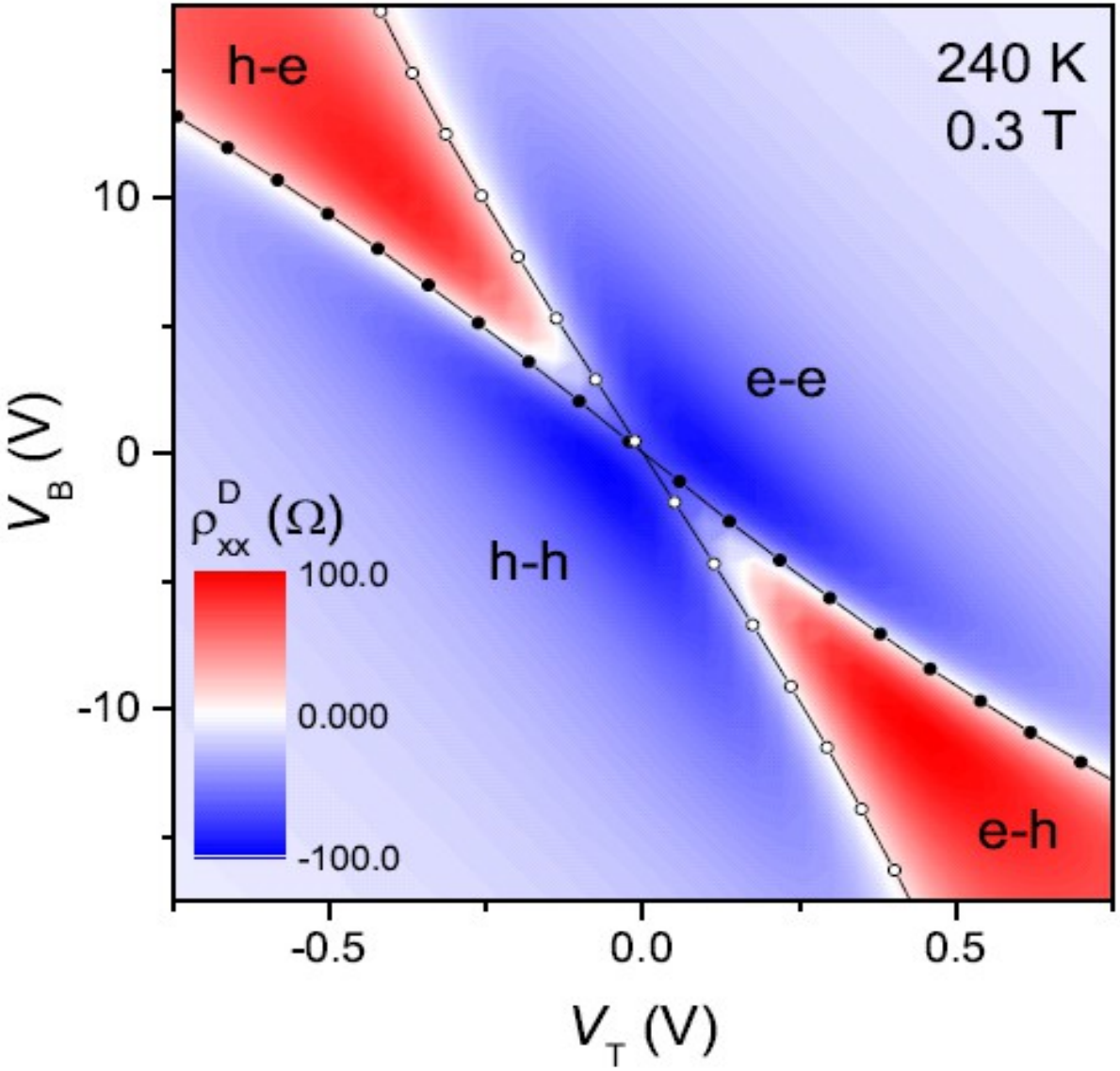
Magnetodrag

Manchester group, Nature Phys. (2012)



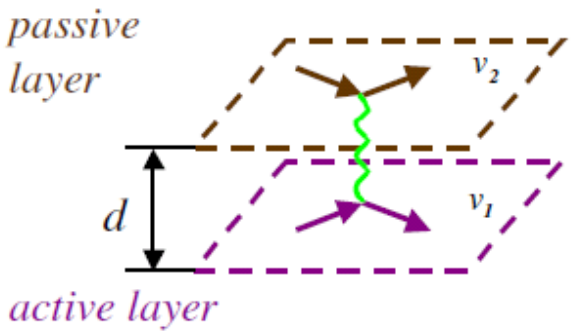


Experiment


h-e
240 K
0.3 T
e-e
h-h
e-h
ρ_{xx}^{D} (Ω)
100.0
0.000
-100.0
10
0
-10
V_B (V)
-0.5
0.0
0.5
V_T (V)

Drude-like model in magnetic field

momentum transfer rate



$$\boldsymbol{v}_2 = 0$$

$$v_1^x = \frac{j_1}{en}$$

equations of motion

- passive layer:

$$\frac{d\boldsymbol{v}_2}{dt} = \frac{e}{m}\boldsymbol{E}_2 + \frac{e}{mc}[\boldsymbol{v}_2 \times \boldsymbol{B}] - \frac{\boldsymbol{v}_2 - \boldsymbol{v}_1}{\tau_D} - \frac{\boldsymbol{v}_2}{\tau}$$

- active layer:

$$\frac{d\boldsymbol{v}_1}{dt} = \frac{e}{m}\boldsymbol{E}_1 + \frac{e}{mc}[\boldsymbol{v}_1 \times \boldsymbol{B}] - \frac{\boldsymbol{v}_1 - \boldsymbol{v}_2}{\tau_D} - \frac{\boldsymbol{v}_1}{\tau}$$

active layer

- no magneto-resistance

$$E_1^x = \frac{m}{e^2 n}\left[\frac{1}{\tau} + \frac{1}{\tau_D}\right] j_1^x \quad \Rightarrow \quad \rho_{xx}^{11} \simeq \frac{m}{e^2 n \tau}$$

- classical Hall effect

$$E_1^y = \frac{B}{nec} j_1^x \quad \Rightarrow \quad \rho_{yx}^{11} = \frac{B}{nec}$$

passive layer

- no magnetodrag

$$E_2^x = \frac{m}{e^2 n \tau_D} j_1^x \quad \Rightarrow \quad \rho_{xx}^{12} = \frac{m}{e^2 n \tau_D}$$

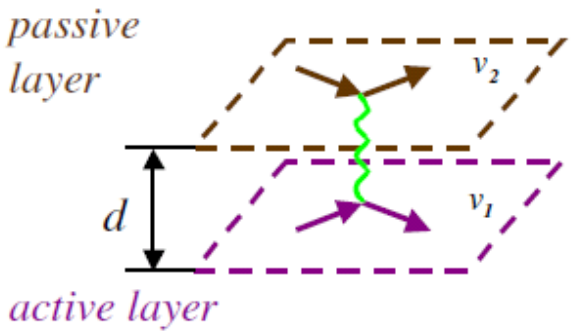
- no Hall drag

$$E_2^y = 0$$

No effect in a single-band model!

Drude-like model in magnetic field

momentum transfer rate



$$\boldsymbol{v}_2 = 0$$

$$v_1^x = \frac{j_1}{en}$$

equations of motion

- passive layer:

$$\frac{d\boldsymbol{v}_2}{dt} = \frac{e}{m}\boldsymbol{E}_2 + \frac{e}{mc}[\boldsymbol{v}_2 \times \boldsymbol{B}] - \frac{\boldsymbol{v}_2 - \boldsymbol{v}_1}{\tau_D} - \frac{\boldsymbol{v}_2}{\tau}$$

- active layer:

$$\frac{d\boldsymbol{v}_1}{dt} = \frac{e}{m}\boldsymbol{E}_1 + \frac{e}{mc}[\boldsymbol{v}_1 \times \boldsymbol{B}] - \frac{\boldsymbol{v}_1 - \boldsymbol{v}_2}{\tau_D} - \frac{\boldsymbol{v}_1}{\tau}$$

active layer

- no magneto-resistance

$$E_1^x = \frac{m}{e^2 n}\left[\frac{1}{\tau} + \frac{1}{\tau_D}\right] j_1^x \quad \Rightarrow \quad \rho_{xx}^{11} \simeq \frac{m}{e^2 n \tau}$$

- classical Hall effect

$$E_1^y = \frac{B}{nec} j_1^x \quad \Rightarrow \quad \rho_{yx}^{11} = \frac{B}{nec}$$

passive layer

- no magnetodrag

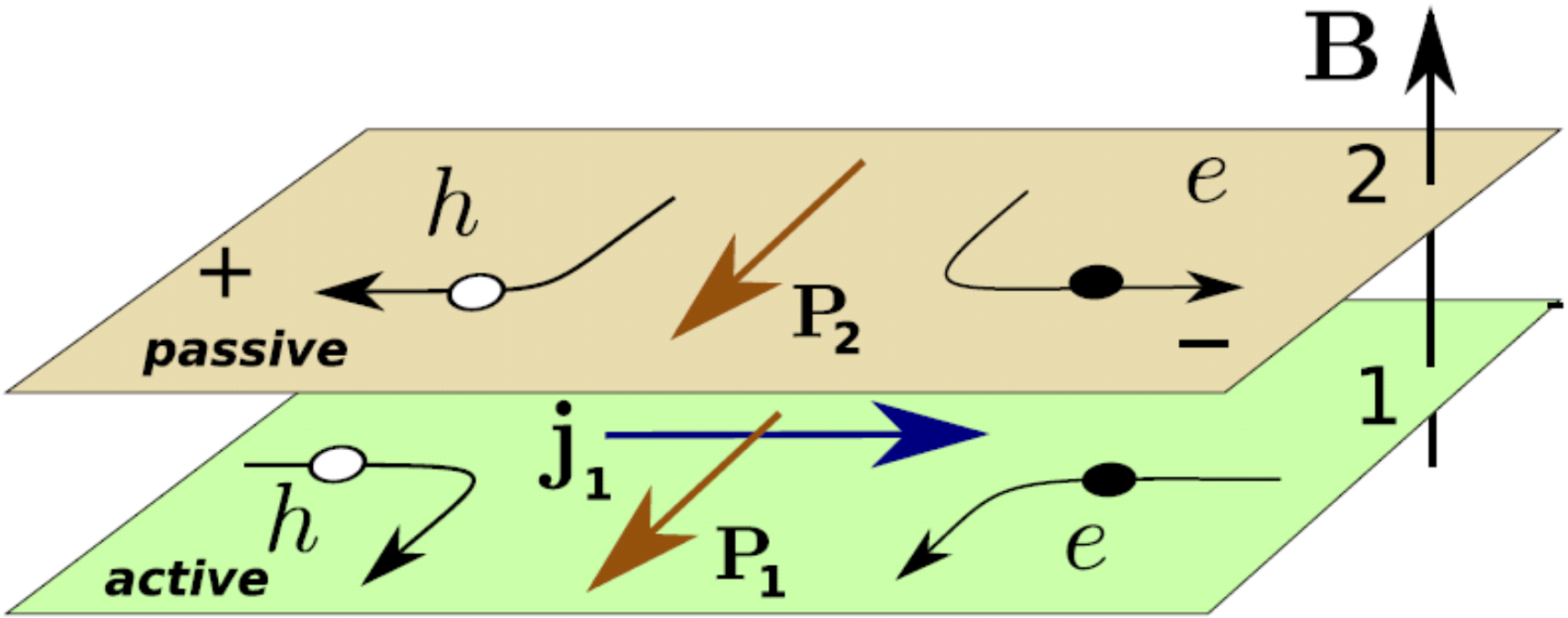
$$E_2^x = \frac{m}{e^2 n \tau_D} j_1^x \quad \Rightarrow \quad \rho_{xx}^{12} = \frac{m}{e^2 n \tau_D}$$

- no Hall drag

$$E_2^y = 0$$

No effect in a single-band model!

Why is drag finite at the double Dirac point?



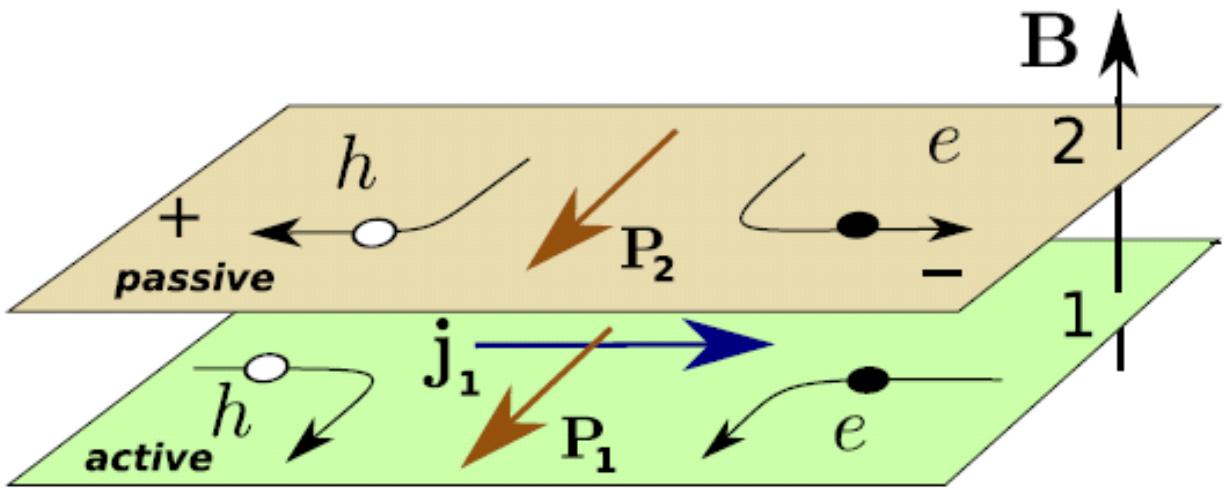
infinite sample

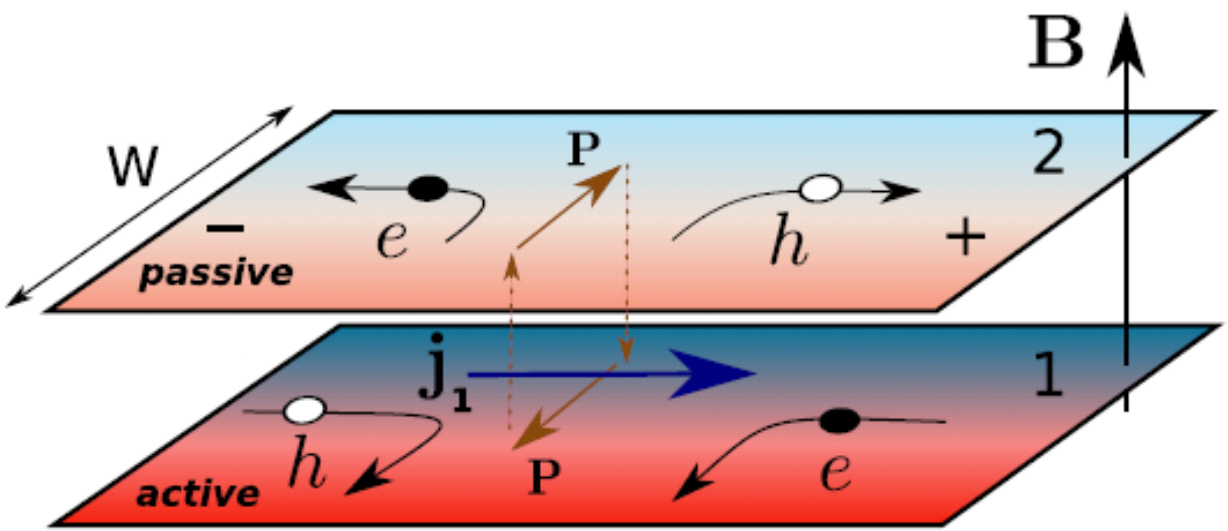
Four fluids (e1, h1 and e2, h2)

Opposite Lorentz forces for electrons and holes: charge and quasiparticle currents noncollinear

``macroscopic'' vs ``mesoscopic'' samples:

energy-escape length (phonons) important





Drude-like description (infinite sample)

$$
\begin{aligned}
e\boldsymbol{E}_1 + e[\boldsymbol{v}_{1e} \times \boldsymbol{B}] &= \boldsymbol{F}_{1e} + e\,\boldsymbol{v}_{1e}/M_1, \\
-e\boldsymbol{E}_1 - e[\boldsymbol{v}_{1h} \times \boldsymbol{B}] &= \boldsymbol{F}_{1h} + e\,\boldsymbol{v}_{1h}/M_1, \\
e\boldsymbol{E}_2 + e[\boldsymbol{v}_{2e} \times \boldsymbol{B}] &= \boldsymbol{F}_{2e} + e\,\boldsymbol{v}_{2e}/M_2, \\
-e\boldsymbol{E}_2 - e[\boldsymbol{v}_{2h} \times \boldsymbol{B}] &= \boldsymbol{F}_{2h} + e\,\boldsymbol{v}_{2h}/M_2.
\end{aligned}
$$

$M =$ mobility

$F =$ friction force

$$
\rho_{xx}^{D} = \frac{\hbar\gamma}{e^2}\,\frac{B^2 M_1 M_2}{1 + \hbar\gamma\,\rho_0(M_1 + M_2)/e}, \qquad n_i = 0
$$

positive drag

Drude-like description (finite sample)

$$\boldsymbol{j}_i = e(n_{ie}\boldsymbol{v}_{ie} - n_{ih}\boldsymbol{v}_{ih}), \qquad \boldsymbol{P}_i = n_{ie}\boldsymbol{v}_{ie} + n_{ih}\boldsymbol{v}_{ih}$$

$$n_i = n_{ie} - n_{ih} \qquad \rho_i = n_{ie} + n_{ih}$$

$$\boldsymbol{F}_{1a} = -\boldsymbol{F}_{2a} = \hbar\gamma\,(\boldsymbol{P}_1 - \boldsymbol{P}_2)$$

$$\begin{aligned} -K_1\boldsymbol{\nabla}\rho_1 + en_1\boldsymbol{E}_1 + [\boldsymbol{j}_1 \times \boldsymbol{B}] &= \rho_1\boldsymbol{F}_1 + e\boldsymbol{P}_1/M_1, \\ e\rho_1\boldsymbol{E}_1 + e[\boldsymbol{P}_1 \times \boldsymbol{B}] &= n_1\boldsymbol{F}_1 + \boldsymbol{j}_1/M_1, \\ \boldsymbol{\nabla}\boldsymbol{P}_1 = -(\rho_1 - \rho_0)/\tau_{\rm ph} &- (\rho_1 - \rho_2)/(2\tau_Q). \end{aligned}$$

energy loss due to phonons

quasiparticle imbalance relaxation

$$K_i = (\pi\hbar^2 v^2/2)(\partial n_i/\partial\mu_i) = 2T\ln(2\cosh\mu_i/2T)$$

Drude-like description (finite sample)

solution at charge neutrality

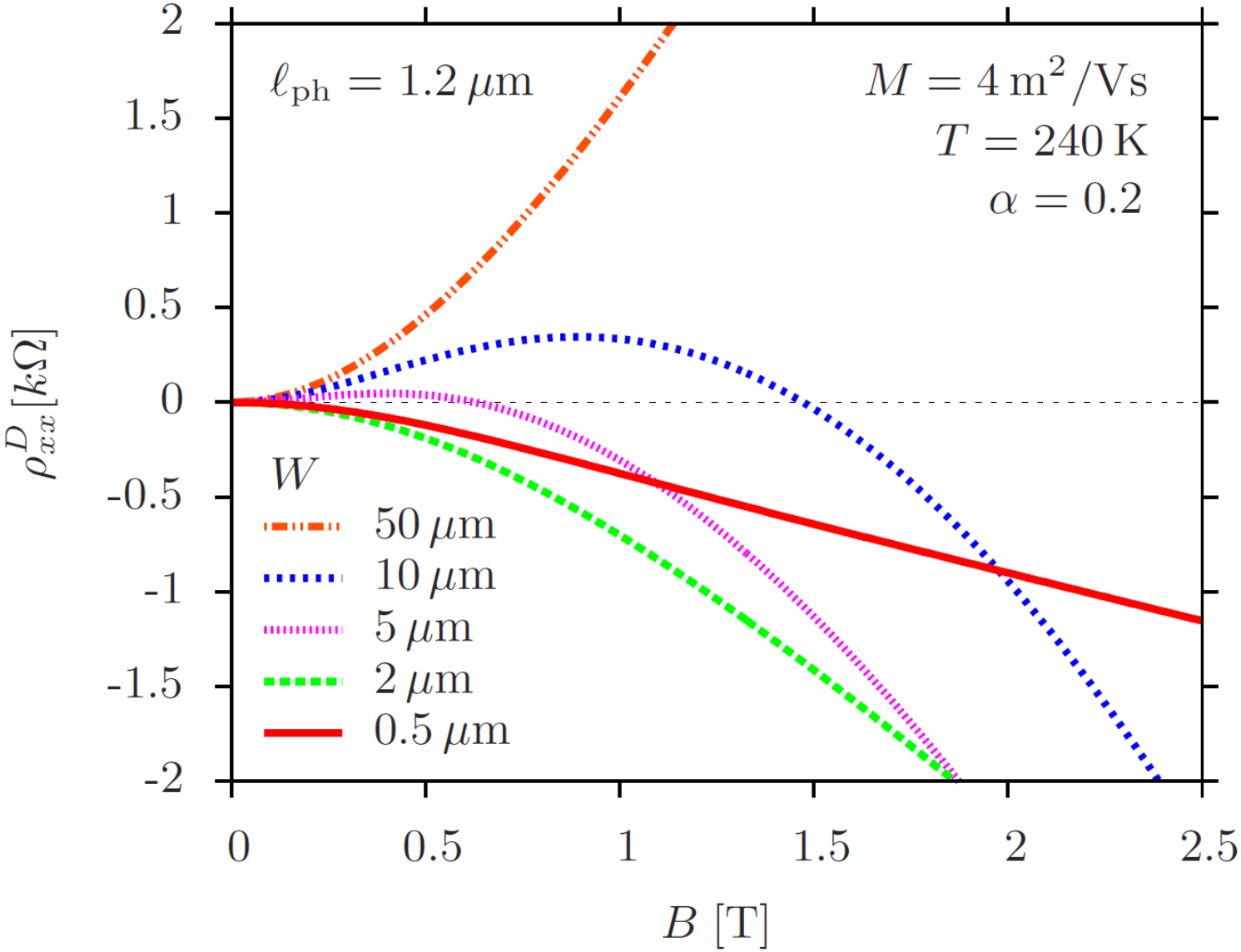
$$\rho_{xx}^{D} = \frac{\rho_{xx}^{(0)}}{2}\left[F(0,0) - F\left(\frac{\hbar\gamma}{e^2\rho_{xx}^{0}}, \frac{\tau_{\rm ph}}{\tau_{\rm Q}}\right)\right],$$

$$F(X,Y) = \frac{1+2X+(MB)^2}{1+2X+(MB)^2\frac{\tanh\Theta(X,Y)}{\Theta(X,Y)}},$$

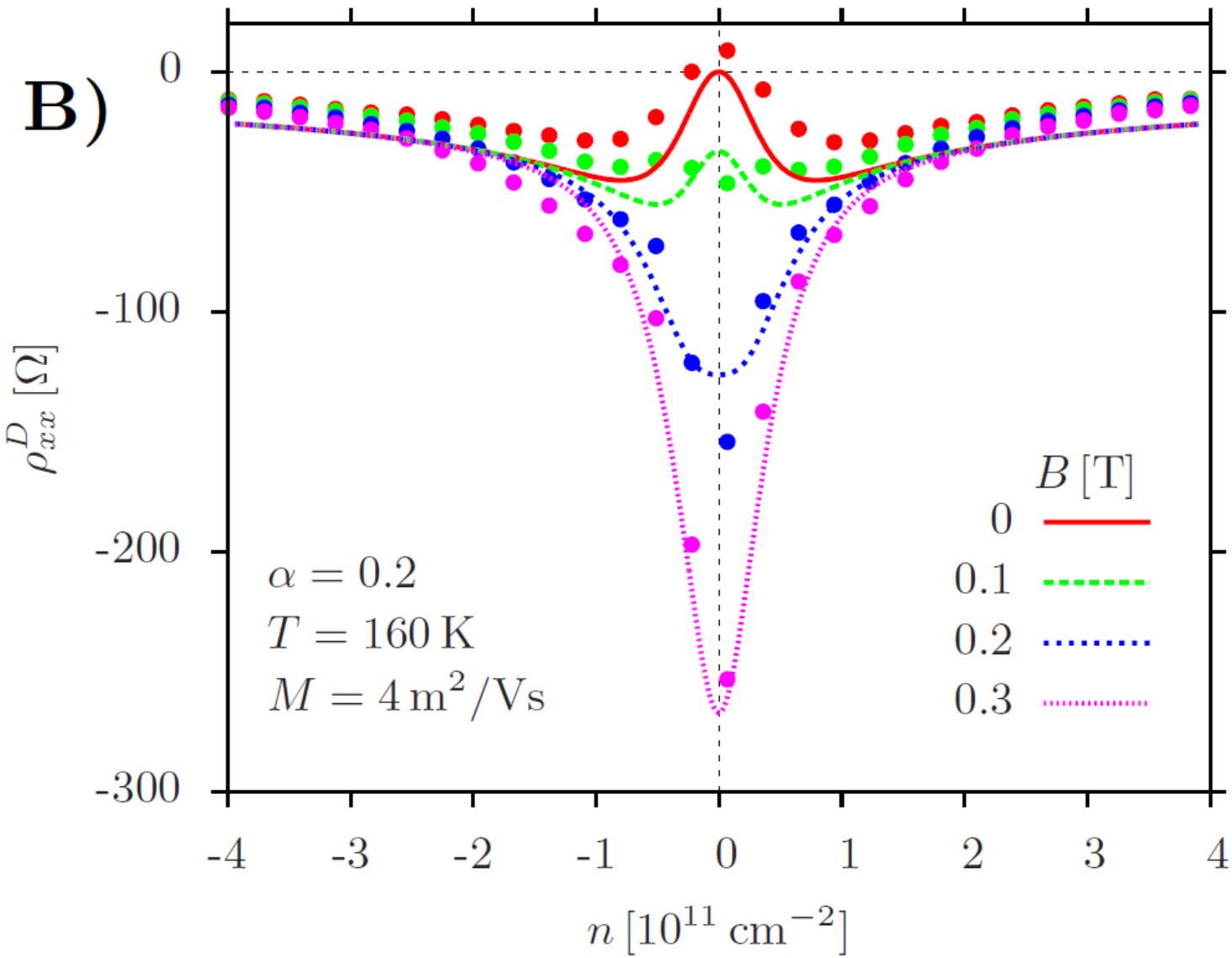
$$\Theta(X,Y) = \frac{W}{2\ell_{\rm ph}}\sqrt{(1+2X+(MB)^2)(1+Y)},$$

- drag sign depends on energy relaxation and sample geometry
- positive drag in “wide” samples
- ***negative drag in “narrow” samples***

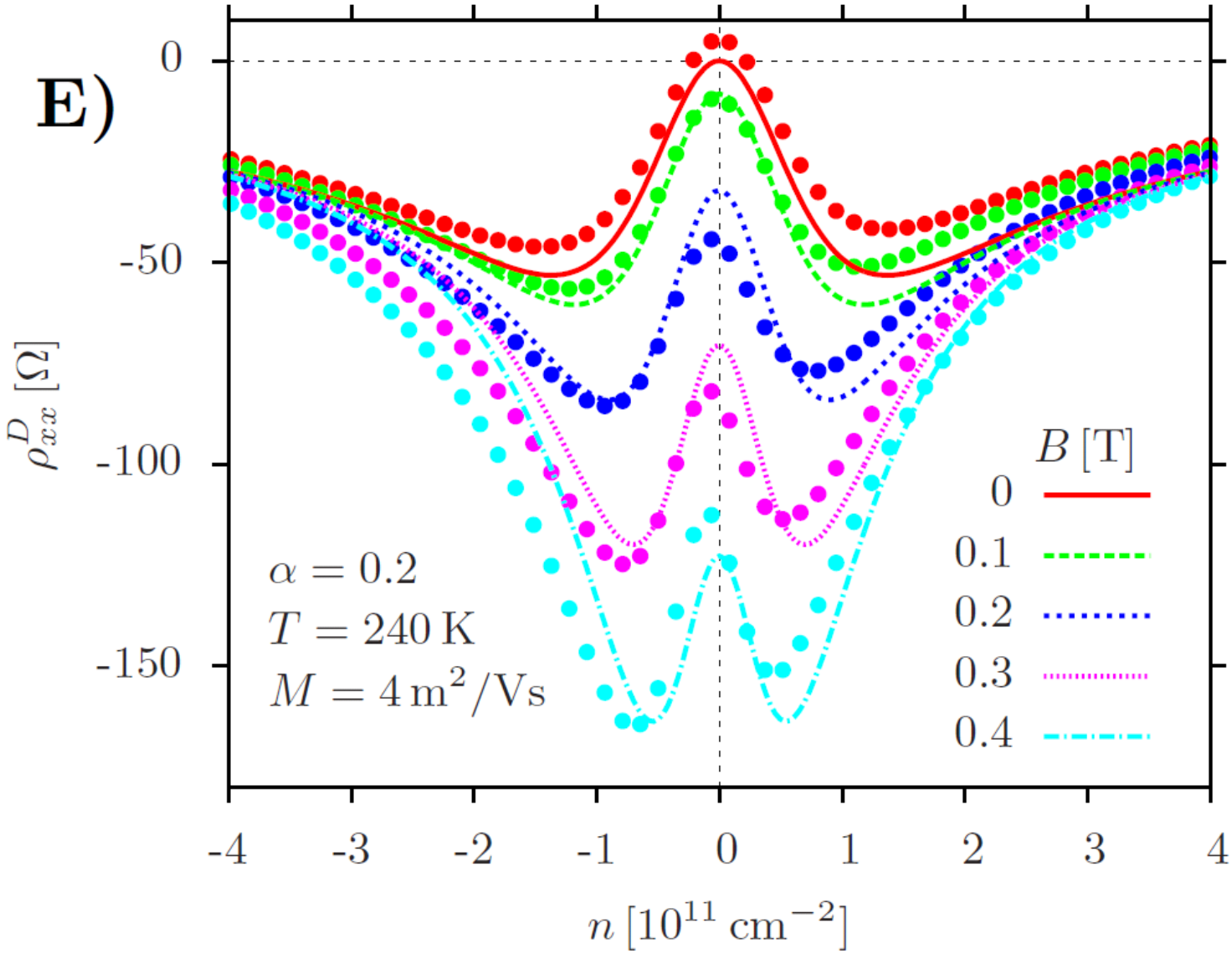
Magnetodrag at the double Dirac point



Magnetodrag: experiment vs theory

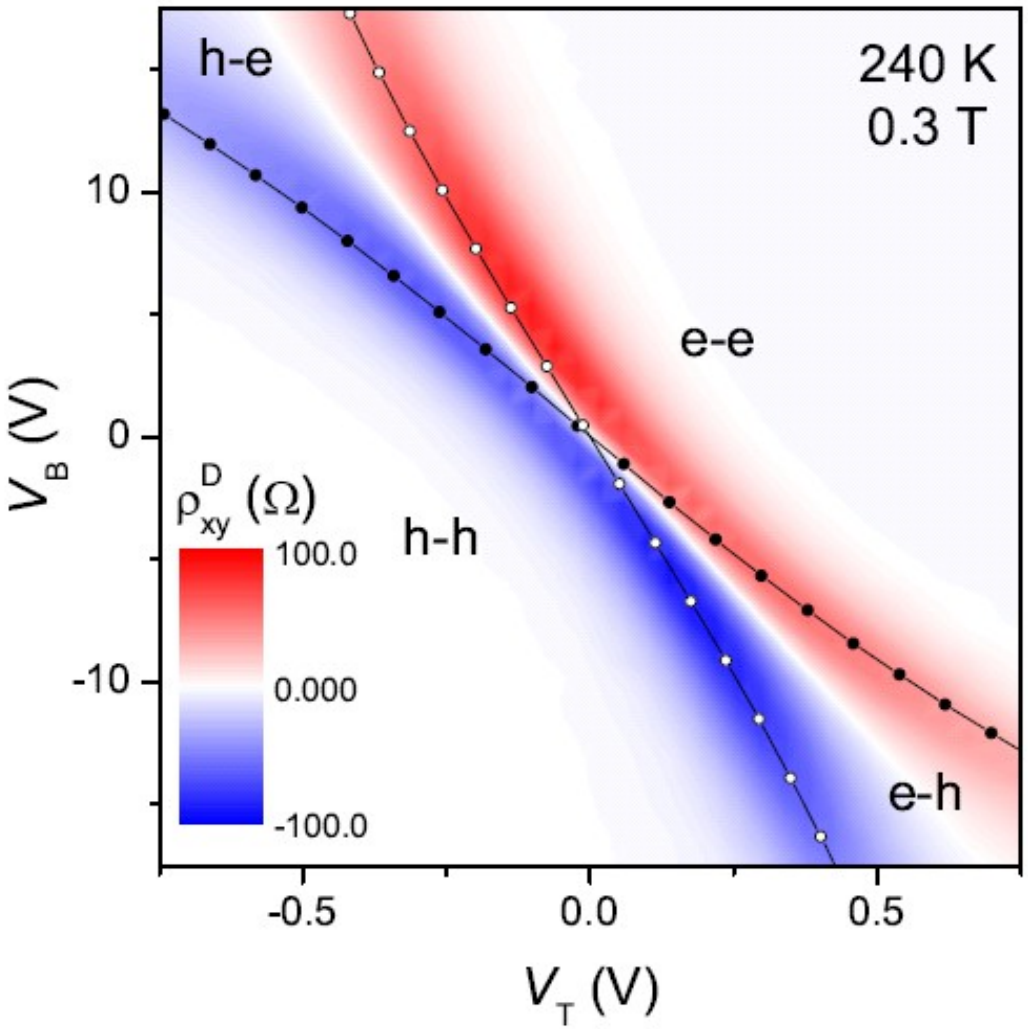


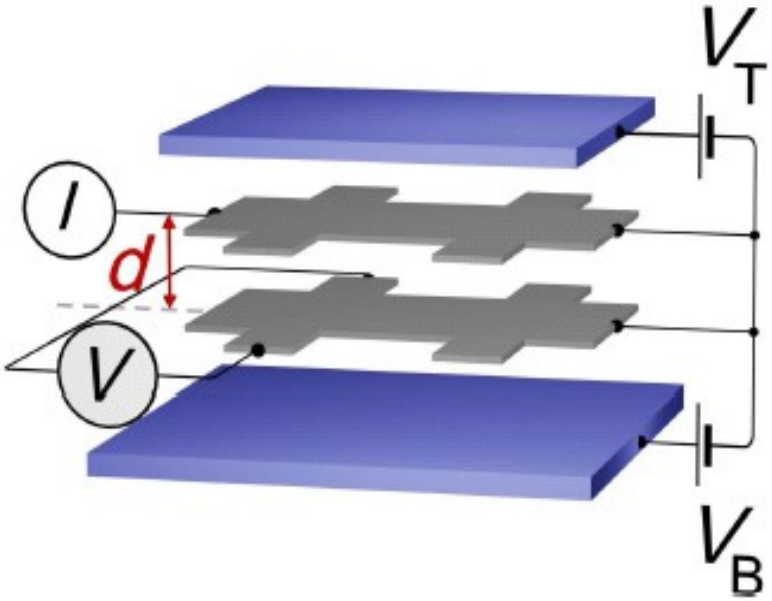
Magnetodrag: experiment vs theory



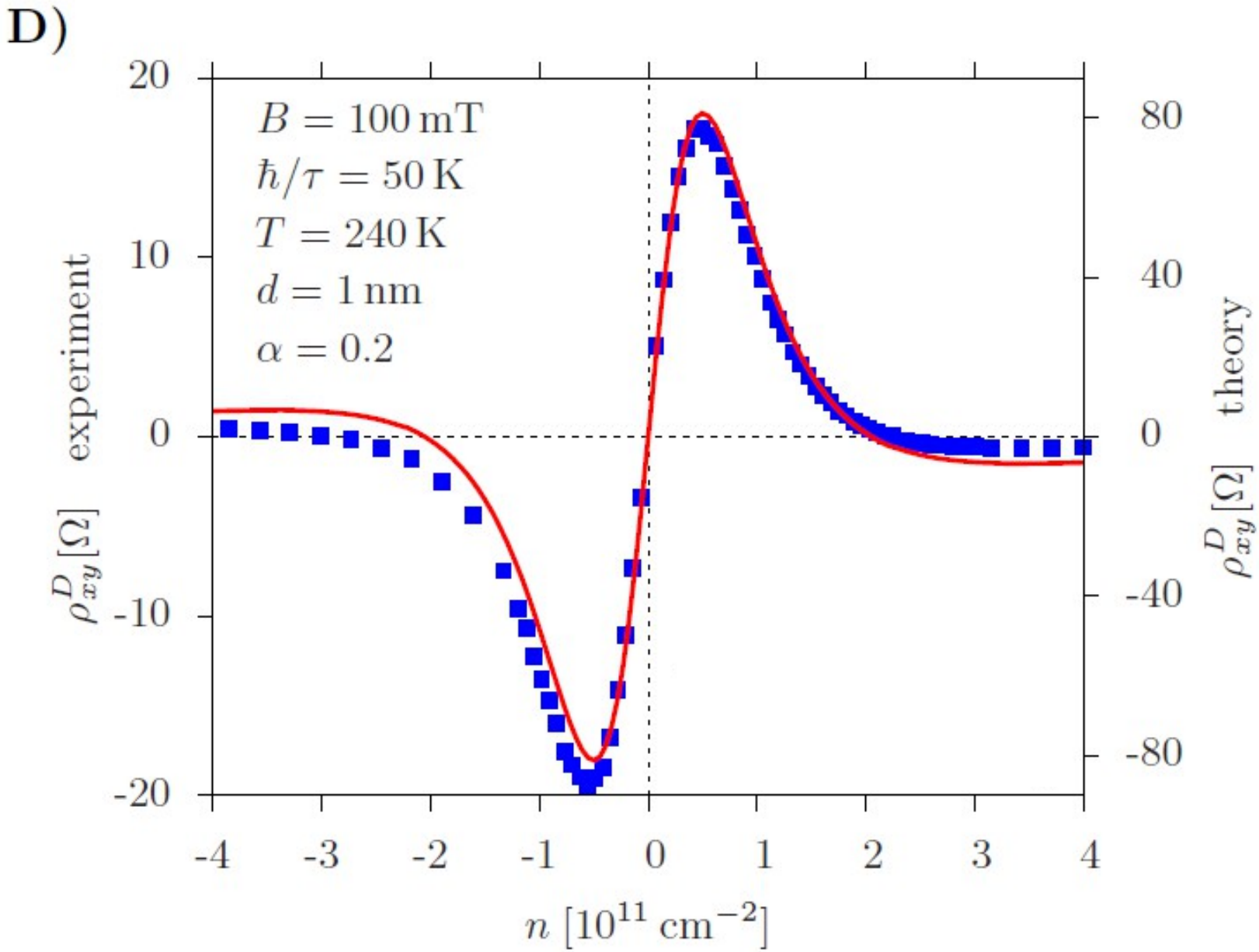
Hall drag

- non-zero in two-band systems, but
  - vanishes at charge neutrality and in the case of oppositely doped layers
- non-trivial density dependence





Hall drag: experiment vs theory



Summary

Coulomb drag in graphene:

- Perturbation theory (disordered graphene)

- Kinetic theory in clean graphene
  (equilibrated drag)

- Peak at the Dirac point
  3rd order drag, drag with correlated disorder

- Giant magnetodrag at the Dirac point
  (four liquid model: positive & negative drag)