

# New results in theory of Ferromagnet/Superconductor hybrid nanostructures

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In collaboration with

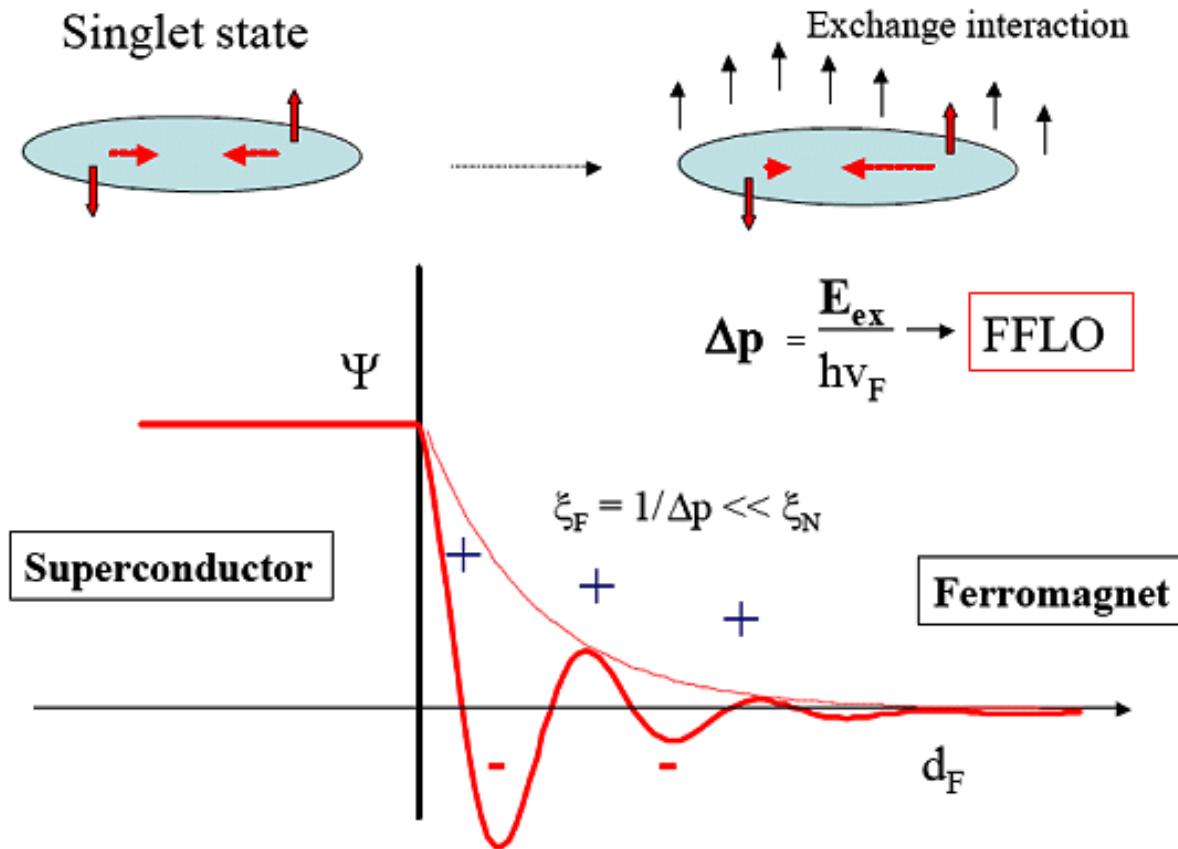
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S. Kawabata (AIST, Tsukuba, Japan)

M.Yu. Kupriyanov (MSU, Russia)

Y. Tanaka and D. Yoshizaki (Nagoya University, Japan)

# *Proximity effect in FS structures*



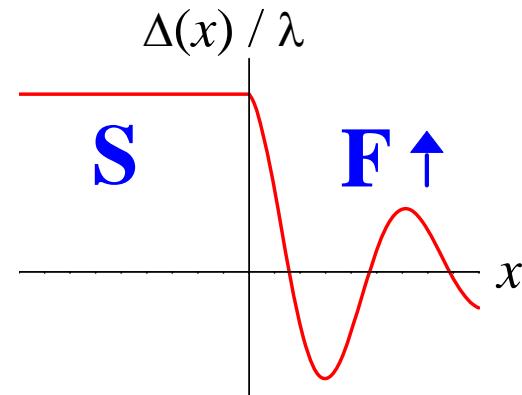
diffusive decay length

$$\xi_F = \sqrt{\frac{\hbar D_F}{E_{ex}}}$$

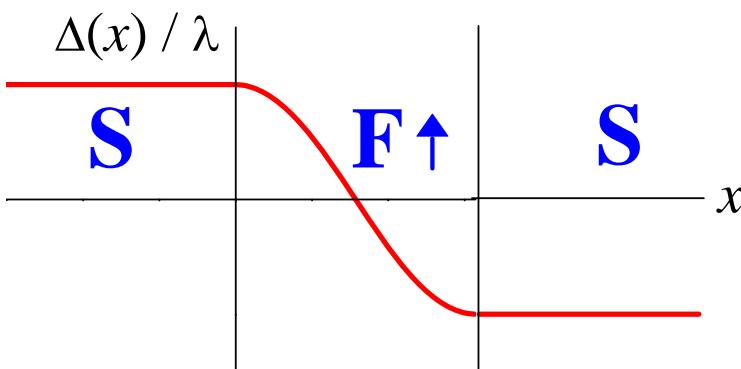
# '0' and ' $\pi$ ' Josephson Junctions

0 junction :  $I = I_C \sin \varphi, \quad I_C > 0$

$\pi$  junction :  $I = -I_C \sin \varphi = I_C \sin(\varphi + \pi)$

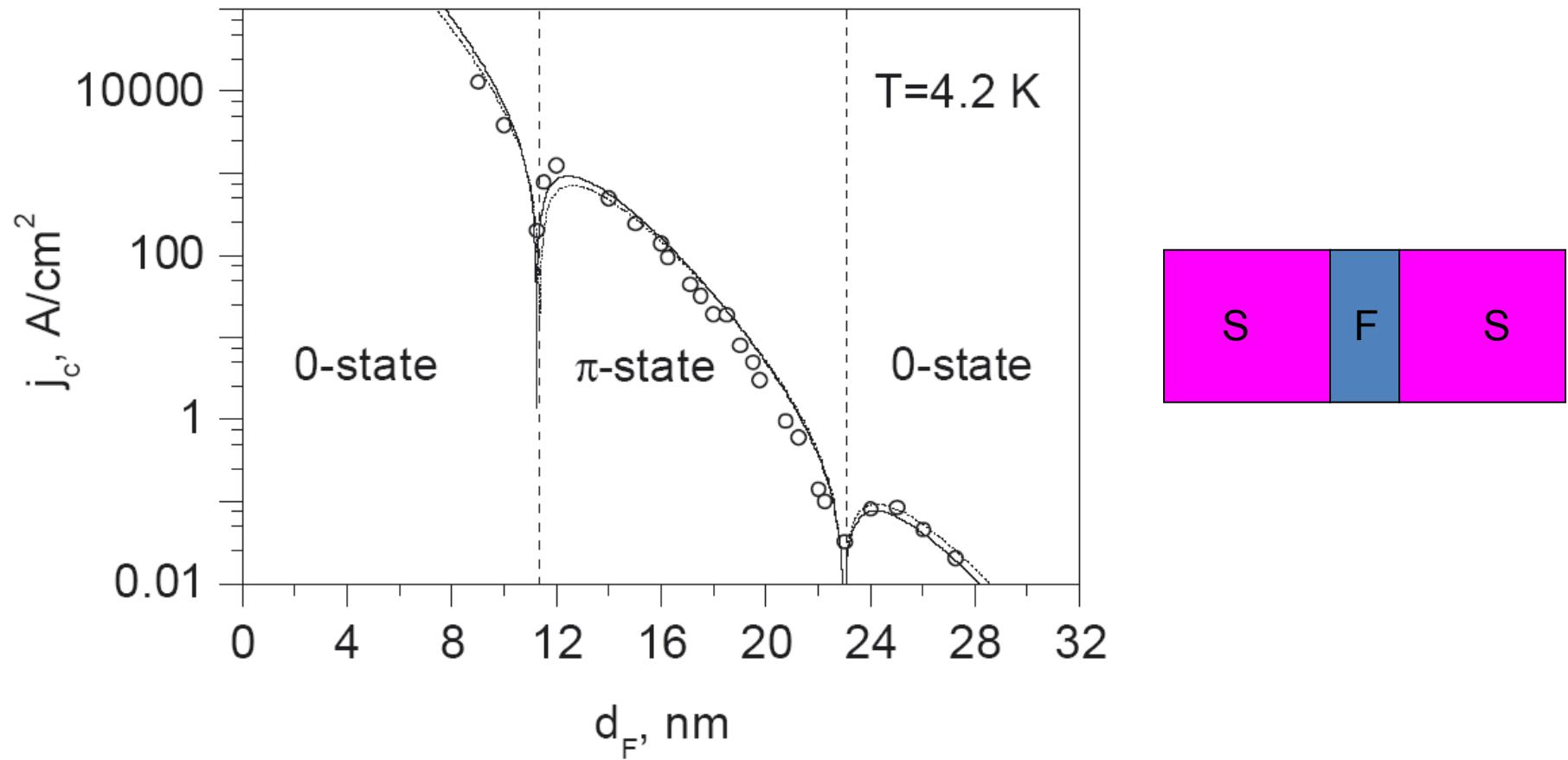


— spatial oscillations of the order parameter in the F layer



— thickness of F layer equals half wave-length of oscillations

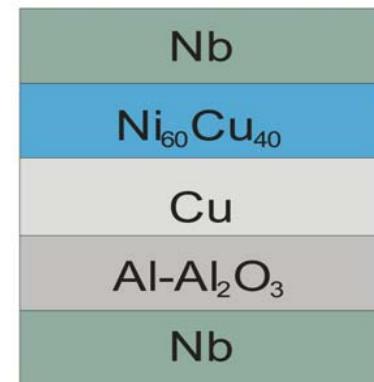
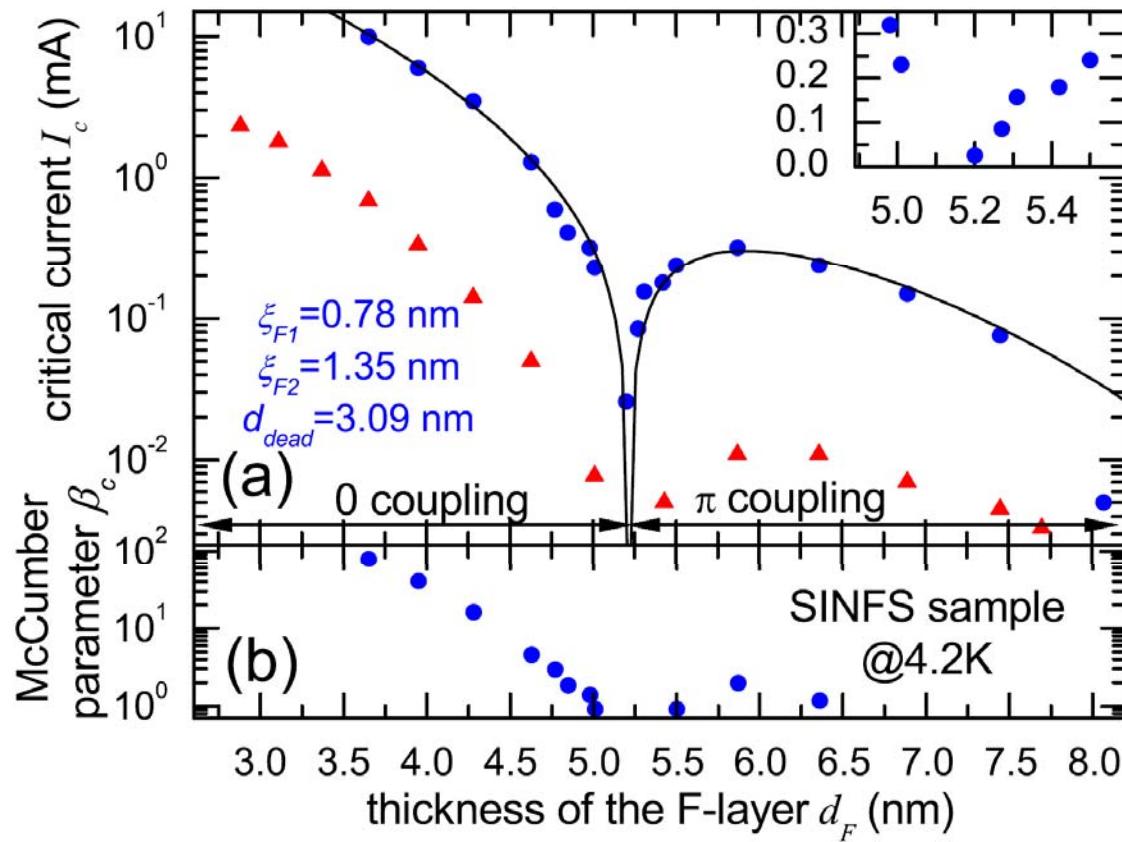
# *SFS $\pi$ -Junctions: experiment*



Oboznov et al., PRL, **96**, 197003 (2006)

# *SIFS $\pi$ -Junctions: experiment*

Weides et al., APL, **89**, 122511 (2006)



# Critical current oscillations

A.S. Vasenko, S. Kawabata, A.A. Golubov, M.Yu. Kupriyanov, and M. Weides  
 PRB 77, 134507 (2008); PRB 84, 024524 (2011)

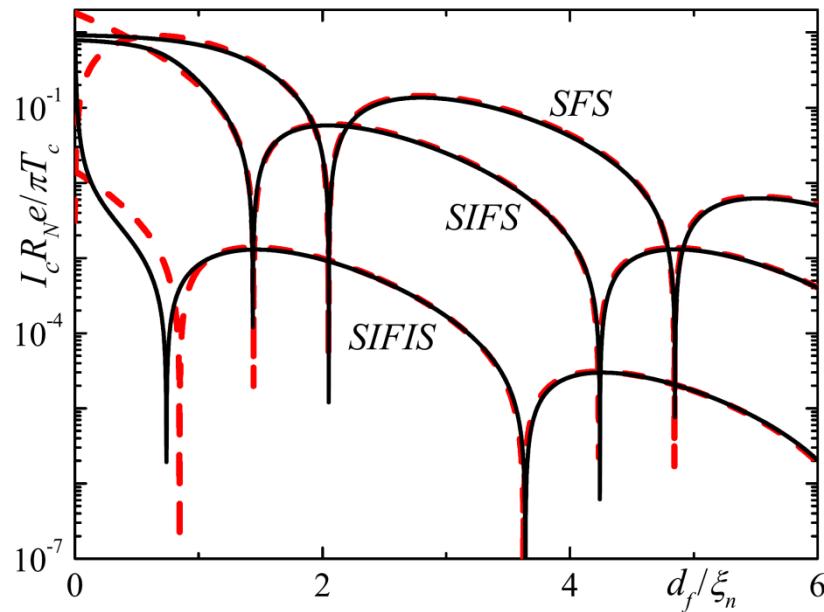


FIG. 2: (Color online) The F layer thickness dependence of the critical current for SFS ( $\gamma_{B1,2} = 0$ ), SIFS ( $\gamma_{B1} = 10^2$ ,  $\gamma_{B2} = 0$ ) and SIFIS ( $\gamma_{B1,2} = 10^2$ ) junctions in the absence of spin-flip scattering. Red dashed lines correspond to the modulus of the analytical results (31),(25) and (29) and black solid lines correspond to the result of the numerical calculation in Sec. IV,  $h = 3\pi T_c$ ,  $T = 0.5T_c$ .

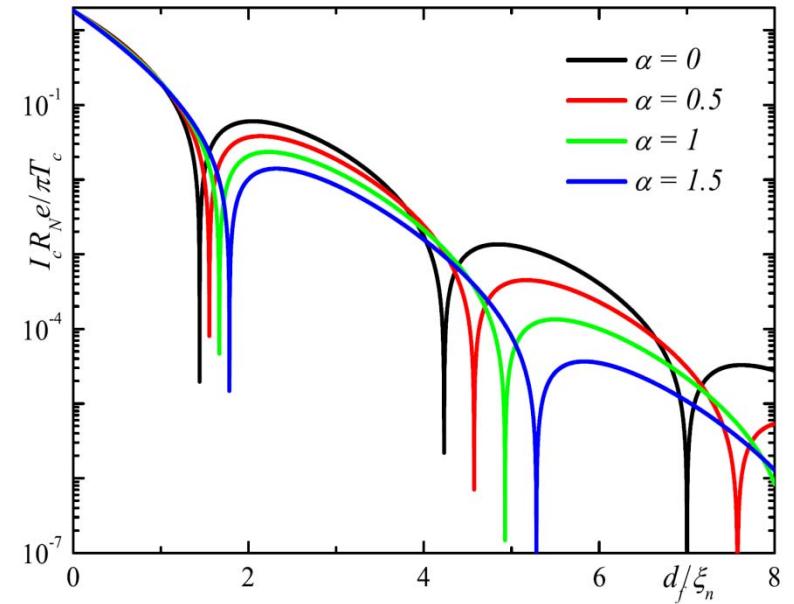


FIG. 3: (Color online) The F layer thickness dependence of the critical current in the SIFS junction [modulus of the Eq. (14)] for different values of  $\alpha = 1/\pi T_c \tau_m$ ,  $h = 3\pi T_c$ ,  $T = 0.5T_c$ .

## Weides et al. experimental data fitting

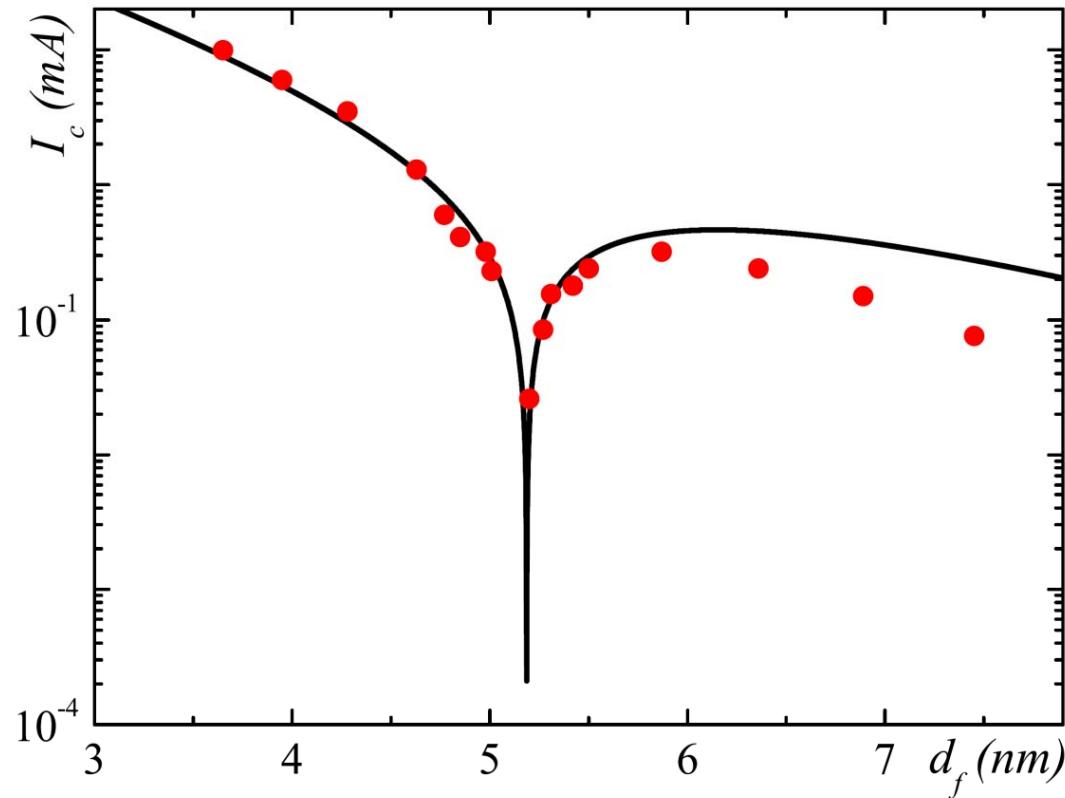
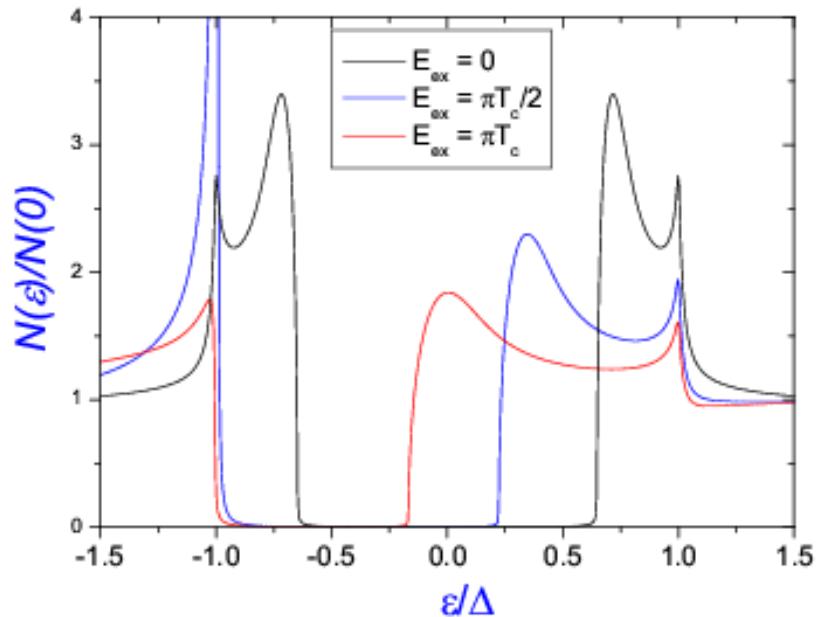
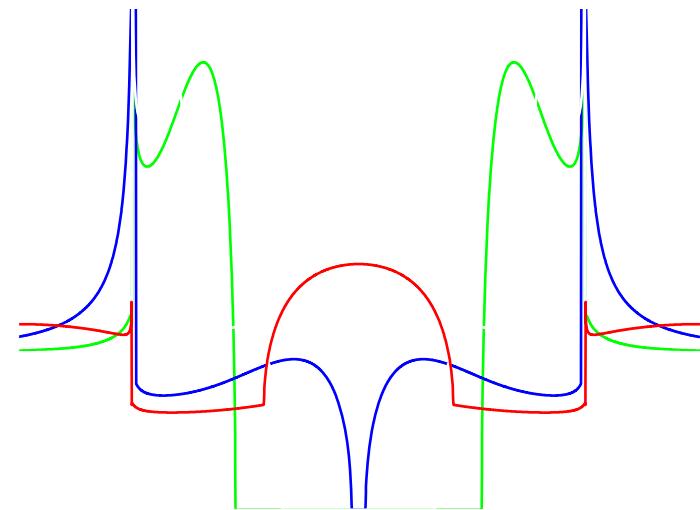


FIG. 5: (Color online) Fit to the experimental data from Ref. 14 for the critical current in a Nb/Al<sub>2</sub>O<sub>3</sub>/Ni<sub>0.6</sub>Cu<sub>0.4</sub>/Nb junction. The fitting parameters are:  $h/k_B = 950\text{ K}$  and  $1/\tau_m = 1.6\text{ h}$ .

# DOS in SF junction: typical behavior



Spin-resolved DoS:  
Zeeman shift of DoS peaks



Total DoS: the sum over both spin subbands.  
Zero energy DoS oscillates as a function of exchange field

# DOS oscillations at Fermi energy

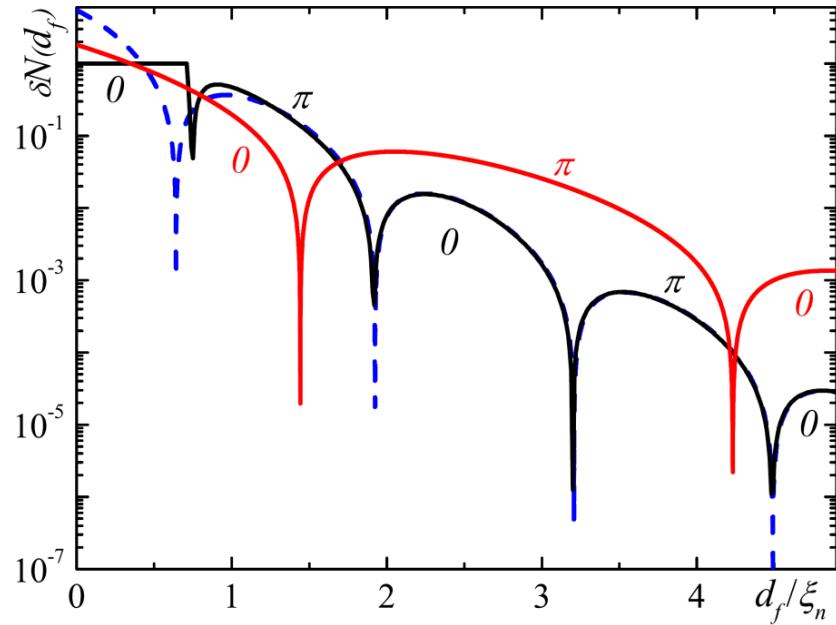
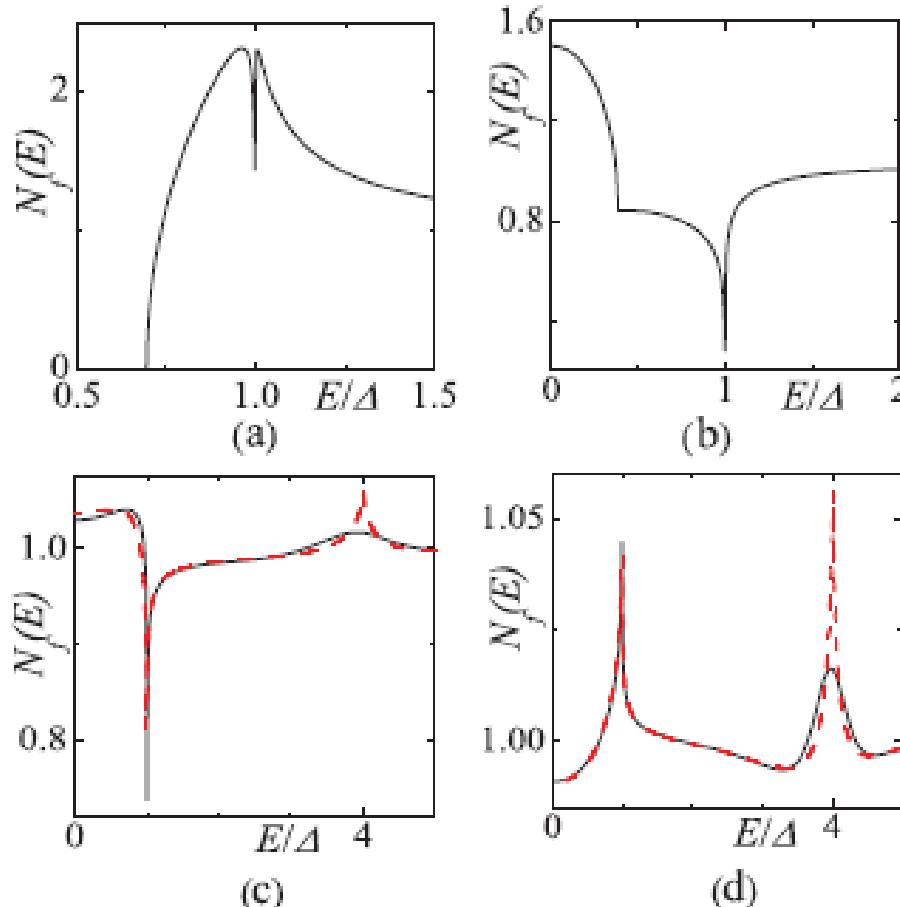


FIG. 8: (Color online) The F-layer dependence of the function  $\delta N(d_f)$  in the absence of spin-flip scattering,  $h = 3\pi T_c$ ,  $T = 0.5T_c$ . Black solid line is a result of the numerical calculation; blue dashed line is calculated with the use of Eq. (41). Red line shows normalized critical current for SIFS junction. Zero and  $\pi$  states defined from  $I_c$  are indicated by red color, while zero and  $\pi$  states defined from DOS are indicated by black color.

$$\delta N(d_f) = |1 - N_0|, \quad N_0 = N(E = 0)$$

$$\delta N(d_f) \approx \frac{32}{3 + 2\sqrt{2}} \left| \exp\left(-\frac{2d_f}{\xi_{f1}}\right) \cos\left(\frac{2d_f}{\xi_{f2}}\right) \right|$$

# DOS in SF electrode of SIFS junctions



$d_f = 0.5(a), 1(b), 2(c), 3(d)$

$$N(E) = [N_{\uparrow}(E) + N_{\downarrow}(E)] / 2,$$

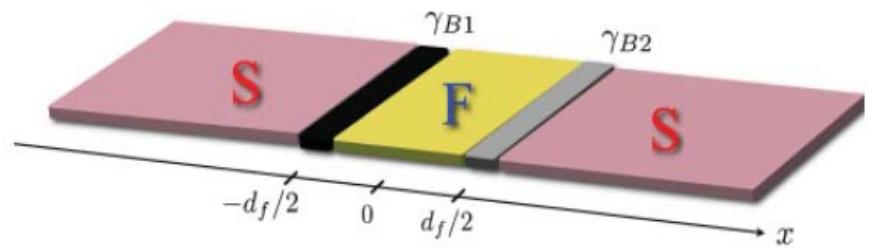
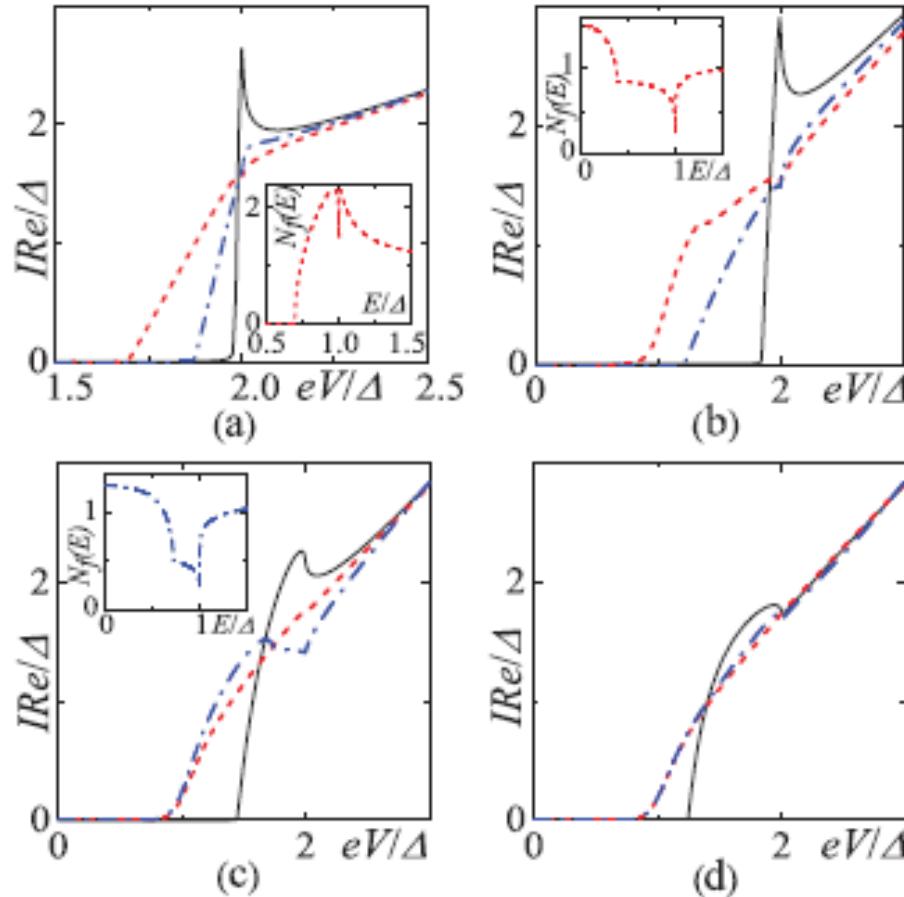
$$N_{\uparrow(\downarrow)}(E) = \text{Re} [\cos \theta_{\uparrow(\downarrow)}(i\omega \rightarrow E + i0)]$$

Peak at  $E=h$  occurs for large F-layer thicknesses, in accordance with the formula

$$N_f \approx 1 - \text{Re} \sum_{\uparrow, \downarrow} \frac{16F^2(E) \exp(-p \frac{2d_f}{\xi_f})}{(\sqrt{(1-\eta^2)F^2(E)+1}+1)^2}$$

**Detailed discussion in the talk by Andrey Vasenko 23-OR-G8**

# I-V characteristics of SIFS junctions

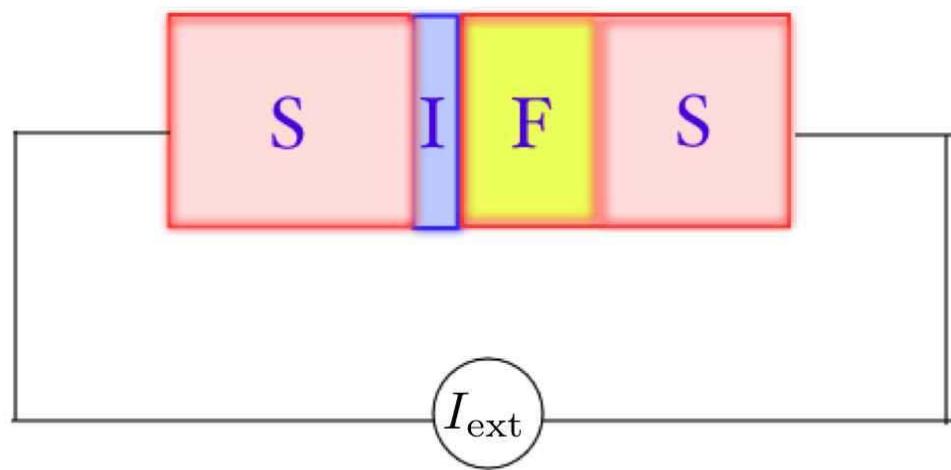


small quasiparticle current  
at low bias => low dissipation

Detailed discussion in the talk by Andrey Vasenko 23-OR-G8

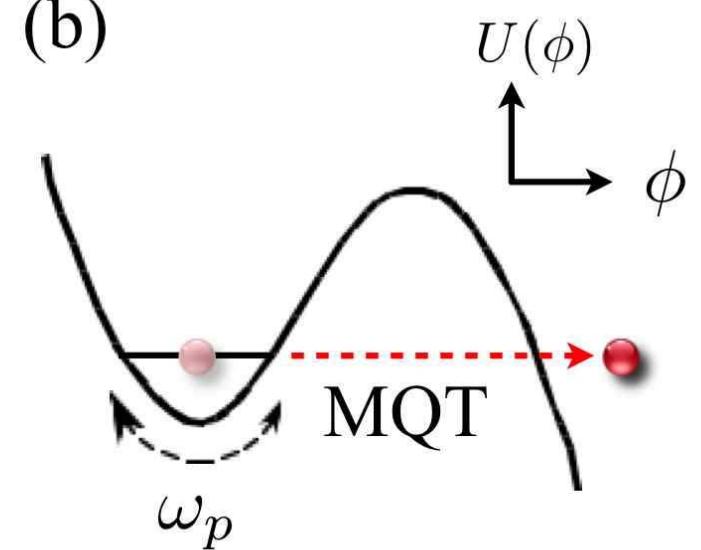
# Macroscopic Quantum Tunneling in SIFS junctions

(a)



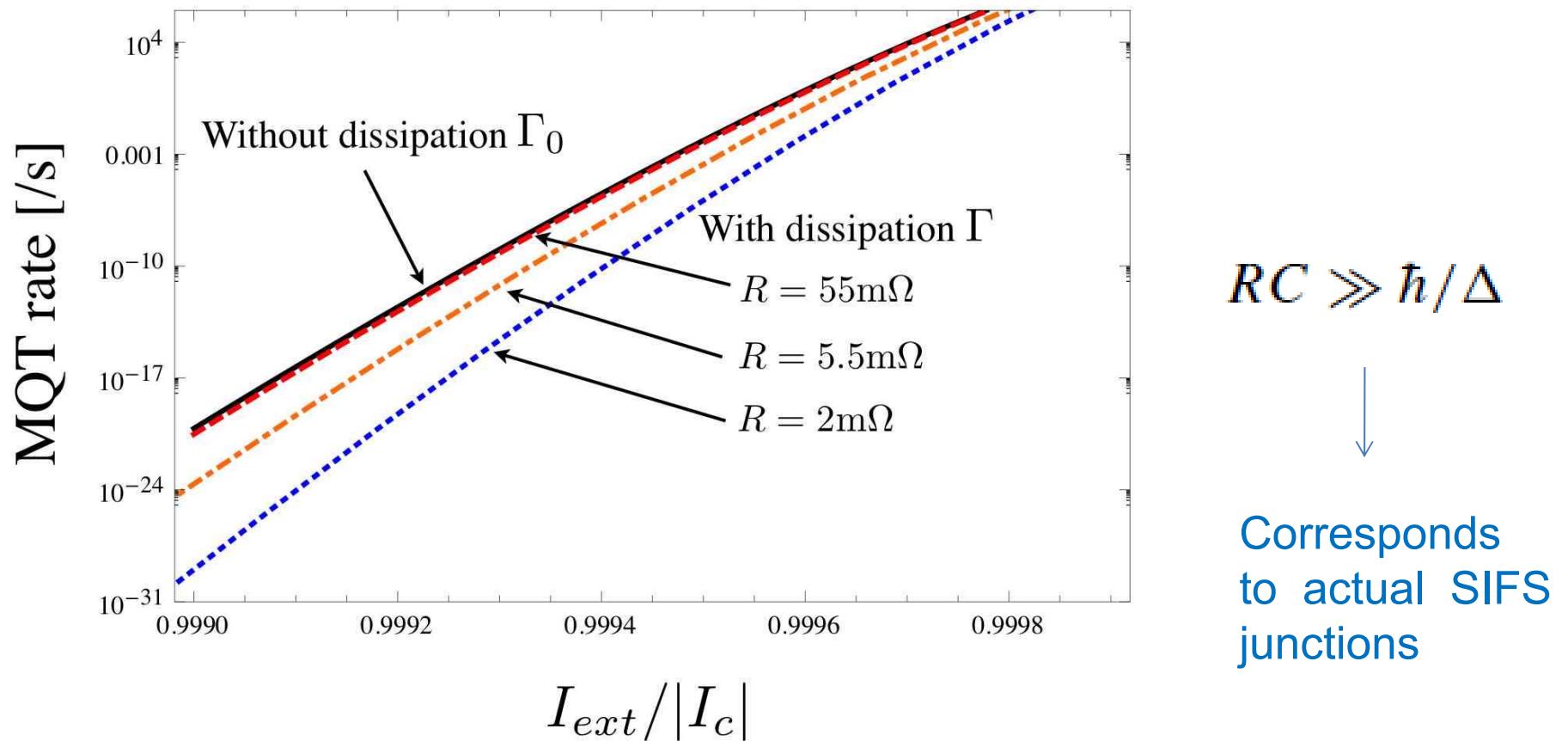
Current biased SIFS Josephson junction

(b)



Potential  $U(\phi)$  vs the phase difference  $\phi$  between two superconductors.  
 $\omega_p$  is the Josephson plasma frequency of the junction

# Macroscopic Quantum Tunneling in SIFS junctions



The MQT escape rate for a current-biased SIFS junction  
Parameters:  $C = 800\text{ pF}$ ,  $\Delta = 1.3\text{ meV}$ , and  $|I_c| = 500\text{ }\mu\text{A}$

## Summary Part I

We have performed theoretical study of critical current,  
DOS oscillations and I-V curves in SIFS tunnel  
Josephson junctions

Experimental data for critical current and  $0-\pi$  transition  
agree well with theory and junction parameters can be  
determined

We show that SIFS junctions are promising for  
application in quantum circuits

# Symmetry of the pair amplitude

Two electron's function = (Orbital)  $\times$  (Spin)  $\times$  ( Time )

$$F(k, R, s_1, s_2, t) = \Phi(k, R) \chi(s_1, s_2) \psi(t)$$

( Fermi-Dirac statistics )

$k$	momentum
$R$	coordinate
$s_1, s_2$	Spin
$t$	Time

Orbital	Spin	Time
even	singlet	even
odd	triplet	even
even	triplet	odd
odd	singlet	odd

Even frequency Cooper pair

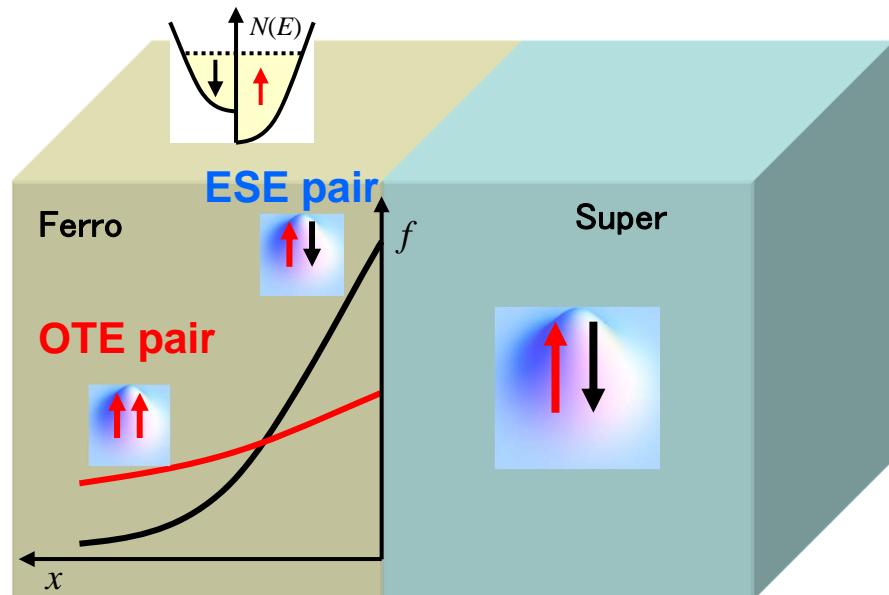
→ BCS Superconductor  
Cuprate Superconductor

→  $^3\text{He}$   $\text{Sr}_2\text{RuO}_4$

Odd frequency Cooper pair

# Background

## Superconductor/Ferromagnet (S-F) junction



F. S. Bergeret, A. F. Volkov, and K. B. Efetov,  
PRL **86**, 4096 (2001)

Interplay between **Cooper pairs with spin-singlet in S and spin-aligned carriers in F**

Possible symmetry of the pairing in F  
(consistent with Pauli's principle)

In the F region, only the **s-wave even-parity pairing state** is allowed due to isotropization by impurity scattering.



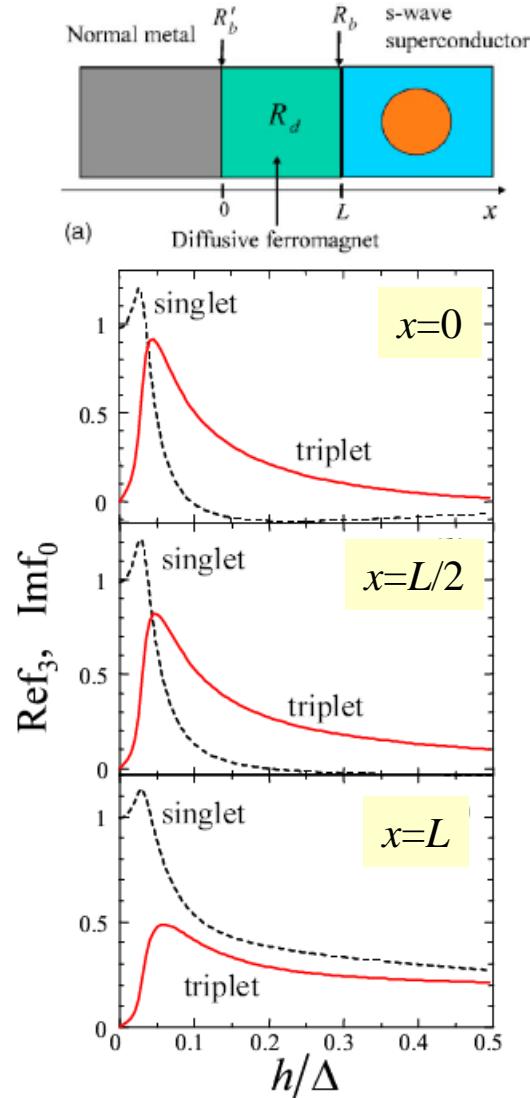
Even frequency spin-singlet s-wave even-parity (ESE) pair



Odd frequency spin-triplet s-wave even-parity (OTE) pair

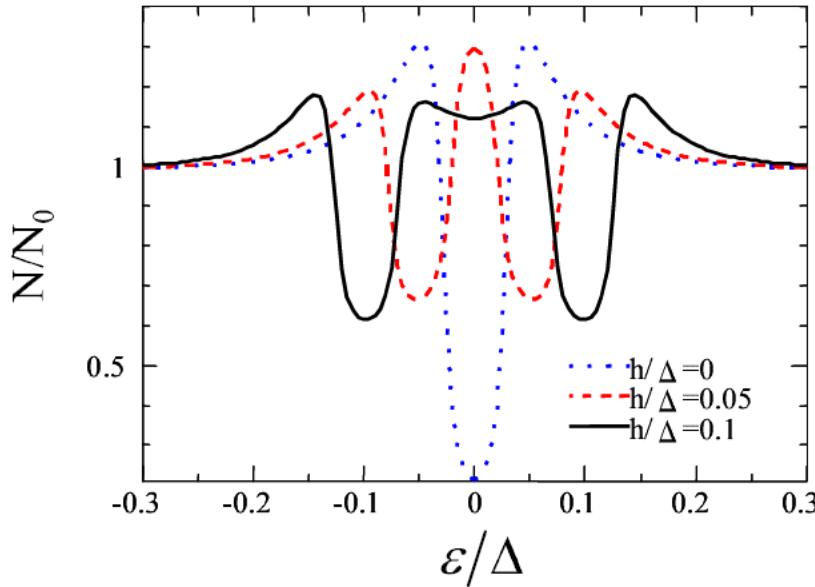
Odd frequency Cooper pair is induced in ferromagnet

# S-F junction (theoretical prediction)



The pair amplitudes  $f_3$  (ESE) and  $f_0$  (OTE) as a function of  $h$  in the F for  $\varepsilon=0$

T. Yokoyama, Y. Tanaka, A.A. Golubov, PRB **75**, 134510 (2007)



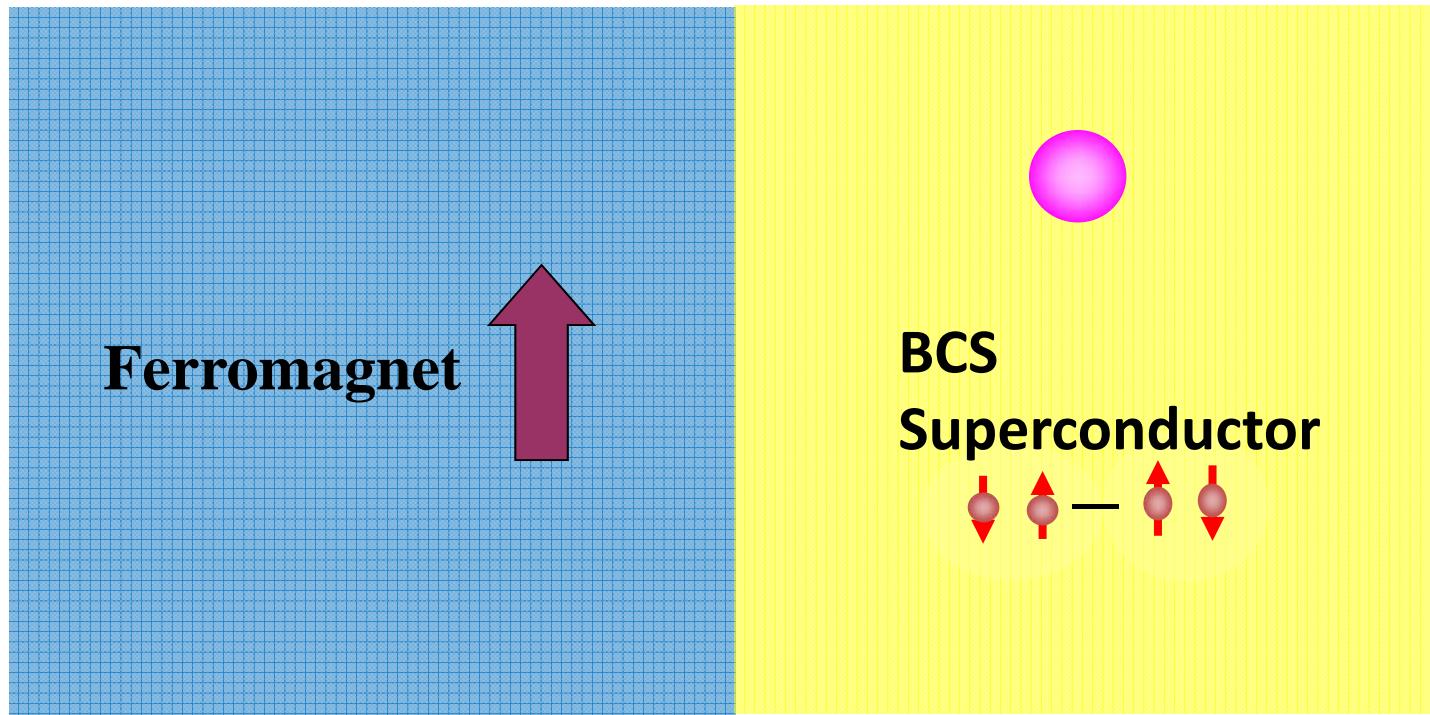
Normalized LDOS as a function of  $\varepsilon$  for  $R_d/R_b=1$  with various  $h/\Delta$

- When the OTE state dominates the ESE state in the F, the resulting LDOS has a zero-energy peak (ZEP).
- The amplitude of the OTE pair wave function near the N/F interface is enhanced at zero energy.

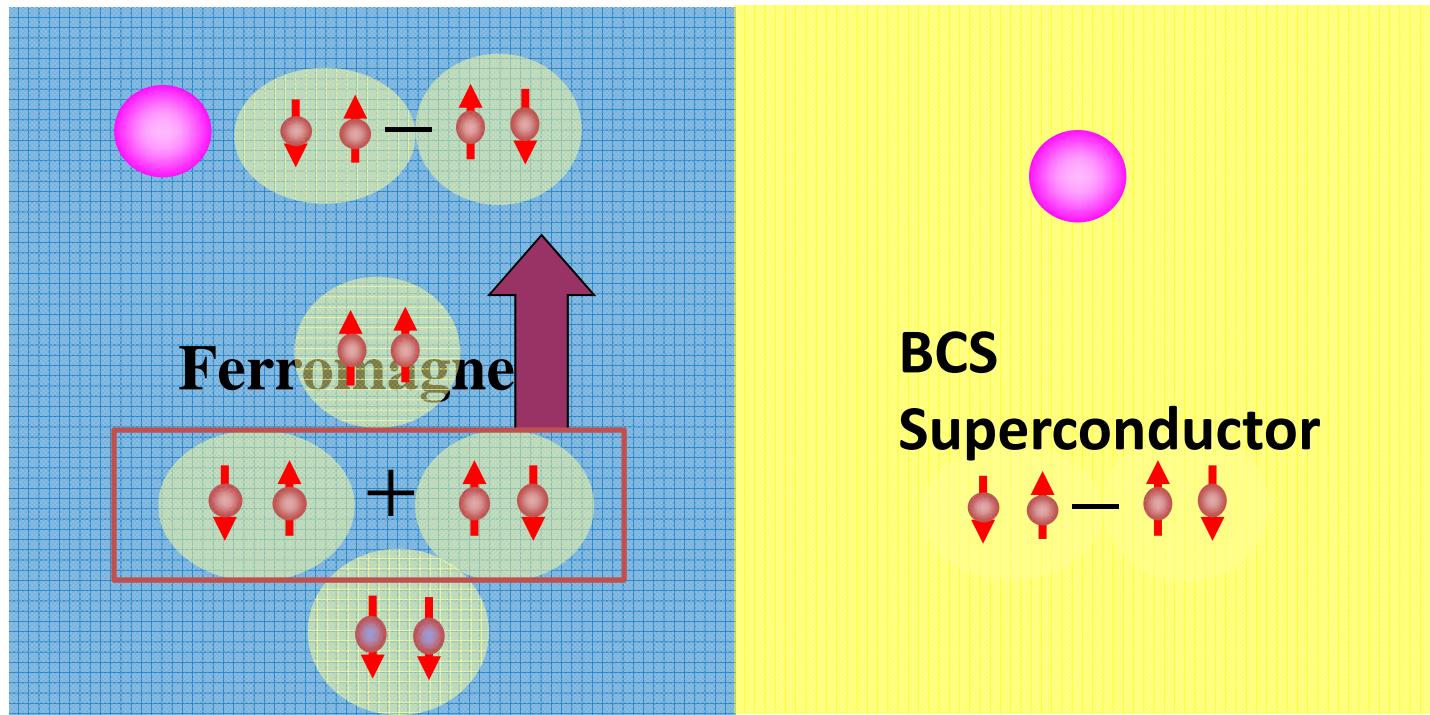


**The observation of ZEP is the evidence that odd-frequency pair exists in F**

# Odd frequency Cooper pair (F/S junction)



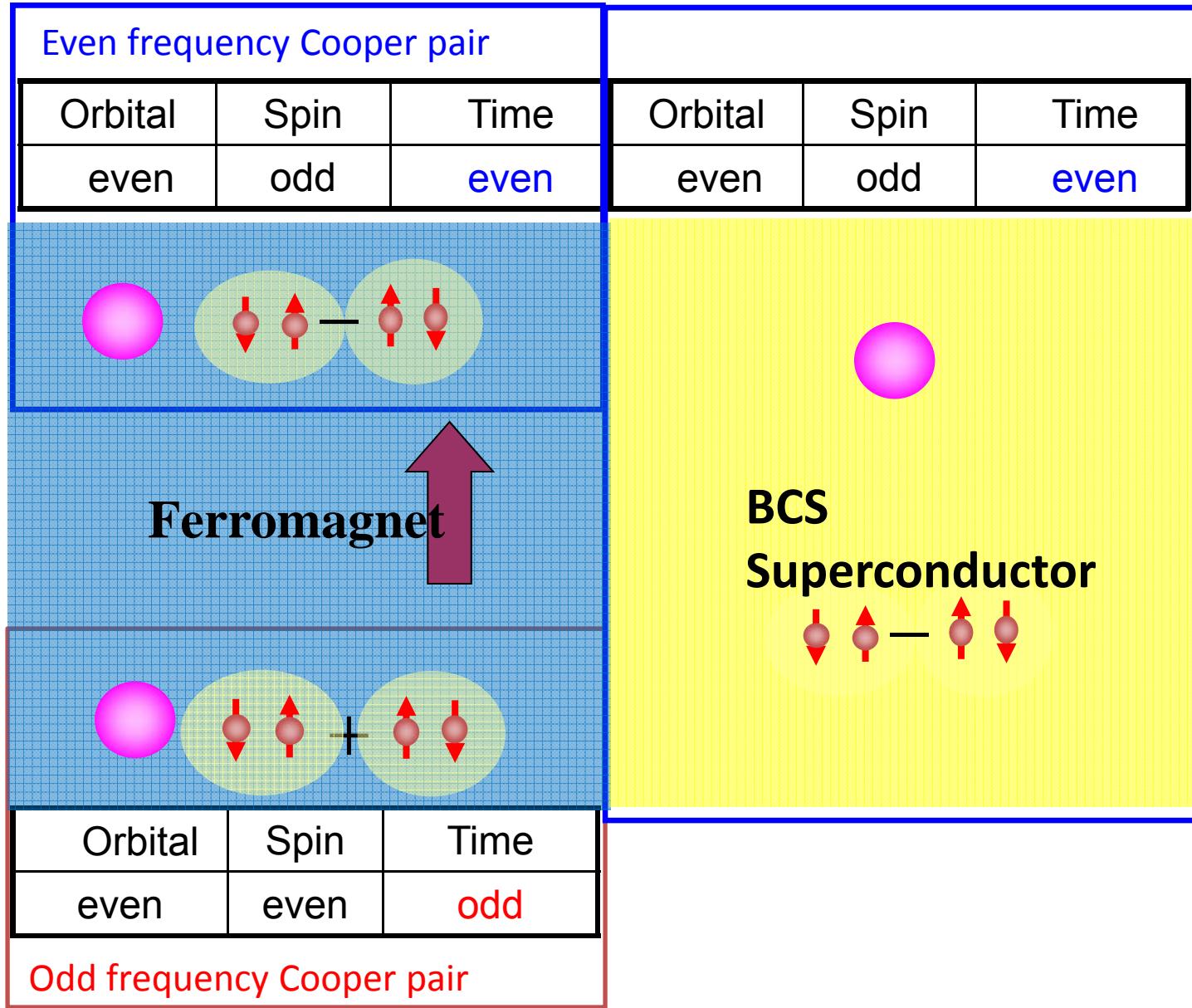
# Odd frequency Cooper pair (F/S junction)



2001 .*Bergeret .et.al*

2001 .*Kadigrobov .et.al*

# Odd frequency Cooper pair (F/S junction)



# Ferromagnet (metal)/superconductor junctions

Ferromagnet

Superconductor

ESE s-wave

All four kinds of pairing  
is possible in ferromagnet  
(Eschrig, 2007)

- (1) Generation of **OSO** pairing by broken inversion (translational) symmetry
- (2) Generation of **OTE** pairing by broken spin rotational symmetry
- (3) Generation of **ETO** pairing both in the presence of broken inversion (translational) symmetry and broken spin rotational symmetry

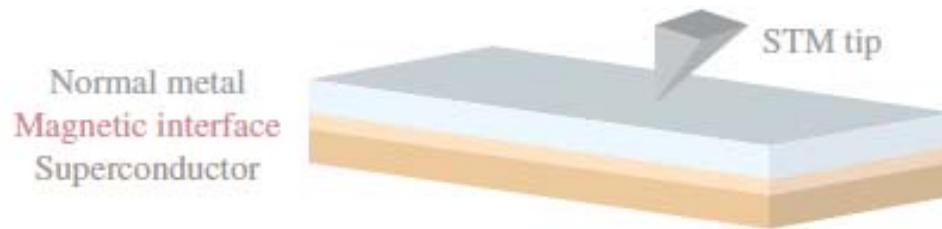
ESE : Even-frequency spin-Singlet Even-parity

ETO : Even-frequency spin-Triplet Odd-parity

OSO : Odd-frequency spin-Singlet Odd-parity

OTE : Odd-frequency spin-Triplet Even-parity

# Induced odd-frequency pairing by magnetic interface



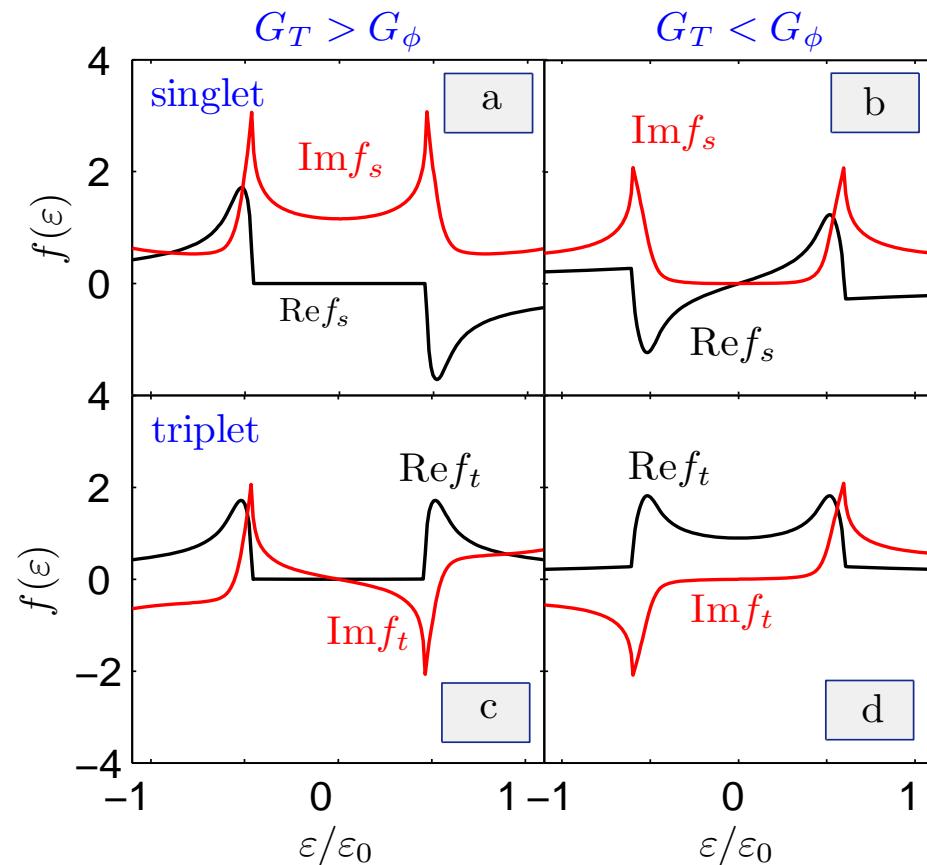
Odd-frequency pairing is induced by **interfacial spin-dependent phase shift**

$G_T$  Interface conductance

$G_\phi$  Mixing conductance

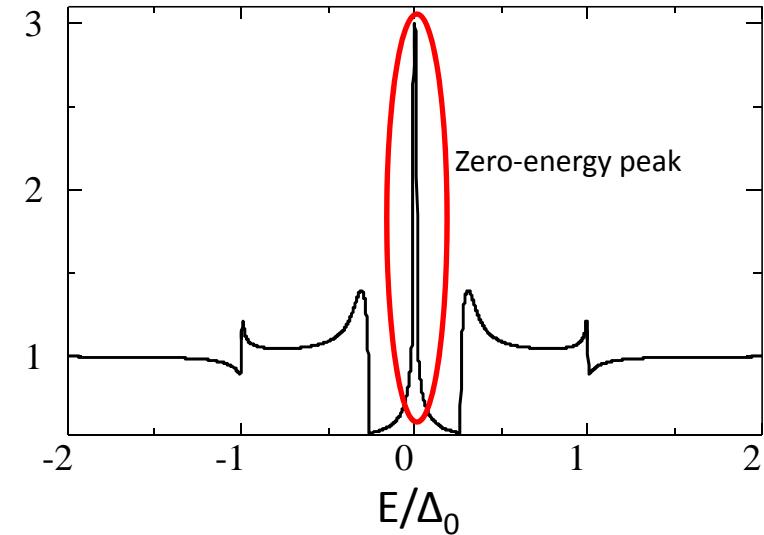
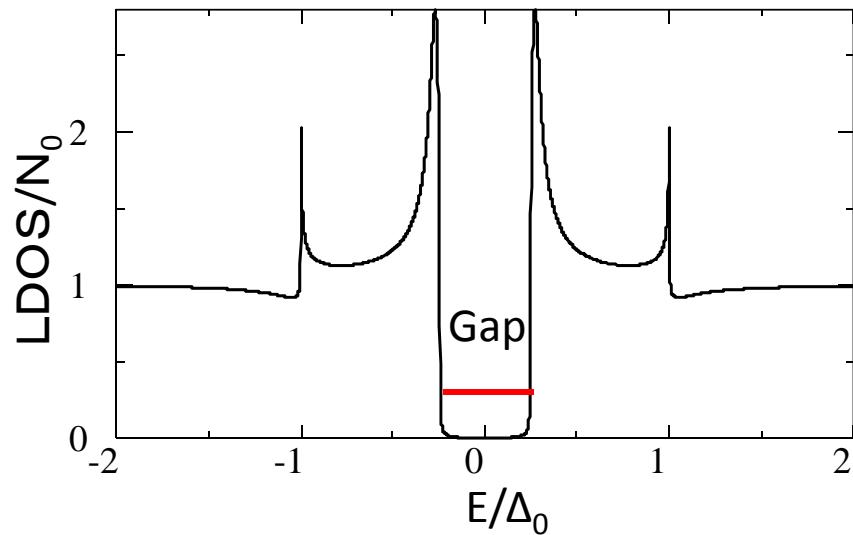
W. Belzig et al. (2002)

A. Brataas et al. (2000)



J. Linder, T. Yokoyama, A. Sudbo, and M. Eschrig, PRL 102, 107008 (2009)

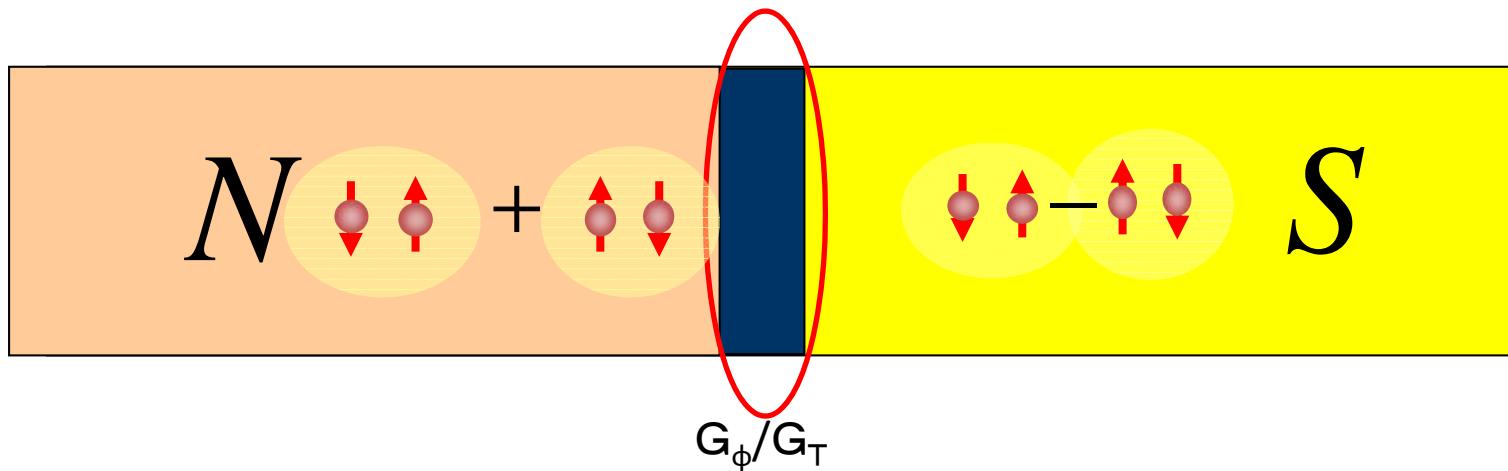
# LDOS



Minigap  
s-wave singlet  
(Even frequency)

Zero-energy peak  
s-wave triplet  
(Odd frequency)

# Odd frequency Cooper pair by spin-active interfaces

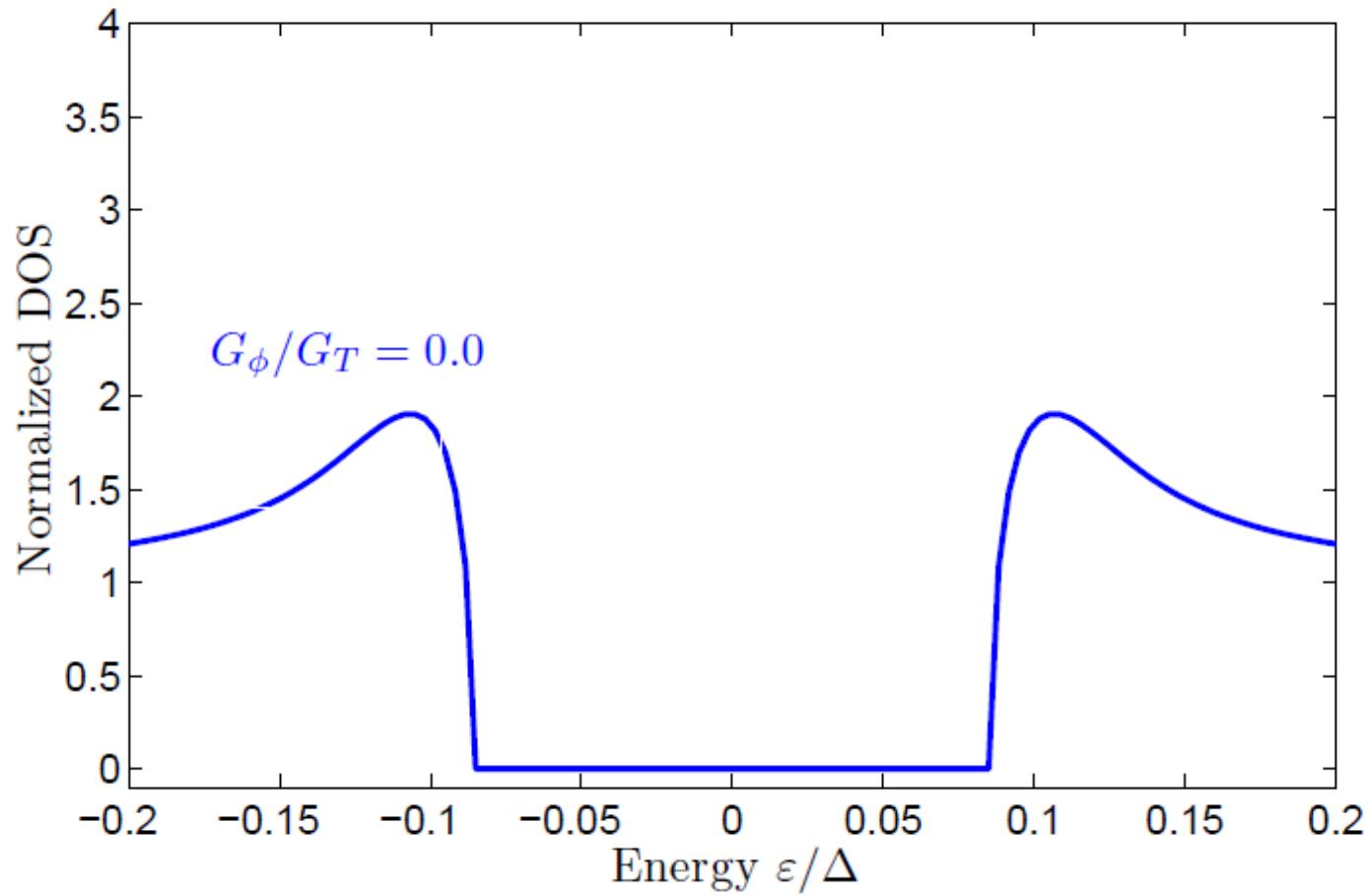


$G_T$  : Normal-state tunnel conductance

$G_\phi$  : Spin-dependent phase-shifts  
occurring at the interface

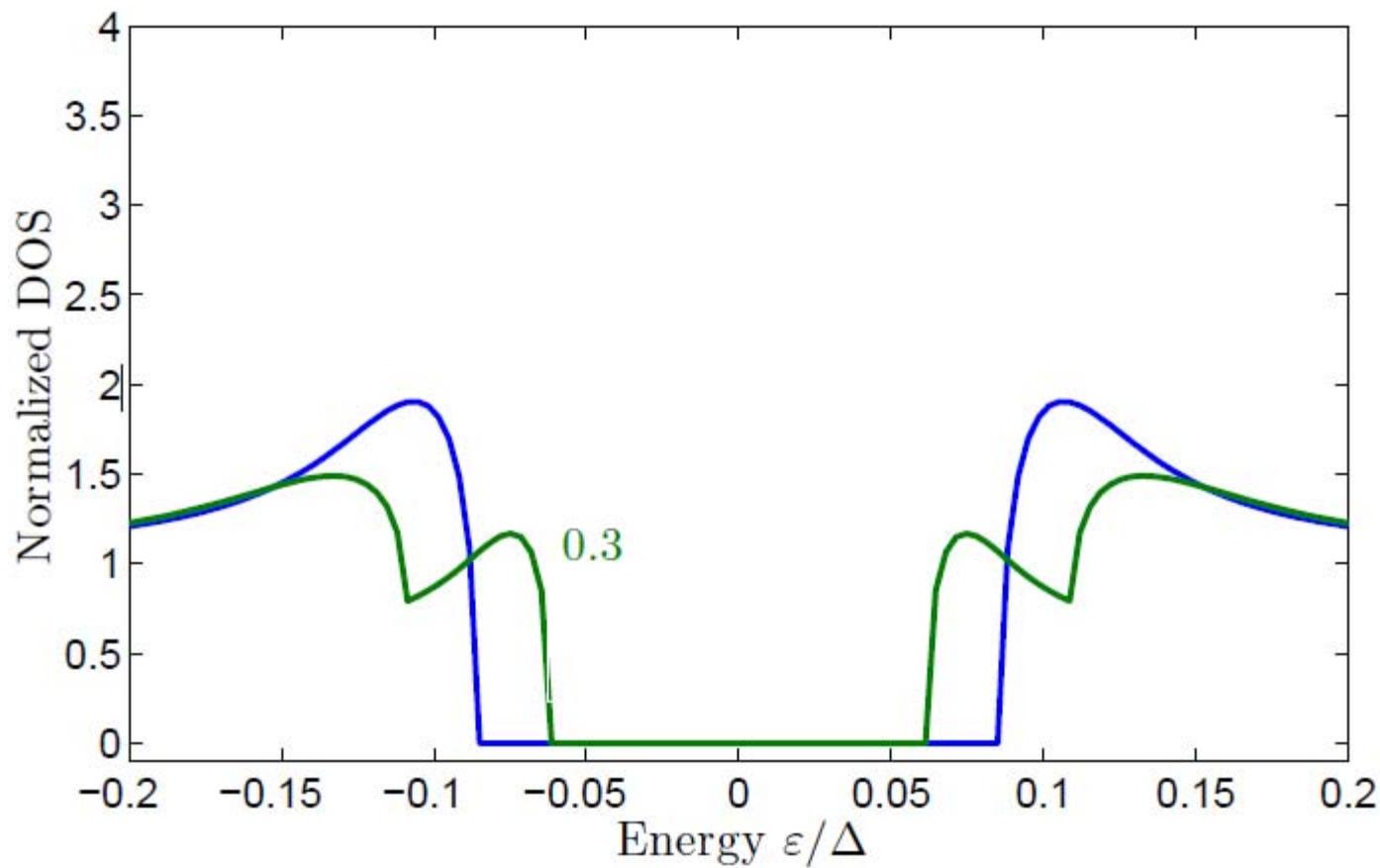
by spin-active  
interfaces

# Change of DOS by spin-active interfaces



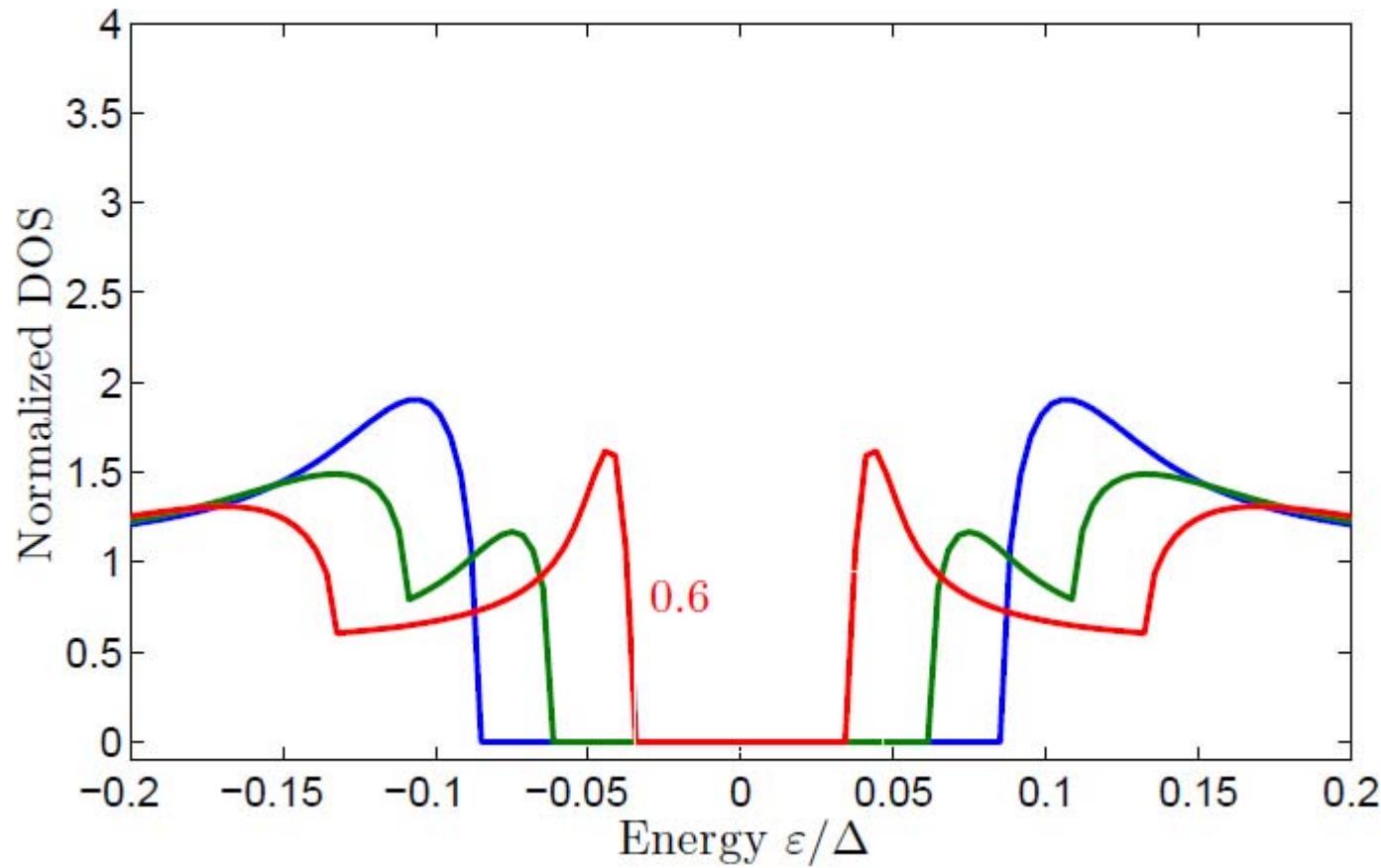
J. Linder, T. Yokoyama, A. Sudbo, and M. Eschrig, PRL 102, 107008 (2009)

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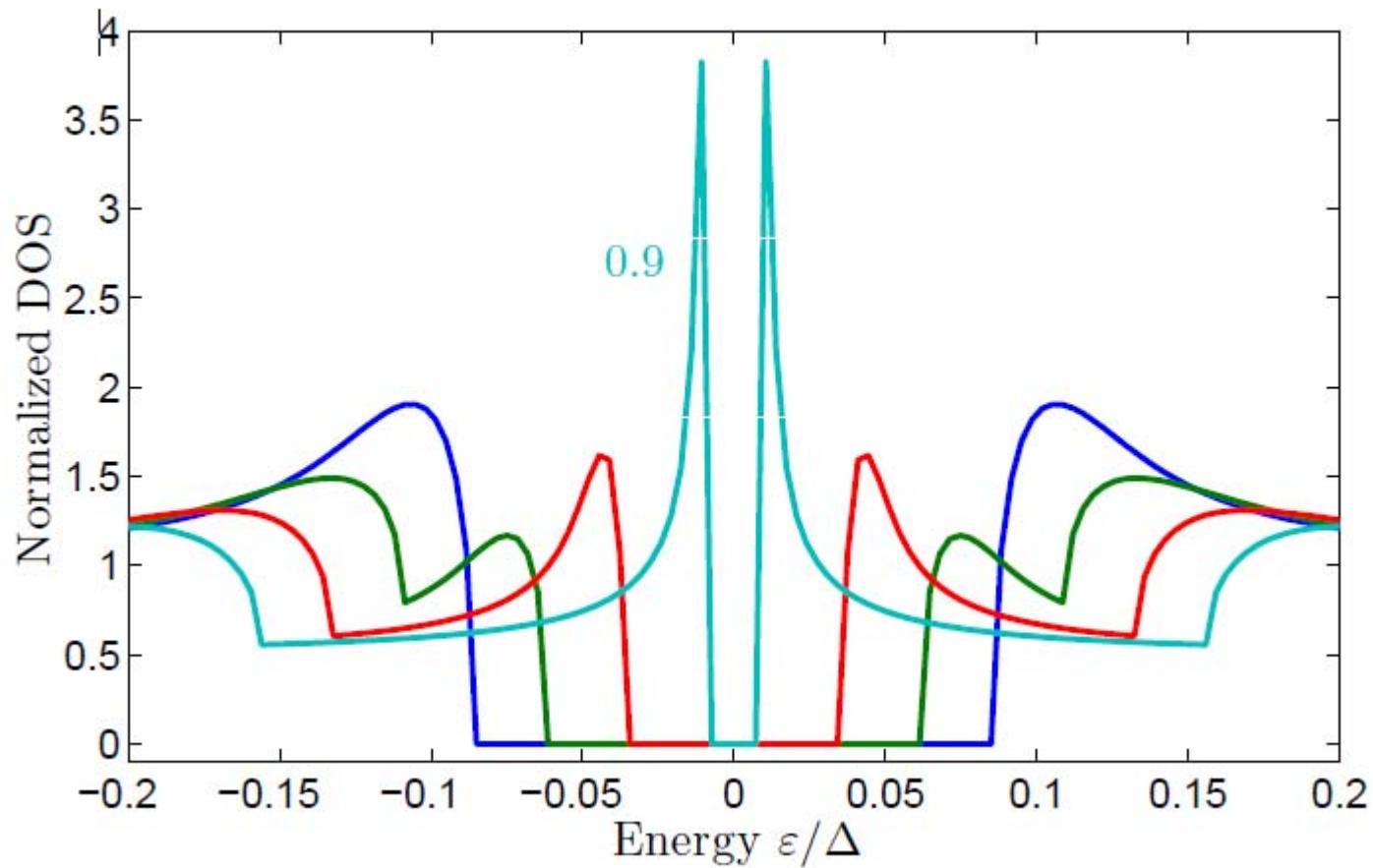
J. Linder, T. Yokoyama, A. Sudbo, and M. Eschrig, PRL 102, 107008 (2009)

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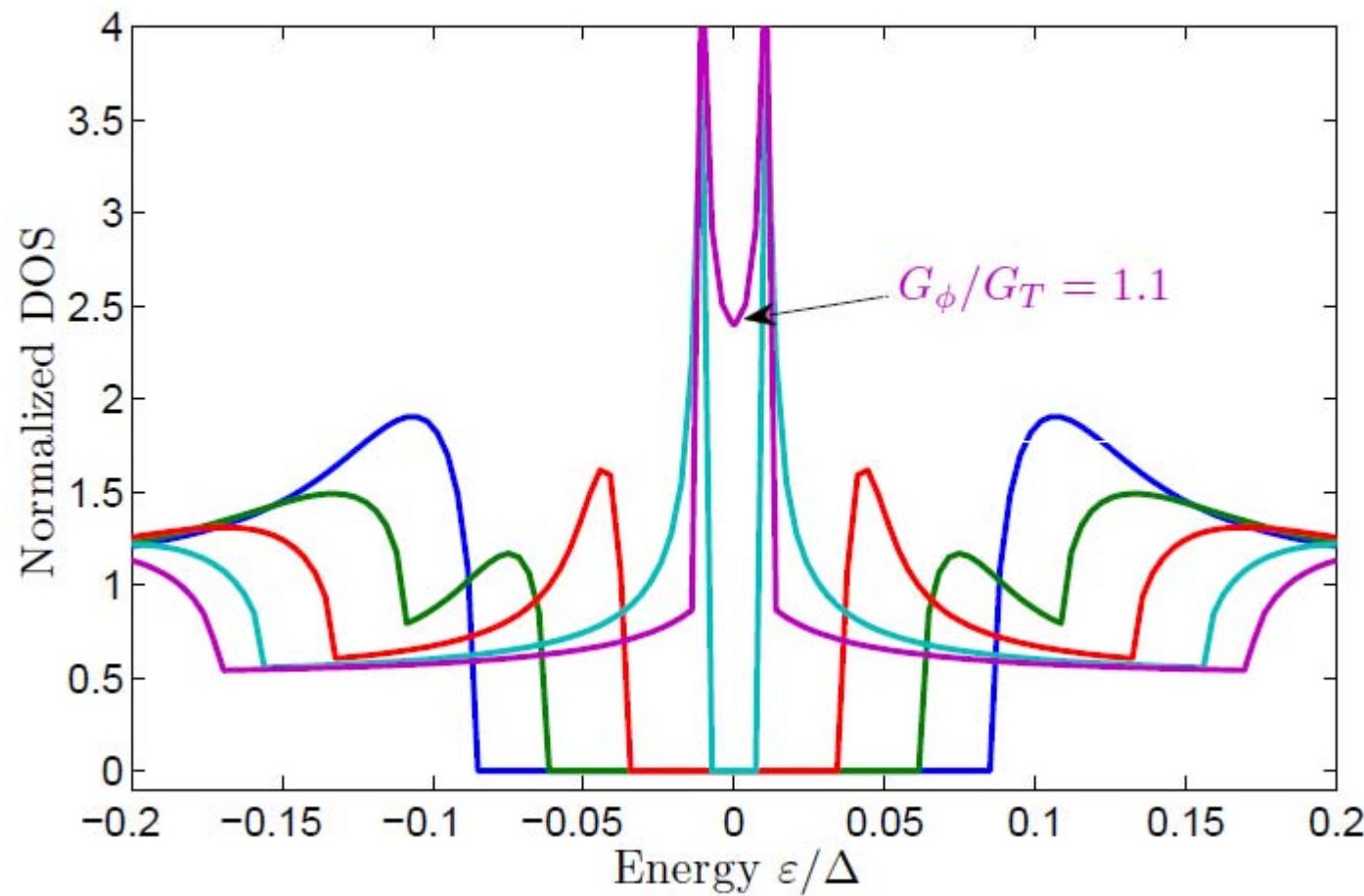
J. Linder, T. Yokoyama, A. Sudbo, and M. Eschrig, PRL 102, 107008 (2009)

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J. Linder, T. Yokoyama, A. Sudbo, and M. Eschrig, PRL 102, 107008 (2009)

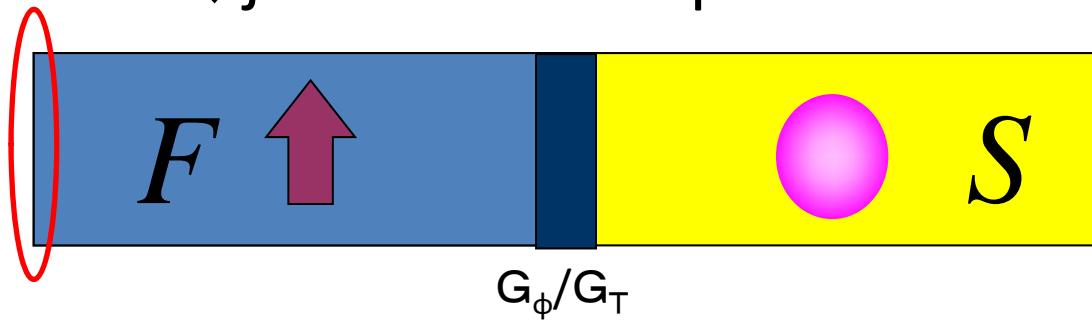
# Change of DOS by spin-active interfaces



J. Linder, T. Yokoyama, A. Sudbo, and M. Eschrig, PRL 102, 107008 (2009)

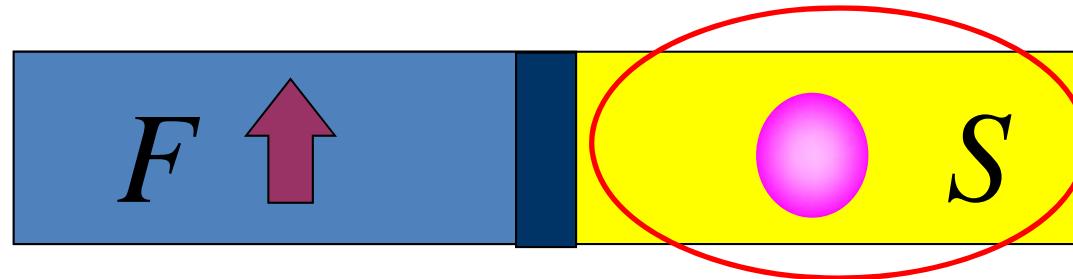
# Model

F/S(s-wave)junction with spin-active interfaces



We study a change of the DOS (LDOS) both in the presence the exchange field from Ferromagnet and spin-active interface

# Method



$\Delta$ : pair potential

The Usadel equation in S ( $x>d$ )

$$\theta_S'' + iE \sin \theta_S + \Delta \cos \theta_S = 0$$

Generalized boundary condition at the FS interface

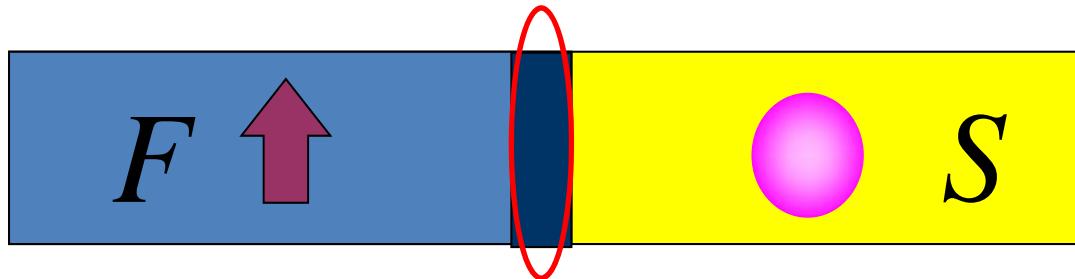
$$\theta_F' = -\frac{1}{\gamma_B} \sin(\theta_F - \theta_S) - i\gamma_\phi \sigma \sin \theta_F$$

F-layer with thickness d locates from  $x=0$  to  $x=d$

The Usadel equation in F

$$\theta_F'' + i(E + \sigma h) \sin \theta_F = 0$$

# Method



The Usadel equation in S ( $x>d$ )

$$\theta_S'' + iE \sin \theta_S + \Delta \cos \theta_S = 0$$

- $\gamma_B$  : Normal state interface resistance
  - $\gamma_\phi$  : Spin dependence of interfacial phase shifts
  - $G_T$  : Normal-state tunnel conductance
  - $G_\phi$  : Spin-dependent phase shifts at the interface
- $$G_\phi/G_T = \gamma_\phi \gamma_B$$

Generalized boundary condition at the FS interface

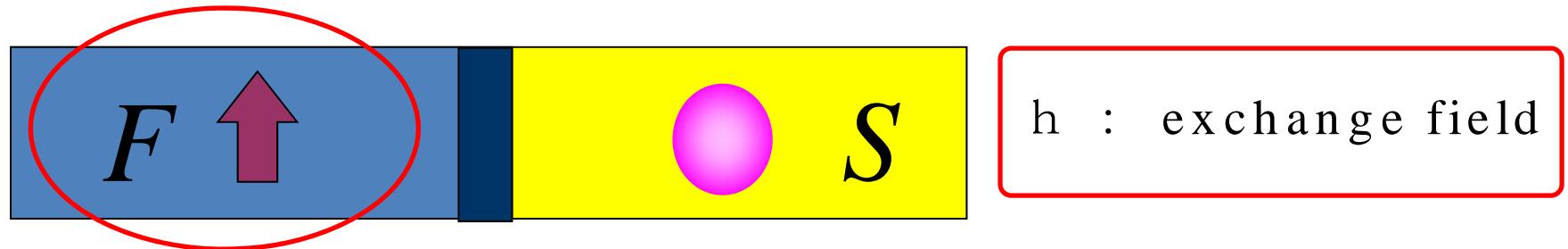
$$\theta_F' = -\frac{1}{\gamma_B} \sin(\theta_F - \theta_S) - i\gamma_\phi \sigma \sin \theta_F$$

F-layer of thickness d occupies the range from  $x=0$  to  $x=d$

The Usadel equation in F

$$\theta_F'' + i(E + \sigma h) \sin \theta_F = 0$$

# Method



The Usadel equation in S ( $x>d$ )

$$\theta_S'' + iE \sin \theta_S + \Delta \cos \theta_S = 0$$

Generalized boundary condition at the FS interface

$$\theta_F' = -\frac{1}{\gamma_B} \sin(\theta_F - \theta_S) - i\gamma_\phi \sigma \sin \theta_F$$

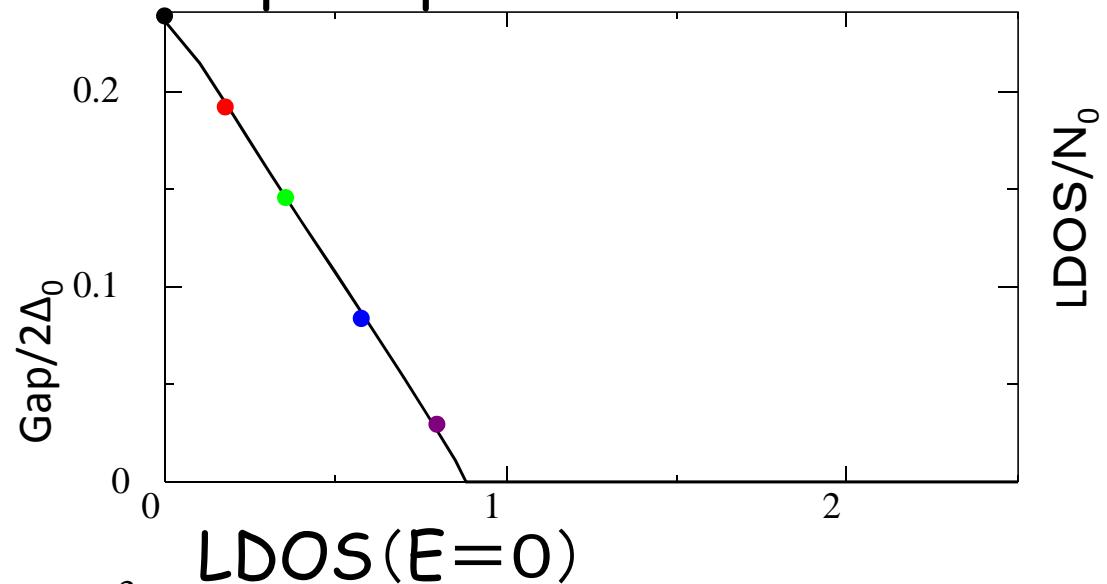
F-layer of thickness d occupies the range from  $x=0$  to  $x=d$

The Usadel equation in F

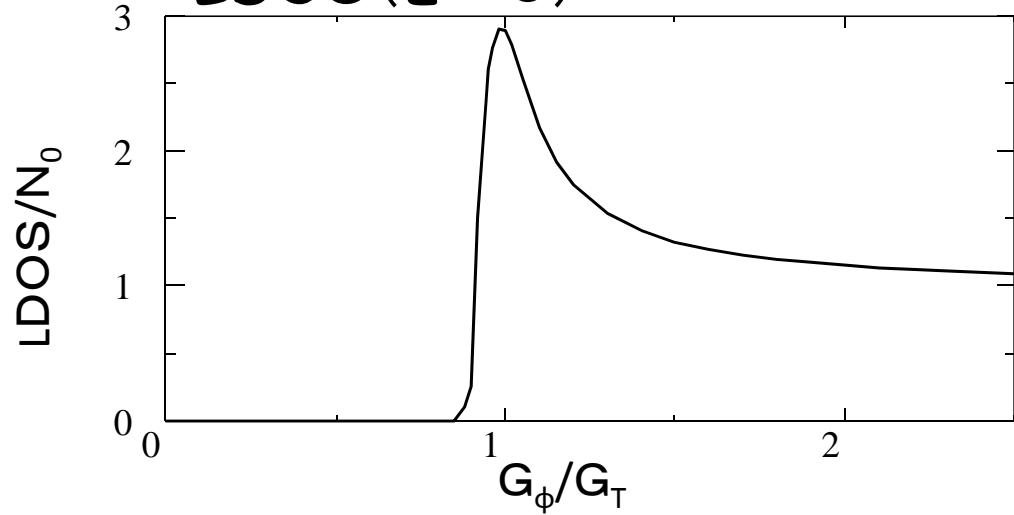
$$\theta_F'' + i(E + \sigma h) \sin \theta_F = 0$$

## *The relation between $G_\phi/G_T$ and DOS*

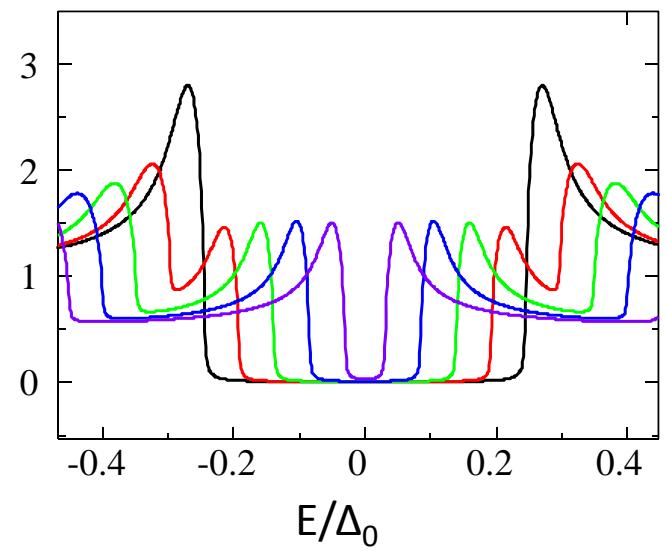
Gap amplitude



$\text{LDOS}(E=0)$

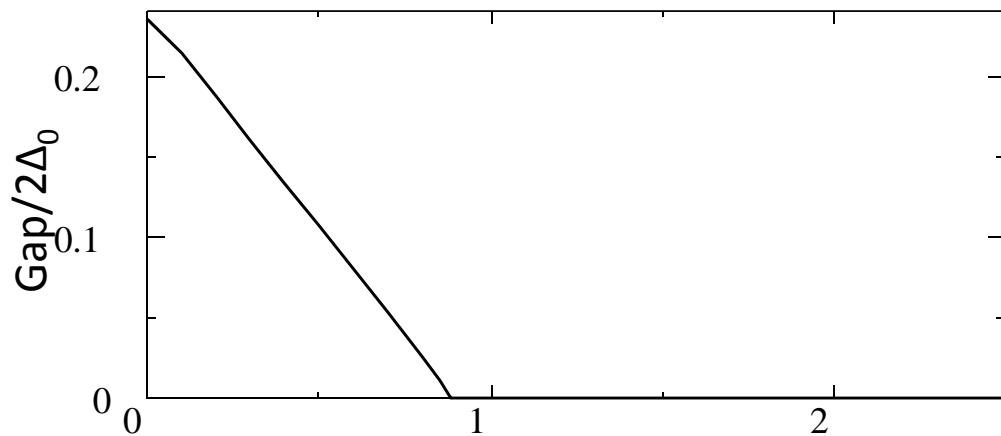


$\text{LDOS}/N_0$

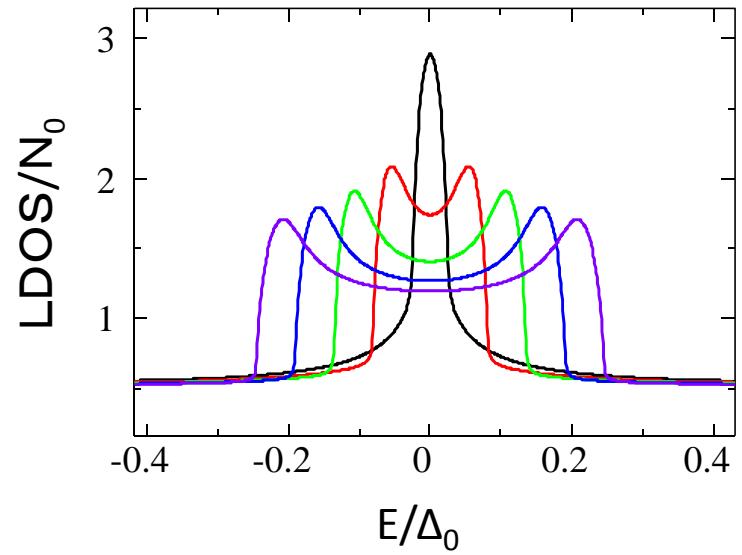
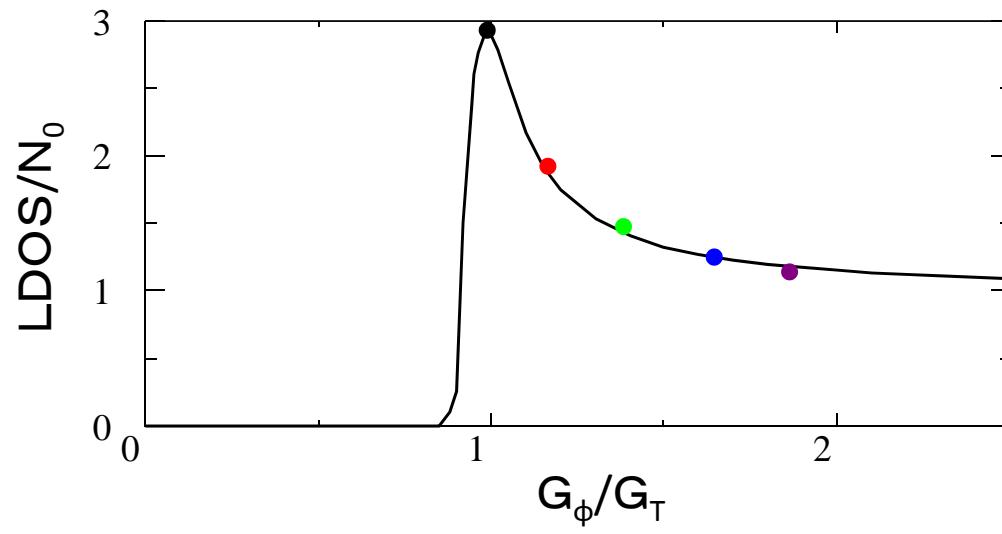


## *The relation between $G_\phi/G_T$ and DOS*

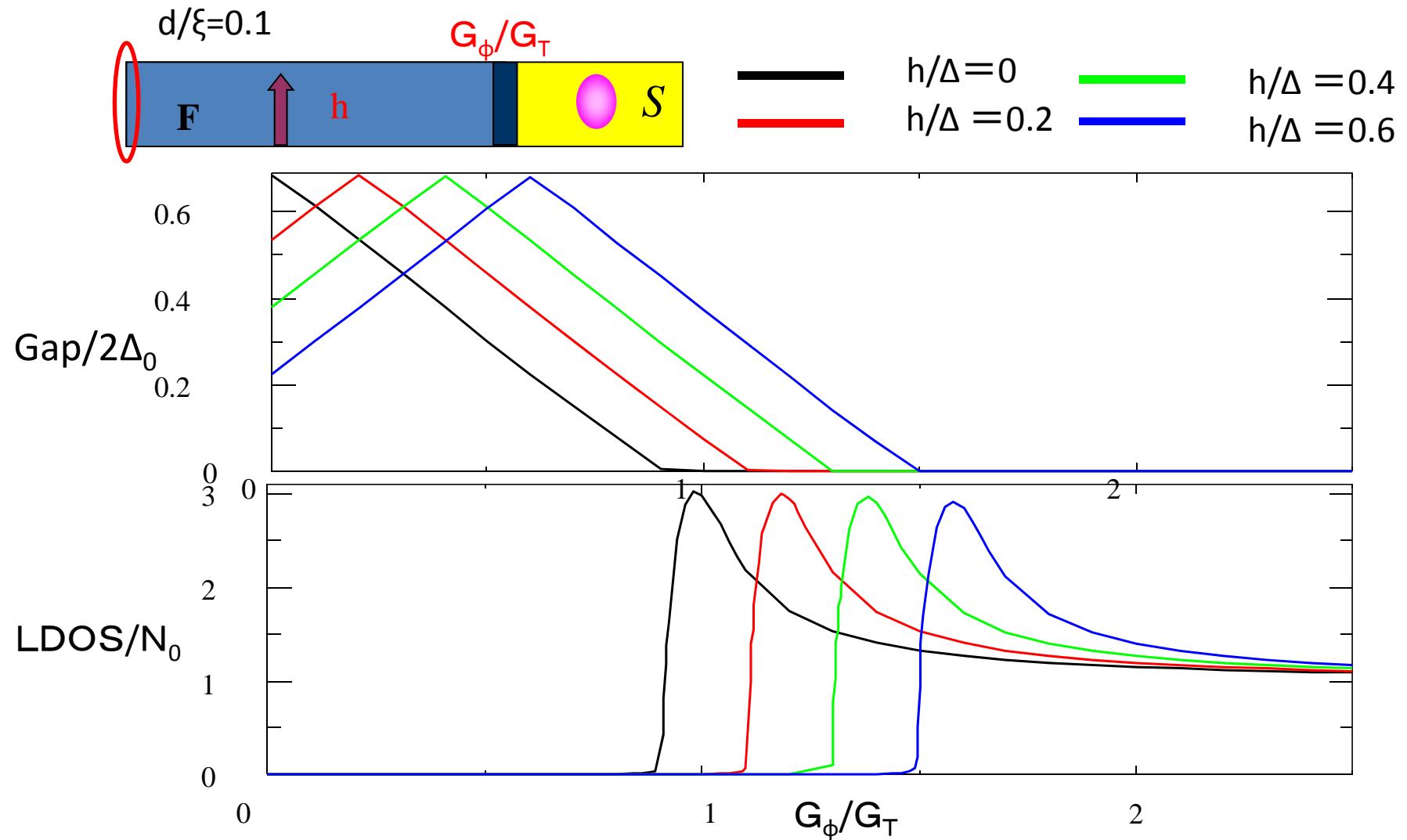
Gap amplitude



LDOS( $E=0$ )

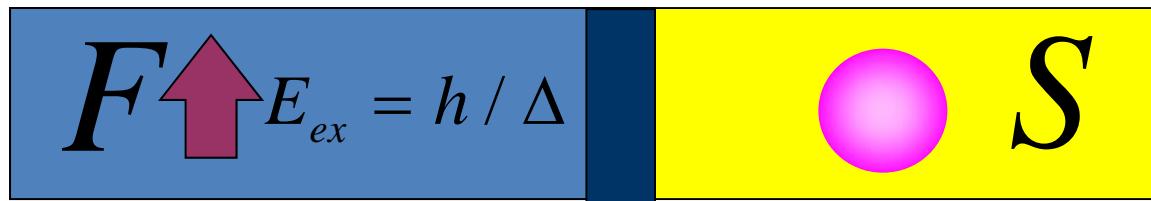


## Change of DOS by $h$ and $G_\phi/G_T$ ( $d/\xi = 0.1$ )



# Crossover point from minigap to zero-energy peak

$$d_0 = d / \xi \quad \gamma_B : \begin{array}{l} \text{Normal state} \\ \text{interface resistance} \end{array}$$



$$G_\phi / G_T$$

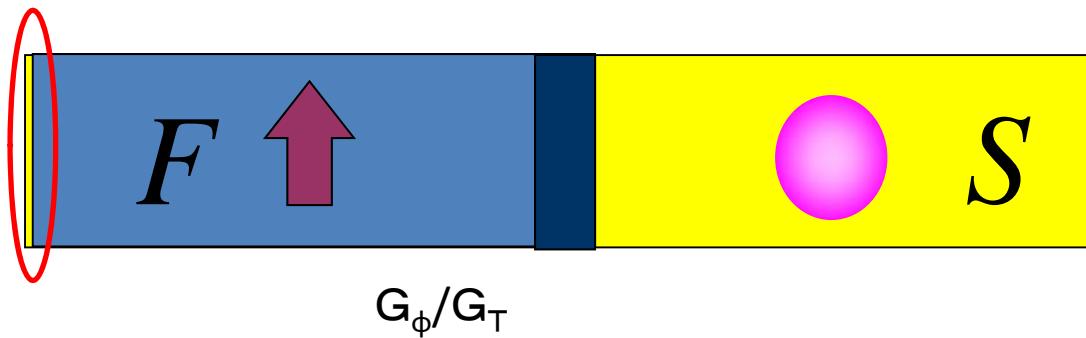
$d/\xi \ll 1$ :

$$LDOS(E = 0) = R e \left( \frac{|E_{ex} \gamma_B d_0 - G_\phi / G_T|}{\sqrt{(E_{ex} \gamma_B d_0 - G_\phi / G_T)^2 - 1}} \right)$$

Crossover point from minigap to zero-energy peak

$$|E_{ex} \gamma_B d_0 - G_\phi / G_T| \geq 1$$

# *Summary Part II*



- The diagram illustrates the relationships between several physical concepts:

  - Ferromagnet effect and spin-active interfaces effect** (blue circle) leads to **Spin triplet Odd frequency cooper pair** (blue arrow).
  - Spin triplet Odd frequency cooper pair** (blue arrow) leads to **LDOS ZEP** (blue arrow).
  - d/ξ << 1 :** (red circle) indicates the **Crossover point from minigap to zero-energy peak (E=0)**.
  - A mathematical condition is shown below the crossover point:  $|E_{ex}\gamma_B d - G_\phi / G_T| \geq 1$