

Odd-Frequency Pairing in Superconducting Heterostructures

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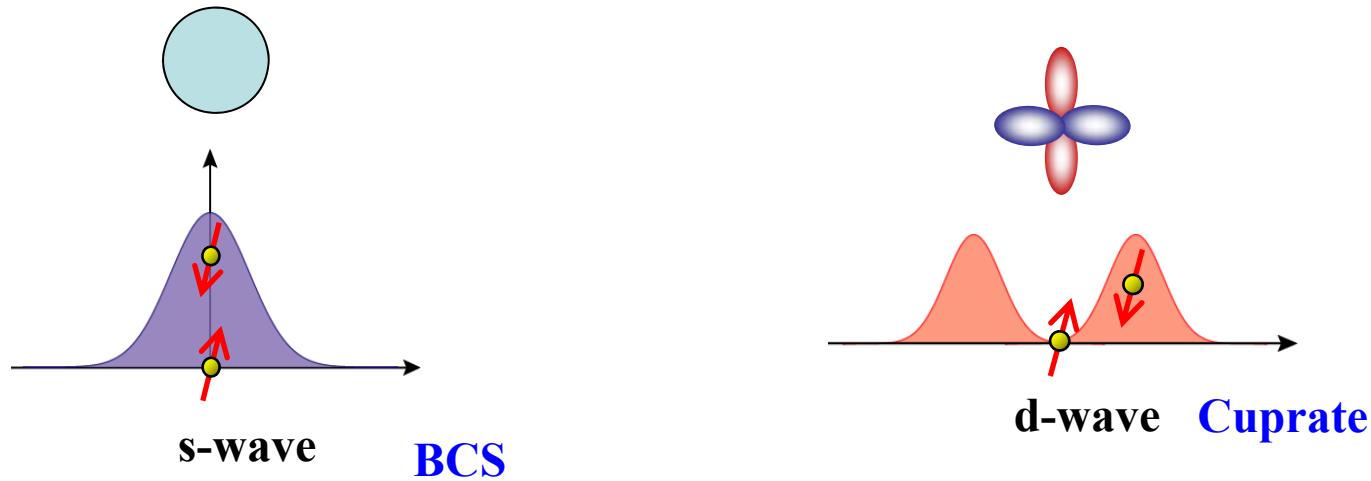
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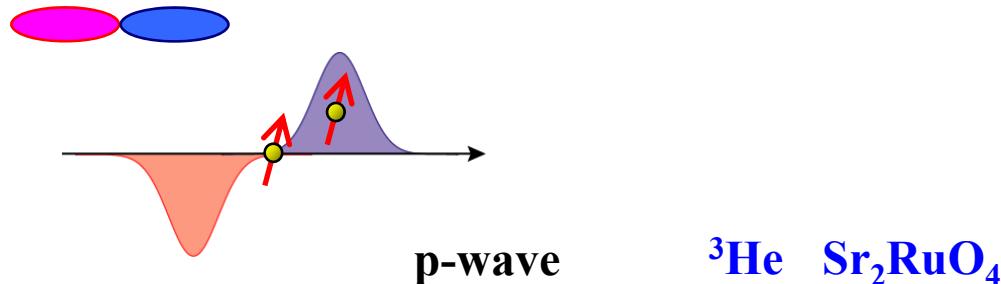
- (1) What is odd-frequency pairing
- (2) Normal metal / Superconductor junctions
- (3) Vortices in p-wave superconductors
- (4) Ferromagnet/Superconductor junctions

Conventional Classification of Symmetry of Cooper pair

Spin-singlet Cooper pair  **Even Parity**



Spin-triplet Cooper pair  **Odd Parity**



Slide 3

t3

tanaka, 7/7/2008

Odd-frequency pairing

Fermi-Dirac statistics

Symmetry of pair wave functions:

$$\mathbf{k} \otimes \sigma \otimes \omega = \text{odd}$$

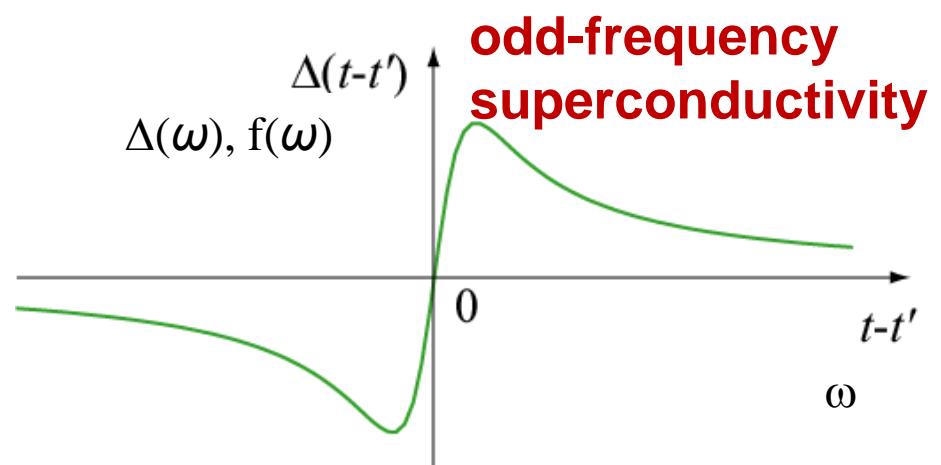
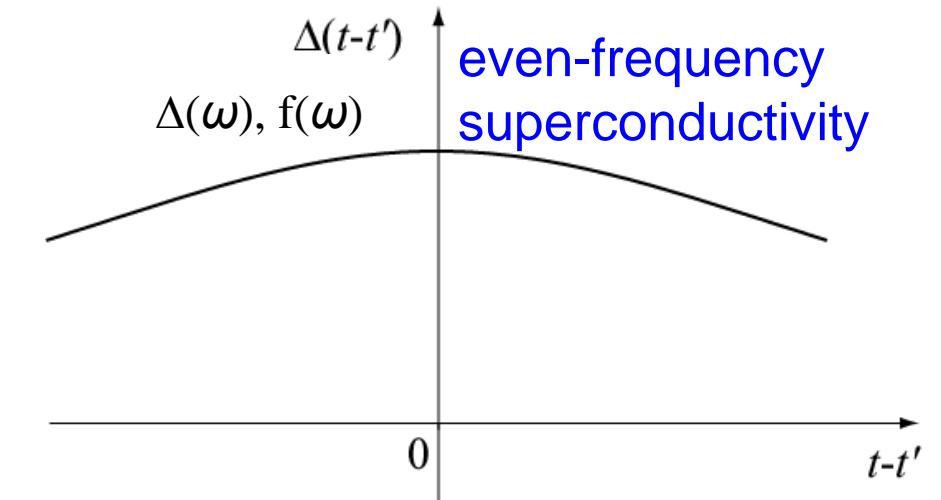
Momentum x Spin x Frequency



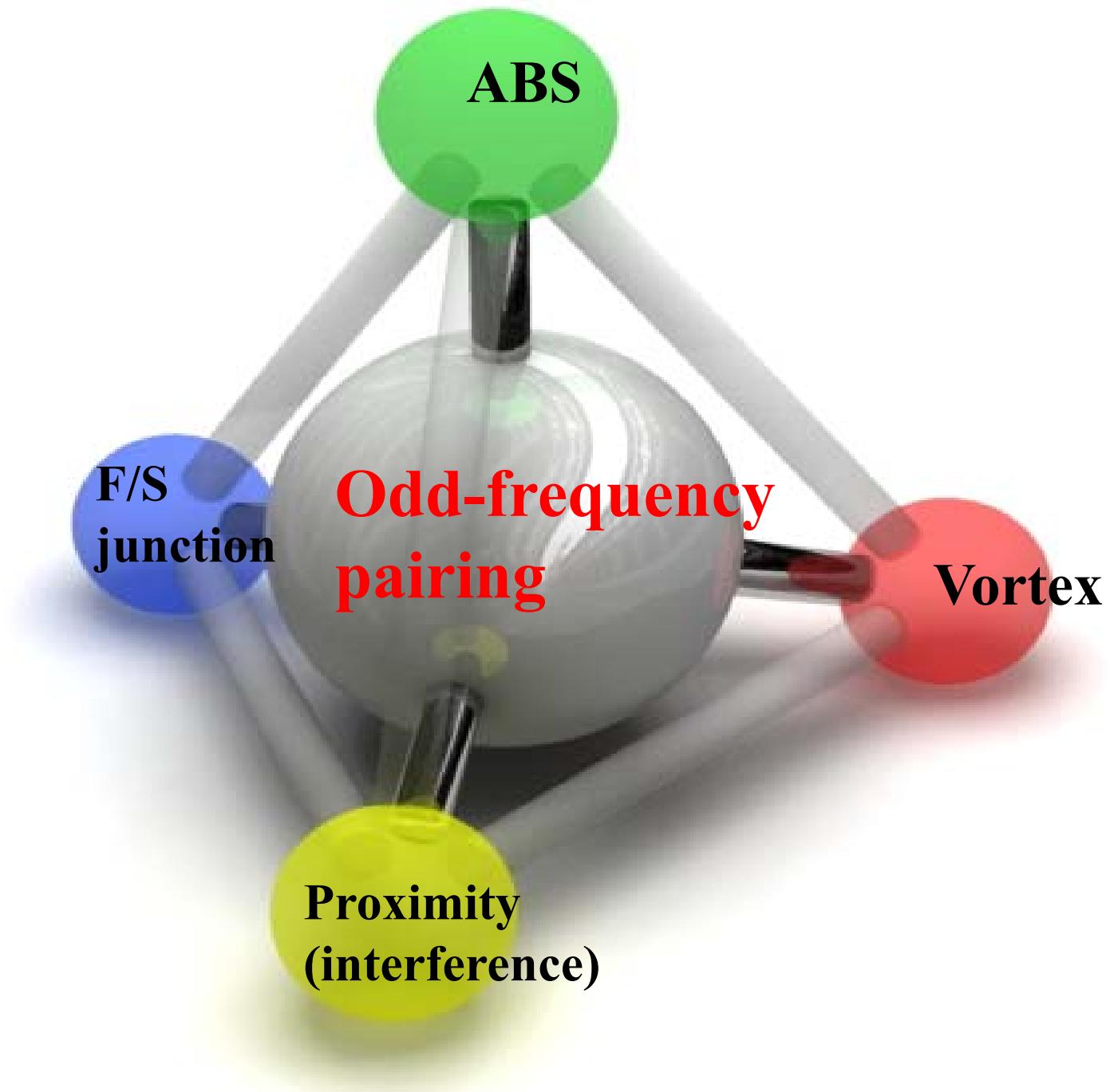
**Berezinskii
(1974):**

Spin-triplet s-wave

**Balatsky&Abrahams
(1992):**
Spin-singlet p-wave



ω , Matsubara frequency



Pair amplitude

Exchange of time

Even-frequency pairing (conventional pairing)

$$F_{\alpha,\beta}(\mathbf{r}_1 t_1, \mathbf{r}_2 t_2) = F_{\alpha,\beta}(\mathbf{r}_1 t_2, \mathbf{r}_2 t_1)$$

Odd-frequency pairing

$$F_{\alpha,\beta}(\mathbf{r}_1 t_1, \mathbf{r}_2 t_2) = -F_{\alpha,\beta}(\mathbf{r}_1 t_2, \mathbf{r}_2 t_1)$$

Symmetry of the pair amplitude

+ symmetric, – anti-symmetric

	Frequency (time)	Spin	Orbital	Total	
ESE	+ (even)	– (singlet)	+ (even)	–	BCS
ETO	+ (even)	+ (triplet)	– (odd)	–	Cuprate
OTE	– (odd)	+ (triplet)	+ (even)	–	^3He
OSO	– (odd)	– (singlet)	– (odd)	–	Sr_2RuO_4

ESE (**Even-frequency spin-singlet even-parity**)

ETO (**Even-frequency spin-triplet odd-parity**)

OTE (**Odd-frequency spin-triplet even-parity**) Berezinskii

OSO (**Odd-frequency spin-singlet odd-parity**) Balatsky, Abrahams

- **Odd-frequency pairing state** is possible in inhomogeneous superconductors even for conventional even-frequency paring in the bulk

Broken spin rotation symmetry or spatial invariance symmetry can induce odd-frequency pairing state:

- **ferromagnet/superconducor junctions:**

Bergeret,Volkov&Efetov, 2001

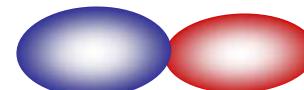
- **non-uniform systems:**

Junctions: Tanaka&Golubov, 2007; Eschrig&Lofwander, 2007

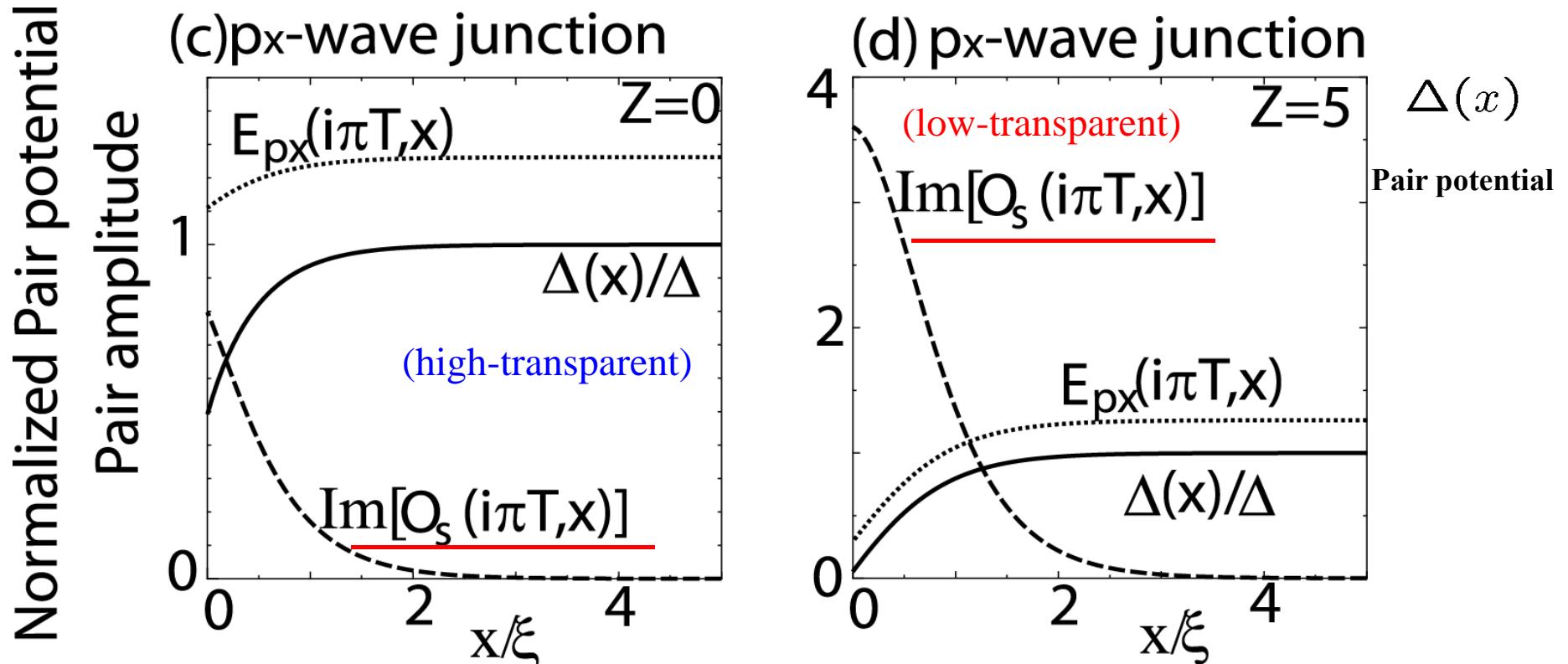
Vortices: Yokoyama *et al.*, 2008; Tanuma *et al.*, 2009)

Normal metal

**spin-triplet p-wave
superconductor**



Symmetry of the bulk pair potential is ETO



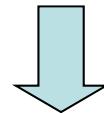
$E_{px}(i\omega_n, x)$ p_x-wave component of ETO pair amplitude
 $O_s(i\omega_n, x)$ s-wave component of OTE pair amplitude

ETO (Even-frequency spin-triplet odd-parity)

OTE (Odd-frequency spin-triplet even-parity) Y. Tanaka, et al PRL 99 037005 (2007)

Underlying physics

Near the interface, **even and odd-parity** pairing states (pair amplitude) can mix due to the **breakdown of the translational symmetry**.

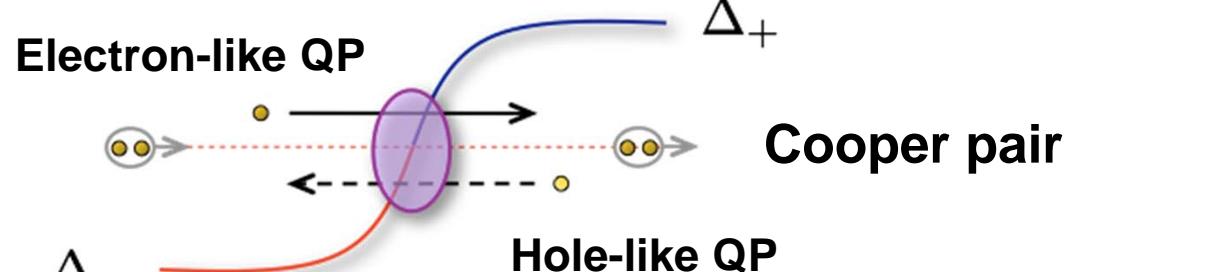


Fermi-Dirac statistics

The interface-induced state (pair amplitude) should be odd in frequency where the bulk pair potential has an **even**-frequency component since there is **no spin flip** at the interface.

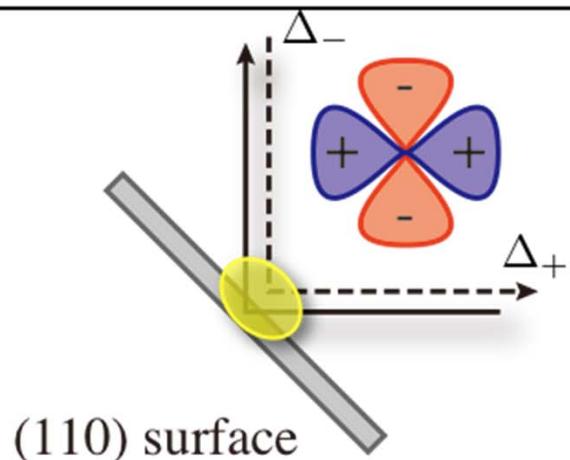
Andreev bound states in inhomogeneous systems are manifestations of odd-frequency pairing amplitude

Andreev bound states



Negative pair potential

Surface: Tanaka et al, 2007

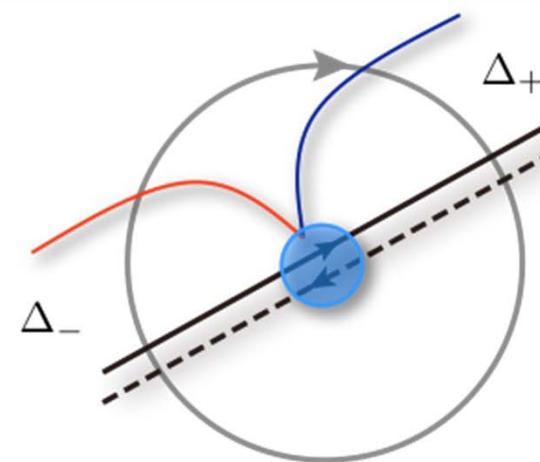


Scattering direction of QP

Positive pair potential

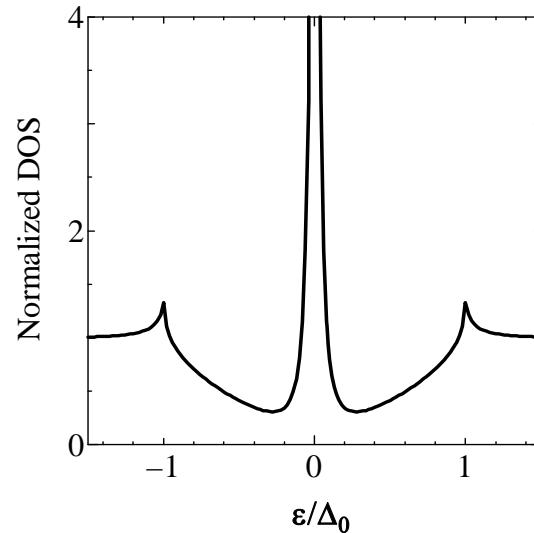
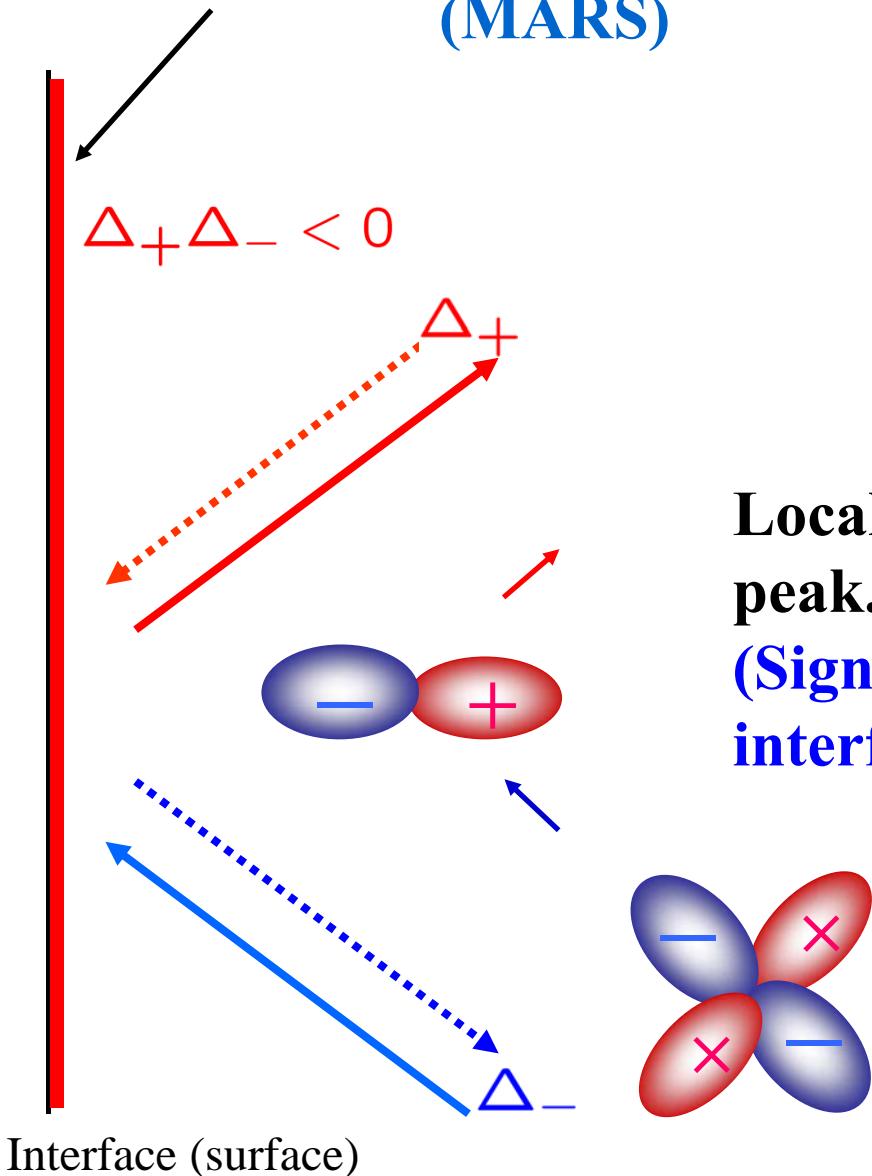
Hole-like QP

Vortex : Tanuma et al, 2009



Phase change due to a vortex

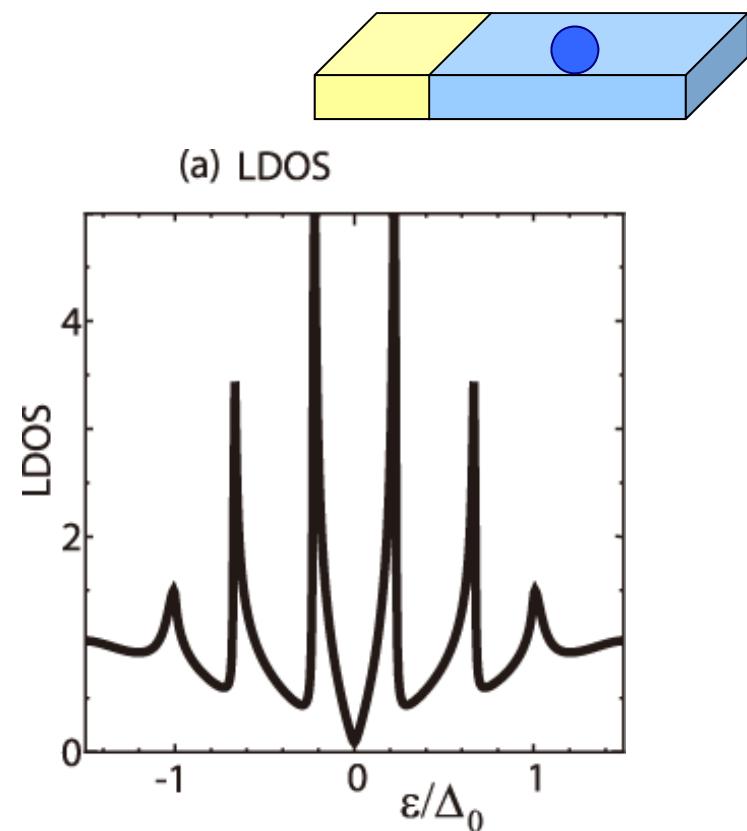
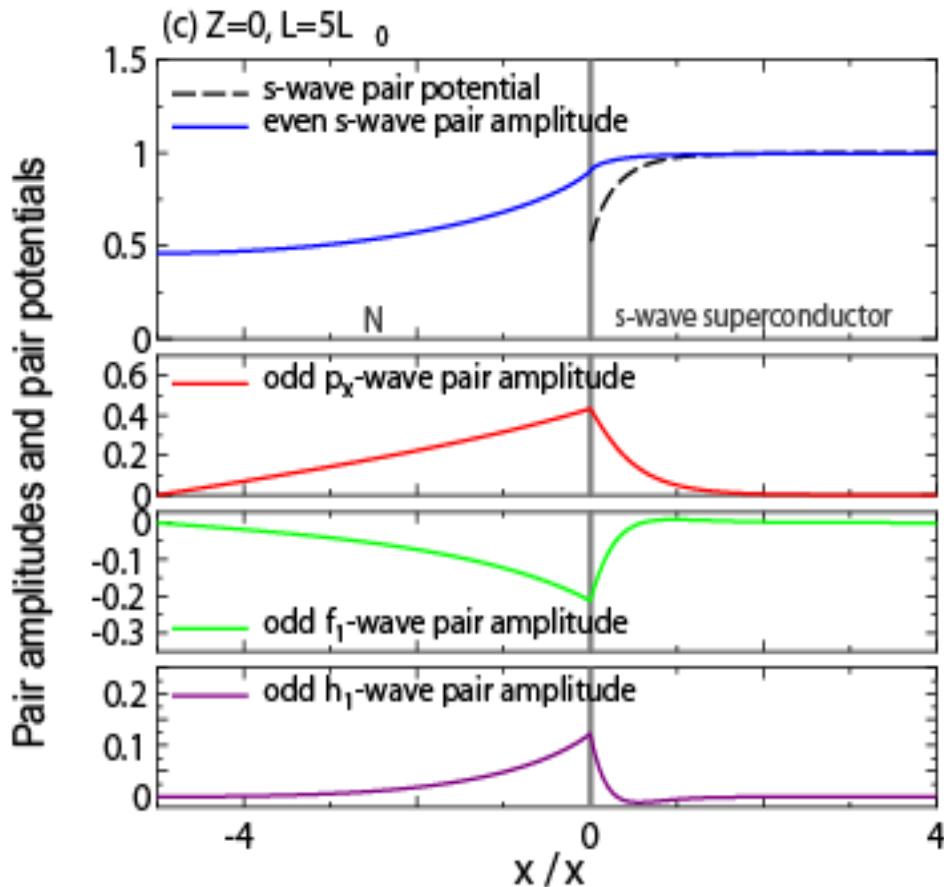
Mid gap Andreev resonant (bound) state (MARS)



Local density of state has a zero energy peak.
(Sign change of the pair potential at the interface)

Tanaka Kashiwaya PRL 74 3451 (1995),
Rep. Prog. Phys. 63 1641 (2000)
Buchholz(1981) Hara Nagai(1986)
Hu(1994) Matsumoto Shiba(1995)
Ohashi Takada(1995)
Hatsugai and Ryu (2002)

Odd-frequency pairing state in N/S junctions (N finite length)



Bounds state are formed in the normal metal

Y. Tanaka, Y. Tanuma and A.A.Golubov, Phys. Rev. B **76**, 054522 (2007)

Ratio of the pair amplitude at the N/S interface and the bound state level

$\Delta_0 \gg \varepsilon$ **Bound states condition (Z=0)**
(McMillan Thomas Rowell)

$$\varepsilon_n = \frac{\pi v_F x}{2L} (n + 1/2), \quad n = 0, 1, 2, \dots$$

$$\left| \frac{f_{1+}^{(N)}(\varepsilon, \theta)}{f_{2+}^{(N)}(\varepsilon, \theta)} \right| = |\tan(\pi/2 + \pi n)| = \infty.$$

Odd-frequency pairing

$$f_{1+}^{(N)}(\varepsilon, \theta)$$

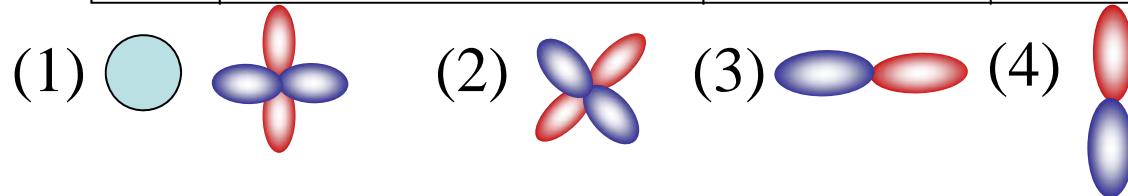
Even-frequency pairing

$$f_{2+}^{(N)}(\varepsilon, \theta)$$

Bound states are due to the generation of the odd-frequency Cooper pair amplitude

Symmetry of the Cooper pair (No spin flip)

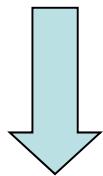
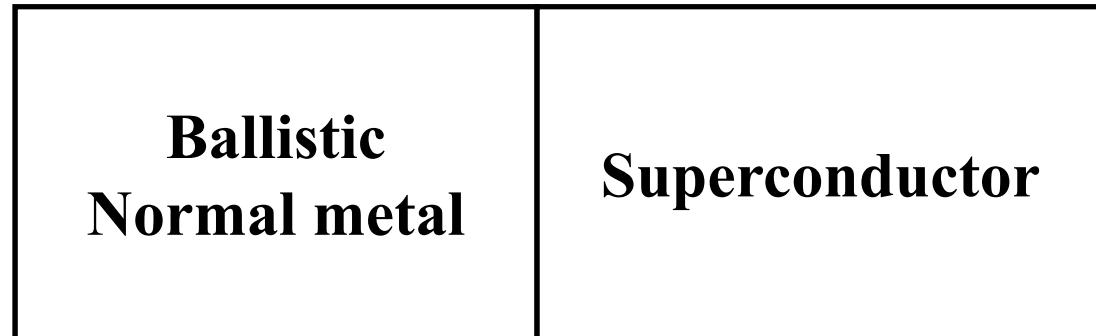
	Bulk state	Sign change (MARS)	Interface-induced symmetry (subdominant component)
(1)	ESE ($s, d_{x^2-y^2}$ -wave)	No	ESE + (OSO)
(2)	ESE (d_{xy} -wave)	Yes	OSO + (ESE)
(3)	ETO (p_x -wave)	Yes	OTE + (ETO)
(4)	ETO (p_y -wave)	No	ETO + (OTE)



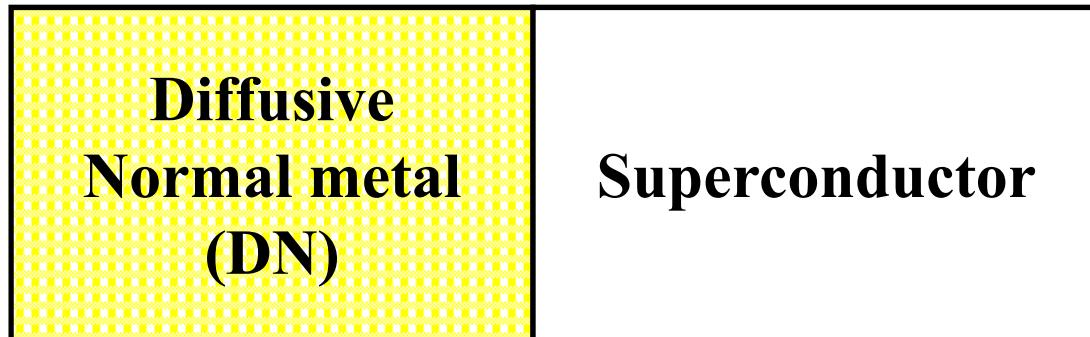
- **ESE (Even-frequency spin-singlet even-parity)**
- **ETO (Even-frequency spin-triplet odd-parity)**
- **OTE (Odd-frequency spin-triplet even-parity)**
- **OSO (Odd-frequency spin-singlet odd-parity)**

Impurity scattering effect

Tanaka and Golubov, PRL. 98, 037003 (2007)



Impurity scattering (isotropic)

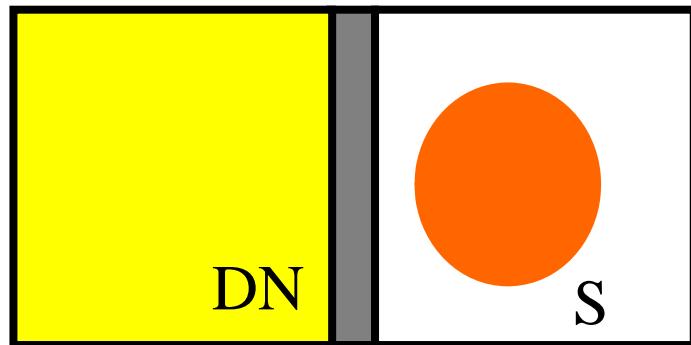


Only s-wave pair amplitude exists in DN

- (1)ESE
- (2)OTE

ESE (Even-frequency spin-singlet even-parity

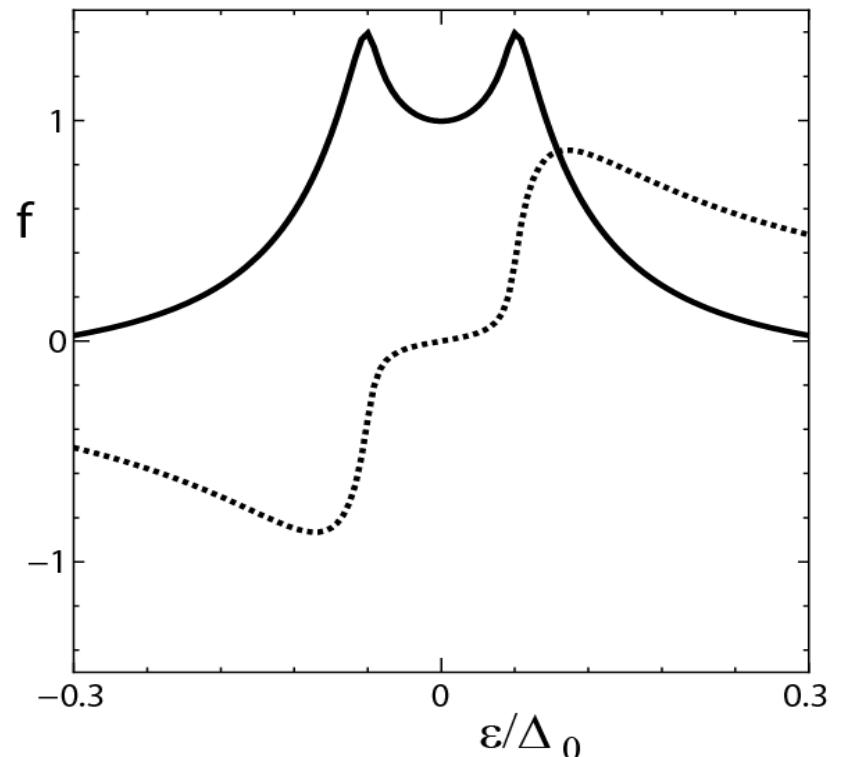
Even frequency spin singlet even parity (ESE) pair potential



$$\text{Real}[f(\varepsilon)] = \text{Real}[f(-\varepsilon)] \\ \text{Imag}[f(\varepsilon)] = -\text{Imag}[f(-\varepsilon)]$$

Even frequency spin singlet
s-wave (**ESE**) pair is induced in DN.

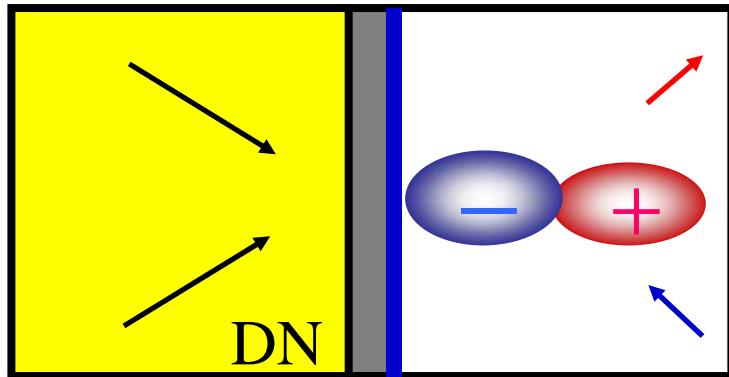
Even frequency spin singlet s-wave



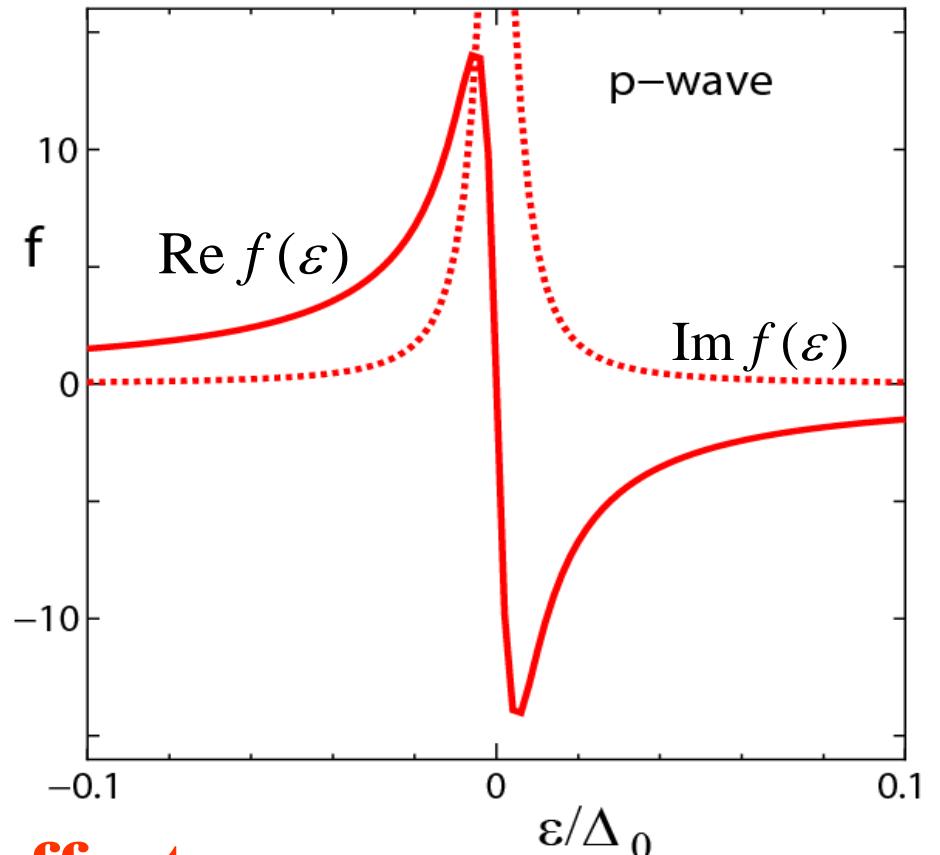
ESE pair /ESE pair potential

Odd frequency spin triplet s-wave (OTE) pair is induced in DN

P_x -wave case



Even frequency spin triple p-wave

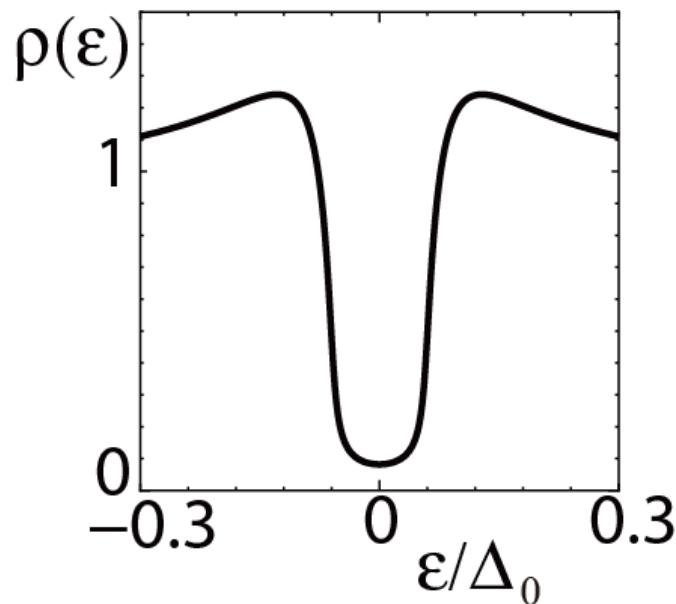
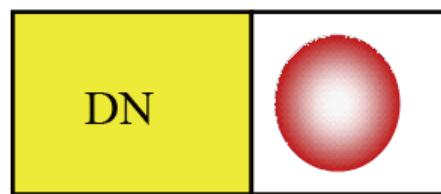


New type of proximity effect

Y.Tanaka, A.A.Golubov, Phys.Rev.Lett. **98**, 037003 (2007)

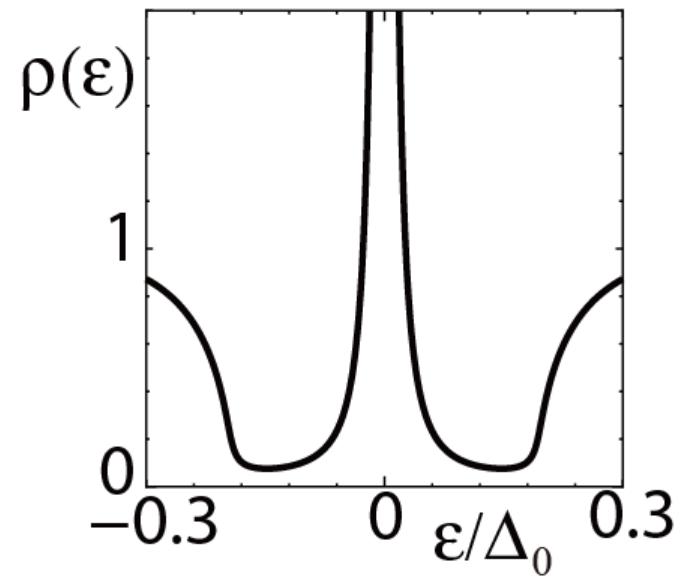
Density of states in DN

(a) s-wave



Conventional proximity effect with
Even-frequency Cooper pair in DN

(b) p_x-wave



Unconventional proximity effect with
Odd-frequency Cooper pair in DN

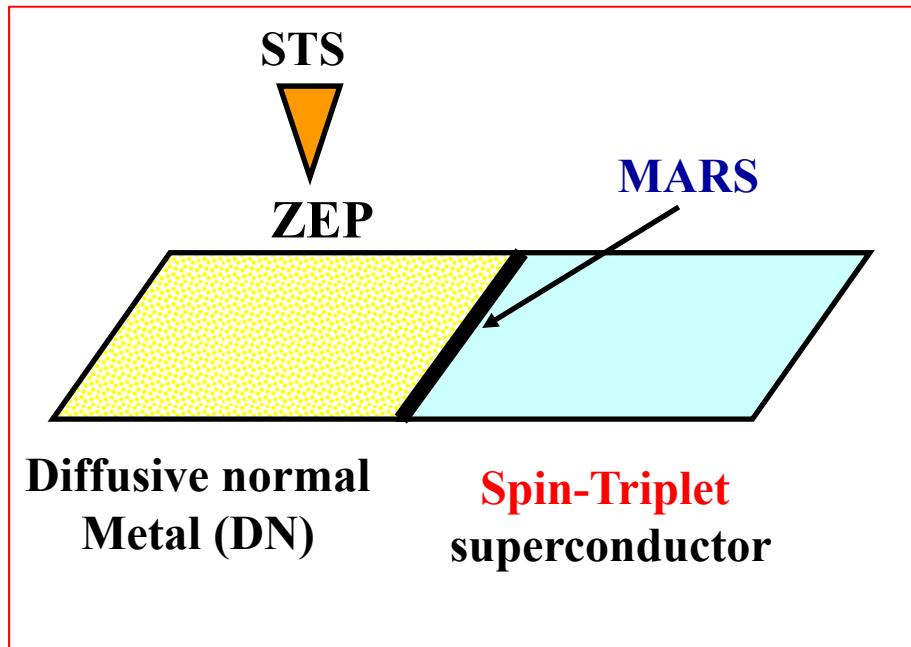
Tanaka&Kashiwaya, 2004

Summary of Proximity effect (diffusive normal metal, *s*-wave pairing state only)

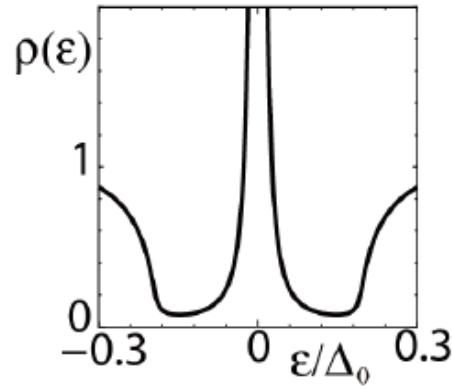
	Symmetry of the pair potential	Induced pair amplitude in DN
(1)	Even frequency spin singlet even parity (ESE)	ESE
(2)	Even frequency spin triplet odd parity (ETO)	OTE
(3)	Odd frequency spin triplet even parity (OTE)	OTE
(4)	Odd frequency spin singlet odd parity (OSO)	ESE

STS experiments

Proximity effect via odd-frequency pairing

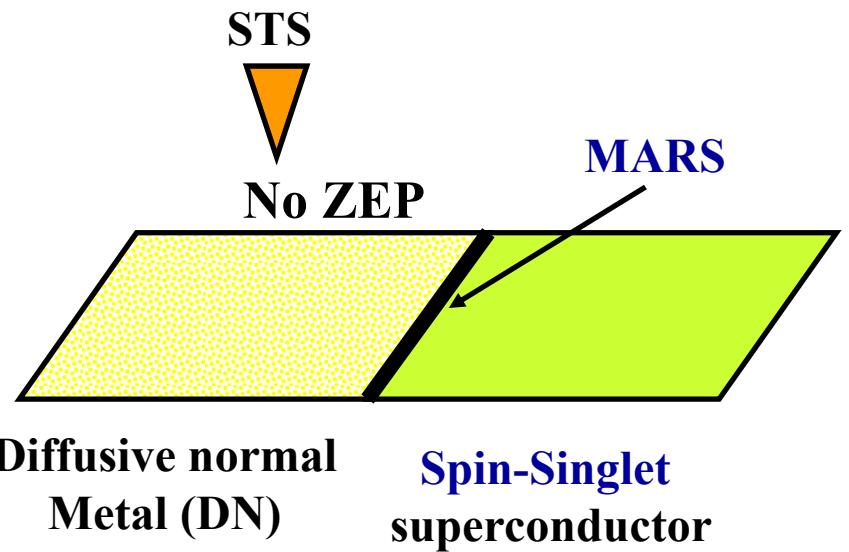


LDOS in DN has a zero energy peak

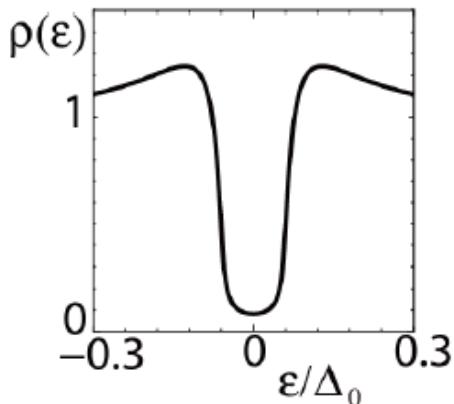


DoS peak is robust again imputiry satterin

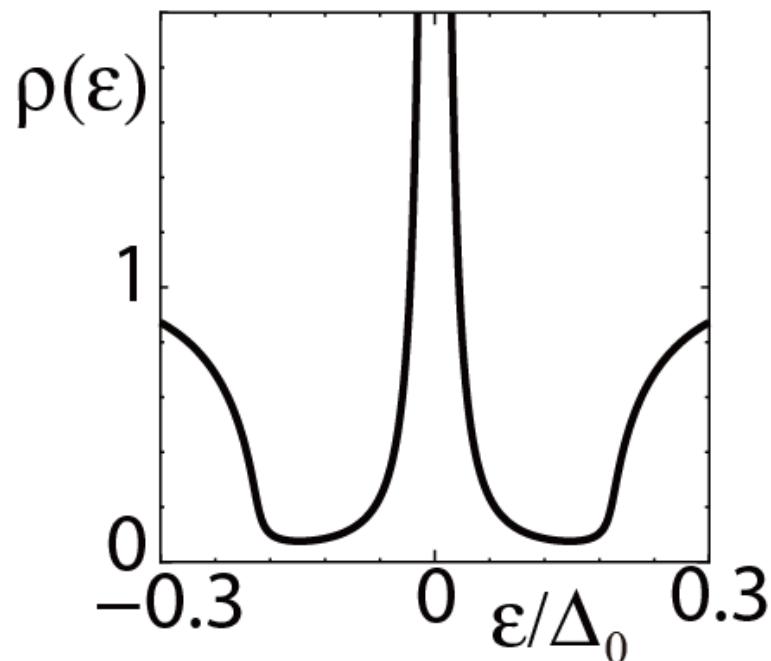
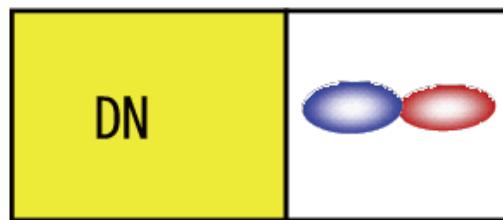
Conventional proximity effect



LDOS in DN has a gap



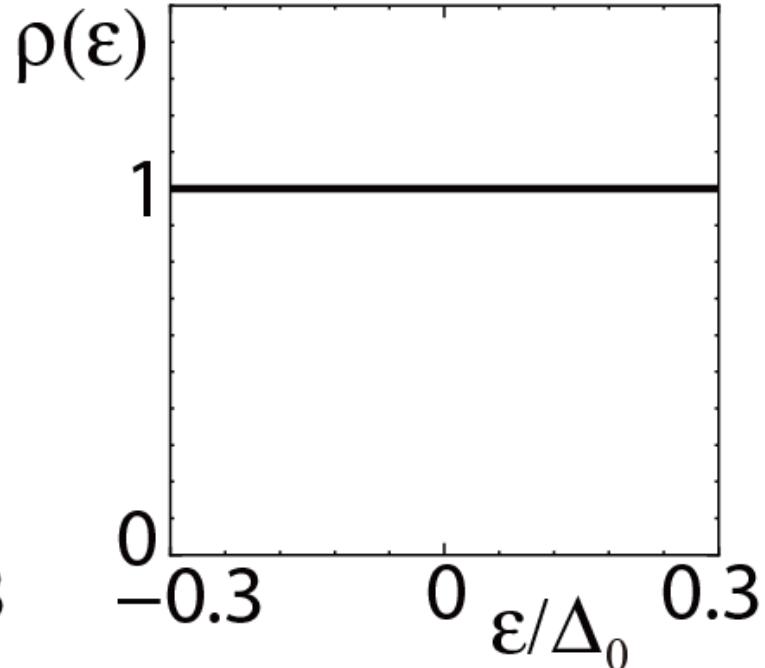
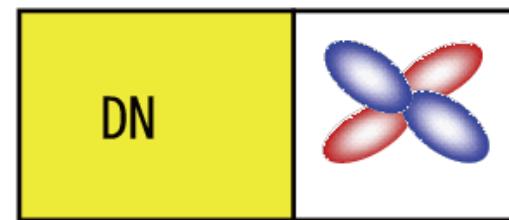
(b) p_x -wave



Unconventional proximity effect

Odd-frequency pairing at the interface includes s-wave component

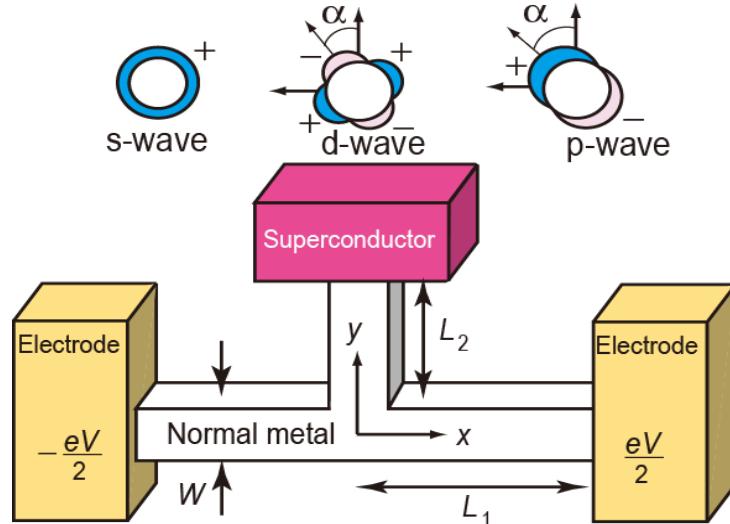
(c) d_{xy} -wave



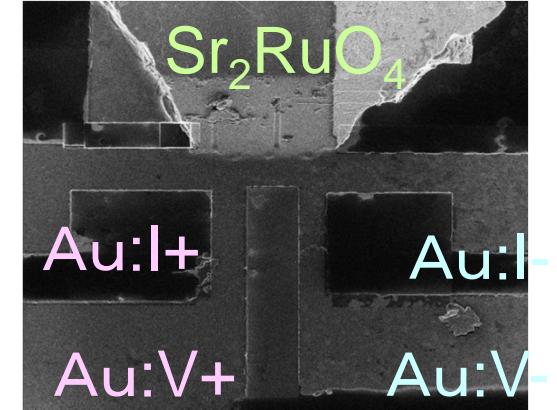
No proximity effect

Odd-frequency pairing at the interface: Odd-parity

How to detect odd-frequency paring amplitude: *measuring electrical conductivity*



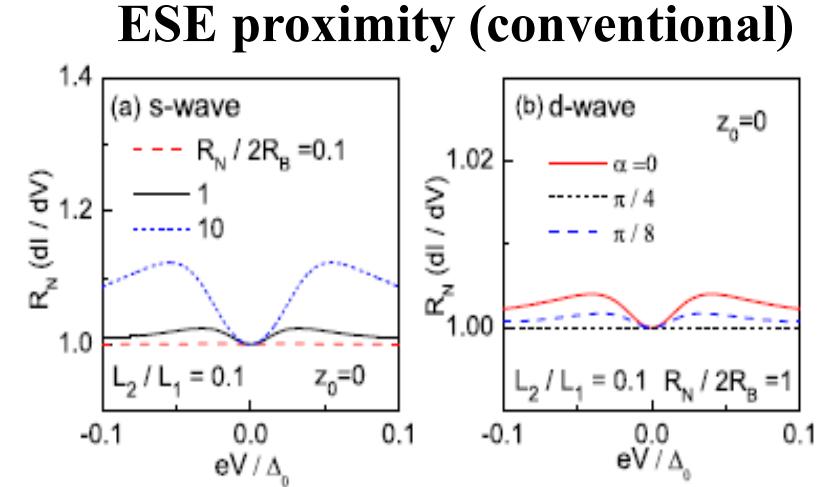
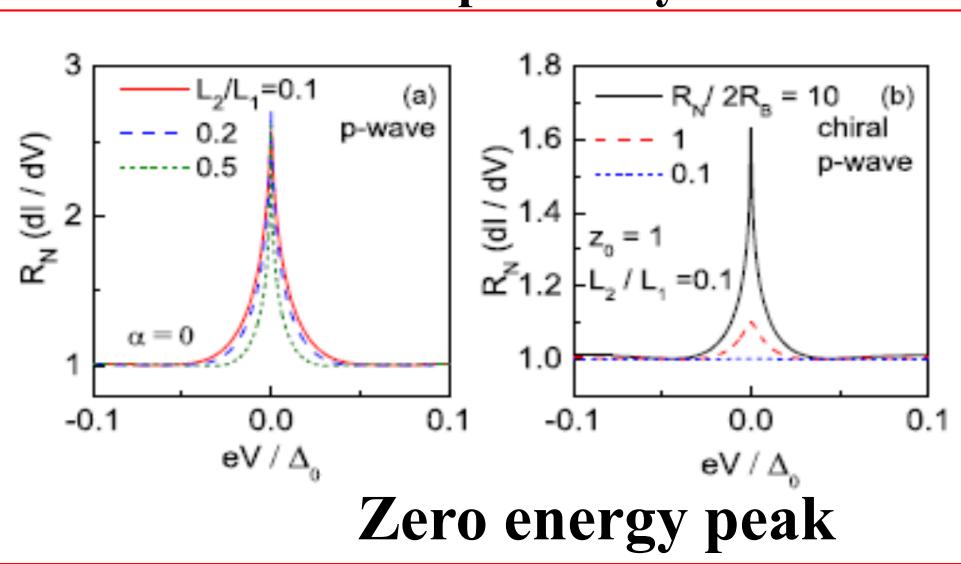
Asano, Tanaka, Golubov, Kashiwaya, PRL 99, 067005 (2007)



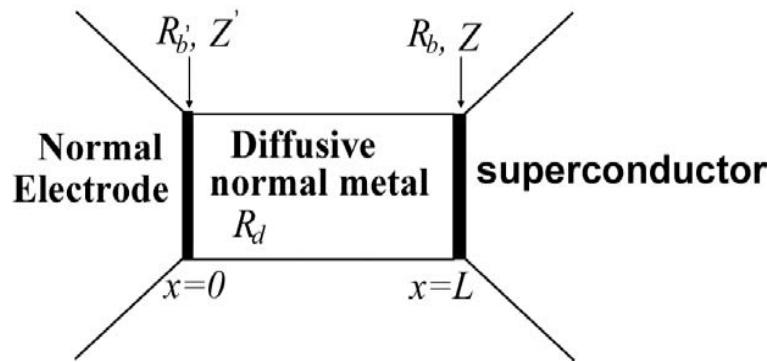
Kashiwaya, Maeno 2007

**OTE (Odd-frequency
spin-triplet even-parity)**

**ESE (Even-frequency
spin-singlet even-parity)**



Anomalous Meissner effect



$$\hat{R}_N(x) = \sin \theta \hat{\tau}_2 + \cos \theta \hat{\tau}_3$$

$$j(x) = \pi e^2 N(0) D T \sum_{\omega_n} \text{Trace}[\hat{\tau}_3 \hat{R}_N(x) [\hat{\tau}_3, \hat{R}_N(x)]] A(x)$$

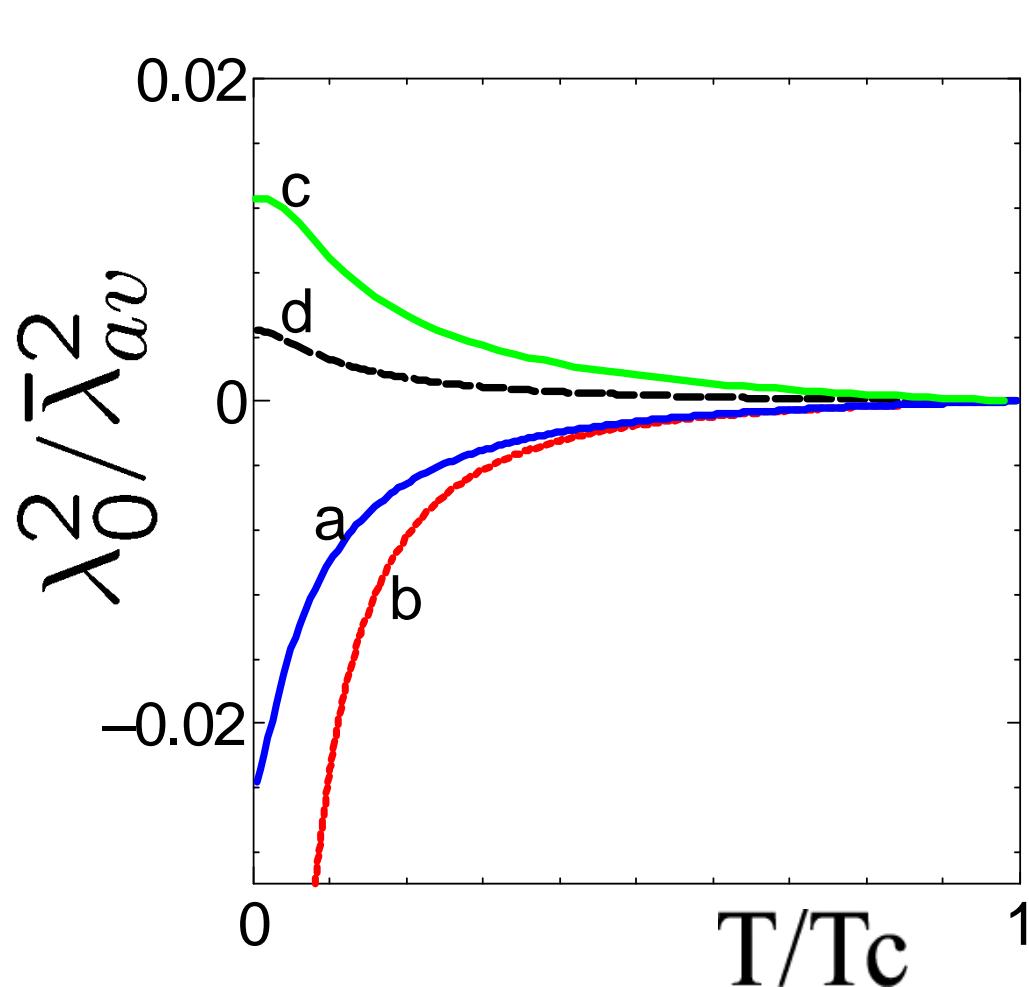
$$H(x) \sim \exp(-x/\lambda(x))$$

$$\frac{1}{\lambda^2(x)} = \frac{T \sum_{\omega_n} \sin^2 \theta(\omega_n)}{\lambda_0^2}, \quad \lambda_0^{-2} = 32\pi^2 e^2 N(0) D T_C$$

$$\bar{\lambda}_{av}^2 = L / \int_0^L \frac{dx}{\lambda^2(x)}$$

Narikiyo and Fukuyama, J. Phys. Soc. Jpn. 58, 4557 (1989)
 Belzig and Bruder PRB 53 5727 (1996)

Temperature dependence of averaged value of local penetration depth

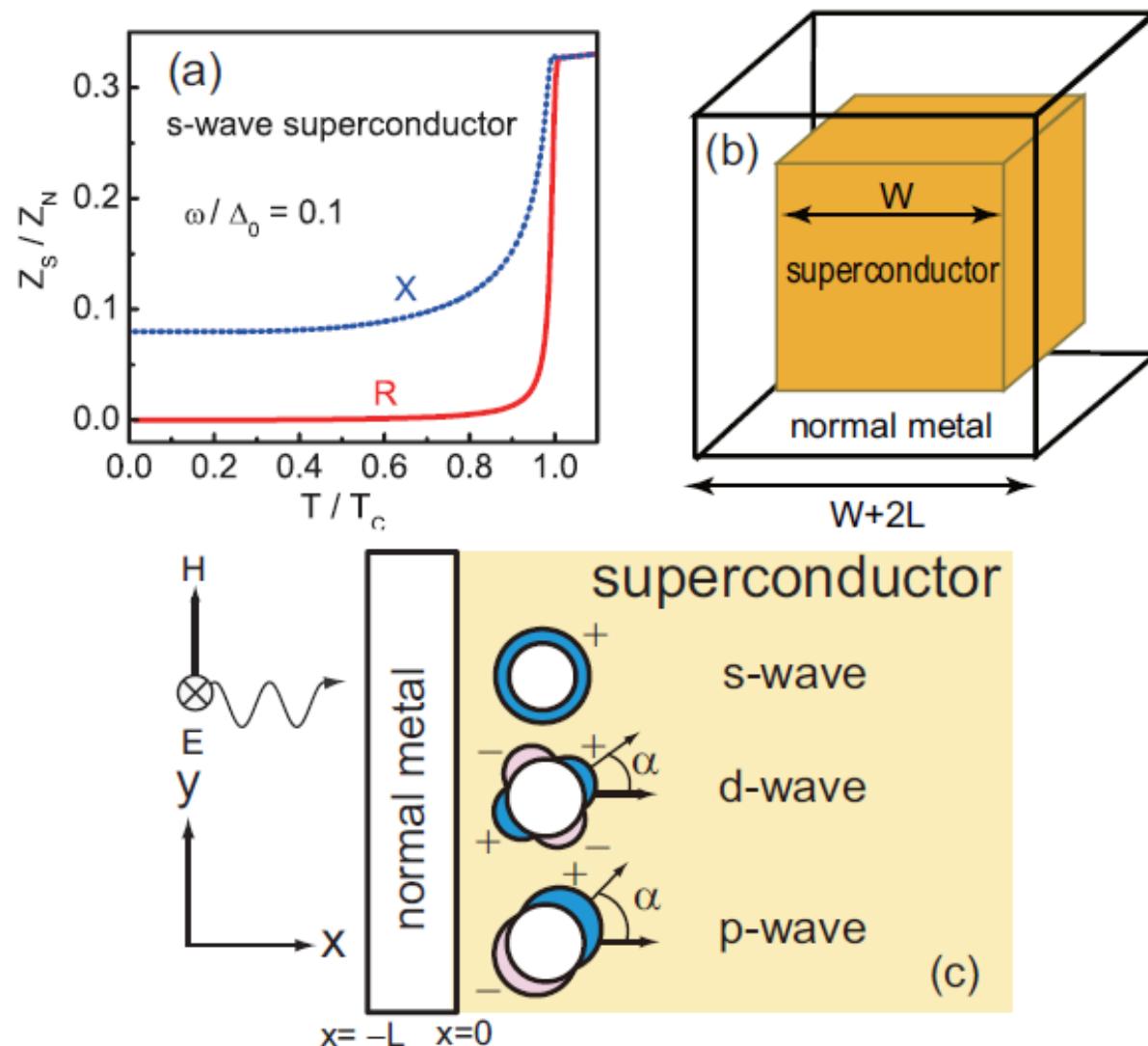


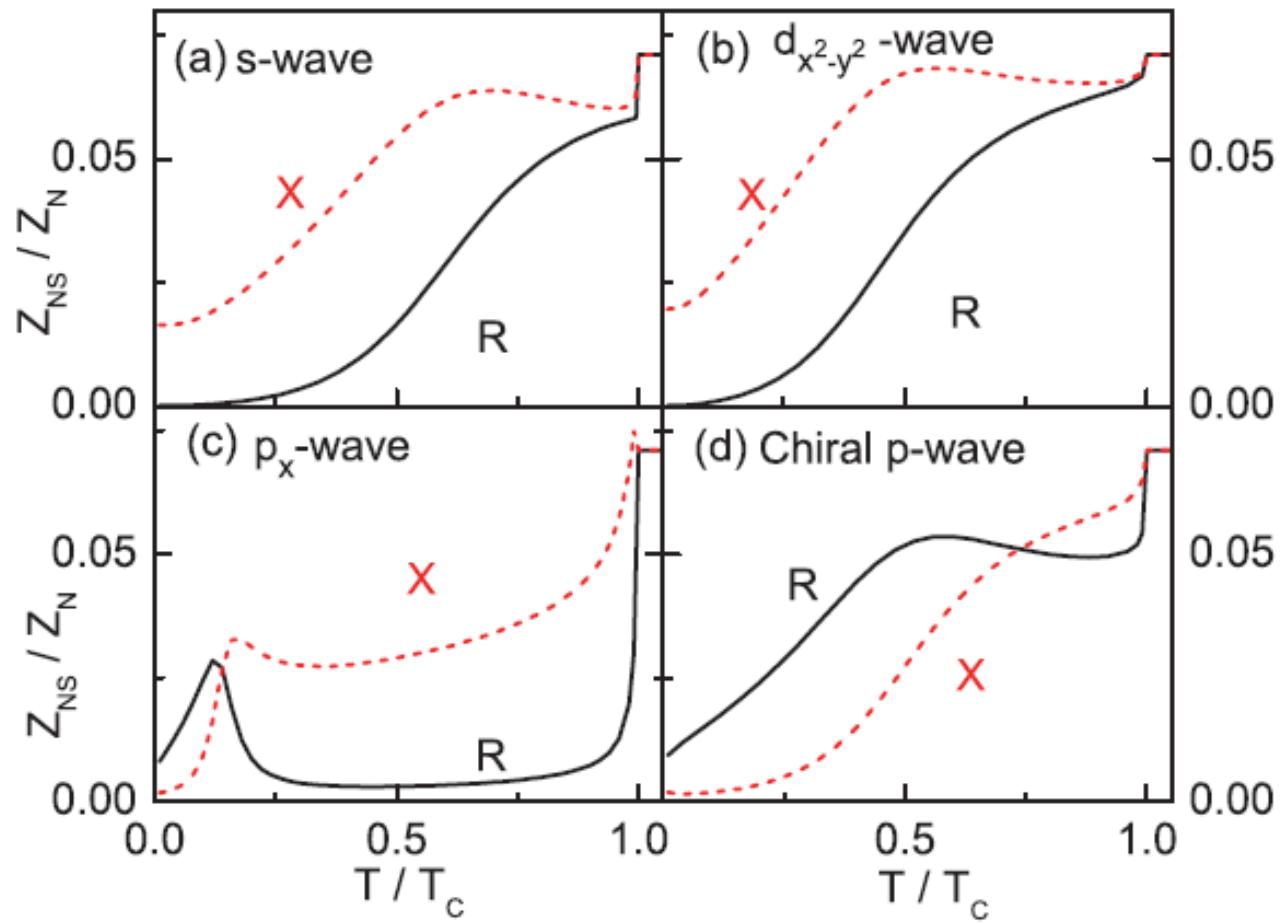
$\bar{\lambda}_{av}$

a purely imaginary number
for spin-triplet junctions

- a: $p_x + ip_y$ -wave
- b: p_x -wave
- c: s -wave
- d: $d_{x^2-y^2} + id_{xy}$ -wave

How to detect anomalous Meissner effect? Surface impedance



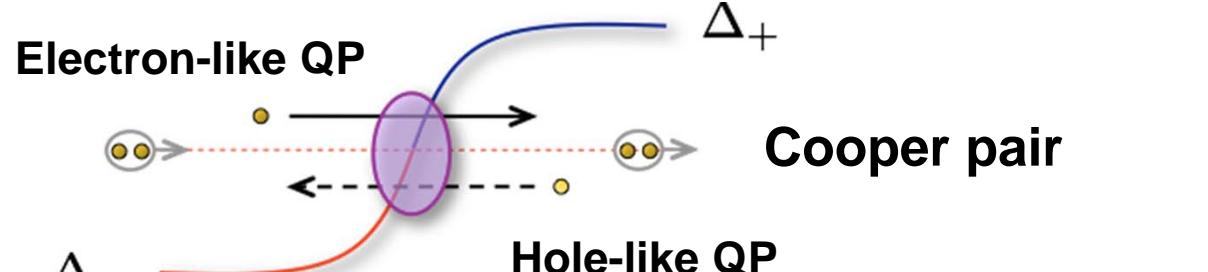


**Robust relation $R < X$ holds in an even-frequency state
This relation violates in an odd-frequency state at low T**

Y.Asano, A.A.Golubov, Ya.V.Fominov, and Y. Tanaka, PRL (2011)

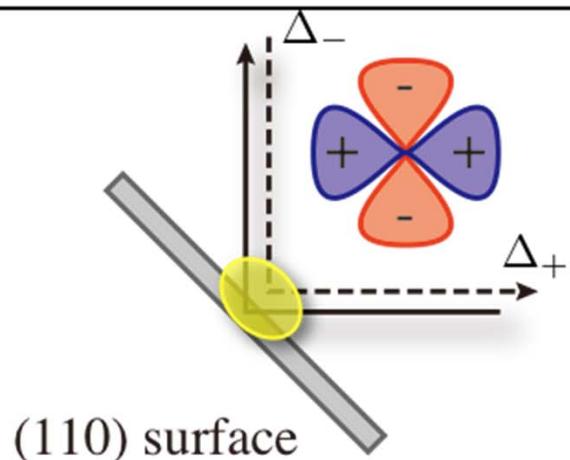
Andreev bound states in inhomogeneous systems are manifestations of odd-frequency pairing amplitude

Andreev bound states



Negative pair potential

Surface: Tanaka et al, 2007

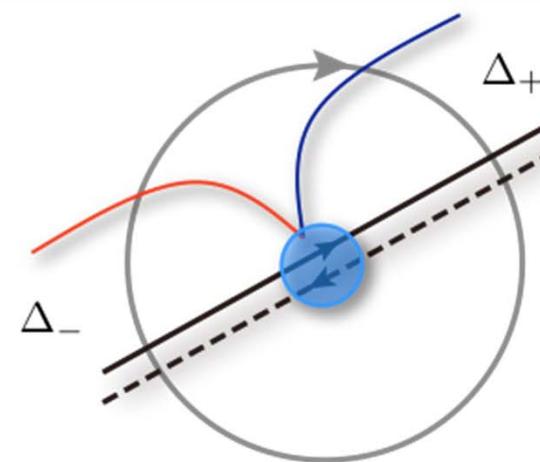


Scattering direction of QP

Positive pair potential

Hole-like QP

Vortex : Tanuma et al, 2009



Phase change due to a vortex

Symmetry of the Cooper pair in a vortex core

l ; angular momentum	m ; vorticity	bulk	Center of the vortex core
Even	Even	ESE (s-wave..)	ESE
Even	Odd	ESE (s-wave..)	OSO
Odd	Even	ETO (chiral p-wave)	ETO
Odd	Odd	ETO (chiral p-wave)	OTE

$$\Delta(\mathbf{r}) = \Delta_0 \exp(il\varphi) \tanh\left(\frac{\sqrt{x^2 + y^2}}{\xi}\right) \left(\frac{x + iy}{\sqrt{x^2 + y^2}}\right)^m$$

ESE (Even-frequency spin-singlet even-parity)

ETO (Even-frequency spin-triplet odd-parity)

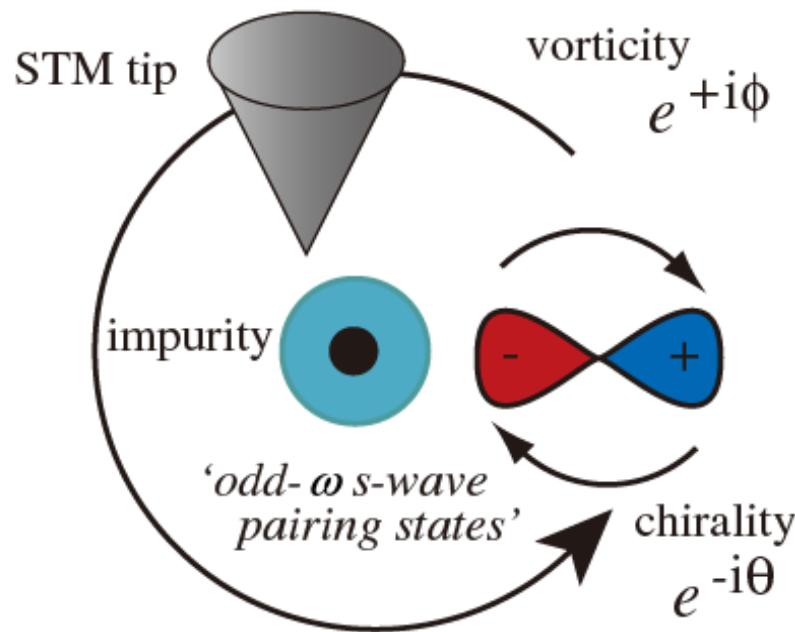
OTE (Odd-frequency spin-triplet even-parity)

OSO (Odd-frequency spin-singlet odd-parity)

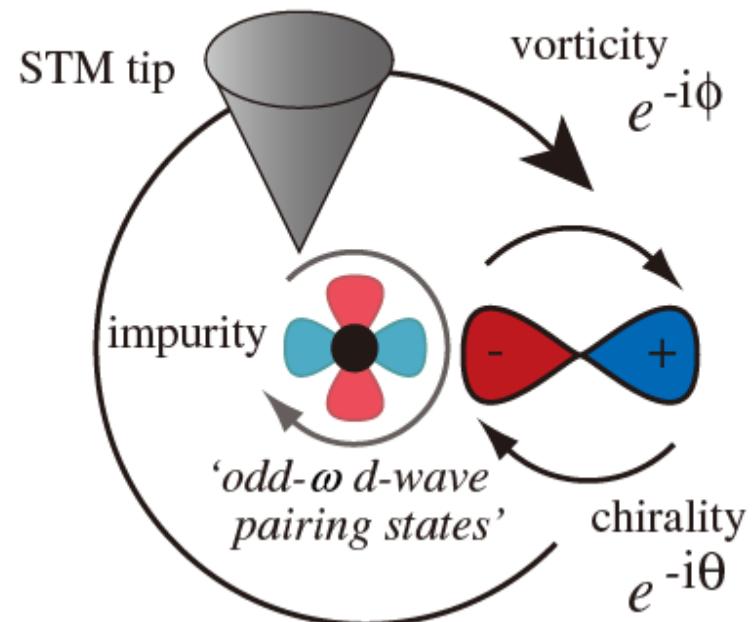
Yokoyama *et al.*, Physical Review B, Vol. 78, 012508, 2008

Difference of the angular momentum of the odd-frequency pair at the core center

(a) Antiparallel chiral p-wave vortex



(b) Parallel chiral p-wave vortex

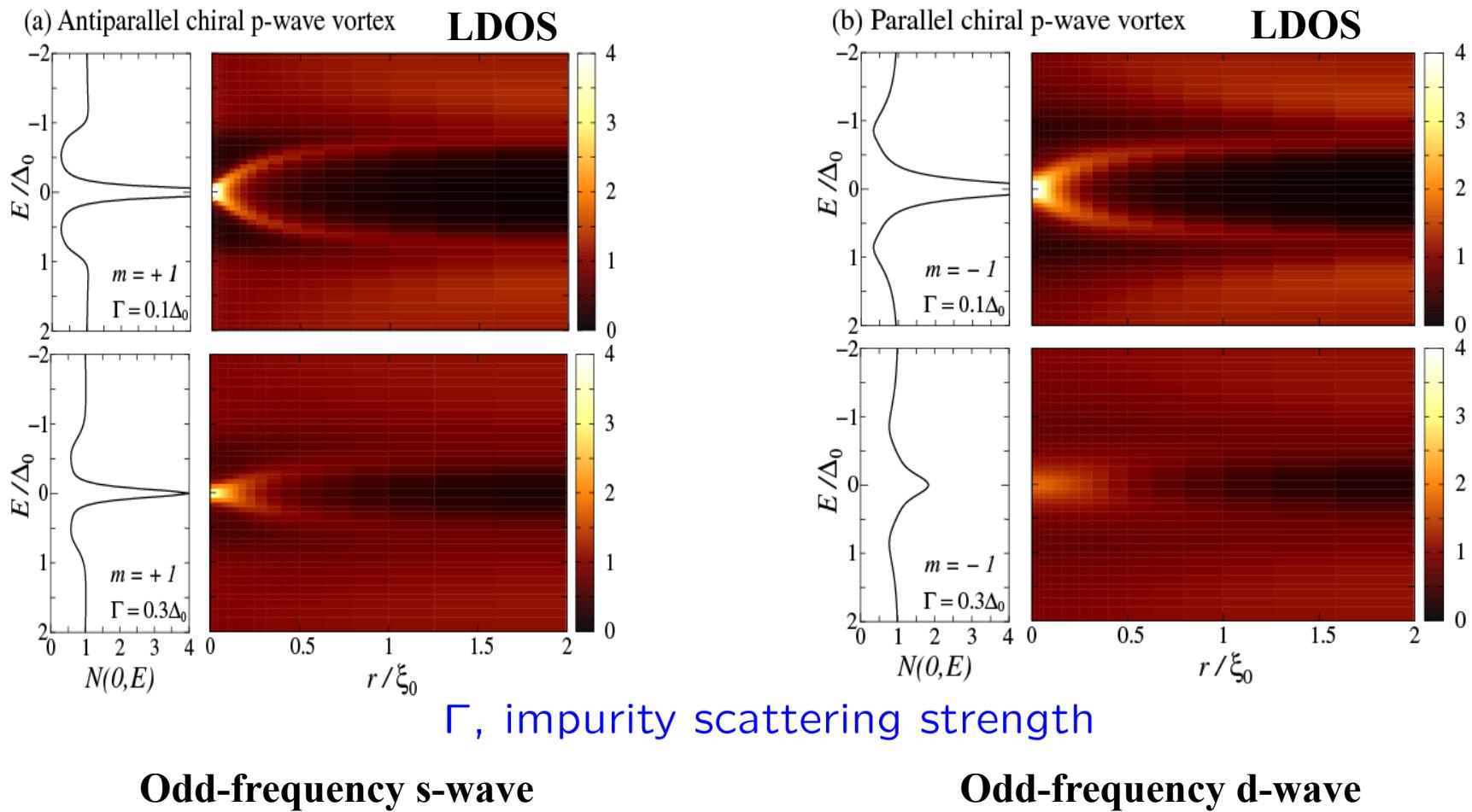


Angular momentum at the center of core; $\textcolor{red}{l+m}$

l : angular momentum

m : vorticity

Impurity effect (Born approximation)

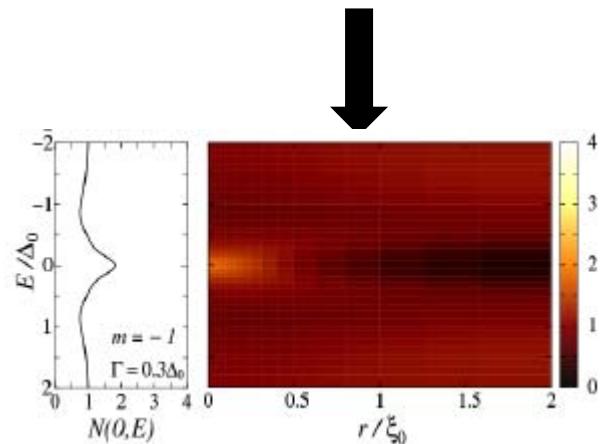


Odd frequency s-wave state; more robust against the impurity scattering Tanuma, Hayashi, Tanaka Golubov, PRL 102 117003 (2009).

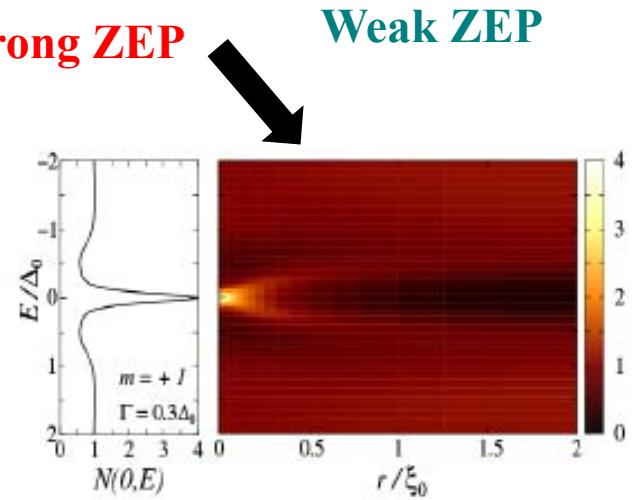
Vortex core spectroscopy in chiral domain

Parallel vortex	Antiparallel vortex	Parallel vortex	Antiparallel vortex
Odd d-wave	Odd s-wave	Odd d-wave	Odd s-wave
$p_x + ip_y$	$p_x - ip_y$	$p_x + ip_y$	$p_x - ip_y$

Weak ZEP



Strong ZEP



Weak ZEP

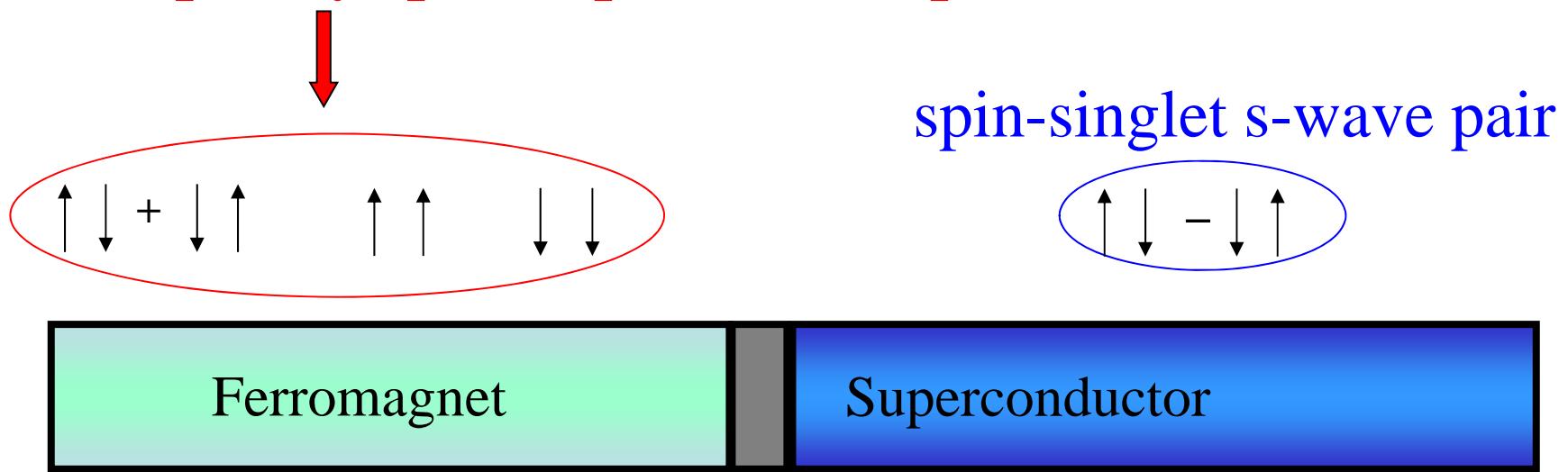
Strong ZEP

Summary (vortices)

- (1) In Abrikosov vortex ($m=1$), only the odd-frequency Cooper pairing is possible at the center of the vortex core.
- (2) Vortex core spectroscopy in chiral p-wave superconductor in the presence of impurity enables one to identify the presence of chirality and the odd-frequency pairing.

Odd-frequency pair amplitude (not pair potential) is generated in ferromagnet junctions

Odd frequency spin-triplet s-wave pair



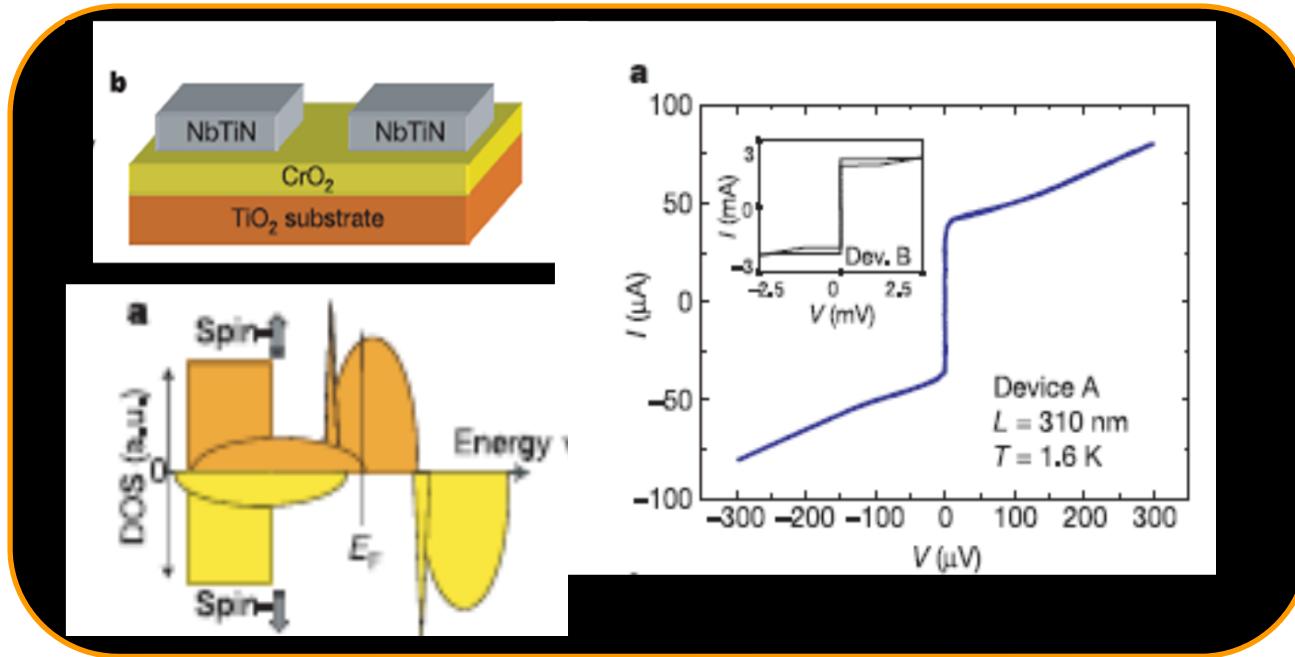
Bergeret, Efetov, Volkov, (2001)

Eschrig, Buzdin, Kadigrobov, Fominov, Radovic

Generation of the odd-frequency pair amplitude
in ferromagnet

Josephson current in S/HM/S

Half metal : CrO₂ Keizer *et.al.*, Nature (2006)



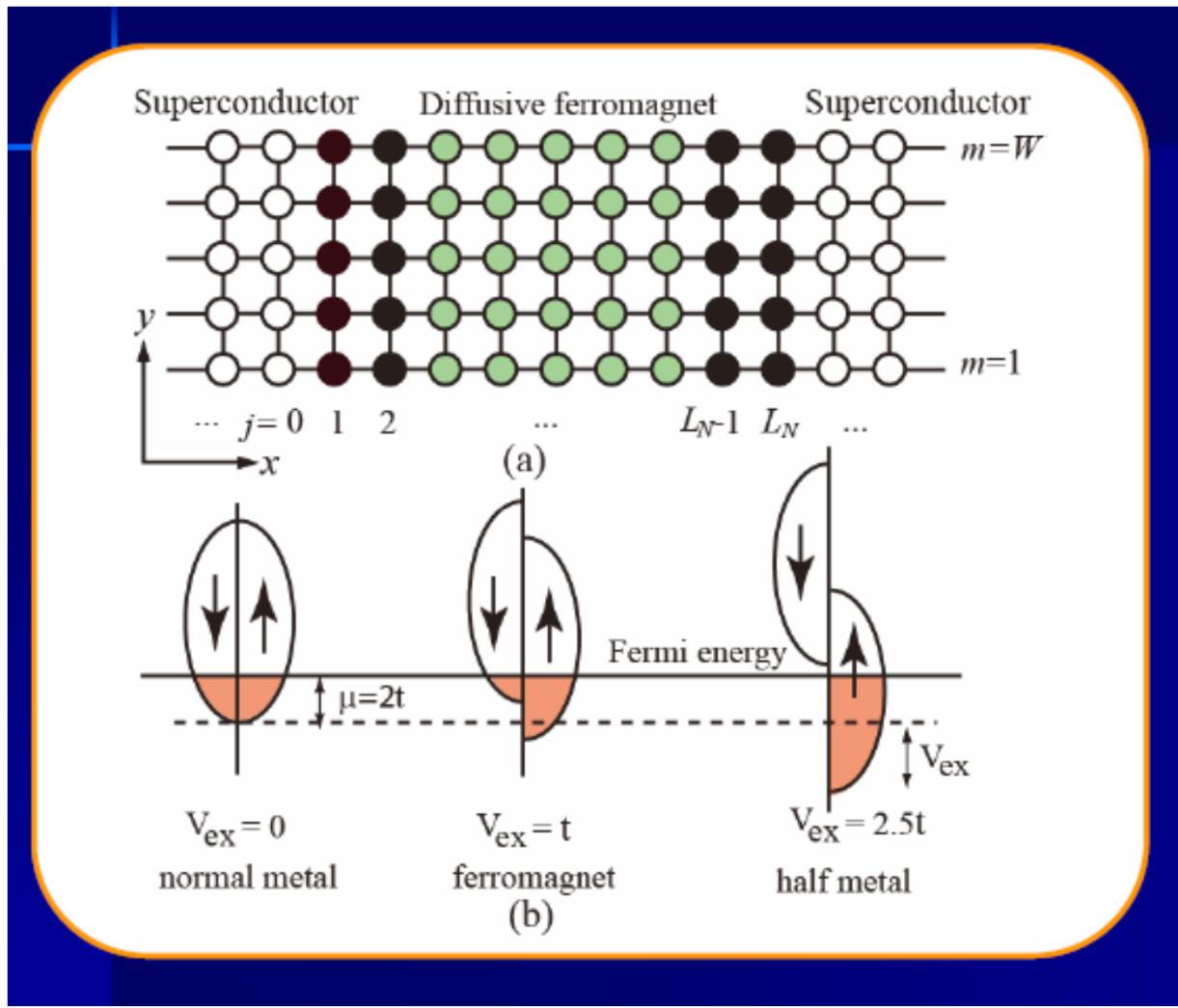
Spin active interface Bergeret et. al., PRL('01),
Kadigrobov et. al., Europhys Lett.('01)

Theory in the **clean limit** Eschrig et. al., PRL('03)

Lofwander and Eschrig, Nature Physics (2008)

Recursive GF

Furusaki, Physica B('92),
Asano, PRB('01)



Advantages

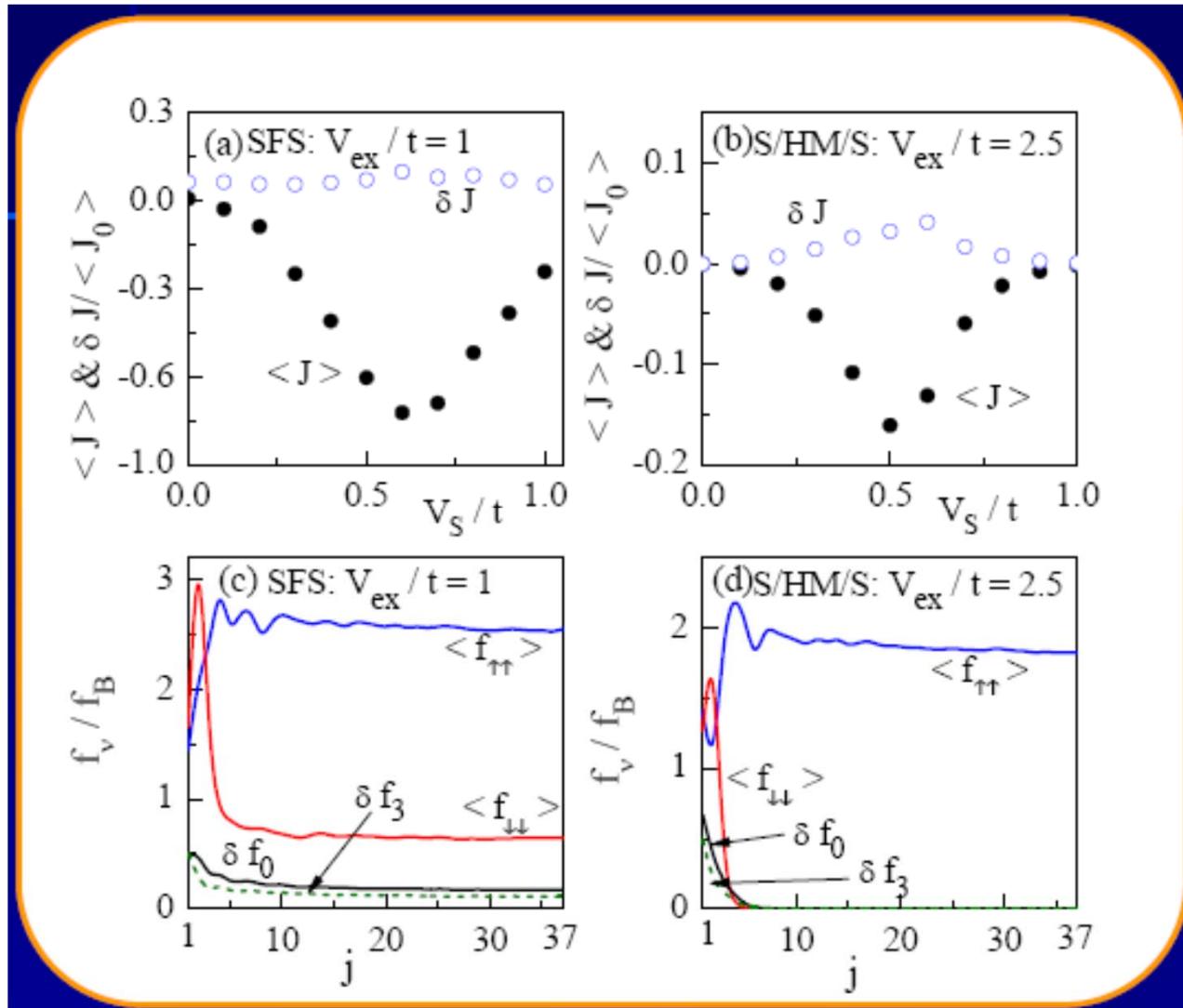
SNS, SFS, S/HM/S

$$\langle J \rangle = \sum_{i=1}^{N_S} J_i$$
$$\delta J = \sqrt{\langle J^2 \rangle - \langle J \rangle^2}$$

Parameters

V_{ex} : exchange
 V_S : spin-flip

Spin active interface



SFS $\langle f_0 \rangle$

$\langle f_{\uparrow\uparrow} \rangle, \langle f_{\downarrow\downarrow} \rangle, \langle f_3 \rangle$

S/HM/S

$\langle f_{\uparrow\uparrow} \rangle$ only

$$\mathbf{V} \cdot \boldsymbol{\sigma} = V_s \sigma_2$$

SFS, S/HM/S

$$\langle J \rangle > \delta J$$

self-averaging

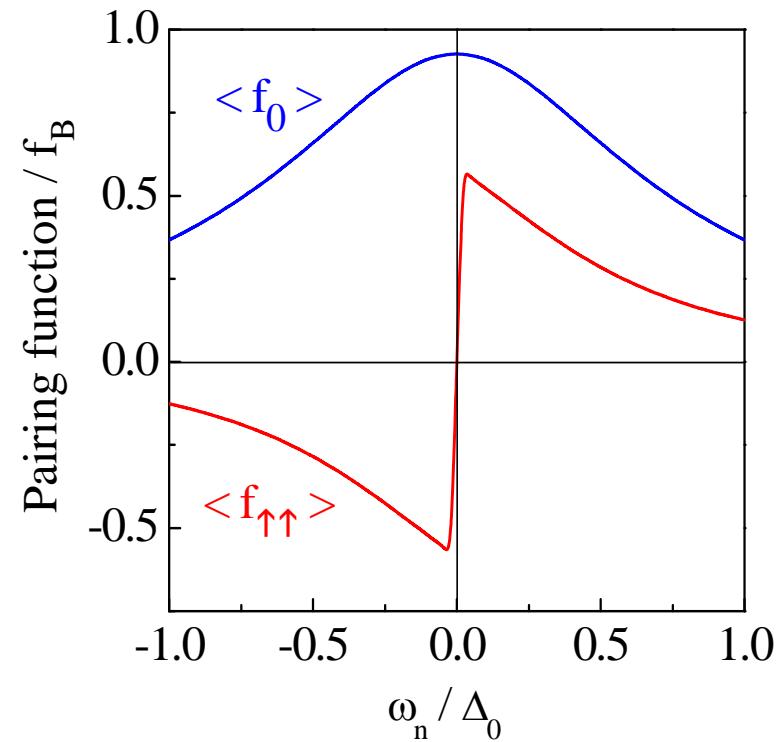
$$\langle f_{\uparrow\uparrow} \rangle \gg \langle f_{\downarrow\downarrow} \rangle, \langle f_0 \rangle, \langle f_3 \rangle$$

$$\langle f_{\uparrow\uparrow} \rangle > \delta f_{\uparrow\uparrow}$$

No sign change

odd-frequency pairs

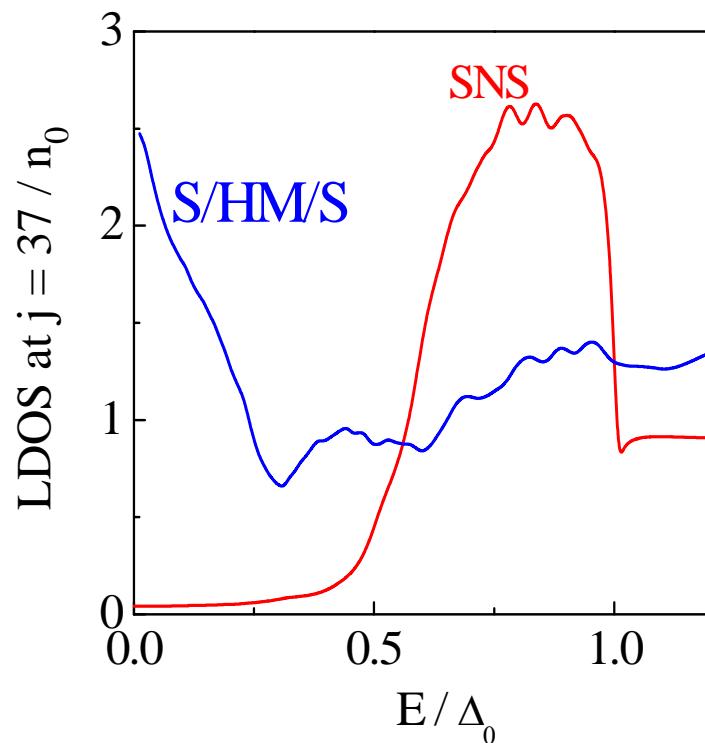
Odd-frequency pairs



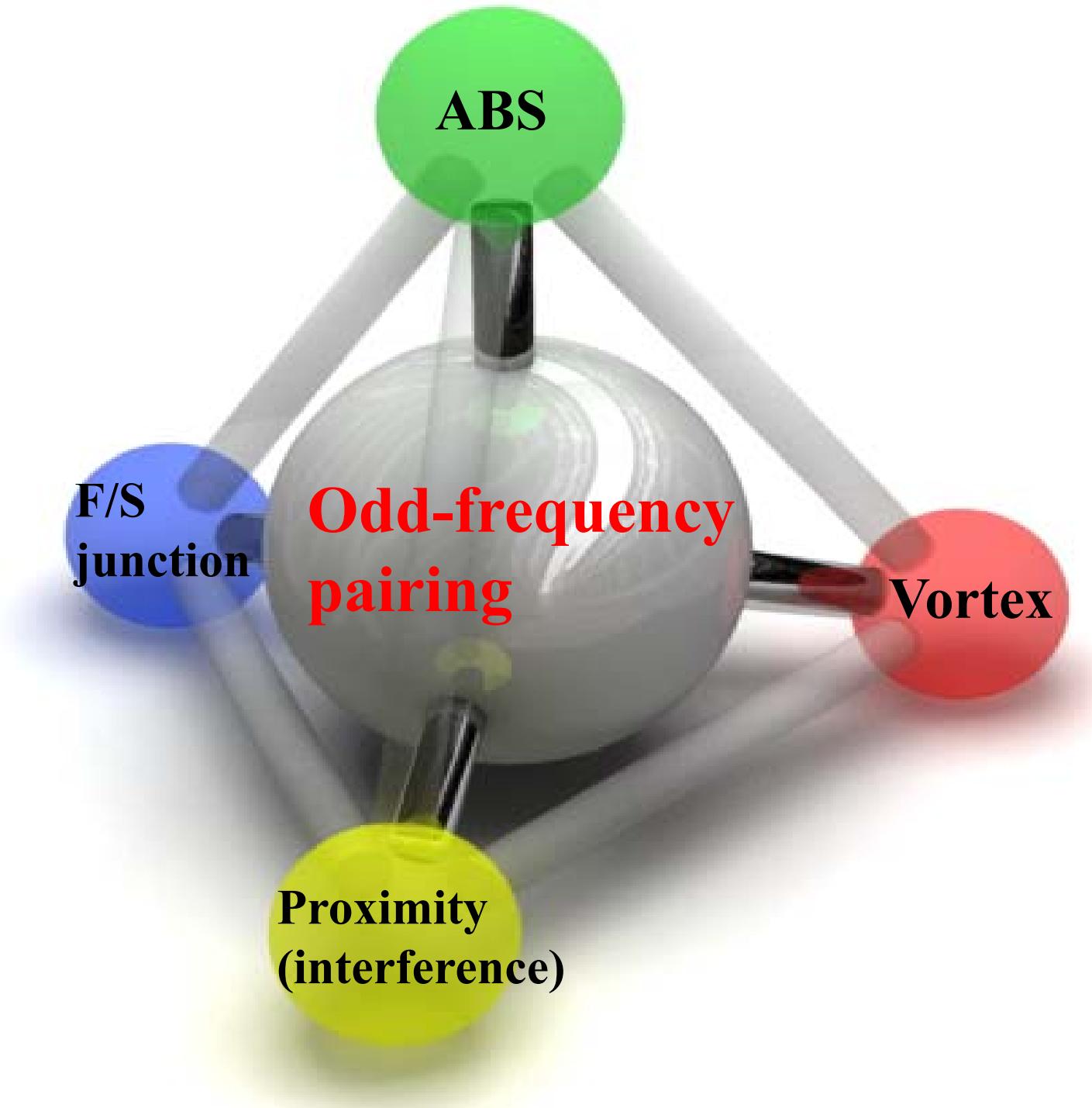
Quasiparticle DOS in Half Metal

SFS $\langle f_0 \rangle$
 $\langle f_{\uparrow\uparrow} \rangle, \langle f_{\downarrow\downarrow} \rangle, \langle f_3 \rangle$
even-odd mix

S/HM/S
 $\langle f_{\uparrow\uparrow} \rangle$ only
pure odd



Zero Energy Peak
can be detected
by tunneling spectroscopy



Summary

- (1) Ubiquitous presence of the **odd-frequency pairs** in **inhomogeneous** systems.
- (2) Low energy Andreev bound states can be expressed in terms of odd-frequency pairing (**proximity effect** and **vortices**).
- (3) The origin of the **anomalous proximity effect** in **DN/spin-triplet p-wave junction** is the generation of the odd-frequency pairing state.
- (4) Odd-frequency triplet pairs are generated in **S/F** junctions.