φ, φ_0, $\varphi_0 \pm \varphi$ Josephson junctons based on a $0-\pi$ SQUID

Edward Goldobin[1],
D. Koelle[1], R. Kleiner[1]

[1] *University of Tübingen, Germany*

Deutsche Forschungsgemeinschaft DFG

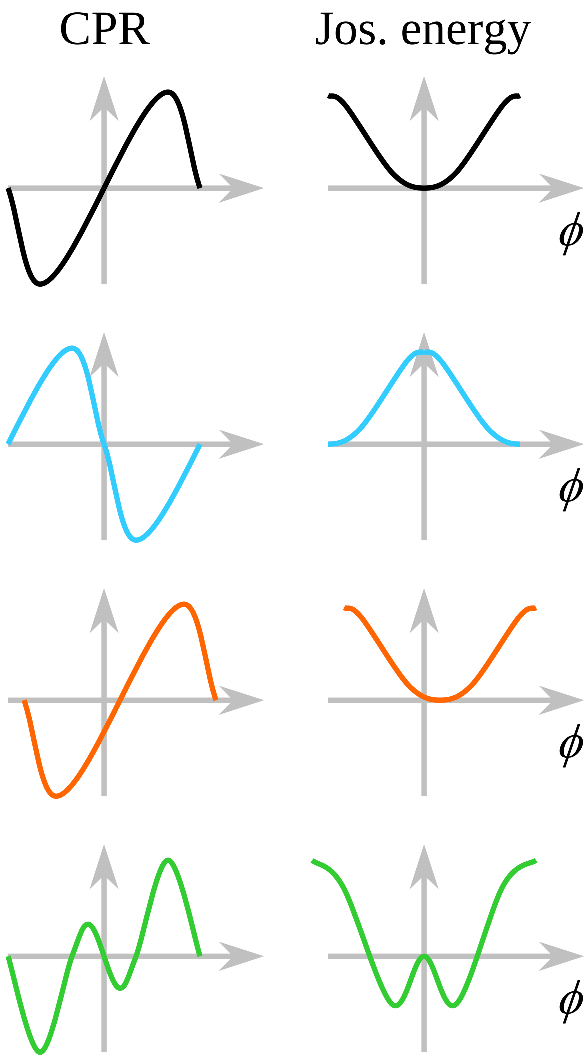
The rough guide to JJ terminology

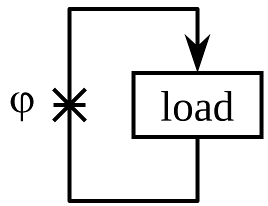
A 0-JJ:
- CPR e.g. $I_s \sim I_c \sin(\phi)$
- ground state: $\phi=0$

A π-JJ:
- CPR e.g. $I_s \sim I_c \sin(\phi-\pi)$
- ground state: $\phi=\pi$

A φ_0-JJ:
- CPR e.g. $I_s \sim I_c \sin(\phi-\varphi_0)$
- ground state: $\phi=\varphi_0$

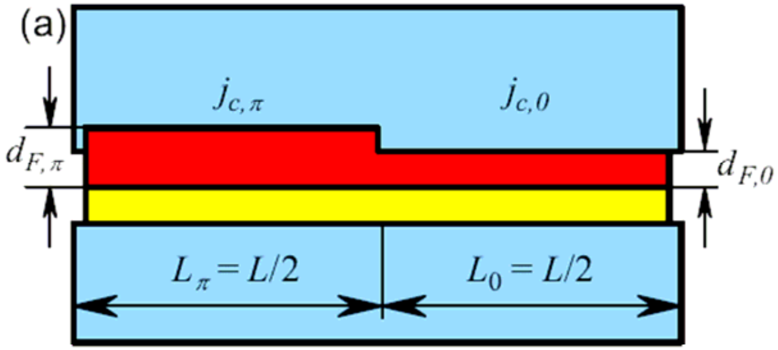
A φ-JJ:
- CPR e.g. $I_s=I_{c1}\sin(\phi)+I_{c2}\sin(2\phi)$
- ground states: $+\varphi$ and $-\varphi$
  $\varphi = \arcsin(-I_{c1}/2I_{c2})$, $I_{c2}<-I_{c1}/2$

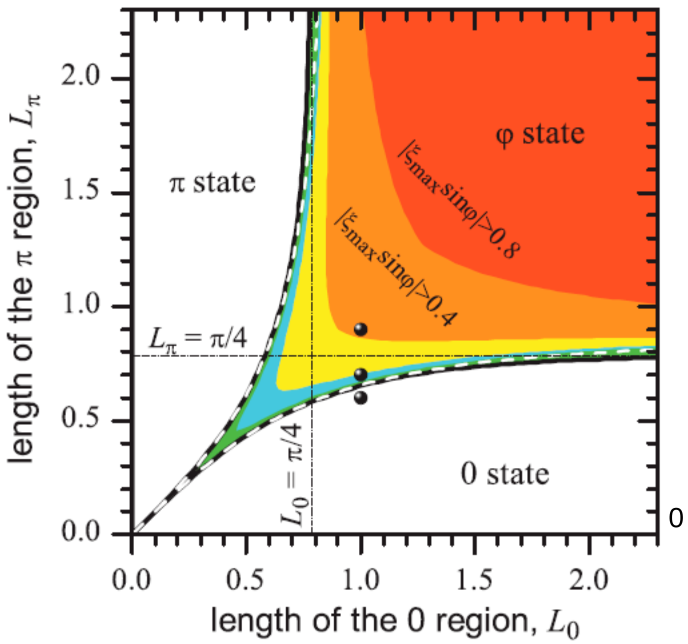




Ortlepp et al., Science **312**, 1495 (2006)
Feofanov et al., Nat. Phys. **6**, 593 (2010)

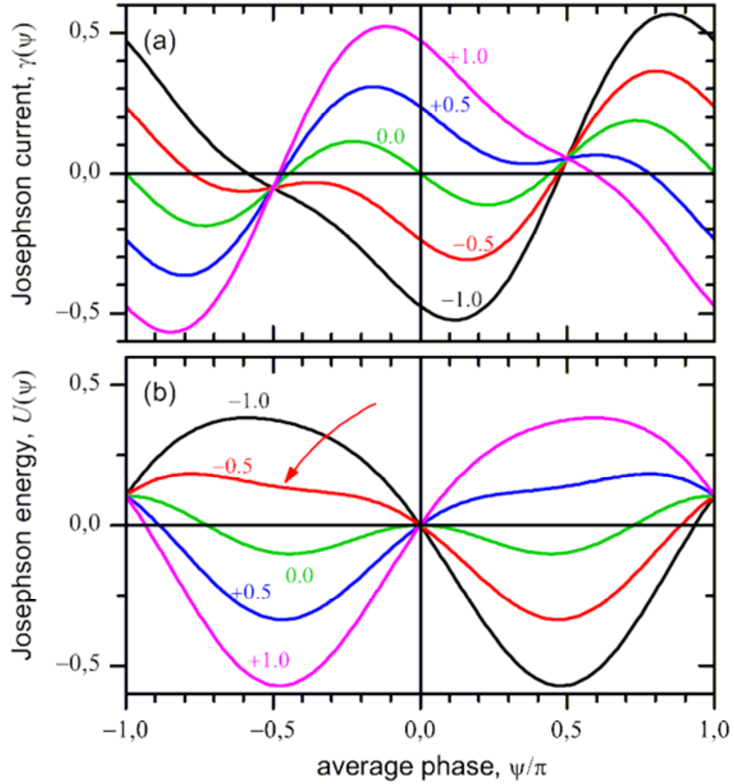
φ JJ made of 0-π JJ





$$\gamma = \langle j_c \rangle \left[\sin\psi + \Gamma_h h \cos\psi + \frac{\Gamma_0}{2} \sin(2\psi) \right].$$

$$U(\psi) = \langle j_c \rangle \left[1 - \cos\psi + \Gamma_h h \sin\psi + \frac{\Gamma_0}{2} \sin^2\psi \right].$$



Proposal: E. Goldobin et al., PRL **107**, 227001 (2011)
Experiment: H. Sickinger et al., PRL **109**, 107002 (2012)
http://www.pro-physik.de/details/news/3790631/Supraleiter_als_Phasenbatterie.htm

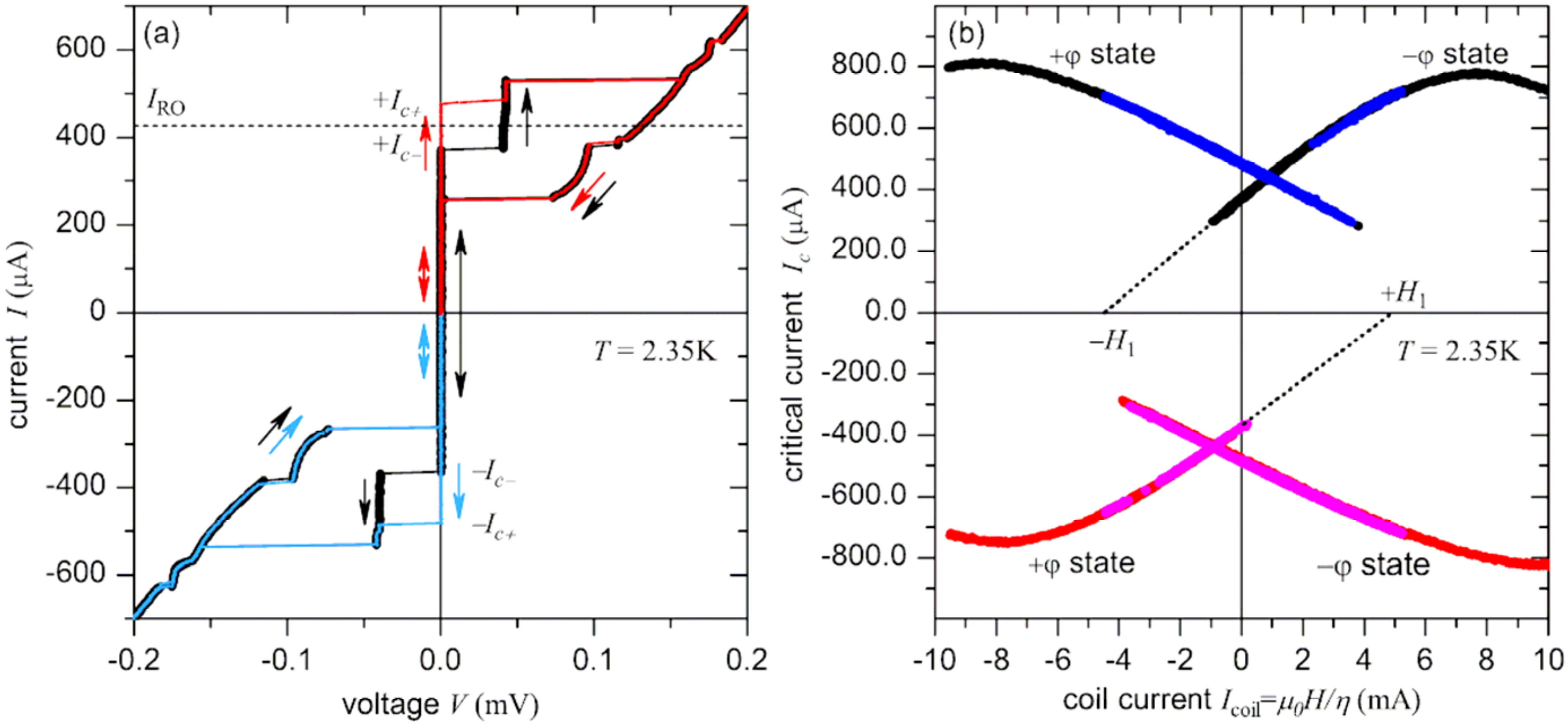
$I_c(H)$

Shifted main minimum

- @ *300*mK < *T* < *4.2*K
- @ T = 2.35 K, φ = 0.45π

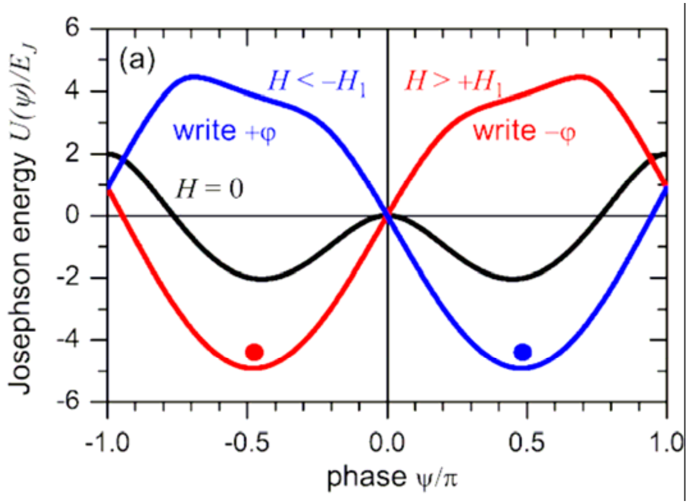
Two branches crossing

- @ T < *3.5* K (low α)
- L-branch = escape from +φ
- R-branch = escape from −φ

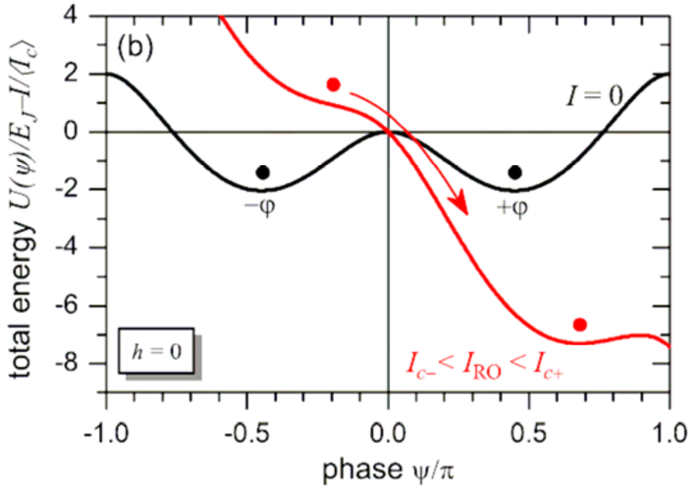


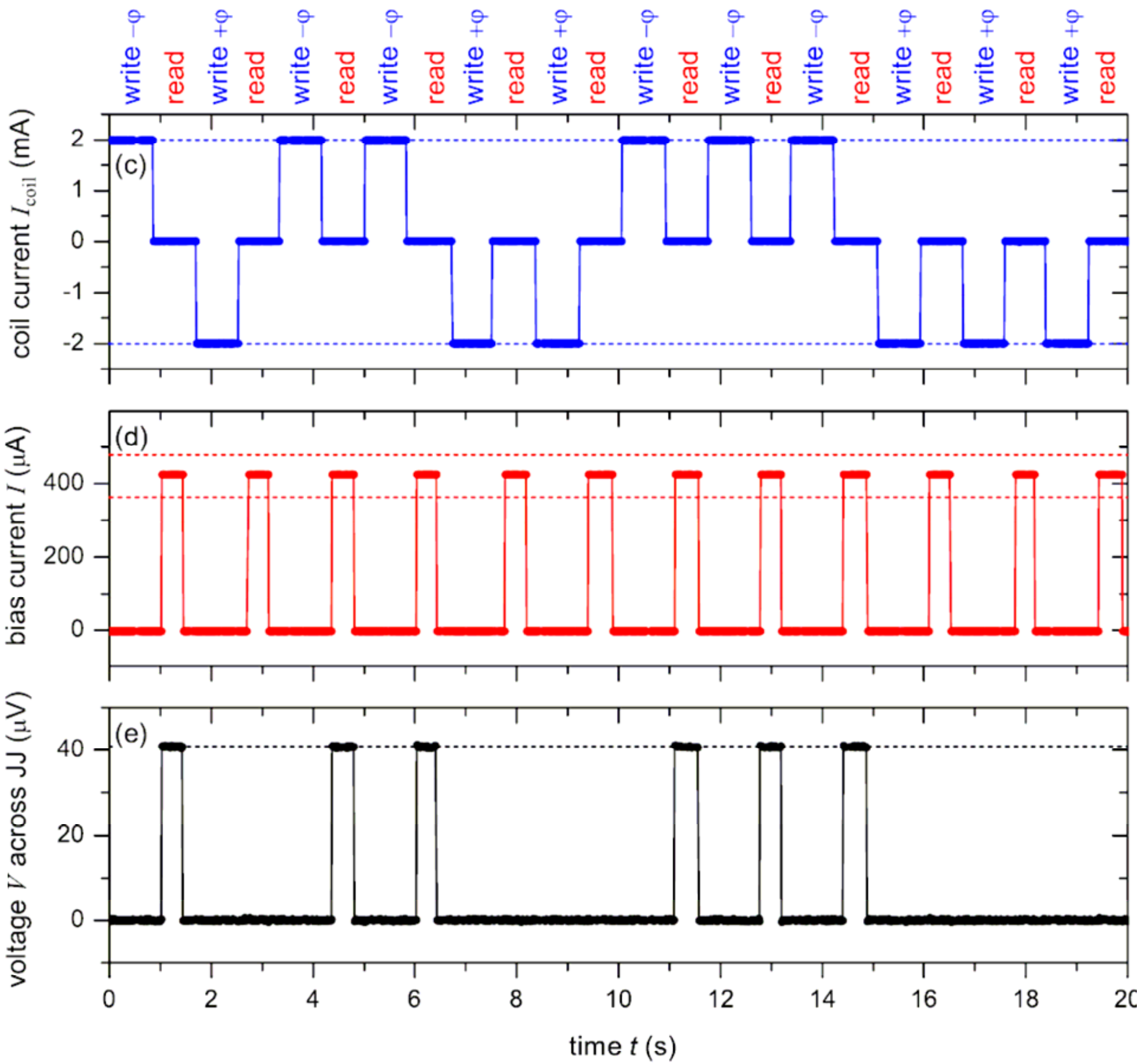
Application: φ bit (memory cell)

Write in

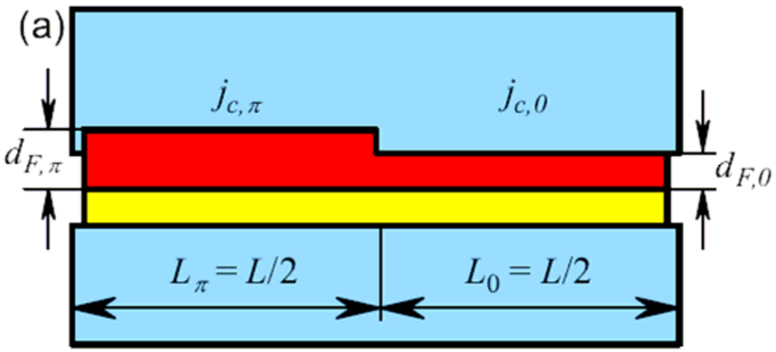


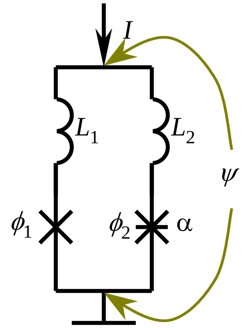
Read out





Problems & Motivation





Problems:

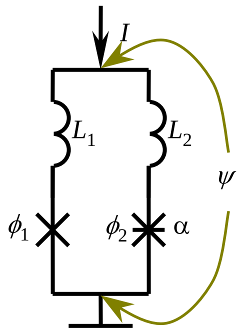
- 0-π JJ is still rather large for any practical application (~100 μm)
- the domain of parameters, where theory works is very slim, or
- domain is large but "average" description is bad (other degrees of freedom become important)

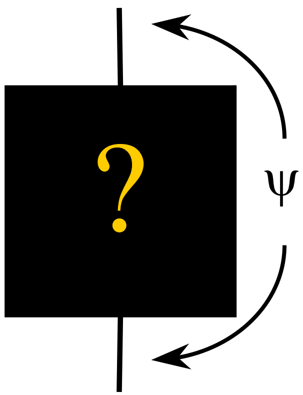
Let's try a **0-π SQUID**:

- discrete version of 0-π JJ
- has non-zero hole area
- point-like JJs
- finite inductance

- was proposed and used as a "**JJ with the magnetic field tunable critical current**"
- Our aim: to calculate CPR and other measurable characteristics

Basic equations





Parameters & assumptions:

$I_{c1} > 0$, $|I_{c2}| < I_{c1}$: $\alpha = I_{c2}/I_{c1}$, i.e. $|\alpha|<1$

L_1 and L_2 are finite and are not very large (see below):

$$\beta_1 = \frac{2\pi I_{c1} L_1}{\Phi_0}, \quad \beta_2 = \frac{2\pi I_{c1} L_2}{\Phi_0}.$$

(note, β_2 is defined via I_{c1}!)

Kirchhof for phases:

$$\begin{aligned} \psi &= \phi_1 + \beta_1 \sin\phi_1 + r_1\phi_e; \\ \psi &= \phi_2 + \alpha\beta_2 \sin\phi_2 - r_2\phi_e, \end{aligned}$$

$(0 < (\beta_1, \beta_2) < 1)$

$$r_1 + r_2 = 1 \;|\; \phi_e = 2\pi f = 2\pi\Phi_e/\Phi_0$$

for purely geometrical inductances (or trivial geometry*)

$$r_1 = \frac{\beta_1}{\beta_\Sigma} = \frac{L_1}{L_1 + L_2}, \quad r_2 = \frac{\beta_2}{\beta_\Sigma} = \frac{L_2}{L_1 + L_2}.$$

Kirchhof for currents:

$$\gamma = \sin\phi_1 + \alpha \sin\phi_2,$$

$$\gamma = I/I_{c1}$$

📖 E. Goldobin et al., arXiv: 1504.05858

CPR and the Josephson energy

Calculation of CPR $\gamma(\psi)$:

- find $\phi_1(\psi)$ from (1a) (unique solution)
- find $\phi_2(\psi)$ from (1b) (unique solution)
- calculate γ from (2).

Total Josephson energy:

$$U(\psi) = U_J(\psi) + U_L(\psi),$$

$$U_J(\psi) = [1 - \cos\phi_1(\psi)] + \alpha[1 - \cos\phi_2(\psi)],$$

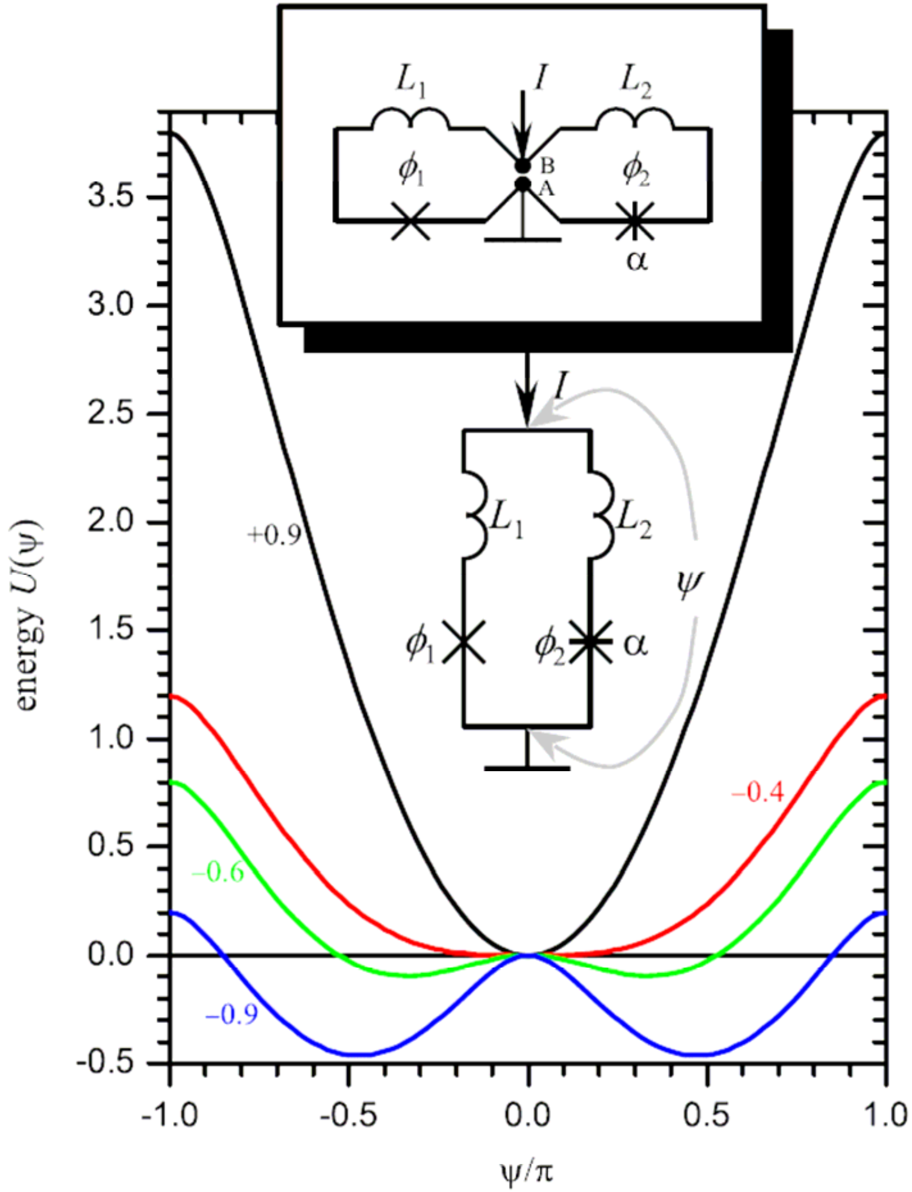
$$U_L(\psi) = \frac{\beta_1}{2}\sin^2\phi_1(\psi) + \frac{\beta_2}{2}\alpha^2\sin^2\phi_2(\psi)$$

By direct substitution:

$$U'(\psi) \equiv \gamma(\psi)$$

Calculation of CPR $U(\psi)$:

- find $\phi_1(\psi)$ from (1a) (unique solution)
- find $\phi_2(\psi)$ from (1b) (unique solution)
- calculate U_J, U_L, U (see the above eqs).

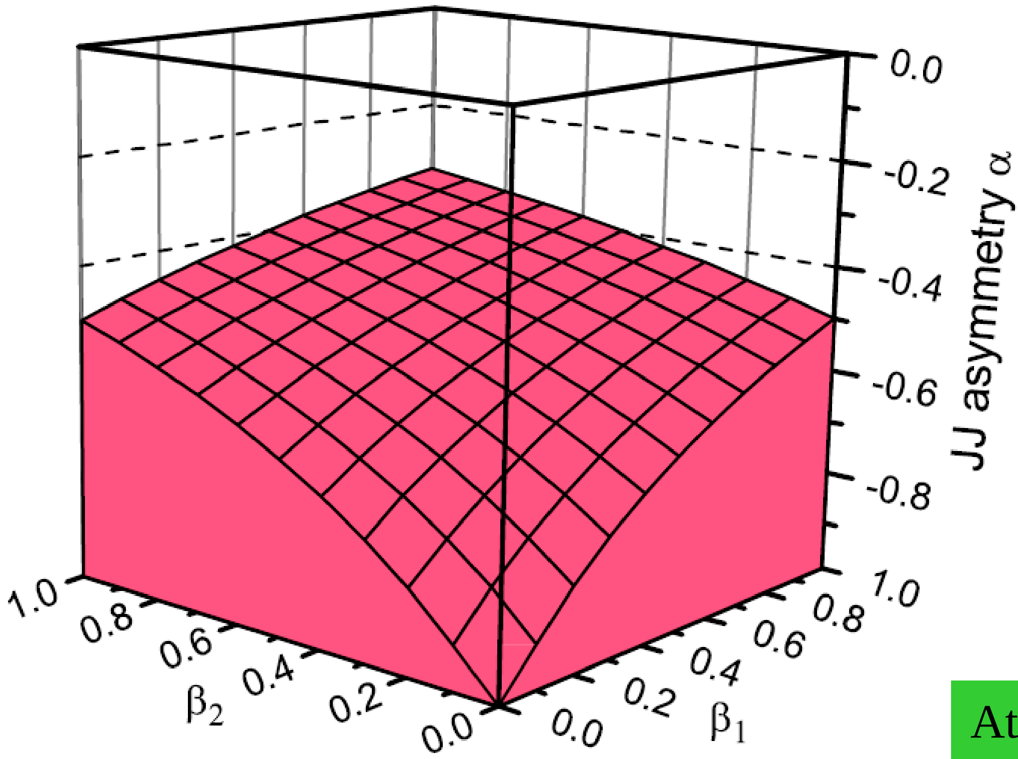


$\beta_1 = \beta_2 = 0.7$ $(\alpha_c \approx -0.417)$

Bifurcation point. φ-domain

As α changes, the transition to φ-domain takes place when: $U''(0) = \gamma'(0) = 0$

$$\alpha_c = \frac{-1}{1+\beta_1+\beta_2} = \frac{-1}{1+\beta_\Sigma} = \frac{-1}{1+\pi\beta_L}.$$



At $\beta_{1,2} \to 0$,
domain (in α direction)
shrinks linearly!

$\beta_\Sigma = 0$ never gives φ domain

CPR at finite field ($\phi_e > 0$)

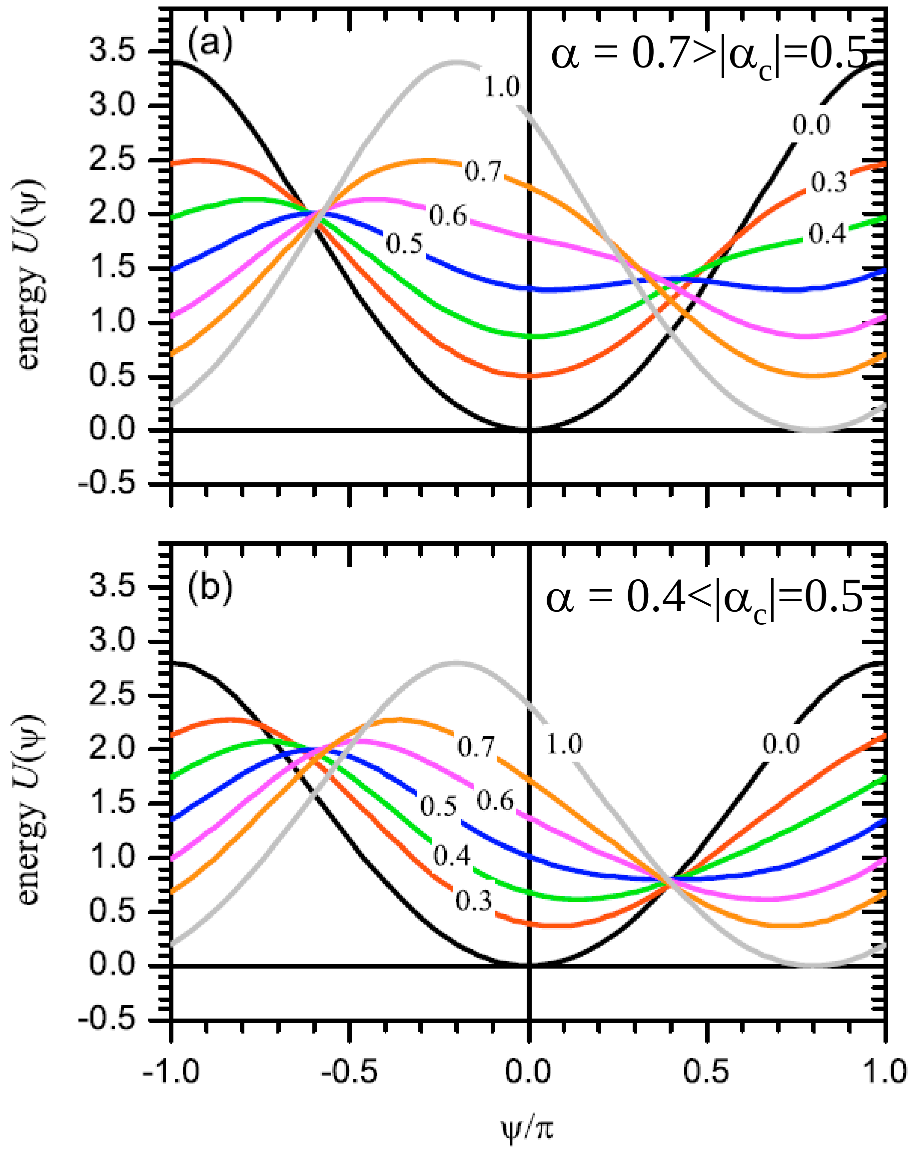
Apply half-integer flux (ϕ_1 & γ = const):

$$\begin{aligned} \phi_e^{\text{new}} &= \phi_e + n\pi; & (10a)\\ \phi_2^{\text{new}} &= \phi_2 + n\pi; & (10b)\\ \alpha^{\text{new}} &= (-1)^n \alpha; & (10c)\\ \psi^{\text{new}} &= r_1 n\pi; & (10d)\\ U^{\text{new}} &= U + [(-1)^n - 1]\alpha, & (10e) \end{aligned}$$

For example:

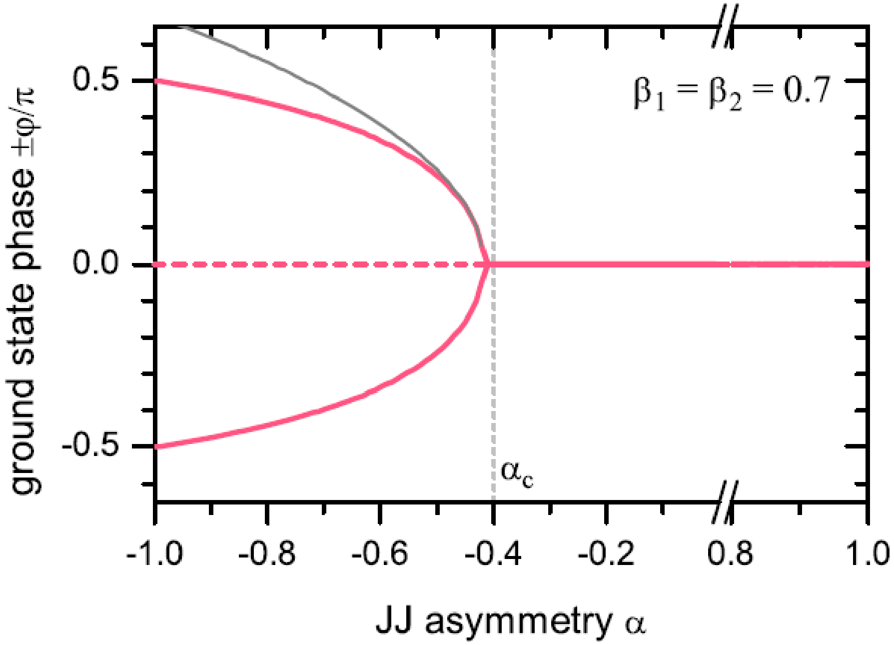
- n=1 turns 0-0 SQUID with $\alpha > |\alpha_c|$ (0-JJ) into a 0-π SQUID with $\alpha_{new} < \alpha_c$ (φ-JJ)
- n=even shifts CPR by $r_1 \pi n$, which is NOT a multiple of 2π.

$\beta_1 = 0.4,\ \beta_2 = 0.6$



Ground state phase

$$\gamma(\varphi) = 0.$$



Maximum phase:

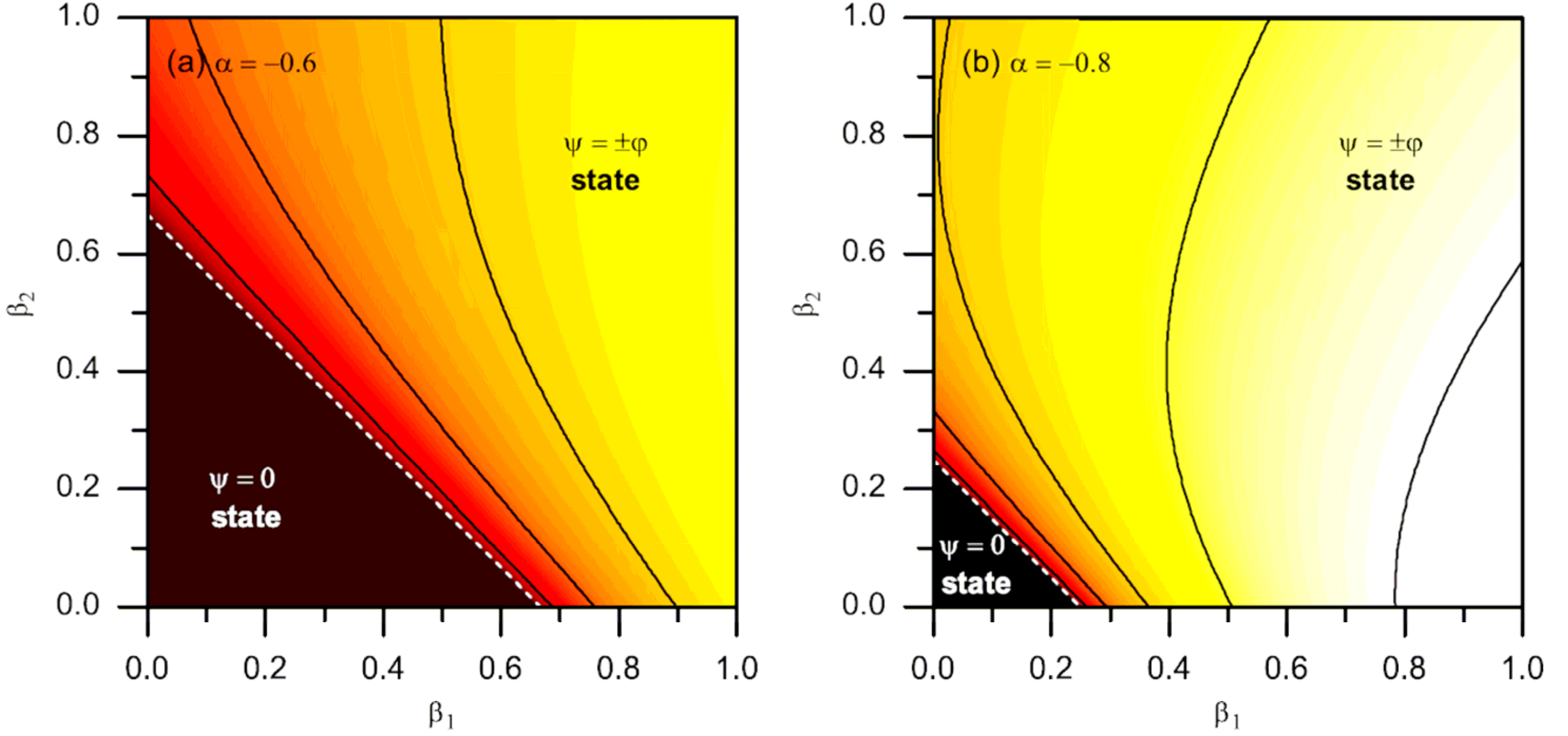
$$\varphi_{\max} = \frac{\pi}{2} + y\frac{\beta_1 - \beta_2}{\beta_1 + \beta_2},$$

$$2y = (\beta_\Sigma)\cos(y),$$

$\phi_{max} = \pi/2 - 0.739$ (β_1=0, β_2=1)
$\phi_{max} = \pi/2 + 0.739$ (β_1=1, β_2=0)

$$y^\star = \cos(y^\star)$$

Lines of constant phase

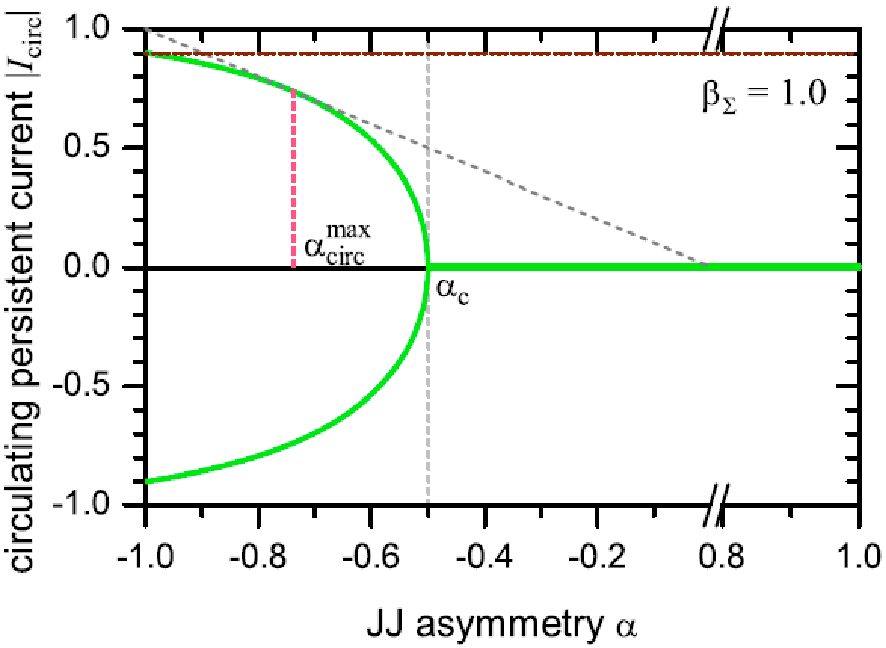


Persistent current

Persistent current and self-generated flux

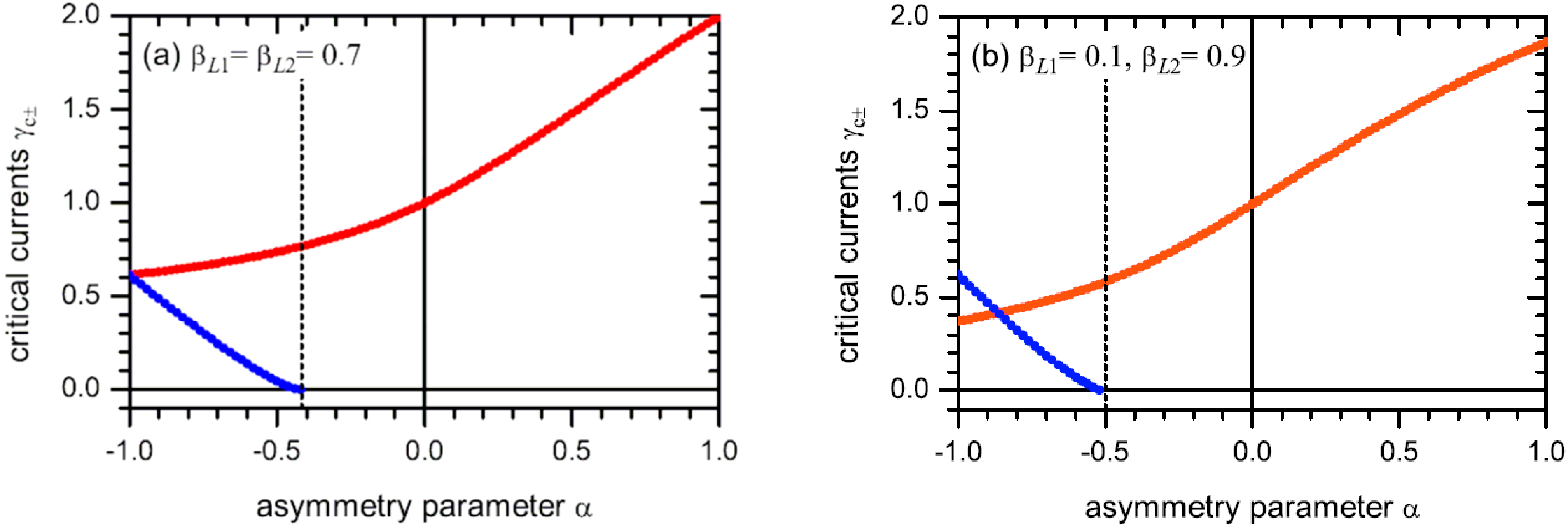
$$2\pi\frac{\Phi}{\Phi_0} = \beta_1 \sin\phi_1(\psi) - \alpha\beta_2 \sin\phi_2(\psi) = \phi_2(\psi) - \phi_1(\psi) - \phi_e.$$

ϕ_1, ϕ_2, Φ, I_{circ} depend only on $\beta_1+\beta_2$

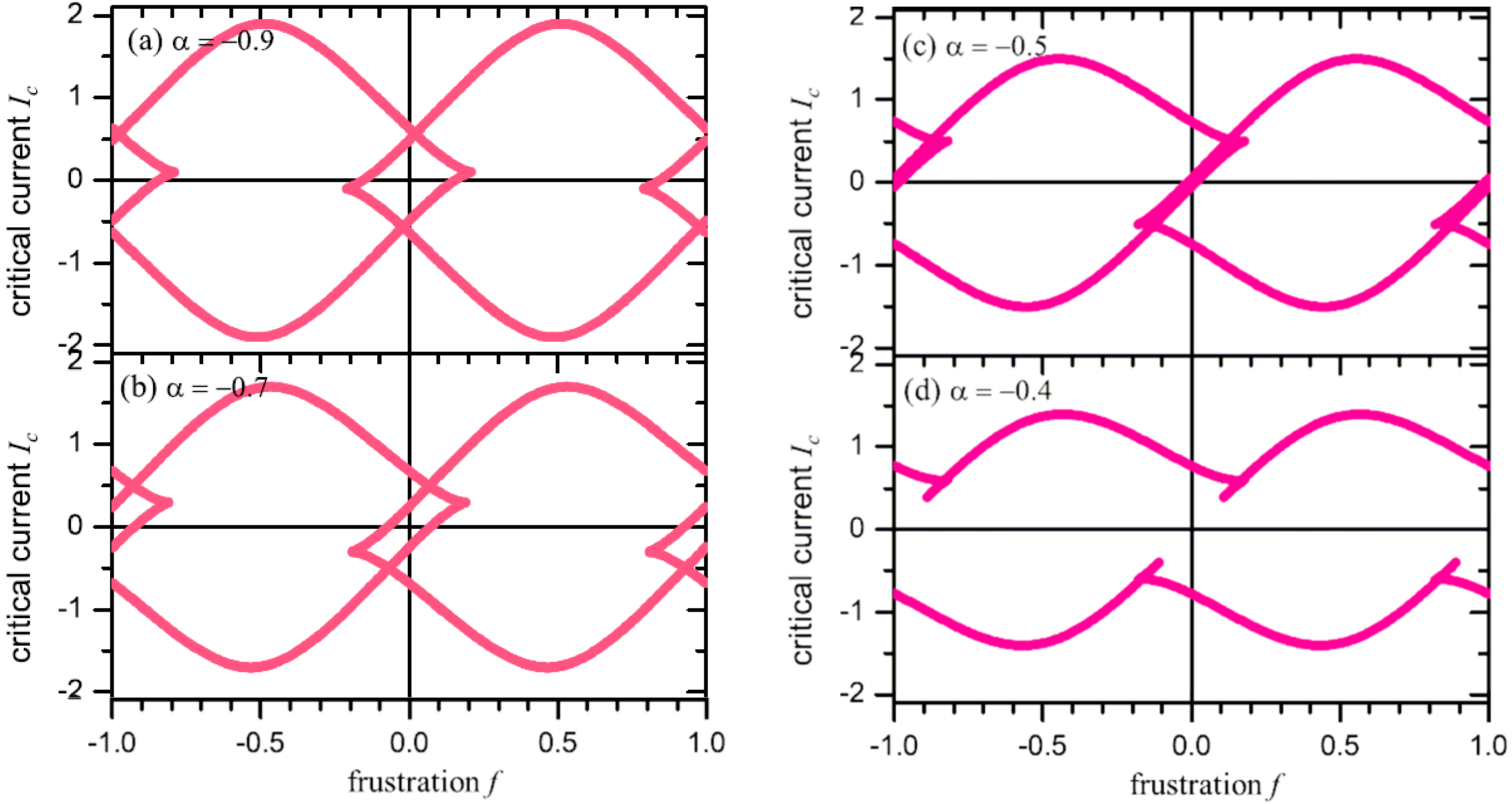


Persistent current =α if: $\alpha\beta_\Sigma + \arcsin(\alpha) + \frac{\pi}{2} = 0$

Two critical currents



Ic(H)



(a) α = −0.9
(b) α = −0.7
(c) α = −0.5
(d) α = −0.4
critical current I_c
frustration f
2
1
0
-1
-2
-1.0
-0.5
0.0
0.5
1.0

Summary

Summary:

- Using asymmetric 0-π SQUID with small inductance you can create:
  - φ JJ if $\alpha<\alpha_c$
  - mag. field: turn 0-JJ into φ_0 ($\alpha<|\alpha_c|$) or into $\varphi_0 \pm \varphi$ ($\alpha>|\alpha_c|$) JJs
- Advantages:
  - JJ can be made small
  - has much bigger φ-domain

arXiv: 1504.05858

All about φ-JJs:

- Proposal (theory): E. Goldobin et al., PRL **107**, 227001 (2011)
- Extended theory: A. Lipman et al., PRB **90**, 184502 (2014)
- Experiment: H. Sickinger et al., PRL **109**, 107002 (2012)
- φ-bit demonstration: E. Goldobin et al., APL **102**, 242602 (2013)
- Butterfly effect: E. Goldobin et al., PRL **111**, 057004 (2013)
- Properties of φ JJs: E. Goldobin et al., PRB **76**, 224523 (2007)
- Ratchet: <to be published>
- Retrapping experiment: <to be published>