מכון ויצמן למדע
WEIZMANN INSTITUTE OF SCIENCE

INELASTIC SCATTERIN IN LUTTINGER MODELS: A NEW PARADIGMATIC MODEL

with

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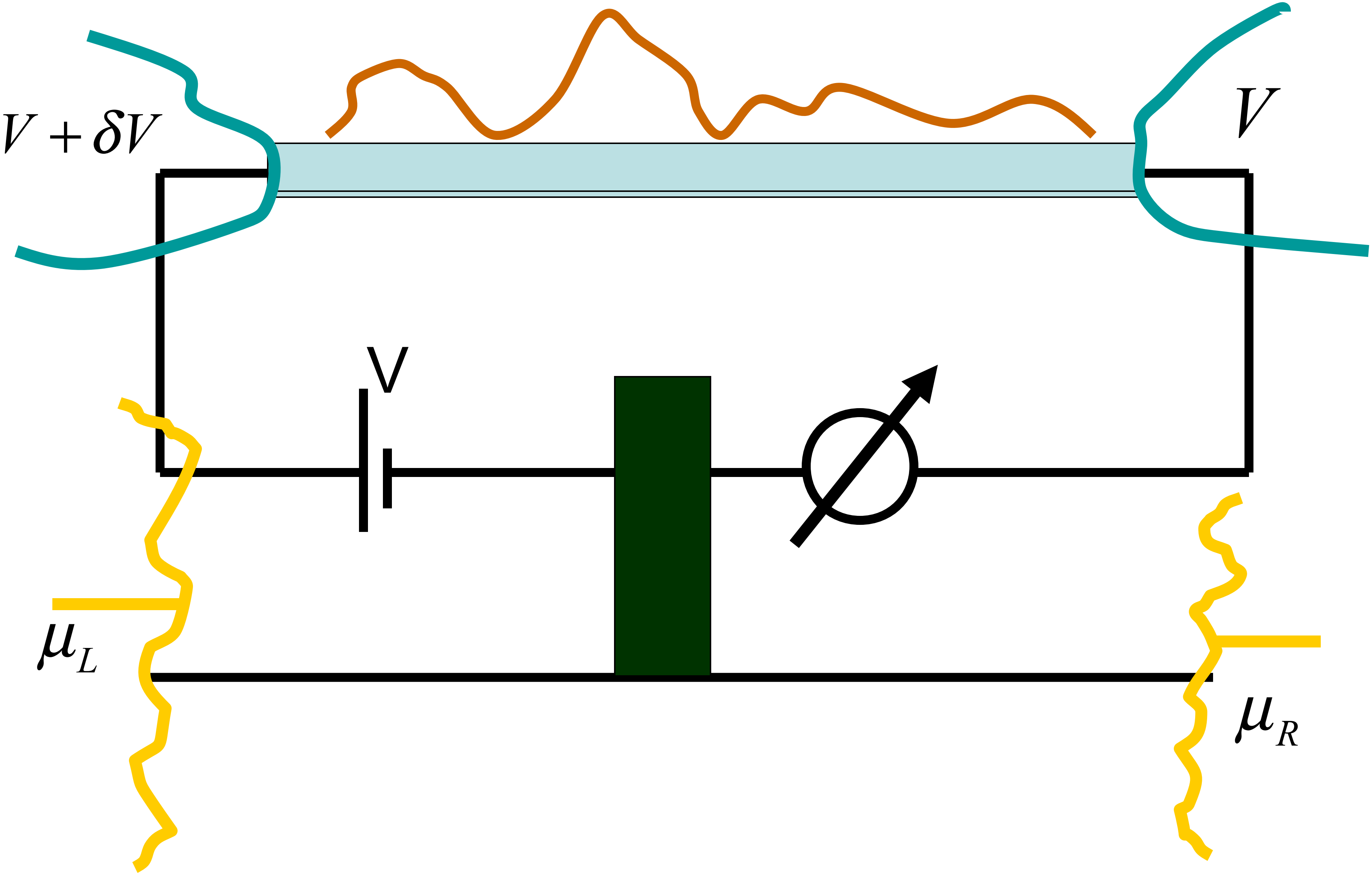
INTRODUCTION: Noise & Transport in 1d

Buttiker-Landauer picture

Kane-Fisher paradigm

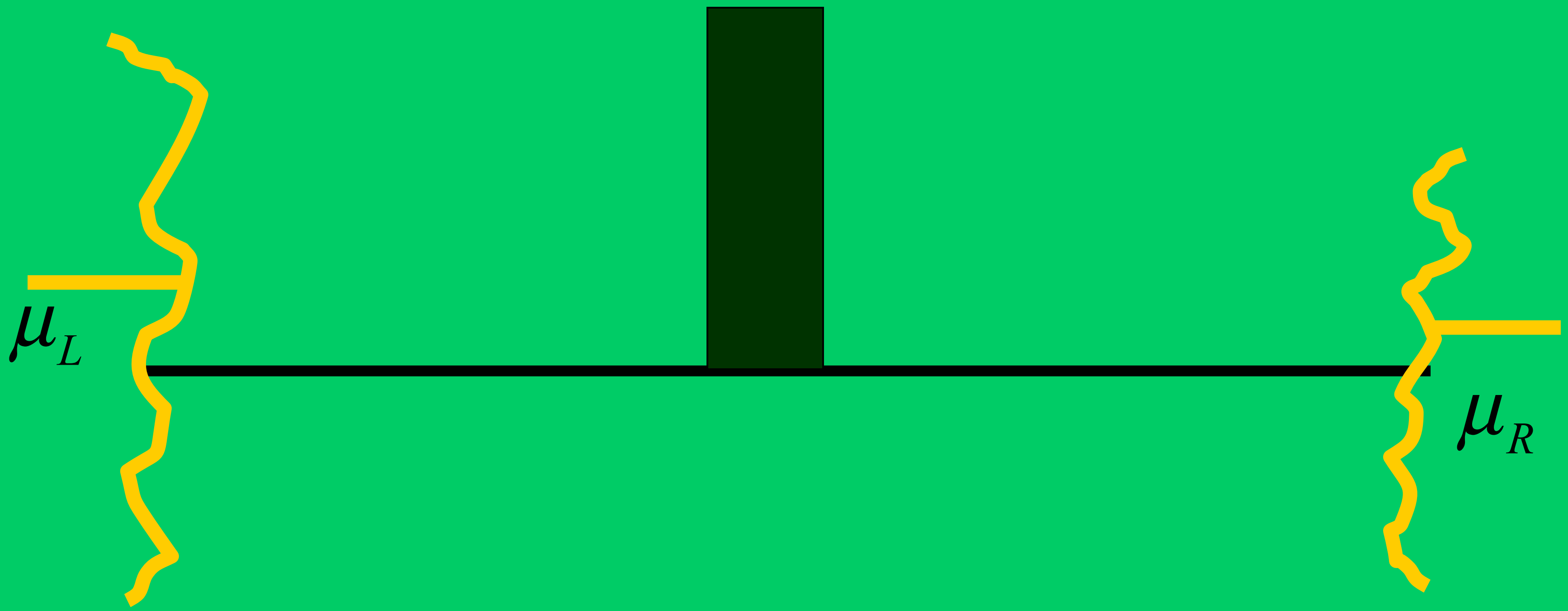
SHOT NOISE (quantum mechanical)



within the Landauer picture (coherent QM scatterers):

thermal noise ⇒ reservoirs

shot noise ⇒ backscattering (partitioning)



$$\langle I_L I_L \rangle > 0;\ \langle I_R I_R \rangle > 0;\ \langle I_L I_R \rangle < 0$$

$$\text{F.T.} \quad < I(t) g I(t+\tau) >= 2e < I >$$

non-equilibrium shot noise

$$S_2^{shot}(\omega \quad 0) = 2(\# e)\langle I \rangle$$

FANO FACTOR

disorder, interaction, fractional charge …

for fermi sea electrons:

$$S_2 = 2e\bar{I}(1 - T)$$

cross- current : negative

$$S_{LL} = S_{RR} = -S_{LR} = -S_{RL}$$

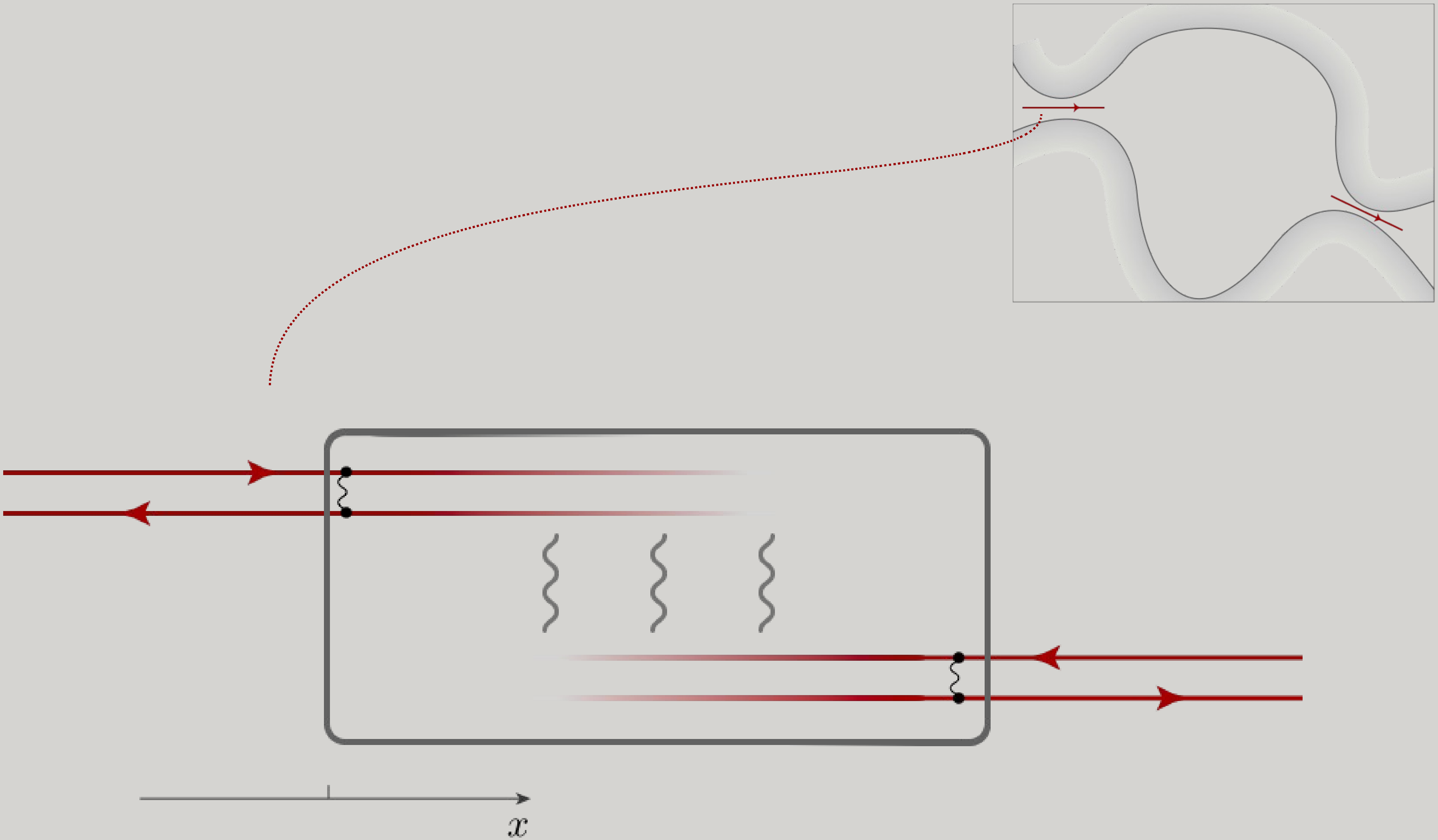
partitioning of beam ⇒ **shot noise**

OUTLINE:

- Matveev model
- generalized model
- scaling analysis
- transport properties

Matveev model

Matveev model (1995)



▷ model describes interplay of backscattering and charging

▷ different dot/lead contacts correlated by charging interaction

▷ ensuing transport mechanism: inelastic cotunneling (cf. Kamenev, Glazman, aa, 04)

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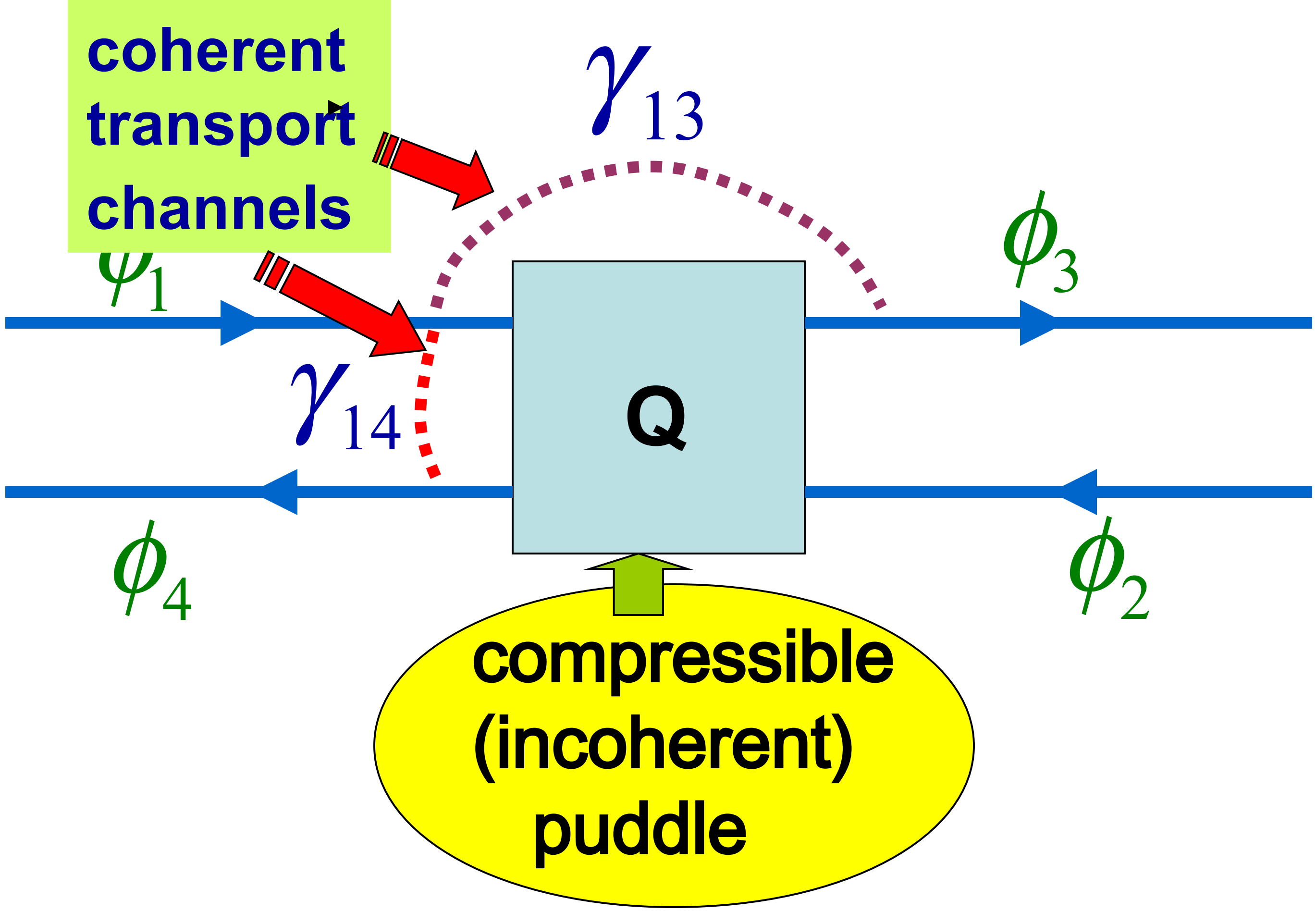
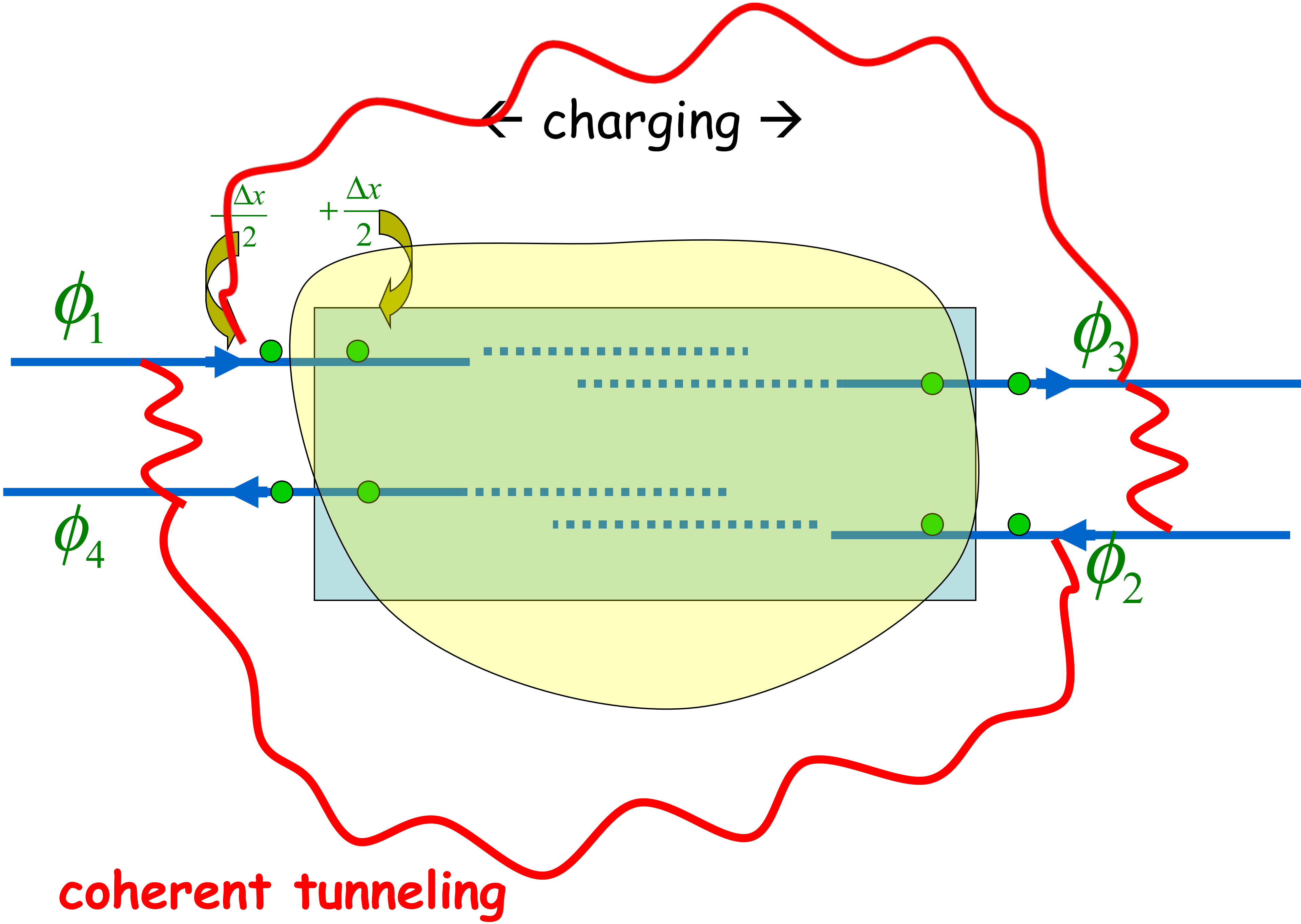
Kane/Fisher (coherent) and Matveev (incoherent) approach describe complementary channels of transport through a constriction.

Many realistic setups may allow for a combination of both.

generalization



ADD ELASTIC QP HOPPING



← charging →
$-\frac{\Delta x}{2}$
$+\frac{\Delta x}{2}$
ϕ_1
ϕ_3
ϕ_4
ϕ_2
coherent tunneling

include scattering channels between all incoming/outgoing modes.

▷ each chiral wire represented by '1/2' degree of freedom – phase space of model contains four (real) modes.

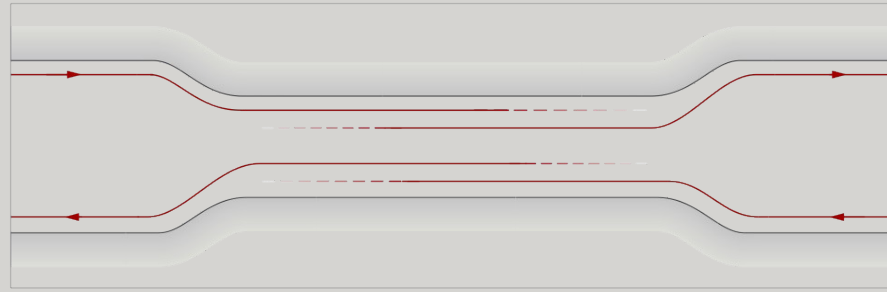
▷ combination of four scattering operators and charging modes represent three modes massive,

▷ one survivor mode is protected by (charge conservation) symmetry.

▷ this mode represents the sum of the incoming currents which, by charge conservation, equals the sum of the outgoing currents.

▷ prevalence of this mode entails that all transport coefficients between incoming and outgoing wires asymptote to value 1/2.

- four chiral modes



fixes distribution

$$S_0[\cdot] \Sigma \frac{1}{2\Sigma\Sigma} \sum_{l\Sigma 1}^{4} \sum dxdt\, \cdot_{l,q} \Sigma \Sigma_x^2 \Sigma s_l \Sigma_x \Sigma_t \Sigma_{,c} \Sigma S_{\text{dist}}[\cdot_q]$$

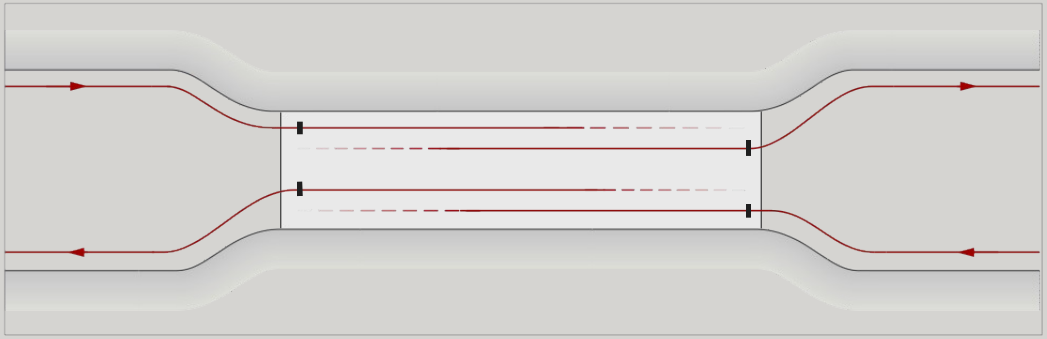
$s_l \Sigma (\Sigma / \Sigma)1$ for $l \Sigma 1, 2 / 3, 4$

$$S_l^{(0)}[\phi] = \frac{\hbar}{2} \int \frac{dq}{2\pi} \int \frac{d\omega}{2\pi}\, \phi^T_{i,q,\omega}\, \hat{G}^{-1}_{q,\omega}\, \phi_{i,-q,-\omega}$$

- charge in the dot

$$Q \,\Sigma \sum_l s_l \dots_l$$

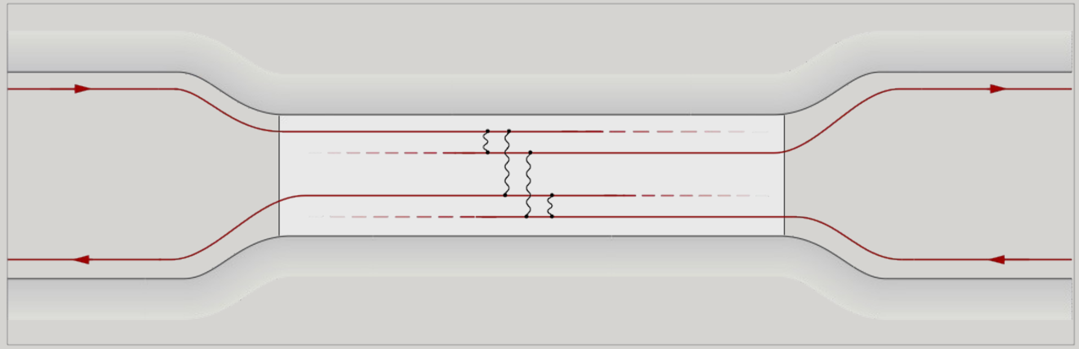
$$\Sigma \dots_l(0)$$



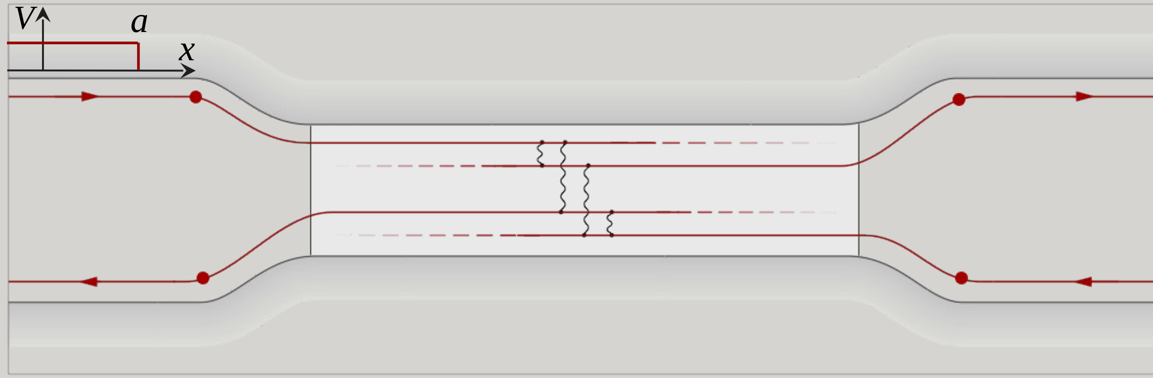
- charging action

$$S_c[\dots] \,\Sigma\, \frac{1}{2C} \sum dt\, Q_c Q_q$$

- add quasi–particle hopping at scattering hot-spots



$$S_{\text{scatt}}[\cdot] \sum \sum_{i\sum 1,2} \sum_{o\sum 3,4} \sum_{s\sum\sum} \cdot_{i,o} \sum dt \cos \sum \cdot_{i,c} \sum \cdot_{o,c} \sum \frac{s}{2} \sum \cdot_{i,q} \sum \cdot_{o,q} \sum$$



- couple wire #1 to voltage kink

$$S_V[\cdot] \Sigma \sum dt \sum_{1,q}(a) \Sigma \cdot_{1,q}(\Sigma\Sigma) \sum V(t)$$

- current at observation points: $j_l \Sigma \frac{1}{2\Sigma} \Sigma_x \cdot_l$ (assuming gauge $A_1 \Sigma 0$)

- strategy: integrate over field fluctuations outside tunneling hot spots
- effective action:

$$S \Sigma S_{\mathrm{diss}} \Sigma S_c \Sigma S_{\mathrm{scatt}} \Sigma S_V$$

$$S_{\mathrm{diss}}[\phi] \Sigma \frac{1}{2\Sigma\Sigma} \sum_l \sum dt\, \phi_{l,c} \Sigma_t \phi_{l,q} \Sigma S_{\mathrm{dist}}[\phi_q]$$

$$S_c[\phi] \Sigma \frac{1}{2C} \sum dt\, (\phi_1 \Sigma \phi_2 \Sigma \phi_3 \Sigma \phi_4)_c (\phi_1 \Sigma \phi_2 \Sigma \phi_3 \Sigma \phi_4)_q$$

$$S_{\mathrm{scatt}}[\phi] \Sigma \sum_{i \Sigma 1,2} \sum_{o \Sigma 3,4} \sum_{s \Sigma \Sigma} \psi_{i,o} \sum dt\, \cos \Sigma \phi_{i,c} \Sigma \phi_{o,c} \Sigma \frac{s}{2} \Sigma \phi_{i,q} \Sigma \phi_{o,q} \Sigma$$

$$S_V[\phi] \Sigma \sum dt\, \phi_{1,q} V$$

discussion of the action

- action contains one exact zero mode (by gauge invariance)

$$\Sigma_0 \Sigma \frac{1}{\sqrt{2}}(\cdot_1 \Sigma \cdot_2 \Sigma \cdot_3 \Sigma \cdot_4)$$

$$S \Sigma S_{\mathrm{diss}} \Sigma S_c \Sigma S_{\mathrm{scatt}} \Sigma S_V$$

$$S_{\mathrm{diss}}[\cdot] \Sigma \frac{1}{2\Sigma\Sigma}\sum_l \sum dt\, \cdot_{l,c} \Sigma \cdot_{l,q} \Sigma S_{\mathrm{dist}}[\cdot_q]$$

$$S_c[\cdot] \Sigma \frac{1}{2C}\sum dt\,(\cdot_1 \Sigma \cdot_2 \Sigma \cdot_3 \Sigma \cdot_4)_c(\cdot_1 \Sigma \cdot_2 \Sigma \cdot_3 \Sigma \cdot_4)_q$$

$$S_{\mathrm{scatt}}[\cdot] \Sigma \sum_{i\Sigma 1,2}\sum_{o\Sigma 3,4}\sum_{s\Sigma\Sigma} \cdot_{i,o} \sum dt\, \cos \Sigma \cdot_{i,c} \Sigma \cdot_{o,c} \Sigma \frac{s}{2} \Sigma \cdot_{i,q} \Sigma \cdot_{o,q} \Sigma$$

$$S_V[\cdot] \Sigma \sum dt\, \cdot_{1,q} V$$

- express action in terms of the linear combinations

$$\Sigma_0 \Sigma \frac{1}{\sqrt{2}}(\cdot_1 \Sigma \cdot_2 \Sigma \cdot_3 \Sigma \cdot_4)$$ average of incoming and outgoing current

$$\Sigma_1 \Sigma \frac{1}{\sqrt{2}}(\cdot_1 \Sigma \cdot_2 \Sigma \cdot_3 \Sigma \cdot_4)$$ total charge in scattering region

$$\Sigma_2 \Sigma \cdot_1 \Sigma \cdot_2$$ difference of incoming currents

$$\Sigma_3 \Sigma \cdot_3 \Sigma \cdot_4$$ difference of outgoing currents

- express action in terms of the linear combinations

$$S \sum S_{\text{diss}} \sum S_c \sum S_{\text{scatt}} \sum S_V$$

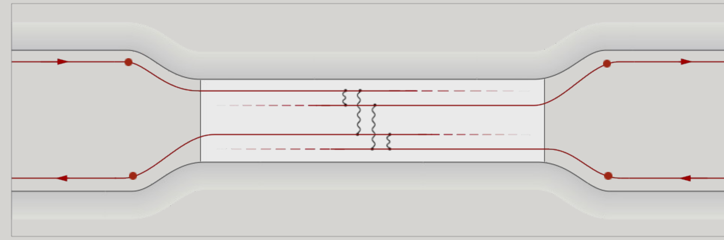
$$S_{\text{diss}}[\leadsto] \sum \frac{1}{2\sum\sum} \sum_a \sum dt \leadsto_{a,c} \sum_t \leadsto_{a,q} \sum S_{\text{dist}}[\leadsto_q]$$

$$S_c[\leadsto] \sum \frac{1}{C} \sum dt \leadsto_{1,c} \leadsto_{1,q}$$

$$S_{\text{scatt}}[\leadsto] \sum S_{\text{scatt}}[\leadsto_1, \leadsto_2, \leadsto_3]$$

$$S_V[\leadsto] \sum \sum dt \frac{1}{\sqrt{8}} (\leadsto_0 \sum \leadsto_1) \sum \frac{1}{2} \leadsto_2 \sum$$

transport



▷ after integration over bulk field fluctuations and discarding massive modes: moments of currents in incoming/outgoing leads become correlation functions of zero mode.

▷ average outgoing current:

distribution function

$$I_{o}(\Sigma) \Sigma 2i \Sigma \langle \sim_{c,\Sigma} \Sigma F_{\Sigma} \sim_{q,\Sigma} \rangle \Sigma \frac{\Sigma}{2 \Sigma} \frac{V_{\Sigma}}{2}$$

▷ equals 1/2 of the response current of an uninterrupted wire.

▷ corrections scale as $\Sigma T^{2(\Sigma^{1}\Sigma 1)}$

▷ low temperature limit of scattering center represents (1/2,1/2) ‘beam splitter’.

- current correlators

$$X_{ll'} \Sigma \Sigma j_{l,c,\Sigma} j_{l'\Sigma c,\Sigma\Sigma} \Sigma\Sigma \Sigma \xi_{l,\Sigma} \xi_{l'\Sigma\Sigma}\Sigma$$

$$\xi_{l,\Sigma} \Sigma \frac{1}{2\Sigma}\left(\Sigma_{x_l} G(x_l \Sigma\Sigma x_l)(G(0))^{\Sigma 1} \xi_0\right)_{c,\Sigma} \Sigma \frac{\Sigma}{2\Sigma} V_{\Sigma\, l,1}$$

Green function between system and observation point

- noise correlators $S_{ll'} \Sigma X_{ll'} \Sigma \langle I_l \rangle \langle I_{l'} \rangle$

$$S_{1,1,\Sigma} \Sigma 2 \Sigma\Sigma\Sigma F_{\Sigma}$$

$$S_{o,o,\omega} = 2\pi\nu\, \omega F_\omega$$

$$S_{1,o,\omega} = 2\pi\nu \frac{\omega F_\omega}{2}, \qquad o = 3,4$$

all other noise correlators vanish.

Equilibrium Noise

intra-wire noise $X_{ll,\omega} = \frac{\nu}{2\pi}\omega F_\omega$

outgoing/outgoing $X_{oo',\omega} = 0,$

incoming/outgoing $X_{io,\omega} = \frac{\nu}{4\pi}\omega F_\omega$

incoming/incoming $X_{ii',\omega} = 0.$

Shot Noise

no intra shot noise

no cross-correlation shot noise

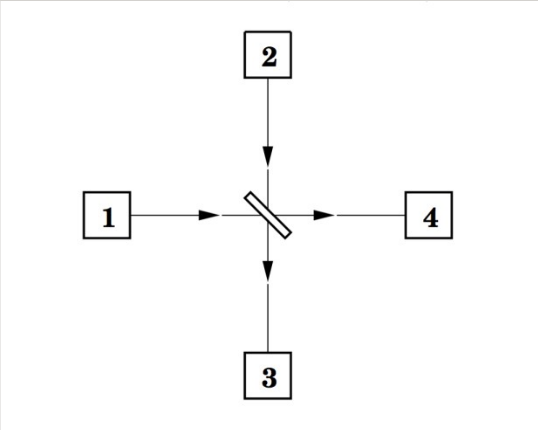
no partitioning noise/ no negative cross correlations

cf: shot noise in quantum diffractive beam splitters

▷ cross–correlations between different outgoing leads generically negative (partition noise)

$$S_{ll',\omega} = \frac{1}{2}\int dt\, e^{i\omega t}\,\langle I_l(t)I_{l'}(0) + I_l(0)I_{l'}(t)\rangle$$

negative for $l \neq l'$ (cf. Blanter & Büttiker, 99.)



summary

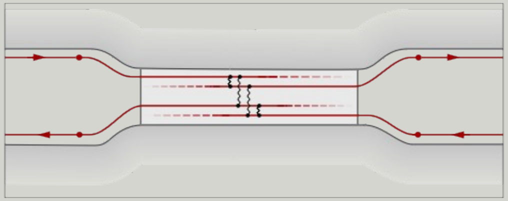
▷ conspiracy of elastic and inelastic scattering channels stabilizes (1/2,1/2) low frequency fixed point.

▷ a robust phenomenon,

▷ however, realization of scaling limit depends on device setup.

p-n graphene; QHE strip + puddle

▷ asymptotic transport regime exhibits no shot noise (absence of partition noise.)



THANK YOU