



FRACTIONAL CHARGE & FRACTIONAL STATISTICS & HANBURY BROWN & TWISS INTERFEROMETRY WITH ANYONS

with

G. Campagnano (WIS), O. Zilberberg (WIS)

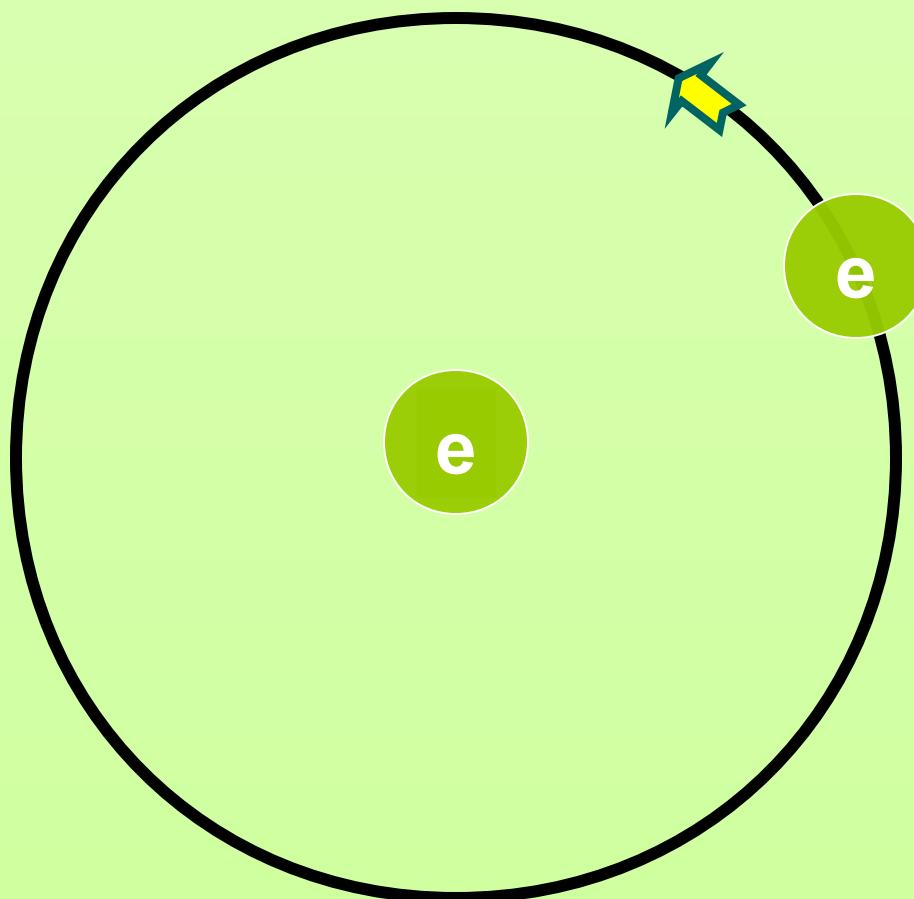
I.Gornyi (KIT)

also

D.E. Feldman (Brown) and A. Potter (MIT)

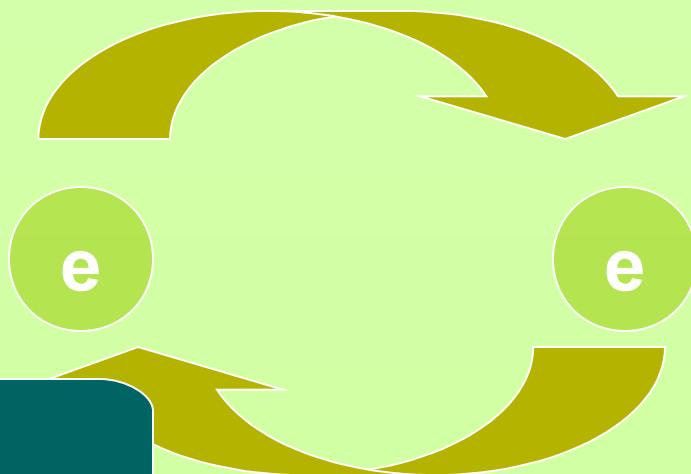


QUANTUM STATISTICS (Fermions, Bosons)

Ψ $e^{i2\pi n}$ Ψ 

QUANTUM STATISTICS OF PARTICLES

identical



e.g.
electrons

Indistinguishable particles

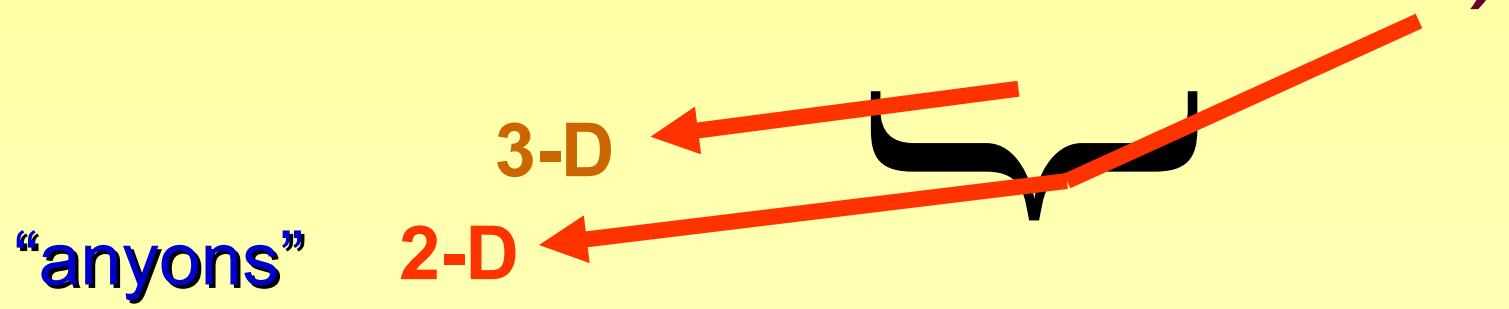
fermions

$$\Psi(\text{particle 1}, \text{particle 2}) = -\Psi(\text{particle 2}, \text{particle 1})$$

bosons

$$\Psi(\text{particle 1}, \text{particle 2}) = \Psi(\text{particle 2}, \text{particle 1})$$

QUANTUM STATISTICS (Fermions, Bosons,... Other)



Leinaas, Myrheim 1977; F. Wilczek 1982

(+ cyclic 1-D)

the fractional quantum Hall effect

Laughlin's ground state wave function:

$$\Psi_{1/m} = \prod_{j < k} (z_j - z_k)^m \exp\left(-\frac{1}{4} \sum_l |z_l|^2\right)$$

quasi-hole localized at z_0

$$\Psi_{1/m}^{+z_0} = (\dots) \prod_l (z_l - z_0) \Psi_{1/m}$$

a short digression: **BERRY (GEOMETRIC) PHASE**

closed contour:

$$\Psi \quad e^{i\alpha} \Psi$$

$$\alpha = E(t)dt + \gamma$$



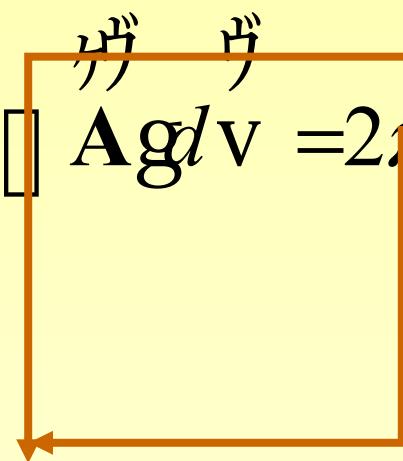
Berry phase

$$\gamma = i \square \left\langle \Psi(t) \mid \frac{d}{dt} \Psi(t) \right\rangle$$

Laughlin wave function + qh

Berry Phase (constant density) : $-2\pi \nu \frac{\phi}{\phi_0}$

Aharonov-Bohm Phase

$$(e^* / hc) \oint \mathbf{A} d\mathbf{v} = 2\pi(e^* / e) \frac{\phi}{\phi_0}$$

$$e^* = \nu e$$

Berry Phase

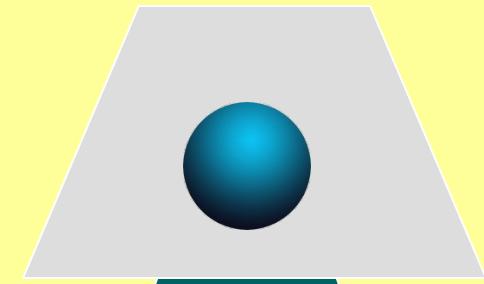
→ an extra term

$$-2\pi \nu \frac{\phi}{\phi_0} - 2\pi \nu$$

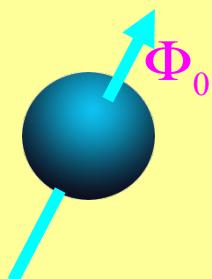
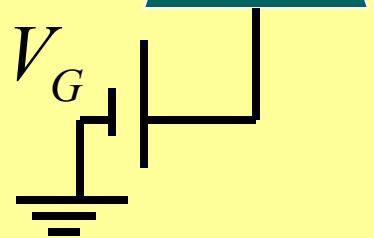


$$\Theta_{stat} = \nu$$

TWO OPTIONS to VISUALIZE a qh -
RESULT IS THE SAME



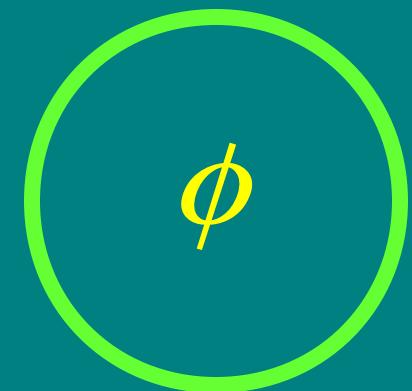
gate voltage generated qh



flux generated qh

ANDERSON:

***WHAT IS THE Aharanov-Bohm PERIODICITY
WITH $e^* = e/3$?***



$$E_n = \frac{2\hbar^2\pi^2}{mL^2} \left(n - \frac{\phi}{\phi_0}\right)^2 = \\ = \frac{2\hbar^2\pi^2}{mL^2} \left(n - \frac{e\phi}{hc}\right)^2$$

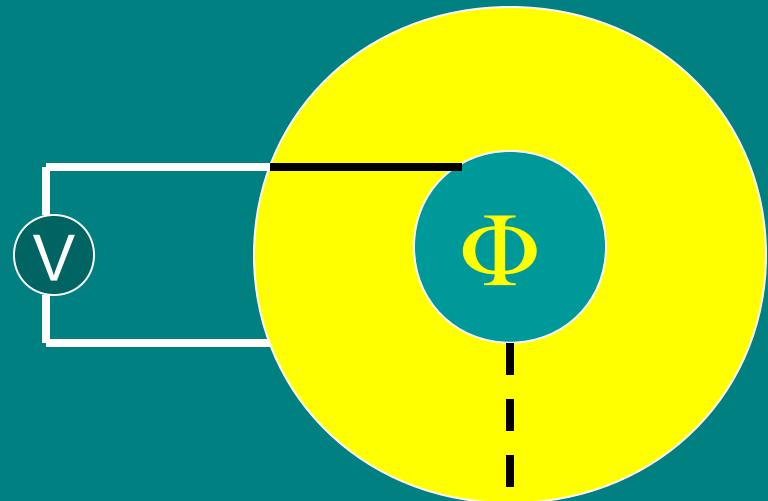
$$e - e^* = \frac{e}{3}$$

periodicity

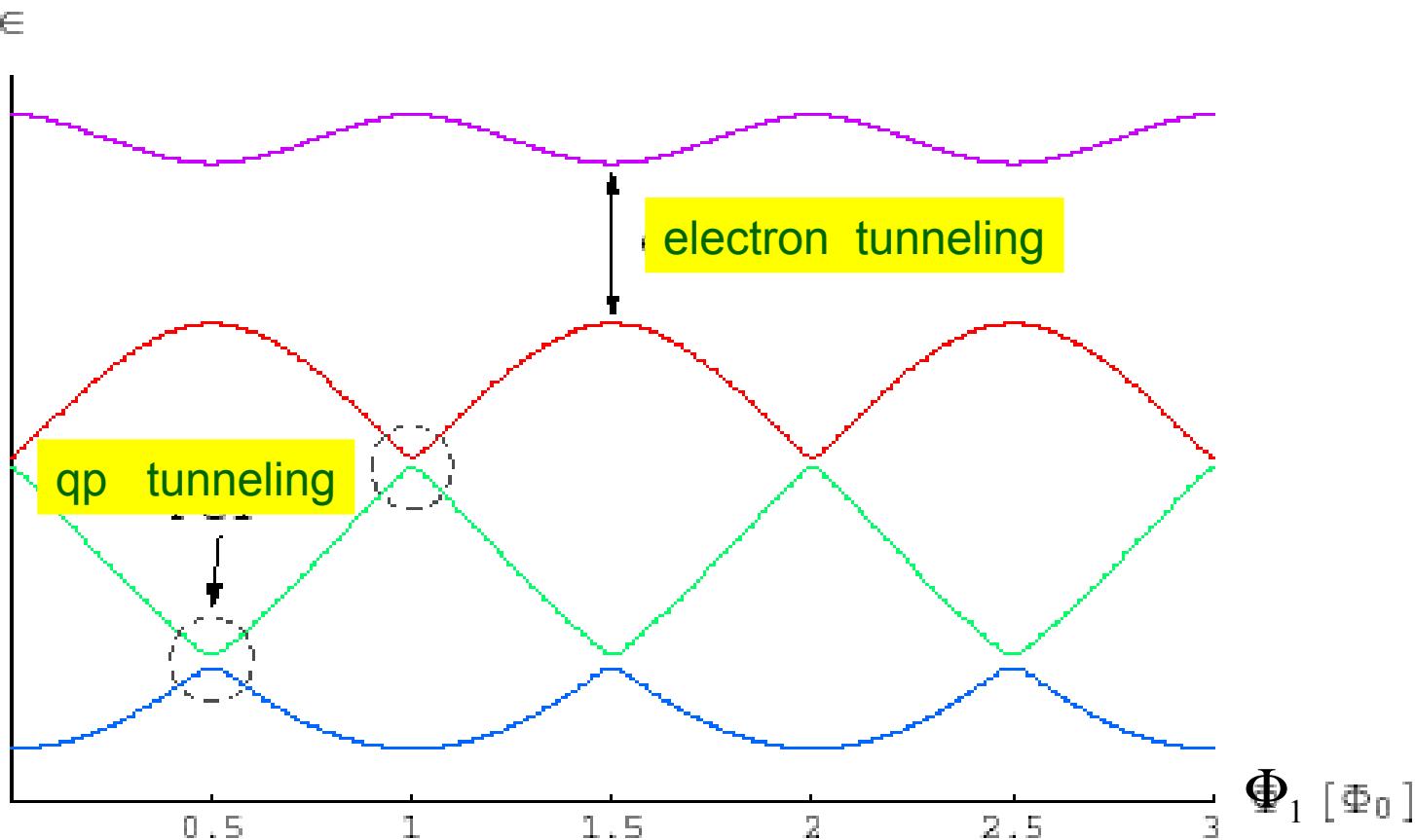
$$\phi_0 = \frac{hc}{e} \quad 3\phi_0 ?$$

*But that would contradict
Byers-Yang ??*

$$\nu = \frac{1}{3}$$



Y.G. & D.J. Thouless

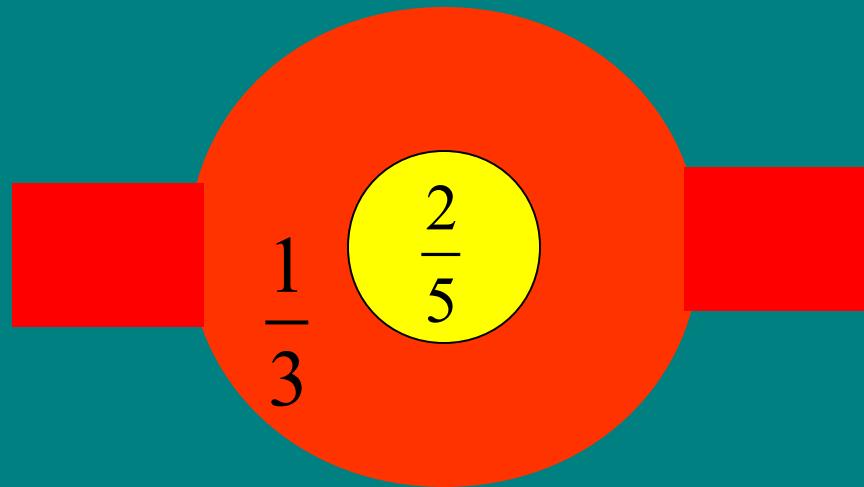


periodicity $\Phi_0 \rightarrow$ quasi-particle tunneling

periodicity $3\Phi_0 \rightarrow$ electron tunneling

(cf. Gefen and Thouless)

CRUCIAL QUESTION: MATRIX ELEMENTS



oscillations with adding flux= $5 \Phi_0$
or charge= $2e$
to central island

NOTE :

$$e^* = \nu e$$

$$\Theta_{stat} = 2\pi \nu$$

Chamon, Freed, Kivelson, Sondhi, Wen (97);
(Fabry-Perot interferometer)

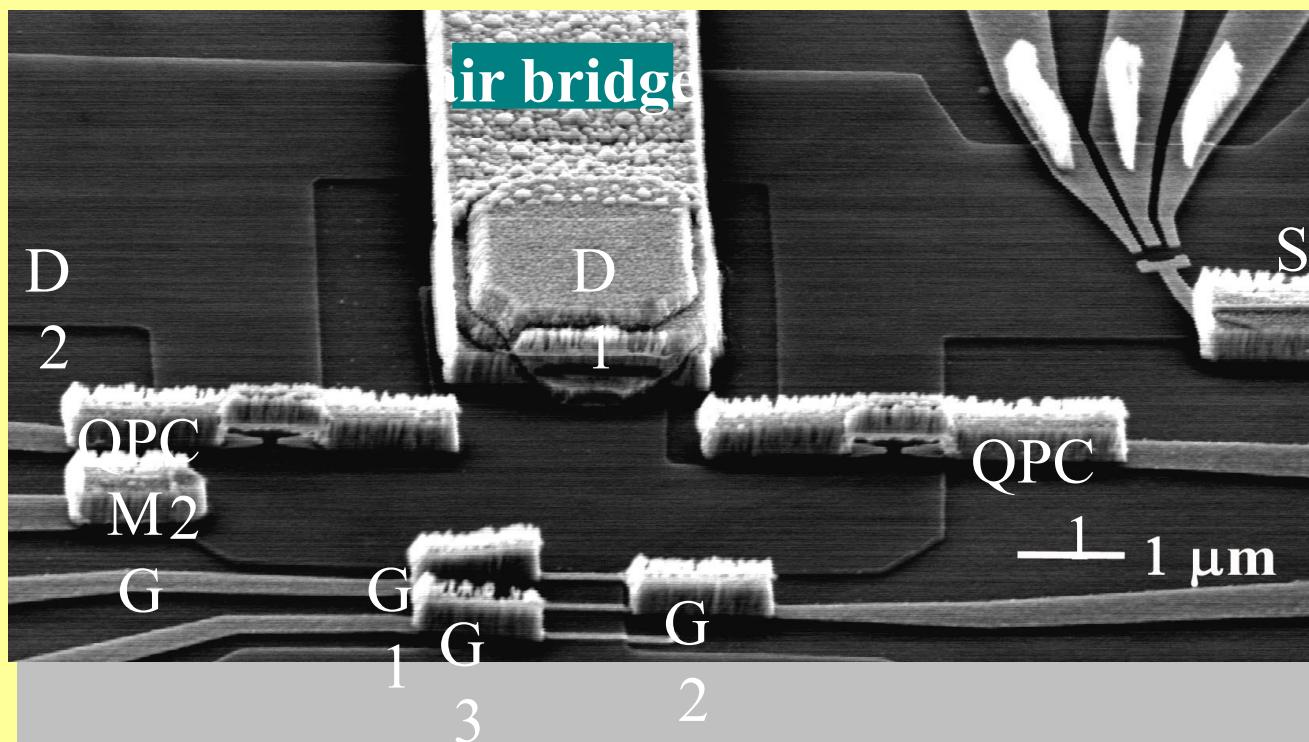
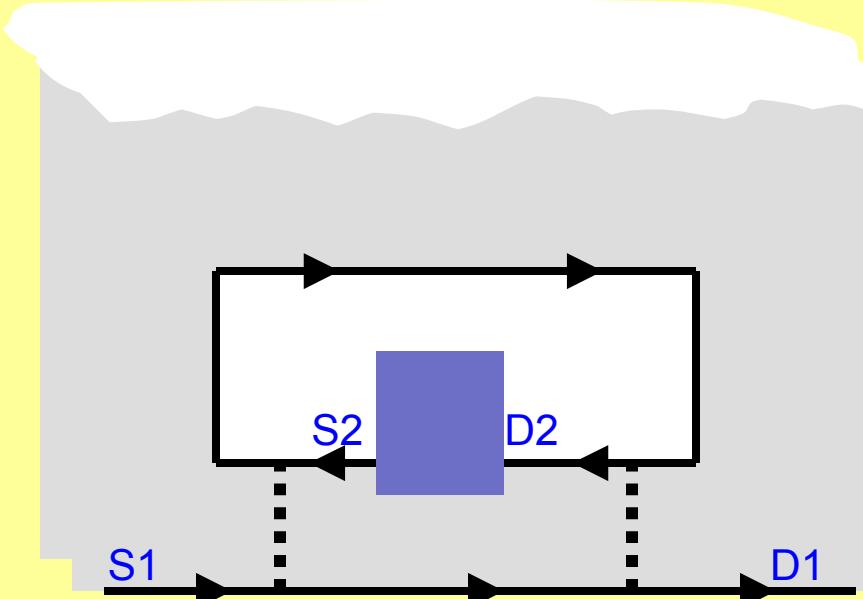
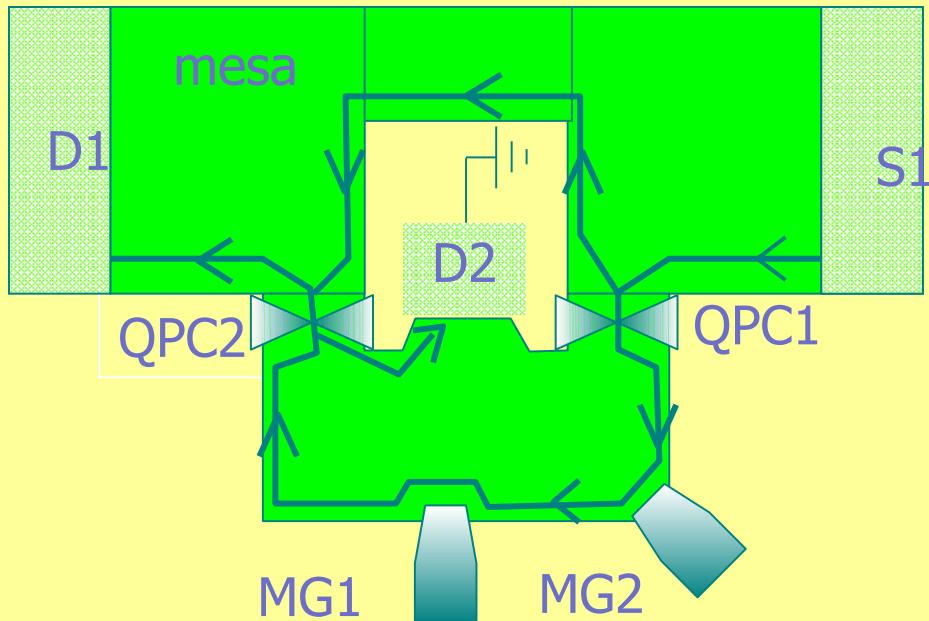
T. Martin ...; Kane; A. Stern...;
Stone...; Kim

Fradkin, Nayak, Tsvelik, Wilczek;
Das Sarma, Freedman, Nayak;
Stern Halperin; Kitaev Shtengel;
Hou, Chamon; Fradkin et al ...

Law, Feldman, Y.G. :

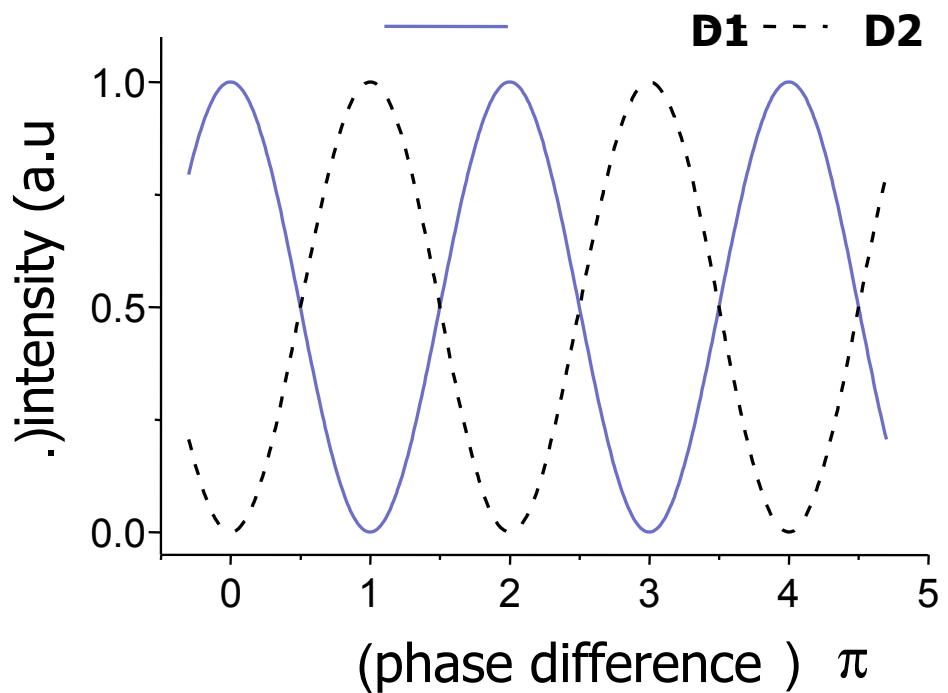
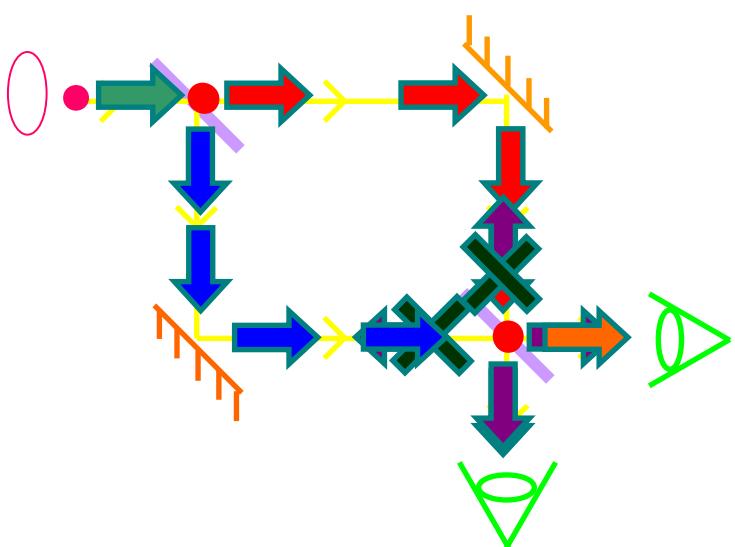
MACH-ZEHNDER INTERFEROMETER

possibly the leading candidate



Neder, Heiblum
et al

Mach-Zehnder Photonic Interferometer



$$T_{S \rightarrow D_2} = |t_{BS1} t_{BS2} + r_{BS1} r_{BS2} e^{i\Delta\Phi}|^2 = T_0 + T_1 \cos \Delta\Phi$$

Visibility T_1 / T_0



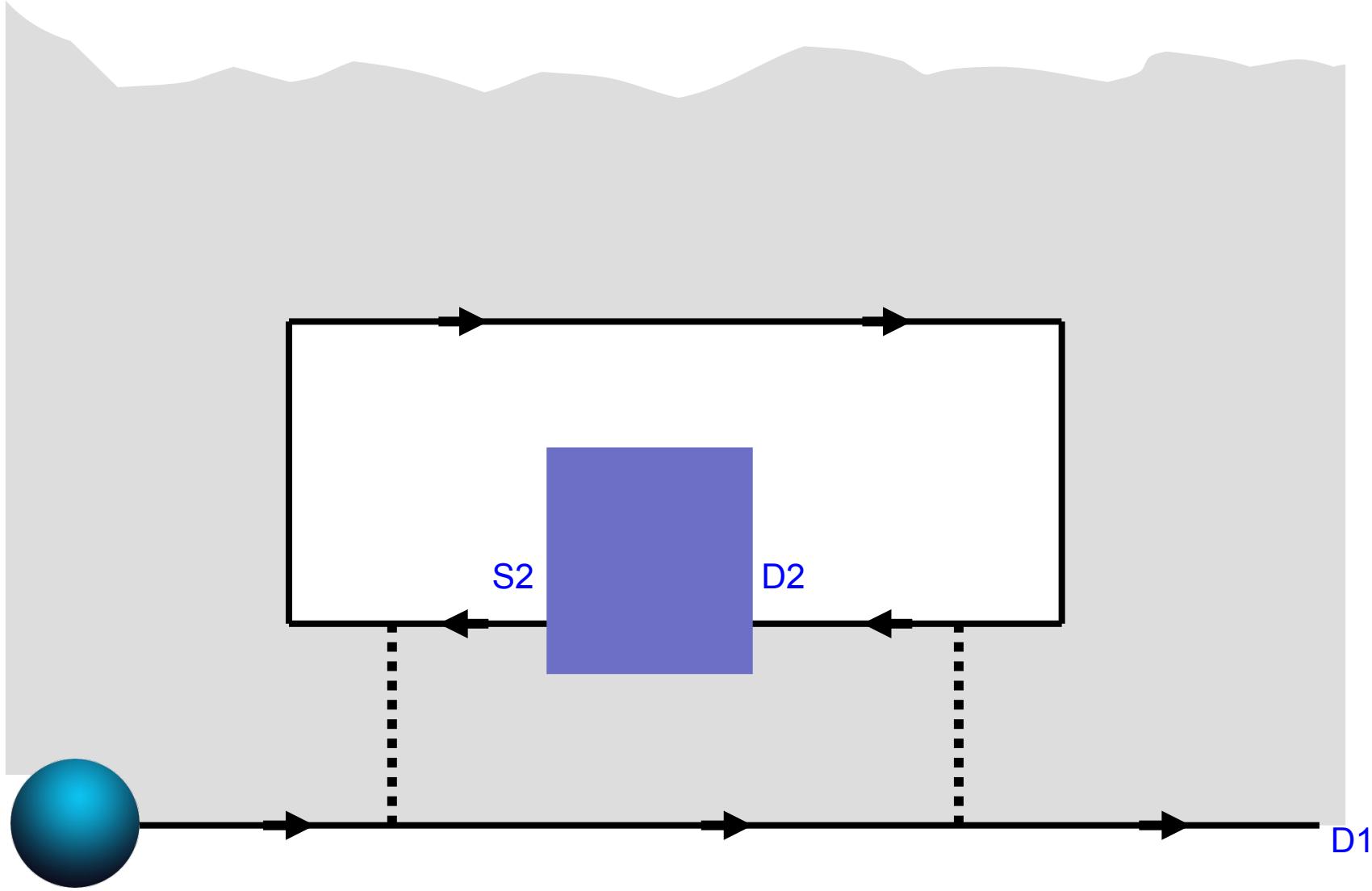
why is a Mach-Zehnder interferometer a detector for (anyonic) quantum statistics?

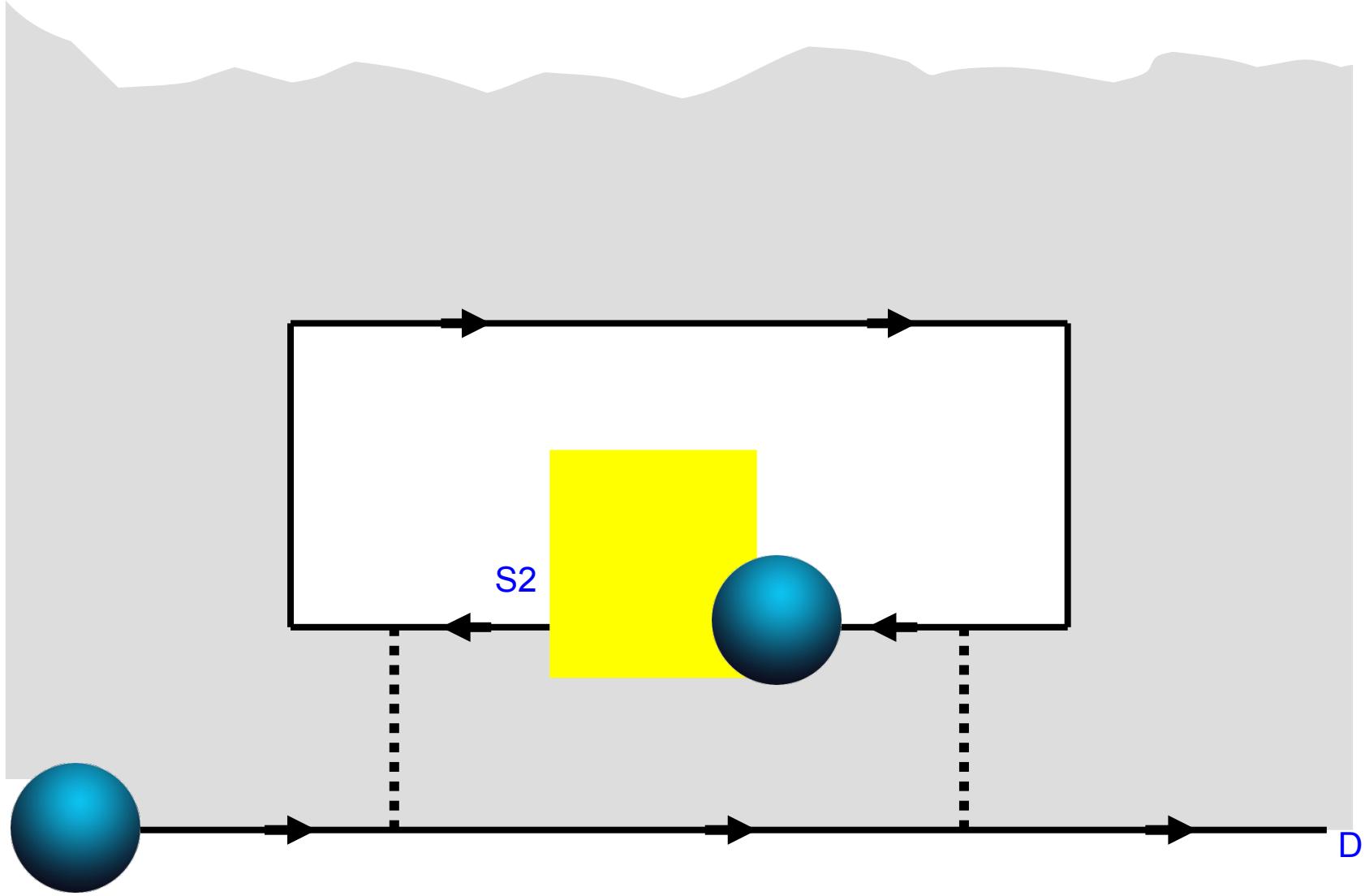


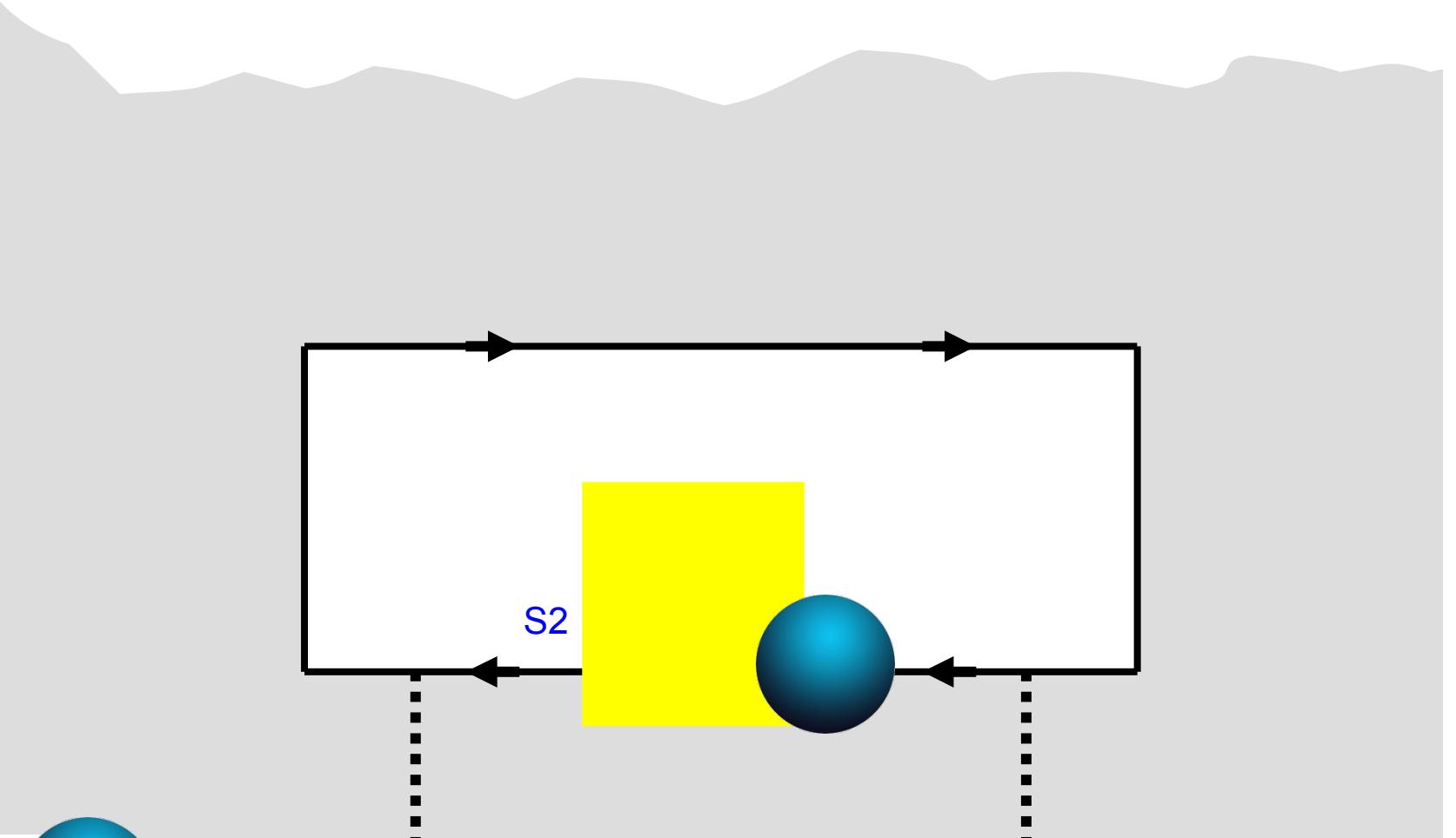
why is it a good detector?



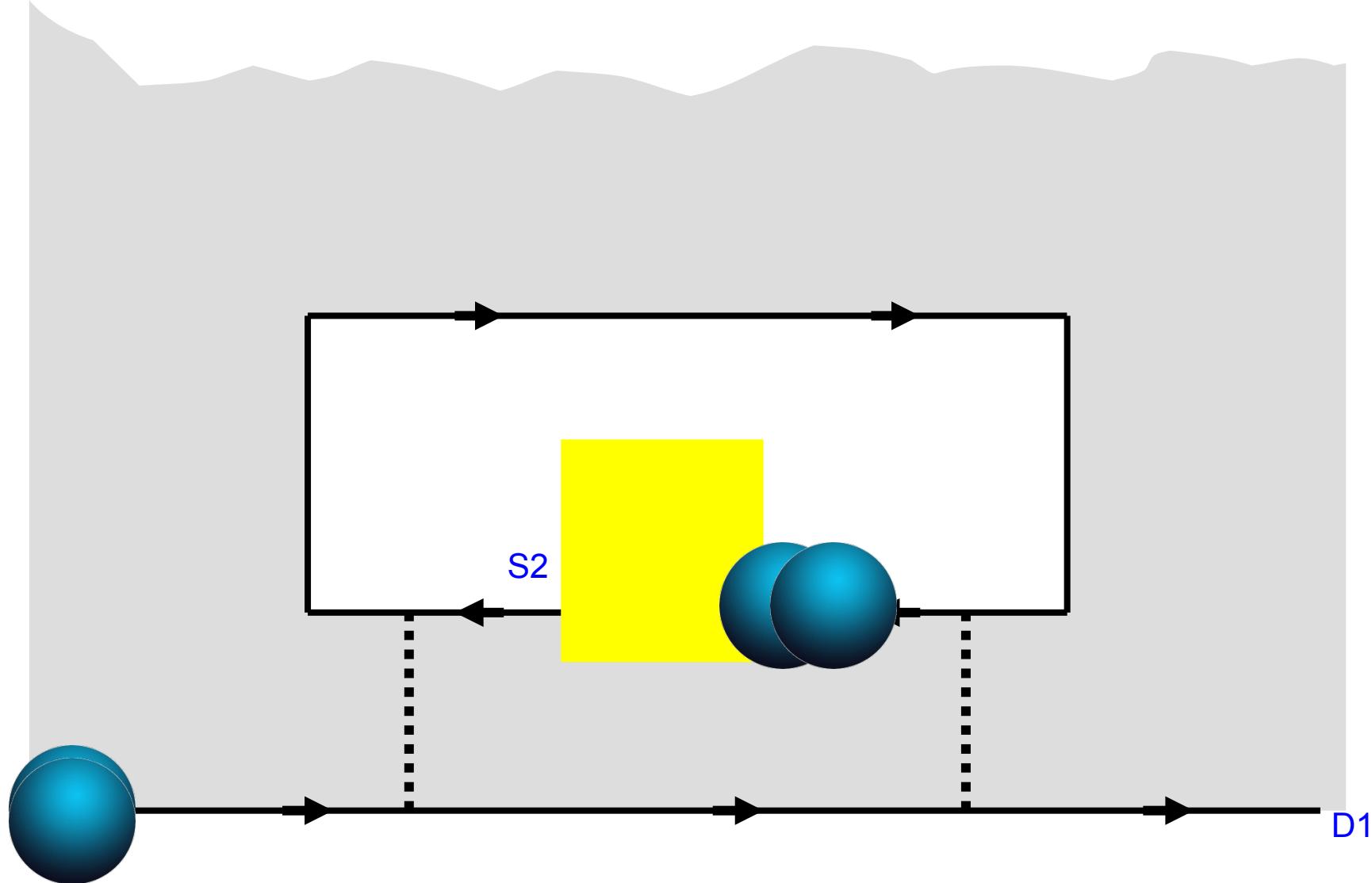
what are the signatures of anyonic statistics?







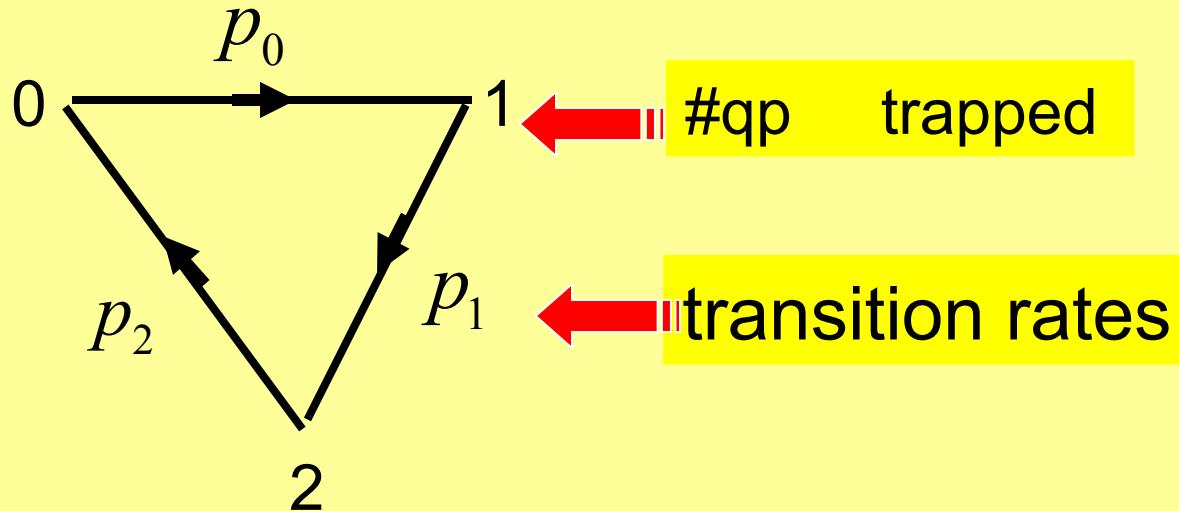
D1



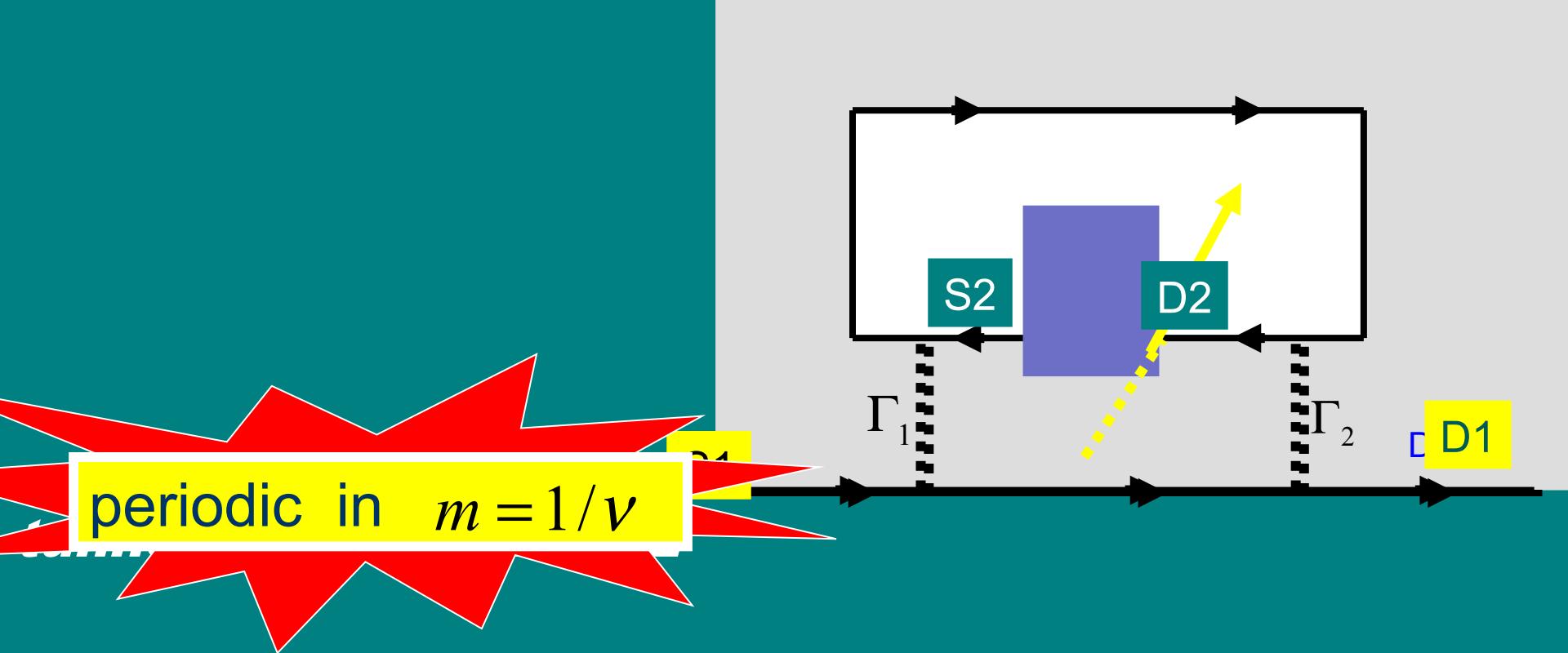
- ★ why is a Mach-Zehnder interferometer a detector for (anyonic) quantum statistics?
- ★ why is it a good detector?
- ★ what are the signatures of anyonic statistics?

$$\nu = 1/3$$

zero temp. kinetic eq.



(a more detailed analysis: Keldysh)



$$p_n = \tilde{c}_1(|\Gamma_1|^2 + |\Gamma_2|^2) + (\tilde{c}_2 \Gamma_1 \Gamma_2^* \exp(-i\phi) + c.c.)$$

$$\phi = 2\pi\nu\frac{\Phi}{\Phi_0} = 2\pi\nu\Phi/\Phi_0 + 2\pi\nu n$$

Aharonov-Bohm of
fractional charge

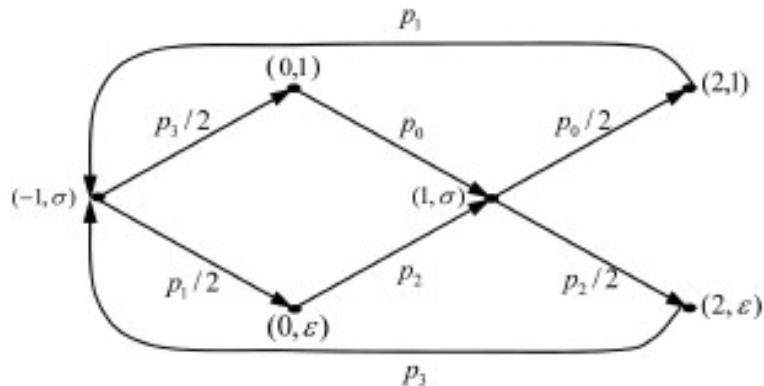
n statistical
fluxoids

WHY IS THIS ROBUST against fluctuations (of q.p., area ...) ?

dynamical averaging over q.p. #

“you

more

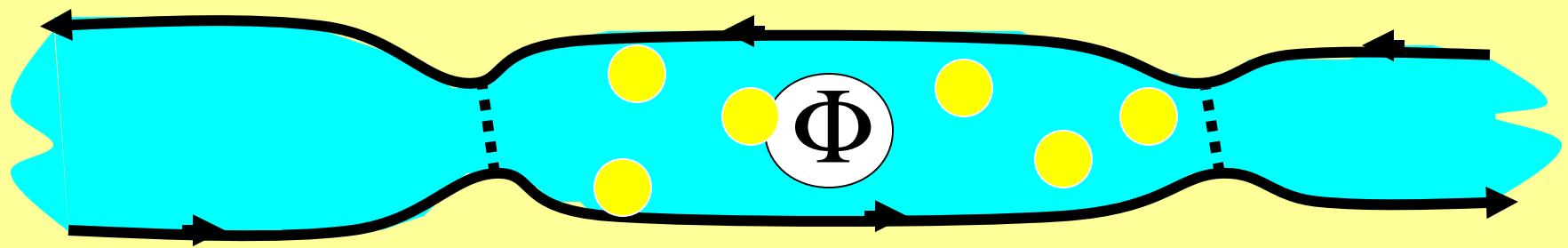


“me”

non-Abelian
Stern

compare with Fabry-Perot interferometer

Chamon, Freed, Kivelson, Sondhi, and Wen (1997)

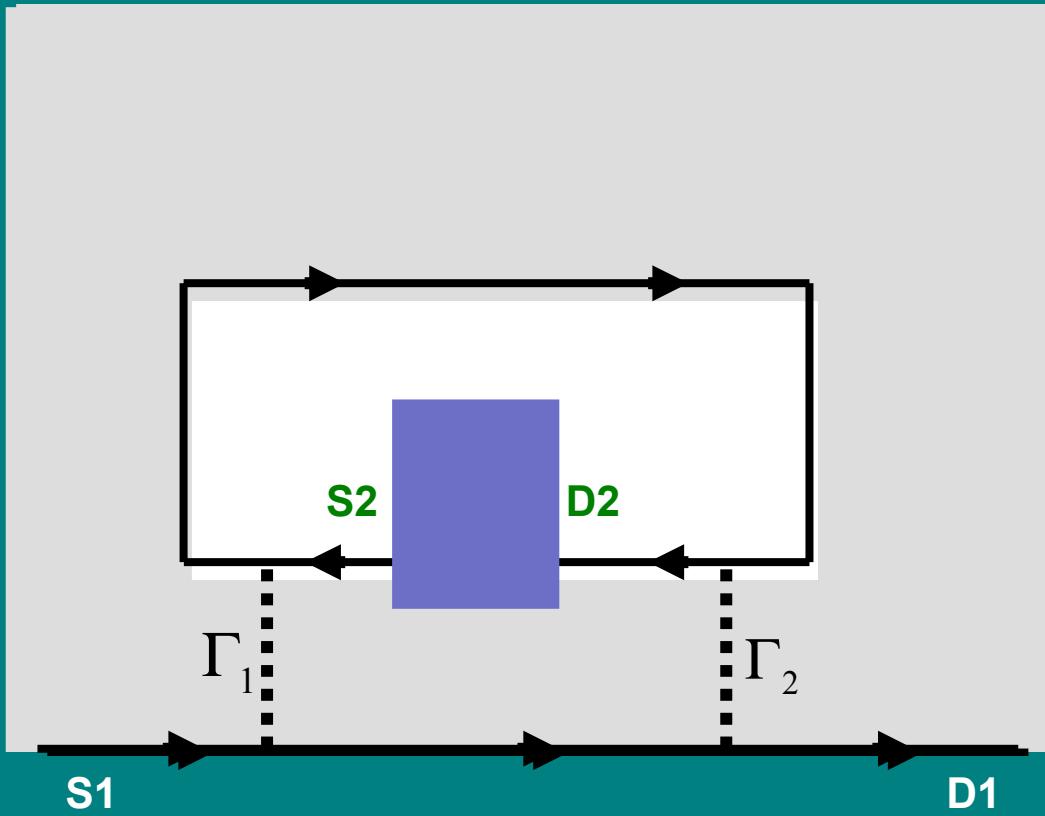


sensitivity to # of qp's

* why is a Mach-Zehnder interferometer a detector for (anyonic) quantum statistics?

* why is it a good detector?

* what are the signatures of anyonic statistics?



$$I = I_0 + I_\Phi$$

*generalization to finite T, V
(kinetic, Keldysh)*

$$\cancel{[I_\Phi(\Gamma_2) - I_\Phi(\Gamma_2 = 0)]} = [I_0(\Gamma_2) - I_0(\Gamma_2 = 0)]^{\pi/\Theta}$$

NOISE... (*kinetic eq. ; zero temp*) also Keldysh

$$S = \overline{\delta Q^2} / t$$

$$\frac{1}{\nu} N \text{ tunneling events } \bar{t} = N \left(\frac{1}{p_0} + \frac{1}{p_1} + \frac{1}{p_2} \right)$$

average square of each tunneling event: $\overline{\delta t_n^2} = \frac{2}{p_n^2}$

fluctuations of total time: $\overline{\delta t^2} = N \frac{1}{p_n^2} \frac{1}{\bar{I}} \frac{1}{p_n^2}$

$\frac{\delta Q}{\text{charge } Q_0} \frac{\text{noise}}{= Ne \text{ transmitted after } t = \bar{t} + \frac{1}{\bar{I}} \delta t} \frac{S = e \bar{I}}{\frac{1}{\bar{I}}} \frac{1}{p_n^2}$

charge transmitted after \bar{t} $Q_0 - \bar{I} \delta t_n$

FANO factor $> \nu$

maximum value = 1 (one of the p_n is small)

non- Abelian case: max. Fano factor > 1

Abelian: invariance under $V \rightarrow -V$

*Non- Abelian : asymmetry under $V \rightarrow -V$
 $(V > T)$*

OTHER

APPROACHES ...

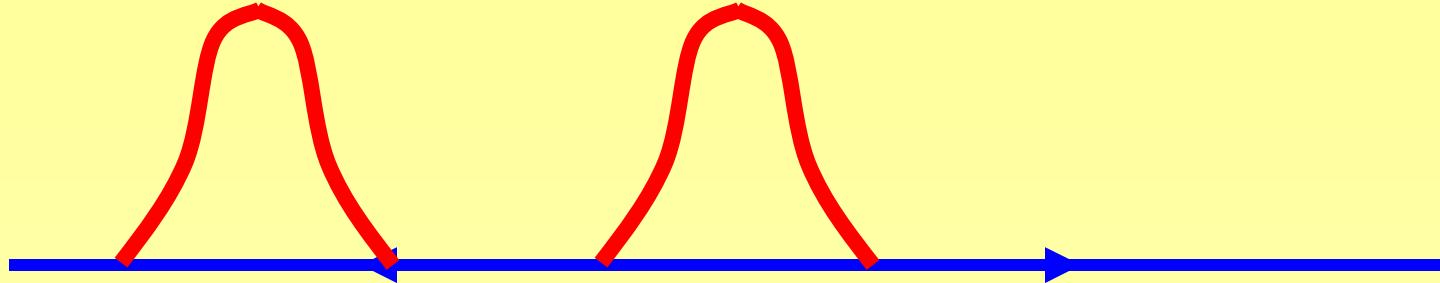
- ★ topological
- ★ algebraic

few more details



*WHAT DOES IT MEAN
TO MEASURE THE STATISTICAL PHASE?*

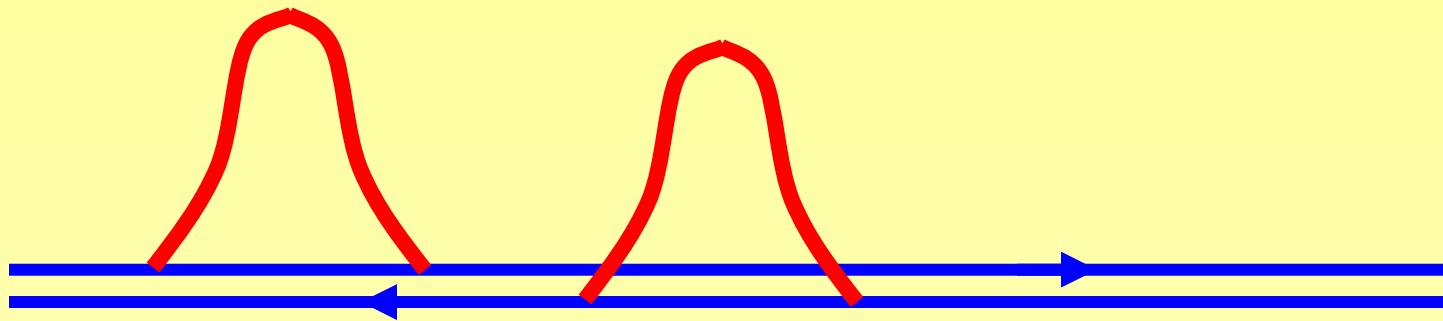
accepted definition...
OBSERVABLES THAT *explicitly*
DEPEND ON KLEIN FACTORS



$$\Psi \quad -\Psi$$

**same branch (edge) --
Bosonic fields guarantee antisymmetrization**

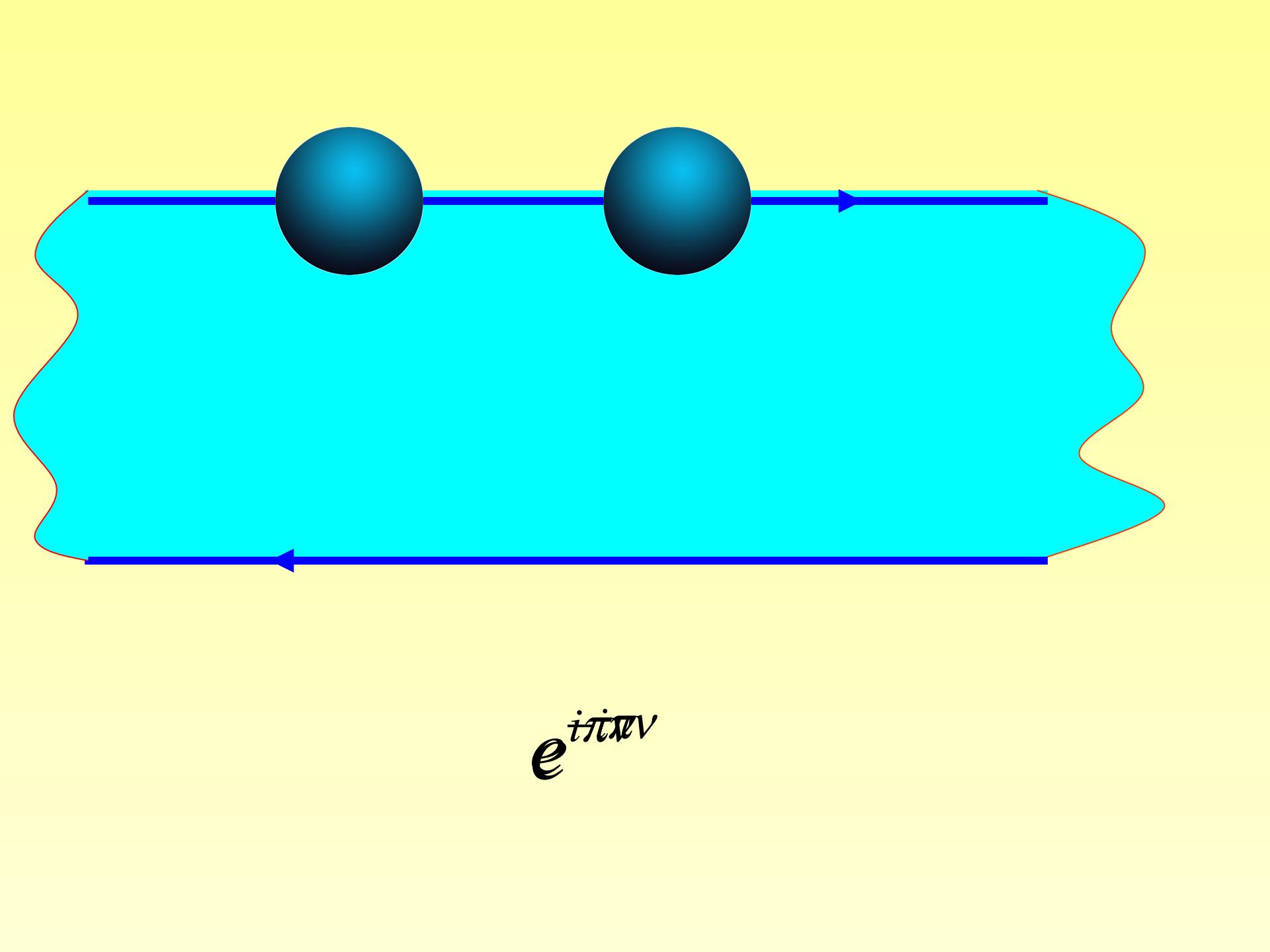
OPPOSITE BRANCHES (EDGES) ...



WE NEED KLEIN FACTORS

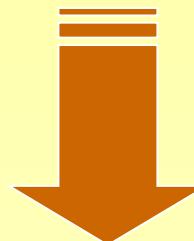
$$\Psi_\eta = \text{K}^\eta K_\eta \ f(N_\eta) \ e^{-i\phi_\eta}$$

$$\{K_\eta, K_{\eta'}\} = 2\delta_{\eta\eta'}$$



$$e^{i\pi\alpha v}$$

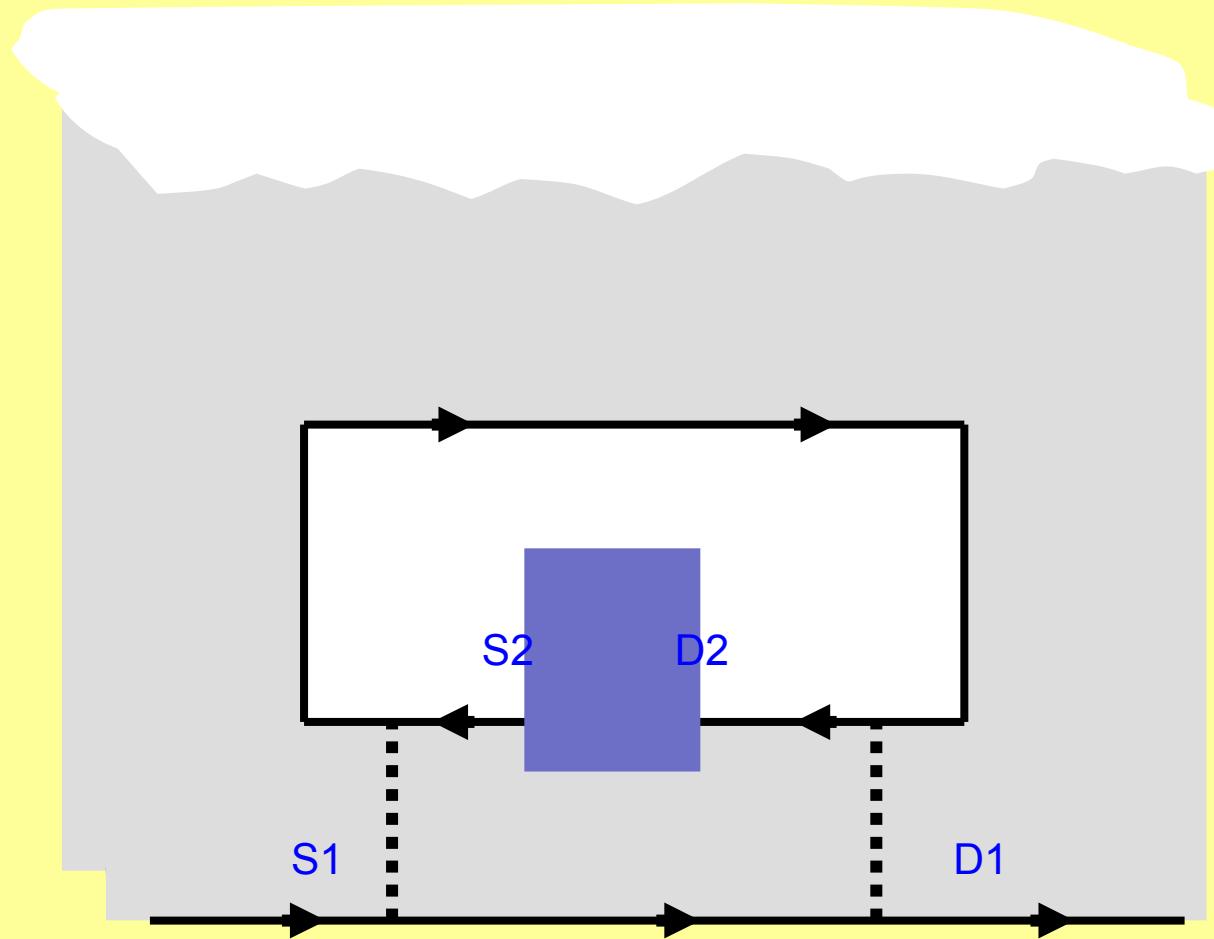
**NO UNIQUE / CONSISTENT WAY TO ASSIGN A
KLEIN FACTOR TO A
FRACTIONALLY CHARGED QUASI-PARTICLE**

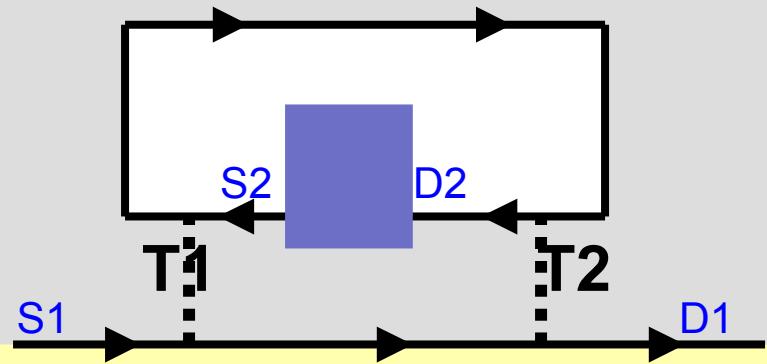


**ASSIGN A KLEIN FACTOR TO A
TUNNELING OPERATOR**

cf. Kane 2003

see also Ponomarenko & Averin





$$L = -\frac{1}{4\pi} \int dx dt \sum_{k=1,2} [\partial_t \phi_k \partial_x \phi_k + v(\partial_x \phi_k)^2] - \int dt (T_1 + T_2),$$

$$T_1 = K_1 \Gamma_1 \exp(i[\phi_1(0, t) - \phi_2(0, t)] + h.c) \quad T_2 = K_2 \Gamma_2 \exp(i[\phi_1(0, t) - \phi_2(0, t)] + h.c)$$

$$\Gamma_1 : \exp(-iVt)$$

$$\Gamma_2 : \exp(-iVt) \exp(i2\pi V\Phi/\Phi_0)$$

$$\kappa_1 \kappa_2 = \exp(-2\pi\nu i) \kappa_2 \kappa_1; \kappa_1 \kappa_2^+ = \kappa_2^+ \kappa_1 \exp(2\pi\nu i).$$

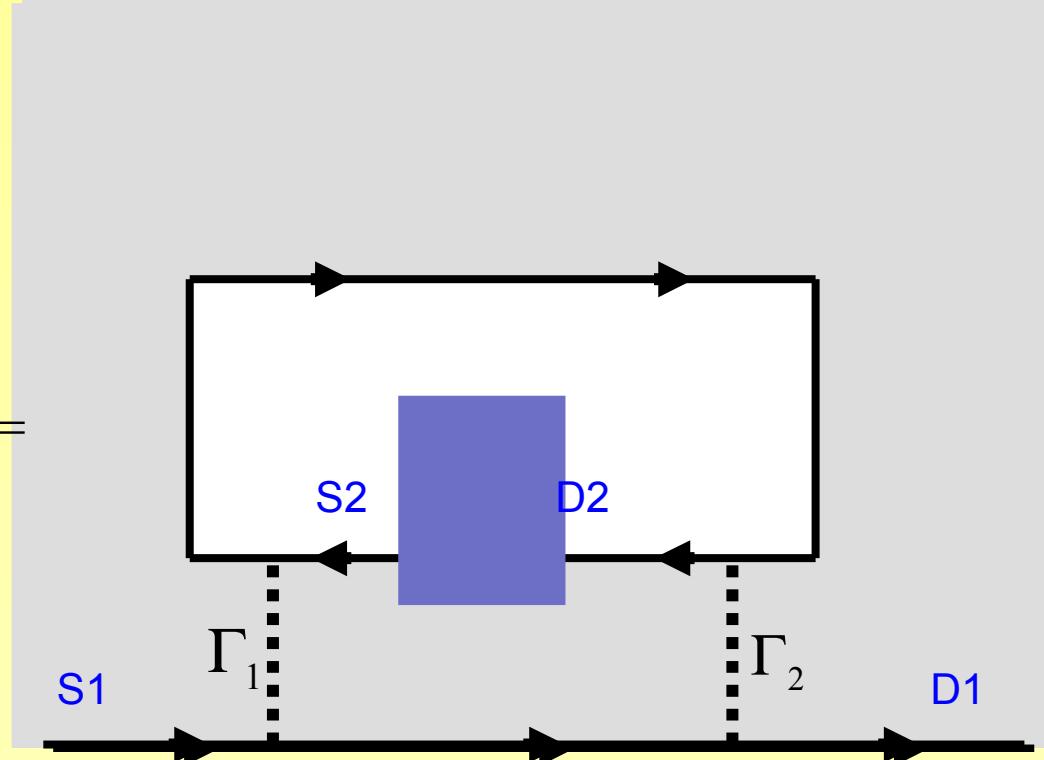
$$\kappa_1 \kappa_2 = \exp(-2\pi\nu i) \kappa_2 \kappa_1; \kappa_1 \kappa_2^+ = \kappa_2^+ \kappa_1 \exp(2\pi\nu i).$$

$$\text{Tr}[(K_1^\dagger)^{n1} (K_1)^{m1} (K_2^\dagger)^{n2} (K_2)^{m2}] =$$

$$\text{Tr}[(K_2^\dagger)^{n2} (K_2)^{m2} (K_1^\dagger)^{n1} (K_1)^{m1}]$$

$$(K_1^\dagger)^{n1} (K_1)^{m1} (K_2^\dagger)^{n2} (K_2)^{m2} =$$

$$\exp[i2\pi\nu \underbrace{(n1-m1)^2}_{\text{INTEGER}}] (K_2^\dagger)^{n2} (K_2)^{m2} (K_1^\dagger)^{n1} (K_1)^{m1}$$



INTEGER

Summary:

$$[I_\Phi(\Gamma_2) - I_\Phi(\Gamma_2 = 0)] \cancel{=} [I_0(\Gamma_2) - I_0(\Gamma_2 = 0)]^{\pi/\Theta}$$

FANO factor : Abelian : max 1
 non-Abelian: >1

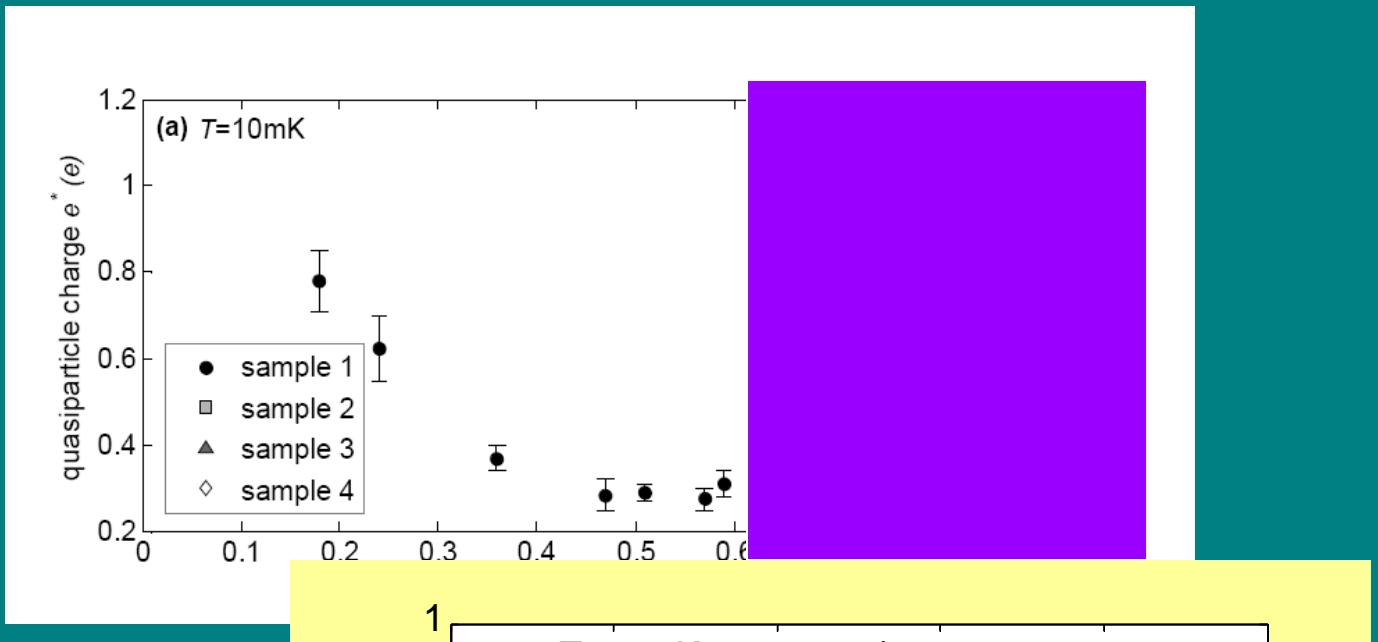
Abelian: invariance under $V \rightarrow -V$
Non-Abelian : asymmetry under $V \rightarrow -V$
($V > T$)

MZI : robust

experimental doldrums

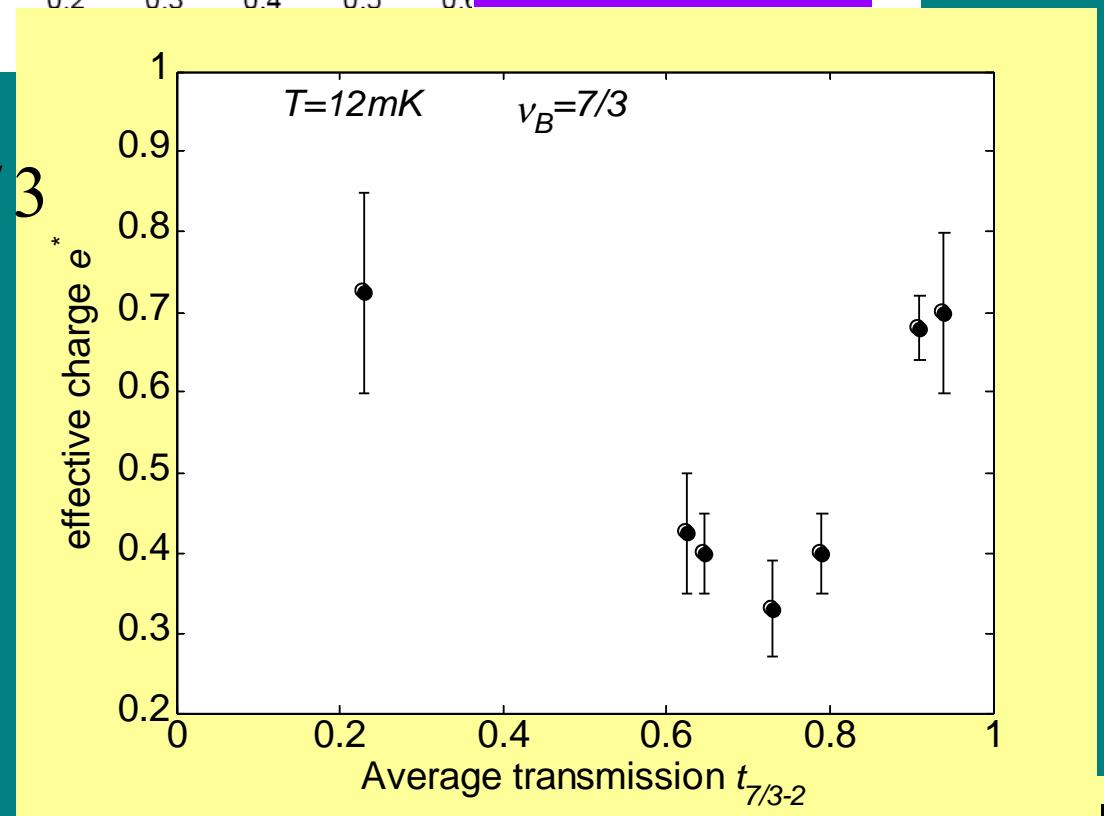
- ★ **very difficult to obtain interferometry signal**
(see however Willett et al for FP)
- ★ **even the measured fractional charge is a puzzle**

$\nu = 5/2$



$\nu = 7/3$

similar results for $\nu = 1/3$



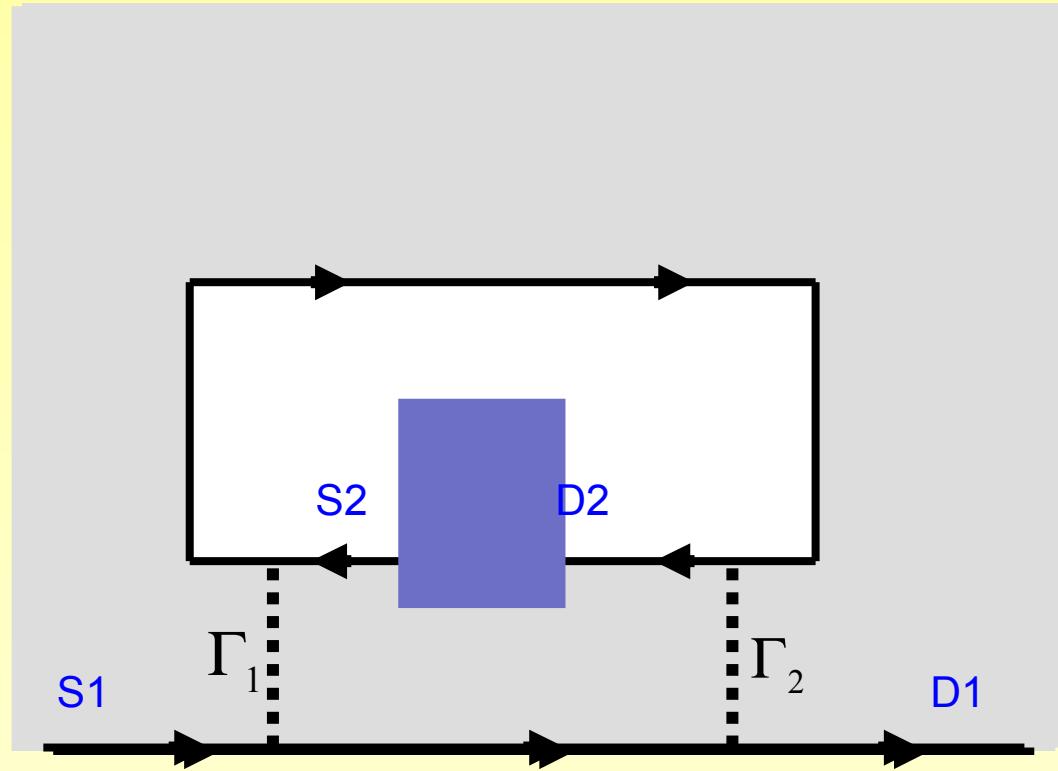
$$I = I_0 + I_\Phi$$

$T > eV$ (linear response)

$$G_\Phi(T) : [G_0(T)]^{1/\nu}$$

$T < eV$

$$I_\Phi(V) : [I_0(V)]^{(3\nu-2)/(2\nu^2-\nu)}$$



$$[I_0(\Gamma_2) - I_0(\Gamma_2 = 0)] = [I_\Phi(\Gamma_2) - I_\Phi(\Gamma_2 = 0)]^{2\nu}$$

SUMMARY

$$[I_0(\Gamma_2) - I_0(\Gamma_2 = 0)] = [I_\Phi(\Gamma_2) - I_\Phi(\Gamma_2 = 0)]^{2\nu}$$

DEPENDS ON KLEIN FACTORS



- temperature
- dephasing
- visibility vs. arm asymmetry

END OF INTRODUCTION

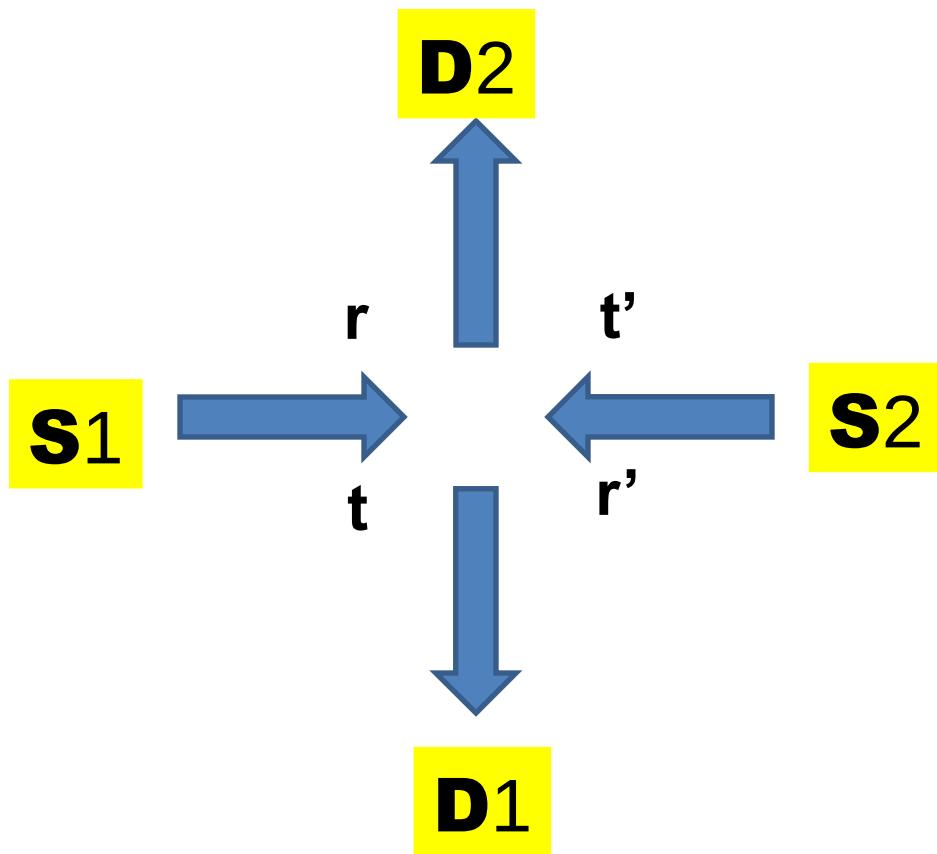
HANBURY BROWN & TWISS INTERFEROMETRY WITH ANYONS

one particle: interferometry (e.g. with AB flux)

two particles: entanglement; quantum statistics
(of identical particles)

$$T = |\mathbf{t}|^2$$

$$R = |\mathbf{r}|^2$$



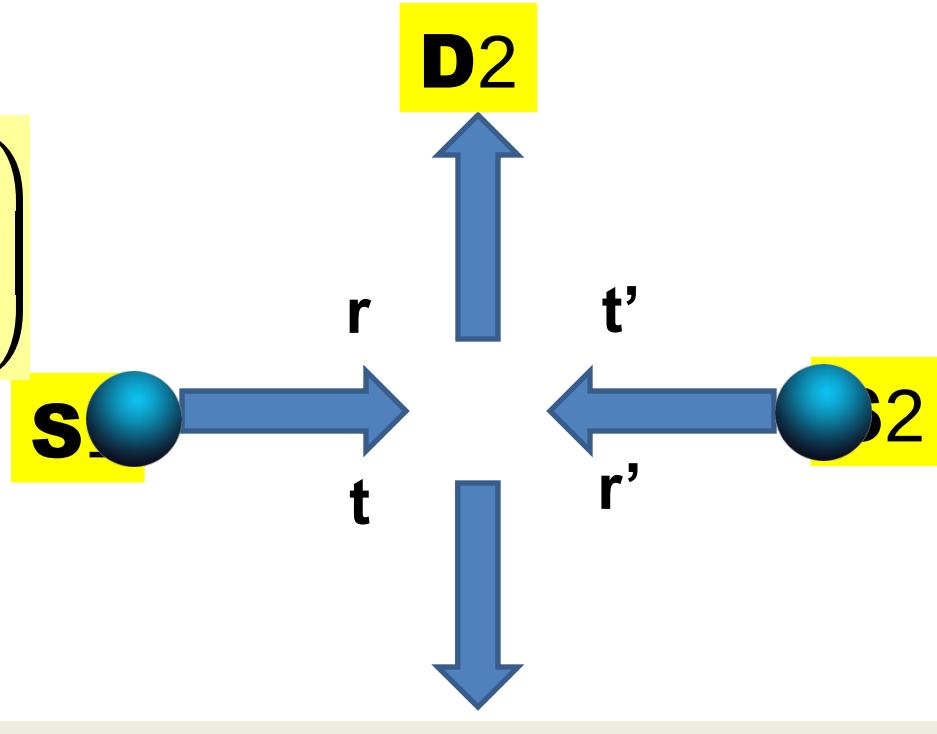
	$P(2,0)$	$P(1,1)$	$P(0,2)$
Classical	RT	$R^2 + T^2$	RT

Fermions:

$$\begin{pmatrix} b_{D1} \\ b_{D2} \end{pmatrix} = \hat{S} \begin{pmatrix} a_{S1} \\ a_{S2} \end{pmatrix}$$

$$\hat{n}_1 = b_{D1}^\dagger b_{D1}$$

$$\hat{n}_2 = b_{D2}^\dagger b_{D2}$$



$$T = |t|^2$$

$$R = |r'|^2$$

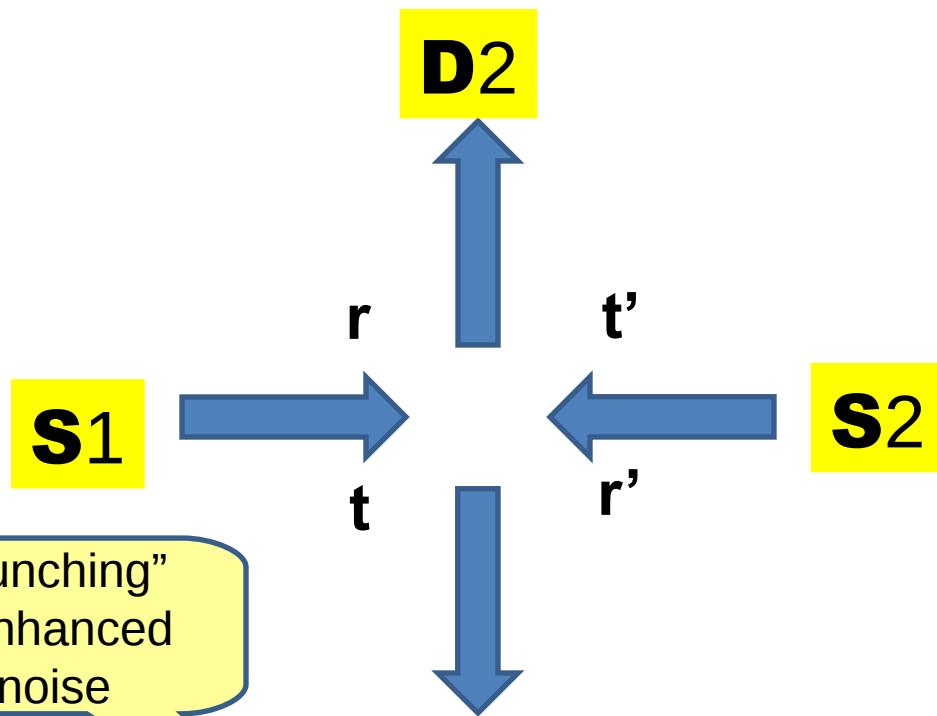
$$P(1,1) = \langle \Psi | \hat{n}_1 \hat{n}_2 | \Psi \rangle = \langle 0 | a_{S2} a_{S1} b_{D1}^\dagger b_{D1} b_{D2}^\dagger b_{D2} a_{S1}^\dagger a_{S2}^\dagger | 0 \rangle$$

$$P(1,1) = (T + R)^2 = 1$$

	P(2,0)	P(1,1)	P(0,2)
Fermions	0	1	0

$$T = |t|^2$$

$$R = |r|^2$$



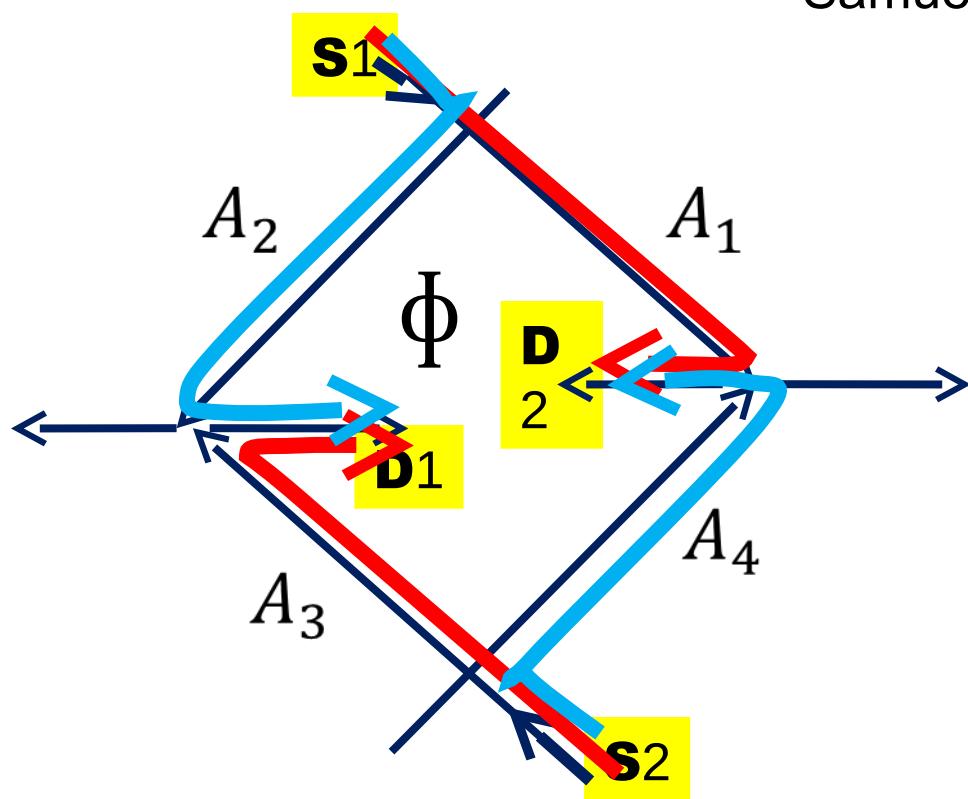
“bunching”
=enhanced noise

“anti-bunching”
=reduced noise

	$P(2,0)$	$P(1,1)$	$P(0,2)$
Classical	RT	$R^2 + T^2$	RT
Bosons	$2RT$	$R^2 + T^2 - 2RT$	$2RT$
Fermions	0	1	0

... adding another handle: A-B flux

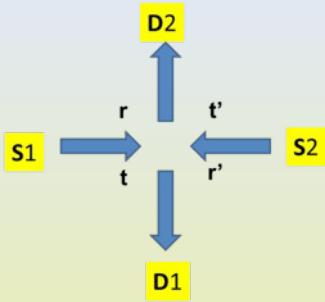
Samuelson, Sukhorukov, Buttiker 2004



$$\langle I_{D1} I_{D2} \rangle : |A_1 + A_4|^2 |A_2 + A_3|^2$$

→ $A_1 \quad A_4^* \quad A_3 \quad A_2^*$

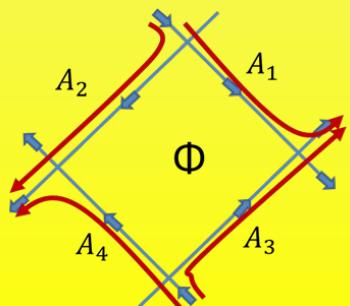
SUMMARY Fermions/Bosons



HBT

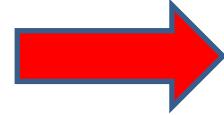
$\left\{ \begin{array}{ll} \text{fermions} & \text{anti-bunching} \\ \text{bosons} & \text{bunching} \end{array} \right.$

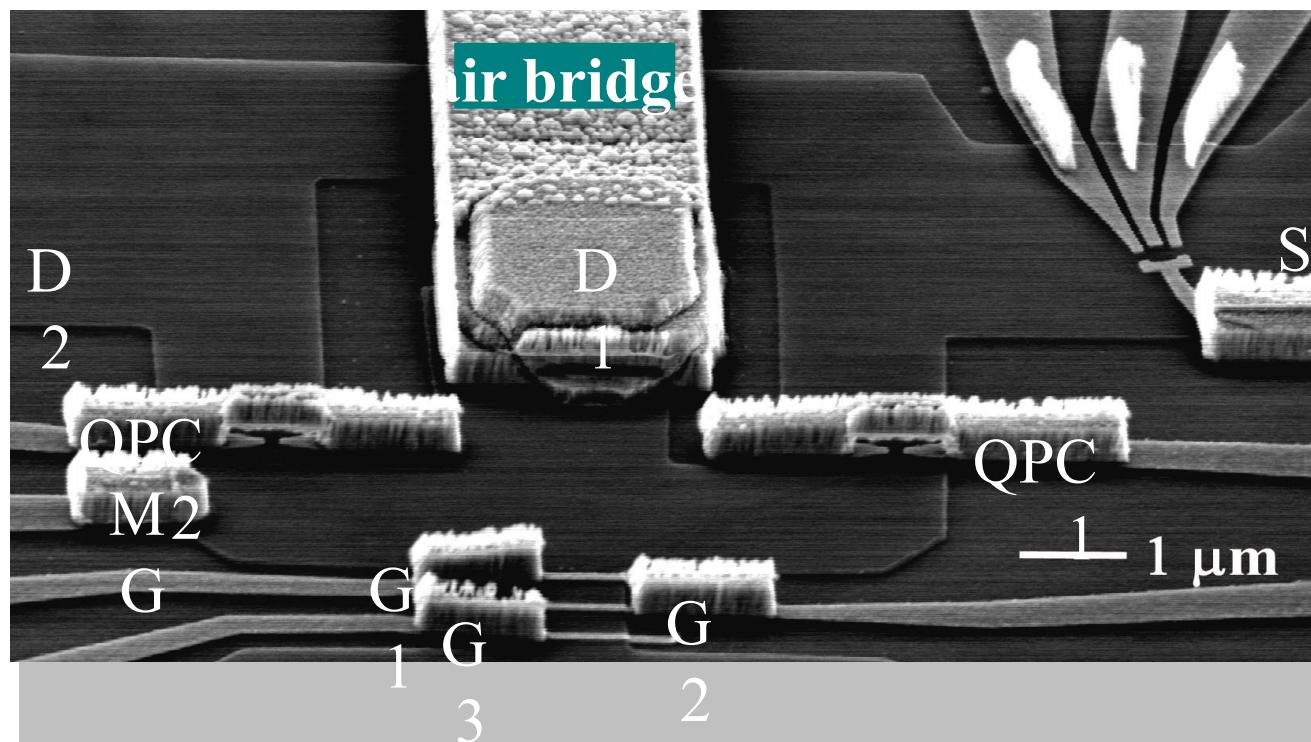
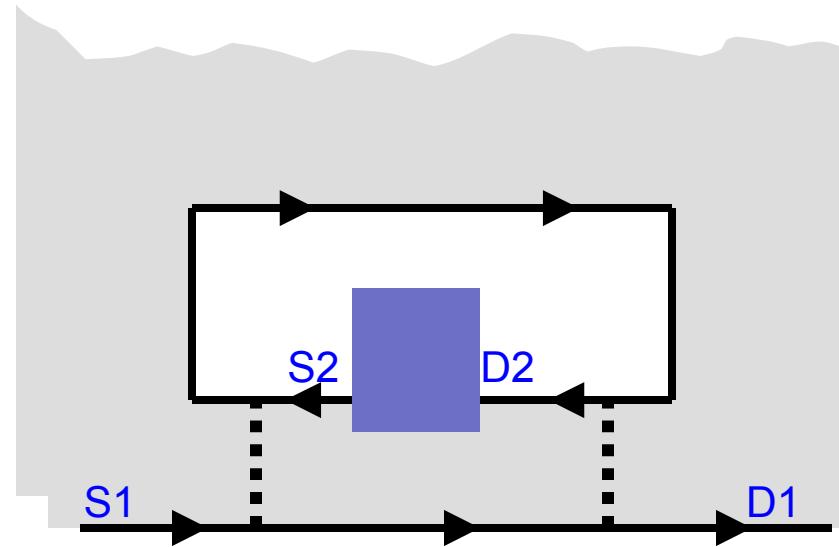
HBT with AB



$\left\{ \begin{array}{ll} \text{fermions} & \cdot \# \cos(2\pi \frac{\phi}{\phi_0}) \\ \text{bosons} & \end{array} \right.$

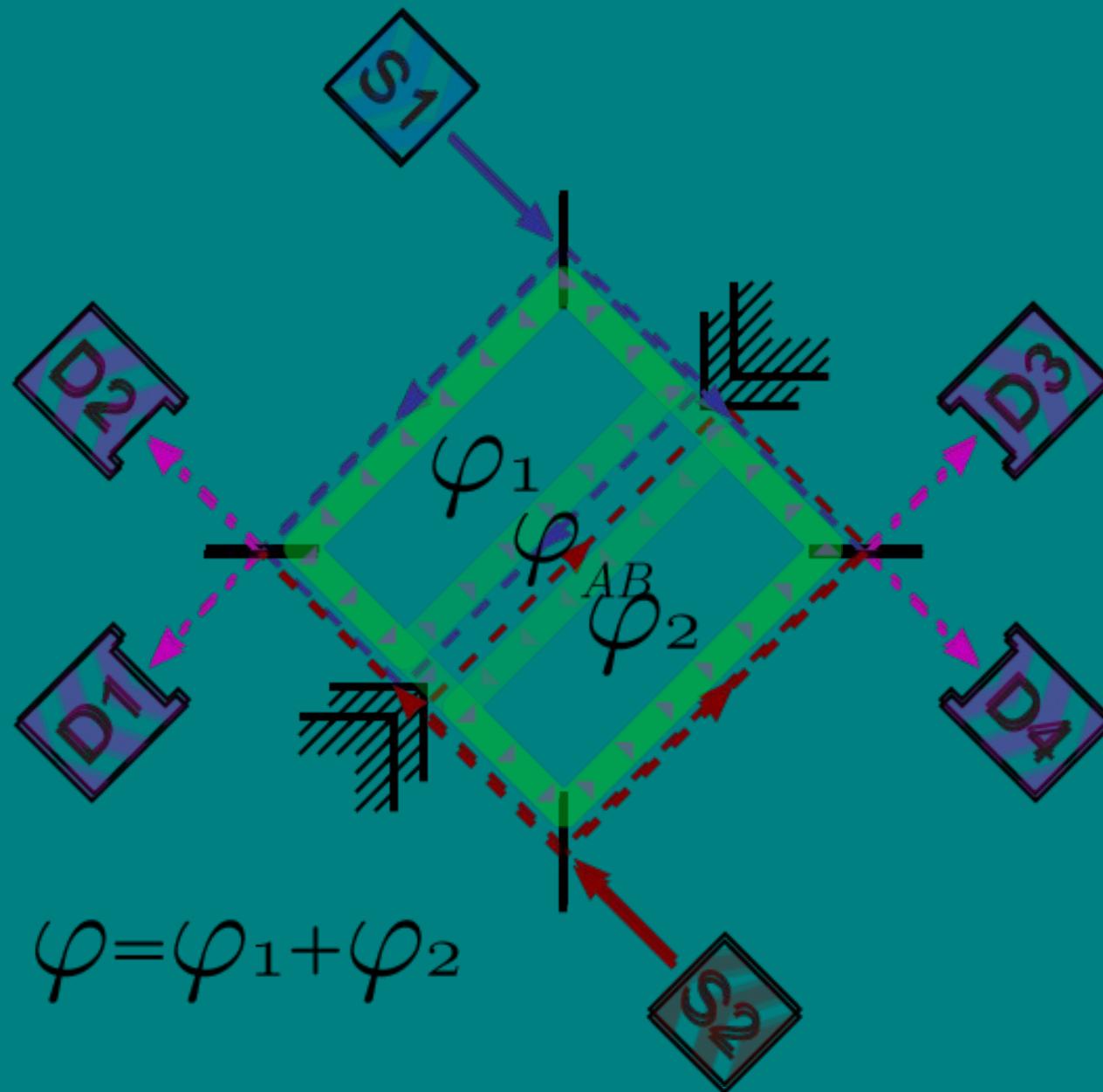
realization of HBT + AB

Mach-Zehnder interferometer 
2 Mach-Zehnder interferometers



Heiblum *et al*

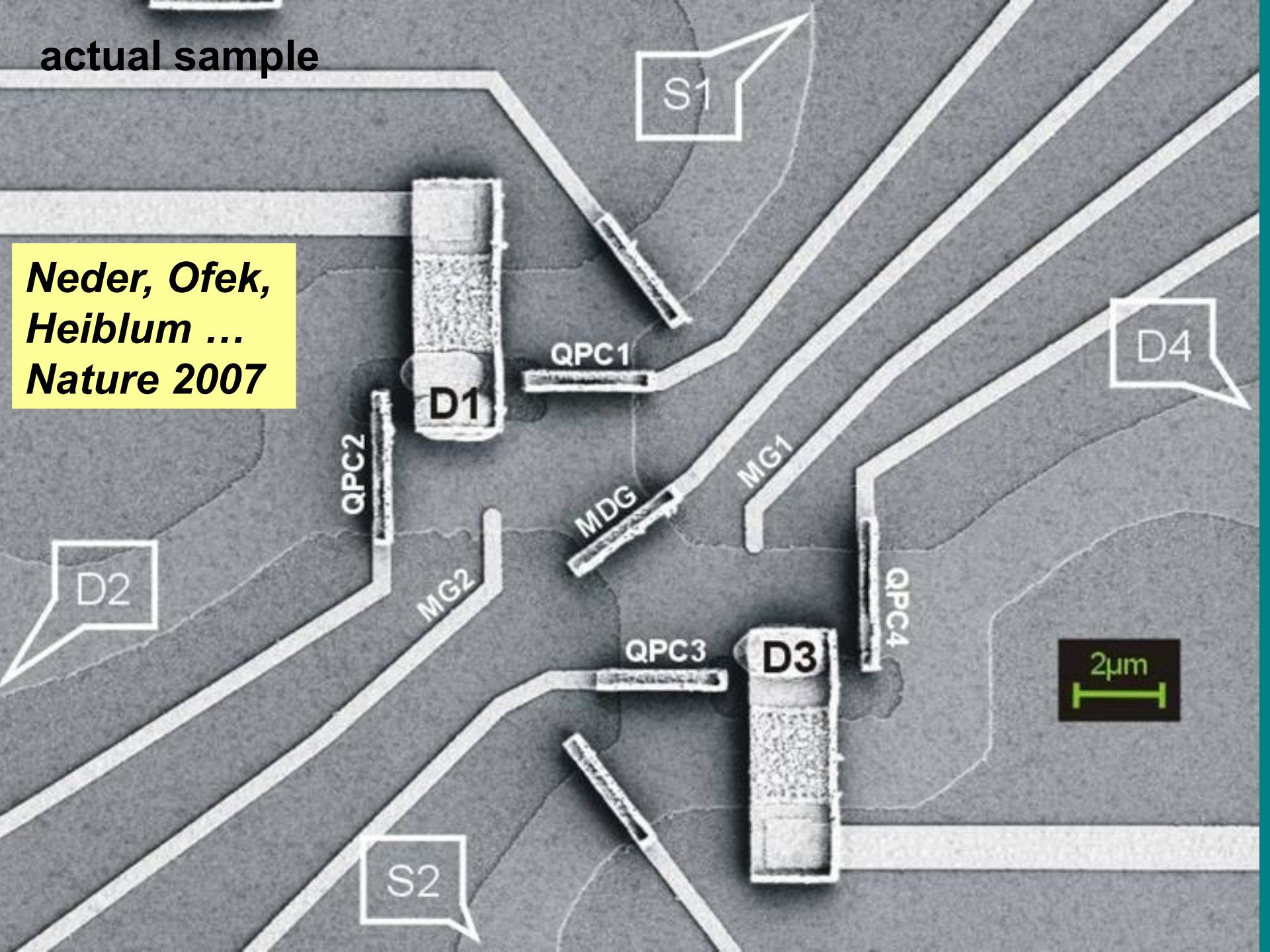
introducing 'two-particle' interference



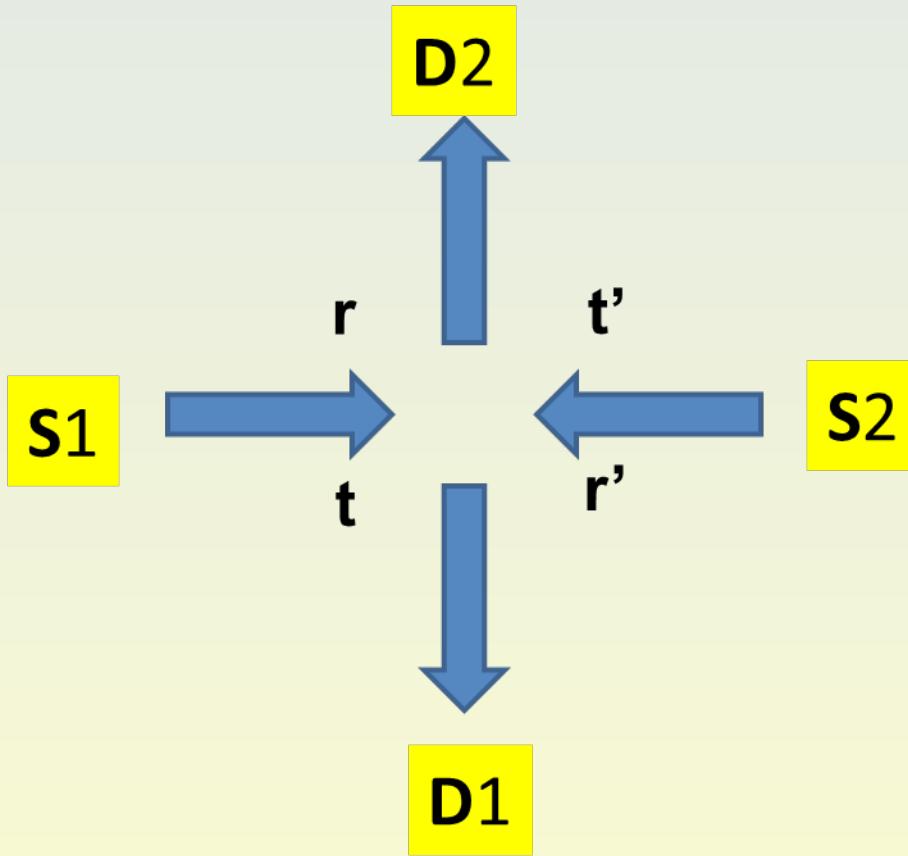
Neder, Ofek,
Heiblum, ...

actual sample

**Neder, Ofek,
Heiblum ...
Nature 2007**

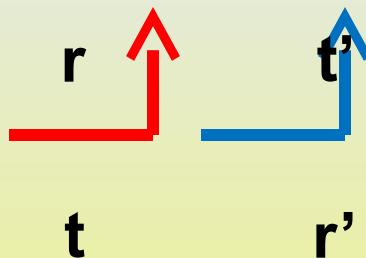


A CARICATURE WITH (Abelian) ANYONS



A CARICATURE WITH (Abelian) ANYONS

$P(2,0)$

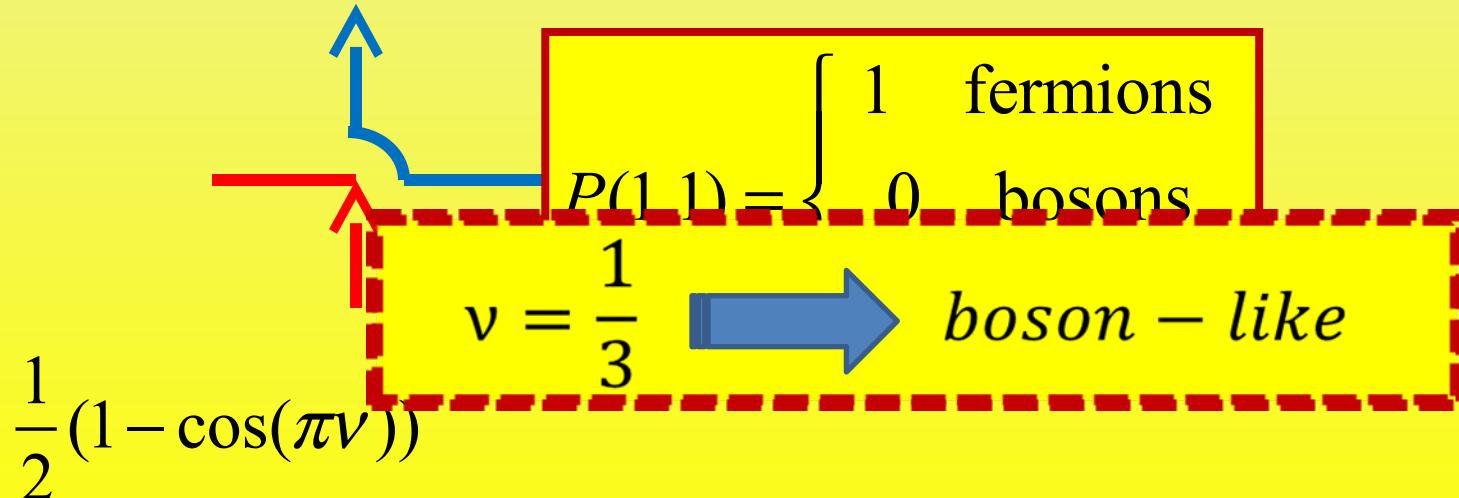


$$t = t' = \frac{1}{\sqrt{2}}$$

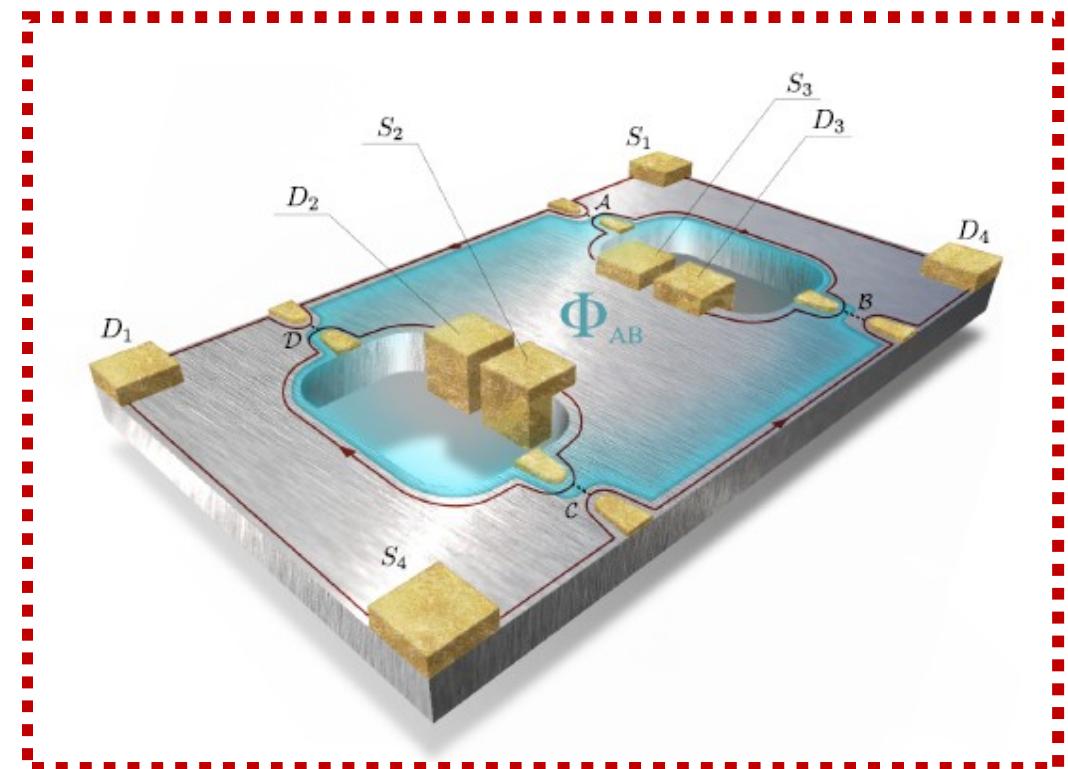
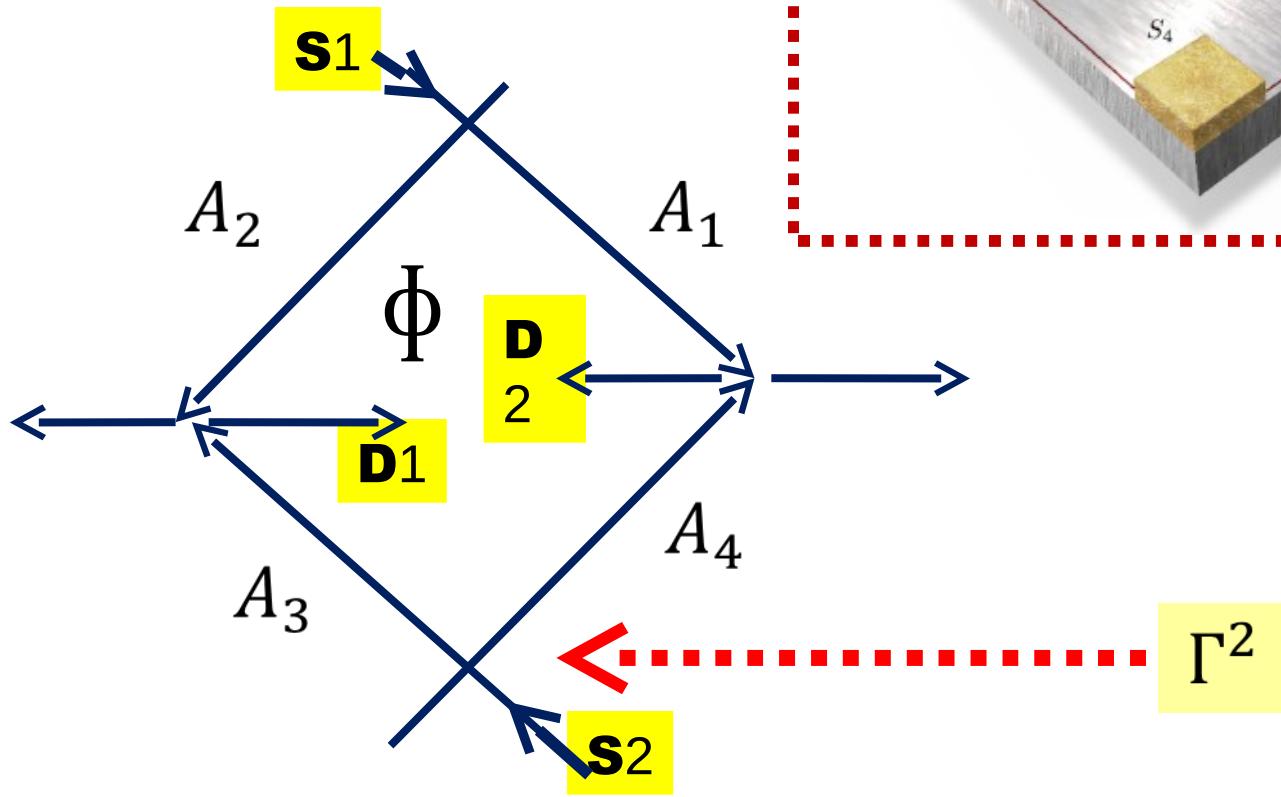
$$r = r' = i \frac{1}{\sqrt{2}}$$

$$\frac{1}{4}(1 + \cos(\pi v)) P(2,0) + P(1,1) + P(0,2) = 1$$

$P(1,1)$

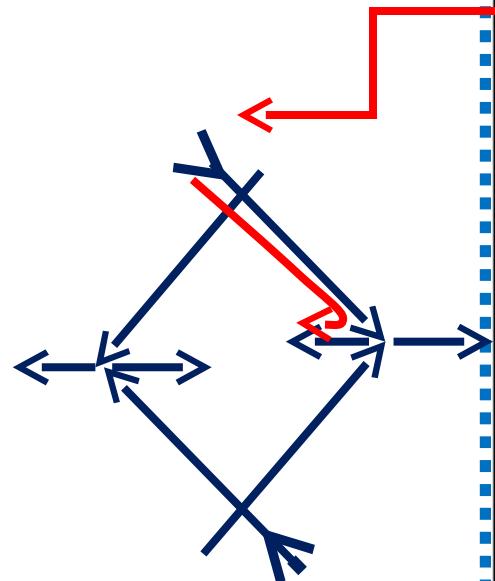


$$\frac{1}{2}(1 - \cos(\pi v))$$

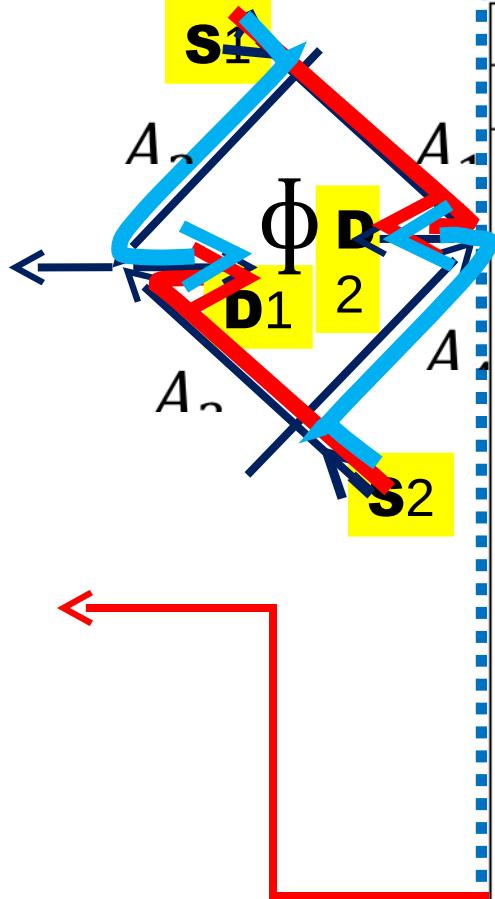


$$\Gamma^2$$

One & Two Qp processes



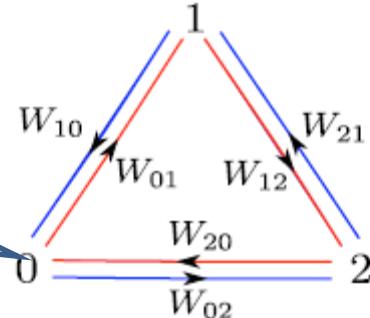
Elementary processes							
Process ζ	Order	(j, f)	D_1	D_2	D_3	D_4	Flux factor $\kappa_j^{(\zeta)}$
$(1, \mathcal{A}, 0)$	Γ^2	$(j, j + 1)$	-1	0	1	0	1
$(1, \mathcal{B}, 0)$	Γ^2	$(j, j + 1)$	0	0	1	-1	1
$(1, \mathcal{C}, 0)$	Γ^2	$(j, j + 1)$	0	1	0	-1	1
$(1, \mathcal{D}, 0)$	Γ^2	$(j, j + 1)$	-1	1	0	0	1
$(2, \mathcal{A}, 0)$	Γ^4	$(j, j + 2)$	-2	0	2	0	1
$(2, \mathcal{B}, 0)$	Γ^4	$(j, j + 2)$	0	0	2	-2	1
$(2, \mathcal{C}, 0)$	Γ^4	$(j, j + 2)$	0	2	0	-2	1
$(2, \mathcal{D}, 0)$	Γ^4	$(j, j + 2)$	-2	2	0	0	1
$(2, \mathcal{AB}, 0)$	Γ^4	$(j, j + 2)$	-1	0	2	-1	1
$(2, \mathcal{CD}, 0)$	Γ^4	$(j, j + 2)$	-1	2	0	-1	1
$(2, \mathcal{AD}, 0)$	Γ^4	$(j, j + 2)$	-2	1	1	0	1
$(2, \mathcal{BC}, 0)$	Γ^4	$(j, j + 2)$	0	1	1	-2	1
$(2, \mathcal{ABCD}, \Phi_{\text{tot}}(j))$	Γ^4	$(j, j + 2)$	-1	1	1	-1	$\cos[\frac{2\pi(\Phi_{AB}+j\Phi_0)}{3\Phi_0}]$
$(1, \mathcal{ABCD}, \Phi_{\text{tot}}(j))_1$	Γ^4	$(j, j + 1)$	0	0	1	-1	$\cos[\frac{2\pi(\Phi_{AB}+j\Phi_0)}{3\Phi_0}]$
$(1, \mathcal{ABCD}, \Phi_{\text{tot}}(j))_2$	Γ^4	$(j, j + 1)$	-1	1	0	0	$\cos[\frac{2\pi(\Phi_{AB}+j\Phi_0)}{3\Phi_0}]$



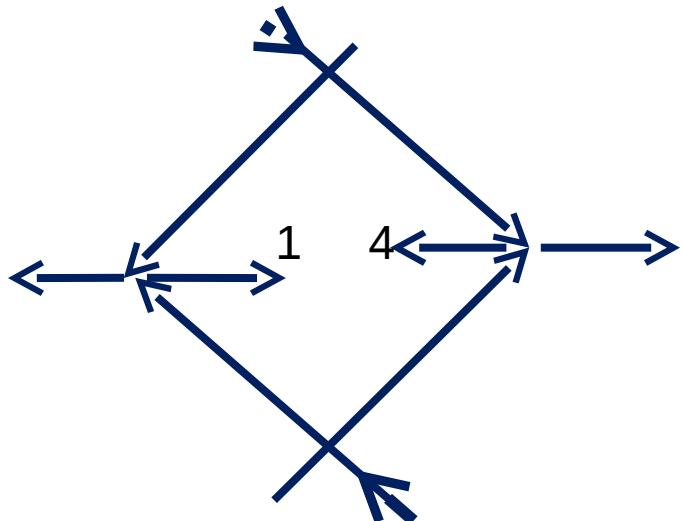
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$(1, \mathcal{D}, 0)$	Γ^2	$(j, j + 1)$	-1	1	0	0	1
$(2, \mathcal{A}, 0)$	Γ^4	$(j, j + 2)$	-2	0	2	0	1
$(2, \mathcal{B}, 0)$	Γ^4	$(j, j + 2)$	0	0	2	-2	1
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$(1, \mathcal{ABCD}, \Phi_{\text{tot}}(j))_2$	Γ^4	$(j, j + 1)$	-1	1	0	0	$\cos[\frac{2\pi(\Phi_{AB} + j\Phi_0)}{3\Phi_0}]$

RATE EQUATION TREATMENT: three-state kinetics

number of
trapped
fluxes
 $\text{mod}(3)$



$$S_{14} = \ll I_1 I_4 \gg$$



**focus on AB component of
current-current correlation:
two questions:**

- ★ *What is the order in Γ of leading harmonics?*
- ★ *What is its sign?*



What is the order in Γ of leading harmonics?

What is its sign?

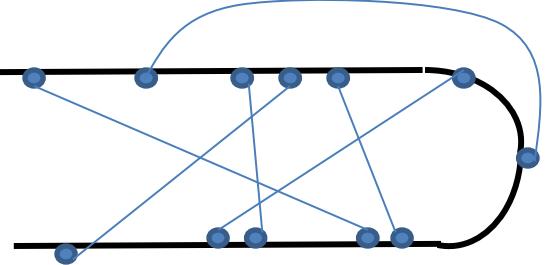
order of Γ^4 vanishes !!

$$S_{14}^{AB} \propto \sum_{if} \cos\left[\frac{2\pi}{3}(\Phi_{AB} + i\Phi_0)/\Phi_0\right] = 0$$

lowest non-trivial order (Byers-Yang OK!) Γ^{12} !!? ??

$$\sum_{\eta,\eta_1,...,\eta_{10}=\pm 1}\eta_1\dots \eta_{10}\int_{-\infty}^{+\infty}dtdt_1\dots dt_{10}e^{-i\nu eV(t+t_1+t_2+t_3+t_4-t_5-t_6-t_7-t_8-t_9-t_{10})}\\e^{2\pi i\nu[\Phi_t(t^{-\eta})+\Phi_t(t^{\eta_3}_3)+\Phi_t(t^{\eta_4}_4)]/\Phi_0}\Big\{\frac{s_{-\eta,\eta_3}(0,t-t_3)s_{-\eta,\eta_4}(0,t-t_4)s_{\eta_3,\eta_4}(0,t_3-t_4)}{s_{-\eta,\eta_5}(-L_1,t-t_5)s_{-\eta,\eta_6}(-L_1,t-t_6)s_{-\eta,\eta_7}(-L_1,t-t_7)}\\ \frac{s_{\eta_5,\eta_6}(0,t_5-t_6)s_{\eta_5,\eta_7}(0,t_5-t_7)s_{\eta_6,\eta_7}(0,t_6-t_7)}{s_{\eta_3,\eta_5}(-L_1,t_3-t_5)s_{\eta_3,\eta_6}(-L_1,t_3-t_6)s_{\eta_3,\eta_7}(-L_1,t_3-t_7)}\\ \frac{1}{s_{\eta_4,\eta_5}(-L_1,t_4-t_5)s_{\eta_4,\eta_6}(-L_1,t_4-t_6)s_{\eta_4,\eta_7}(-L_1,t_4-t_7)}\\ \frac{s_{\eta,\eta_1}(0,-t_1)s_{\eta,\eta_2}(0,-t_2)s_{\eta_1,\eta_2}(0,t_1-t_2)}{s_{\eta,\eta_5}(-L_2,-t_5)s_{\eta,\eta_6}(-L_2,-t_6)s_{\eta,\eta_7}(-L_2,-t_7)}\\ \frac{s_{\eta_5,\eta_6}(0,t_5-t_6)s_{\eta_5,\eta_7}(0,t_5-t_7)s_{\eta_6,\eta_7}(0,t_6-t_7)}{s_{\eta_1,\eta_5}(-L_2,t_1-t_5)s_{\eta_1,\eta_6}(-L_2,t_1-t_6)s_{\eta_1,\eta_7}(-L_2,t_1-t_7)}\\ \frac{1}{s_{\eta_2,\eta_5}(-L_2,t_2-t_5)s_{\eta_2,\eta_6}(-L_2,t_2-t_6)s_{\eta_2,\eta_7}(-L_2,t_2-t_7)}\\ \frac{s_{-\eta,\eta_3}(0,t-t_3)s_{-\eta,\eta_4}(0,t-t_4)s_{\eta_3,\eta_4}(0,t_3-t_4)}{s_{-\eta,\eta_8}(-L_3,t-t_8)s_{-\eta,\eta_9}(-L_3,t-t_9)s_{-\eta,\eta_{10}}(-L_3,t-t_{10})}\\ \frac{s_{\eta_8,\eta_9}(0,t_8-t_9)s_{\eta_8,\eta_{10}}(0,t_8-t_{10})s_{\eta_9,\eta_{10}}(0,t_9-t_{10})}{s_{\eta_3,\eta_8}(-L_3,t_3-t_8)s_{\eta_3,\eta_9}(-L_3,t_3-t_9)s_{\eta_3,\eta_{10}}(-L_3,t_3-t_{10})}\\ \frac{1}{s_{\eta_4,\eta_8}(-L_3,t_4-t_8)s_{\eta_4,\eta_9}(-L_3,t_4-t_9)s_{\eta_4,\eta_{10}}(-L_3,t_4-t_{10})}\\ \frac{s_{\eta,\eta_1}(0,-t_1)s_{\eta,\eta_2}(0,-t_2)s_{\eta_1,\eta_2}(0,t_1-t_2)}{s_{\eta,\eta_8}(-L_4,-t_8)s_{\eta,\eta_9}(-L_4,-t_9)s_{\eta,\eta_{10}}(-L_4,-t_{10})}\\ \frac{s_{\eta_8,\eta_9}(0,t_8-t_9)s_{\eta_8,\eta_{10}}(0,t_8-t_{10})s_{\eta_9,\eta_{10}}(0,t_9-t_{10})}{s_{\eta_1,\eta_8}(-L_4,t_1-t_8)s_{\eta_1,\eta_9}(-L_4,t_1-t_9)s_{\eta_1,\eta_{10}}(-L_4,t_1-t_{10})}\\ \frac{1}{s_{\eta_2,\eta_8}(-L_4,t_2-t_8)s_{\eta_2,\eta_9}(-L_4,t_2-t_9)s_{\eta_2,\eta_{10}}(-L_4,t_2-t_{10})}\Big\}^{\frac{1}{3}}$$

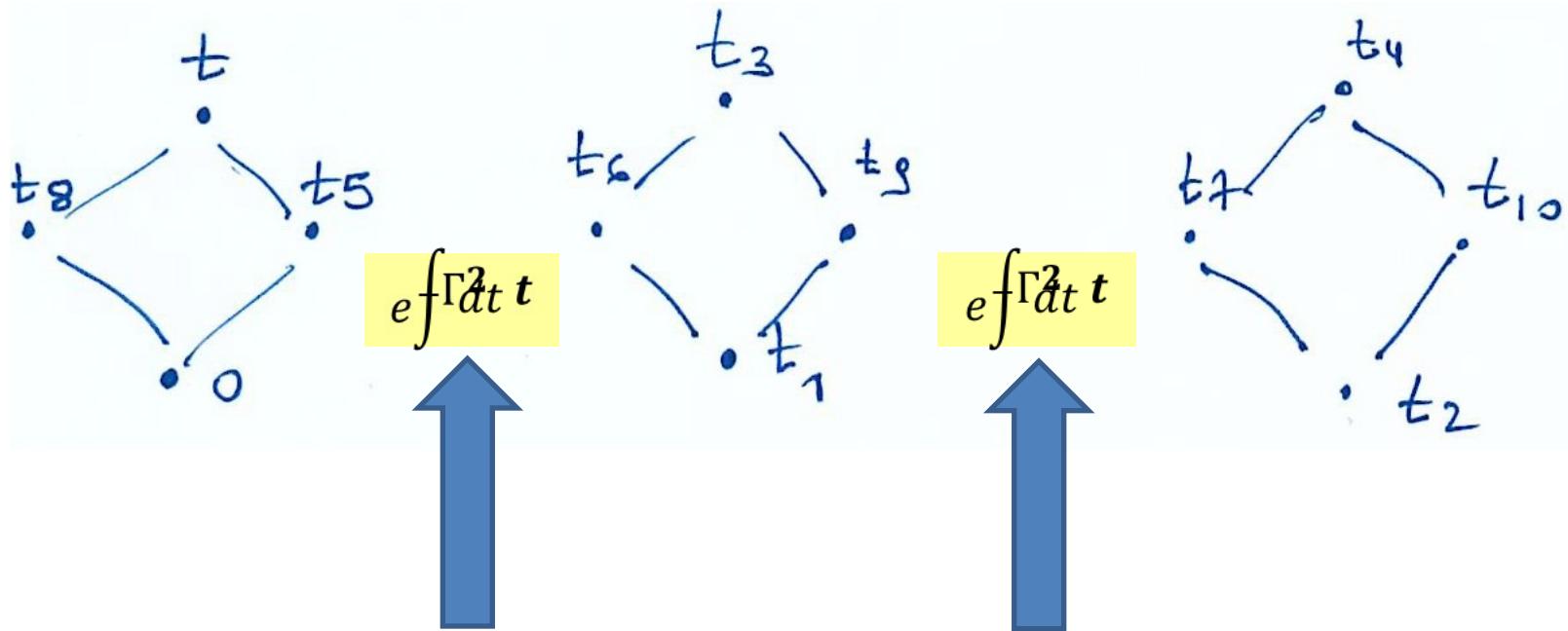
$$\ll I_1\left(t=0\right)I_4(t)\gg$$



$$s_{\eta,\eta'}(x,t)=\frac{\beta}{\pi}\sinh\left\{\frac{\pi}{\beta}\chi_{\eta,\eta'}(t)[t-x]-i\epsilon\right\}$$

$$\chi_{\eta,\eta'}(t)=\frac{\eta+\eta'}{2}sgn(t)-\frac{\eta-\eta'}{2}.$$

$\ell_{thermal} \ll \text{length of interferometer arms}$



$$S_{14}^{AB} = \frac{|\Gamma_a \Gamma_b \Gamma_c \Gamma_d|^3 \Omega^3 \cos[2\pi(\Phi_{AB}/\Phi_0)]}{6(|\Gamma_a|^2 + |\Gamma_b|^2 + |\Gamma_c|^2 + |\Gamma_d|^2)^2 \gamma^2}$$

calculated by
Keldysh

What is the order in Γ of leading harmonics?

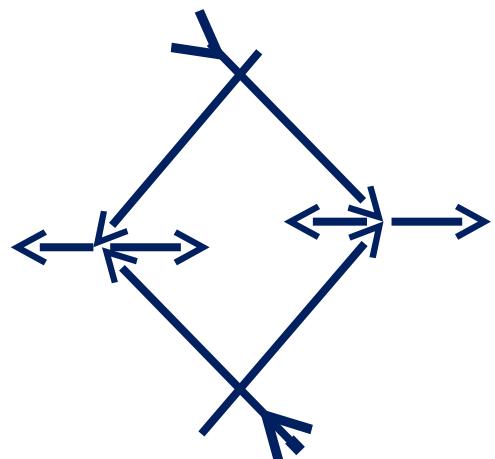


What is its sign?

$$S_{14}^{AB} = \frac{|\Gamma_a \Gamma_b \Gamma_c \Gamma_d|^3 \Omega^3 \cos[2\pi(\Phi_{AB}/\Phi_0)]}{6(|\Gamma_a|^2 + |\Gamma_b|^2 + |\Gamma_c|^2 + |\Gamma_d|^2)^2 \gamma^2}$$

positive → boson-like

in agreement with the caricature



SUMMARY:

1. current-current non-analytic
in single-particle rate
2. as function of Γ boson-like \rightarrow
fermion -like

THANK YOU