Spin Phenomena in Elastic Scattering of ^{6}He and ^{8}He off Protons

Ch. Elster

A. Orazbayev, S.P. Wepper

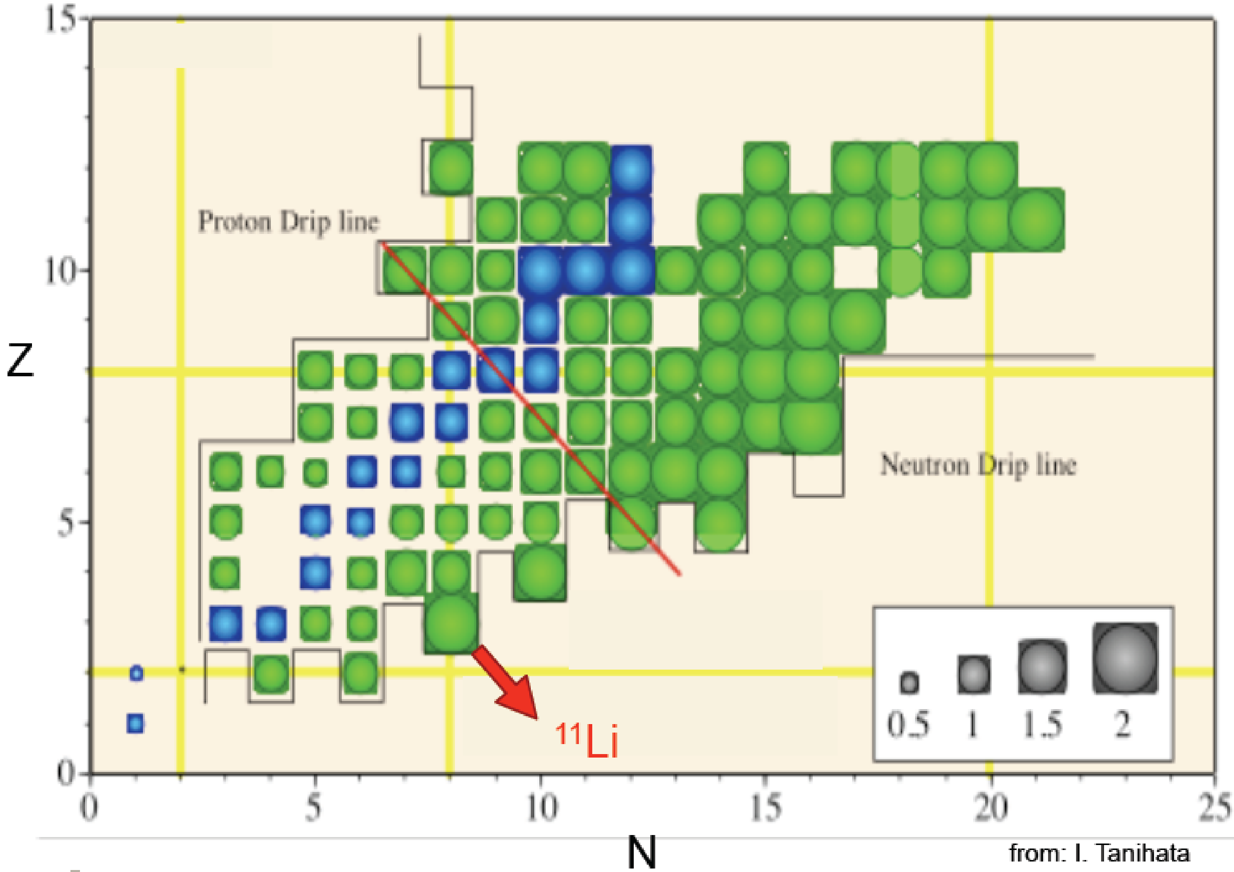
Supported by: U.S. DOE & TORUS

Department of physics + astronomy

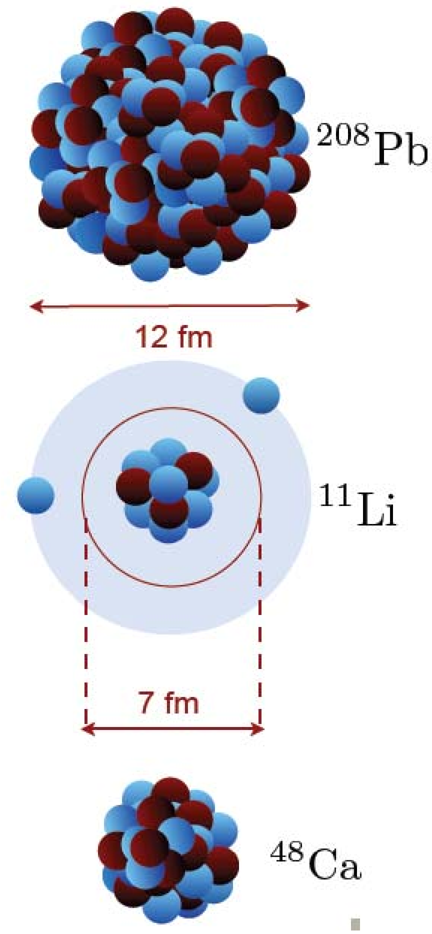
OHIO UNIVERSITY

Nuclear Sizes:

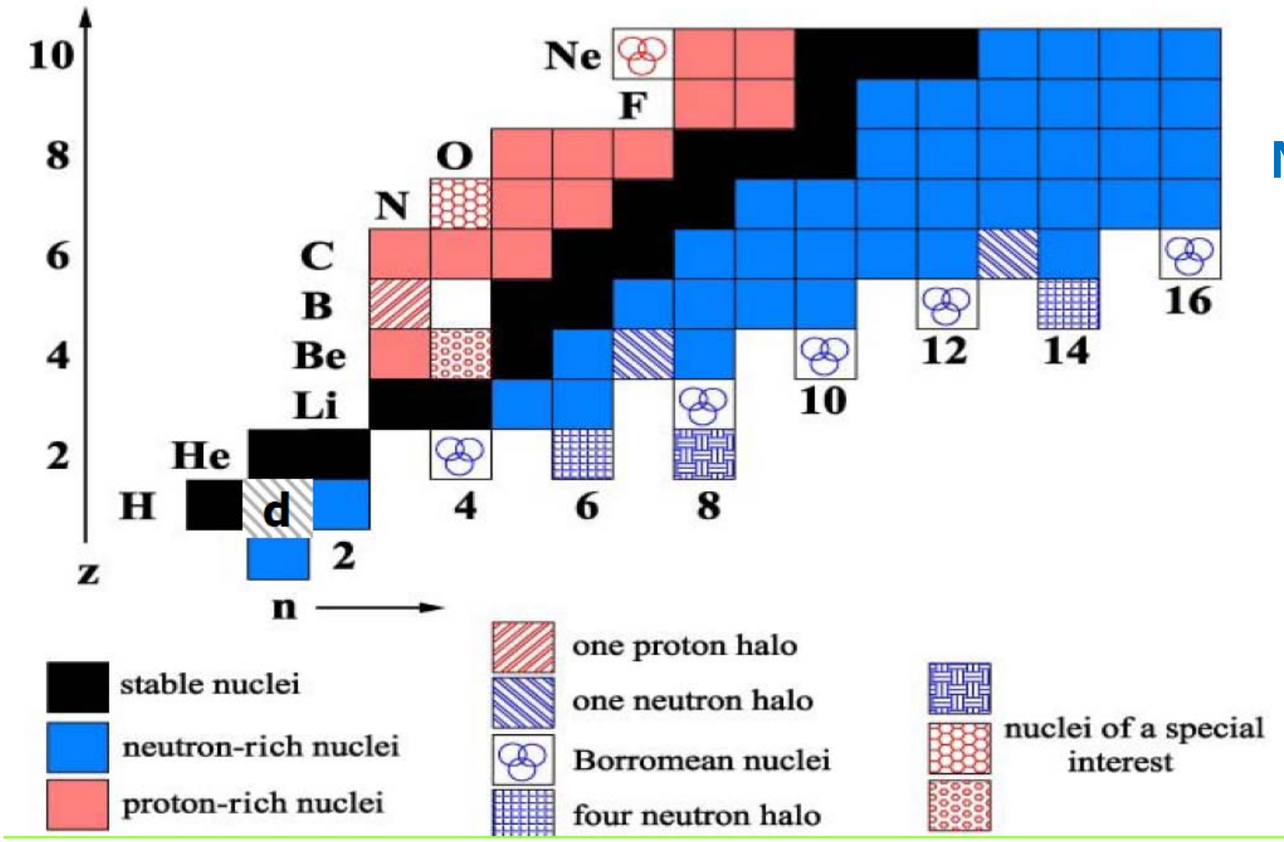
Nuclear radius for stable nuclei: $R_N \sim r_0 A^{1/3}$ with $r_0 \sim 1.2$ fm



from: I. Tanihata



Nuclear Chart for Light Nuclei



Neutron Rich Nuclei

Large n/p ratio

Halo	n/p
^{6}He	2
^{8}He	3
^{11}Li	2.66
^{12}C	1

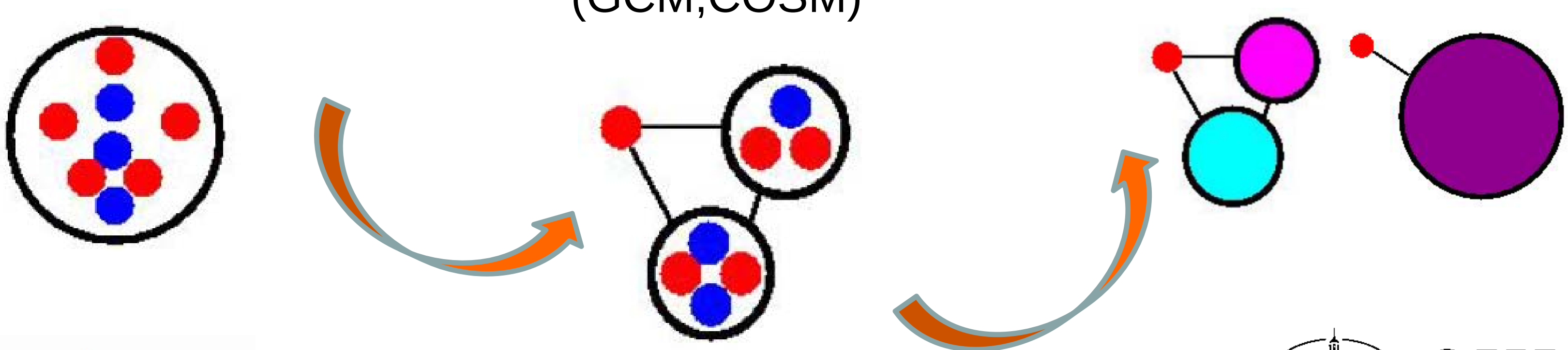
Why are they interesting?

- Their behavior deviates from nuclei in the valley of stability.
- Test our understanding of structure and reactions.
- Enormous progress in experimental information.
- Structure of light nuclei is accessible to different theoretical descriptions

Microscopic Models
(GFMC,NCSM)

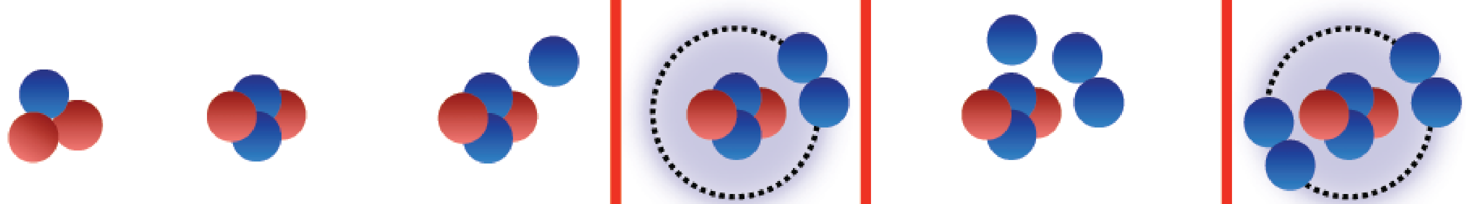
Cluster Models (semi-microscopic)
(GCM,COSM)

Few-Body Models
(Inert constituents)



The Helium Isotopes

$^{3}\mathrm{He}$	$^{4}\mathrm{He}$	$^{5}\mathrm{He}$	$^{6}\mathrm{He}$	$^{7}\mathrm{He}$	$^{8}\mathrm{He}$	....
bound	bound	unbound	bound halo	unbound	bound halo	



Borromean system

lives 806 ms

Most exotic nucleus
"on earth"

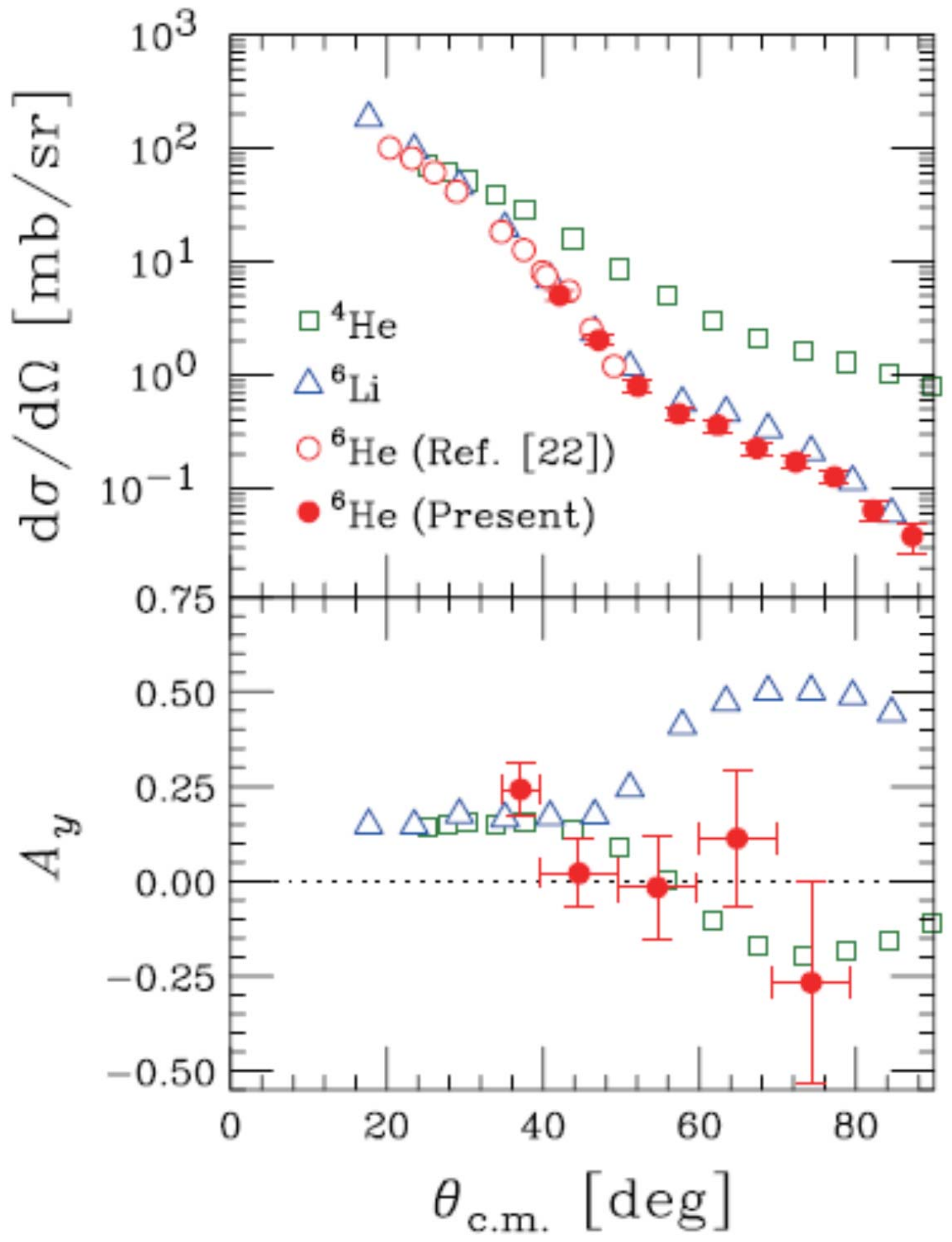
$$\frac{N}{Z} = 3$$

lives 108 ms

RIKEN:
$^{6}He(p,p)^{6}He$
@ 71 MeV

S. Sakaguchi et al.

Phys.Rev. C84 (2011) 024604



For the first time : spin-observable measured for a halo-nucleus

physics + astronomy

RIKEN: ^{6}He(p,p)^{6}He

S. Sakaguchi et al.

Phys.Rev. C84 (2011) 024604

Quote:

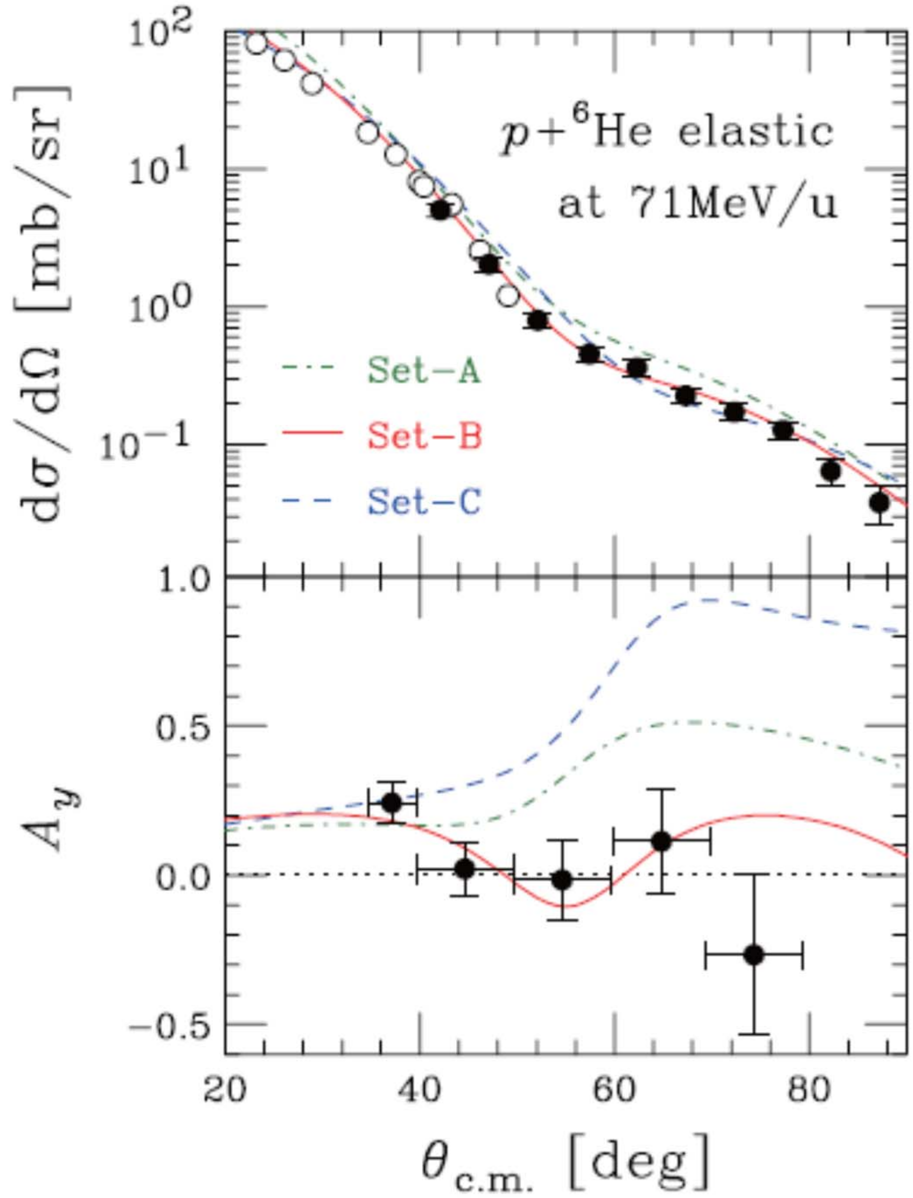
We adopted a standard Woods-Saxon optical potential with a spin-orbit term of the Thomas form:

$$U_{\mathrm{OM}}(R) = -\ V_0\, f_r(R) - i\, W_0\, f_i(R) + 4i\, a_{id}\, W_d\, \frac{d}{dR} f_{id}(R) + V_s\, \frac{2}{R}\, \frac{d}{dR} f_s(R)\, \boldsymbol{L} \cdot \boldsymbol{\sigma}_p + V_{\mathrm{C}}(R) \quad (1)$$

with

$$f_x(R) = \left[1 + \exp\left(\frac{R - r_{0x} A^{1/3}}{a_x}\right)\right]^{-1} \quad (2)$$

$$(x = r, i, id, \text{ or } s).$$



physics + astronomy

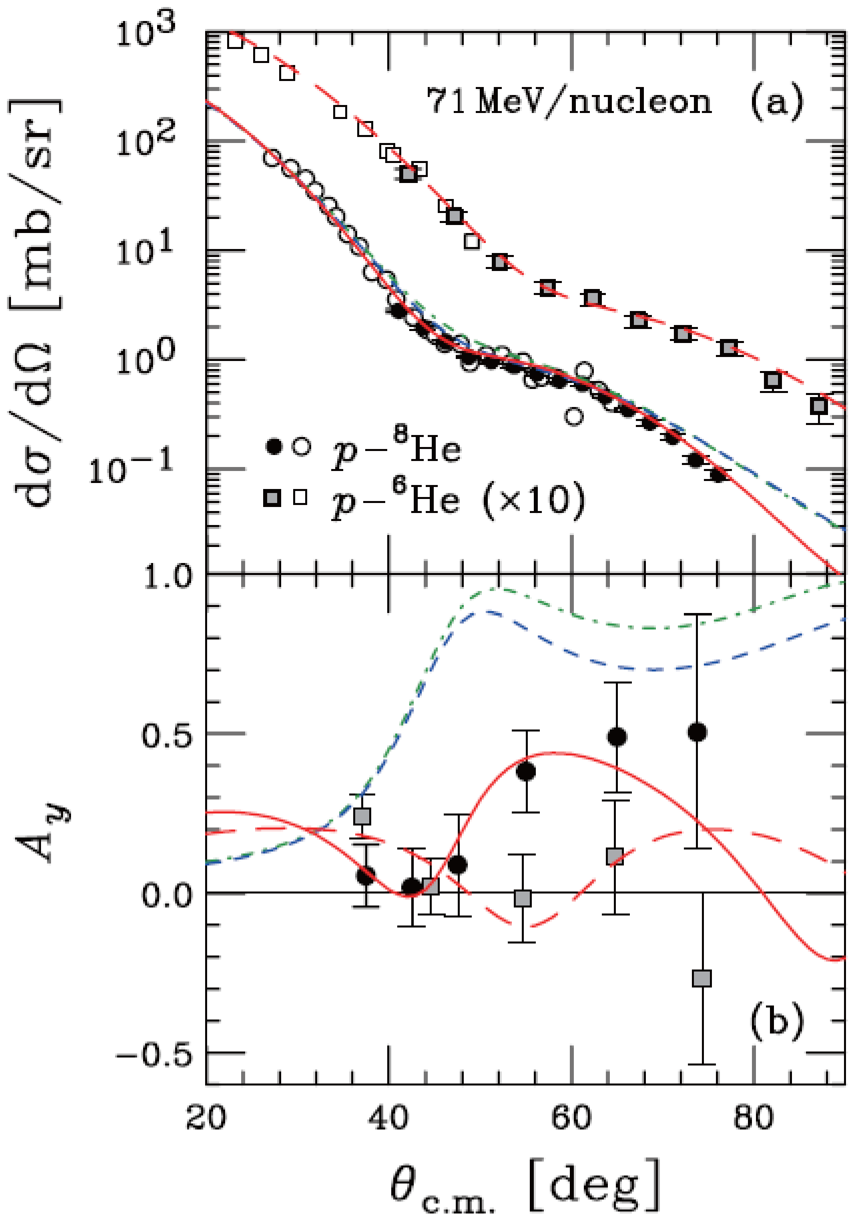
RIKEN:
$^{6}He(p,p)^{6}He$
and
$^{8}He(p,p)^{8}He$

S. Sakaguchi et al.

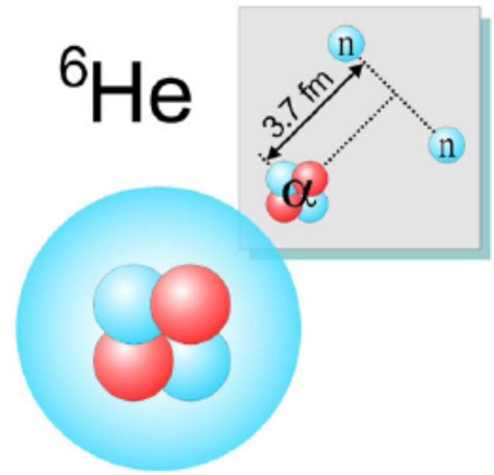
PRC 87, 021601(R) (2013)

Analyzing Powers of ^{6}He and ^{8}He behave differently!

A new A_y puzzle ?



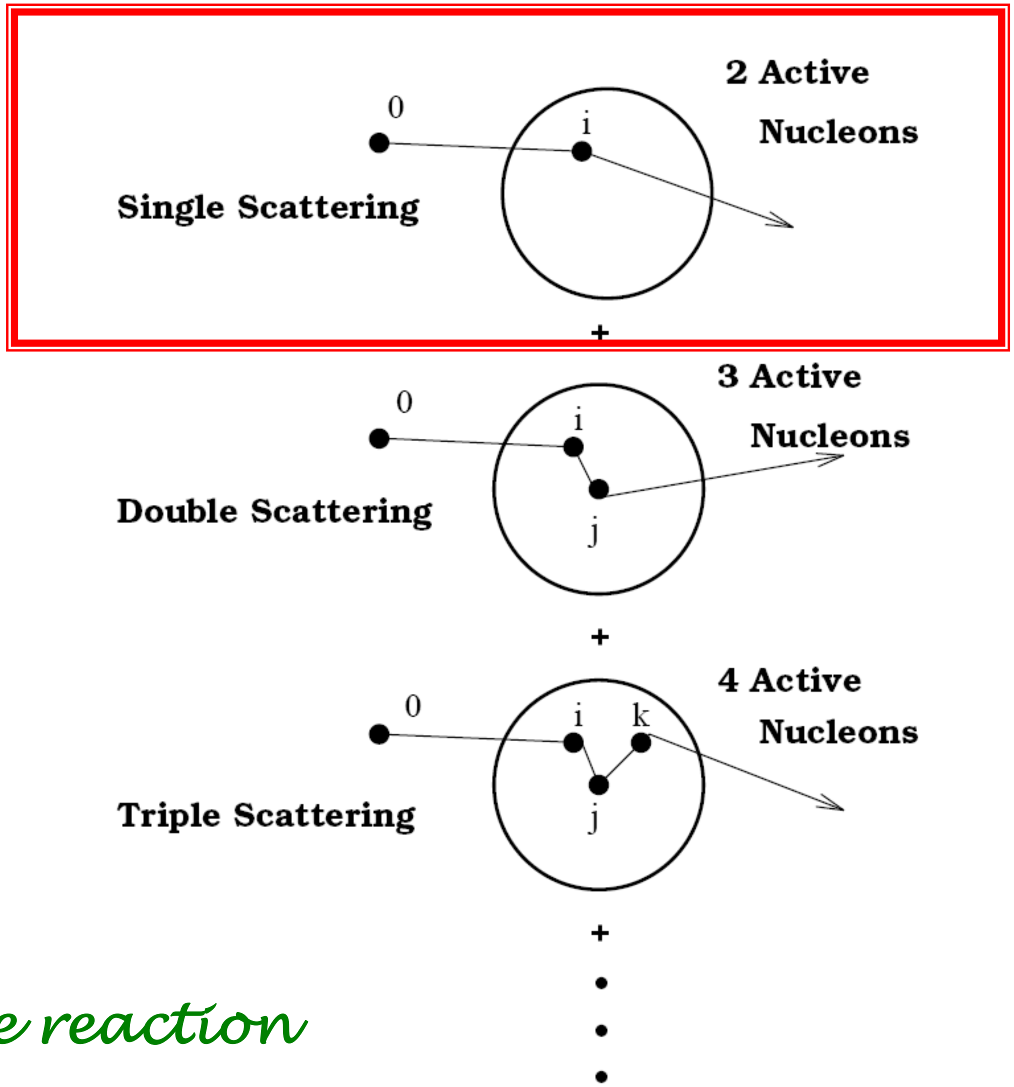
Challenges for ^{6}He
(and similar exotic nuclei)



- **^{6}He is spin-0 nucleus**
  - **NOT a closed shell nucleus**
- **^{6}He is loosely bound nucleus**
  - **with cluster structure: core + 2 neutrons**

Traditional microscopic optical potentials do NOT consider those properties

p+A Scattering:
multiple scattering problem



Spectator Expansion:

Written down by

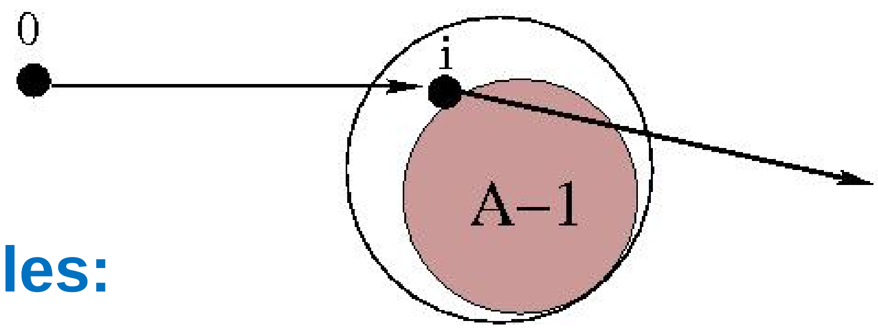
Siciliano, Thaler (1977)

Picklesimer, Thaler (1981)

Expansion in:

- *particles active in the reaction*
- *Antisymmetrized in active particles*

Single Scattering



In principle:

Three-body problem with particles:

o – i – (A-1)-core

o – i : NN interaction

i – (A-1) core : e.g. mean field force

If projectile energy sufficiently high:

Mean field force negligible ➔

Impulse approximation

Phenomenological Optical Potentials parameterize single scattering term

Microscopic Optical Potentials

Folding Models” for closed shell nuclei (~1990s)

- **Watson Multiple Scattering**
  - Elster, Weppner, Chinn, Thaler, Tandy, Redish
    - **Separation of p-A and n-A optical potential**
    - Based on NN t-matrix as interaction input
    - **Treating of interaction with (A-1)-core via mean field and as implicit three-body problem**
- **Kerman-McManus-Thaler (KMT)**
  - Crespo, Johnson, Tostevin, Thompson
    - Based on NN t-matrix as input
    - **Couple explicitly to (A-1) core**
    - **Introduce cluster ansatz for halo targets within coupled channels**
- **G-matrix folding**
  - Arellano, Brieva, Love
    - Based on a g-matrix folding with local density approximation
  - Picked up by Amos, Karataglidis and extended to exotic nuclei

Scattering: Lippmann-Schwinger Equation

- LSE: $T = V + V\,G_0\,T$
- Hamiltonian: $H = H_0 + V$
- Free Hamiltonian: $H_0 = h_0 + H_A$
  - h_0: kinetic energy of projectile '0'
  - H_A: target hamiltonian with $H_A\,|\Phi\rangle = E_A\,|\Phi\rangle$
- V: interactions between projectile '0' and target nucleons 'i' $V = \Sigma^A_{i=0}\, v_{0i}$
- Propagator is (A+1) body operator
  - $G_0(E) = (E - h_0 - H_A + i\varepsilon)^{-1}$

Elastic Scattering:

- In- and Out-States have the target in ground state Φ_0
- Projector on ground state $P = |\Phi_0\rangle\langle\Phi_0|$
  - **With $1=P+Q$ and $[P,G_0]=0$**
- For elastic scattering one needs
- $$P\,T\,P = P\,U\,P + P\,U\,P\,G_0(E)\,P\,T\,P$$
- Or $\boxed{\begin{array}{l} T = U + U\,G_0(E)\,P\,T \\ U = V + V\,G_0(E)\,Q\,U \quad \Leftarrow \text{ “optical potential”} \end{array}}$

Single Scattering: $U^{(1)} \approx \Sigma^{A}_{i=0}\, \tau_{0i}$ **(1st order)**

with $\boxed{\tau_{0i} = v_{0i} + v_{0i}\, G_0(E)\, Q\, \tau_{0i}}$

$$\boxed{\tau_{0i} = v_{0i} + v_{0i}\, G_0(E)\, Q\, \tau_{0i}}$$

- $G_0(E) = (E - h_0 - H_A + i\varepsilon)^{-1}$ == (A+1) body operator
  - Standard **"impulse approximation":** average over H_A
  - $\rightarrow G_0(e)$ ==: two body operator
- Handle operator **Q**
  - Define "two-body" operator $\mathbf{t_{0i}^{free}}$ by
  - $\boxed{t_{0i}^{free} = v_{0i} + v_{0i}\, G_0(e)\, t_{0i}^{free}}$
  - and relate via integral equation to τ_{oi}
  - $\boxed{\tau_{oi} = t_{0i}^{free} - t_{0i}^{free}\, G_0(e)\, \tau_{oi} \quad \text{[integral equation]}}$
  - **Important for keeping correct iso-spin character of optical potential**
  - $\boxed{U^{(1)} = \Sigma^{A}_{i=1}\, \tau_{oi} =: N\, \tau_n + Z\, \tau_p}$

Chinn, Elster, Thaler, PRC47, 2242 (1993)

"First order Watson optical potential"

$$U^{(1)} = \Sigma^{A}_{i=1} \tau_{oi} =: \Sigma^{N}_{i=1} \tau_{n} + \Sigma^{P}_{i=1} \tau_{p}$$

- Important for treating $N \neq Z$ nuclei
- Sensitive to proton vs. neutron scattering
- In general
  - $t_{pp} \neq t_{np}$ **and** $\rho_p \neq \rho_n$
- These differences enter in a non-linear fashion into first order Watson optical potential

$$\tau_\alpha = t_\alpha - t_\alpha G_0^\alpha(e) \tau_\alpha , \quad \alpha = n,p$$

- **This formulation allows a more complicated structure of the optical potential and e.g. a cluster ansatz**

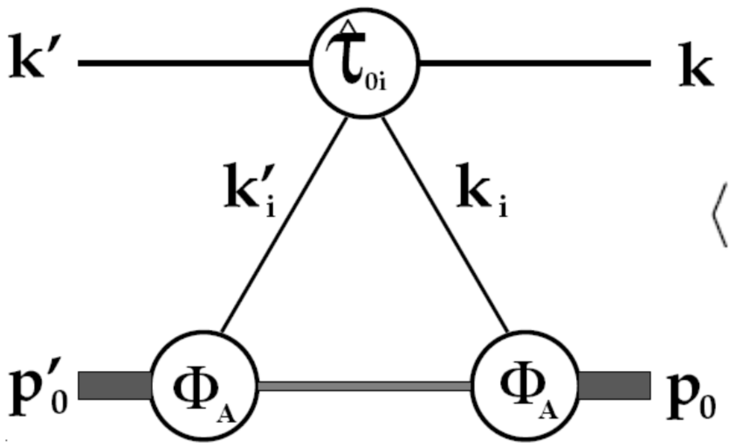
More explicit:

P:= projector on ground state

- Elastic scattering :
- First order Watson O.P.:

$$T_{el} = PUP + PUPG_0(E)PT_{el}.$$

$$\langle \mathbf{k}'|\langle\phi_A|PUP|\phi_A\rangle\mathbf{k}\rangle \equiv U_{el}(\mathbf{k}',\mathbf{k}) = \sum_{i=n,p} \langle\mathbf{k}'|\langle\phi_A|\hat{\tau}_{0i}(\mathcal{E})|\phi_A\rangle\mathbf{k}\rangle$$



$$\langle\hat{\tau}_{01}\rangle \equiv \langle\mathbf{k}'|\langle\phi_A|\hat{\tau}_{01}(\mathcal{E})|\phi_A\rangle\mathbf{k}\rangle$$

Proton scattering:

$$U_{el}(\mathbf{k}',\mathbf{k}) = Z\langle\hat{\tau}_{01}^{pp}\rangle + N\langle\hat{\tau}_{01}^{np}\rangle$$

Department of physics + astronomy

OHIO UNIVERSITY

Calculate: $$\langle\hat{\tau}_{01}\rangle \equiv \langle\mathbf{k}'|\langle\phi_A|\hat{\tau}_{01}(\mathcal{E})|\phi_A\rangle\mathbf{k}\rangle$$

Jacobi Coordinates

$$\langle\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3\mathbf{k}_4...\mathbf{k}_A|\phi_A\rangle = \delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4...+\mathbf{k}_A-\mathbf{p}_0)\langle\boldsymbol{\zeta}_1\boldsymbol{\zeta}_2\boldsymbol{\zeta}_3\boldsymbol{\zeta}_4...\boldsymbol{\zeta}_{\mathbf{A-1}}|\phi_A\rangle.$$

$$\langle\hat{\tau}_{01}\rangle = \int\prod_{j=1}^{A}d\mathbf{k}'_j \int\prod_{l=1}^{A}d\mathbf{k}_l\ \langle\phi_A|\zeta'_1\zeta'_2\zeta'_3\zeta'_4...\zeta'_{A-1}\rangle\delta(\mathbf{p}'-\mathbf{p}'_0)\ \langle\mathbf{k}'\mathbf{k}'_1|\hat{\tau}_{01}(\mathcal{E})|\mathbf{k}\mathbf{k}_1\rangle$$
$$\prod_{j=2}^{A}\delta(\mathbf{k}'_j-\mathbf{k}_j)\delta(\mathbf{p}-\mathbf{p}_0)\ \langle\zeta_1\zeta_2\zeta_3\zeta_4...\zeta_{A-1}|\phi_A\rangle, \tag{2.48}$$

With single particle density matrix :

$$\rho(\boldsymbol{\zeta}'_1,\boldsymbol{\zeta}_1) \equiv \int\prod_{l=2}^{A-1}d\boldsymbol{\zeta}'_l\int\prod_{j=2}^{A-1}d\boldsymbol{\zeta}_{\mathbf{j}}\langle\phi_A|\zeta'_1\zeta'_2\zeta'_3\zeta'_4...\zeta'_{A-1}\rangle\ \langle\zeta_1\zeta_2\zeta_3\zeta_4...\zeta_{A-1}|\phi_A\rangle.$$

$$\boxed{\begin{aligned}\langle\hat{\tau}_{01}\rangle &= \int d\boldsymbol{\zeta}'_1\int d\boldsymbol{\zeta}_1\langle\mathbf{k}'\ \boldsymbol{\zeta}'_1+\frac{\mathbf{p}'_0}{A}|\hat{\tau}_{01}(\mathcal{E})|\mathbf{k}\ \boldsymbol{\zeta}_1+\frac{\mathbf{p}_0}{A}\rangle\ \rho(\boldsymbol{\zeta}'_1,\boldsymbol{\zeta}_1)\\ &\delta(\frac{A-1}{A}\mathbf{p}'_0-\boldsymbol{\zeta}'_1-\frac{A-1}{A}\mathbf{p}_0+\boldsymbol{\zeta}_1).\end{aligned}}$$

physics + astronomy — Department of

OHIO UNIVERSITY

Calculate:

$$\langle \hat{\tau}_{01} \rangle \equiv \langle \mathbf{k}' | \langle \phi_A | \hat{\tau}_{01}(\mathcal{E}) | \phi_A \rangle | \mathbf{k} \rangle$$

Jacobi Coordinates

$$\langle \mathbf{k}_1 \mathbf{k}_2 \mathbf{k}_3 \mathbf{k}_4 ... \mathbf{k}_A | \phi_A \rangle = \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_4 ... + \mathbf{k}_A - \mathbf{p}_0) \langle \zeta_1 \zeta_2 \zeta_3 \zeta_4 ... \zeta_{A-1} | \phi_A \rangle .$$

$$\langle \hat{\tau}_{01} \rangle = \int \prod_{j=1}^{A} d\mathbf{k}'_j \int \prod_{l=1}^{A} d\mathbf{k}_l \, \langle \phi_A | \zeta'_1 \zeta'_2 \zeta'_3 \zeta'_4 ... \zeta'_{A-1} \rangle \delta(\mathbf{p}' - \mathbf{p}'_0) \langle \mathbf{k}' \mathbf{k}'_1 | \hat{\tau}_{01}(\mathcal{E}) | \mathbf{k} \mathbf{k}_1 \rangle$$

$$\prod_{j=2}^{A} \delta(\mathbf{k}'_j - \mathbf{k}_j) \delta(\mathbf{p} - \mathbf{p}_0) \, \langle \zeta_1 \zeta_2 \zeta_3 \zeta_4 ... \zeta_{A-1} | \phi_A \rangle \qquad (2.48)$$

With single particle density matrix :

$$\rho(\zeta'_1, \zeta_1) \equiv \int \prod_{j=2}^{A-1} d\zeta'_j \int \prod_{j=2}^{A-1} d\zeta_j \langle \phi_A | \zeta'_1 \zeta'_2 \zeta'_3 \zeta'_4 ... \zeta'_{A-1} \rangle \, \langle \zeta_1 \zeta_2 \zeta_3 \zeta_4 ... \zeta_{A-1} | \phi_A \rangle .$$

$$\boxed{\begin{aligned} \langle \hat{\tau}_{01} \rangle = \int d\zeta'_1 \int d\zeta_1 \langle \mathbf{k}' \, \zeta'_1 + \frac{\mathbf{p}'_0}{A} | \hat{\tau}_{01}(\mathcal{E}) | \mathbf{k} \, \zeta_1 + \frac{\mathbf{p}_0}{A} \rangle \, \rho(\zeta'_1, \zeta_1) \\ \delta(\frac{A-1}{A}\mathbf{p}'_0 - \zeta'_1 - \frac{A-1}{A}\mathbf{p}_0 + \zeta_1). \end{aligned}}$$

Not quite the whole story!

Where is the nucleon spin?

General Single Particle Density Matrix

Wave function ~ $\Phi_0(i) \sim f_l(i)\, Y_l^m(i)\, \chi_S^{(i)}$

Single particle density matrix

$$\rho_{I\,M_I,I\,M_I'}(i,i') \sim \sum_{k_l\,q_l;\,k_s\,q_s;\,k\,q} \begin{bmatrix} I & K & I \\ M_I' & q & M_I \end{bmatrix} \left\langle \Psi_I \,||\, \rho_{kq} \,||\, \Psi_I \right\rangle \chi^{ll'}_{k_l q_l}(i,i')\, f_l(i)\, f^*_{l'}(i')$$

$$\left\langle S\, m_s \,|\, \tau^{(i)}_{k_s q_s}(S) \,|\, S'\, m_s' \right\rangle \begin{bmatrix} k_l & k_s & k \\ q_l & q_s & q \end{bmatrix} \begin{Bmatrix} l & l' & k_l \\ s & s & k_s \\ j & j' & k \end{Bmatrix}$$

Auxiliary tensor operator

$$\tau^{(i)}_{k_s,q_s}\left(S = \tfrac{1}{2}\right): \quad \begin{aligned} \tau^{(i)}_{00} &= 1 \\ \tau^{(i)}_{10} &= 2s_z \\ \tau^{(i)}_{1,\pm1} &= \mp\frac{2}{\sqrt{2}}(S_x \pm iS_y) \end{aligned}$$

$$\chi^{ll'}_{k_l\,q_l}(i,i') = \sum_{l\,l'} (-1)^{l'-\lambda'} \begin{bmatrix} l & l' & k_l \\ \lambda & -\lambda' & q_l \end{bmatrix} Y_{l\lambda}(i)\, Y^*_{l'\lambda'}(i')$$

Orbital angular momentum

Case $k_s = 0$ $\quad \left\langle \Phi_0 \middle| \mathbf{1}^{(i)} \middle| \Phi_0 \right\rangle$

s-shell

$$\rho_{0\,0,0\,0}(i,i') \sim \left\langle \Psi_s \,||\, \rho_{00} \,||\, \Psi_s \right\rangle \chi^{00}_{00}(i,i')\, f_s(i)\, f^*_s(i') \underbrace{\left\langle S\, m_s \,\middle|\, \tau^{(i)}_{00}(S) \,\middle|\, S'\, m'_s \right\rangle}^{=1}$$

p-shell

$$\rho_{0\,0,0\,0}(i,i') \sim \sum_{k_l} \left\langle \Psi_p \,||\, \rho_{kq} \,||\, \Psi_p \right\rangle \chi^{11}_{k_l 0}(i,i')\, f_p(i)\, f^*_p(i')$$

$$\left\langle S\, m_s \,\middle|\, \overbrace{\tau^{(i)}_{00}}^{=1}(S) \,\middle|\, S'\, m'_s \right\rangle \begin{Bmatrix} 1 & 1 & k_l \\ \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{3}{2} & \frac{3}{2} & k \end{Bmatrix}$$

Scalar density

Case k_s = 1 $\left\langle \Phi_0 \middle| \boldsymbol{\sigma}^{(i)} \middle| \Phi_0 \right\rangle$

p-shell

$$\rho_{0\,0,0\,0}(i,i') \sim \sum_{k_l\ q_l;\ 1\ q_s;\ k\ q} \left\langle \Psi_0 \,||\, \rho_k \,||\, \Psi_0 \right\rangle \chi^{11}_{k_l q_l}(i,i')\ f_p(i)\ f^*_p(i')$$
$$\left\langle S\ m_s \,\middle|\, \tau^{(i)}_{1 q_s}(S) \,\middle|\, S'\ m'_s \right\rangle \begin{bmatrix} k_l & 1 & k \\ q_l & q_s & q \end{bmatrix} \begin{Bmatrix} 1 & 1 & k_l \\ \frac{1}{2} & \frac{1}{2} & 1 \\ \frac{3}{2} & \frac{3}{2} & k \end{Bmatrix}$$

with

$$\tau^{(i)}_{10} = 2s_z$$
$$\tau^{(i)}_{1,\pm 1} = \mp \frac{2}{\sqrt{2}}(S_x \pm iS_y)$$

For closed shell nuclei this term is zero

Department of physics + astronomy

OHIO UNIVERSITY

Expectation Values for struck target nucleon

$$I_1 = \frac{1}{8\pi^2} \int d\hat{\zeta}\; d\hat{\zeta}'\; \Phi_0(\hat{\zeta}')\; \boldsymbol{\sigma}^{(i)} \cdot \hat{\mathbf{n}}\; \Phi_0(\hat{\zeta})\; \delta(\hat{\zeta} \cdot \hat{\zeta}' - \cos\alpha_{\zeta\zeta'})$$

$$I_2 = \frac{1}{8\pi^2} \int d\hat{\zeta}\; d\hat{\zeta}'\; \Phi_0(\hat{\zeta}')\; \boldsymbol{\sigma}^{(i)} \cdot \hat{\mathbf{q}}\; \Phi_0(\hat{\zeta})\; \delta(\hat{\zeta} \cdot \hat{\zeta}' - \cos\alpha_{\zeta\zeta'})$$

$$I_3 = \frac{1}{8\pi^2} \int d\hat{\zeta}\; d\hat{\zeta}'\; \Phi_0(\hat{\zeta}')\; \boldsymbol{\sigma}^{(i)} \cdot \hat{\mathbf{P}}\; \Phi_0(\hat{\zeta})\; \delta(\hat{\zeta} \cdot \hat{\zeta}' - \cos\alpha_{\zeta\zeta'})$$

Momentum Variables:

$$\mathbf{q} = \mathbf{k}' - \mathbf{k} = \frac{A}{A-1}(\zeta - \zeta')$$

$$\mathbf{P} = \frac{\zeta + \zeta'}{2}; \quad \mathbf{K} = \frac{\mathbf{k} + \mathbf{k}'}{2}$$

Expectation Values for struck target nucleon

$$I_1 = \frac{1}{8\pi^2} \int d\hat{\zeta}\; d\hat{\zeta}'\; \Phi_0(\hat{\zeta}')\; \boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{n}}\; \Phi_0(\hat{\zeta})\; \delta(\hat{\zeta} \cdot \hat{\zeta}' - \cos\alpha_{\zeta\zeta'})$$

$$I_2 = \frac{1}{8\pi^2} \int d\hat{\zeta}\; d\hat{\zeta}'\; \Phi_0(\hat{\zeta}')\; \boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{q}}\; \Phi_0(\hat{\zeta})\; \delta(\hat{\zeta} \cdot \hat{\zeta}' - \cos\alpha_{\zeta\zeta'})$$

$$I_3 = \frac{1}{8\pi^2} \int d\hat{\zeta}\; d\hat{\zeta}'\; \Phi_0(\hat{\zeta}')\; \boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{P}}\; \Phi_0(\hat{\zeta})\; \delta(\hat{\zeta} \cdot \hat{\zeta}' - \cos\alpha_{\zeta\zeta'})$$

Result:

$$\Psi_p^*(\hat{\zeta}')\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{n}}\Psi_p(\hat{\zeta}) = -i\frac{2}{9\pi^{3/2}\nu_p^{5/2}}\,|\zeta \times \zeta'|\, exp\left(-\frac{\zeta^2}{2\nu_p} - \frac{\zeta'^2}{2\nu_p}\right)$$

$$\Psi_p^*(\hat{\zeta}')\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{q}}\Psi_p(\hat{\zeta}) = 0; \qquad \Psi_p^*(\hat{\zeta}')\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{P}}\Psi_p(\hat{\zeta}) = 0$$

Model Ansatz for ground states of ^{6}He and ^{8}He

HO

$$\text{s-shell: } f_{00\frac{1}{2}}(\boldsymbol{\zeta}) = (2\pi)^{3/2}\sqrt{\frac{4}{\sqrt{\pi}\nu_s^{3/2}}}\, exp\left(-\frac{\zeta^2}{2\nu_s}\right)\, \mathcal{Y}_0^{\frac{1}{2},m_j}$$

$$\text{p-shell: } f_{01\frac{3}{2}}(\boldsymbol{\zeta}) = (2\pi)^{3/2}\sqrt{\frac{4}{\sqrt{\pi}\nu_p^{3/2}}}\sqrt{\frac{2}{3}}\frac{\zeta}{\sqrt{\nu_p}} exp\left(-\frac{\zeta^2}{2\nu_p}\right) \mathcal{Y}_1^{\frac{3}{2},m_j}(\hat{\boldsymbol{\zeta}})$$

With change of variables

$$\mathbf{q} = \mathbf{k}' - \mathbf{k} = \frac{A}{A-1}(\boldsymbol{\zeta} - \boldsymbol{\zeta}')$$

$$\mathbf{P} = \frac{\boldsymbol{\zeta} + \boldsymbol{\zeta}'}{2}; \quad \mathbf{K} = \frac{\mathbf{k} + \mathbf{k}'}{2}$$

$k_\zeta=0$

$$\rho_s(\mathbf{q}, \mathbf{P}) = \frac{1}{\pi^{3/2}\nu_s^{3/2}}\, exp\left(-\frac{P^2}{\nu_s} - \left(\frac{A-1}{2A}\right)^2 \frac{q^2}{\nu_s}\right),$$

$$\rho_p(\mathbf{q}, \mathbf{P}) = \frac{2}{3}\frac{1}{\pi^{3/2}\nu_p^{5/2}}\left(P^2 - \left(\frac{A-1}{2A}\right)^2 q^2\right) exp\left(-\frac{P^2}{\nu_p} - \left(\frac{A-1}{2A}\right)^2 \frac{q^2}{\nu_p}\right)$$

$k_\zeta=1$

$$\tilde{\rho}_p(\mathbf{q}, \mathbf{P}) = N_p\,\frac{2}{9}\frac{(-i)}{\pi^{3/2}\nu_p^{5/2}}\left(\frac{A-1}{2A}\right)|\mathbf{q}\times\mathbf{P}|\, exp\left(-\frac{P^2}{\nu_p} - \left(\frac{A-1}{2A}\right)^2 \frac{q^2}{\nu_p}\right)$$

Determination of Oscillator Parameters

Charge Radius

$$\langle r_{ch}^2 \rangle = \frac{\int d^3r\ \Phi_s^*(\mathbf{r})\ r^2\ \Phi_s(\mathbf{r})}{\int d^3r\ \Phi_s^*(\mathbf{r})\ \Phi_s(\mathbf{r})}$$

$$\nu_s = \frac{3}{2\ \langle r_{ch}^2 \rangle}$$

Matter Radius

$$\langle r_{mat}^2 \rangle = \frac{\int\ d^3\mathbf{r}\ \Phi_{s+p}^{*\,6,8\mathrm{He}}\ r^2\ \Phi_{s+p}^{6,8\mathrm{He}}}{\int\ d^3\mathbf{r}\ \Phi_{s+p}^{*\,6,8\mathrm{He}}\ \Phi_{s+p}^{6,8\mathrm{He}}}$$

$$\nu_p^{^6\mathrm{He}} = \frac{5}{6\ \langle r_{mat}^2 \rangle - 4\ \langle r_{ch}^2 \rangle}$$

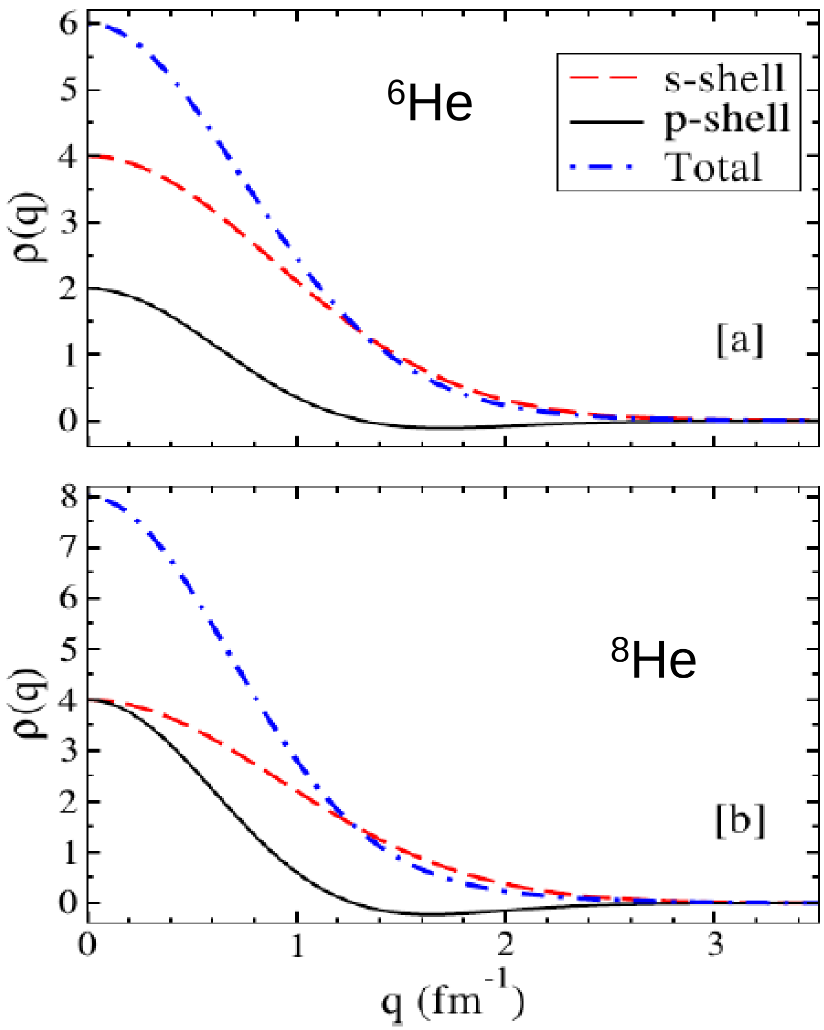
$$\nu_p^{^8\mathrm{He}} = \frac{5}{4\ \langle r_{mat}^2 \rangle - 2\ \langle r_{ch}^2 \rangle}$$

^{A}He	r_{ch}(fm)	r_{mat}(fm)	ν_s (fm^{-2})	ν_p (fm^{-2})
^{6}He	1.955	2.333	0.393	0.289
^{8}He	1.885	2.53	0.422	0.270

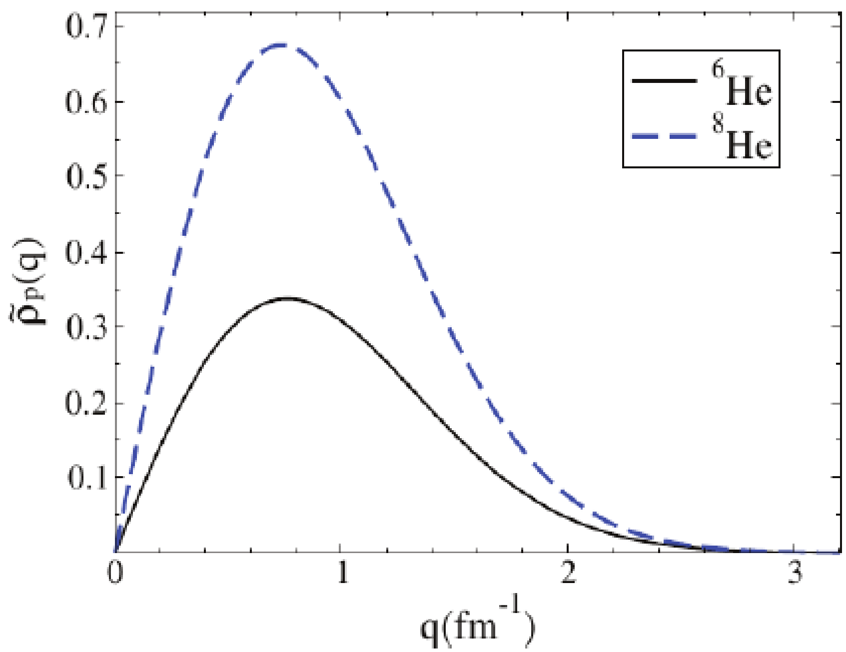
Mueller et al PRL 99(2007), Tanihata et al Phys.Let. B289(1992), Alkhazov et al Nucl. Phys. A712(2002), Brodeur et al Phys. Rev. Lett. 108 (2012)

Diagonal Densities

$k_s=0$



$k_s=1$



Reminder: calculate $\langle \hat{\tau}_{01} \rangle \equiv \langle \mathbf{k}' | \langle \phi_A | \hat{\tau}_{01}(\mathcal{E}) | \phi_A \rangle \mathbf{k} \rangle$

NN t-matrix in Wolfenstein representation:

Projectile "*0*" : plane wave basis
Struck nucleon "*i*" : target basis

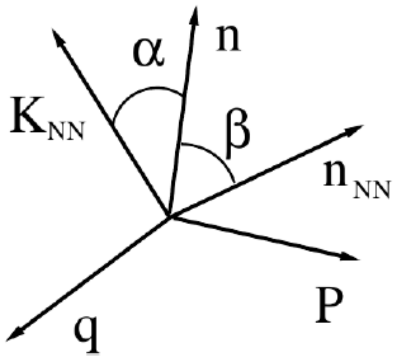
$$\begin{aligned}
\overline{\mathrm{M}}(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E}) = {} & A(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E})\, \mathbf{1} + i \cdot C(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E})\, (\boldsymbol{\sigma}^{(0)} \otimes \mathbf{1} + \mathbf{1} \otimes \boldsymbol{\sigma}^{(i)}) \cdot \hat{\mathrm{n}}_{NN} \\
& + M(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E})(\boldsymbol{\sigma}^{(0)} \cdot \hat{\mathrm{n}}_{NN}) \otimes (\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{n}}_{NN}) \\
& + (G(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E}) - H(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E}))(\boldsymbol{\sigma}^{(0)} \cdot \hat{\mathrm{q}}) \otimes (\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{q}}) \\
& + (G(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E}) + H(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E}))(\boldsymbol{\sigma}^{(0)} \cdot \hat{\mathrm{K}}_{NN}) \otimes (\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{K}}_{NN}) \\
& - \\
& + D(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E}) \left[(\boldsymbol{\sigma}^{(0)} \cdot \hat{\mathrm{q}}) \otimes (\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{K}}_{NN}) + (\boldsymbol{\sigma}^{(0)} \cdot \hat{\mathrm{K}}_{NN}) \otimes (\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{q}}) \right]
\end{aligned}$$

Off-shell

Couple to k_s=1 in single particle density matrix

physics + astronomy (Department of) OHIO UNIVERSITY 1804

Projection to NN frame



$$\cos\alpha = \hat{\mathbf{n}} \cdot \hat{\mathbf{K}}_{NN}$$

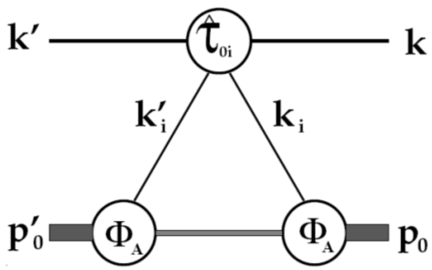
$$\cos\beta = \hat{\mathbf{n}} \cdot \hat{\mathbf{n}}_{NN}$$

$$\Psi_p^*(\hat{\zeta}')\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathbf{n}}_{NN}\Psi_p(\hat{\zeta}) = -i\frac{2}{9\pi^{3/2}\nu_p^{5/2}}\,|\zeta \times \zeta'|\, exp\left(-\frac{\zeta^2}{2\nu_p} - \frac{\zeta'^2}{2\nu_p}\right)\,\cos\beta$$

$$\Psi_p^*(\hat{\zeta}')\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathbf{q}}\Psi_p(\hat{\zeta}) = 0$$

$$\Psi_p^*(\hat{\zeta}')\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathbf{K}}_{NN}\Psi_p(\hat{\zeta}) = -i\frac{2}{9\pi^{3/2}\nu_p^{5/2}}\,|\zeta \times \zeta'|\, exp\left(-\frac{\zeta^2}{2\nu_p} - \frac{\zeta'^2}{2\nu_p}\right)\,\cos\alpha$$

First order Watson Optical Potential:



$$\langle \mathbf{k}'|\langle\phi_A|PUP|\phi_A\rangle\mathbf{k}\rangle \equiv U_{el}(\mathbf{k}',\mathbf{k}) = \sum_{i=n,p}\langle\mathbf{k}'|\langle\phi_A|\hat{\tau}_{0i}(\mathcal{E})|\phi_A\rangle\mathbf{k}\rangle$$

$$\overline{\mathrm{M}}(\mathbf{q},\mathbf{K}_{NN},\mathcal{E}) = A(\mathbf{q},\mathbf{K}_{NN},\mathcal{E})\,\mathbf{1} + i\cdot C(\mathbf{q},\mathbf{K}_{NN},\mathcal{E})\,(\boldsymbol{\sigma}^{(0)}\otimes\mathbf{1} + \mathbf{1}\otimes\boldsymbol{\sigma}^{(i)})\cdot\hat{\mathbf{n}}_{NN}$$

$k_s = 0$

Central :

$$U_A(\mathbf{q},\mathbf{K}) = \sum_{i=s,p}\int d^3\mathbf{P}\,A\left(\mathbf{q},\,\frac{1}{2}\left(\frac{A+1}{A}\mathbf{K}-\mathbf{P}\right),\,\mathcal{E}\right)\,N_i\,\rho_i\left(\mathbf{P}-\frac{A-1}{2A}\mathbf{q},\,\mathbf{P}+\frac{A-1}{2A}\mathbf{q}\right)$$

Spin-Orbit :

$$i\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathbf{n}}_{NN}\,U_C(\mathbf{q},\mathbf{K}) = i\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathbf{n}}_{NN}\sum_{i=s,p}\int d^3\mathrm{P}\,C\left(\mathbf{q},\,\frac{1}{2}\left(\frac{A+1}{A}\mathbf{K}-\mathbf{P}\right),\,\mathcal{E}\right)$$
$$N_i\,\rho_i\left(\mathbf{P}-\frac{A-1}{2A}\mathbf{q},\,\mathbf{P}+\frac{A-1}{2A}\mathbf{q}\right)$$

Optical Potential for Valence Neutrons of ^{6}He

± indicate spin-flip amplitudes

$$U_{val}(q, \mathcal{K}) =$$

$$N \int t_{2B}(q, \mathcal{K}, p, p')\, \rho^{neutron}_{j=\frac{3}{2}, l=1}(q, \mathcal{K}, p, p')\, d^3p\, d^3p' =$$

$$N \int \left(f_{j=\frac{3}{2}, l=1}(p)\, f_{j=\frac{3}{2}, l=1}(p')\right)_{neutron} \Bigg((\mathcal{A} \pm \mathcal{C}) \left(\frac{\pi}{2}\, P_{l=1}(\mathrm{Cos}[\gamma])\right)$$

$$+ \left(\frac{i\, \pi\, \mathrm{Sin}[\gamma]}{6}\right) \Bigg(\mathcal{C}\, \mathrm{Cos}[\beta] \pm \mathcal{M}\, \mathrm{Cos}[\beta] +$$

$$(\mathcal{G} + \mathcal{H}) \left(\frac{1}{2\, |\mathcal{K}_{nn}|} \left(|k_{nn}| + |k'_{nn}|\, e^{(\mp i \gamma_{nn})}\right) \mathrm{Cos}[\alpha]\right) +$$

$$\mathcal{D} \left(\frac{1}{|q_{nn}|} \left(-|k_{nn}| + |k'_{nn}|\, e^{(\mp i \gamma_{nn})}\right) \mathrm{Cos}[\alpha]\right)\Bigg)\Bigg)\, d^3p\, d^3p',$$

Give zero contribution integrated with p-3/2 states !

Optical Potential for ^{6}He with all terms from the valence neutrons ($p_{3/2}$ shell)

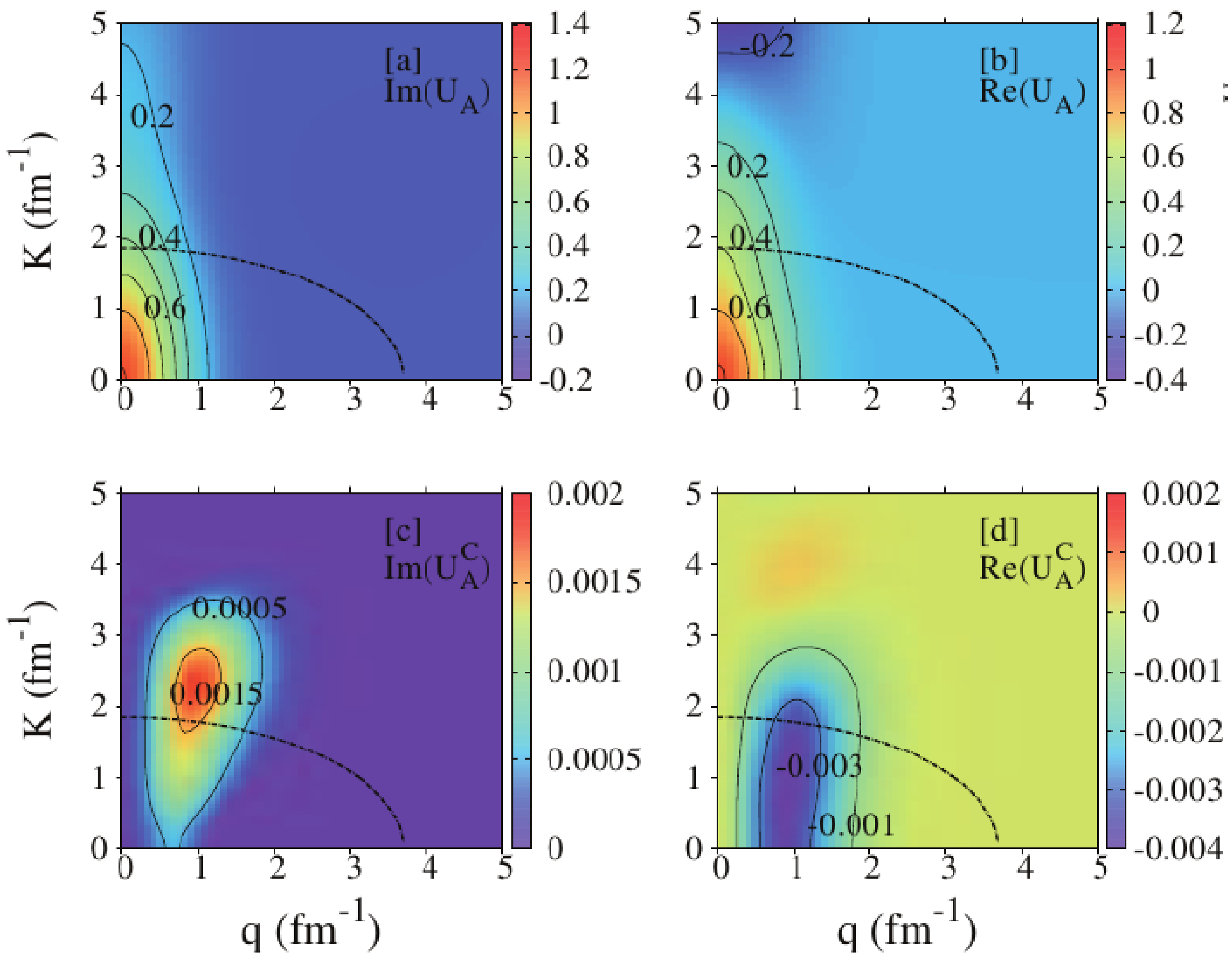
$$U_{^6He}(\mathbf{q},\mathbf{K}) = \sum_{i=N,P} U_{core}(\mathbf{q},\mathbf{K}) + U_{val}(\mathbf{q},\mathbf{K})$$

$$\begin{aligned} U_{val_{central}} &= U_A(\mathbf{q},\mathbf{K}) + U_A^C(\mathbf{q},\mathbf{K}) \\ U_{val_{spin-orbit}} &= U_C(\mathbf{q},\mathbf{K}) + U^M(\mathbf{q},\mathbf{K}) \end{aligned}$$

Similar for ^{8}He

Central Terms

[MeV fm^3]



- $k_s=0$
- $U_A(q,K)$

- $k_s=1$
- $U_A^C(q,K)$

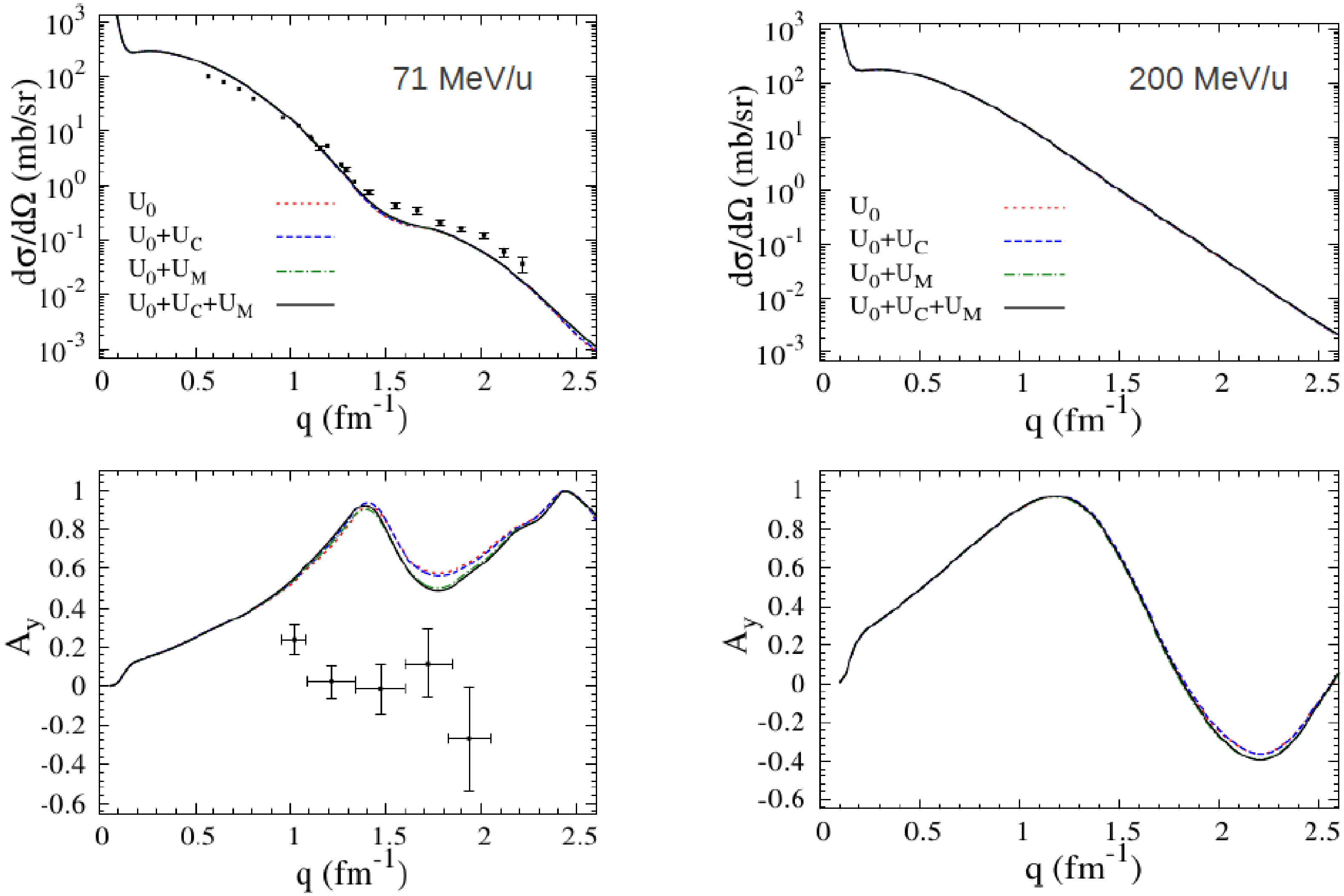
$$\left(\frac{A-1}{2A}\right)^2 q^2 + K^2 = k_0^2$$

On-shell condition

Department of physics + astronomy

OHIO UNIVERSITY

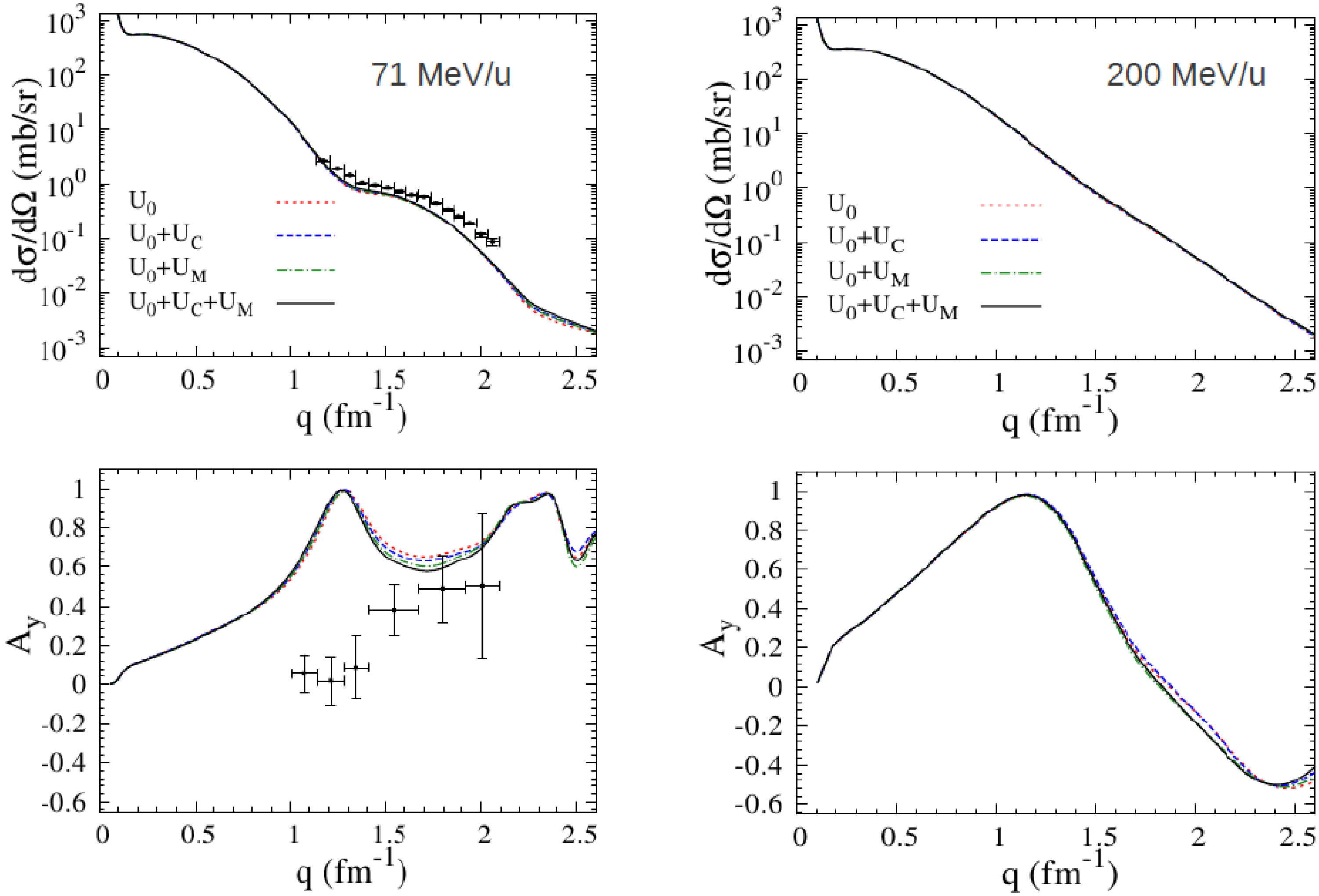
Scattering Observables for ^{6}He



physics + astronomy

OHIO UNIVERSITY

Scattering Observables for ^{8}He



physics + astronomy

OHIO UNIVERSITY

Asymptotic behavior of the p-shell wave function

HO $p_{3/2}$-shell $\qquad \Psi_{01}(\zeta) = (2\pi)^{3/2}\sqrt{\frac{4}{\sqrt{\pi}\nu_p^{3/2}}}\sqrt{\frac{2}{3}}\frac{\zeta}{\sqrt{\nu_p}} exp\left(-\frac{\zeta^2}{2\nu_p}\right)\mathcal{Y}_1^m(\hat{\zeta})$

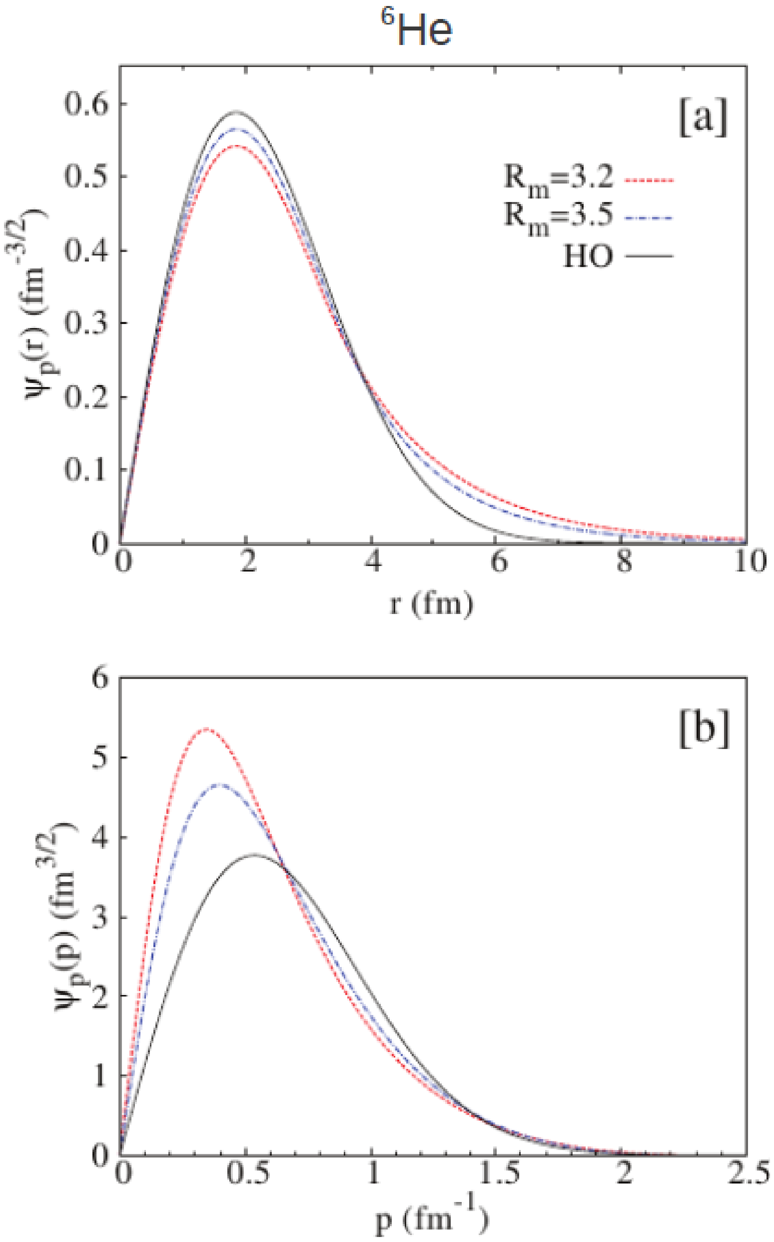
Halo nuclei ~ large extension $\qquad \psi_e(r) = B\ \exp(-\mu\, r)$

Matching: equate functions and logarithmic derivatives (2 parameters)

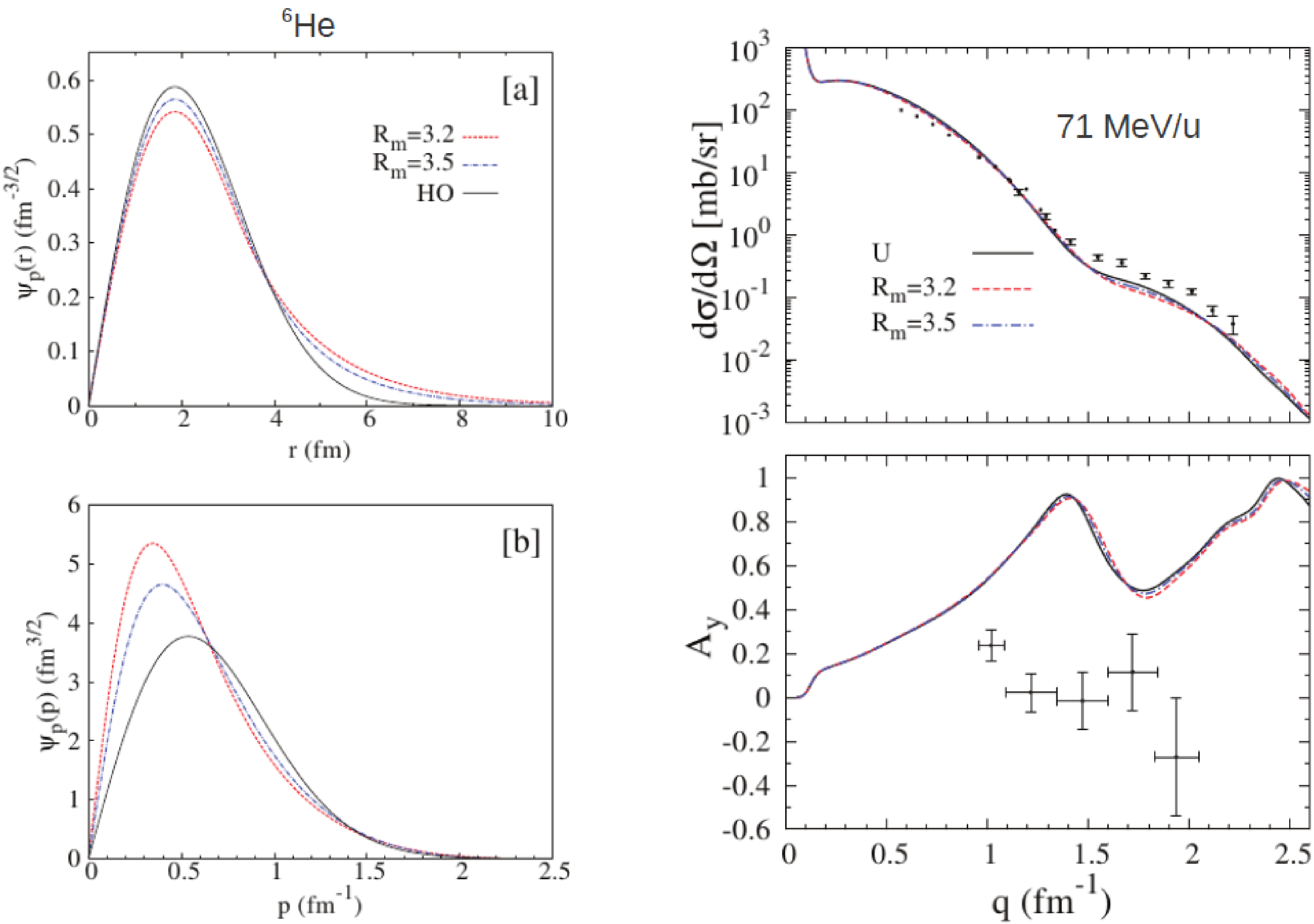
Normalize to the number of particles

R_m(fm)	mass% s-shell	mass% p-shell	μ (fm^{-1})	B fm^{-3}	norm in p-shell	r_m (fm) not norm.	r_m (fm) normalized
2.8	89.56	52.5	0.453	0.833	3.048	3.37	2.89
3.2	95.46	68.6	0.613	1.347	2.354	2.687	2.55
3.4	97.17	75.51	0.689	1.732	2.213	2.548	2.472
3.5	97.79	78.57	0.727	1.970	2.166	2.501	2.444
3.6	98.28	81.38	0.763	2.246	2.129	2.465	2.421
3.8	98.99	86.2	0.836	2.936	2.076	2.414	2.389

Effect of “tail” modifications



Effect of “tail” modifications



Reminder:

Relation of charge and matter radii to oscillator parameters

Charge Radius

$$\langle r^2_{ch} \rangle = \frac{\int d^3r \, \Phi_s^*(\mathbf{r}) \, r^2 \, \Phi_s(\mathbf{r})}{\int d^3r \, \Phi_s^*(\mathbf{r}) \, \Phi_s(\mathbf{r})}$$

$$\nu_s = \frac{3}{2 \, \langle r^2_{ch} \rangle}$$

Matter Radius

$$\langle r^2_{mat} \rangle = \frac{\int \, d^3\mathbf{r} \, \Phi^{*^{6,8}\mathrm{He}}_{s+p} \, r^2 \, \Phi^{6,8\mathrm{He}}_{s+p}}{\int \, d^3\mathbf{r} \, \Phi^{*^{6,8}\mathrm{He}}_{s+p} \, \Phi^{6,8\mathrm{He}}_{s+p}}$$

$$\nu_p^{^6\mathrm{He}} = \frac{5}{6 \, \langle r^2_{mat} \rangle - 4 \, \langle r^2_{ch} \rangle}$$

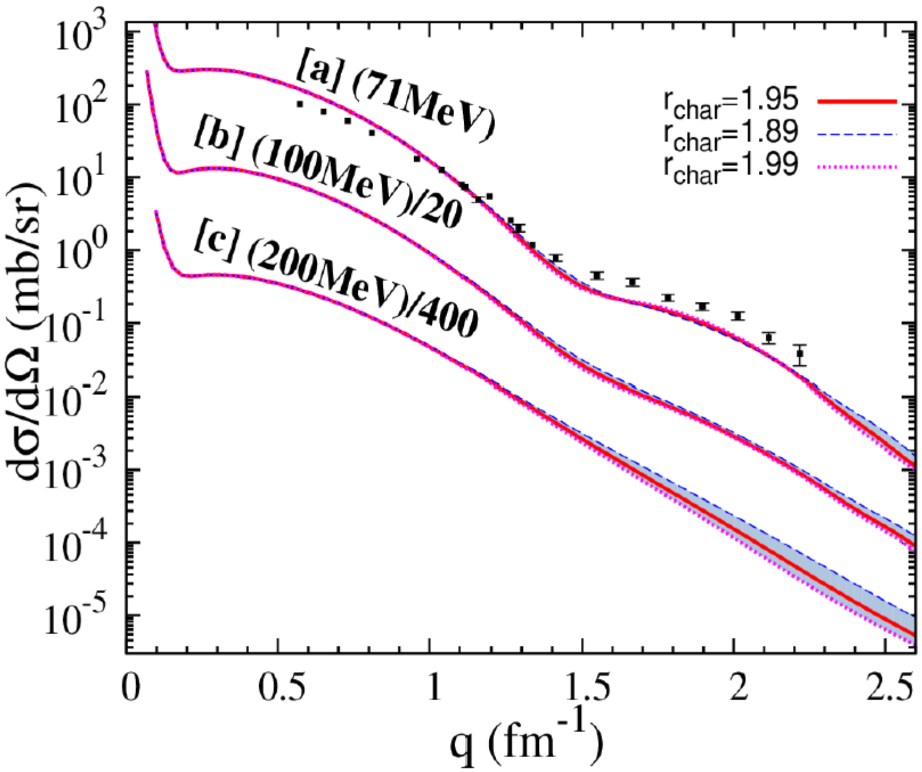
$$\nu_p^{^8\mathrm{He}} = \frac{5}{4 \, \langle r^2_{mat} \rangle - 2 \, \langle r^2_{ch} \rangle}$$

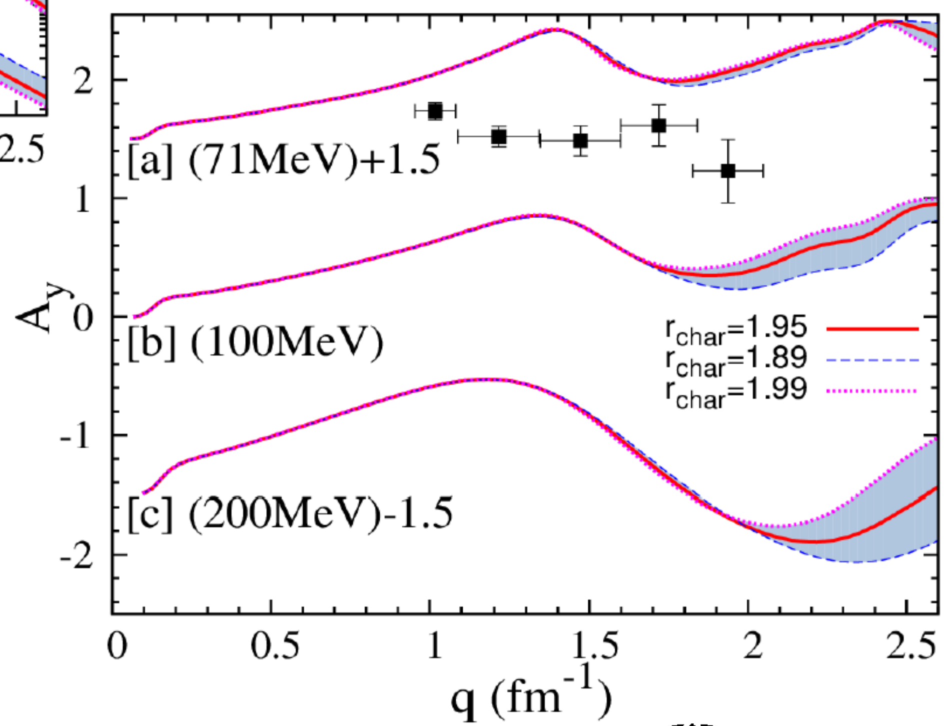
Variations of the charge and matter radii in ^{6}He

r_{ch}(fm)	r_m(fm)	ν_s (fm^{-2})	ν_p (fm^{-2})
1.894	**2.33**	0.422	0.231
1.996	**2.33**	0.376	0.301
1.955	2.24	0.392	0.337
1.955	2.602	0.392	0.197

Kaki et al. PRC 86, 044601 (2012), Mueller et al PRL 99(2007), Tanihata et al Phys.Let. B289(1992), Alkhazov et al Nucl. Phys. A712(2002), Bacca et al. Phys. Rev. C86(2012)

Variation of Charge Radius in ^6He





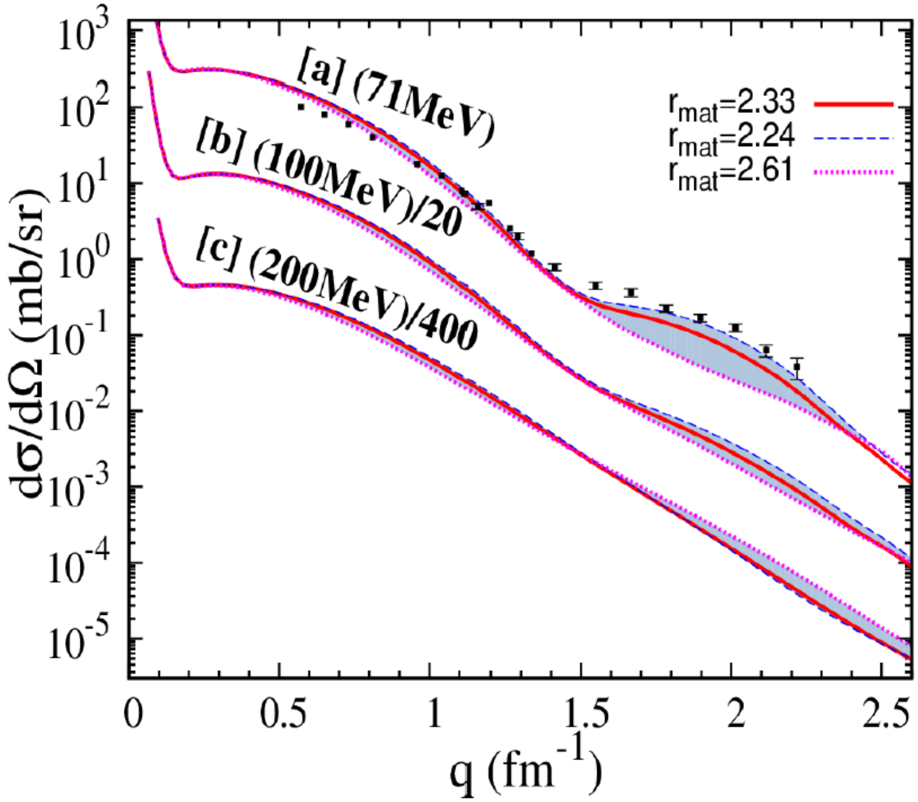
R_{mat}=2.33

R_{char}=1.89 (min)

R_{char}=1.95 (aver)

R_{char}=1.99 (max)

Variation of Matter Radius in ^{6}He

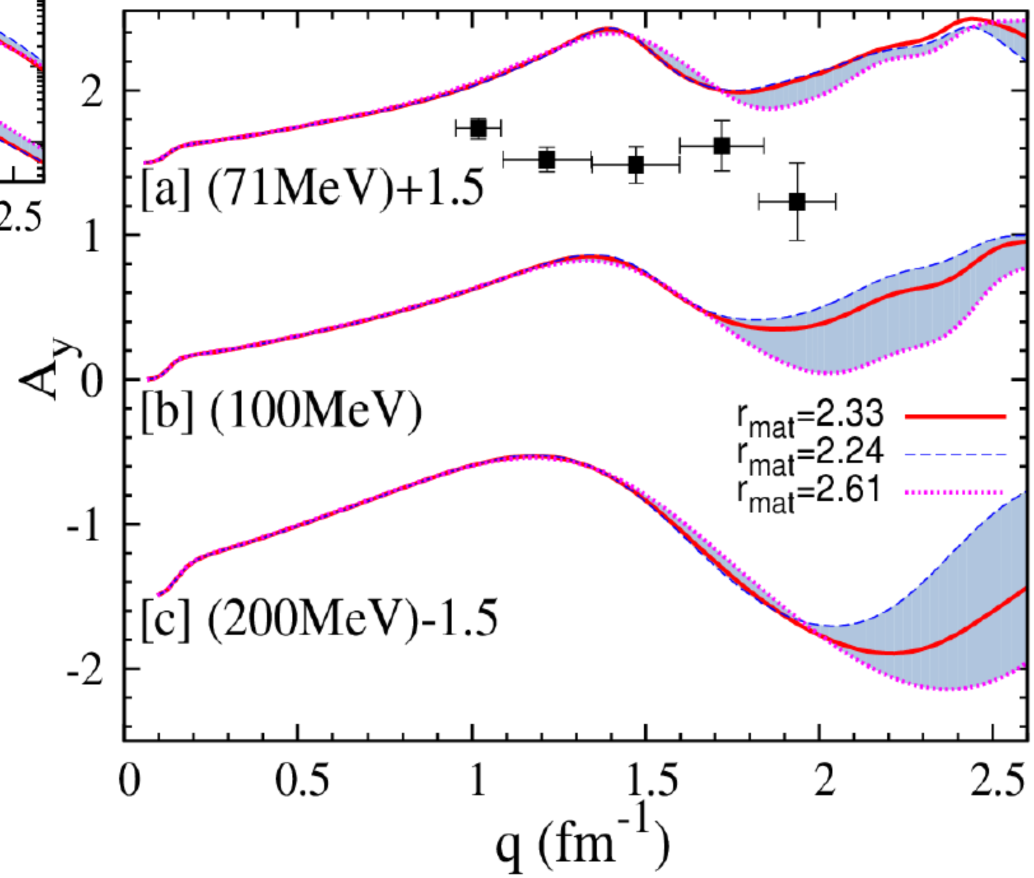


R_{char}=1.95

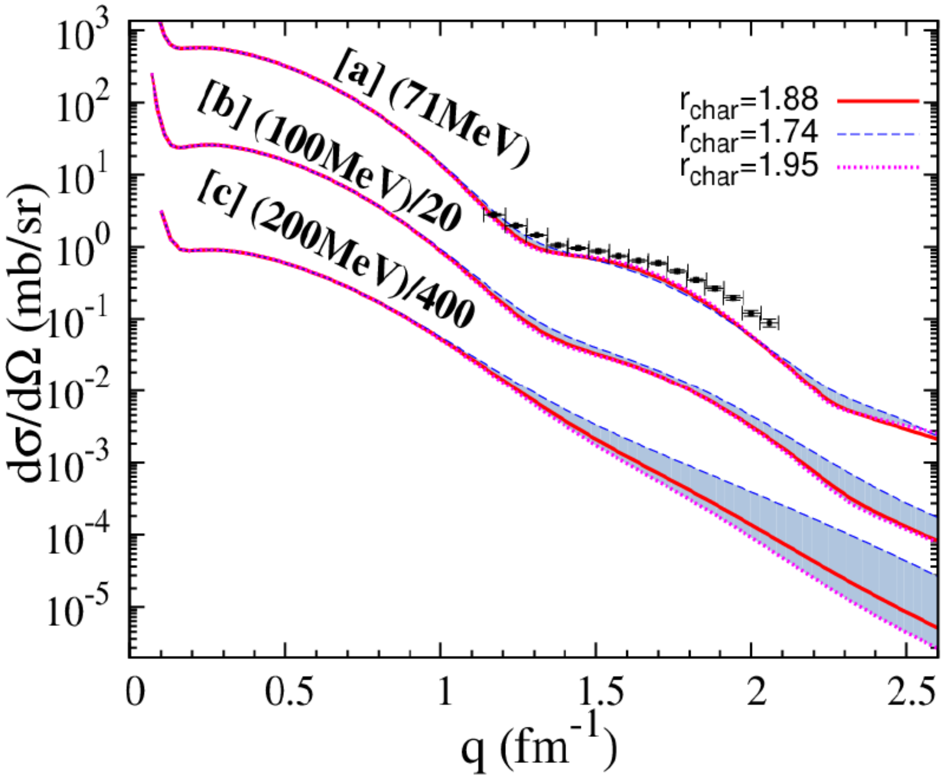
R_{mat}=2.24 (min)

R_{mat}=2.33 (aver)

R_{mat}=2.61 (max)



Department of physics + astronomy



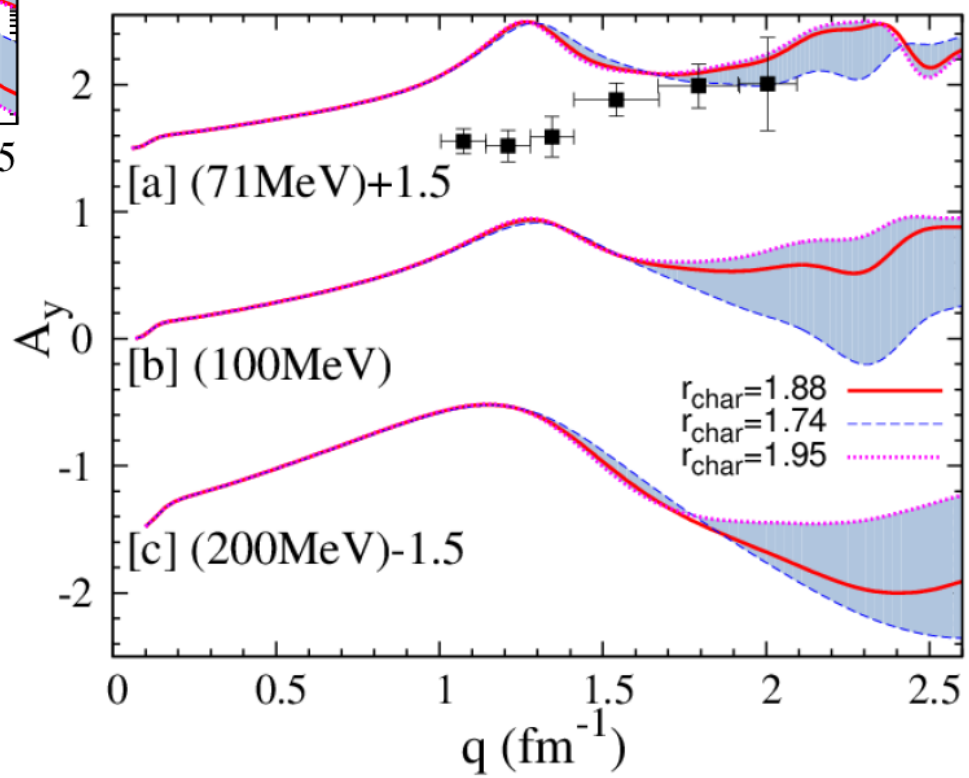
Variation of Charge Radius in ^8He

R_{mat}=2.5

R_{char}=1.74 (min)

R_{char}=1.88 (aver)

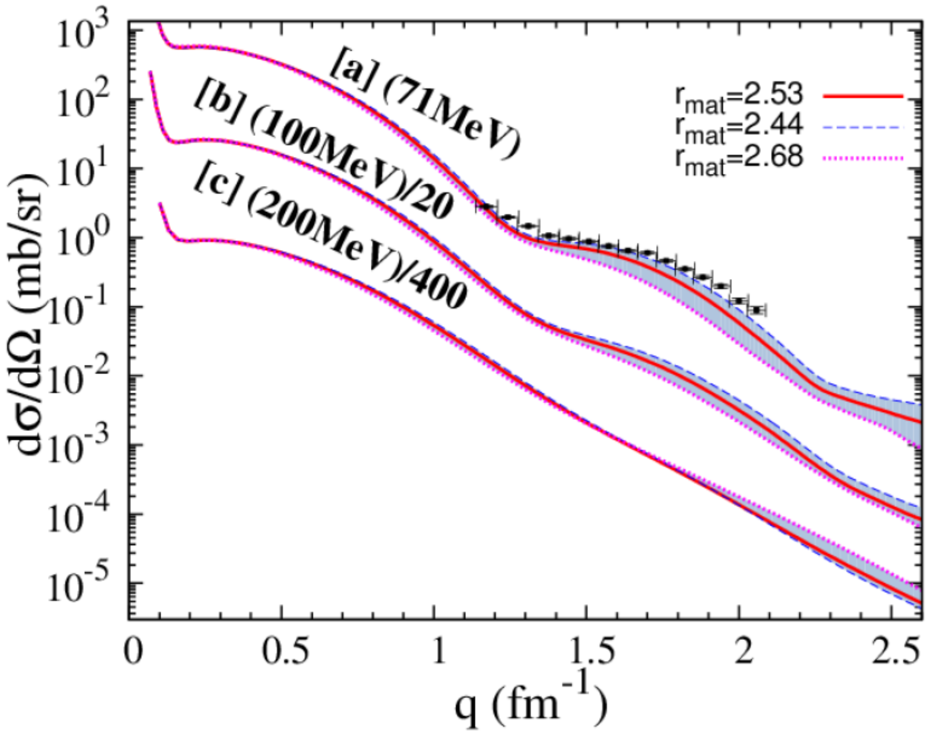
R_{chart}=1.95 (max)



Department of physics + astronomy

1804 UNIVERSITY

Variation of Matter Radius in ^{8}He

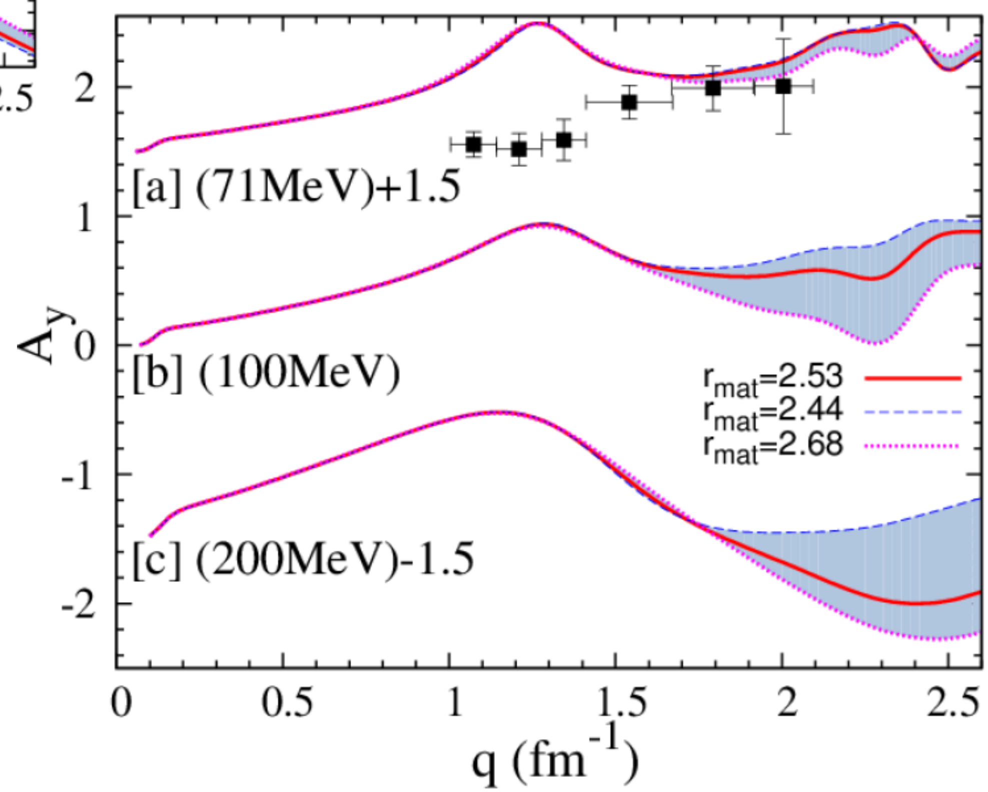


R_{char}=1.88

R_{mat}=2.44 (min)

R_{mat}=2.53 (aver)

R_{mat}=2.68 (max)

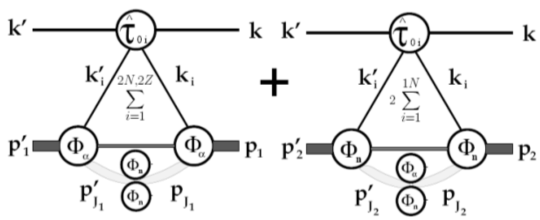


Summarizing and Reflecting

- **^{6}He and ^{8}He are not closed shell nuclei.**
- **Single particle density matrix has spin independent and spin dependent parts.**
- **In a microscopic first order optical potential all amplitudes of the NN t-matrix contribute.**
- **First calculation:**
  - HO ansatz with filled s-shell (alpha core) and valence neutrons in $p_{3/2}$ shell (COSM)
  - Valence neutrons: additional central and spin orbit contribution (M amplitude)
  - Additional contributions small effect in scattering observables
- **Additional results:**
  - At energies considered long distance behavior of valence wavefunction no effect on observables
  - Observables sensitive to variations in matter and charge radius
- **Further observations:**
  - Valence neutrons in $p_{3/2}$ shell → same angular momentum
  - Transition $p_{3/2}$-$d_{3/2}$ → Wolfenstein G+H and D contribute

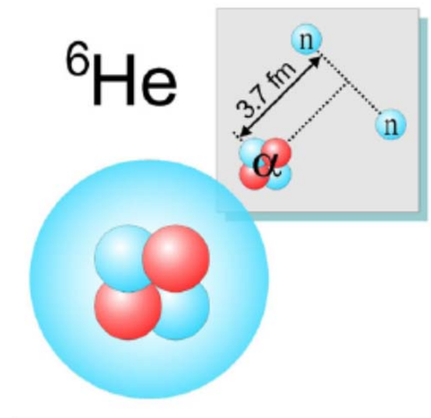
Summarizing and Reflecting

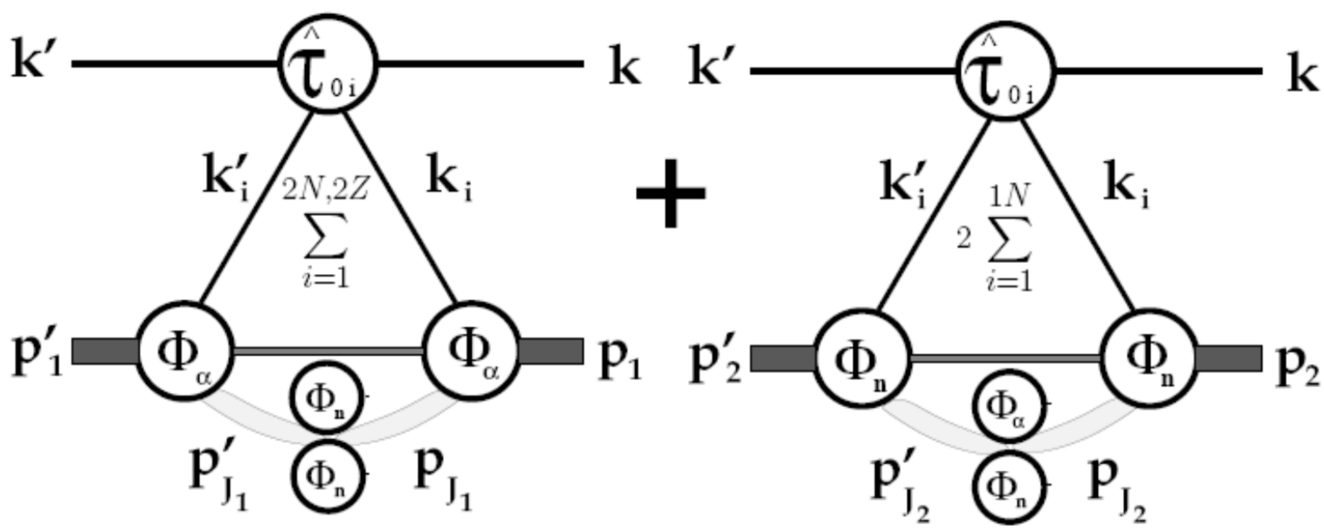
- **^{6}He is a loosely bound nucleus**
- **Is a cluster ansatz (3-body model) viable?**
- **Weppner, Elster: microscopic cluster optical potential**
  - Optical potential for core correlated with valence neutrons and neutrons correlated with core



  - Core – valence neutron correlation = additional degree of freedom
  - Only microscopic calculation with negative A_y at larger angles
    - Check to be done: cluster ansatz for ^{8}He
  - Correlations also visible in cross section at forward angles
- **^{6}He (and ^{8}He) are challenging nuclei to describe**

Optical Potential for ^{6}He as cluster α+n+n





Weppner, Elster, PRC 85, 044617 (2012)

Cluster Folding Optical Potential (n+n+α)

Jacobi momenta

$$\mathbf{p}_{j_i} = \frac{1}{A}(A_{s_i}\mathbf{p}_i - A_i\mathbf{p}_{\mathbf{s}_i})$$

Correlation Density

$$\rho_{corr}(\mathbf{p}_{j_1}, \mathbf{p}_{j_1}') \equiv \int \prod_{l=2}^{N_c} d\mathbf{p}_{j_l}' \int \prod_{m=2}^{N_c} d\mathbf{p}_{j_m} \langle \phi_A | \mathbf{p}_{\mathbf{j}_1}' \mathbf{p}_{\mathbf{j}_2}' \cdots \mathbf{p}_{\mathbf{j}_{N_c}}' \rangle \, \langle \mathbf{p}_{\mathbf{j}_1} \mathbf{p}_{\mathbf{j}_2} \cdots \mathbf{p}_{\mathbf{j}_{N_c}} | \phi_A \rangle$$

$p_{3/2}$ HO state

Cluster optical potential

$$U_{el}(\mathbf{q}, \mathbf{K}) = \sum_{c=1,N_c} \sum_{i=n_c,p_c} \int d\mathbf{P} \, d\mathcal{P}_{j_c} \, \rho_{corr}(\mathcal{P}_{j_c})$$

$$\hat{\tau}_{0i}\left(\mathbf{q}, \frac{1}{2}\left(\frac{A+1}{A}\mathbf{K} - \mathbf{P}\right), \mathcal{E}\right) \, \rho_{ci}\left(\mathbf{P} - \frac{A-1}{2A}\mathbf{q}, \mathbf{P} + \frac{A-1}{2A}\mathbf{q}\right)$$

Cluster folding potential for ^{6}He+p

$$
{}^{6\mathrm{He}}U_{el}(\mathbf{q},\mathbf{K}) = U_\alpha + 2U_n =
$$

$$
\sum_{i=n,p} \int d\mathbf{P}\, d\mathcal{P}_{j_\alpha}\, \rho_{corr}(\mathcal{P}_{j_\alpha})\, \hat{\tau}_{0i} \left(\mathbf{q}, \frac{1}{2}\left(\frac{A+1}{A}\mathbf{K}-\mathbf{P}\right), \mathcal{E}\right)\, \rho_{\alpha i}\left(\mathbf{P}-\frac{A-1}{2A}\mathbf{q}, \mathbf{P}+\frac{A-1}{2A}\mathbf{q}\right)
$$

$$
+2\int d\mathbf{P}\, d\mathcal{P}_{j_n}\, \rho_{corr}(\mathcal{P}_{j_n})\, \hat{\tau}_{0n}\left(\mathbf{q}, \frac{1}{2}(\frac{A+1}{A}\mathbf{K}-\mathbf{P}), \mathcal{E}\right)\, \rho_n\left(\mathbf{P}-\frac{A-1}{2A}\mathbf{q}, \mathbf{P}+\frac{A-1}{2A}\mathbf{q}\right).
$$

For calculation:

NN t-matrix: Nijmegen II potential

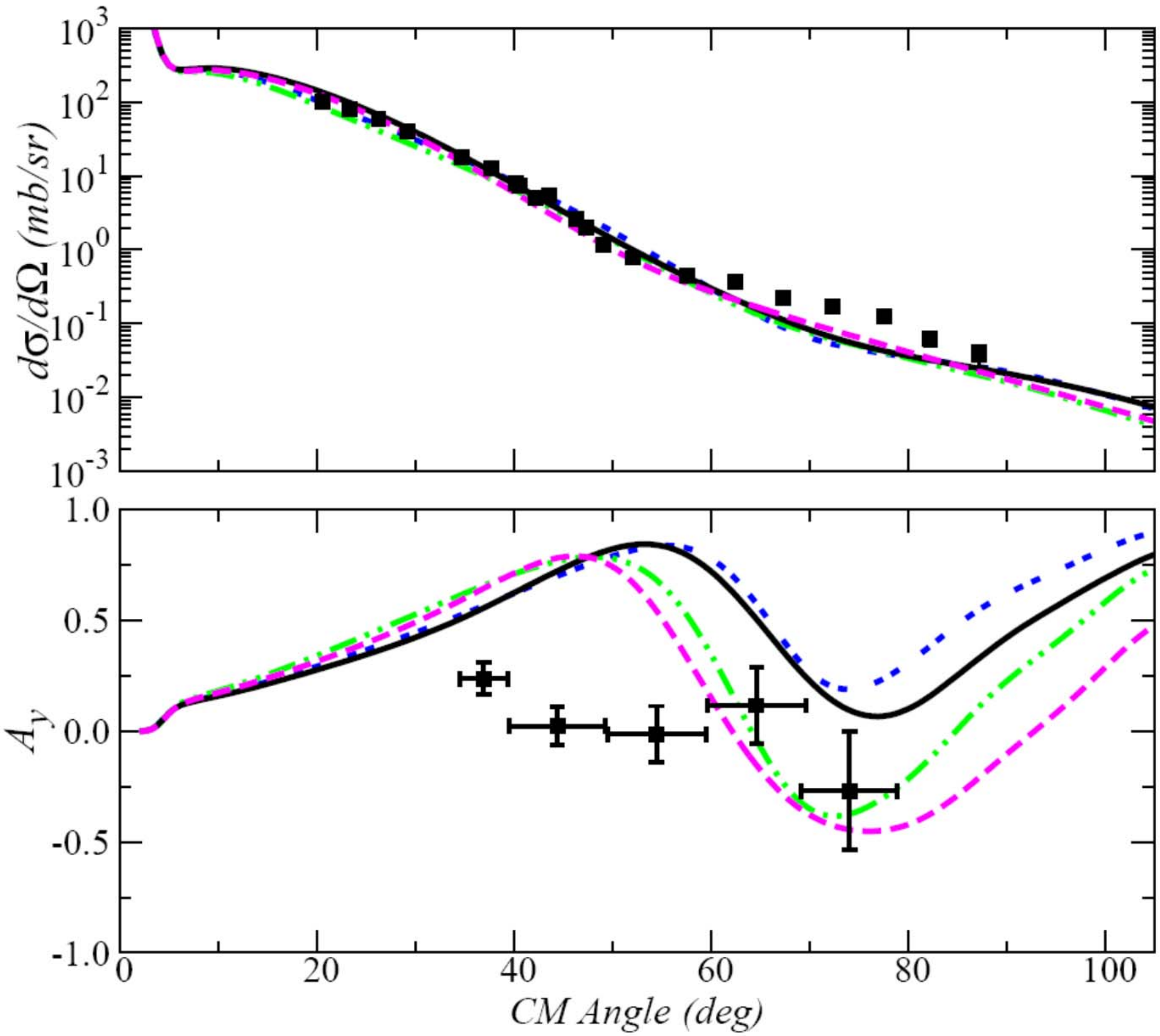
Densities:

COSMA density == s & p- shell harmonic oscillator wave functions

Fitted to give rms radius of ^{6}He (older value)

and for ^{4}He: Gogny density with coupling to medium

^{6}He (p,p) ^{6}He @ 71 MeV



COSMA single particle OP

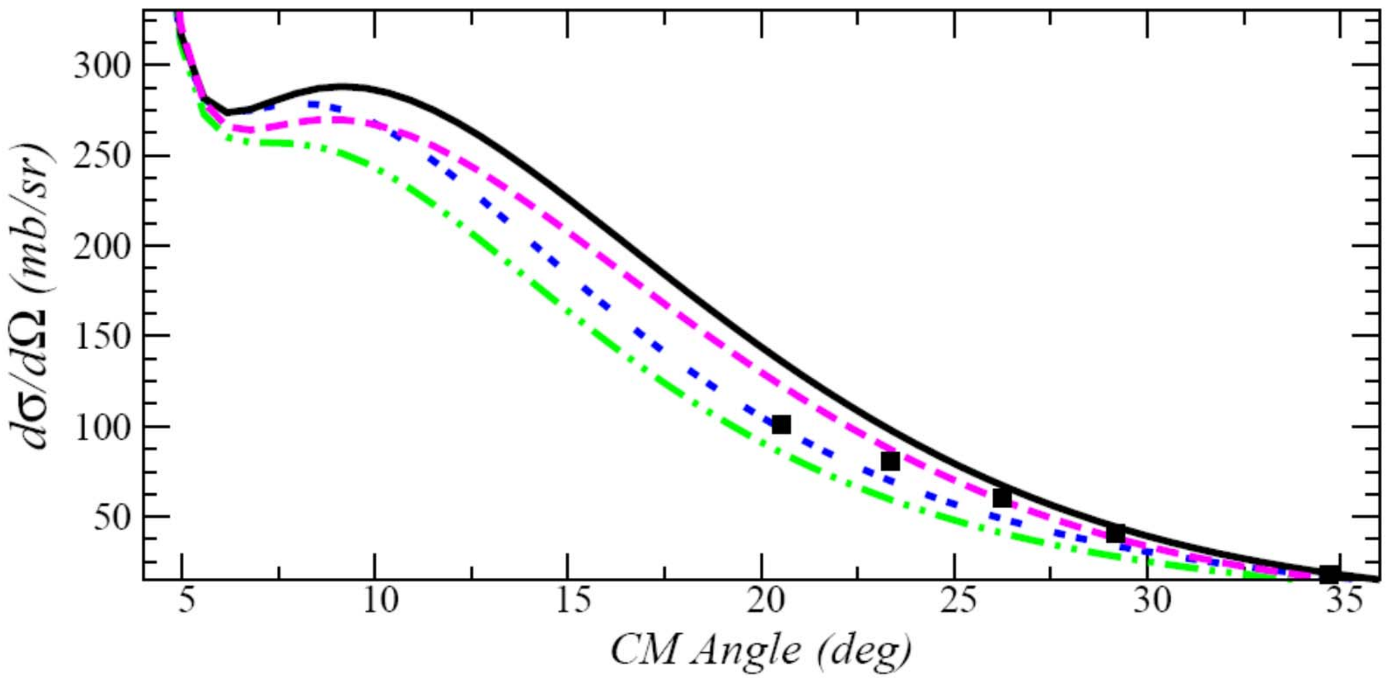
COSMA cluster OP

α- HFB

n - COSMA

α - HFB
n – COSMA
no correlations

^{6}He (p,p) ^{6}He @ 71 MeV



COSMA single particle OP

COSMA cluster OP

α- HFB

n - COSMA

α - HFB
n – COSMA
no correlations