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EWSB in Conformal TC Models

(доклад в ИЯИ РАН, 18.04.2011)

- Most people think SM is an effective theory
- $\Lambda_{UV} \gg \Lambda_{EW} \sim \text{TeV} \Rightarrow$ see renormalizable Lagrangian at low energies
- Higher dim. ($\Delta > 4$) operators are Λ_{UV} - suppressed

SM global symmetries "accidental"

$$\mathcal{L}_{SM} + \frac{1}{\Lambda_{UV}^2}\mathcal{O}_{\not B} + \frac{1}{\Lambda_{UV}}\mathcal{O}_{\not L} + \frac{1}{\Lambda_{UV}^2}\mathcal{O}_{\not F}$$

↳ proton decay ↳ ν masses ↳ FCNC

$\Rightarrow$ smallness (or non-observation) of these effects.

- Higgs sector is a blemish on this picture

$|H|^2$ is 'relevant' ($\Delta = 2 < 4$).

$\mu_H^2 \ll \Lambda_{UV}^2$ needs "Experimenter" to finetune to reach ferromagnetic critical point, (like finetuning $T \to T_c$)

aka $\boxed{\text{Hierarchy Problem}}$

Ways out

- by symmetry: Change the theory above Λ_{EW} so that $|H|^2$ transforms nontrivially under a global symmetry (SUSY, or shift symmetry in PGB Higgs scenario)

$\Rightarrow$ coefficient can be naturally small

- $\boxed{\text{Today}}$ by dynamics: Change the theory so that $|H|^2$ is irrelevant above Λ_{EW}.

Ancient example: Technicolor (Weinberg, Susskind 79)

EW sector to replace Higgs:

Above Λ_{EW} a new as. free gauge group $SU(N_{TC})$ with N_{TF} families of 'techniquarks' (Minimal $N_{TC} = 3$, $N_{TF} = 2$). Coupling runs. $\Lambda_{IR} \sim e^{16\pi^2/g_{UV}^2}\Lambda_{UV}$ - confinement scale (dim. transmutation). Global chiral symmetry $SU(N_{TF})_L \times SU(N_{TF})_R$ breaks spontaneously to $SU(N_{TF})_{diag}$ via χSB

Assuming $SU(2)_W \times U(1)_Y \subset SU(N_{TF})_L \times SU(N_{TF})_R$ gauges

(this can be achieved by giving techniquarks $SU(2)_W \times U(1)_Y$ quantum numbers, e.g.

(Q_{1L}, Q_{2L}) - $SU(2)_W$ doublet, $Y=0$

Q_{1R} $Y = 1/2$, Q_{2R}, $Y = -1/2$, $SU(2)$ singlets)

$\Rightarrow$ EW symmetry breaks and $\Lambda_{EW} \sim \Lambda_{IR}$.

Main TC problem: Flavor

What is the UV origin of IR SM Yukawa couplings $\bar f f H$? Must come from $(\bar f f)(\bar Q Q)$ $(H \sim \bar Q Q)$ interactions (ETC)

Any dynamics which generates these will also generate $(\bar f_i f_j)(\bar f_k f_l)$ with roughly same size coefficients. Coefficients of these FCNC ops are experimentally bounded $\frac{1}{\Lambda_F^2}(\bar f_i f_j)(\bar f_k f_l)$, $\Lambda_F > 10^{3\div 4}$ TeV (from K physics)

$$\frac{1}{\Lambda_F^2}(\bar f f)(\bar Q Q) \to m_f \lesssim \Lambda_{IR}\left(\frac{\Lambda_{IR}}{\Lambda_F}\right)^2 \sim 10^{-3}\ \text{GeV}$$

Not good

Walking TC [Holdom '86]

Assume TC does not confine straight away but enters a strongly coupled near scale-inv. fixed point at which $H \sim \bar Q Q$ has a large anomalous dimension γ, $d_H = 3-\gamma$.

Then looking from down up, coefficient of operator $\frac{m_f}{\Lambda_{IR}^d}(\bar f f)H$ at some scale becomes $= \frac{1}{\Lambda_{UV}^{d-1}}$

where the last eq. determines UV scale at which this operator becomes strong: $\frac{m_f}{\Lambda_{IR}^d} \sim \frac{1}{\Lambda_{UV}^{d-1}}$. $m_f \sim \Lambda_{IR}\left(\frac{\Lambda_{IR}}{\Lambda_F}\right)^{d-1}$

Using truncated Schwinger-Dyson eqn, people concluded that $d \simeq 2 \Rightarrow m_f \sim$ GeV cannot reproduce top mass

Conformal TC [Luty, Okui 2004].

Free yourself from thinking about gauge dynamics which sits at ultra-high scales and is not observable in near future. Think in terms of the fixed point.

What makes it viable:

a) $d_H \leq 1+\varepsilon$, ε has to be small to reproduce top mass $m_t \sim \Lambda_{IR}\left(\frac{\Lambda_{IR}}{\Lambda_F}\right)^{\varepsilon} \Rightarrow \varepsilon \lesssim 0.3$ is needed

b) Dimension of $|H|^2$ has to stay above (or close to) 4.

We don't want to throw baby out with water

How does EW symmetry breaks?

Case a) $\dim |H|^2 = 4-\delta < 4$

At UV scale we have perturbation:

$$\mathcal{L} \supset \mu_{UV}^{\delta}|H|^2, \quad \mu_{UV} = c\Lambda_{UV}$$

It is not natural to suppose that $c \lll 1$ but $c = 0.1$ say could be plausible

At which IR scale this perturbation becomes $O(1)$?

What is $|H|^2$?

$$H^\dagger(x)\times H(0) = \frac{1}{|x|^{2d_H}}\left(1+\alpha|x|^{d_S}\cdot S(0)+\dots\right)$$

$S \equiv |H|^2$ – lowest scalar operator which is singlet under the global $SU(2)\times SU(2)$

Dimensions can jump.

$$\Lambda_{IR}^{\delta} = c\,\Lambda_{UV}^{\delta} \to \Lambda_{IR} = c^{1/\delta}\Lambda_{UV}$$

E.g. For $\delta = 0.3$ and $c = 0.1$ $\Lambda_{IR} = 10^{-3}\Lambda_{UV}$ becomes natural. Exponential hierarchies can be generated from algebraic (~ 0.1) ones. This is not so different from dim. transmutation (also there had to assume $g_{UV} \ll 1$)

~~Case (a)~~ Notice that top loop generates a contribution to $|H|^2$ coefficient. This contribution can be computed via conformal pert. theory

$$\mathcal{L} = \mathcal{L}_{CFT} + \frac{g}{\Lambda_{UV}^{d}}(\bar{Q}_L t_R)H + \text{h.c.}$$

$$e^{i\mathcal{L}} \supset \frac{g^2}{\Lambda_{UV}^{2d}}\int d^4x\, d^4y\, (\bar{Q}_L t_R H)(x)\,(\bar{Q}_L t_R H)^\dagger(y)$$

$$H(x)\times H^\dagger(y) \sim \frac{c_{HH^\dagger S}}{|x-y|^{2d_H - d_S}}\, S(y)$$

integrating over $|x-y| > \Lambda_{UV}^{-1}$ get

$$\delta\mu_{UV} \sim c_{HH^\dagger S}\, g^2\, \Lambda_{UV}^{\delta}$$

Case (b) $\dim |H|^2 > 4$

In this case we have to assume that there is a strongly relevant operator which is not singlet so that its coefficient can be naturally small.

$\boxed{EX}$ QCD in conf. window ~~select~~ with $N_F > 2$ flavors. Decouple $N_F - 2$ flavors by giving them a mass.

- Experimental signals

a) Imagine that dimensions in H sector are generated by coupling it to a truly strong sector

$$\mathcal{L} = |DH|^2 + c\,\mathcal{O}\cdot H^\dagger$$

To make contribution $O(\varepsilon)$ to H's dimension

$c \sim 4\pi\sqrt{\varepsilon}$ (assume $H^\dagger H$ large dimension is an accident)

$$\mathcal{L}_{eff} = |DH|^2 + \frac{\Lambda^4}{16\pi^2}\,\mathcal{F}\left(\frac{4\pi\sqrt{\varepsilon}H}{\Lambda}, \frac{D_\mu}{\Lambda}\right)$$

(in the spirit of NDA) $\Lambda = \Lambda_{IR}$

$$\mathcal{L}_{eff} \sim |DH|^2 - m_h^2|H|^2 - \frac{1}{4}\lambda|H|^4$$

$$m_h^2 \sim \varepsilon\Lambda^2, \quad \lambda \sim 16\pi^2\varepsilon^2$$

$$\langle H\rangle \sim \frac{\Lambda}{4\pi\sqrt{\varepsilon}}$$

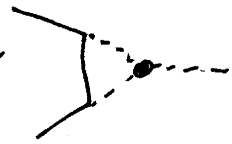
•) Leading correction to WWh coupling

$$\mathcal{L}_{eff} \supset \frac{\Lambda^4}{16\pi^2}\left(\frac{g_2 W}{\Lambda}\right)^2 \frac{4\pi\sqrt{\varepsilon}\, h}{\Lambda} \sim (\varepsilon g_2^2 v) W_\mu^2 h$$

$\Rightarrow O(\varepsilon)$ correction to WWh coupling

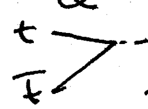
•) $O(100\%)$ corrections to cubic h^3 coupling

$$\left[\frac{\Lambda^4}{16\pi^2} V\left(\frac{h}{v}\right)\right]$$

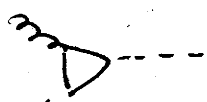
$\delta y_t \sim$  $\sim \dfrac{y_t^2}{16\pi^2}\,\dfrac{\Lambda^4}{16\pi^2}\left(\dfrac{1}{v}\right)^3 \dfrac{1}{\underset{\substack{\uparrow \\ \text{IR-dominated}}}{m_h}} \sim \dfrac{y_t^2}{4\pi}\cdot\varepsilon$

Not for the LHC.

b) More model-independent signals:

Expect that CFT coupling $\overline{Q}\, t\, H_{CFT}$ will give rise to ~~production~~ vertices  ← scalar resonances TeV-scale transforming according to 3 or 1 of $SU(2)_{diag}$

$(2, 2) \Rightarrow 3 \oplus 1$

Then gg production 

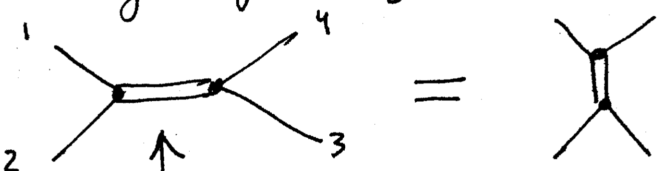
Apart from decays back to $t\bar{t}$

- Singlets decay to $\pi\pi$ ($= W_L W_L$) $\rightarrow$ broad (like σ in QCD)
- Triplet scalars cannot decay to $\pi\pi$] $\underbrace{\text{decay to } \pi\pi\pi \text{ or to } \pi W_T}_{\text{can be comparable due to strong dynamics.}}$ (like $A \to Zh$ in 2HDM)

How can we test theoretical consistency of this idea?
[Rattazzi, S. R., Tonni, Vichi 2008 ...]

Conformal bootstrap [Polyakov 1974]

Crossing symmetry constraints on CFTs.



• dimensions, spins of exchanged fields
• (squares of) OPE coefficients

four-point function

$$\langle \varphi(x_1)\, \varphi(x_2)\, \varphi(x_3)\, \varphi(x_4) \rangle = \frac{1}{x_{12}^{2d_\varphi}\, x_{34}^{2d_\varphi}} \underbrace{\sum_{\Delta, \ell} C_{\Delta,\ell}^2\, g_{\Delta,\ell}(u,v)}_{G(u,v)}$$

$$u = \frac{X_{12}^2 X_{34}^2}{X_{13}^2 X_{24}^2}, \quad v = u_{2 \leftrightarrow 4}$$

$g_{\Delta,\ell}(u,v)$ — explicitly known functions (~hypergeometrics)

Crossing: $G(u,v) = \left(\frac{u}{v}\right)^{d_\varphi} G(v,u)$.

OPE: $\varphi(x)\, \varphi(0) = \frac{1}{x^{2d_\varphi}} + \sum C_{\Delta,\ell}\, K(x)\, O_{\Delta,\ell}(0)$

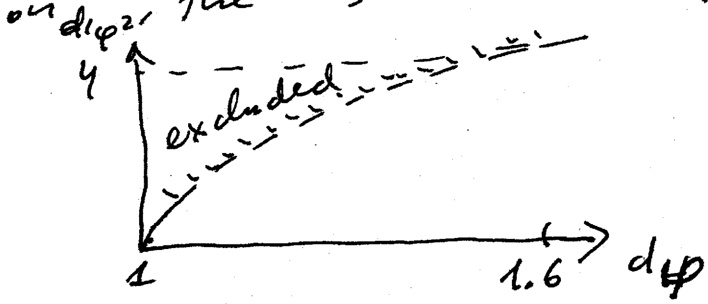
known functions

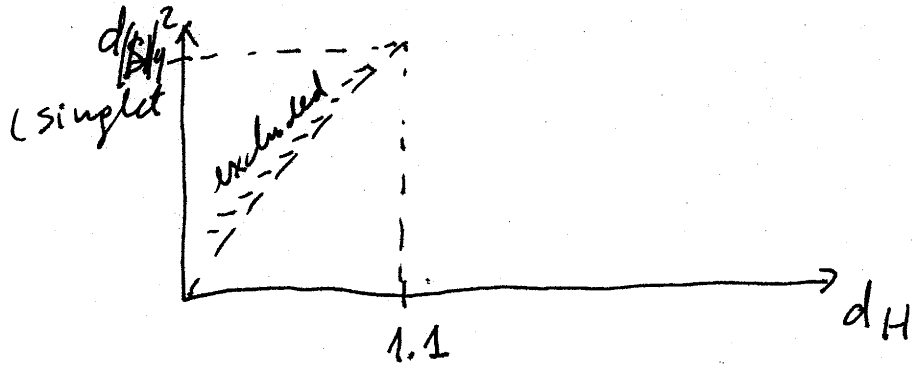
Sum rule

$$\boxed{1 = \sum C_{\Delta,\ell}^2\, F_{\Delta,\ell}(u,v)}$$

$$F_{\Delta,\ell} = \frac{v^{d_\varphi}\, g_{\Delta,\ell}(u,v) - u^{d_\varphi}\, g_{\Delta,\ell}(v,u)}{u^{d_\varphi} - v^{d_\varphi}}.$$

This equation imposes nontrivial constraints on the structure of conformal theory

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