High energy physics and gravity: Common grounds and perspectives

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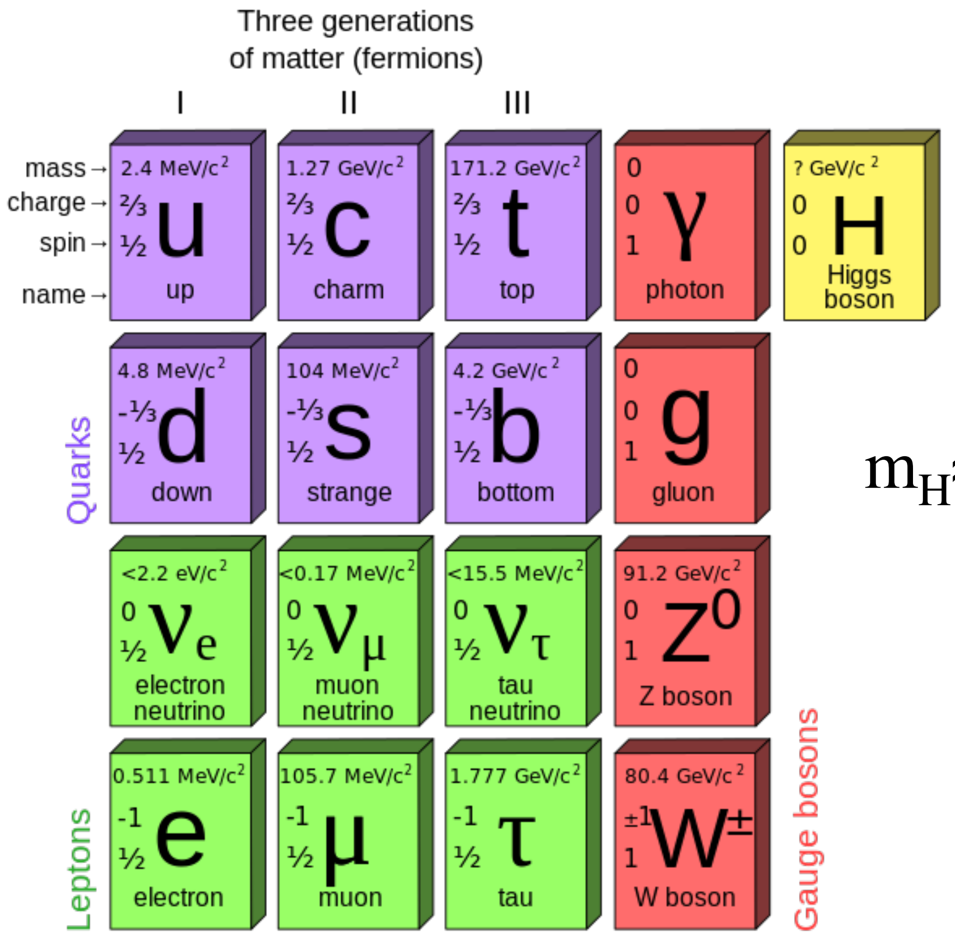
Effective action for GR

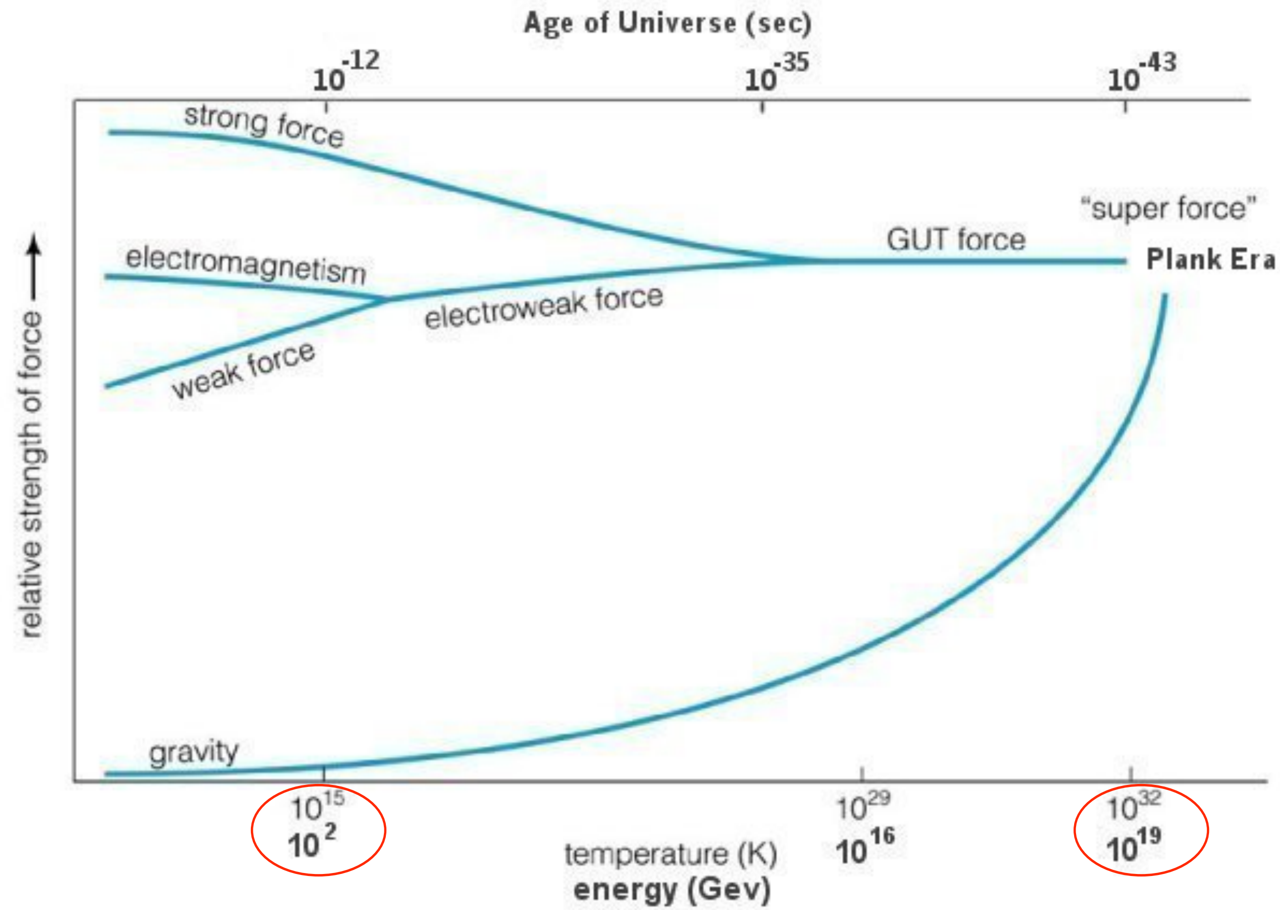
- How can we describe general relativity quantum mechanically?
- Well known issues with linearized GR: it is not renormalizable.
- This is the reason d'être of string theory, loop quantum gravity etc…
- How much can we understand using QFT techniques?
- We have good reasons to think that length scales smaller than the Planck scale are not observables due to the formation of small black holes.
- Effective field theories might be all we need to discuss physics at least up to the Planck scale.

Why quantize gravity? Unification…

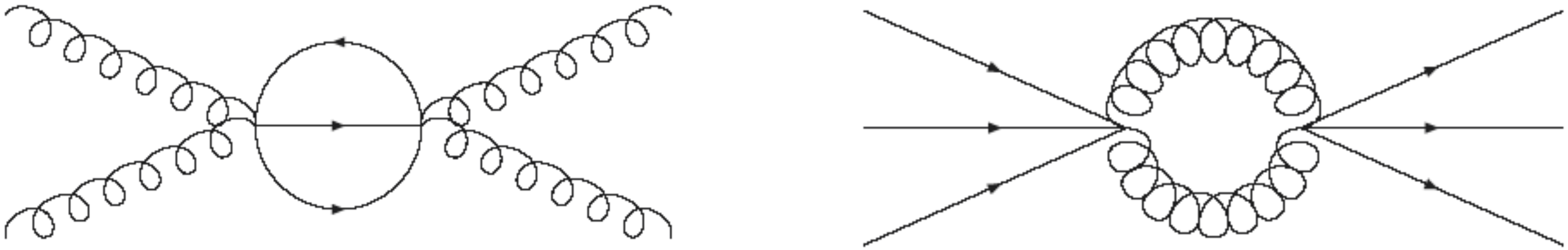


$m_H \approx 125$ GeV

Grand unification?



• Besides the beauty of grand unification, there are formal problems as well e.g.



• We thus have very good reasons to believe that gravity must be quantized.

Effective action for GR

- I am going to assume general covariance (diffeomorphism invariance)
- Quantum gravity has only 2 dofs namely the massless graviton (which has 2 helicity states).
- We know the particle content of the "matter theory" (SM, GUT, inflation etc).
- We can write down an effective action for quantum gravity.

Effective action for GR

- This program was started by Feynman in the 60's using linearized GR.

- Try to find/calculate observables

- Try to find consistency conditions which could guide us on our path towards a quantization of GR.

Effective action for GR coupled to known matter

- We can describe any theory of quantum gravity below the Planck scale using effective field theory techniques:

$$S=\int d^4x\,\sqrt{-g}\left[\left(\frac{1}{2}M^2+\xi H^\dagger H\right)\mathcal{R}-\Lambda_C^4+c_1\mathcal{R}^2+c_2\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}+\mathcal{L}_{SM}+\mathcal{O}(M_\star^{-2})\right]$$

- Electroweak symmetry breaking:

$$(M^2+\xi v^2)=M_P^2$$

- Several energy scales:
  - $\Lambda_C \sim 10^{-12}$ GeV cosmological constant
  - M_P or equivalently Newton's constant $G = 1/(8\pi\, M_P^2)$

$$M_P=2.4335\times10^{18}\ \mathrm{GeV}$$

  - $M_\star$ energy scale up to which one trusts the effective theory

- Dimensionless coupling constants ξ, c_1, c_2

What values to expect for the coefficients?

- It all depends whether they are truly new fundamental constants or whether the operators are induced by quantum gravitational effects.
  - If fundamental constants, they are arbitrary
  - If induced by quantum gravity we can estimate their magnitude.

- Usually induced dimension four operators are expected to be small

$$\exp(-M_P/\lambda)$$

λ is some low energy scale

- However, $\xi H^\dagger H \mathcal{R}$ translates into $\xi H^\dagger H h \Box h / M_P^2$ in terms of the graviton h.

- It is thus a dimension 6 operators and the nonminimal coupling should be of order unity.

What values to expect for the coefficients?

- In terms of the graviton h. $\mathcal{R}^2$-type operators lead to

$$h\Box h h\Box h/M_P^4$$

- We thus expect the coefficients of these operators to be $O(1)$.

- Naturalness arguments would imply $M_\star \sim \Lambda_C$. However, there is no sign of new physics at this energy scale.

- The Higgs boson seems to be the 2nd nail in the coffin for the naturalness argument.

Something special about the Higgs boson

- It can be coupled in a nonminimal way to gravity.

$$S \supset \int d^4x \sqrt{-g}\, \xi H^\dagger H \mathcal{R},$$

- This is a dimension 4 operator: we'll assume that it is a fundamental constant of nature.

- Is there any bound on its value?

The decoupling effect

- Let's consider the SM with a nonminimal coupling to R

$$S = -\int d^4x \sqrt{-g}\left[\left(\frac{1}{2}M^2 + \xi H^\dagger H\right) R - (D^\mu H)^\dagger (D_\mu H) + \mathcal{L}_{SM} + \mathcal{O}(M_P^{-2})\right]$$

- We can always go from the Jordan frame to the Einstein frame

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$$

$$\tilde{g}^{\mu\nu} = \Omega^{-2} g^{\mu\nu}\,, \quad \sqrt{-\tilde{g}} = \Omega^d \sqrt{-g}\,.$$

$$R = \Omega^2\left[\tilde{R} - 2(n-1)\tilde{\Box}\omega - (n-1)(n-2)\tilde{g}^{\mu\nu}\partial_\mu\omega\partial_\nu\omega\right]$$

$$\omega \equiv \ln\Omega\,, \quad \tilde{\Box}\omega = \frac{1}{\sqrt{-\tilde{g}}}\partial_\mu(\sqrt{-\tilde{g}}\,\tilde{g}^{\mu\nu}\partial_\nu\omega)$$

$$\Omega^2 = (M^2 + 2\xi H^\dagger H)/M_P^2$$

The decoupling effect

- In the Einstein frame, the action reads

$$S = -\int d^4x\,\sqrt{-\tilde{g}}\left[\frac{1}{2}M_P^2\tilde{R} - \frac{3\xi^2}{M_P^2\Omega^4}\partial^\mu(H^\dagger H)\partial_\mu(H^\dagger H) - \frac{1}{\Omega^2}(D^\mu H)^\dagger(D_\mu H) + \frac{\mathcal{L}_{SM}}{\Omega^4}\right]$$

- One notices that the Higgs boson kinetic term is not canonically normalized. We need to diagonalize this term.

- Let me now use the unitary gauge

$$H = \frac{1}{\sqrt{2}}(0, h+v)^\top$$

- The Planck mass is defined by

$$(M^2 + \xi v^2) = M_P^2$$

The decoupling effect

- To diagonalize the Higgs boson kinetic term:

$$\frac{d\chi}{dh} = \sqrt{\frac{1}{\Omega^2} + \frac{6\xi^2 v^2}{M_P^2 \Omega^4}}$$

- To leading order in Ω^{-1} $\Omega^2 = (M^2 + 2\xi H^\dagger H)/M_P^2$

$$h = \frac{1}{\sqrt{1+\beta}}\chi \qquad \beta = 6\xi^2 v^2 / M_P^2$$

The decoupling effect

- The couplings of the Higgs boson to particles of the SM are rescaled! E.g.

$$y h \bar{\psi} \psi \to \frac{y}{\sqrt{1+\beta}} \chi \bar{\psi} \psi$$

- For a large nonminimal coupling, the Higgs boson decouples from the Standard Model:

$$\xi^2 \gg M_P^2 / v^2 \simeq 10^{32}$$

The decoupling effect

- The decoupling can also be seen in the Jordan frame:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

$$\mathcal{L}^{(2)} = -\frac{M^2 + \xi v^2}{8} \left(h^{\mu\nu}\Box h_{\mu\nu} + 2\partial_\nu h^{\mu\nu}\partial^\rho h_{\mu\rho} - 2\partial_\nu h^{\mu\nu}\partial_\mu h^\rho_\rho - h^\mu_\mu \Box h^\nu_\nu\right) + \frac{1}{2}(\partial_\mu h)^2 + \xi v(\Box h^\mu_\mu - \partial_\mu\partial_\nu h^{\mu\nu})h.$$

$$h = \frac{1}{\sqrt{1+\beta}}\chi,$$

$$h_{\mu\nu} = \frac{1}{M_P}\tilde{h}_{\mu\nu} - \frac{2\xi v}{M_P^2\sqrt{1+\beta}}\bar{g}_{\mu\nu}\chi.$$

same renormalization factor!

Bound on the nonminimal coupling from the LHC

- The LHC experiments produce fits to the data assuming that all Higgs boson couplings are modified by a single parameter (arXiv:1209.0040 [hep-ph]):

$$\kappa = 1/\sqrt{1+\beta}$$

- In the narrow width approximation, one finds:

$$\begin{aligned}\sigma(ii \to h \to ff) &= \sigma(ii \to h) \cdot \mathrm{BR}(h \to ff) \\ &= \kappa^2\, \sigma_{\mathrm{SM}}(ii \to h) \cdot \mathrm{BR}_{\mathrm{SM}}(h \to ff).\end{aligned}$$

Bound on the nonminimal coupling from the LHC

- Current LHC data allows to bound

$$\mu = \sigma/\sigma_{\rm SM} = \quad 1.4 \pm 0.3 \qquad \text{ATLAS}$$

$$0.87 \pm 0.23 \qquad \text{CMS}$$

- Combining these two bounds one gets:

$$\mu = 1.07 \pm 0.18$$

- which excludes

$$|\xi| > 2.6 \times 10^{15} \text{ at the } 95\% \text{ C.L.}$$

Atkins & xc, PRL 110 (2013) 051301

Bound on the nonminimal coupling from the LHC

- At a 14 TeV LHC with an integrated luminosity of 300 fb^{-1}, could lead to an improved bound on the nonminimal coupling:

$$|\xi| < 1.6 \times 10^{15}$$

- while an ILC with a center of mass energy of 500 GeV and an integrated luminosity of 500 fb^{-1}, could give

$$|\xi| < 4 \times 10^{14}$$

- It seems tough to push the bound below this limit within the foreseeable future.

Other dimensionless couplings: What do experiments tell us?

- In 1977, Stelle has shown that one obtains a modification of Newton's potential at short distances from R^2 terms

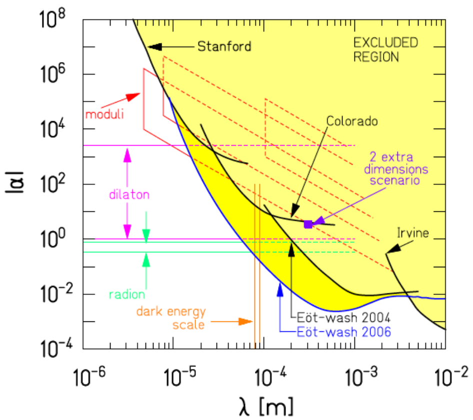
$$\Phi(r) = -\frac{Gm}{r}\left(1 + \frac{1}{3}e^{-m_0 r} - \frac{4}{3}e^{-m_2 r}\right) \qquad m_0^{-1} = \sqrt{32\pi G\left(3c_1 - c_2\right)}$$

$$m_2^{-1} = \sqrt{16\pi G c_2}$$

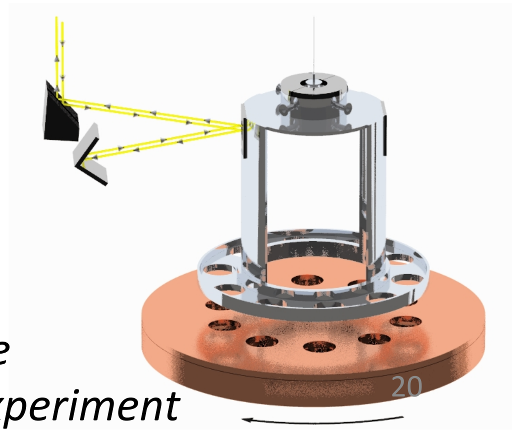
$$V(r) = -G_N \frac{m_1 m_2}{r}\left[1 + \alpha \exp\left(-r/\lambda\right)\right]$$

c_1 and $c_2 < 10^{61}$

xc, Hsu and Reeb (2008)



NB: Bound has improved by 10 order of magnitude since Stelle's paper!



Schematic drawing of the Eöt-Wash Short-range Experiment

Can better bounds be obtained in astrophysics?

- Bounds on Earth are obtained in weak curvature, binary pulsar systems are probing high curvature regime.

- Approximation: Ricci scalar in the binary system of pulsars by G M/(r^3c^2) where M is the mass of the pulsar and r is the distance to the center of the pulsar.

- But: if the distance is larger than the radius of the pulsar, then the Ricci scalar vanishes. This is a rather crude estimate.

Can better bounds be obtained in astrophysics?

- Let me be optimistic and assume one can probe gravity at the surface of the pulsar. I take r=13.1km and M=2 solar masses.

- I now request that the R^2 term should become comparable to the leading order Einstein-Hilbert term ($1/2\, M_P^2 R$)

- One could reach bounds of the order of 10^{78} only on c_1 or c_2

- Such limits are obviously much weaker that those obtained on Earth.

What do we know about $M_\star$?

(energy scale up to which one trusts the effective theory)

First data: black hole formation

Second: perturbative unitarity considerations

Black Holes as probe of strong gravity

- Formation of small black holes in the collisions of particles would be a signal of strong gravity.

- LHC

- Cosmic rays

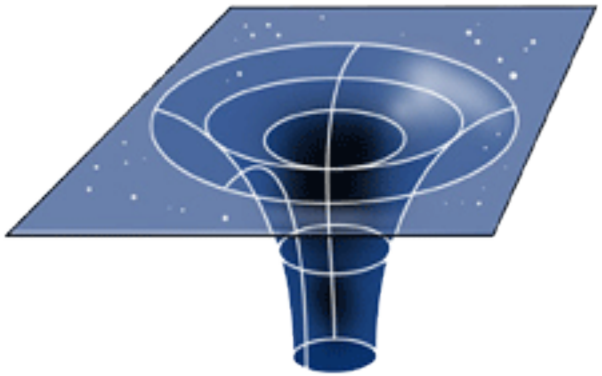
A brief review on the formation of black holes

When does a black hole form?

This is well understood in general relativity with symmetrical distribution of matter:

$$c^2 d\tau^2 = \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{r_s}{r}} - r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$r_s = \frac{2GM}{c^2}$$



But, what happens in particle collisions at extremely high energies?

Small black hole formation

(in collisions of particles)

- In trivial situations (e.g. vacuum), one can solve explicitly Einstein's equations e.g. Schwarzschild metric.
- In more complicated cases one can't solve Einstein equations exactly and one needs some other criteria.
- Hoop conjecture (Kip Thorne): if an amount of energy E is confined to a ball of size R, where R < E, then that region will eventually evolve into a black hole.

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- In trivial situations (e.g. vacuum), one can solve explicitly Einstein's equations e.g. Schwarzschild metric.
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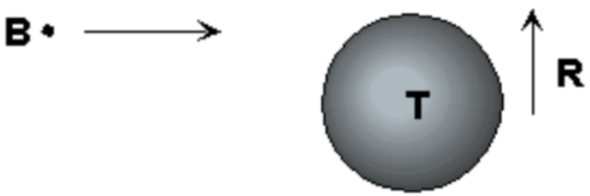
- Hoop conjecture (Kip Thorne): if an amount of energy E is confined to a ball of size R, where R < E, then that region will eventually evolve into a black hole.
- Cross-section for semi-classical BHs (closed trapped surface constructed by Penrose; D'Eath & Payne; Eardley & Giddings):

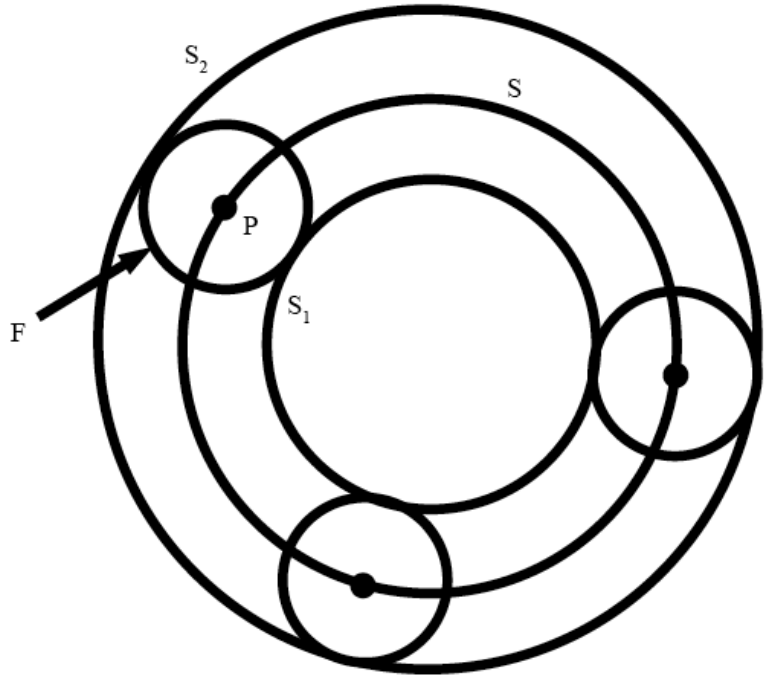
$$\hat{\sigma} \approx \pi r_s^2 \qquad r_s(M_{\rm BH}) = \frac{1}{M_D}\left[\frac{M_{\rm BH}}{M_D}\right]^{\frac{1}{1+n}}\left[\frac{2^n \pi^{(n-3)/2}\Gamma\left(\frac{n+3}{2}\right)}{n+2}\right]^{\frac{1}{1+n}}$$



$$r_s = \frac{2GM}{c^2}$$

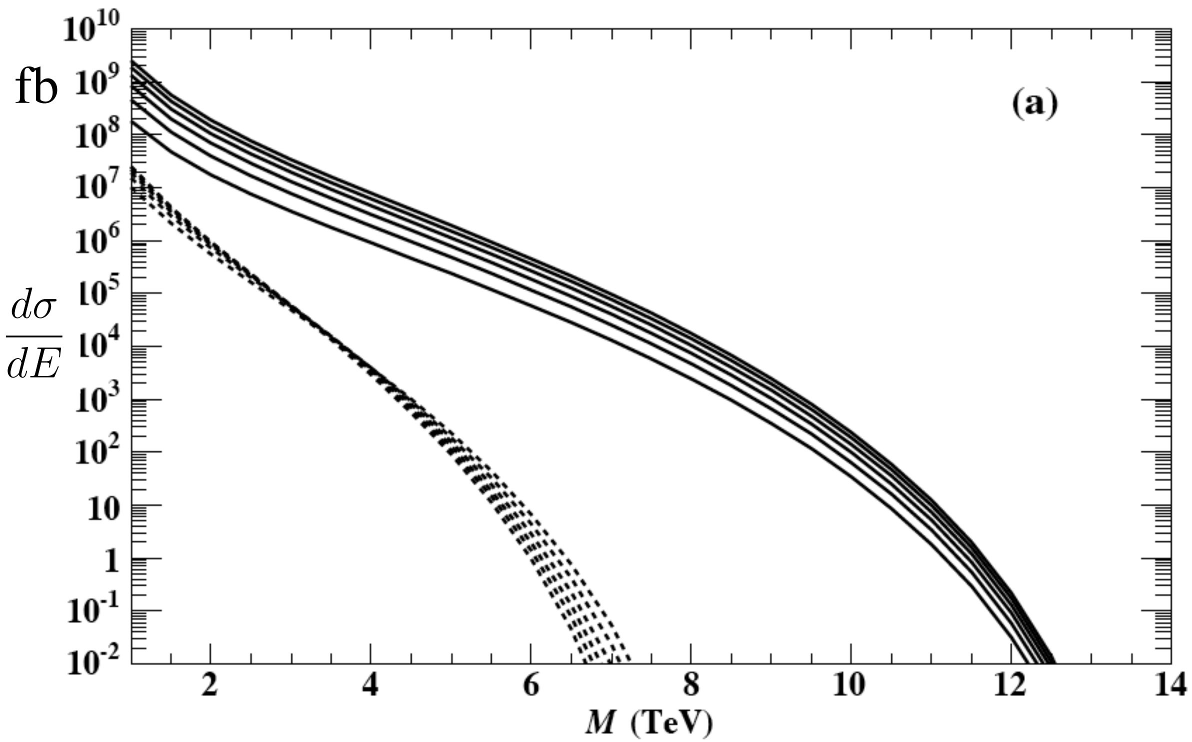
The cross section for point-like particles colliding with a sphere is just the area of the sphere projected onto the transverse plane, that is, a circular disk of radius R.

- A CTS is a compact spacelike two-surface in space-time such that outgoing null rays perpendicular to the surface are not expanding.



- At some instant, the sphere S emits a flash of light. At a later time, the light from a point P forms a sphere F around P, and the envelopes S_1 and S_2 form the ingoing and outgoing wavefronts respectively. If the areas of both S_1 and S_2 are less than of S, then S is a closed trapped surface.

Small BHs @ LHC
(studied by Anchordoqui et al. and many other people)



$$\sigma \sim \frac{1}{M_D^2}\left(\frac{\sqrt{s}}{M_D}\right)^{\frac{2}{1+n}}$$

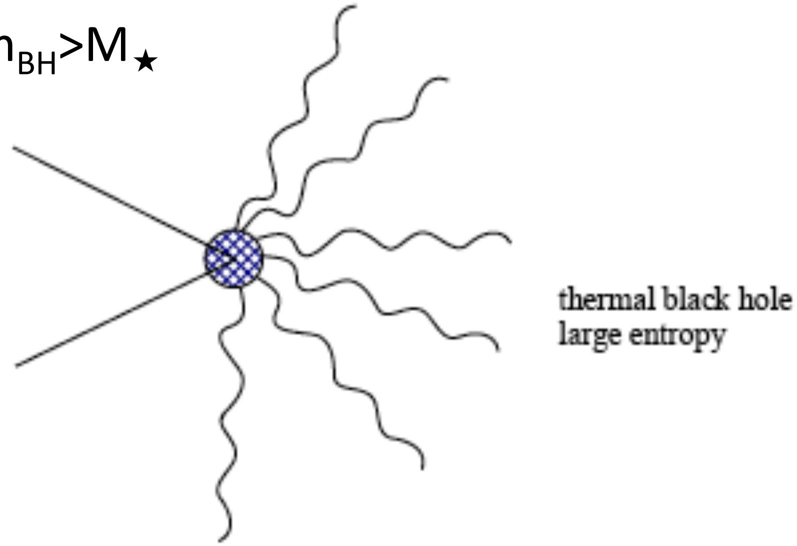
For partons, σ increases with energy but note that PDFs go so fast to zero that they dominate. In other words quantum black holes dominate!

σ(pp->BH+X), M_D=1 TeV

This shows the significance of the inelasticity in BH production

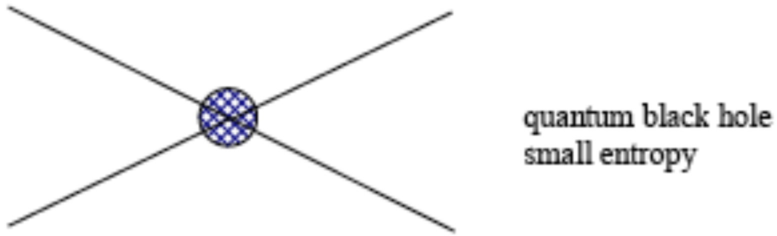
Semi-classical (thermal) versus quantum black hole: calculate the entropy!

$m_{BH} > M_\star$



$$S = \frac{1+n}{2+n} \frac{M_{BH}}{T_{BH}}$$

$m_{BH} \sim M_\star$



$$\langle N \rangle \propto \left(\frac{M_{BH}}{M_\star} \right)^{\frac{n+2}{n+1}}$$

Keep in mind that the CTS construction only works for $m_{BH} \gg M_\star$

Assumptions on Quantum Black Holes decays

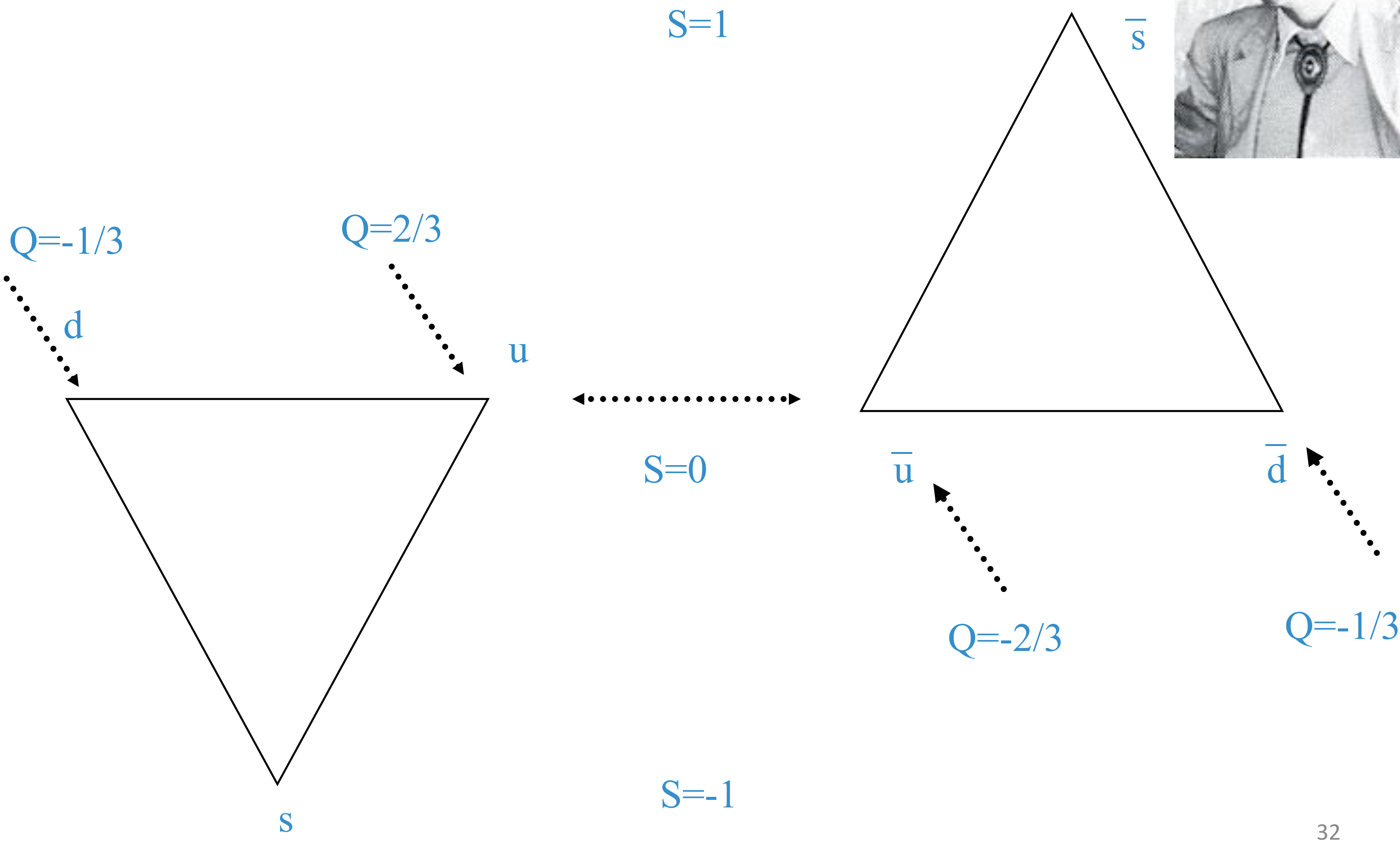
- Gauge invariance is preserved (conservation of $U(1)$ and $SU(3)_C$ charges)

- Global charges can be violated. Lepton flavor is not conserved. Lorentz invariance could be broken or not.

- Gravity is democratic.

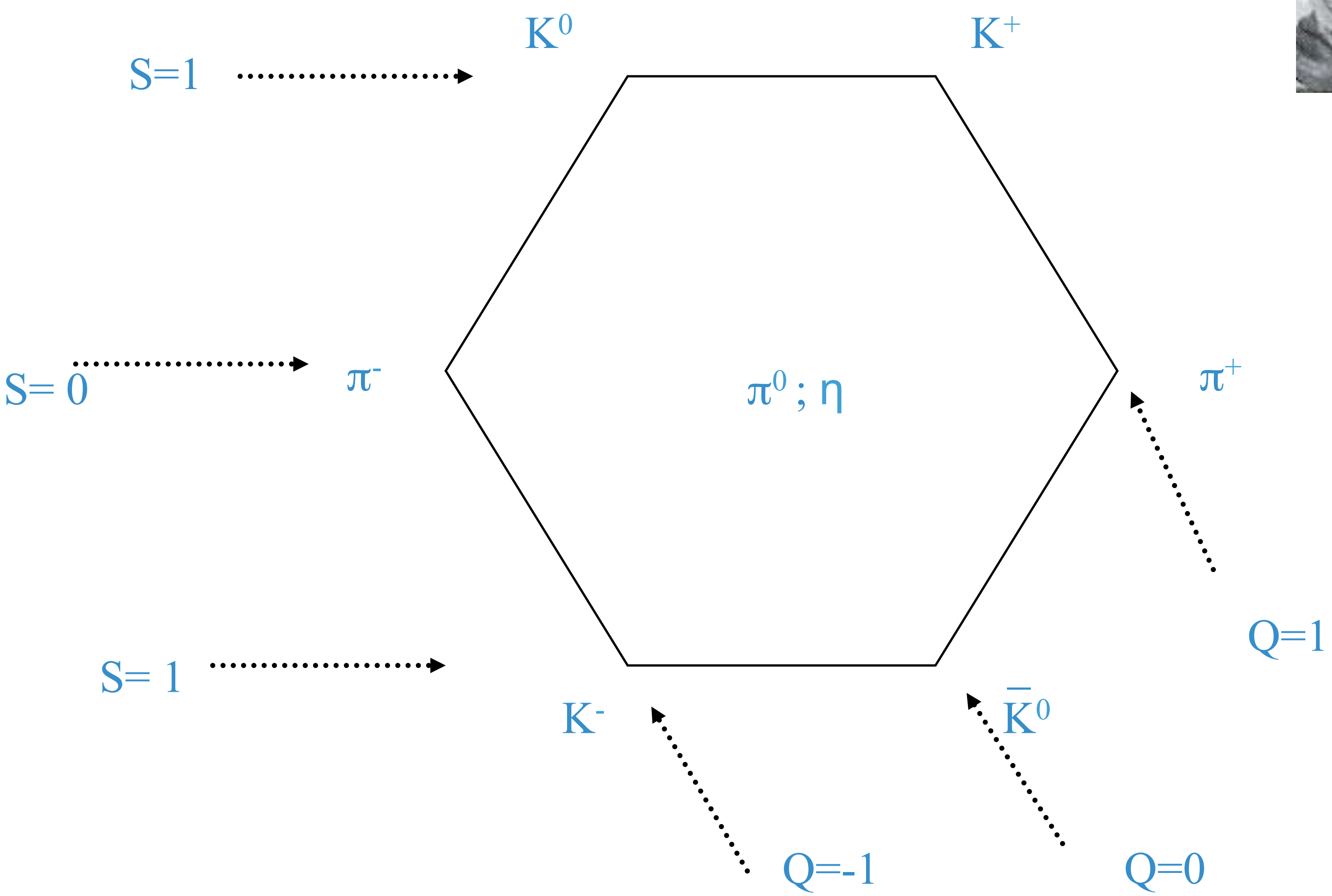
- We can think of quantum black holes as gravitational bound states.

The Quark Model (1964)

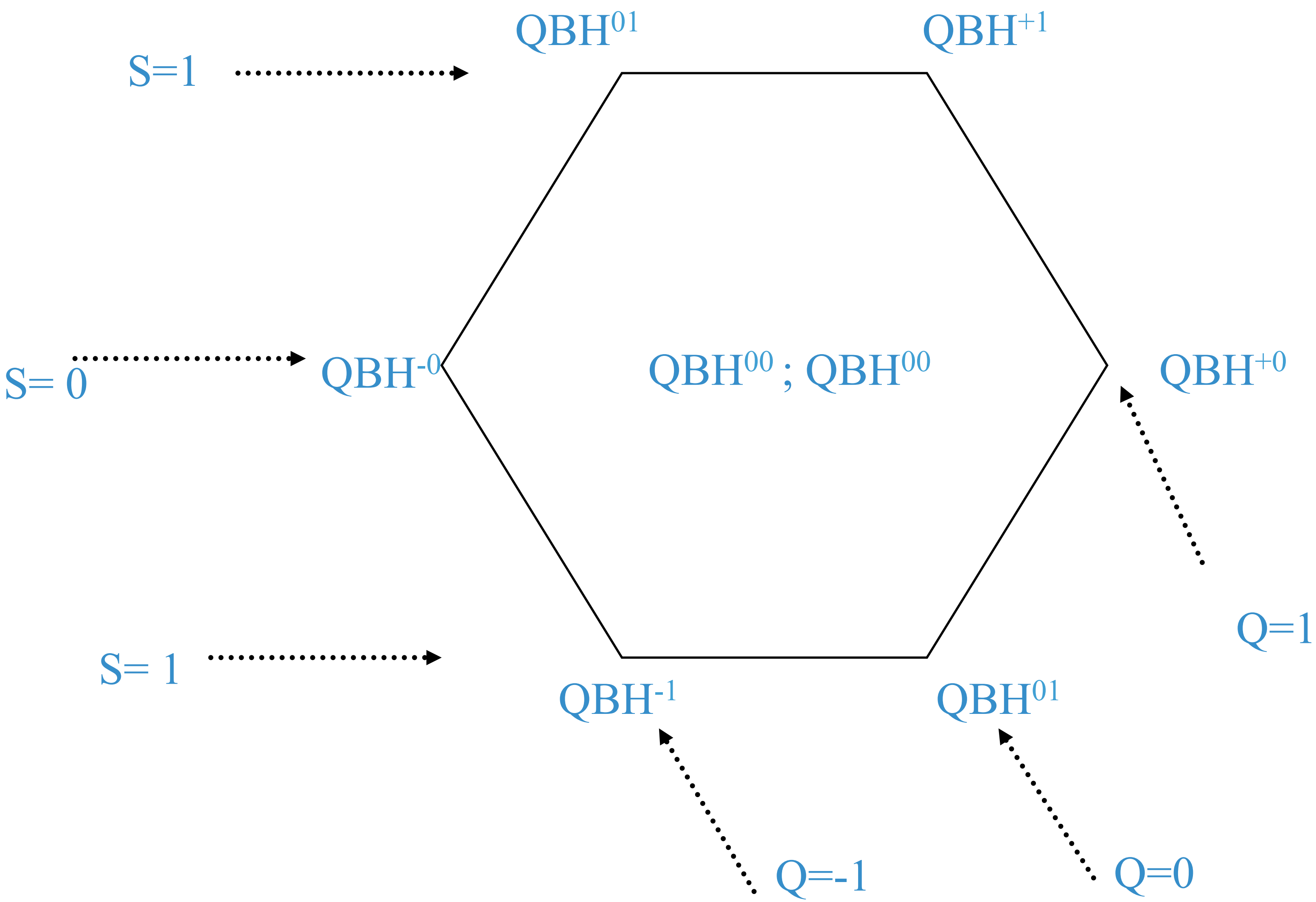


THE EIGHTFOLD WAY (1961)

The Meson Octet



The Quantum Black Hole Octet



QCD for Quantum Black Holes

XC, W. Gong & S. Hsu

- Quantum Black Holes are classified according to representations of $SU(3)_C$ and $U(1)$.
- For LHC the following Quantum Black Holes are relevant:

$$\mathbf{3} \times \overline{\mathbf{3}} = \mathbf{8} + \mathbf{1}$$
$$\mathbf{3} \times \mathbf{3} = \mathbf{6} + \overline{\mathbf{3}}$$
$$\mathbf{3} \times \mathbf{8} = \mathbf{3} + \overline{\mathbf{6}} + \mathbf{15}$$
$$\mathbf{8} \times \mathbf{8} = \mathbf{1}_S + \mathbf{8}_S + \mathbf{8}_A + \mathbf{10} + \overline{\mathbf{10}}_A + \mathbf{27}_S$$

- They can have non-integer QED charges.
- They can carry a $SU(3)_C$ charge.

Bounds (orders of magnitude) on $M_\star$

n	1	2	3	4	5	6
Gravity exp.	10^7 km	0.2mm				0.1 fm
LEP2/ Tevatron		1 TeV	1 TeV	1 TeV	1 TeV	1 TeV
LHC		~5 TeV	~5 TeV	~5 TeV	~5 TeV	~5 TeV
Astro. SN +NS		10^3 TeV	10^2 TeV	5 TeV	none	none
Cosmic rays	1 TeV	1 TeV	1 TeV	1 TeV	1 TeV	1 TeV

Summary of current status of GR coupled to SM

- We can describe any theory of quantum gravity below the Planck scale using effective field theory techniques:

$$S = \int d^4x \sqrt{-g} \left[\left(\frac{1}{2} M^2 + \xi H^\dagger H \right) \mathcal{R} - \Lambda_C^4 + c_1 \mathcal{R}^2 + c_2 \mathcal{R}_{\mu\nu} \mathcal{R}^{\mu\nu} + \mathcal{L}_{SM} + \mathcal{O}(M_\star^{-2}) \right]$$

- Planck scale $(M^2 + \xi v^2) = M_P^2$ $M_P = 2.4335 \times 10^{18}$ GeV
- $\Lambda_C \sim 10^{-12}$ GeV; cosmological constant.
- $M_\star >$ few TeVs from QBH searches at LHC and cosmic rays.
- Dimensionless coupling constants ξ, c_1, c_2
  - c_1 and $c_2 < 10^{61}$ [xc, Hsu and Reeb (2008)]

    R^2 inflation requires $c_1 = 5 \times 10^8$ (Faulkner et al. astro-ph/0612569]).
  - $\xi < 2.6 \times 10^{15}$ [xc & Atkins, 2013]

    Higgs inflation requires $\xi \sim 10^4$.

What do we know about $M_{\star}$?

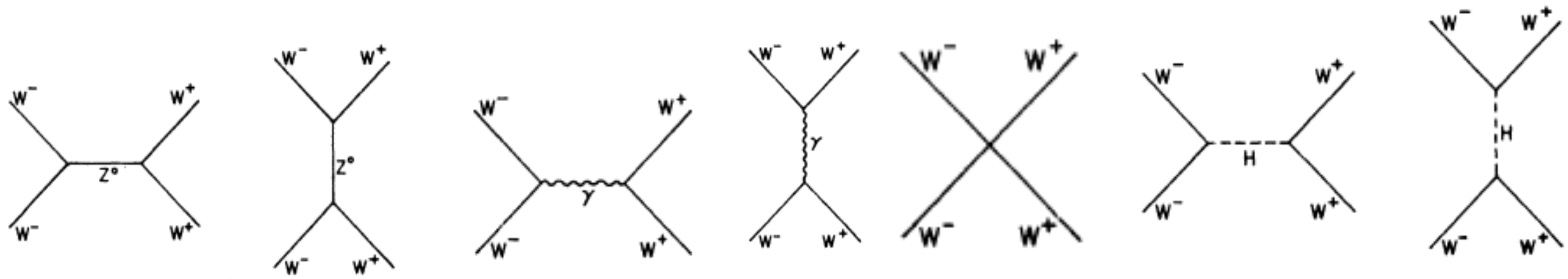
(energy scale up to which one trusts the effective theory)

First data: black hole formation

Second: perturbative unitarity considerations

Unitarity in quantum field theory

- Follows from the conservation of probability in quantum mechanics.
- Implies that amplitudes do not grow too fast with energy.
- One of the few theoretical tools in quantum field theory to get information about the parameters of the model.
- Well known example is the bound on the Higgs boson's mass in the Standard Model (m<790 GeV).



Let us consider gravitational scattering of the particles included in that model (s-channel, we impose different in and out states) (Han & Willenbrock 2004)

$\to$	$s'\bar{s}'$	$\psi'_+\bar{\psi}'_-$	$\psi'_-\bar{\psi}'_+$	$V'_+V'_-$	$V'_-V'_+$
$s\bar{s}$	$-2\pi G_N s(1/3 d^2_{0,0} - 1/3(1+12\xi)^2 d^0_{0,0})$	$-2\pi G_N s\sqrt{1/3}\, d^2_{0,1}$	$-2\pi G_N s\sqrt{1/3}\, d^2_{0,-1}$	$-4\pi G_N s\sqrt{1/3}\, d^2_{0,2}$	$-4\pi G_N s\sqrt{1/3}\, d^2_{0,-2}$
$\psi_+\bar{\psi}_-$	$-2\pi G_N s\sqrt{1/3}\, d^2_{1,0}$	$-2\pi G_N s d^2_{1,1}$	$-2\pi G_N s d^2_{1,-1}$	$-4\pi G_N s\, d^2_{1,2}$	$-4\pi G_N s\, d^2_{1,-2}$
$\psi_-\bar{\psi}_+$	$-2\pi G_N s\sqrt{1/3}\, d^2_{-1,0}$	$-2\pi G_N s d^2_{-1,1}$	$-2\pi G_N s d^2_{-1,-1}$	$-4\pi G_N s 2\, d^2_{-1,2}$	$-4\pi G_N s 2\, d^2_{-1,-2}$
V_+V_-	$-4\pi G_N s\sqrt{1/3}\, d^2_{2,0}$	$-4\pi G_N s\, d^2_{2,1}$	$-4\pi G_N s\, d^2_{2,-1}$	$-8\pi G_N s\, d^2_{2,2}$	$-8\pi G_N s\, d^2_{2,-2}$
V_-V_+	$-4\pi G_N s\sqrt{1/3}\, d^2_{-2,0}$	$-4\pi G_N s\, d^2_{-2,1}$	$-4\pi G_N s\, d^2_{-2,-1}$	$-8\pi G_N s\, d^2_{-2,2}$	$-8\pi G_N s\, d^2_{-2,-2}$

$$\mathcal{A} = 16\pi \sum_J (2J+1) a_J d^J_{\mu,\mu'} \qquad |\mathrm{Re}\; a_J| \leq 1/2$$

Let us look at J=2 partial wave

$$a_2 = -\frac{1}{320\pi}\frac{s}{\bar{M}_P^2}N \qquad N = 1/3N_s + N_\psi + 4N_V$$

One gets the bound:

$$E^*_{\rm CM} \leq \bar{M}_P\sqrt{\frac{160\pi}{N}}$$

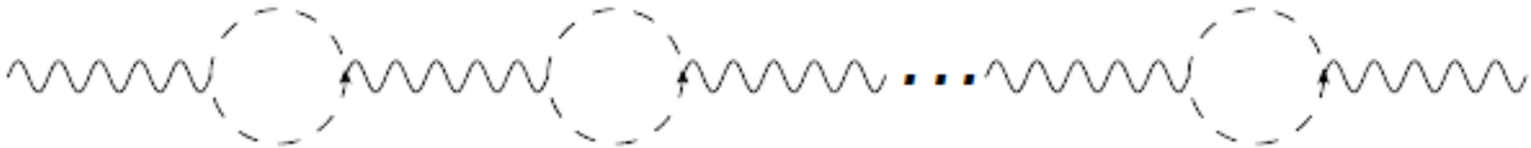
For large *N*, unitarity can be violated well below the Planck mass.

From the J=0 partial wave, one gets

$$\Lambda \simeq \bar{M}_P/\xi$$

Self-healing of unitarity

- Aydemir, Anber & Donoghue argued that the effective theory heals itself.
- In the case of linearized gravity coupled to the SM, resum:



- in the large N limit, keeping NG_N small. One obtains a resummed graviton propagator

$$iD^{\alpha\beta,\mu\nu}(q^2) = \frac{i\left(L^{\alpha\mu}L^{\beta\nu} + L^{\alpha\nu}L^{\beta\mu} - L^{\alpha\beta}L^{\mu\nu}\right)}{2q^2\left(1 - \frac{NG_N q^2}{120\pi}\log\left(-\frac{q^2}{\mu^2}\right)\right)}$$

$$N = N_s + 3N_f + 12N_V$$

$$L^{\mu\nu}(q) = \eta^{\mu\nu} - q^\mu q^\nu / q^2$$

- One can check explicitly

$$|A_{dressed}|^2 = \mathrm{Im}(A_{dressed})$$

Self-healing of unitarity nonminimal coupling

- One can also resum the infinite series of 1-loop polarization diagrams

- In the large ξ and N limits but keeping N ξG_N small, I get

$$iD^{\alpha\beta\mu\nu}_{dressed} = -\frac{i}{2s}\frac{L^{\alpha\beta}L^{\mu\nu}}{\left(1-\frac{sF_1(s)}{2}\right)}$$

$$F_1(q^2) = -N_s G_N \xi^2 \log\left(\frac{-q^2}{\mu^2}\right)$$

- The dressed amplitude fulfills exactly

$$|A_{dressed}|^2 = \mathrm{Im}(A_{dressed})$$

XC & Casadio 2014

Comments

- In linearized GR, the effective theory self-heals itself.
- In the large N limit one finds poles in the resummed graviton propagator: sign of strong interaction.
- The positions of these poles depend on the number of fields
- One finds

$$
\begin{aligned}
q_1^2 &= 0, \\
q_2^2 &= \frac{1}{G_N N} \frac{120\pi}{W\left(\frac{-120\pi M_P^2}{\mu^2 N}\right)}, \\
q_3^2 &= (q_2^2)^*,
\end{aligned}
$$

Comments

- It is tempting to interpret these poles as black hole precursors.
- In the SM

$$N_s = 4,\ N_f = 45,\ \text{and}\ N_V = 12$$

- We thus find

$$(7 - 3i) \times 10^{18}\ \text{GeV}\ \text{and}\ (7 + 3i) \times 10^{18}\ \text{GeV}$$

- The first one corresponds to a state with mass $p_0^2 = (m - i\Gamma/2)^2$

$$7 \times 10^{18}\ \text{GeV}$$

- And width

$$6 \times 10^{18}\ \text{GeV}$$

Comments

- Our interpretation is similar to the sigma-meson case which can be identified as the pole of a resummed scattering amplitude in the large N limit of chiral perturbation theory.

- This resummed amplitude is an example of self-healing in chiral perturbation theory.

- In low energy QCD, the position of the pole does correspond to the correct value of the mass and width of the sigma-meson.

Comments

- Note that the 2^{nd} pole has the wrong sign for particle: it is a ghost

$$(7+3i)\times 10^{18}\ \mathrm{GeV}$$

- Acausal effects: connection to black hole information paradox? Could be canceled by e.g. Lee and Wick's mechanism.

- Non local effects

$$S=\int d^4x\sqrt{g}\left[R\log\left(\frac{\Box}{\mu^2}\right)R\right]$$

- Can these effects soften singularities?

Comments

- With our interpretation in mind, an interesting picture emerges.
- Self-healing in the case of gravitational interactions implies unitarization of quantum amplitudes via quantum black holes.
- As the center of mass energy increases so does the mass of the black hole and it becomes more and more classical.
- This is nothing but classicalization.
- What we call Planck scale (first QBH mass/cut off for the EFT) is now a dynamical quantity which depends on the number of fields.
- The effective theory certainly breaks down at the Planck scale.
- Self-healing makes the link between several concepts that had been proposed previously.

Comments

- Let's think about perturbative unitarity again.

- We are taught that a breakdown of perturbative unitarity is a sign of new physics or strong dynamics.

- In the case of quantum gravity in the large N, we have identified the strong dynamics as quantum black holes: this is not a surprise.

- More surprising is the case of a large nonminimal coupling of scalars to R, here we found a resummed propagator that does not have poles beyond the one at $q^2=0$.

- Unitarity is restored by the self-healing mechanism without new physics or strong dynamics.

Gravity leads to non-local effects in Matter

XC, Croon & Fritz (2015)

- Let's reconsider the resummed graviton propagator

$$iD^{\alpha\beta,\mu\nu}(q^2) = \frac{i\left(L^{\alpha\mu}L^{\beta\nu} + L^{\alpha\nu}L^{\beta\mu} - L^{\alpha\beta}L^{\mu\nu}\right)}{2q^2\left(1 - \frac{NG_N q^2}{120\pi}\log\left(-\frac{q^2}{\mu^2}\right)\right)}$$

- Using this propagator we can now calculate the dressed amplitude for the gravitational scattering of 2 scalar fields.
- The tree-level amplitude has been known for a long time:

$$\begin{aligned} A_{tree} \quad &= \quad 16\pi G\left(m^4\left(\frac{1}{s}+\frac{1}{t}+\frac{1}{u}\right) + \frac{1}{2s}(2m^2+t)(2m^2+u)\right. \\ &\left. + \frac{1}{2t}(2m^2+s)(2m^2+u) + \frac{1}{2u}(2m^2+s)(2m^2+t)\right) \end{aligned}$$

Gravity leads to non-local effects in Matter

- Let me rewrite the dressed propagator as

$$iD^{\alpha\beta,\mu\nu}(q^2) = \frac{P^{\alpha\beta,\mu\nu}(q^2)}{1+f(q^2)}, \qquad f(q^2) = -\frac{NG_N q^2}{120\pi}\log\left(-\frac{q^2}{\mu^2}\right).$$

- We find the Taylor expended dressed amplitude:

$$A_{dressed} = A_{tree} + A^{(1)} + \dots$$

$$\begin{aligned}A^{(1)} = \frac{2}{15}G_N^2 N\Bigg(m^4\left(\log\left(-\frac{stu}{\mu^6}\right)\right) &+ \log\left(-\frac{s}{\mu^2}\right)(2m^2+t)(2m^2+u) + \log\left(-\frac{t}{\mu^2}\right)(2m^2+s)(2m^2+u) \\ &+ \log\left(-\frac{u}{\mu^2}\right)(2m^2+s)(2m^2+t)\Bigg).\end{aligned}$$

Gravity leads to non-local effects in Matter

- It is easy to see that $A^{(1)}$ can be obtained from this effective operator:

$$O_8 = \frac{2}{15} G_N^2 N \left(\partial_\mu \phi(x) \partial^\mu \phi(x) - m^2 \phi(x)^2\right) \log\left(-\frac{\Box}{\mu^2}\right) \left(\partial_\nu \phi(x) \partial^\nu \phi(x) - m^2 \phi(x)^2\right)$$

- This is a non-local operator, we need to make sense of the log term to obtain a causal theory (Espiru et al. (2005), Donoghue &El-Menoufi (2014) and Barvinsky et al in the 80's.)

$$\begin{aligned} S &= \int d^4x d^4y \sqrt{-g} \left(\frac{1}{16\pi G_N} R - \frac{1}{2} \partial_\mu \phi(x) \partial^\nu \phi(x) + \frac{m^2}{2} \phi^2 \right. \\ &\quad + \left(\frac{2}{15} G_N^2 N \right) \times \\ &\left. \left(\left(\partial_\mu \phi(x) \partial^\mu \phi(x) + m^2 \phi(x)^2\right) \int d^4y \sqrt{-g(y)} \langle x | \log\left(-\frac{\Box}{\mu^2}\right) | y \rangle \left(\partial_\nu \phi(y) \partial^\nu \phi(y) - m^2 \phi(y)^2\right) \right) \right) \end{aligned}$$

Gravity leads to non-local effects in Matter

- One can define the interpolating function:

$$\mathcal{L}(x,y) = \langle x|\log\left(-\frac{\Box}{\mu^2}\right)|y\rangle$$

- which can be evaluate $\log(x) \approx -1/\epsilon + \ x^\epsilon/\epsilon$

$$\begin{aligned} -\langle x|\frac{1}{\epsilon}|y\rangle + \langle x|\frac{(\Box/\mu^2)^\epsilon}{\epsilon}|y\rangle &= -\frac{1}{\epsilon}\delta(x-y) + \frac{1}{\epsilon}\frac{2\pi^2}{\mu^{2\epsilon}}\int d^4k k^{2+2\epsilon}\frac{1}{|x-y|}J_1(k|x-y|) \\ &\sim -\frac{1}{\epsilon}\delta(x-y) - \frac{8\pi^2}{\mu^{2\epsilon}}\frac{1}{|x-y|^{4+2\epsilon}}, \end{aligned}$$

- For a purely time-dependent problem one has

$$\mathcal{L}(t,t') = -2\lim_{\epsilon\to 0}\left(\frac{t-t'-\epsilon}{t-t'} + \delta(t-t')(\log(\mu\epsilon)+\gamma\right)$$

Gravity leads to non-local effects in Matter

- We have seen that the non-local effects observed in gravity feeds back into matter.

- This is compatible with our interpretation of the poles of the resummed propagators as quantum black holes (black hole precursors) which are extended objects.

- The new higher dimensional operators have an approximate shift symmetry

$$\phi \to \phi + c, \text{ where } c \text{ is a constant}$$

- which is broken explicitly by the mass of the scalar field.

- This is interesting for models of inflation.

Gravity leads to non-local effects in Matter

- Are there any observational consequences of this short distance non-locality?
- The effect is suppressed by powers of the Planck scale, one can see that it leads to a small non-Gaussianities even for a single scalar inflation model.
- However the effect is too small to be observable.
- Let's considering the following Lagrangian

$$
\begin{aligned}
L(x) \;&=\; X + \frac{m^2}{2}\phi^2(x) + \\
&+\frac{8}{15}G_N^2 N\left(X(x) + \frac{m^2}{2}\phi^2(x)\right)\int d^4y\sqrt{-g(y)}\mathcal{L}(x,y)\left(X(y) + \frac{m^2}{2}\phi^2(y)\right)
\end{aligned}
$$

$$
X(x) = -1/2\partial_\mu\phi(x)\partial^\mu\phi(x) \qquad X(y) = -1/2\partial_\mu\phi(y)\partial^\mu\phi(y)
$$

Gravity leads to non-local effects in Matter

- We can calculate the speed of sound:

$$c_s^2 = \frac{L(x)_{,X(x)}}{L(x)_{,X(x)} + 2X(x)L(x)_{,X(x)X(x)}} \approx 1 - \frac{32}{15}X(x)G_N^2 N$$

- which remarkably to leading order does not depend on the specific form of the nonlocal function.

- GR coupled to a single scalar field thus predict a small amount from non-Gaussianity, but with a speed of sound very close to one.

- Non-locality is a generic feature of quantum field theory coupled to GR.

QBHs in Effective Field Theory

- We have seen that the QBHs like objects naturally appear in the context of EFT.

- These QBHs have a mass close to the Planck mass and a width of the same order.

- What if their width was 0, i.e. what if they were stable objects?

- We can still use the classification we have introduced for quantum black holes with a finite width for these objects.

What if the lightest black holes are stable?

- Remnants are usually dismissed for two reasons:
  - They could be pair produced at low energy experiments
  - They could impact massively low energy experiments via loops.
- Both arguments do not necessarily apply!
  - First one is actually just wrong for obvious reasons, there is a step function in energy for BH production:

$$\sigma = \pi r_S^2 \theta(s - M_{BH}^2)$$

  - The second argument (loop) deserves more scrutiny.

What if the lightest black holes are stable?

- Let me consider a spin-0 quantum black hole in a loop

$$I = \int_0^\Lambda d^4p \frac{1}{p^2 - M_{BH}^2 + i\epsilon}$$

- For momenta smaller than the BH mass, this integral behaves as

$$\Lambda^4 / M_{BH}^2$$

- Since we are looking at low energy experiments, the cutoff is much smaller than the BH mass.
- For one QBH the effect is small, however…

What if the lightest black holes are stable?

- The spectrum of quantum gravity contains potentially a large number of states. The effect can thus be large.
- For a continuous mass spectrum, one has

$$I = \int_{M_{BH,l}}^{M_{BH,h}} \frac{\Lambda^4}{M^2} \rho(M) dM$$

- with a mass density given by $\rho(M) = NM^{-1}$
- where N is the number of QBH states. We thus find

$$I_{continuous} = \int_{M_{BH,l}}^{M_{BH,h}} \frac{\Lambda^4}{M^2} \rho(M) dM \sim \frac{\Lambda^4 (M_{BH,h}^2 - M_{BH,l}^2)}{M_{BH,h}^2 M_{BH,l}^2} N.$$

- which for a large N (infinite) is indeed a large effect.

Remnants and the information paradox

- In the case of remnants as a solution to the information paradox, it is argued that there is a large multiplicity factor ***M*** arising from a sum over all the possible quantum numbers of the black holes contributing in the loop.

- This is the standard argument against the resolution of the black hole information paradox based on remnants.

- Let's revisit this question.

QBHs in loops

- Let's look again at

$$I_{continuous} = \int_{M_{BH,l}}^{M_{BH,h}} \frac{\Lambda^4}{M^2}\rho(M)dM \sim \frac{\Lambda^4(M_{BH,h}^2 - M_{BH,l}^2)}{M_{BH,h}^2 M_{BH,l}^2}N.$$

- What should we take for

$$M_{BH,h} \text{ and } M_{BH,l}$$

  ?

- The lightest black hole produced cannot have a mass below the Planck mass.

$$M_{BH,l} \sim M_P$$

- On the other hand, we know that black holes with mass 5 to 20 times M_P are semi-classical objects.
- In contrast to QBHs, they are thus unlike particles which typically only couple to a few other particles. I thus identify

$$M_{BH,h} \text{ with 5-20 } M_P$$

QBHs in loops

- We thus find

$$I_{continuous} = \frac{\Lambda^4}{M_P^2} N \mathcal{M}$$

- Since $\Lambda \ll$ MP as we are interested in low energy experiments, the number of state N and the potentially large multiplicity ***M*** are the source of potential large contributions to low energy physics observables.

- Is there a way out?

Quantization of the mass spectrum

- An obvious solution to the large (actually infinite) factor N is that the spectrum of quantum black holes with masses up to 5-20 M_P is quantized.

- This is perfectly reasonable as we have strong arguments in favor of a quantization of space-time in terms of the Planck scale.

- If we assume that the mass spectrum is quantized in terms of M_P then $N = 5 - 20$ and is not a large factor.

What about the large multiplicity?

- That's difficult to answer without a theory of quantum gravity.
- QBHs must carry local gauge charges, otherwise they would mediate anomalies.
- However, this leads to a small group theoretical factor in the loop calculation.
- The problem are global charges: the usual assumption is that one can differentiate between two remnants using their global charges.
- There must thus be a large number of remnants if they solve the information paradox
- This is not so clear though. Information (global charges) could be contained within the Schwarzschild radius and thus not observable to an outside observer: from a low energy effective theory there is no way to differentiate the two QBHs carrying different global charges, there are thus one state. This does not lead to a large multiplicity.

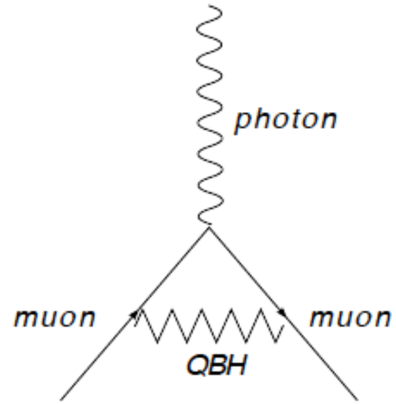
What do experiments tell us?

- Unless fundamental symmetries (e.g. Lorentz or chiral symmetry) are violated by quantum black holes, their effect is not large.

- Let's look at g-2 of the muon, current data allows to probe Λ_{NP} of the order of 2 TeV.

- We expect

$$N\frac{e}{2}\frac{m_\mu}{16\pi^2 \bar{M}_P^2}\bar{\psi}\sigma_{\mu\nu}\psi F^{\mu\nu}$$



- And thus extract the number of QBHs allowed by data

$$N < 16\pi^2 M_P^2/\Lambda_{NP}^2 \sim 10^{32}$$

Remnants as dark matter?

- It is an old idea worth reconsidering given the lack of BSM physics at the LHC.
- Planckian QBHs would be a viable dark matter candidate.
- Very low density: we will never find them, at least with current ideas used to look for dark matter.
- As we have seen, they are compatible with all of low energy data.
- The problem is that they could be over produced (Barrow, Copeland and Liddle (1992)). The problem is very similar to that of monopoles.
- The details depend on the cosmological scenario.

Grand unification through gravitational effects

[xc, Hsu and Reeb (2008,2010)]

- Generically speaking there are many dimension five operators:

$$\mathcal{L} = \frac{c_i}{4M_{Pl}} H_i^{ab} G^a_{\mu\nu} G^{b\mu\nu}$$

Could be obtained by integrating out e.g. QBHs.

- Modified unification condition:

$$\begin{aligned}\alpha_G &= (1+\epsilon_1)\,\alpha_1(M_X) = (1+\epsilon_2)\,\alpha_2(M_X) \\ &= (1+\epsilon_3)\,\alpha_3(M_X)\ .\end{aligned}$$

$$\epsilon_1 = \frac{\epsilon_2}{3} = -\frac{\epsilon_3}{2} = \frac{\sqrt{2}}{5\sqrt{\pi}} \frac{c\eta}{\sqrt{\alpha_G}} \frac{M_X}{\hat{M}_{\rm Pl}}$$

- Unification without supersymmetry can easily be obtained.
- Unification scale is typically quite high and potentially close to the Planck mass.
- No problem with proton decay.
- Nice feature of non-SUSY unification: avoid Landau pole above the unification scale.

Yukawa couplings

- Dimension 5 terms in SU(5)

Ψ and f are fermion fields in $\mathbf{10}$ and $\mathbf{5}$ respectively
scalar fields in the $\mathbf{24}$ and $\mathbf{5}$ representations

$$\begin{aligned}\mathcal{O}_5 &= \frac{a_1}{\hat{\mu}_\star}\{\phi_{mn}\bar{f}^{mk}H^l_k\Psi^n_l\} \\ &+ \frac{a_2}{\hat{\mu}_\star}\{\phi_{mn}H^{mk}\bar{f}^l{}_k\Psi^n_l\} \\ &+ \frac{a_3}{\hat{\mu}_\star}\varepsilon^{mnpql}\{\Psi_{mn}\Psi_{pq}H_k\phi^k_l\},\end{aligned}$$

- New unification condition:

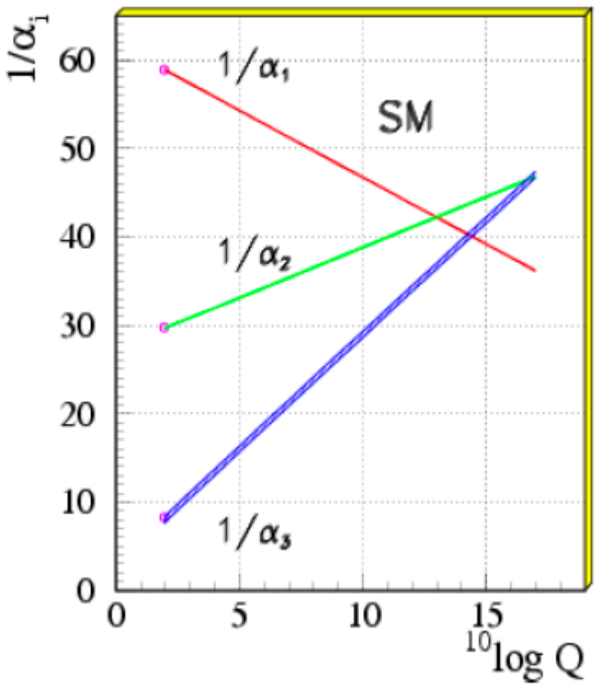
$$m_d(M_X)[1+\frac{3}{2}\zeta_1-\zeta_2] = m_e(M_X)[1+\frac{3}{2}\zeta_1+\frac{3}{2}\zeta_2]$$

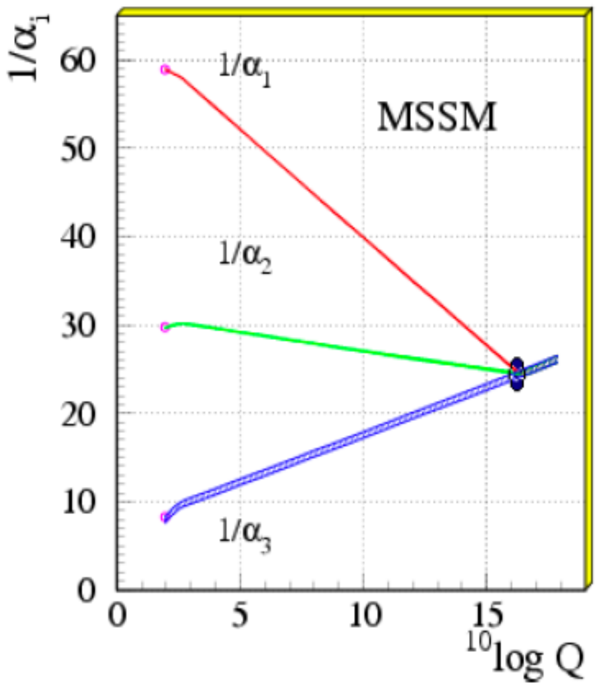
$$\zeta_i = \frac{-2\sqrt{2}}{5G_d g_u}\frac{M_X}{\bar{M}_{Pl}}a_i\eta$$

[xc and Yang (2011)]

Unification of the couplings of the Standard Model?

One of LEP's most impressive result





Standard Model does not work

But the minimal Supersymmetric (SUSY) Standard Model works beautifully

This is not quite correct because of quantum gravity!

Quantum Gravity and GUT

- Quantum gravity can help to unify the gauge couplings and Yukawa couplings.

- It spoils predictions done using low energy data.

- LEP does not favor SUSY unification: Extrapolation from low energy data is too naïve.

- If no BSM is discovered, gravity induced unification should be taken very seriously

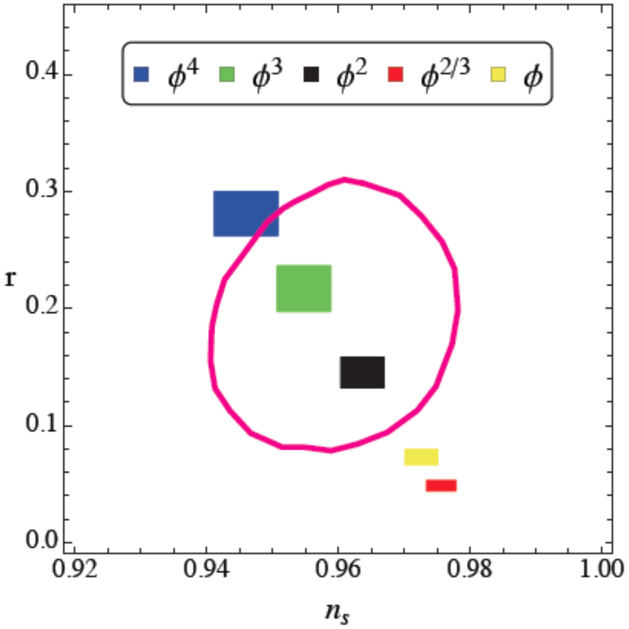
- Impossible to make any prediction without knowing the full details of the unification group and symmetry breaking pattern.

Quantum Gravity and models of inflation

- Effective action for the inflaton:

$$S = \int d^x \sqrt{-g} \left(\frac{\bar{M}_P^2}{2} R + f(\phi) F(R, R_{\mu\nu}) + g^{\mu\nu} \partial_\mu \phi \partial^\nu \phi + V_{ren}(\phi) + \sum_{n=5}^{\infty} c_n \frac{\phi^n}{\bar{M}_P^{n-4}} \right)$$

$$V_{ren} \supset v^3 \phi + m^2 \phi^2 + \lambda_3 \phi^3 + \lambda_4 \phi^4$$



Predictions for various polynomial forms of V_{ren} with $N \in [50, 60]$.
The pink circle corresponds to the 95% CL from BICEP2 (used for illustration only!).

Quantum Gravity and models of inflation

- In inflationary models, one often focuses on one specific term and one sets the remaining Wilson coefficients to zero (or advocate a shift symmetry).

- However in quantum field theory, with the exception of dimension three and four operators higher dimensional operators will be generated by quantum corrections.

- The Wilson coefficients of dimension 3&4 operators can be tiny as seen before, however those of higher dimensional operators are expected to be order unity.

Quantum Gravity effects on chaotic inflation (CI)

- We consider the potential

$$V(\tilde{\phi}) = \bar{M}_P^4 \left(\tilde{m}^2 \tilde{\phi}^2 + c_n \tilde{\phi}^n\right) ,$$

- with

$$\tilde{\phi} = \phi/\bar{M}_P \ , \tilde{m} = m/\bar{M}_P \ .$$

- For illustration let's take the dimension 6 operator

$$c_6 = \alpha_m \tilde{m}^2 \to V(\tilde{\phi}) = \bar{M}_P^4 \tilde{m}^2 \tilde{\phi}^2 \left(1 + \alpha_m \tilde{\phi}^4\right)$$

- Effective theory is valid if

$$|\alpha_m| \tilde{\phi}^4 < 1$$

Quantum Gravity effects on chaotic inflation (CI)

- The higher-dimensional operator term modifies the slow-role conditions:

$$\epsilon = \frac{1}{16\pi}\left(\frac{V'(\tilde{\phi})}{V(\tilde{\phi})}\right)^2 = \frac{1}{4\pi}\frac{1}{\tilde{\phi}^2}\left(\frac{1+3\,\alpha_m\tilde{\phi}^4}{1+\alpha_m\tilde{\phi}^4}\right)^2 = \epsilon_{CI} + \frac{\alpha_m\tilde{\phi}^4}{\pi\tilde{\phi}^2} + \mathcal{O}(\alpha_m\tilde{\phi}^4)^3$$

with the usual CI parameter given by $\epsilon_{CI} \;=\; 1/(4\pi\tilde{\phi}^2)$

- The second slow-roll parameter, which is zero in usual CI, reads

$$\eta = \frac{1}{8\pi}\left(\frac{V''(\tilde{\phi})}{V(\tilde{\phi})} - \frac{1}{2}\left(\frac{V'(\tilde{\phi})}{V(\tilde{\phi})}\right)^2\right) \simeq \frac{5}{2\pi\tilde{\phi}^2}\,(\alpha_m\tilde{\phi}^4).$$

Quantum Gravity effects on chaotic inflation (CI)

- The condition for the end of inflation is modified;

$$\tilde{\phi}_E^2 = \frac{1}{4\pi}\left(1 + \frac{\alpha_m}{4\pi}\right)$$

- The number of e-foldings

$$N = 2\sqrt{\pi}\int_{\tilde{\phi}_E}^{\tilde{\phi}_I} \frac{1}{\sqrt{\epsilon}} = 2\pi\tilde{\phi}_I^2\left(1 - \frac{2\,\alpha_m\tilde{\phi}_I^4}{3}\right) - \frac{1}{2} - \frac{5\,\alpha_m}{48\,\pi^2}$$

- value of the field at the beginning of inflation with with N e-foldings,

$$\tilde{\phi}_N^2 \simeq \tilde{\phi}_{N,CI}^2 + \frac{N^3}{12\pi^3}\alpha_m \simeq \frac{N}{2\pi}\left(1 + \frac{N^2\alpha_m}{6\pi^2}\right) \qquad \tilde{\phi}_{N,CI}^2 = \frac{1+2N}{4\pi}$$

Quantum Gravity effects on chaotic inflation (CI)

- The convergence of the effective theory implies

$$|\alpha_m|\tilde{\phi}_N^4 \simeq \frac{N^2|\alpha_m|}{4\pi^2} \lesssim 1 \to |\alpha_m|^{EFT} \lesssim 2\times 10^{-2}$$

- NB: for values of α_m close to this bound, and negative, cancellations could lead to a value of the field below the Planck mass:

$$\phi < \bar{M}_P \text{ for } N \simeq 60$$

- while there is no simultaneous cancellation in the potential:

$$V_N \simeq \frac{\tilde{m}^2 N}{2\pi}\left(1+\frac{5}{3}\frac{N^2\alpha_m}{4\pi^2}\right)$$

Quantum Gravity effects on chaotic inflation (CI)

- The scalar power spectrum is affected as well:

$$P_{\mathcal{R}}^{1/2} = \frac{4\sqrt{24\pi}}{3}\frac{V(\tilde{\phi}_N)^{3/2}}{V'(\tilde{\phi}_N)} \simeq P_{\mathcal{R},\mathcal{CI}}^{1/2}\left(1 - \frac{5}{6}\frac{N^2\alpha_m}{4\pi^2}\right)$$

- where

$$P_{\mathcal{R},\mathcal{CI}}^{1/2} = 2\sqrt{\tfrac{2}{3\pi}}N\tilde{m}$$

- The usual limit on the inflaton mass:

$$\tilde{m} \simeq 4\times 10^{-7} \to m \sim 10^{12}\ \mathrm{GeV}.$$

Quantum Gravity effects on chaotic inflation (CI)

- Finally one obtains the spectral index

$$n_s - 1 = (n_s - 1)_{CI}\left(1 - \frac{5}{3}\frac{N^2\alpha_m}{4\pi^2}\right)$$

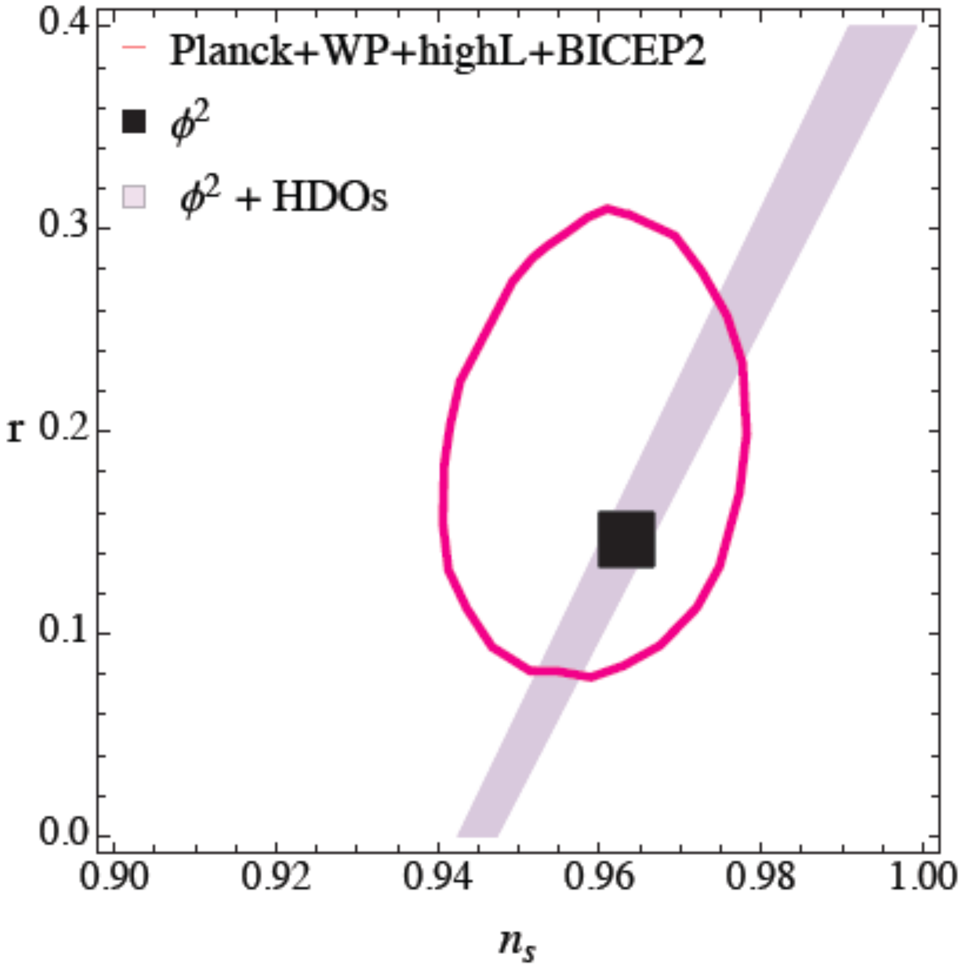
- And the tensor-to-scalar ratio

$$r = r_{CI}\left(1 + \frac{10}{3}\frac{N^2\alpha_m}{4\pi^2}\right)$$

- Which are constrained by BICEP2 (Again for illustration)

$$\alpha_m^{BICEP2} \in [-2, 3] \times 10^{-3}$$

Quantum Gravity effects on chaotic inflation (CI)



Effect of quantum gravity is quite dramatic.

$$\alpha_m^{BICEP2} \in [-2, 3] \times 10^{-3}$$

$$c_6 \lesssim 10^{-9}$$

[xc, and Sanz (2014)]

Quantum Gravity effects on ϕ^4 inflation

Same calculation as before:

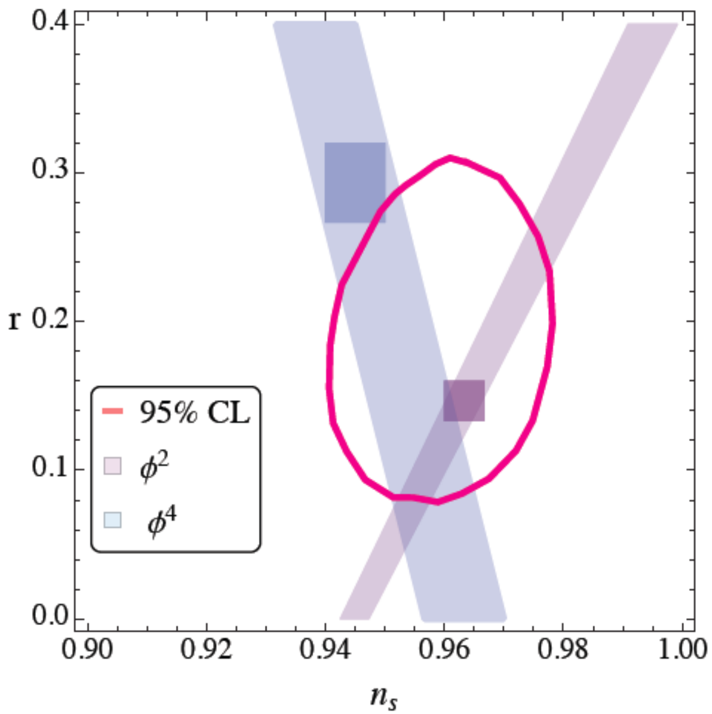
$$V(\tilde{\phi}) = \bar{M}_P^4 \left(\lambda \tilde{\phi}^4 + c_n \tilde{\phi}^n\right)$$

$$c_6 = \alpha_\lambda \lambda \to V(\tilde{\phi}) = \bar{M}_P^4 \lambda \tilde{\phi}^4 \left(1 + \alpha_\lambda \tilde{\phi}^2\right)$$

$$|\alpha_\lambda|^{EFT} \lesssim 0.06$$

$$\begin{aligned} \epsilon &= \epsilon_{\phi^4}\left(1 + \alpha_\lambda \tilde{\phi}^2\right) \\ \eta &= \eta_{\phi^4}\left(1 + 5\alpha_\lambda \tilde{\phi}^2/2\right) \end{aligned}$$

$$\alpha_\lambda^{BICEP} \in [-0.06, 0] \to c_6 < 10^{-15}$$



The effect of higher-dimensional operators on ϕ^4 and ϕ^2 potentials shown in blue and purple respectively.
The darker boxes corresponds to potentials without higher-dimensional operators, and the pink circle is the area of 95% CL from BICEP2.

[xc, and Sanz (2014)]

Quantum Gravity and Dark Matter

- Imagine a hidden DM sector
- Let's study how these particles couple to the SM via gravitational interaction.
- Dim 4 operator (global symmetries likely to broken by QG)

$$\epsilon \bar{L} H D_R,$$

- Lifetime of DM> lifetime of our universe implies

$$\epsilon < 8 \times 10^{-43}$$

- Remember that Wilson coefficients of dim 4 operators can be very small naturally.

Quantum Gravity and Dark Matter

- Dim 5 operator

$$\frac{\epsilon_1}{M_P}\bar{D}H^\dagger \not{D} L$$

- After SSB $\sim \frac{\epsilon_1 v}{M_P}\bar{D}\not{Z}\nu_L,$

- For $\epsilon_1 \sim 1$

$$\tau \sim M_D^{-1} \times 10^{-32} \sim 10^6 \text{ s}$$

- which is much shorter than the lifetime of our universe $\sim 3 \times 10^{17}$ s

Quantum Gravity and Dark Matter

- The same would apply to right handed neutrino DM

$$\frac{\epsilon_2}{M_P}\bar{D}_L \nu_R H^\dagger H,$$

- QG implies that global symmetries protecting DM from decaying must be gauged.

- Then hidden sectors can mix via e.g.

$$\epsilon_3 F_{\mu\nu}\hat{F}^{\mu\nu}$$

but again the Wilson coefficient can be technically small.

Conclusions

- We have discussed a conservative effective action for quantum gravity within several frameworks
  - Standard model
  - Grand Unified Models
  - Inflationary models
  - DM hidden sectors
- We have seen that the effects of quantum gravity can be huge in inflationary models and in grand unified theories.
- They are relatively modest within the standard model (as expected).
- It's tough to probe QG using low energy experiments we have a good chance of testing the symmetries of quantum gravity using inflation.

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Thanks for your attention!

BACK UP SLIDES

A minimal length from QM and GR

Claim: GR and QM imply that no operational procedure exists which can measure a distance less than the Planck length.

Assumptions:

- Hoop Conjecture (GR): if an amount of energy E is confined to a ball of size R, where R < E, then that region will eventually evolve into a black hole.
- Quantum Mechanics: uncertainty relation.

Minimal Ball of uncertainty:

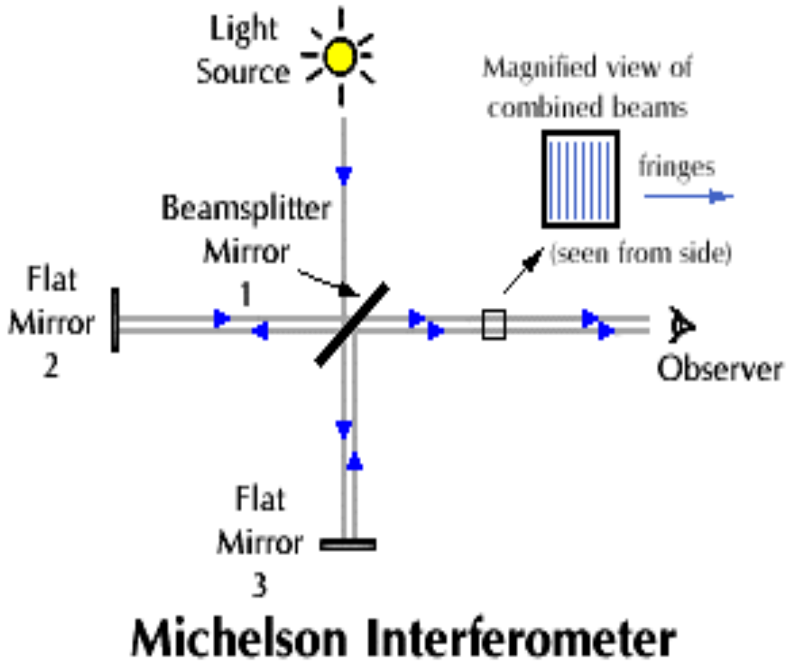
Consider a particle of Energy E which is not already a Black hole.
Its size r must satisfy:

$$r \gtrsim \max\left[1/E\,,\,E\right]$$

where 1/E is the Compton wavele
Hoop Conjecture. We find:

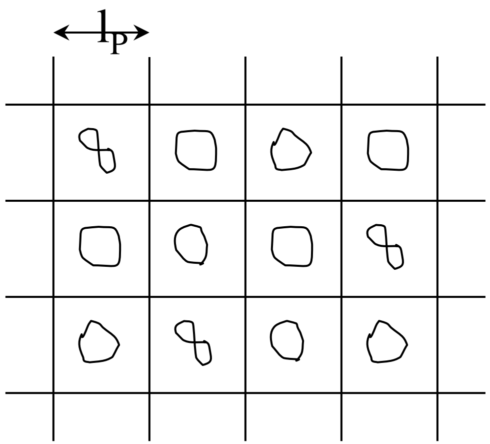
$$r \sim l_P$$

Could an interferometer do better?



Our concrete model:

We assume that the position operator has discrete eigenvalues separated by a distance l_P or smaller.



- Let us start from the standard inequality:

$$(\Delta A)^2(\Delta B)^2 \geq -\frac{1}{4}(\langle[A,B]\rangle)^2$$

- Suppose that the position of a test mass is measured at time t=0 and again at a later time. The position operator at a later time t is:

$$x(t) = x(0) + p(0)\frac{t}{M}$$

- The commutator between the position operators at t=0 and t is

$$[x(0), x(t)] = i\frac{t}{M}$$

- so using the standard inequality we have:

$$|\Delta x(0)||\Delta x(t)| \geq \frac{t}{2M}$$

- At least one of the uncertainties $\Delta x(0)$ or $\Delta x(t)$ must be larger than:

$$\sqrt{t/2M}$$

- A measurement of the discreteness of x(0) requires two position measurements, so it is limited by the greater of $\Delta x(0)$ or $\Delta x(t)$:

$$\Delta x \equiv max\left[\Delta x(0), \Delta x(t)\right] \geq \sqrt{\frac{t}{2M}}$$

- This is the bound we obtain from Quantum Mechanics.

- To avoid gravitational collapse, the size R of our measuring device must also grow such that R > M.
- However, by causality R cannot exceed t.
- GR and causality imply:

$$t > R > M$$

- Combined with the QM bound, they require $\Delta x > 1$ in Planck units or

$$\Delta x > l_P$$

- This derivation was not specific to an interferometer - the result is device independent: no device subject to quantum mechanics, gravity and causality can exclude the quantization of position on distances less than the Planck length.