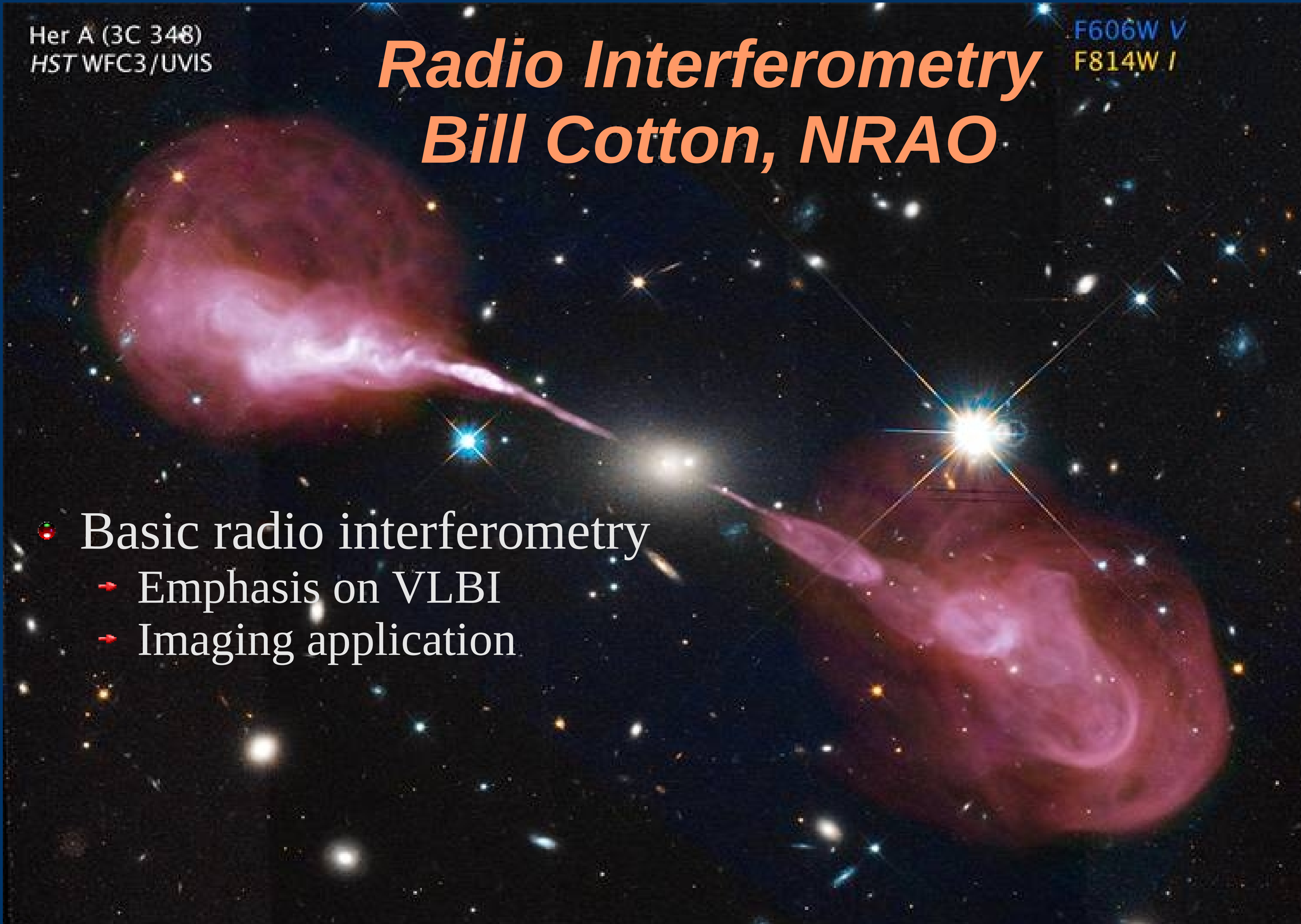
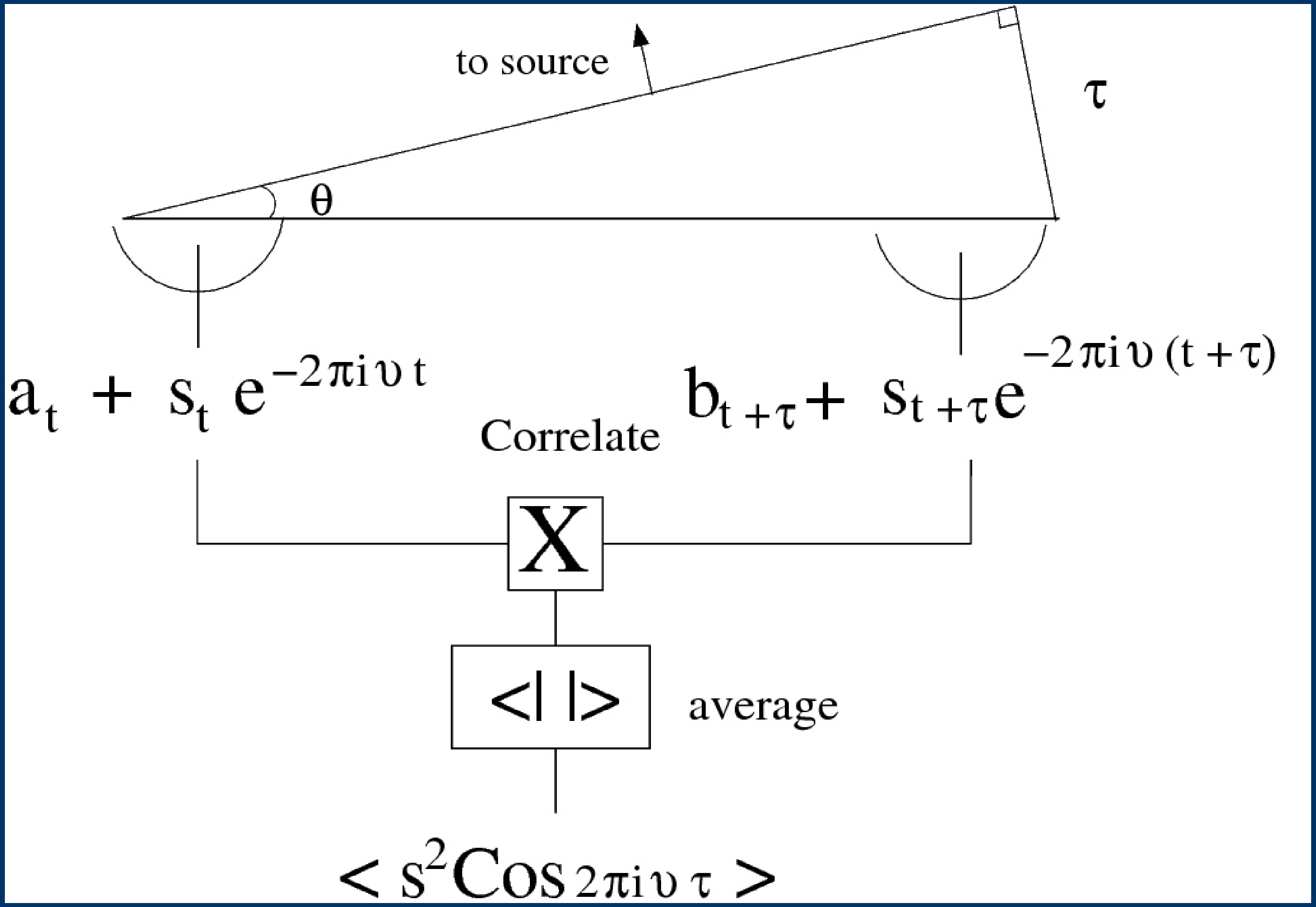
Radio Interferometry
Bill Cotton, NRAO



- Basic radio interferometry
  - Emphasis on VLBI
  - Imaging application

Simplest Radio Interferometer



Monochromatic, point source

Interferometer response

- Adding quarter wavelength to one arm allows measuring phase.
- Interferometer response is:

$$\rho = s^2 \; e^{-2\pi i \nu \tau}$$

Where

$$\tau = \frac{b}{c} \, sin(\theta)$$

Interferometer response, cont'd

$$\rho \;=\; s^2 e^{-2\pi i\nu \frac{b}{c}} \; sin(\theta)$$

substitute

$$\mathrm{u} \;=\; \nu \frac{b}{c}$$

$$\mathrm{l} \;=\; sin(\theta)$$

$$\mathrm{B} \;=\; s^2$$

Response becomes

$$\rho \;=\; B e^{-2\pi i \mathrm{ul}}$$

Extended Sources, limited FOV

Response linear, integrate over source (1D):

$$\rho_{\mathrm{u}} \ = \ \int B_l e^{-2\pi i \mathrm{ul}} \mathrm{dl}$$

This is a Fourier transform, trivially extended to 2D,
Full 3D version is:

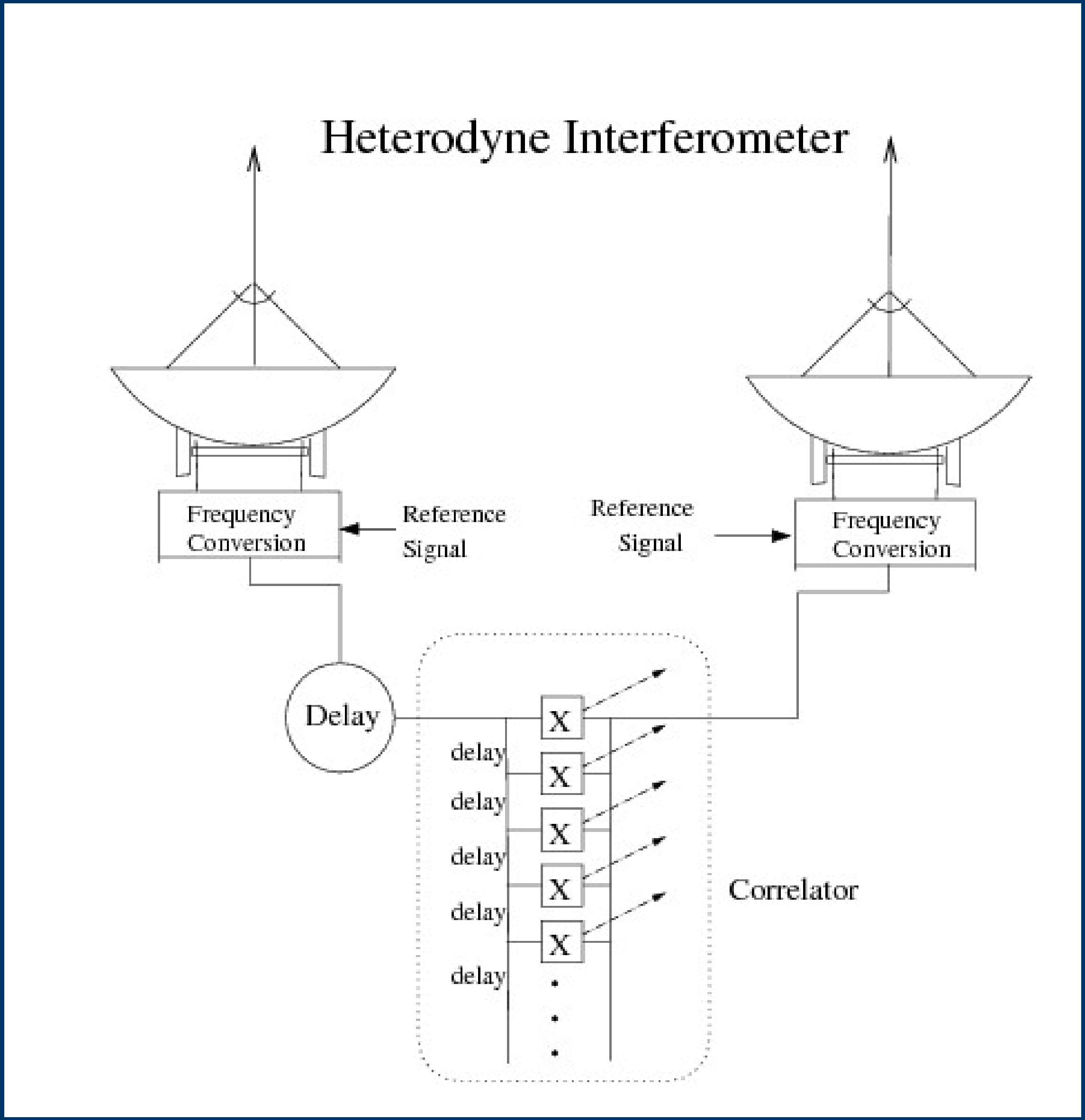
$$\rho_{\mathbf{u},\mathbf{v},\mathbf{w}} \ = \ \int\int A_{l,m} B_{l,m} \ e^{-2j\pi \ \left(ul+vl+w\left(\sqrt{1-l^2-m^2}-1\right)\right)} \ \frac{dl \ dm}{\sqrt{1-l^2-m^2}}$$

Which is not a Fourier transform.

Finite bandwidth

- In practice, real systems have finite bandwidth.
- Integral over $\nu\ \tau$ will decorrelate signal when τ is large.
- Compensate by adding delays to align antennas.
- Measure multiple relative delays among antennas
- Delay and sky frequency related by Fourier transform, allows spectroscopy.

More realistic interferometer



Polarization

- Phase sensitive detectors respond to a single polarization state
- Two orthogonal polarizations are needed to fully sample wavefront.
  - Right and left hand circular polarization
  - Orthogonal linear polarization
- All four correlation products to fully measure polarization.

Circular feeds, 1st order

$$V_{RR} = I + V$$

$$V_{LL} = I - V$$

$$V_{RL} = I(d_{R,i} + d^*_{L,k}) - (jQ - U)e^{-j2\chi_m}$$

$$V_{LR} = I(d_{L,i} + d^*_{R,k}) - (jQ - U)e^{j2\chi_m}$$

I, Q, U, V = Stokes parameters, χ = parallactic angle
d = "leakage" terms

Linear feeds, 1st order

$$
\begin{aligned}
V_{XX} &= I + Q cos 2\chi + U sin 2\chi \\
V_{YY} &= I - Q cos 2\chi - U sin 2\chi \\
V_{XY} &= I(d_{X,i} + d^*_{Y,k}) - Q cos 2\chi + U sin 2\chi + jV \\
V_{YX} &= I(d_{Y,i} + d^*_{X,k}) - Q cos 2\chi + U sin 2\chi - jV
\end{aligned}
$$

I, Q, U, V = Stokes parameters, χ = parallactic angle
d = "leakage" terms

Amplitude Calibration

- Need to calibrate visibility amplitudes into physical units, Janskys, for baseline i-k

$$\mathbf{V}_{\mathbf{ik}}^{\mathbf{obs}} = \rho_{\mathbf{jk}}\,\eta\sqrt{\frac{T_{S_j}T_{S_k}}{K_jK_k}}$$

ρ_{jk} = complex correlation coefficient
η = efficiency factor
T_s = System temperature
K = sensitivity in K/Jy

Full Calibration

Represent electric fields measured by the two detectors:

$$\mathbf{E} = \begin{pmatrix} e_p \\ e_q \end{pmatrix}$$

Measured visibility is: $\mathbf{V}_{\mathbf{ik}}^{\mathbf{obs}} = (\mathbf{J_i} \otimes \mathbf{J_k^*})(\mathbf{E_i} \otimes \mathbf{E_k^*})$

Where J = 2 x 2 complex "Jones" matrix,

$\otimes$ indicates outer product

Full Calibration, cont'd

Jones matrices are determined from observations of known calibrator sources and applied:

$$\mathbf{V}_{\mathrm{ik}}^{\mathrm{cal}} = \left(\mathbf{J}_{\mathrm{i}}^{-1} \otimes \mathbf{J}^{-1}{}_{\mathrm{k}}^{*}\right) \mathbf{V}_{\mathrm{ik}}^{\mathrm{obs}}.$$

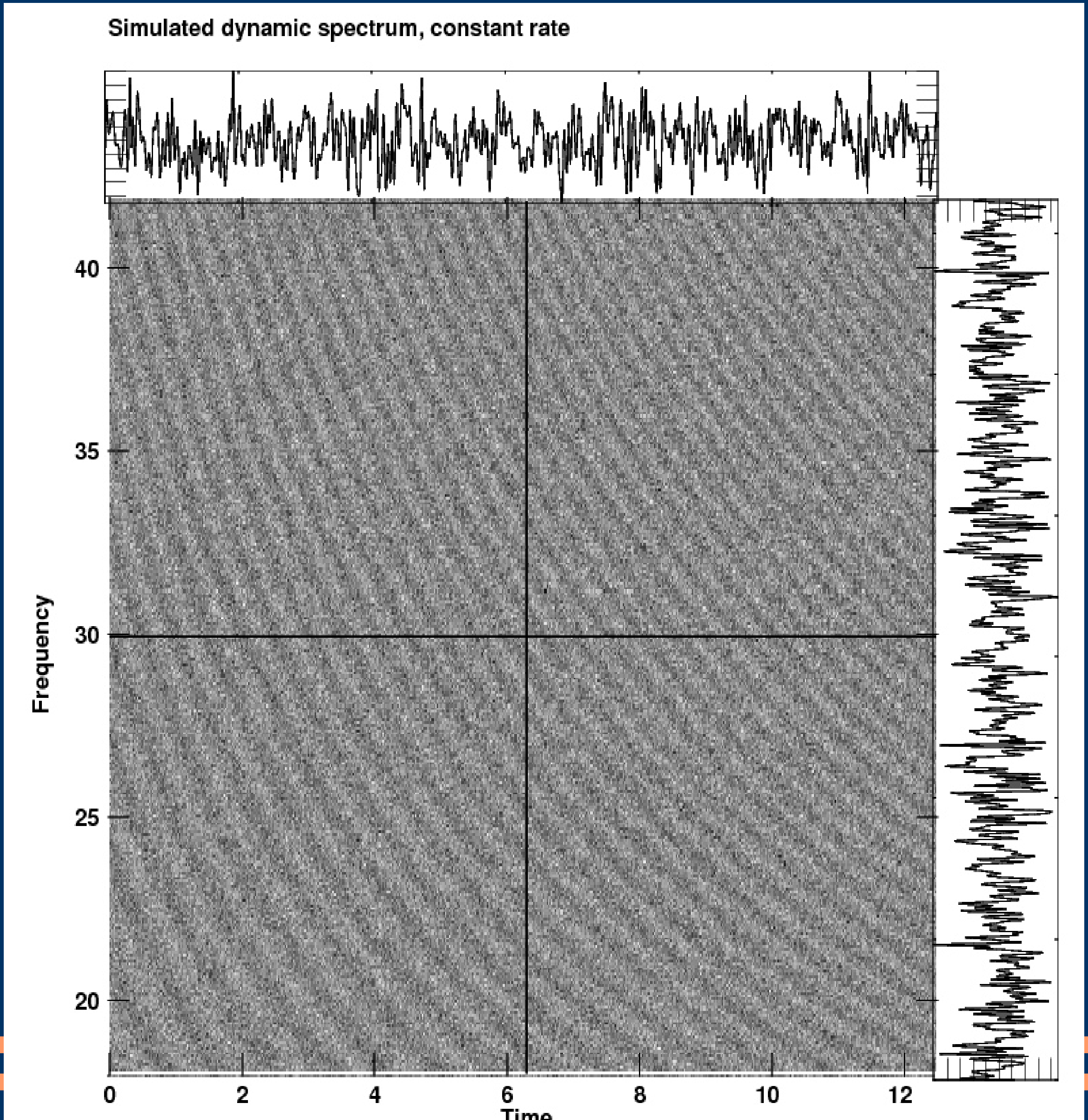
Fringe Fitting

- Especially on long baselines instrumental delays may not be know a priori
  - Atmospheric delays
  - Instrumental delays
  - Clock errors
  - Errors in orbit
- Model phases as linear in time and frequency:

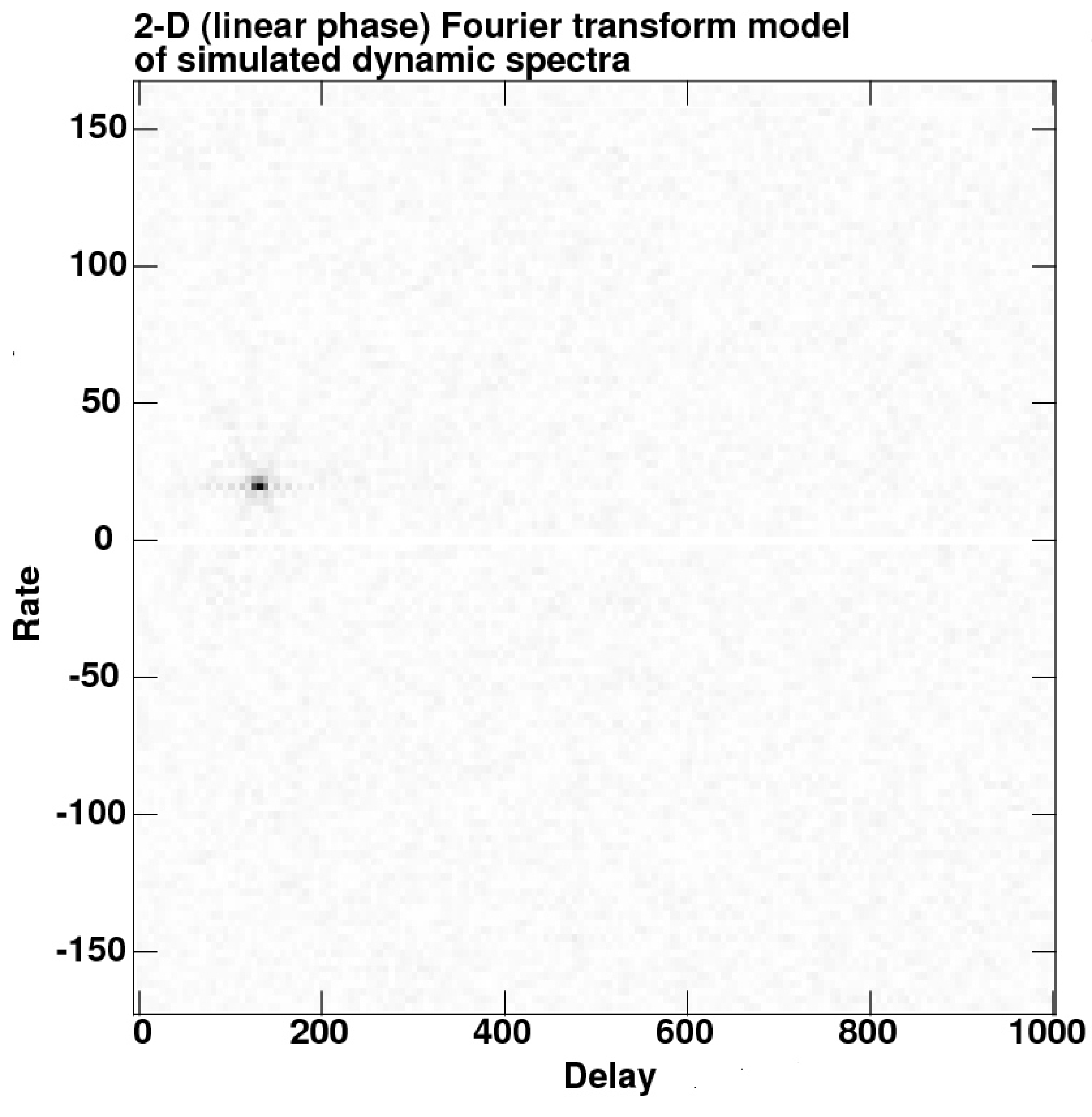
$$\phi_{t,\nu} = \phi_0 + \left(\frac{\partial\phi}{\partial\nu}\Delta\nu + \frac{\partial\phi}{\partial t}\Delta t\right)$$

- Fitting group delay and rate = “fringe fitting”

Time-Frequency data



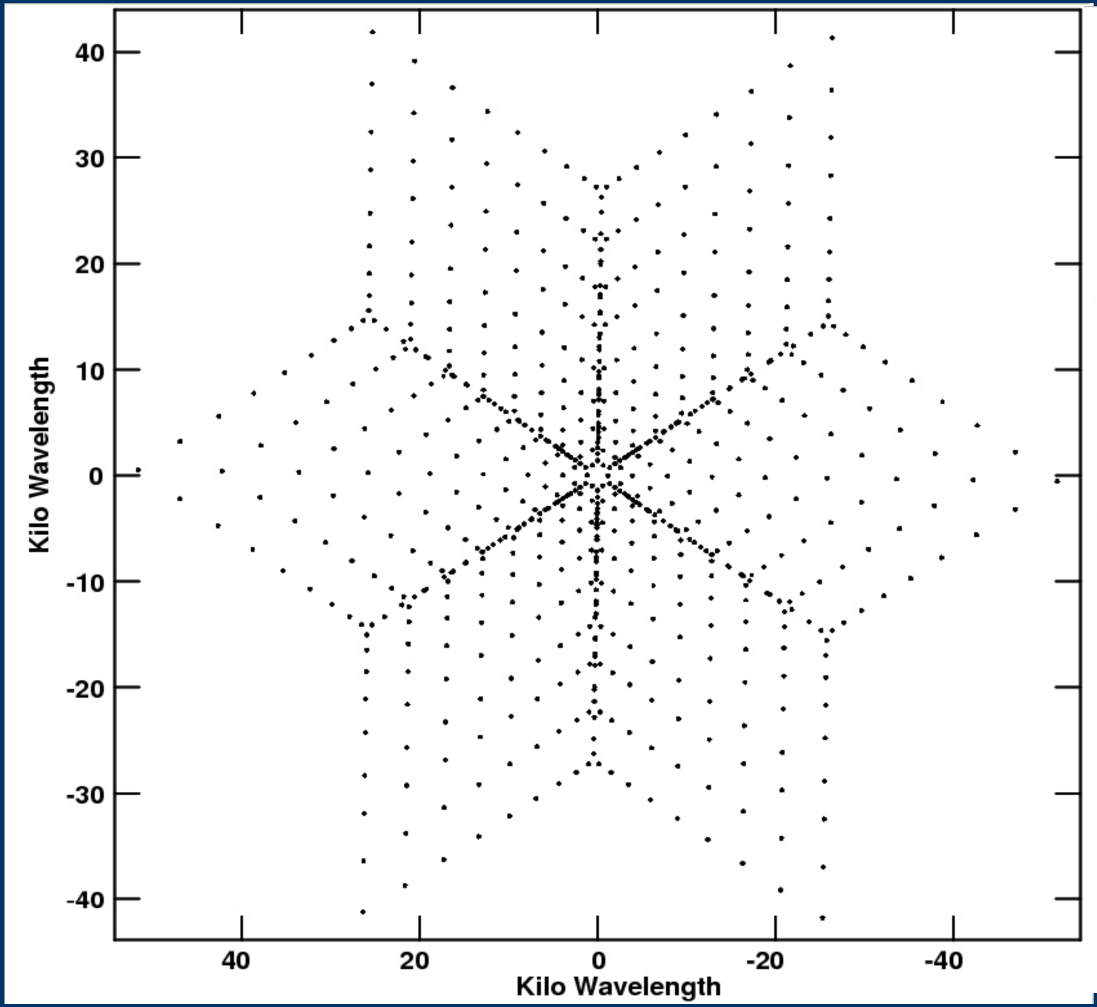
Transformed to Delay - Rate



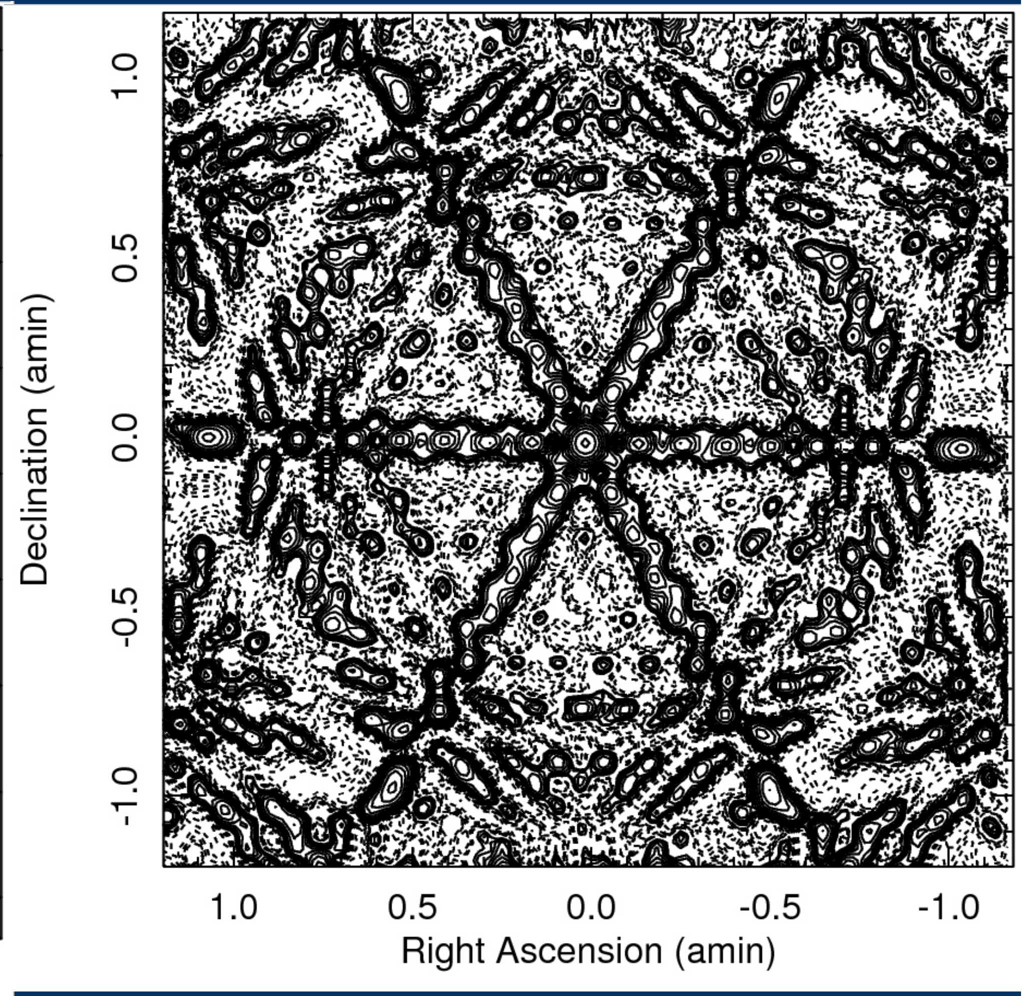
Interferometer arrays and imaging

- Single interferometer samples FT of sky brightness
- Many samples needed for good sampling of UV (Fourier) plane.
- Each pair of antennas gives a measurement
- FT of real function hermetian => conjugate points
- (n * n-1)/2 pair-wise combinations of n antennas.
- Point source response (“dirty beam”) is the Fourier transform of u-v coverage.

VLA UV snapshot coverage and beam



U-V Coverage

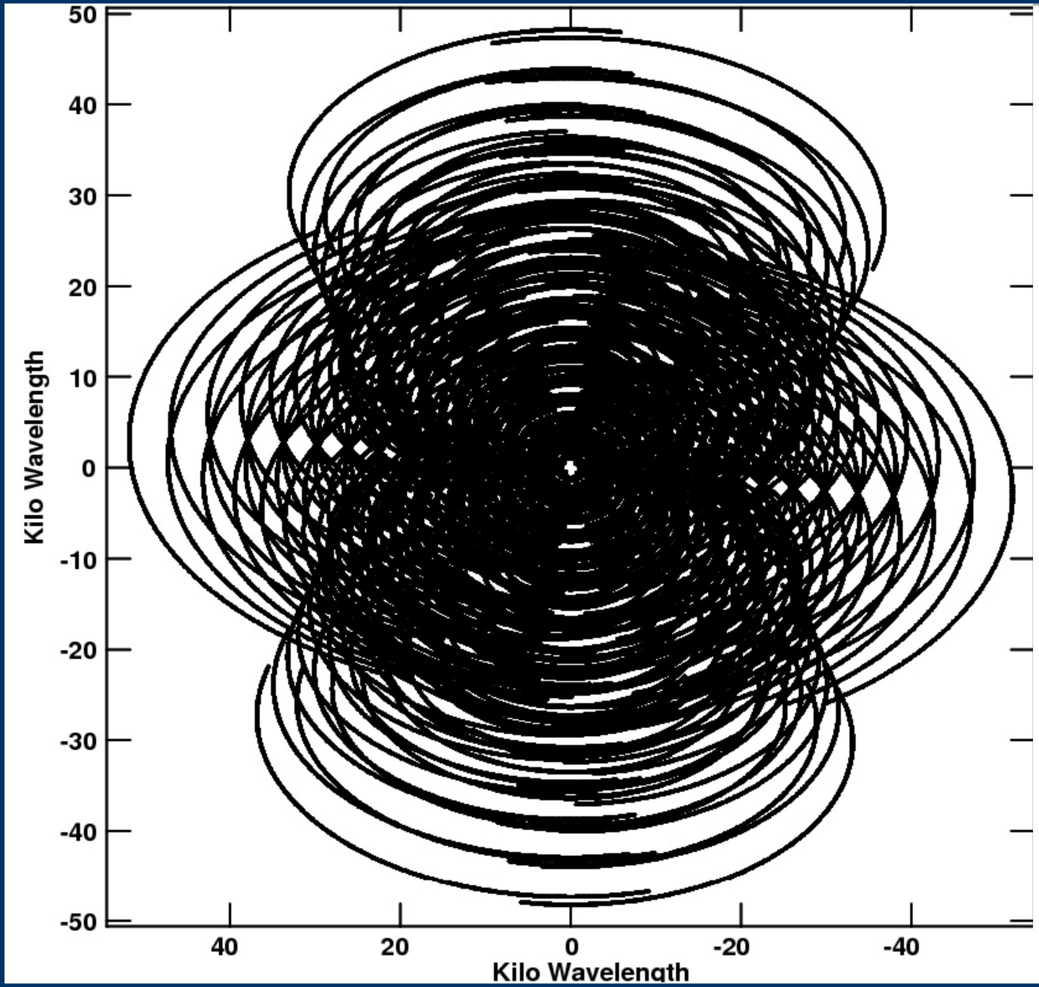


Dirty Beam

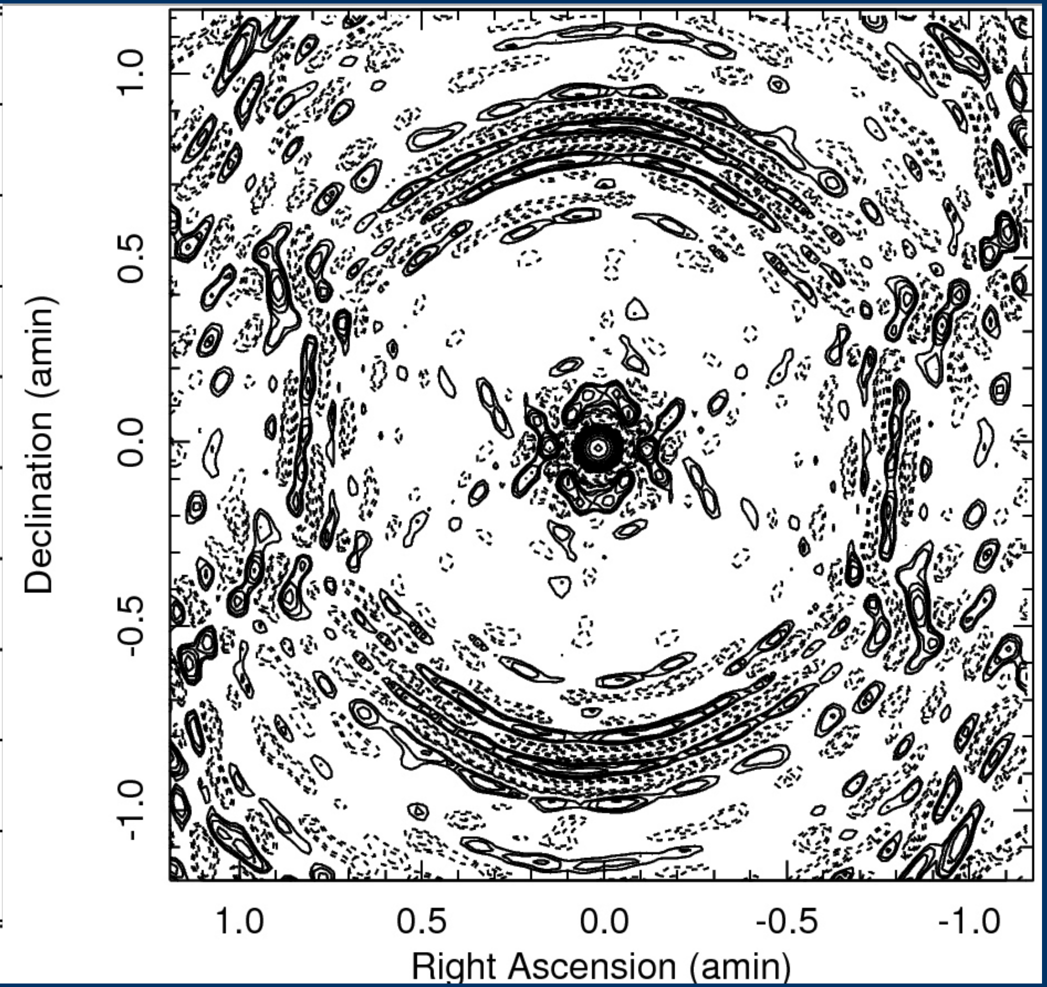
Interferometer arrays and imaging, cont'd

- Interferometer parameters (u,v) vary with earth rotation.
- Improve coverage with time.

VLA 8 Hour Coverage and Beam



U-V Coverage



Dirty Beam

Deconvolution

- Even very good uv coverage has holes
- Can use zero for unmeasured visibilities
- Allows inverting Fourier transform
- Derive “Dirty Image”
- Deconvolution to recover true sky distribution

Deconvolution, cont'd

Define Sampling Function:

$$S_{u,v} \;=\; \begin{cases} weight \text{ of sample where a sample was made} \\ 0 \text{ elsewhere.} \end{cases}$$

Sampled (calibrated) visibility:

$$V'_{u,v} \;=\; V_{u,v}\; S_{u,v}$$

which is defined everywhere,

Inverse Fourier transform of $V'_{u,v}$ gives "Dirty Image", $B'_{l,m}$
Inverse Fourier transform of $S_{u,v}$ gives "Dirty Beam", $D_{l,m}$

Deconvolution, cont'd

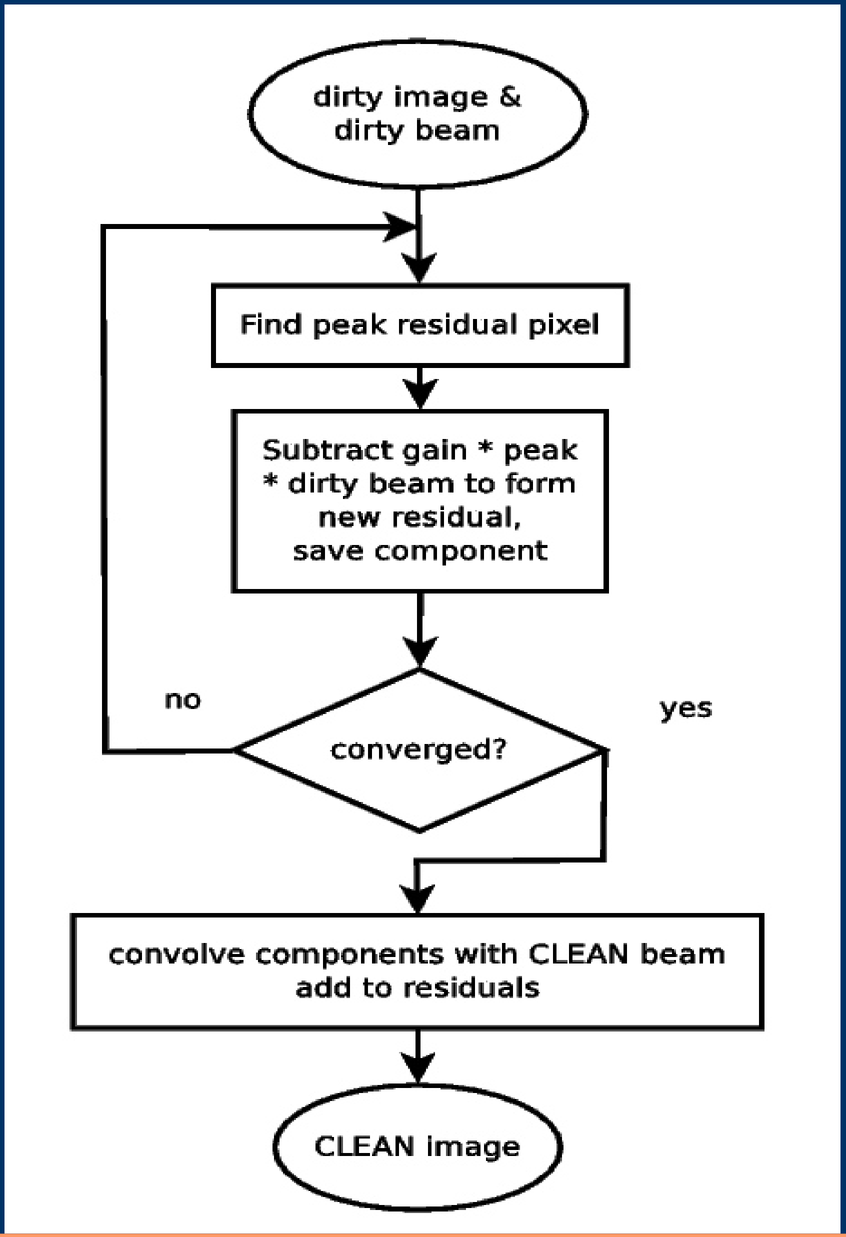
Convolution theorem says:

$$B'_{l,m} = B_{l,m} \star D_{l,m}$$

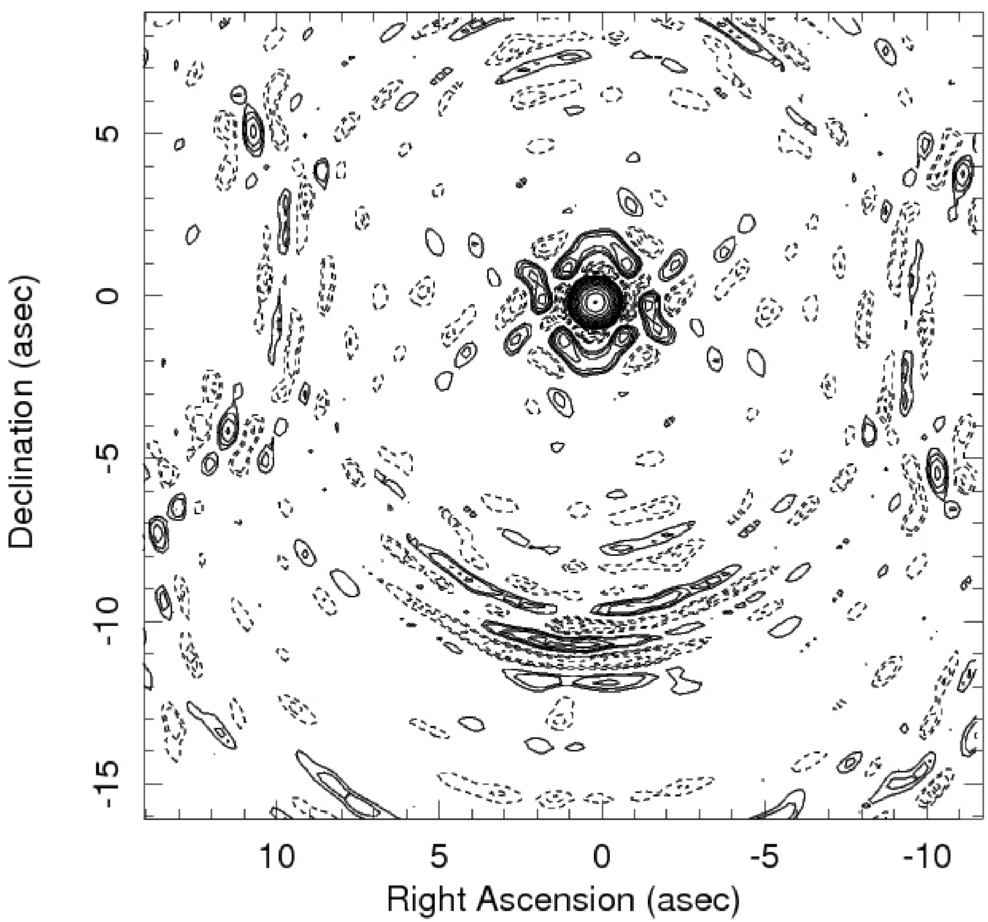
Where $\star$ indicates convolution

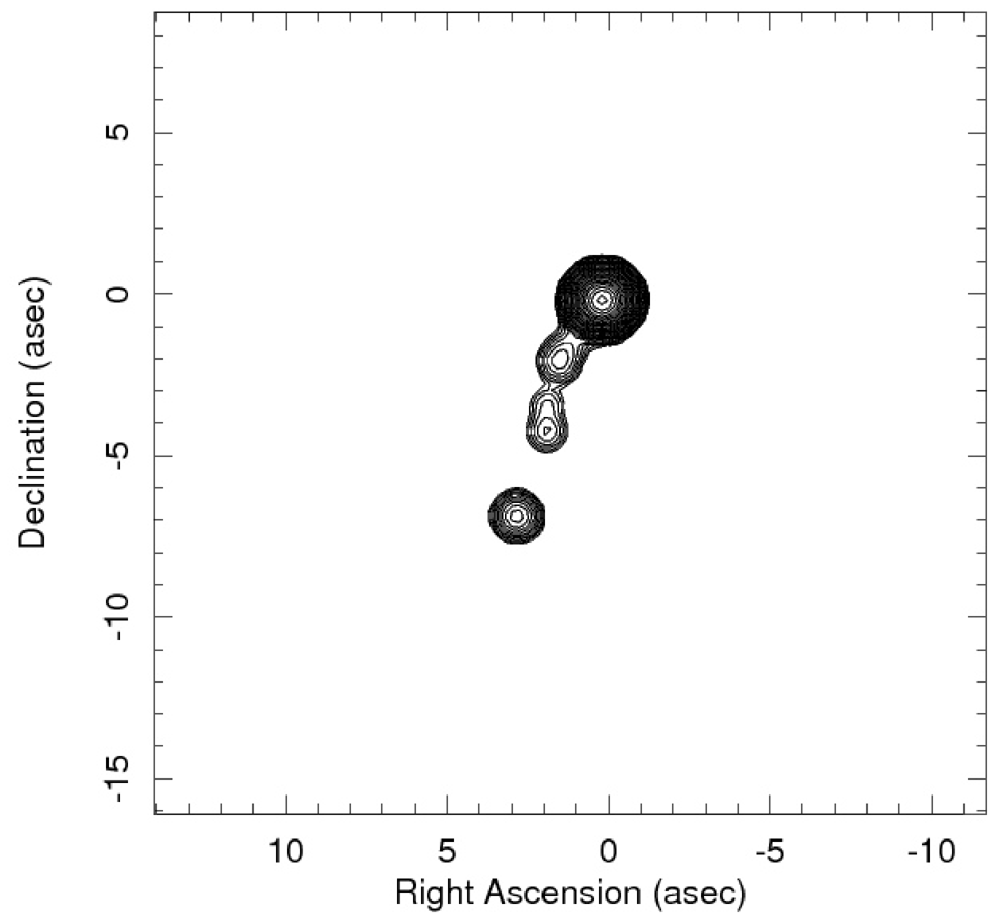
- Deconvolution needed to recover true sky brightness $B_{l,m}$
- Zeroes in $S_{u,v}$ require nonlinear deconvolution

CLEAN Deconvolution variant



CLEAN of Noiseless Simulated Data



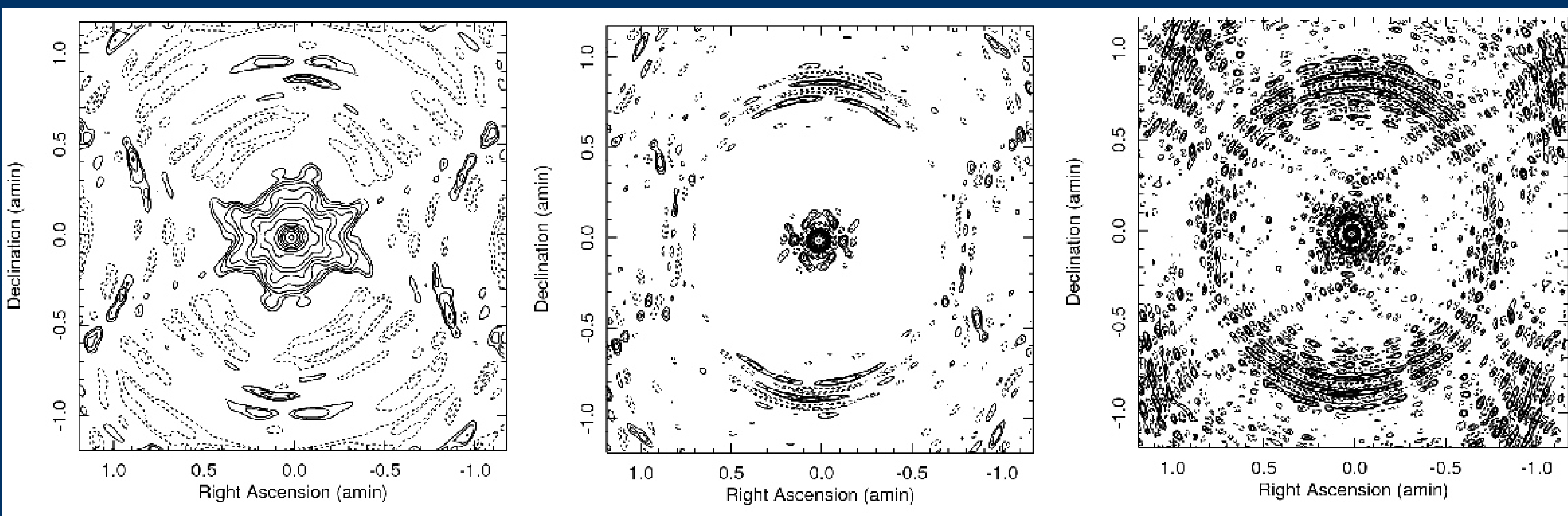


Dirty Image, 8 hr VLA

CLEAN deconvolved image contours 100 times lower

Weighting

- Weights in Sampling function, arbitrary, can be used to modify image
  - “Natural” weight ⇒ best sensitivity
  - “Uniform” weight ⇒ best resolution
  - “Robust” weighting ⇒ in between



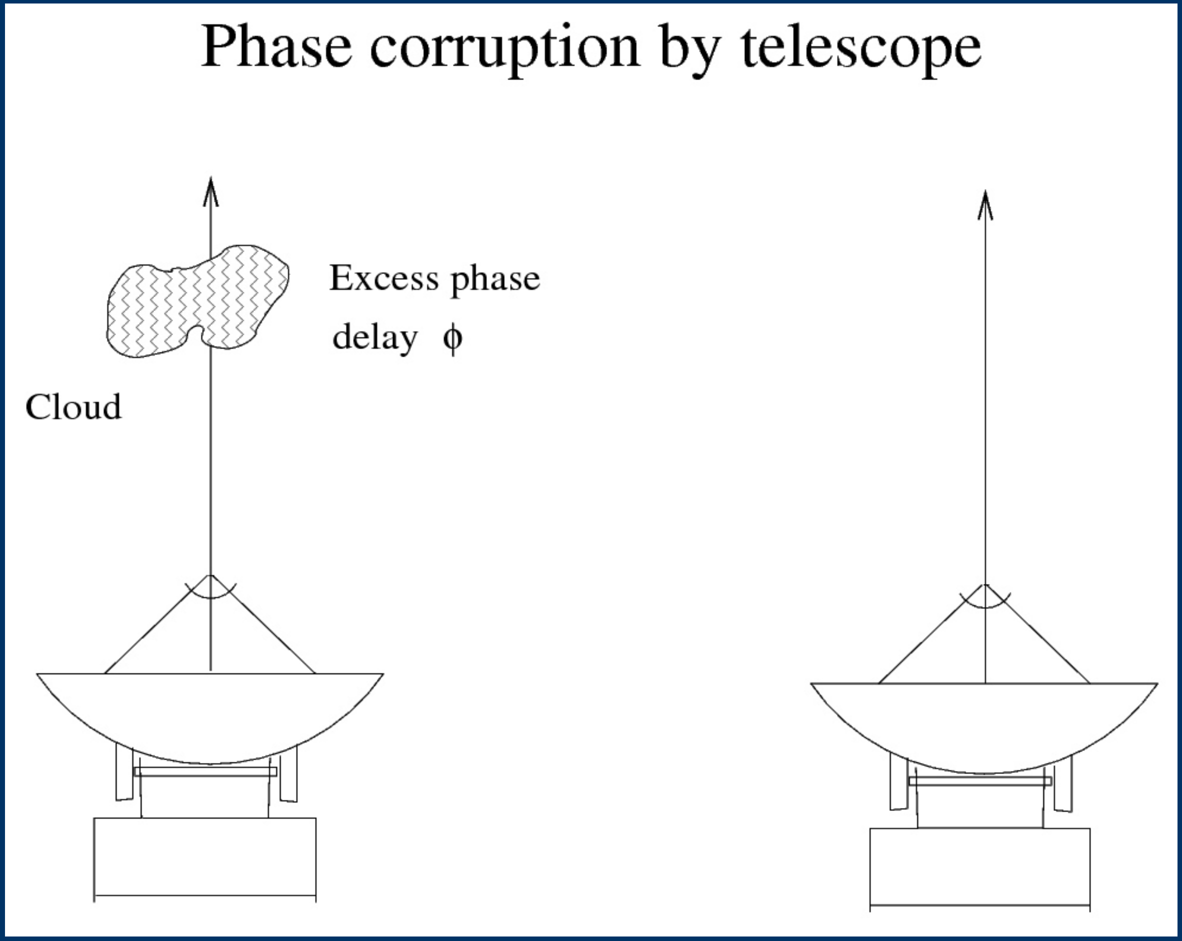
Natural | **Optimum, Robust=0** | **Uniform**

Self calibration

- External calibration approximate
  - Different time
  - Different direction
  - Phase calibration not practical for some VLBI
- If target is bright enough and model is available, it can be used as the calibrator
- Must be detectable in coherence time

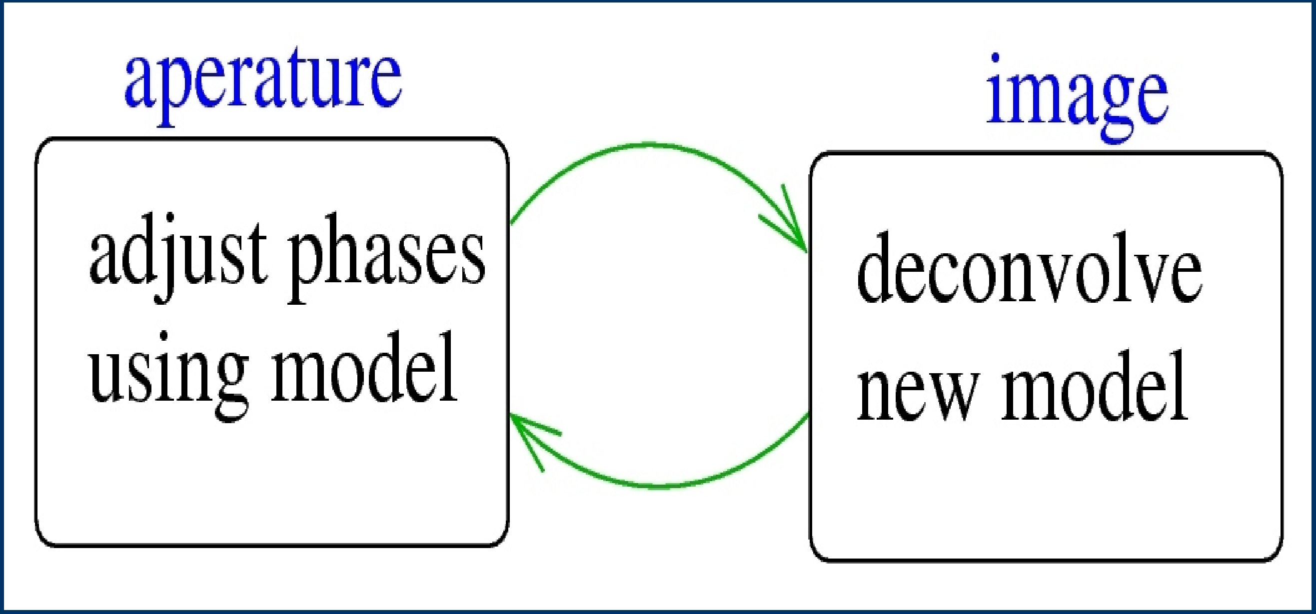
Self calibration, cont'd

- Most calibration errors affect antennas

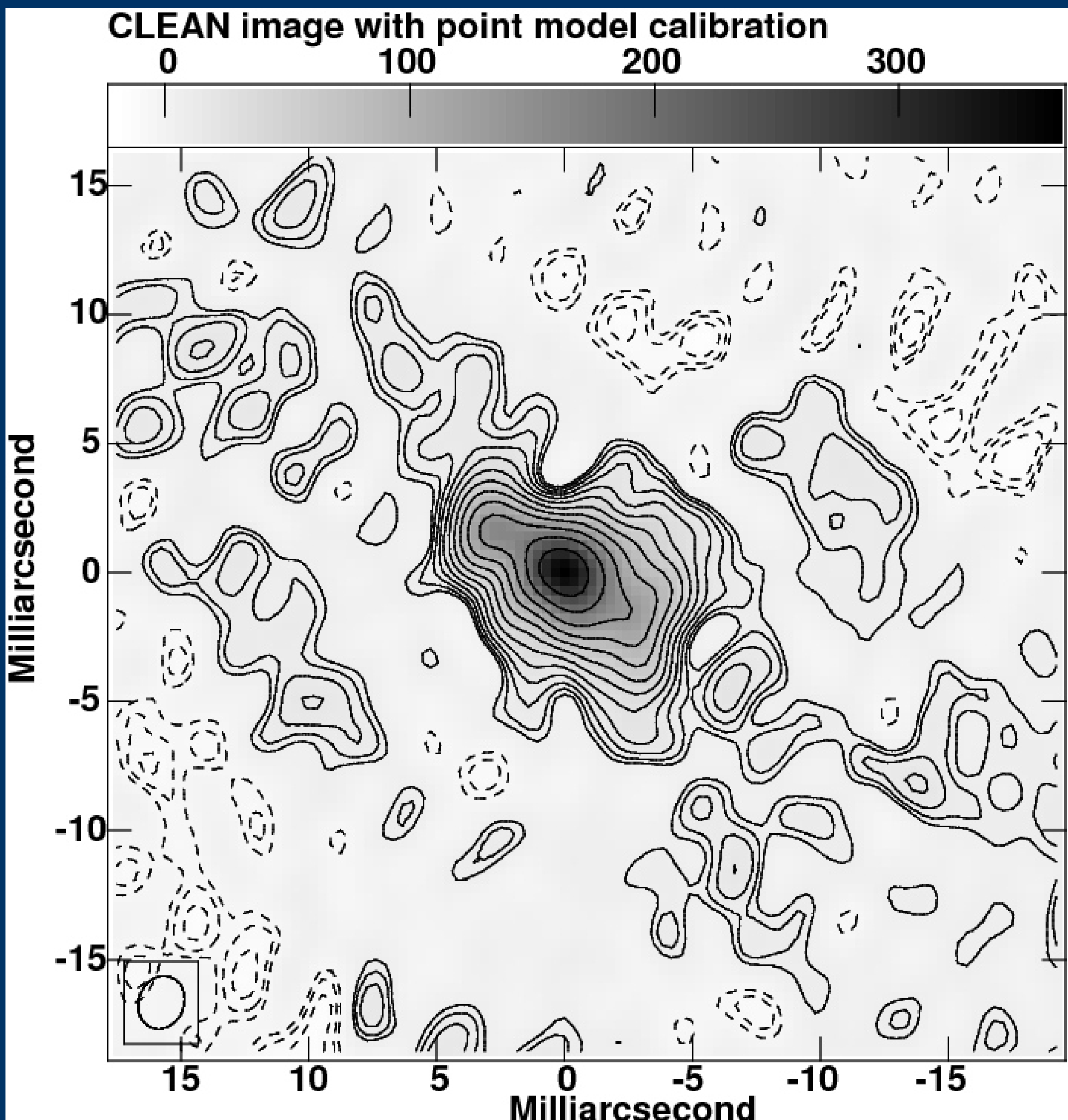


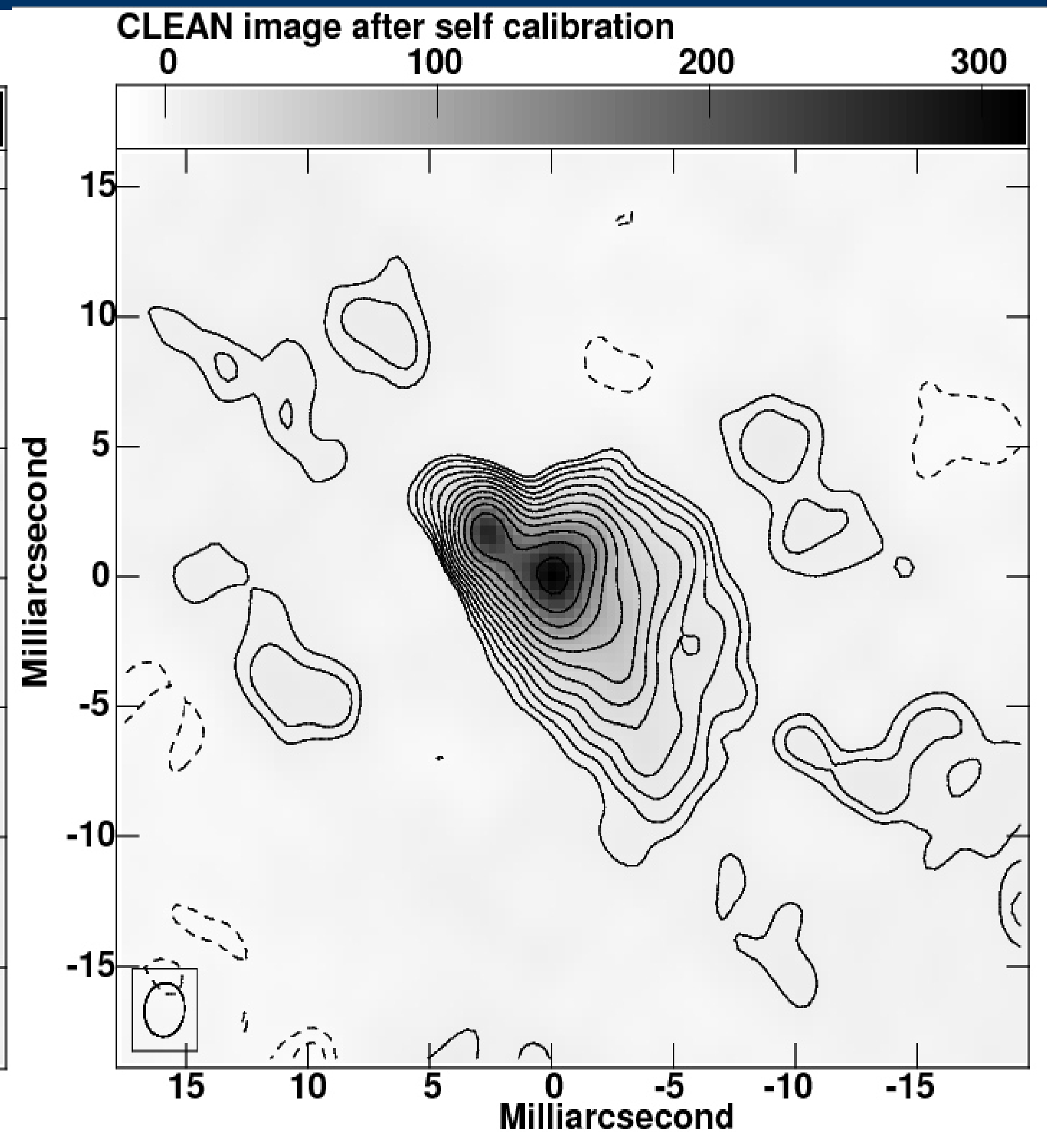
Self calibration, cont'd

- For phase calibration have n-1 unknowns and n*(n-1)/2 measurements.
- Iterate calibration/imaging
  - Initial model uses external calibration if possible
  - VLBI initial point model usually good enough



Self calibration example





Thank you