Magnetic Moment Manipulation by a Josephson Curent

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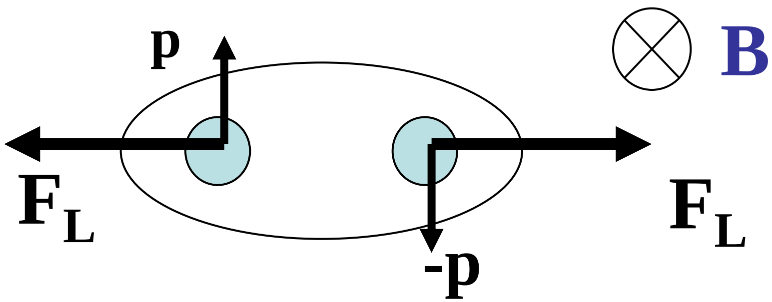
- Josephson π-junction.
- φ_0-junctions.
- Coupling between magnetic moment and Josephson current in φ_0-junctions.
- Magnetic moment precession via a.c. Josephson effect.

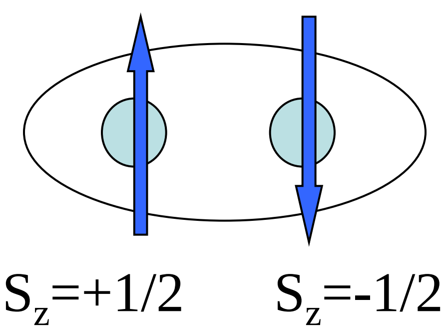
Antagonism of magnetism (ferromagnetism) and superconductivity

- Orbital effect (Lorentz force)



Electromagnetic mechanism (breakdown of Cooper pairs by magnetic field induced by magnetic moment)

- Paramagnetic effect (singlet pair)



$\mu_B H \sim \Delta \sim T_c$

$I\left(\vec{S}\cdot\vec{s}\right)\approx T_c$

Exchange interaction

Superconducting order parameter behavior in ferromagnet

Standard Ginzburg-Landau functional:

$$F = a|\Psi|^2 + \frac{1}{4m}|\nabla\Psi|^2 + \frac{b}{2}|\Psi|^4$$

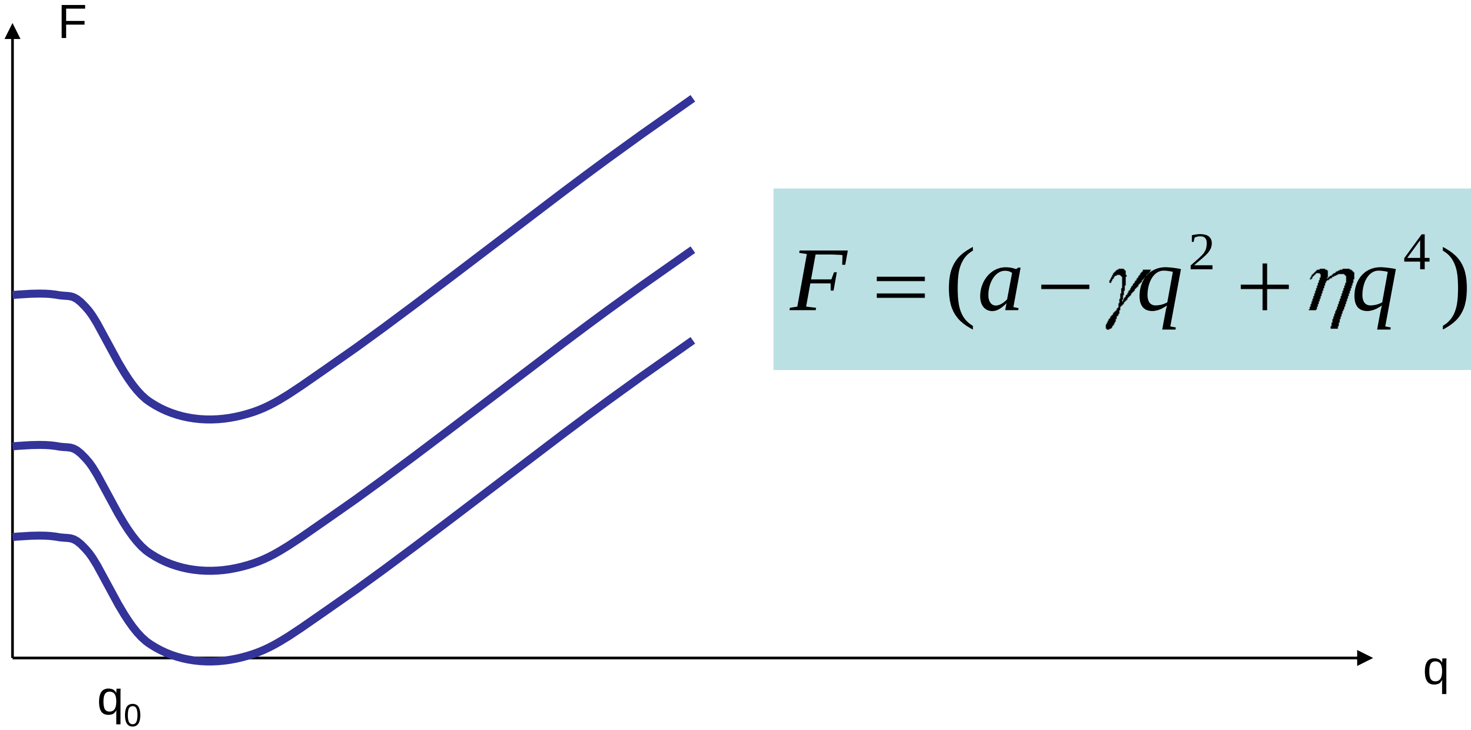
The minimum energy corresponds to Ψ=const

The coefficients of GL functional are functions of internal exchange field h !

Modified Ginzburg-Landau functional ! :

$$F = a|\Psi|^2 - \gamma|\nabla\Psi|^2 + \eta|\nabla^2\Psi|^2 + \ldots$$

The **non-uniform** state Ψ~exp(iqr) will correspond to minimum energy and higher transition temperature



$$F = (a - \gamma q^2 + \eta q^4)|\Psi_q|^2$$

$\Psi \sim \exp(iqr)$ - **Fulde-Ferrell-Larkin-Ovchinnikov state (1964).**

Only in pure superconductors and in the very narrow region.

Proximity effect in a ferromagnet ?

In the usual case (normal metal):

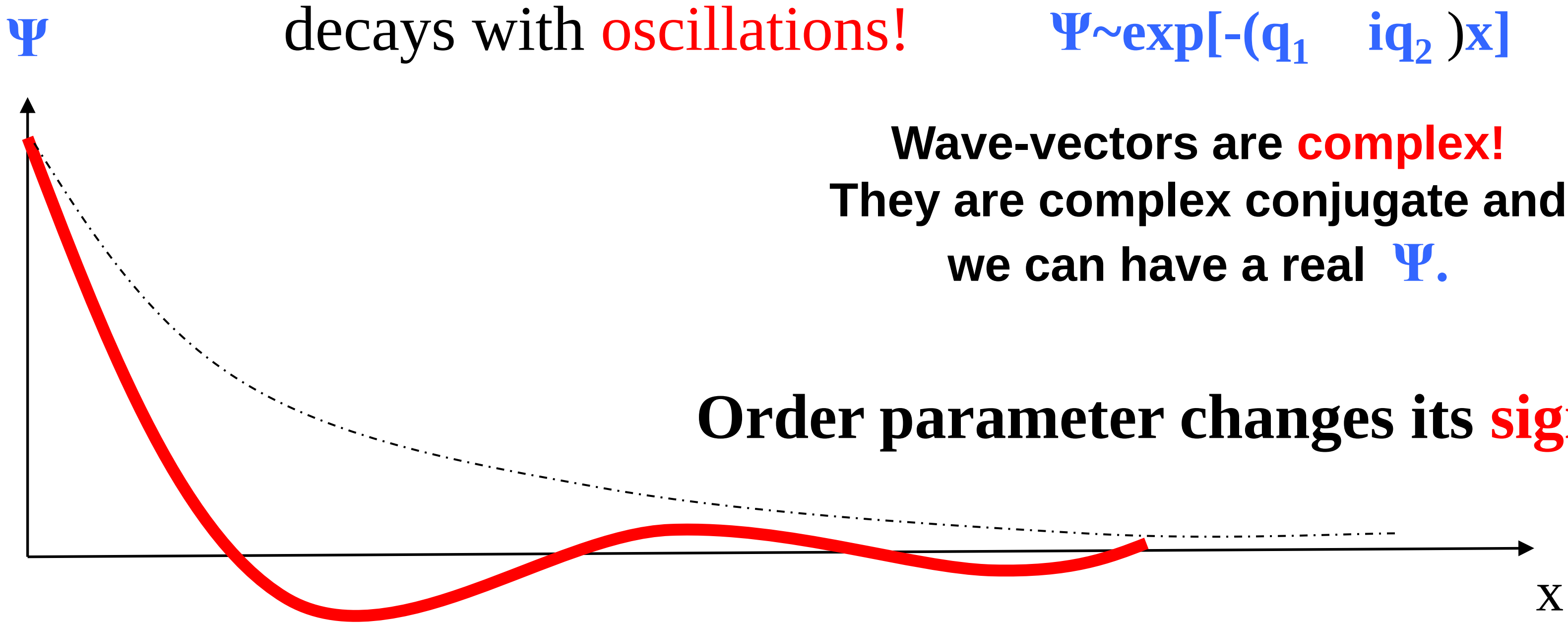
$$a\Psi - \frac{1}{4m}\nabla^2\Psi = 0, \text{ and solution for } T > T_c \text{ is } \Psi \propto e^{-qx}, \text{ where } q = \sqrt{4ma}$$

In **ferromagnet** (in presence of exchange field) the equation for superconducting order parameter is different

$$a\Psi + \gamma\nabla^2\Psi - \eta\nabla^4\Psi = 0$$

Its solution corresponds to the order parameter which decays with oscillations!

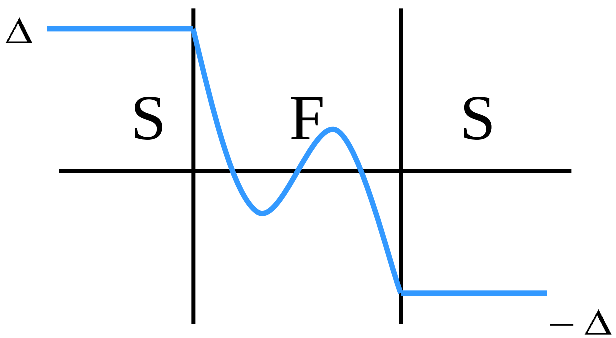
$\Psi \sim \exp[-(q_1 \quad iq_2)x]$



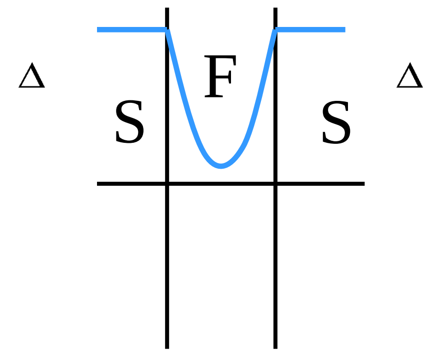
Wave-vectors are complex!
They are complex conjugate and we can have a real Ψ.

Order parameter changes its sign!

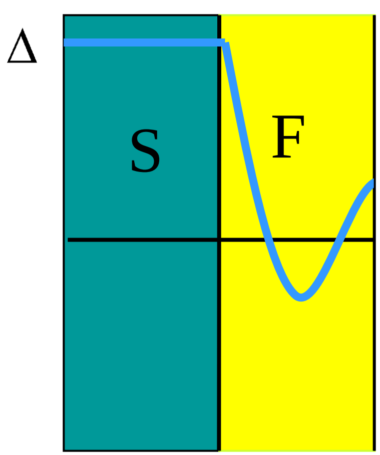
Remarkable effects come from the possible shift of sign of the wave function in the ferromagnet, allowing the possibility of a « π-coupling » between the two superconductors (π-phase difference instead of the usual zero-phase difference)



« π phase »



« 0 phase »



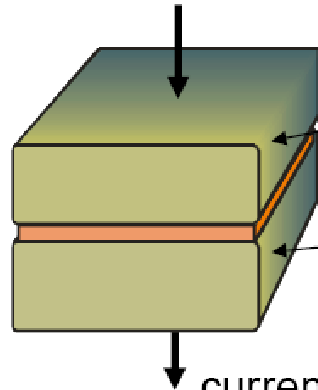
S/F bilayer

$$\xi_f = \sqrt{D_f / h} \propto (1-10)nm$$

h-exchange field,

D_f-diffusion constant

Josephson effect



$$\Psi_1 = |\Psi_1| \exp(i\theta_1)$$

$$\Psi_2 = |\Psi_2| \exp(i\theta_2)$$

$$|\Psi_1| = |\Psi_2|$$

superconducting phase difference: $\varphi = \theta_1 - \theta_2$

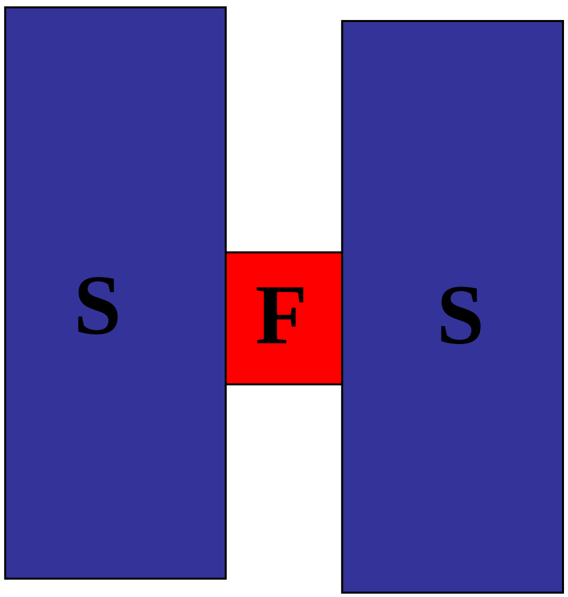
Josephson relations

$$\begin{cases} I_s = I_c \sin\varphi \\ V = \dfrac{\hbar}{2e}\dfrac{d\varphi}{dt} \end{cases}$$

Electromagnetic radiation at the frequency f

$$f = \frac{V}{\Phi_0}$$

S-F-S Josephson junction in the clean/dirty limit



Damping oscillating dependence of the critical current I_c as the function of the parameter $\alpha = hd_F / v_F$ has been predicted.

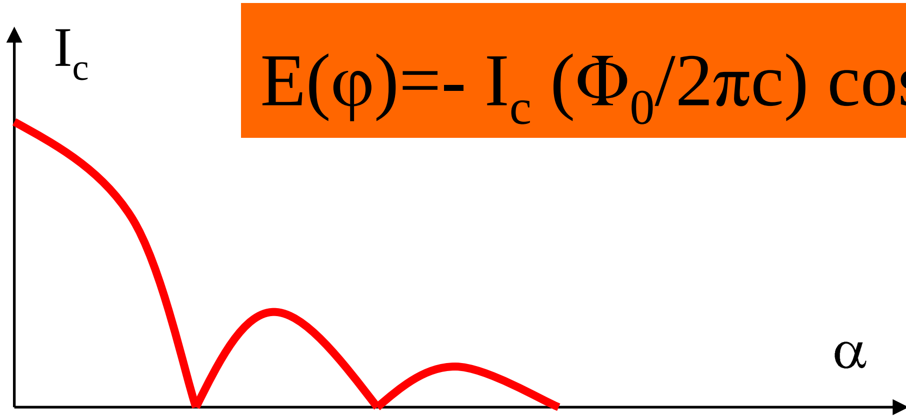
(Buzdin, Bulaevskii and Panjukov, JETP Lett. 81)

h- exchange field in the ferromagnet,

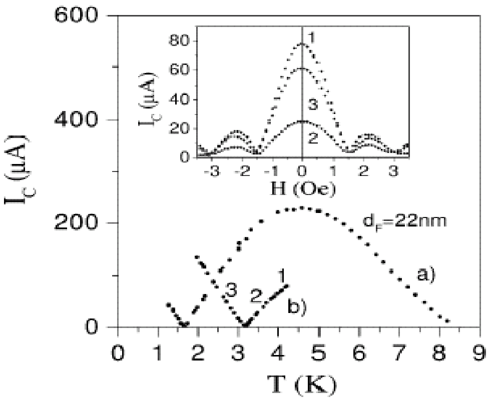
d_F - its thickness

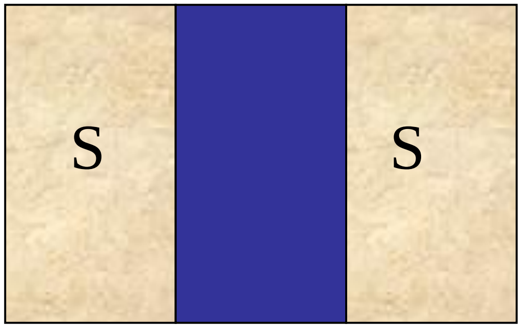
$$E(\varphi) = -\, I_c\, (\Phi_0/2\pi c) \cos\varphi$$

$$J(\varphi) = I_c \sin\varphi$$



The oscillations of the critical current as a function of temperature (for different thickness of the ferromagnet) in S/F/S trilayers have been observed on experiment by Ryazanov et al. 2000, PRL





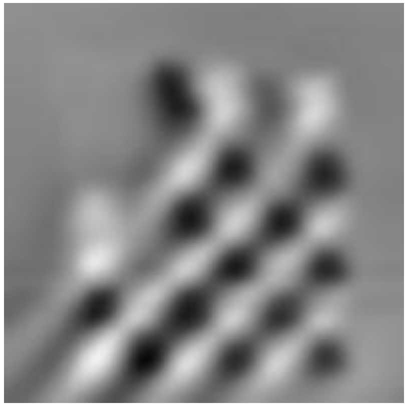
F layer is CuNi alloys

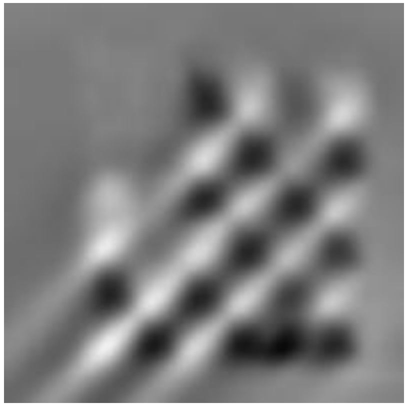
Scanning SQUID Microscope images
(Van Harlingen, Ryazanov et al.)

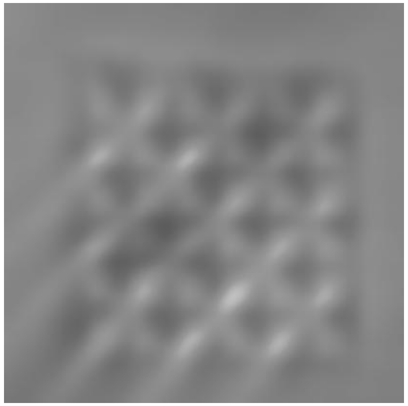
T = 1.7K

T = 2.75K

T = 4.2K

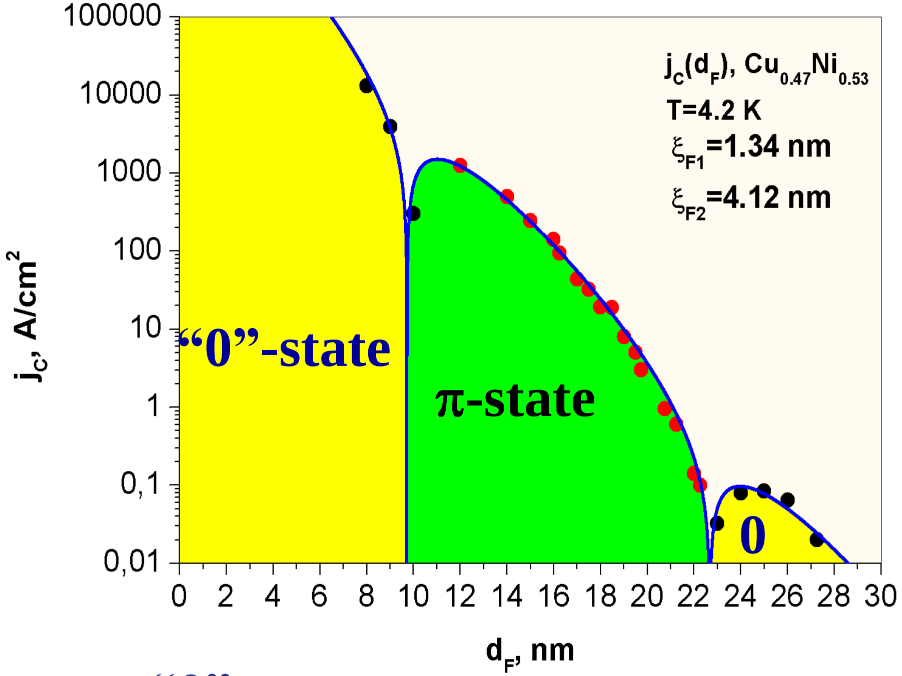






Critical current density vs. F- layer thickness (V.A.Oboznov et al., PRL, 2006)

$$I_c = I_{c0} \exp(-d_F/\xi_{F1}) \left|\cos(d_F/\xi_{F2}) + \sin(d_F/\xi_{F2})\right|$$



$d_F \gg \xi_{F1}$

Spin-flip scattering decreases the decaying length and increases the oscillation period.

$\xi_{F2} > \xi_{F1}$

Nb-$Cu_{0.47}Ni_{0.53}$-Nb

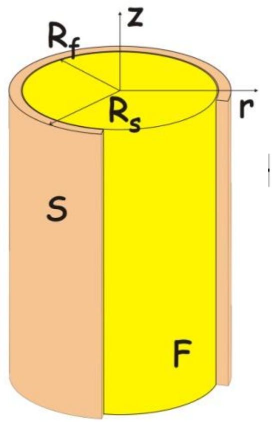
"0"-state

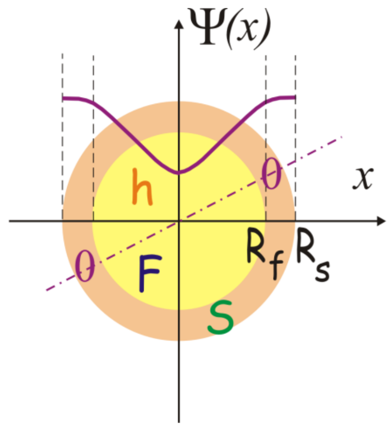
$I = I_c \sin\varphi$

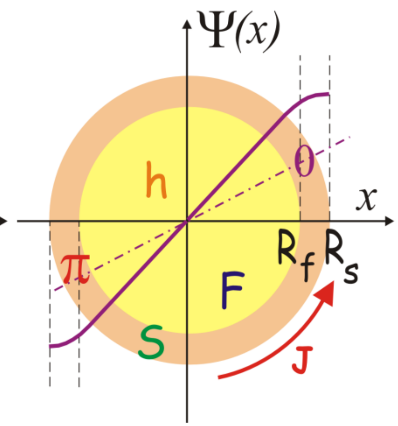
π-state

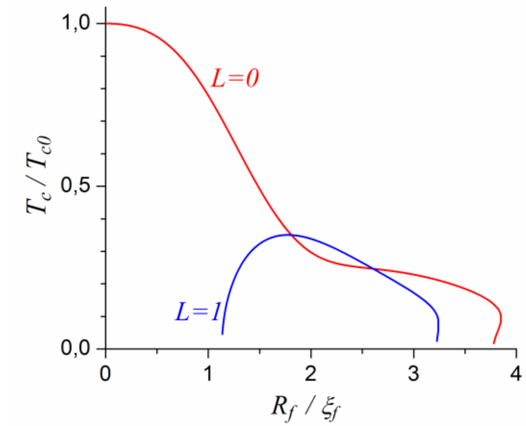
$I = I_c \sin(\varphi + \pi) = -I_c \sin(\varphi)$

Exchange effect vs Orbital effect

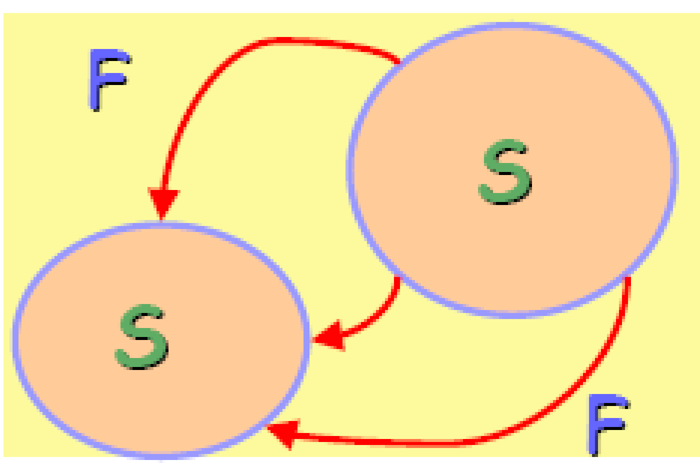


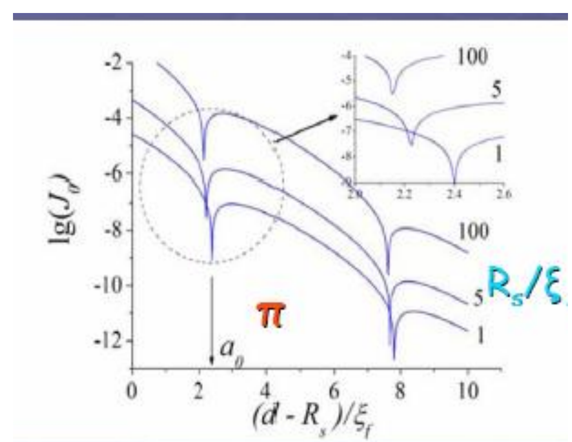






S grains in F matrix





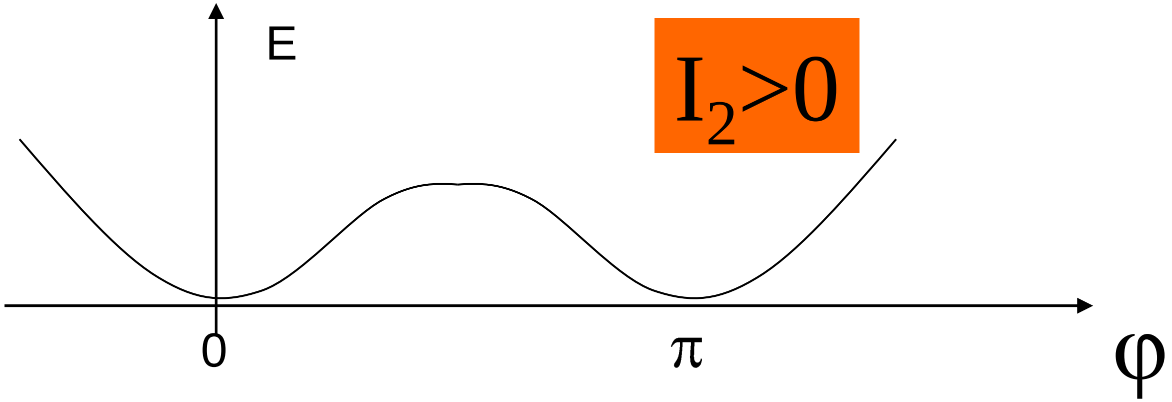
$(h / 2\pi T_c = 100 \; ; \; \tau_s h = 100 \; ; \; \gamma = 100)$

How the transition from 0- to π – state occurs?

$J(\varphi)=I_c \sin\varphi$; $I_c>0$ in the **0- state and**
$I_c<0$ in the **π – state**

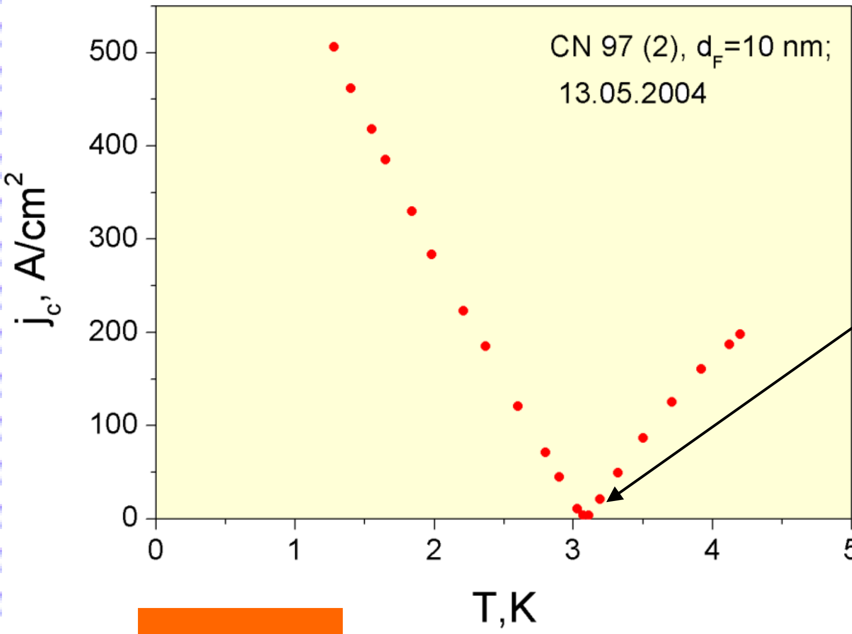
$$J(\varphi)=I_1 \sin\varphi + I_2 \sin 2\varphi$$

$$\text{Energy } E(\varphi)=(\Phi_0/2\pi c)[-I_1 \cos\varphi - (I_2/2)\cos 2\varphi]$$



$I_2>0$

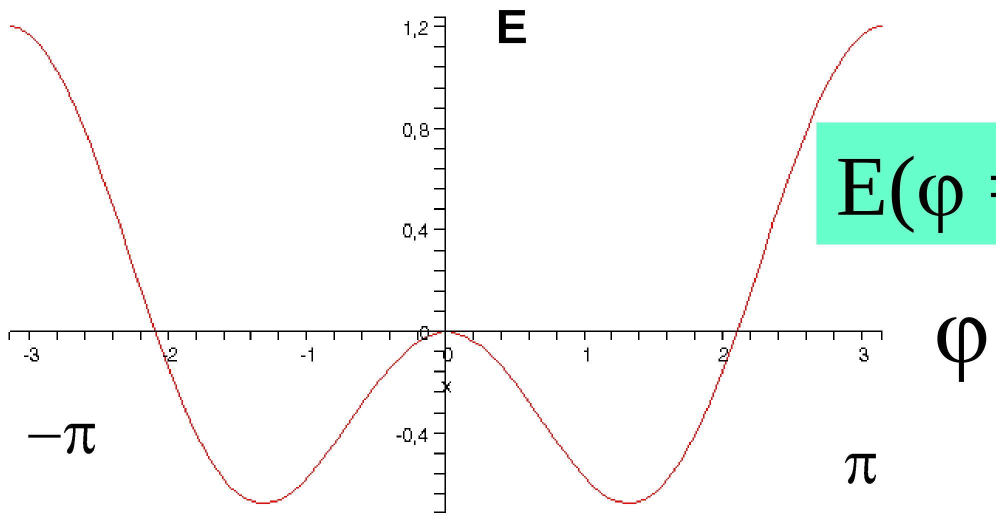
$I_c=|I_2|$



$J(\varphi)=I_2 \sin 2\varphi$

The realization of the equilibrium phase difference $0<\varphi_0<\pi$

$I_2<0$



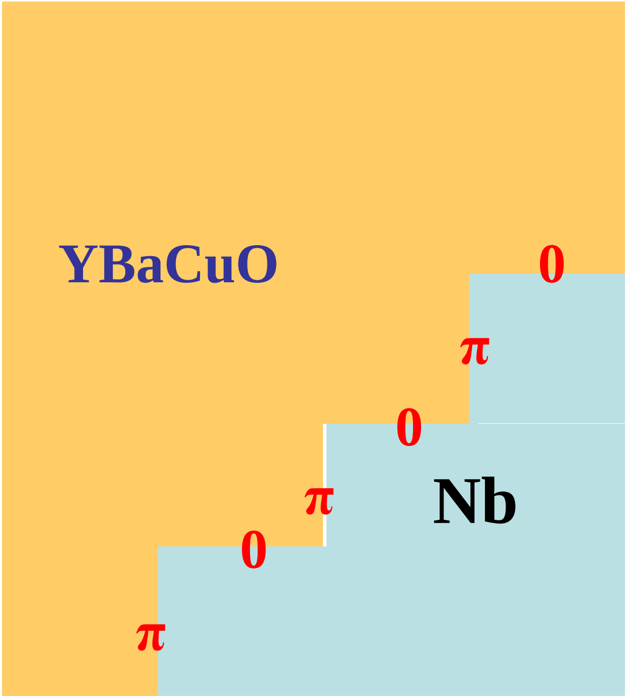
$E(\varphi=\varphi_0)=E(\varphi=-\varphi_0)$

Grain boundaries in YBaCuO

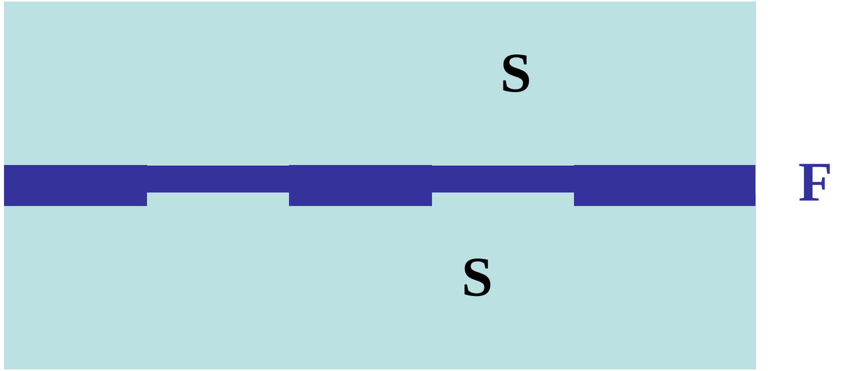
(Manhnhart, van-Harlingen et al. 1995-1996)

YBaCuO-Nb Josephson junctions of zig-zag geometry

(Hilgenkamp, Smilde et al. 2002)



Possibility to fabricate different alternating 0- and π- junctions



Arbitrary equilibrium phase difference: φ- junction

(Buzdin, Koshelev, 2003)

Different mechanism for the φ_0 - Josephson junction realization.

Recently the broken inversion symmetry (BIS) superconductors (like $CePt_3Si$) have attracted a lot of interest.

Very special situation is possible when the weak link in Josephson junction is a non-centrosymmetric magnetic metal with broken inversion symmetry !
Suitable candidates : MnSi, FeGe.

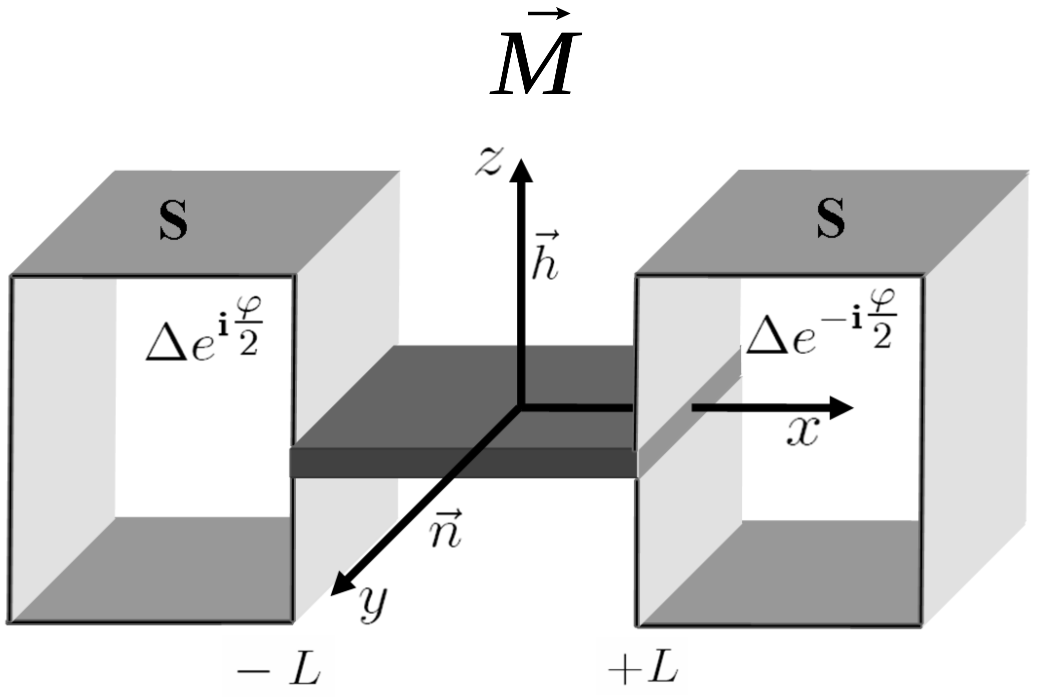
Josephson junctions with time reversal symmetry: $j(-\varphi) = -j(\varphi)$;
i.e. higher harmonics can be observed $\sim j_n \sin(n\varphi)$ –the case of the π junctions.

Without this restriction a more general dependence is possible
$j(\varphi) = j_0 \sin(\varphi + \varphi_0)$.

Rashba-type spin-orbit coupling $\alpha(\vec{\sigma} \times \vec{p}) \cdot \vec{n}$

$\vec{n}$ is the unit vector along the asymmetric potential gradient.

Geometry of the junction with BIS magnetic metal



$$F = a|\Psi|^2 + \gamma|\vec{D}\Psi|^2 + \frac{b}{2}|\Psi|^4 - \varepsilon\vec{n}\,[\vec{h}\times(\Psi(\vec{D}\Psi)^* + \Psi^*(\vec{D}\Psi))],$$

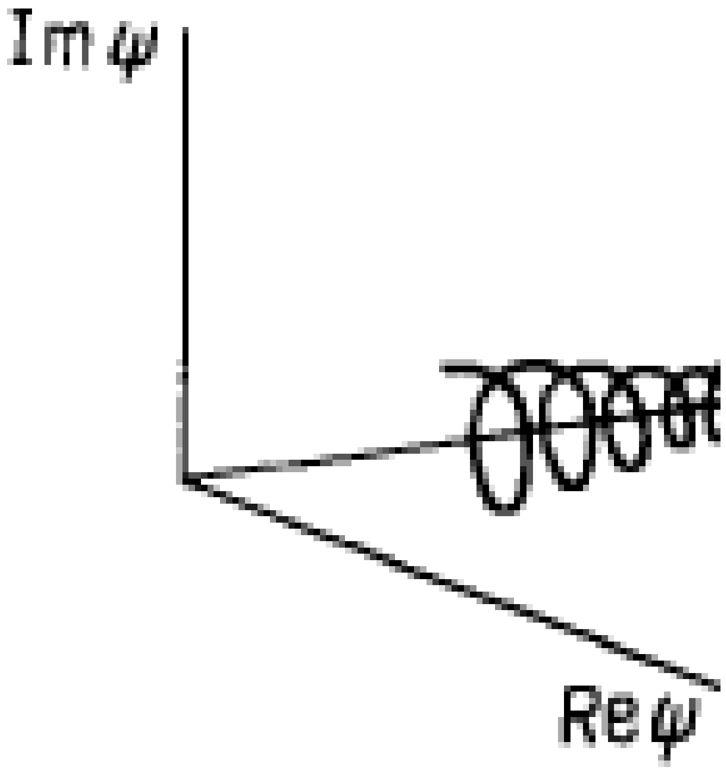
$$D_i = -i\partial_i - 2eA_i$$

$$a\Psi - \gamma\frac{\partial^2\Psi}{\partial x^2} + 2i\varepsilon h\frac{\partial\,\Psi}{\partial x} = 0,$$

$$\Psi \propto \exp(i\tilde{\varepsilon}x)\exp\left(-x\sqrt{\frac{a-a_c}{\gamma}}\right), \quad where\ \tilde{\varepsilon} = \frac{\varepsilon h}{\gamma}$$

φ_0 - Josephson junction (A. Buzdin, PRL, 2008).

$$\Psi \propto \exp(i\tilde{\varepsilon}x)\exp(-x\sqrt{\frac{a-a_c}{\gamma}}), \quad where\ \tilde{\varepsilon} = \frac{\varepsilon h}{\gamma}$$



In contrast with a π junction it is not possible to choose a real Ψ function !

φ_0 Josephson junction

$$j(\varphi) = j_c \sin(\varphi + \varphi_o)$$

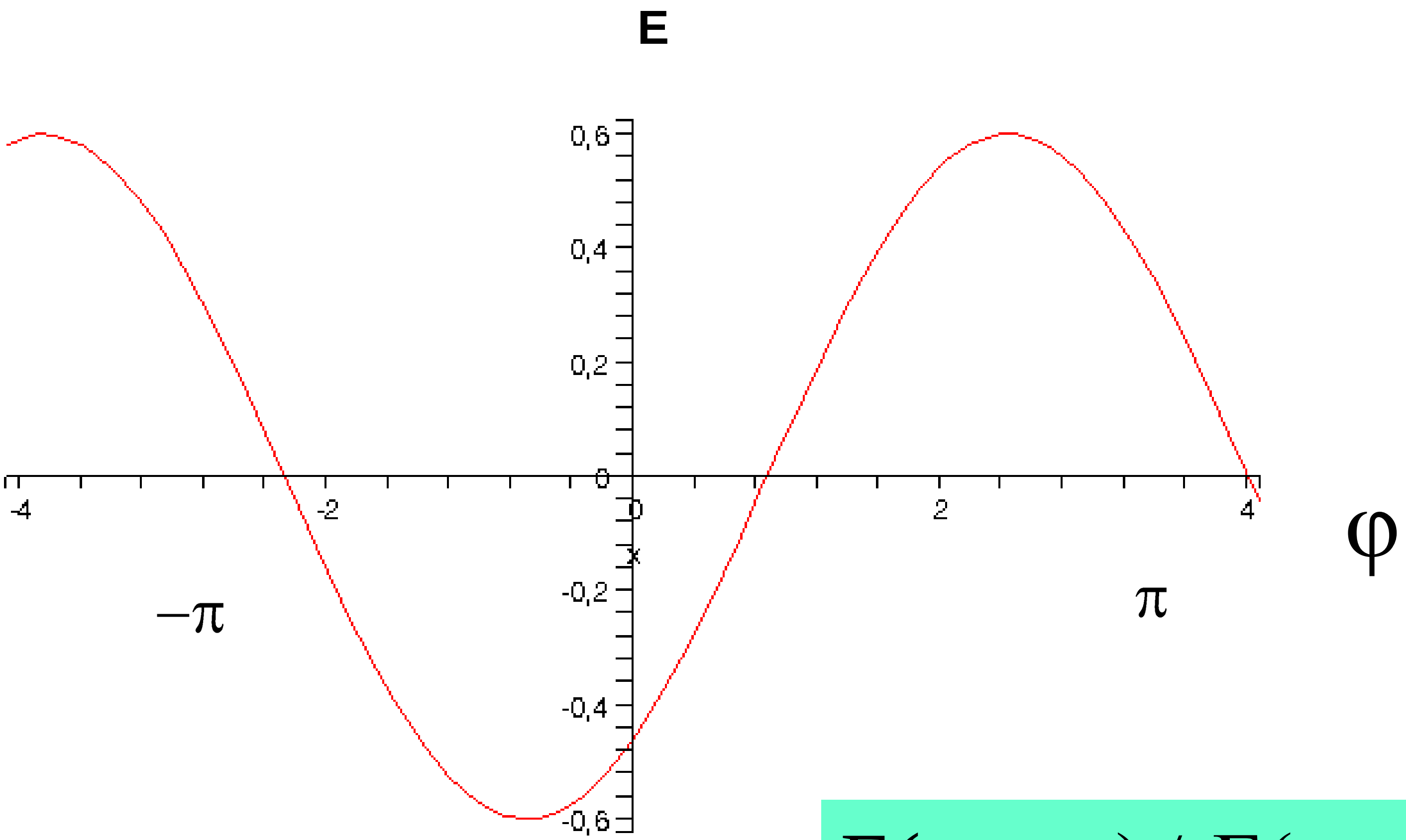
where $$\varphi_o = \frac{2\varepsilon h L}{\gamma}$$

The phase shift φ_0 is proportional to the length and the strength of the BIS magnetic interaction.

The φ_0 Junction is a natural phase shifter.

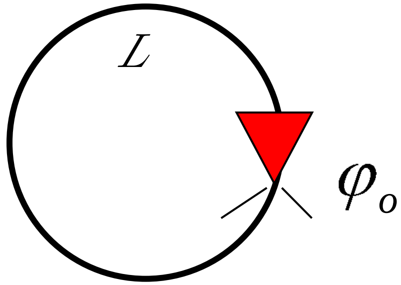
Energy $E_J(\varphi) \sim -j_c \cos(\varphi + \varphi_0)$

$E_J(\varphi) \sim -j_c \cos(\varphi + \varphi_0)$



$E(\varphi = \varphi_0) \neq E(\varphi = -\varphi_0)$

Spontaneous flux (current) in the superconducting ring with φ_0 - junction.



$$E(\varphi) = \frac{j_c}{2e}\left(-\cos(\varphi + \varphi_0) + \frac{k\varphi^2}{2} \right)$$

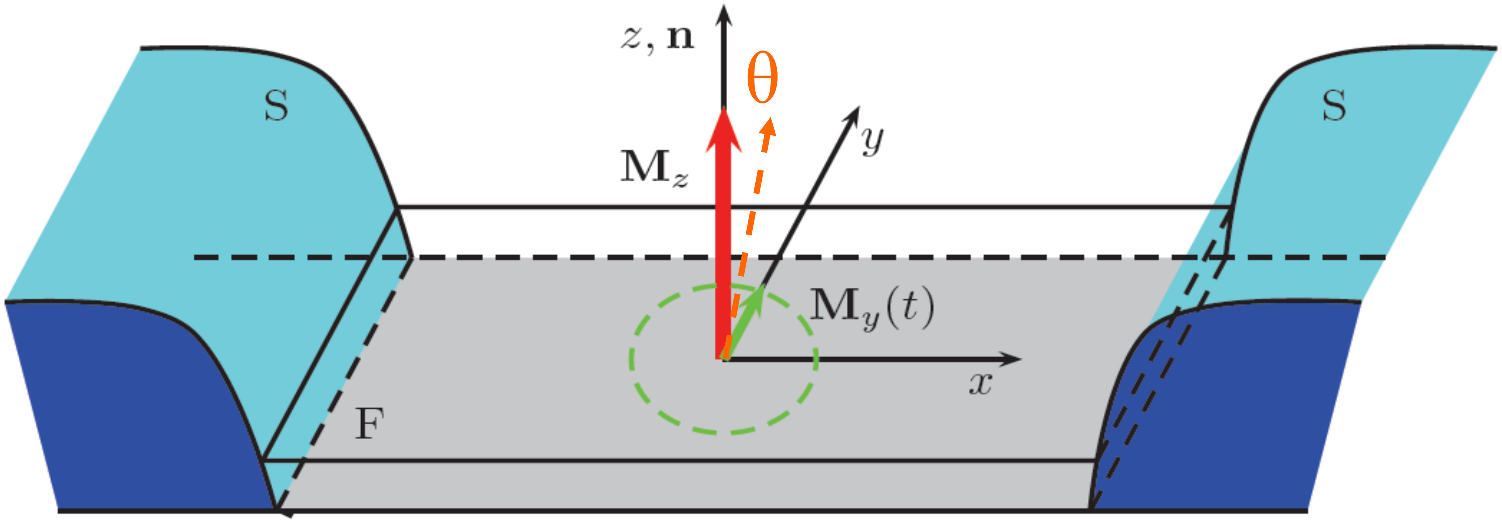
$$k = \frac{c\Phi_0}{2\pi L j_c}$$

In the k<<1 limit the junction generates the flux $\Phi=\Phi_0(\varphi_0/2\pi)$

$$\varphi_o = \frac{2\varepsilon h L}{\gamma}$$

Very important : The φ_0 junction provides a mechanism of a **direct coupling** between supercurrent (superconducting phase) and magnetic moment (z component).

Let us consider the following geometry :



$$\varphi_0 = x \frac{v_{so}}{v_F} \frac{M_y}{M_0}$$

$$\sin\theta = \frac{I}{I_c}\Gamma \quad \text{with } \Gamma = \frac{E_J}{K\mathcal{V}} x \frac{v_{so}}{v_F}$$

voltage-biased Josephson junction

$$\varphi(t) = \omega_J t$$

Magnetic anisotropy (easy z-axis) energy :

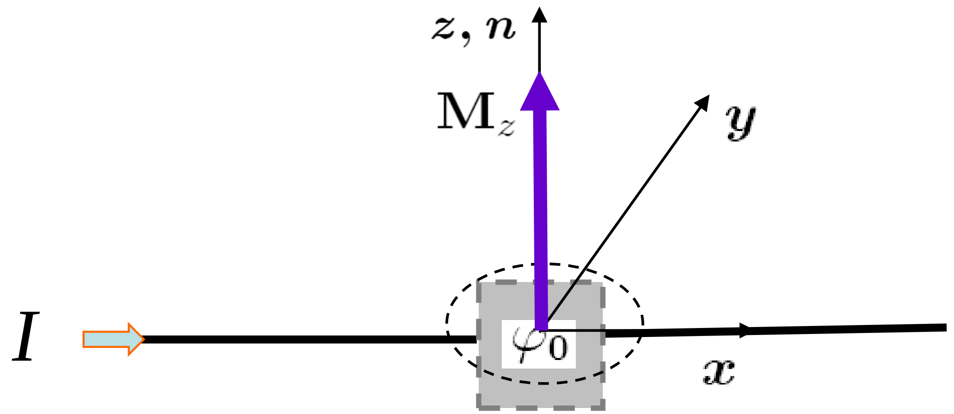
$$E_M = -\frac{K\mathcal{V}}{2}\left(\frac{M_z}{M_0}\right)^2$$

Coupling parameter :

$$\Gamma = \frac{E_J}{K\mathcal{V}} x \frac{v_{so}}{v_F}$$

Weak coupling regime : Γ<1.
Srong coupling regime : Γ>1.

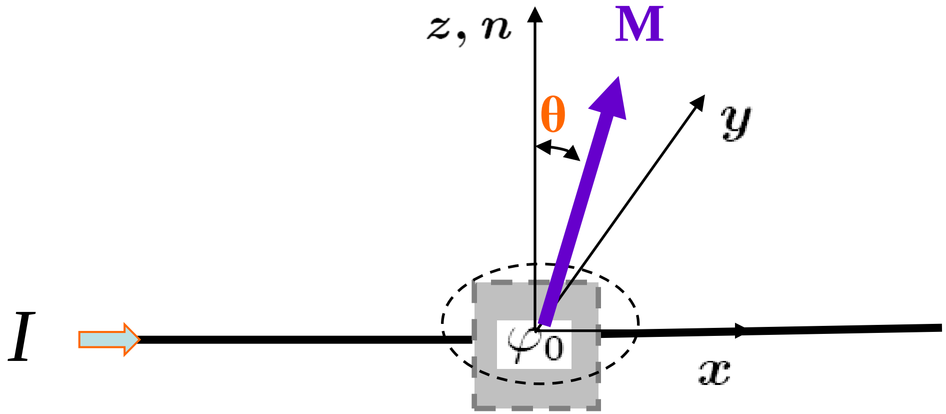
Let us consider first the φ_0 - junction when a constant current $I<I_c$ is applied :



$$\varphi_0 = x\frac{v_{so}}{v_F}\frac{M_y}{M_0}$$

$$E_{tot} = -\frac{\Phi_0}{2\pi}\varphi I + E_s(\varphi, \varphi_0) + E_M(\varphi_0)$$

Minimum energy condition: $\partial_\varphi E_{tot} = \partial_{\varphi_0} E_{tot.} = 0.$



The current provokes rotation of the magnetic moment :

$$\sin\theta = \frac{I}{I_c}\Gamma$$

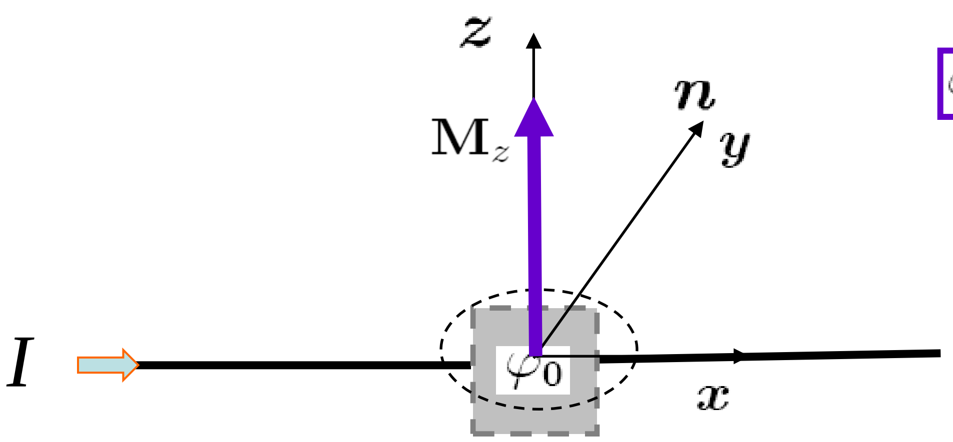
$$M_y = M_0 \sin\theta$$

For the case $\Gamma > 1$ when $I > I_c / \Gamma$ the moment will be oriented along the y-axis.

Applying to the φ_0 - junction a current (phase difference) we can generate the magnetic moment rotation.

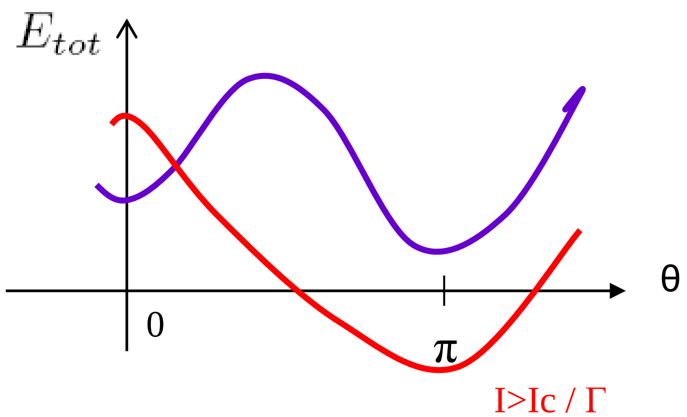
a.c. current -> moment's precession!

What happens if the SO gradient is along **y**-axis?



$$\varphi_0 = x\,(v_{\mathrm{so}}/v_F)\cos\theta$$

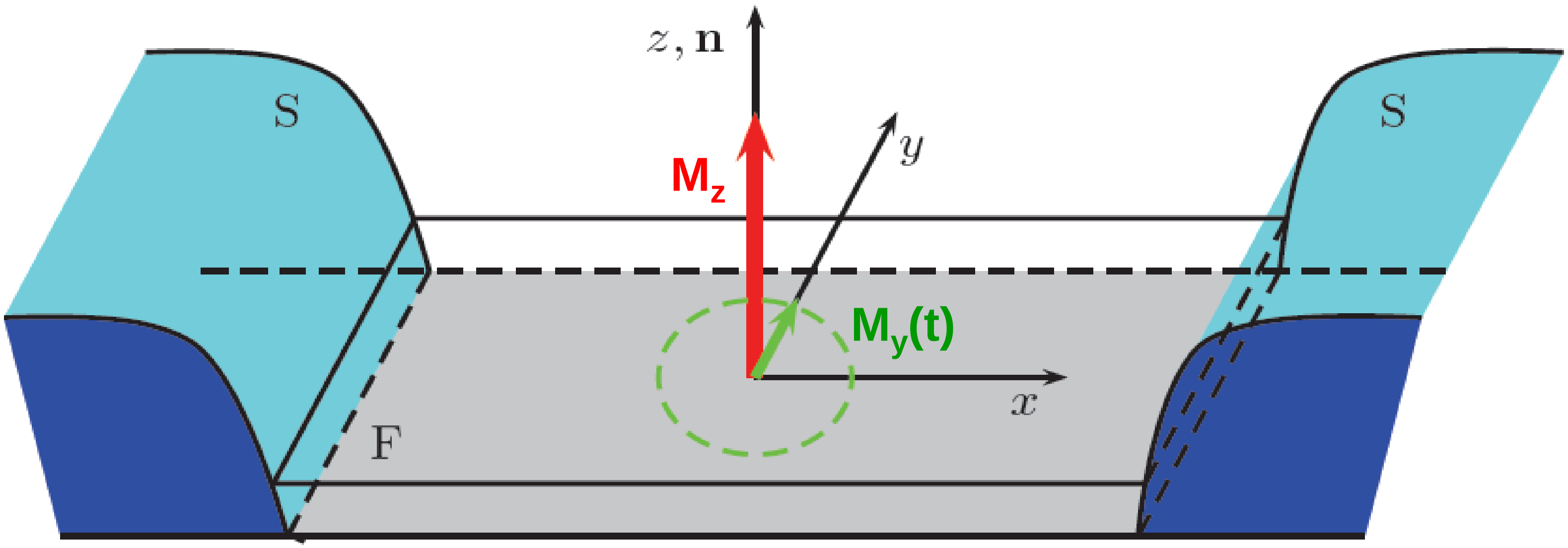
The total energy has two minima θ=(0, π).



The current pulses would provoke the switches of **M** between θ=0 and θ= π orientations. This corresponds to the transition of the junction from $+\varphi_0$ to $-\varphi_0$ state.

Magnetic moment precession – voltage-biased φ_0- junction

$\varphi(t) = \omega_J t$



$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \wedge \mathbf{H}_{\mathrm{eff}} + \frac{\alpha}{M_0}\left(\mathbf{M} \wedge \frac{d\mathbf{M}}{dt}\right),$$

where $\mathbf{H}_{\mathrm{eff}} = -\delta F / \mathcal{V}\delta\mathbf{M}$ is the effective magnetic field

$$\mathbf{H}_{\mathrm{eff}} = \frac{K}{M_0}\left[\Gamma \sin\left(\omega_J t - r\frac{M_y}{M_0}\right)\hat{\boldsymbol{y}} + \frac{M_z}{M_0}\hat{\boldsymbol{z}}\right]$$

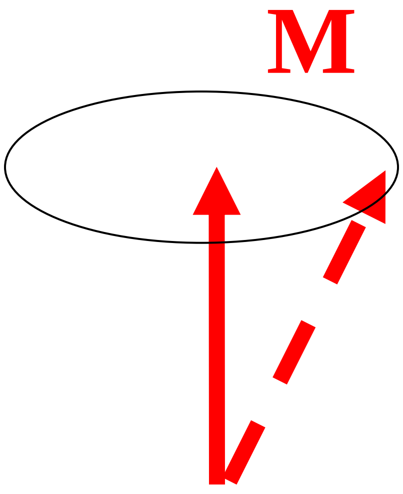
$r = x v_{\mathrm{so}}/v_F$

Landau-Lifshitz equation :

$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \wedge \mathbf{H}_{\text{eff}} + \frac{\alpha}{M_0}\left(\mathbf{M} \wedge \frac{d\mathbf{M}}{dt}\right)$$

$$\mathbf{H}_{\text{eff}} = \frac{K}{M_0}\left[\Gamma \sin\left(\omega_J t - r\frac{M_y}{M_0}\right)\hat{\boldsymbol{y}} + \frac{M_z}{M_0}\hat{\boldsymbol{z}}\right]$$

Magnetic moment precession :



$$\frac{I}{I_c} = \sin \omega_J t + \frac{\Gamma r}{2}\frac{1}{\omega^2 - 1}\sin 2\omega_J t + \ldots,$$

Weak coupling regime : Γ<<1.

$$\omega = \omega_J/\omega_F$$

Without damping

$$m_x(t) = \frac{\Gamma\omega\cos\omega_J t}{1-\omega^2} \text{ and } m_y(t) = -\frac{\Gamma\sin\omega_J t}{1-\omega^2}.$$

$$\frac{I}{I_c} = \sin\omega_J t + \frac{\Gamma r}{2}\frac{1}{\omega^2-1}\sin 2\omega_J t + \ldots,$$

With damping

$$m_y(t) = \frac{\omega_+ - \omega_-}{r}\sin\omega_J t + \frac{\alpha_- - \alpha_+}{r}\cos\omega_J t,$$

$$\omega_\pm = \frac{\Gamma r}{2}\frac{\omega \pm 1}{\Omega_\pm} \text{ and } \alpha_\pm = \frac{\Gamma r}{2}\frac{\alpha}{\Omega_\pm} \quad \text{with } \Omega_\pm = (\omega \pm 1)^2 + \alpha^2.$$

$$I(t) \approx I_c\sin\omega_J t + I_c\frac{\omega_+ - \omega_-}{2}\sin 2\omega_J t + \\ + I_c\frac{\alpha_- - \alpha_+}{2}\cos 2\omega_J t + I_0(\alpha)$$

The current acquires a d..c. component !

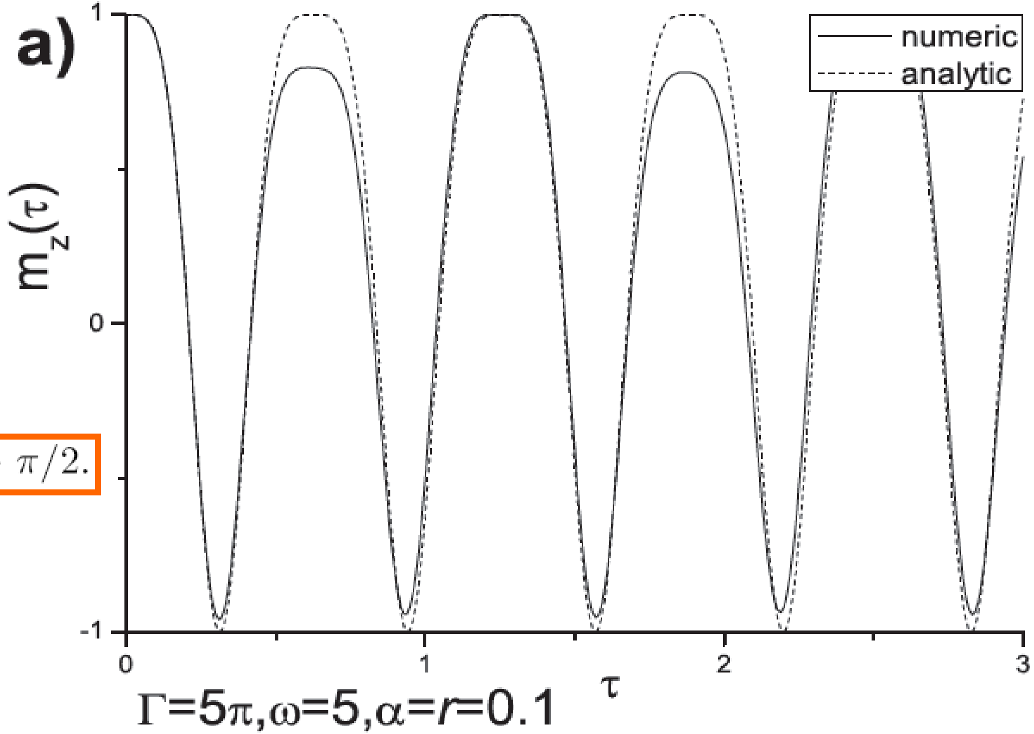
$$I_0(\alpha) = \frac{\alpha\Gamma r}{4}\left(\frac{1}{\Omega_-} - \frac{1}{\Omega_+}\right)$$

Srong coupling regime : Γ>>1.

if r <<1 $r = xv_{\text{so}}/v_F$ then $m_y \approx 0$ and without damping we have :

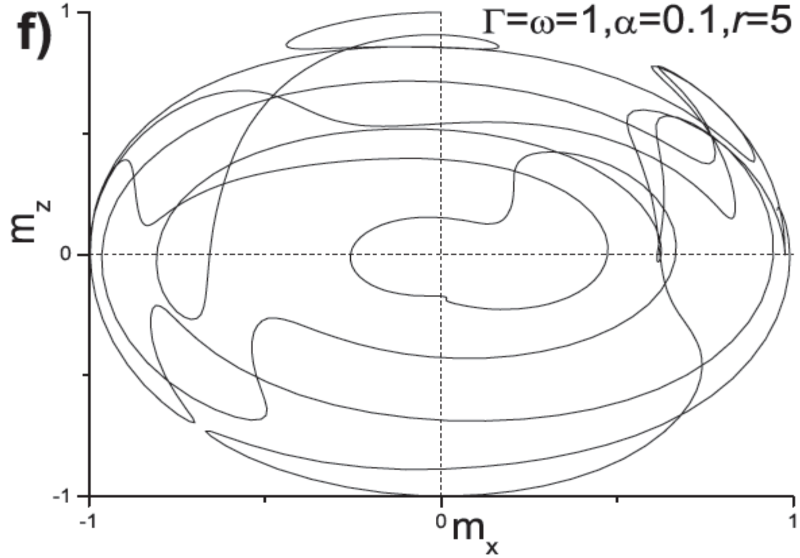
$$\begin{cases} m_x(t) = \sin\left[\frac{\Gamma}{\omega}\left(1-\cos\omega_J t\right)\right] \\ m_z(t) = \cos\left[\frac{\Gamma}{\omega}\left(1-\cos\omega_J t\right)\right] \end{cases}$$

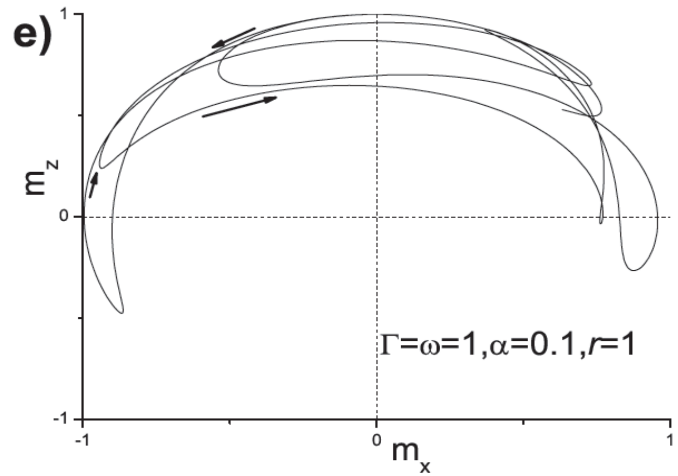
complete reversal being induced by $\Gamma/\omega > \pi/2$.



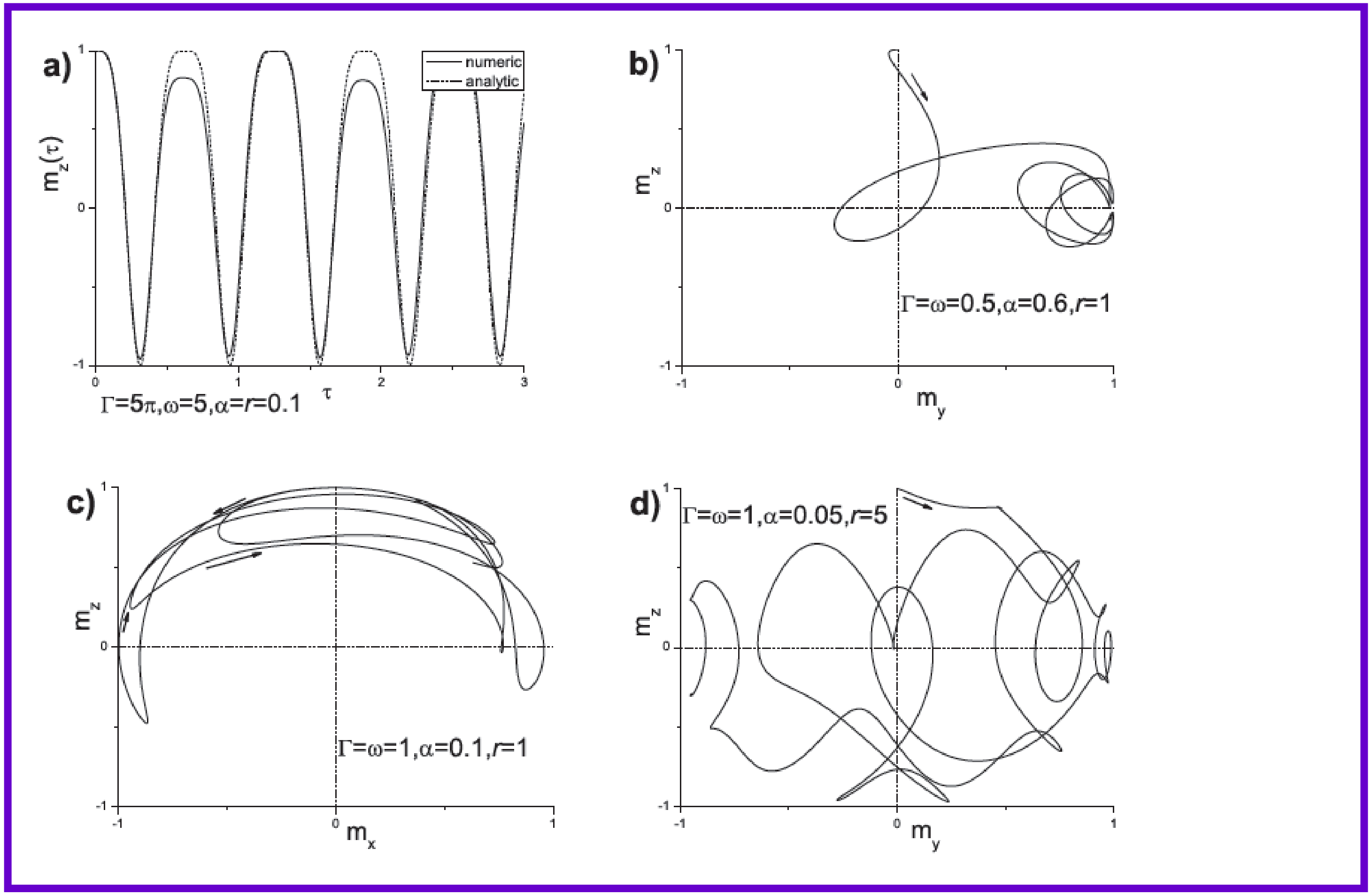
Comparison between analytic results (dashed line) and numerical computation.

Pretty complicated dynamics regimes in general case





Complicated regime of the magnetic dynamics :



For more details – see (F. Konschelle and A. Buzdin, PRL, 2009).

A very rich physics emerges if the φ_0 – junction is exposed to the microwave radiation ω_1.

In addition to the Shapiro steps at ω_J=n ω_1 it will appear the half-interger-steps.

The microwave field may also generate the additional precession with ω_1 frequency.

Dramatic increase of the amplitude of the Shapiro steps near the ferromagnetic resonance would be expected.

Conclusions

- **The BIS (broken inversion symmetry) magnets provide a mechanism of the realization of the novel φ_0 - junctions with the very special properties.**

- **In these φ_0 - junctions a direct (linear) coupling between superconductivity and magnetism is realized. They are the natural phase shifter.**

- **Josephson current permits to switch the orientation of the magnetic moment and a.c. Josephson effect provokes its precession.**

- **The magnetic dynamics of the φ_0 – junctions may be very rich.**