Vortex Molecules in Thin Film of Layered Superconductors

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in collaboration with

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inspired by experiments of **Tonomura** (Hitachi, Japan)

Institut for Solid State Physics, Chernogolovka, February 22, 2011

Outline

◈ **Introduction.**

Repulsion between vortices in superconductors.
Interaction of vortices in thin films. Pearl's effect.
Attraction between vortices in anisotropic superconductors.

◈ **Tilted vortices in thin films of layered superconductors.**
Interaction potential for a vortex pair.

◈ **Interplay between long-range attraction and repulsion.**
Vortex molecules. Multiquanta flux lattices.

Wine Import Company

Japan : Vortex

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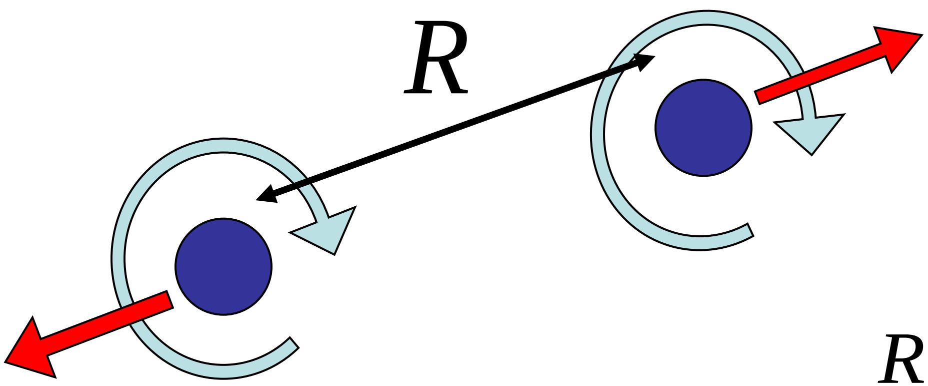
Vortex Wine Aerating Funnel

Interaction between vortices in type-II superconductors: bulk samples

- ◈ **repulsive interaction between vortices**
- ◈ **hexagonal vortex lattice**



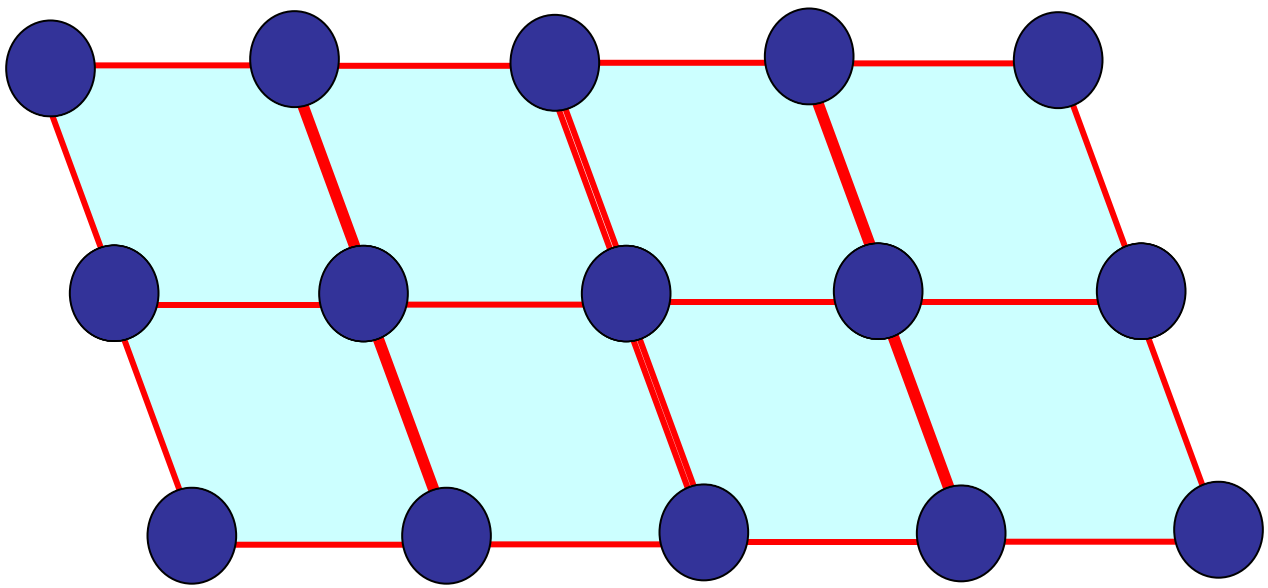
$$\varepsilon_{\text{int}} \approx \frac{\phi_0^2 d}{8\pi^2\lambda^2} K_0\left(\frac{R}{\lambda}\right)$$

$$R \ll \lambda \qquad K_0\left(\frac{R}{\lambda}\right) \approx \ln\left(\frac{\lambda}{R}\right)$$

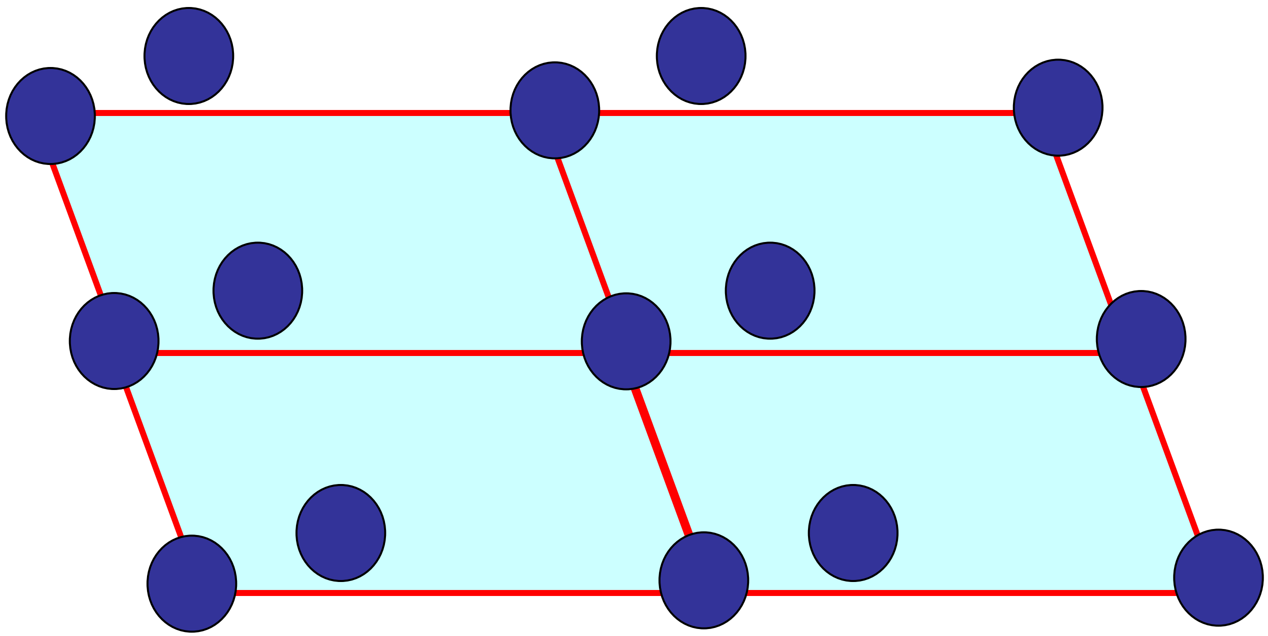
$$R \gg \lambda \qquad K_0\left(\frac{R}{\lambda}\right) \approx \sqrt{\frac{\pi\lambda}{2R}} \cdot e^{-R/\lambda}$$

$$\vec{F} = \frac{\phi_0}{c}\left[\vec{j}, \vec{z}_0\right]$$

Question: may the number of vortices per unit cell be 2 or 3 or …?



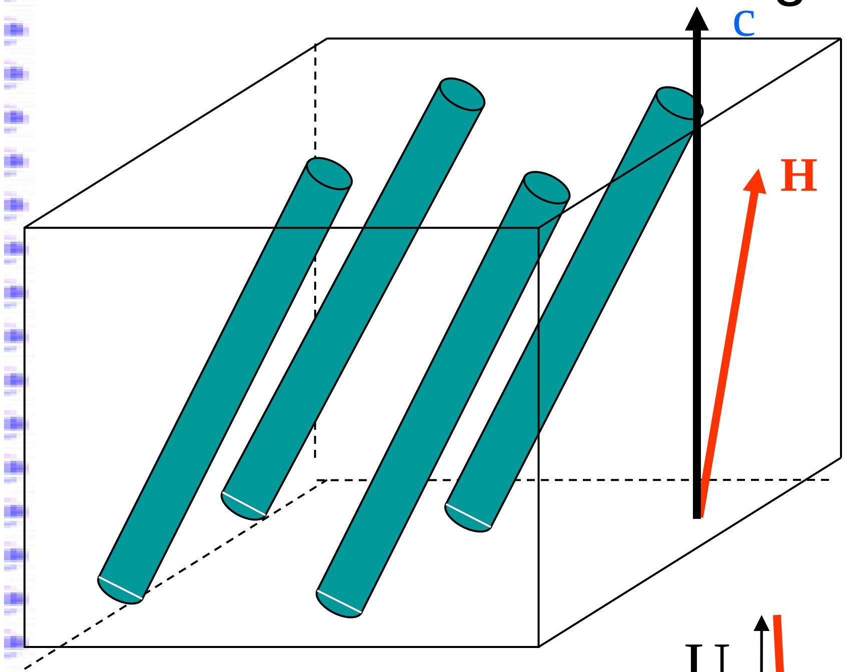
Question: may the number of vortices per unit cell be 2 or 3 or …?



Yes, if there is attraction between vortices !

Example : superconductors with 2 component order parameter.

In the layered superconductors with moderate anisotropy when the field **H** is inclined towards **c**-axis, vortices are no longer directed along the field

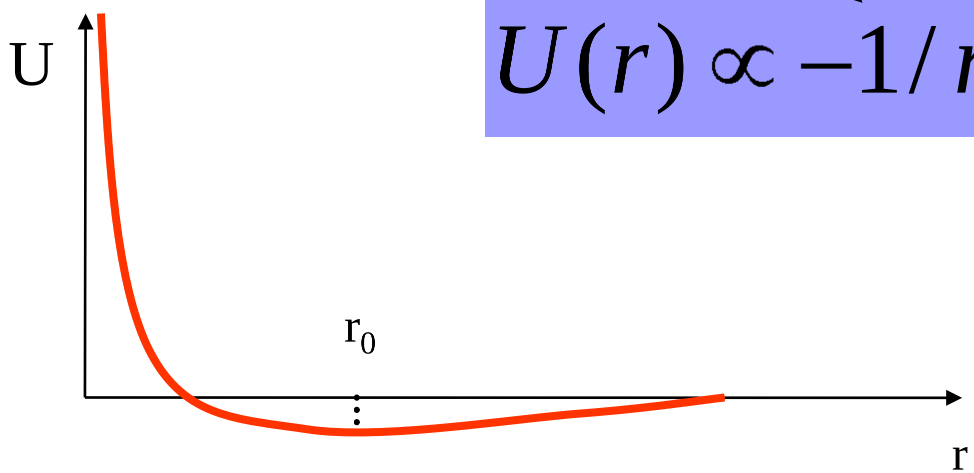


In the plane **H - c** the long range attraction appears!

A. Buzdin and A. Simonov, JETP Lett. (1990).

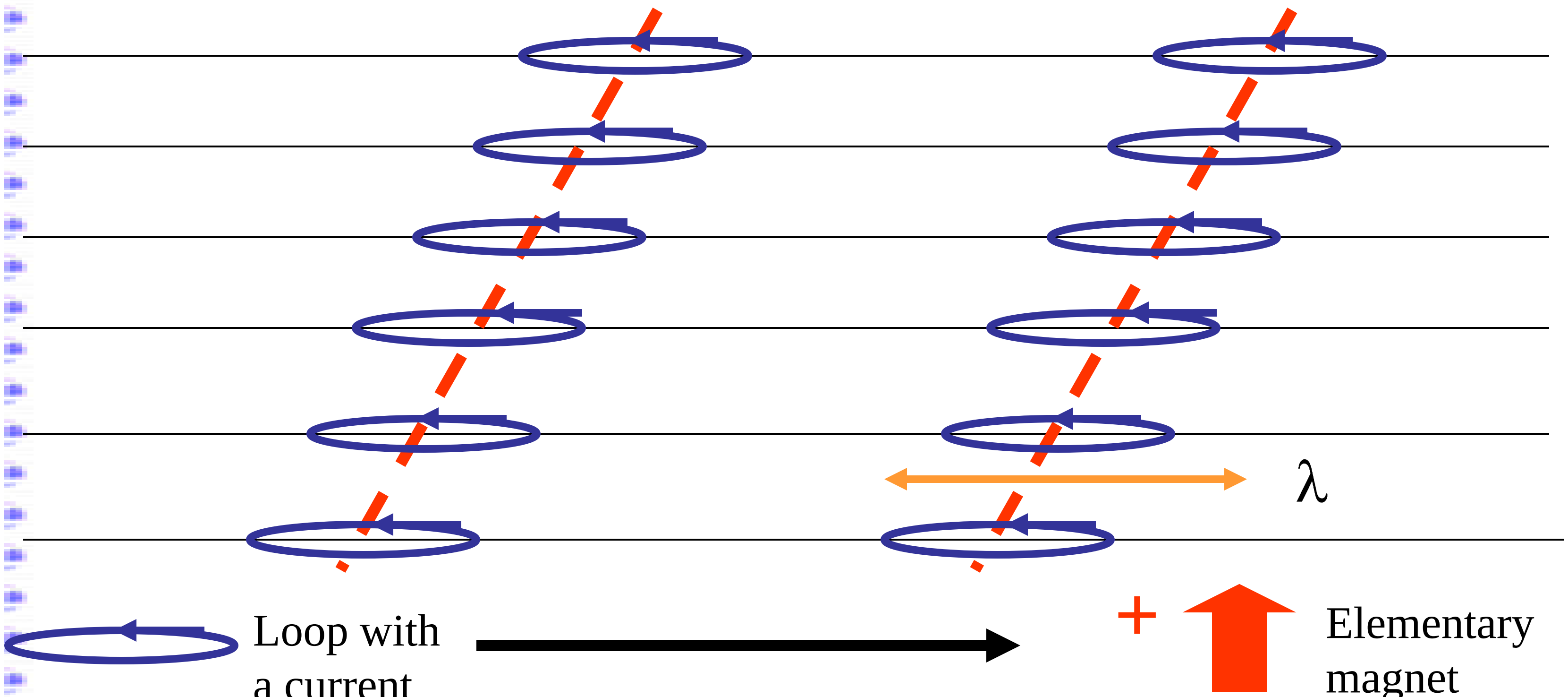
A. Grishin, et al. JETP (1990)

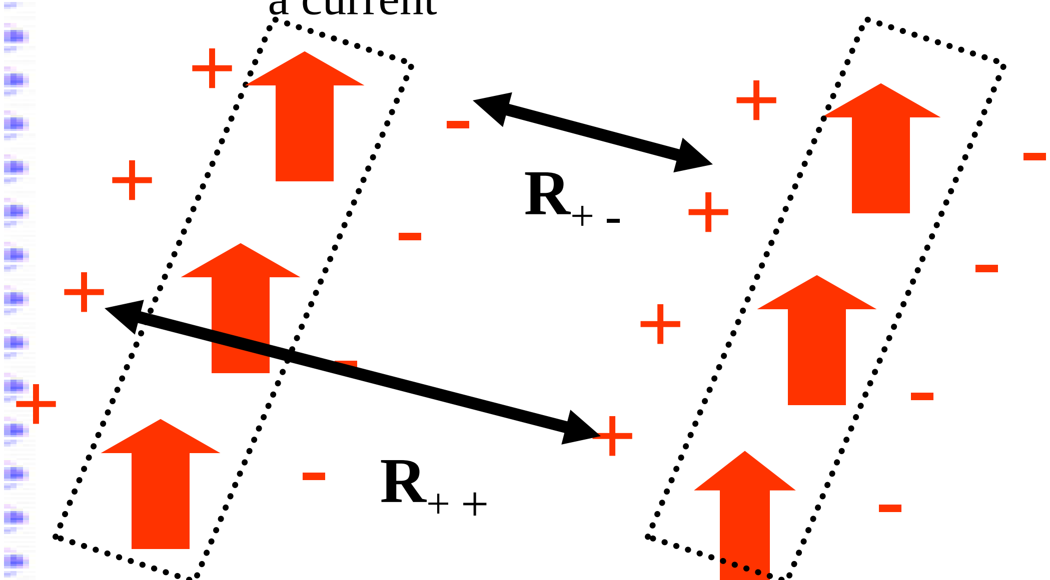
$$U(r) \propto -1/r^2$$



The attraction between tilted vortices leads to the formation of **vortex chains.**

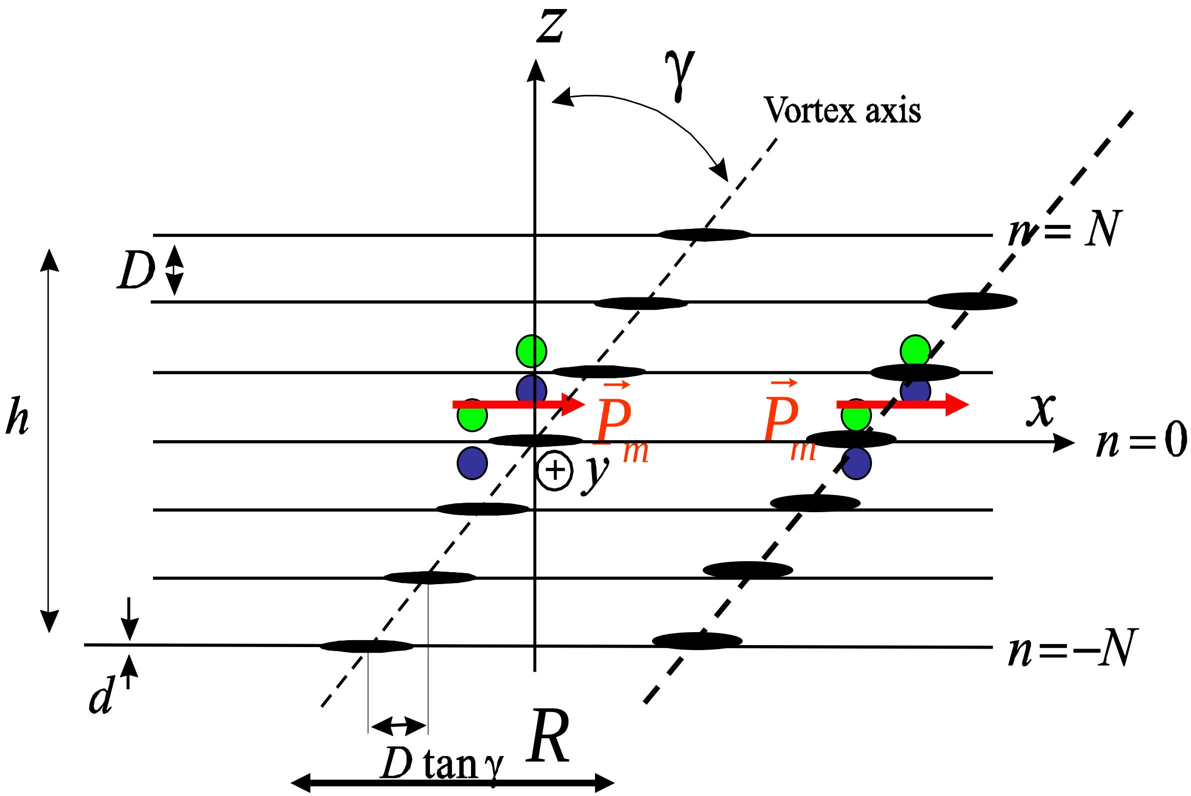
Qualitative explanation of the long range vortex attraction





The distance $\mathbf{R}_{+-} < \mathbf{R}_{++}$,
then we have
the net attraction!

Layered superconductors



Magnetic field in between the layers (at large distances r) = field of 2D $\vec{P}_m$ magnetic dipoles

$$P_m \propto \phi_0 \tan\gamma$$

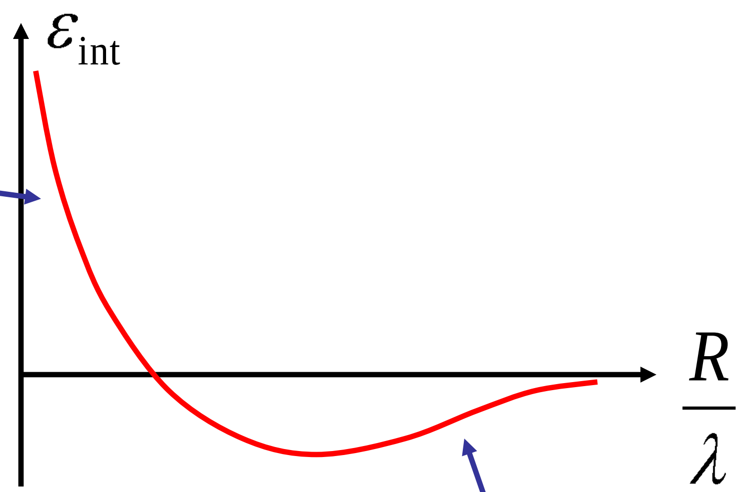
Interaction energy of 2D magnetic dipoles $E_{\text{int}} \propto -\dfrac{P_m^2}{R^2} \propto -\dfrac{\phi_0^2 \tan^2\gamma}{R^2}$

Attraction of vortices in anisotropic superconductors

$$\varepsilon_{\text{int}} \approx \frac{\phi_0^2 d}{8\pi^2\lambda^2} K_0\left(\frac{R}{\lambda}\right)$$

$$R \ll \lambda$$

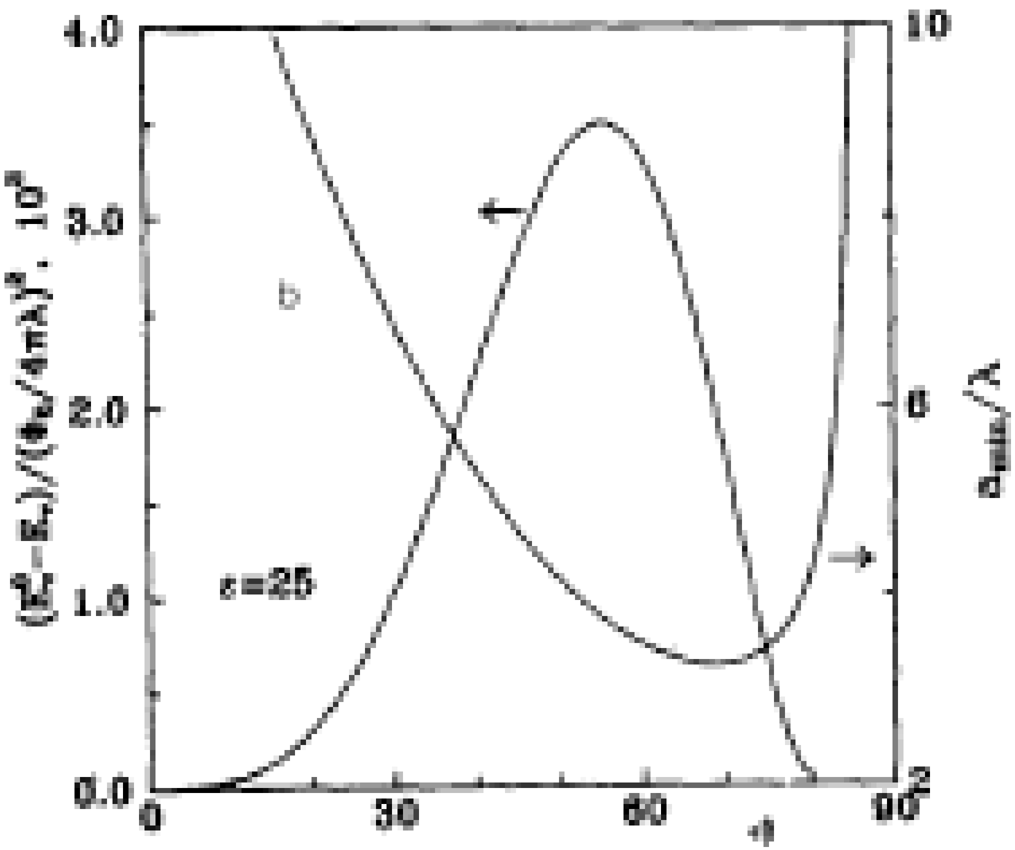
$$K_0\left(\frac{R}{\lambda}\right) \approx \ln\left(\frac{\lambda}{R}\right)$$



$$\varepsilon_{\text{int}} \approx -\frac{\phi_0^2}{8\pi^2}\frac{d\tan^2\gamma}{R^2}$$

$$\mathbf{h}_{\mathbf{k}} = \frac{\Phi_0}{\lambda^2}\,\frac{\lambda^{-2} + (q^2 + Q^2)(1 + \epsilon\nu_z^2)}{(\lambda^{-2} + q^2 + Q^2)\,[\lambda^{-2} + (1+\epsilon)q^2 + (1 + \epsilon\nu_z^2)Q^2\,]}\,, \qquad (2)$$

where $\nu_z^2 = (\hat{\nu}\mathbf{1})^2$, $k_x = \mathbf{Q}$, $k_y = \mathbf{q}$, $\varepsilon = (m_\perp/m_\parallel) - 1 > 0$ and $\lambda^{-2} = 8\pi|\Psi|^2 e^2/mc^2$. The energy of a vortex is given as a function of its orientation by[2-4]



at $r \ll \lambda_\perp$

$U(r) \propto -1/r^2$

Long ranged attraction!

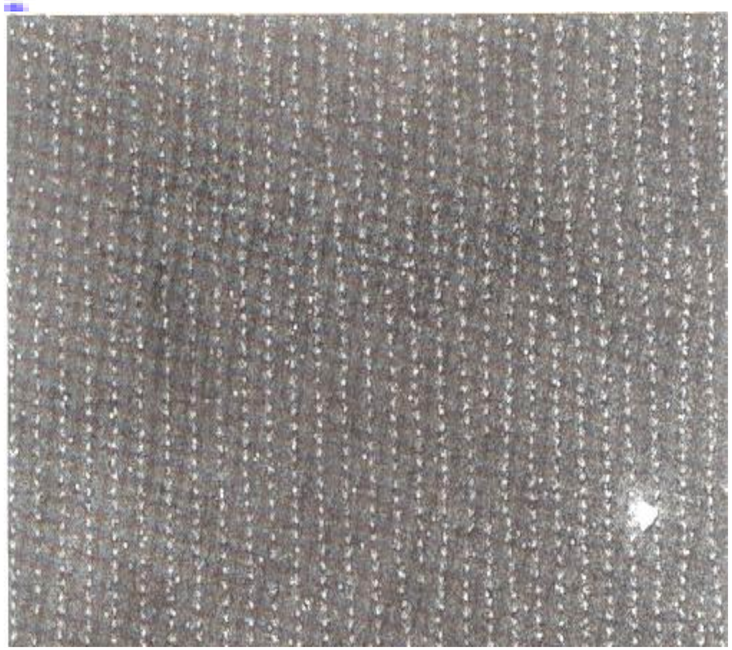
Anisotropy of YBaCuO type

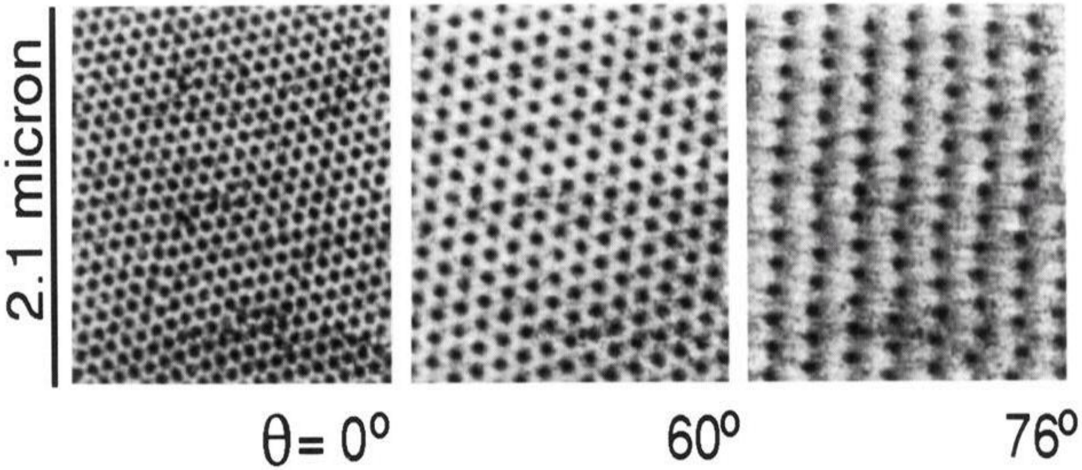
$\varepsilon = m_c/m_{ab} - 1$

The prediction of vortex chain formation has been confirmed later by experiment

SEM micrograph of the vortex chains with a field applied 70° from the **c**-axis in YBaCuO
Images obtained by Gammel et al, Phys. Rev. Lett. (92)

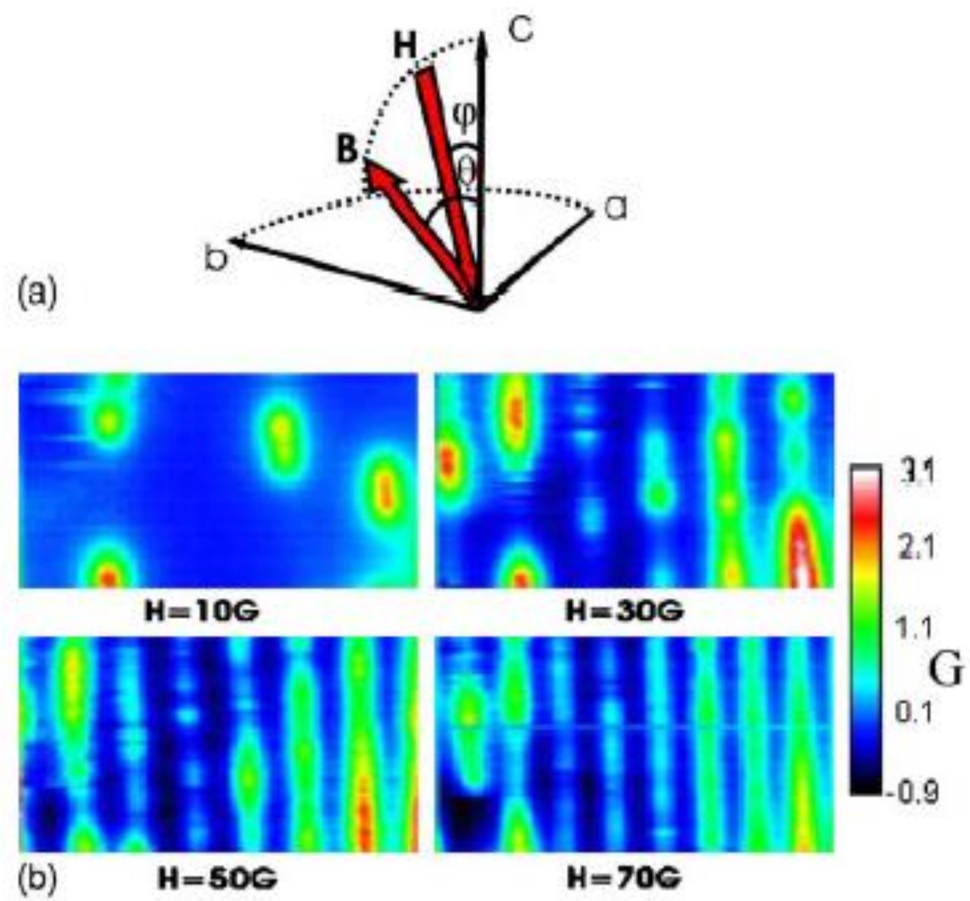
STM conductance image showing vortex locations on the surface of 2H-$NbSe_2$ $\gamma \sim 11$
Images obtained by Hess et al, Phys. Rev. Lett. (92)



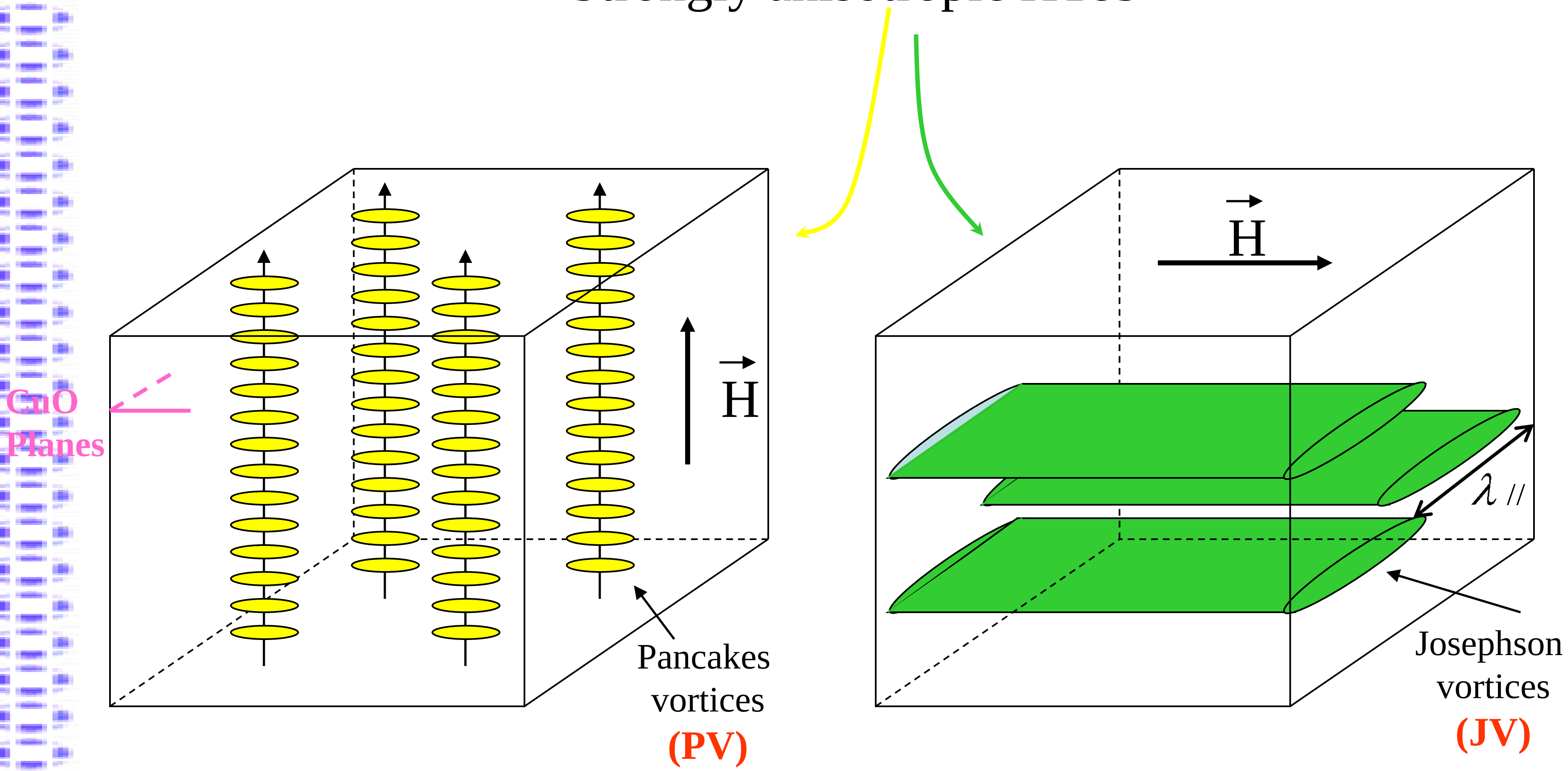


Imaging of the vortex chains in anisotropic triplet(?) superconductor Sr_2RuO_4

By *V. Dolocan et al., PRL, 95, 097004 (2005).*
The image was obtained by scanning μSQUID force microscope.



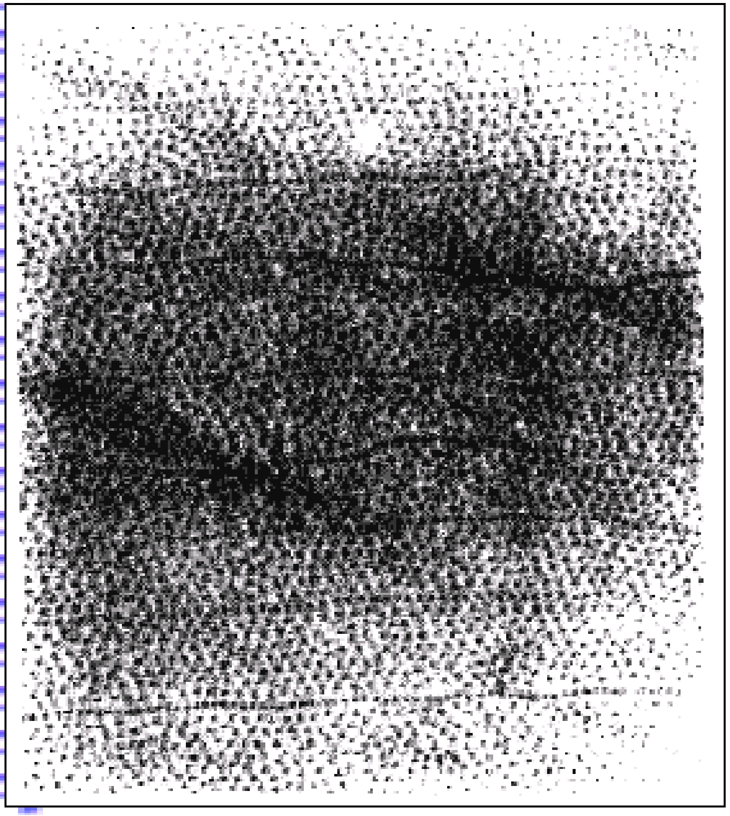
Strongly anisotropic HTcS



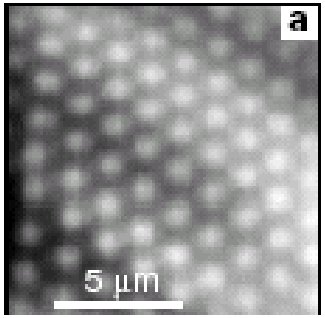
In a first approximation these two types of vortices (PV and JV) are not interacting and form a **crossing lattice**

Taking into account the interaction between JV and PV (A. Koshelev, *Phys. Rev. Lett.* (1999)) it is possible to explain the mixed chain-lattice formation in BSCCO observed by decoration by C. Bolle et al., *Phys. Rev. Lett.* (1991)

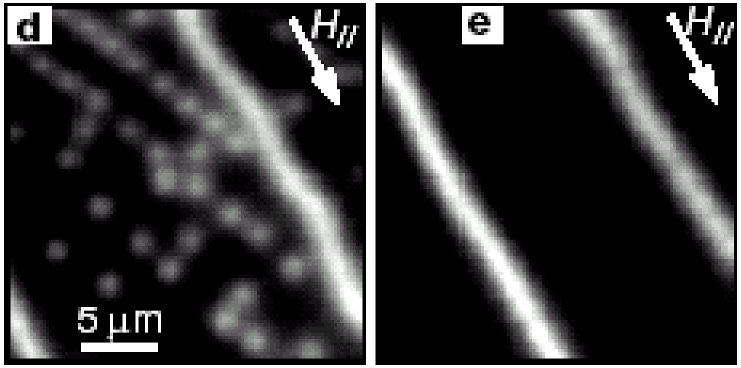
and recently by scanning Hall probe microscopy by A. Grigorenko et al., *Nature* **414**, 728 (2001)



$H_{//} = 21$ Oe
$H_{\perp} = 12$ Oe



$H_{//} = 0$
Classic Abrikosov lattice

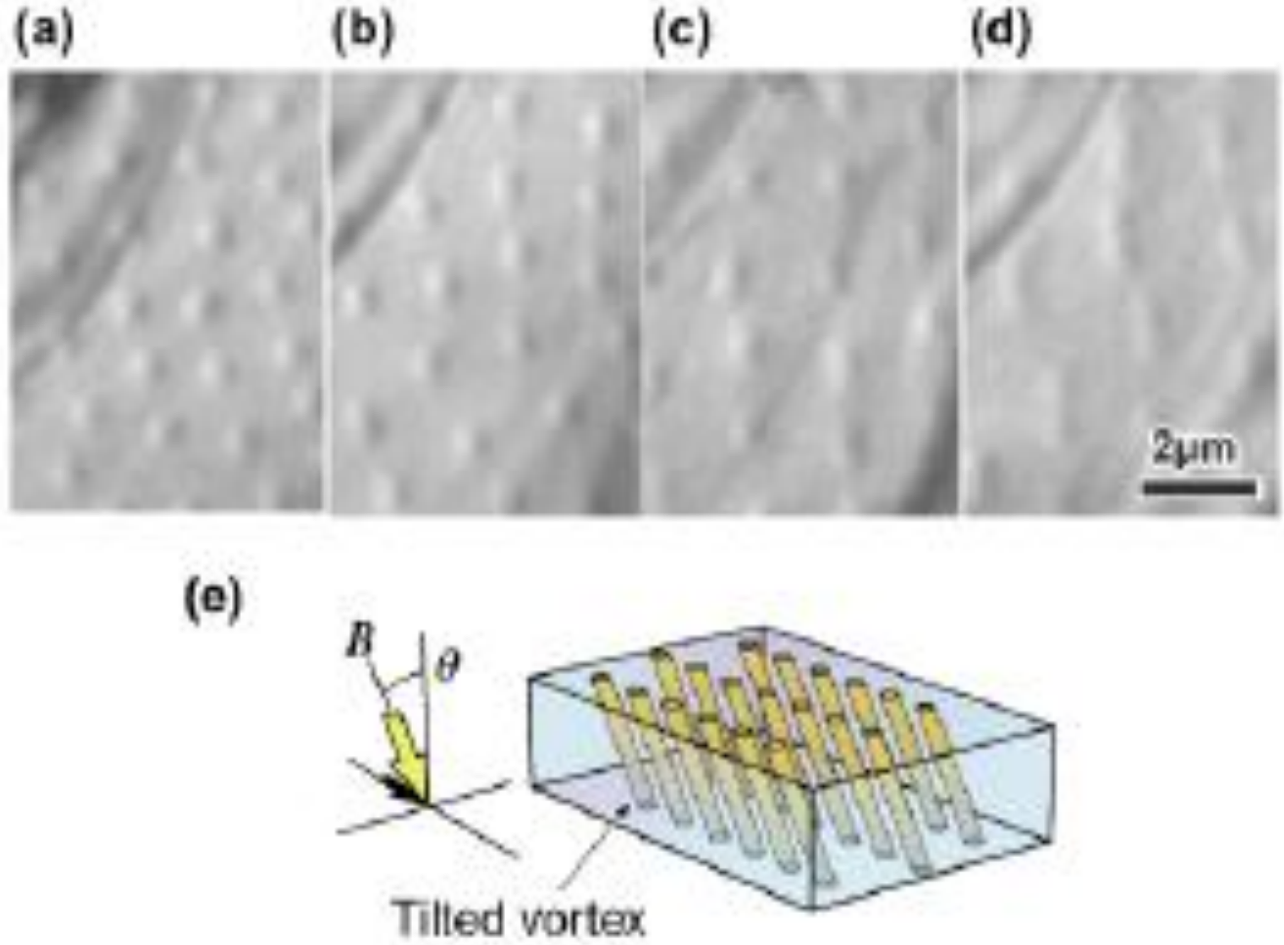


$H_{//} = 27$Oe
$H_{\perp} = 2$Oe

$H_{//} = 40$Oe
$H_{\perp} = 1.2$Oe

Vortex Chains

The very appropriate technique to study the vortices –Lorentz microscopy technique developed by A. Tonomura group.



A.Tonomura et al.
PRL, ***88***, *237001 (2002)*

The Lorentz microscopy technique needs the very thin films.
It is an important point for further discussion!

Recent Lorentz microscopy data of Tonomura's group

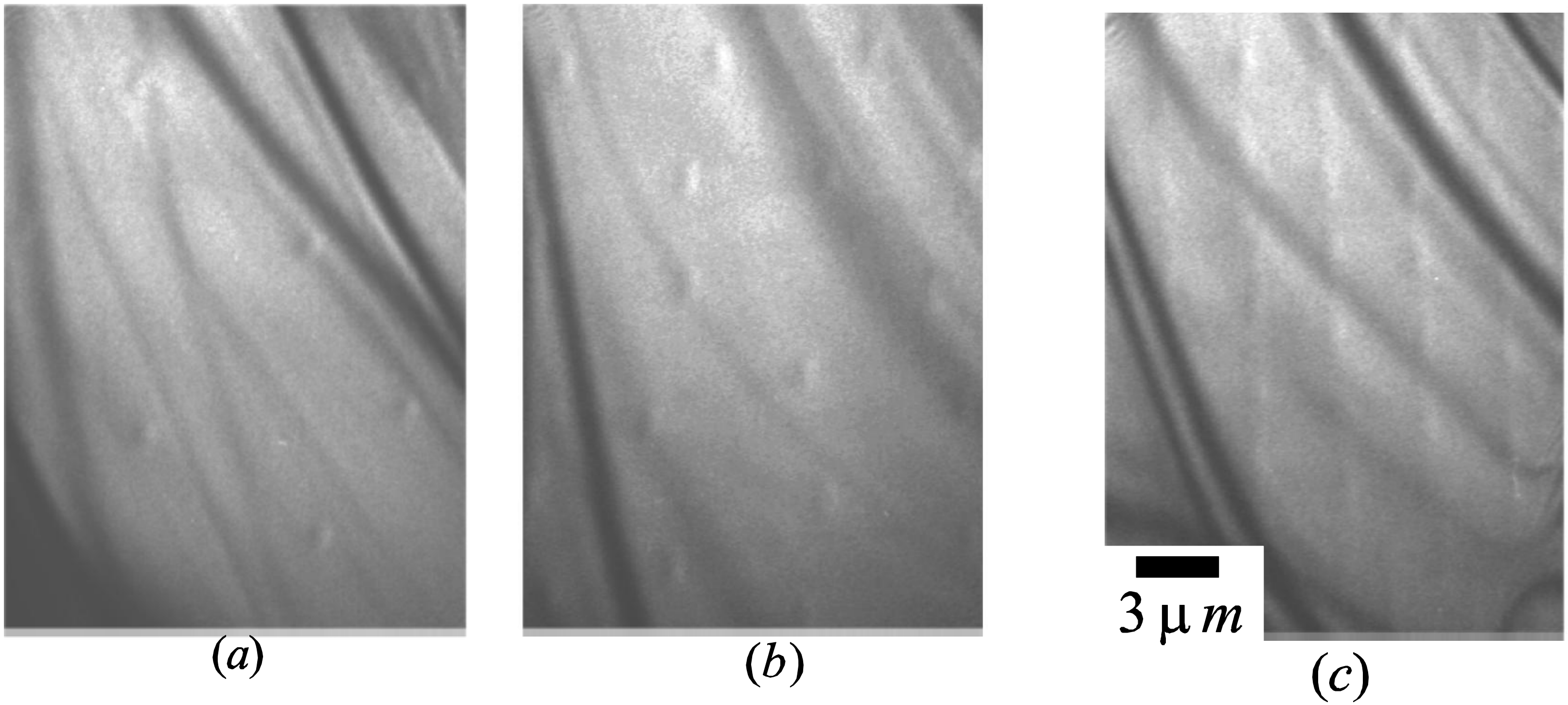
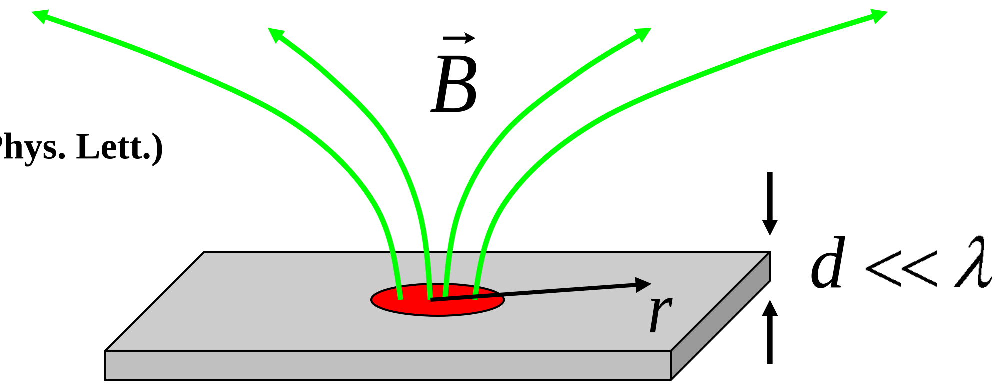


FIG. 3: Lorentz microscopy images for YBCO films at temperature $T = 5.7K$, the field tilting angle $\gamma_H = 85^\circ$ and absolute magnetic field values (a) $B_0 = 10G$, (b)$B_0 = 20G$, (c) $B_0 = 60G$.

No vortex chains at small tilting angles of vortices!

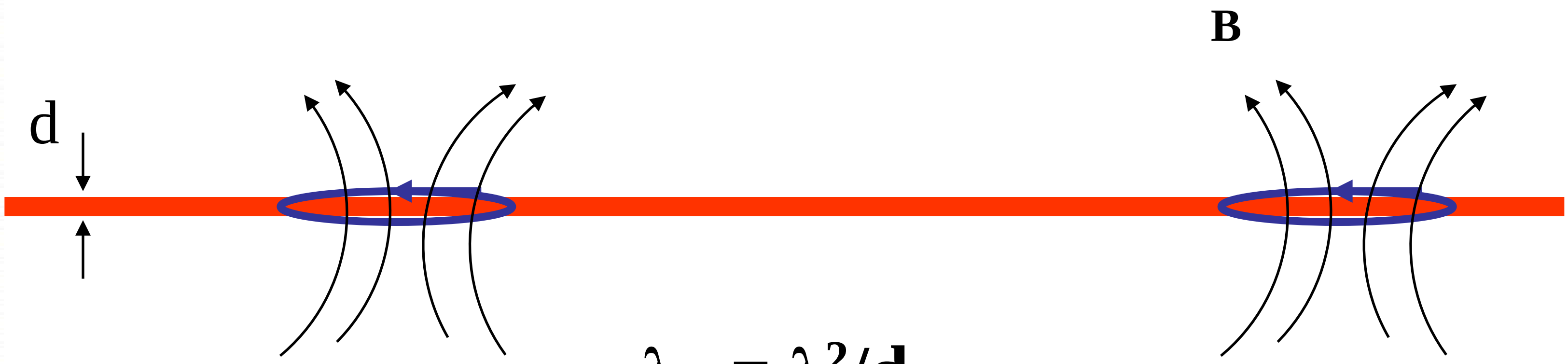
Electrodynamics of vortices in thin films. Pearl's effect.

(J. Pearl, 1964, Appl. Phys. Lett.)



	Current density	**Magnetic field**	**Intervortex interaction potential**
$r << \dfrac{\lambda^2}{d}$	$j \approx \dfrac{c\phi_0 d}{8\pi^2\lambda^2 r}$	$b_z \approx \dfrac{\phi_0 d}{4\pi\lambda^2 r}$	$\varepsilon_{\text{int}} \approx \dfrac{\phi_0^2 d}{8\pi^2\lambda^2}\ln\left(\dfrac{\lambda}{R}\right)$
$r >> \dfrac{\lambda^2}{d}$	$j \approx \dfrac{c\phi_0}{4\pi^2 r^2}$	$b_z \approx \dfrac{2\phi_0\lambda^2}{\pi d r^3}$	$\varepsilon_{\text{int}} \approx \dfrac{\phi_0^2}{4\pi^2 R}$

Tilted vortices in thin superconducting film. Very special case!



$$\lambda_{eff} = \lambda^2/d$$

The intervortex interaction occurs mainly via the space outside the film.

The long ranged intervortex interaction (repulsion) **is not exponential.** At r>> λeff

$$U_{rep} \sim \Phi_0^2/r$$

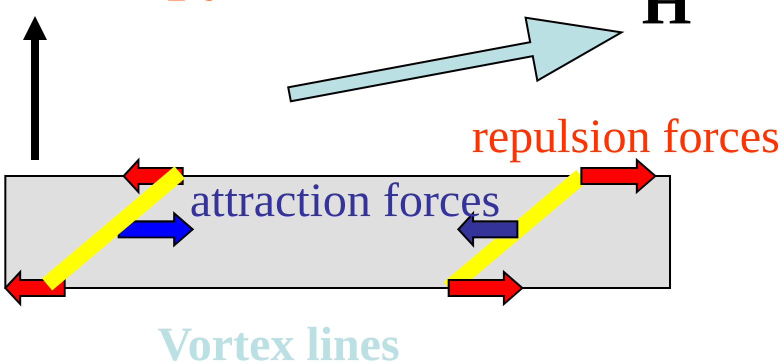
More long ranged than the tilted vortex attraction!

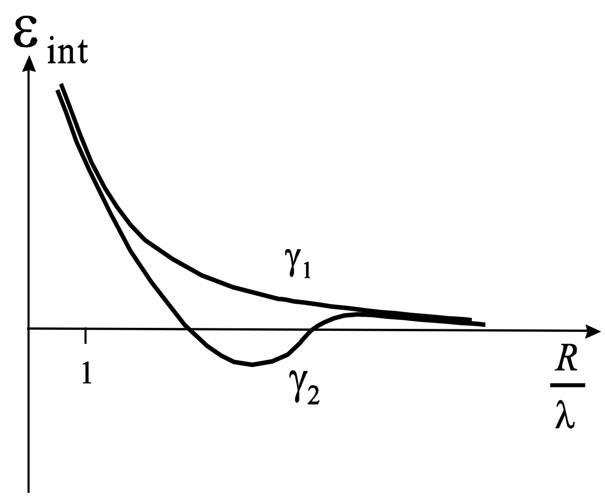
$$U_{att}(r) \propto -1/r^2$$

Interplay between the intervortex attraction and repulsion in thin films

The Lorentz microscopy technique is the technique
“Par excellence” to study the vortex interaction in the regime d~λ !

Anisotropy axis





Surface contribution to the intervortex repulsion

$$\varepsilon_{\text{int}} \approx \frac{\phi_0^2}{8\pi^2}\left(-\frac{d_{eff}\tan^2\gamma}{R^2}+\frac{2}{R}\right)$$

Effective film thickness = $d_{eff} = d - 2\lambda_{||}\tanh\dfrac{d}{2\lambda_{||}}$

Layered system of finite thickness: London model

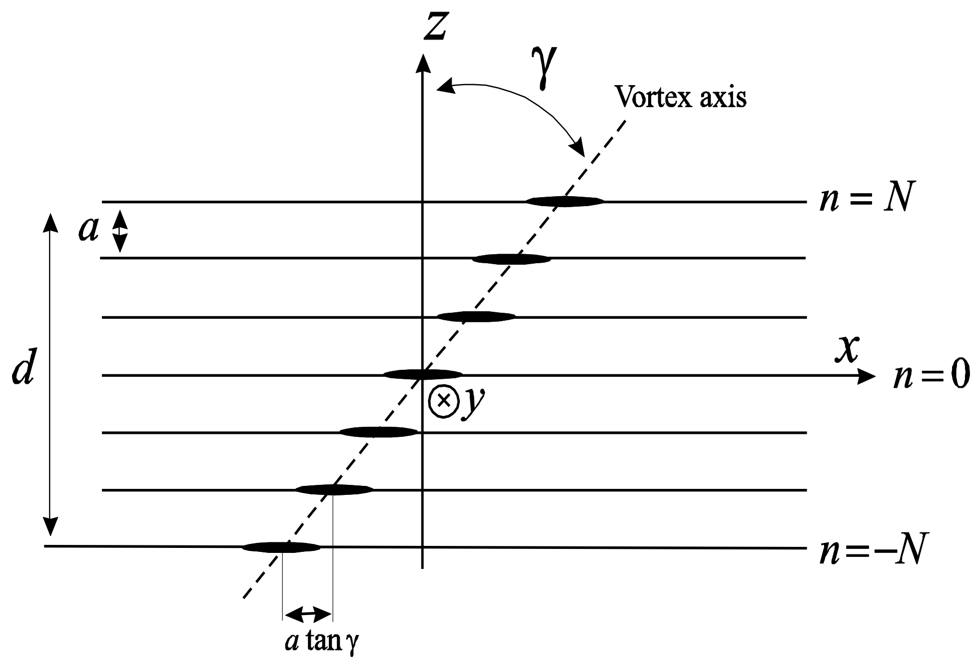
$$R << \lambda_{\perp}$$

$$rot\,rot\vec{A} = \frac{4\pi}{c}\vec{j} = \frac{a}{\lambda_{||}^{2}}\sum_{n=-N}^{N}\left(\vec{\Phi}_n(\vec{r}) - \vec{A}\right)\delta(z - na)$$

$$\vec{\Phi}_n(\vec{r}) = \vec{\Phi}(\vec{r} - \vec{r}_n)$$

$$\vec{\Phi}(\vec{r}) = \frac{\phi_0}{2\pi}\frac{[\vec{z}_0\vec{r}]}{r^2}$$

$$\vec{r}_n = \vec{x}_0 \cdot na \cdot \tan\gamma$$



Interaction potential for a vortex pair

$$\vec{j}_n = \frac{ca}{4\pi\lambda_{||}^2}\left(\Phi_n(\vec{q})\, e^{i\vec{q}\vec{r}_n} - \vec{A}_n(\vec{q})\right) = \frac{ca}{4\pi\lambda_{||}^2} f_n(\vec{q})\,\Phi(\vec{q})$$

$$f_n(\vec{q}) + \frac{a}{2q\lambda_{||}^2}\sum_{m=-N}^{N} e^{-|n-m|qa} f_m = e^{i\vec{q}\vec{r}_n}$$

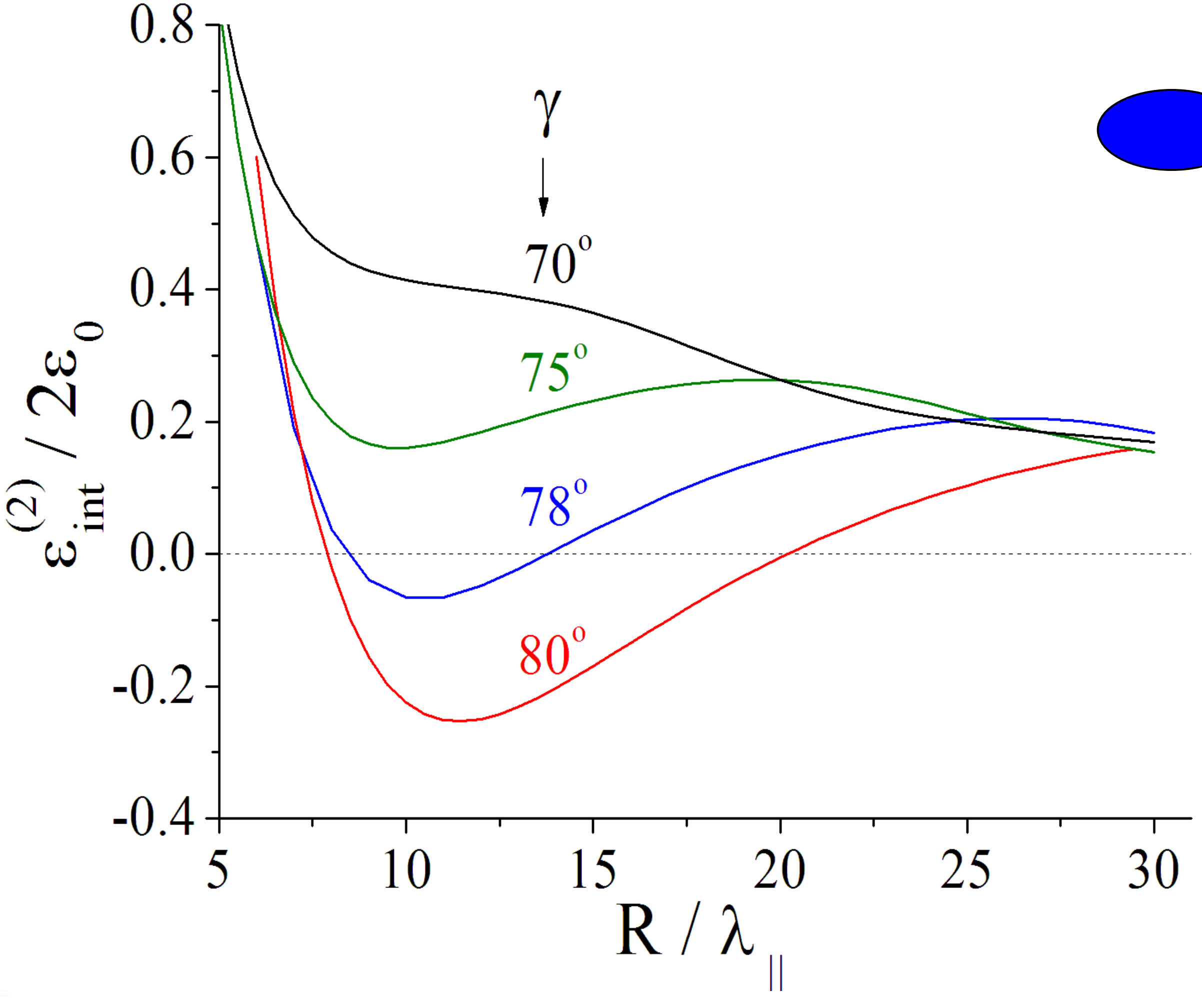
Image method for pancakes in anisotropic superconductors: V.Pudikov (Physica C, 1993).
Continuum approach : G. Carneiro and E.H. Brandt (PRB, 2000).

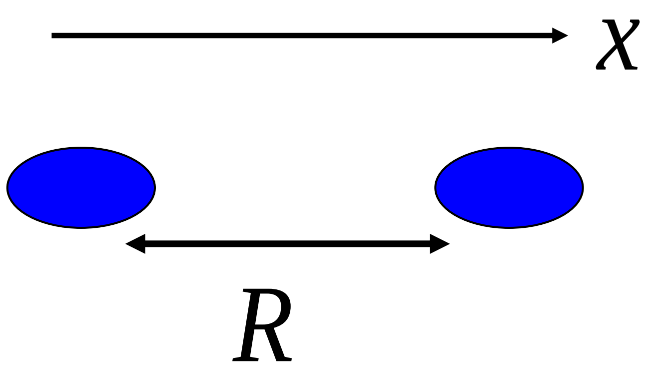
$$\varepsilon_{int} = \frac{\phi_0^2}{16\pi^3\lambda_{||}}\int \frac{d^2q}{q^2}\cos\frac{\mathbf{qR}}{\lambda_{||}}\left(\frac{h}{d_{||}}\frac{p^2+k^2}{1+p^2} + \frac{2(1-k^2)(k(1-p^2)\sinh L + (k^2-p^2)(\cosh L - \cos(2pL)) + 2kp\sin(2pL))}{\sqrt{1+q^2}(1+p^2)^2(2k\cosh L + (1+k^2)\sinh L)}\right)$$

$$L = \frac{d}{\lambda_{||}}\sqrt{1+q^2}\ ,\ k = \frac{q}{\sqrt{1+q^2}}\ ,\ p = \frac{q_x\tan\gamma}{\sqrt{1+q^2}}\ .$$

$$\varepsilon_{\text{int}} \approx \frac{\phi_0^2}{8\pi^2}\left(-\frac{d_{eff}\tan^2\gamma}{R^2} + \frac{2}{R}\right)$$

Interplay between the intervortex attraction and repulsion in thin films: interaction potential for a vortex pair

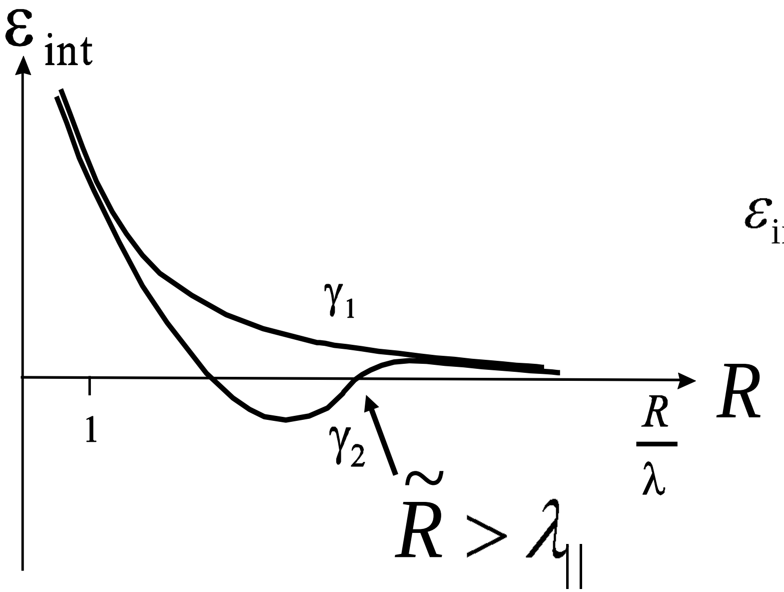




$$d = 3\lambda_{||}$$

$$\varepsilon_0 = \frac{\phi_0^2}{8\pi^3\lambda_{||}}$$

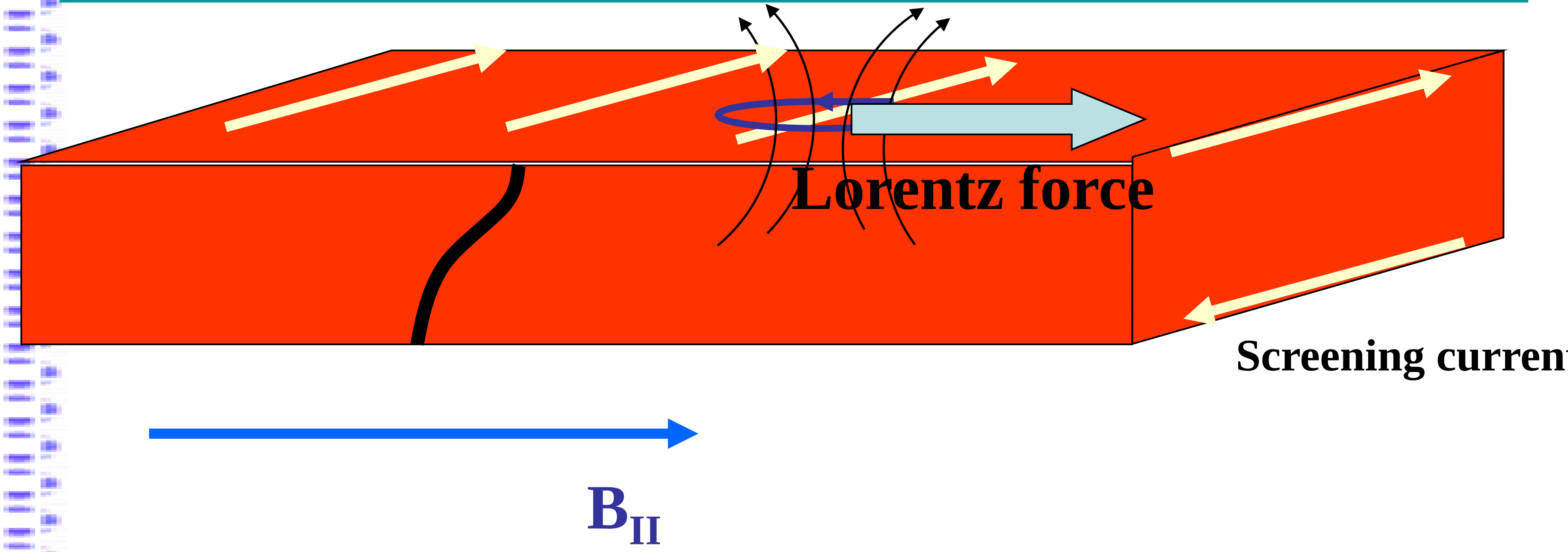
Condition of existence of vortex chains:

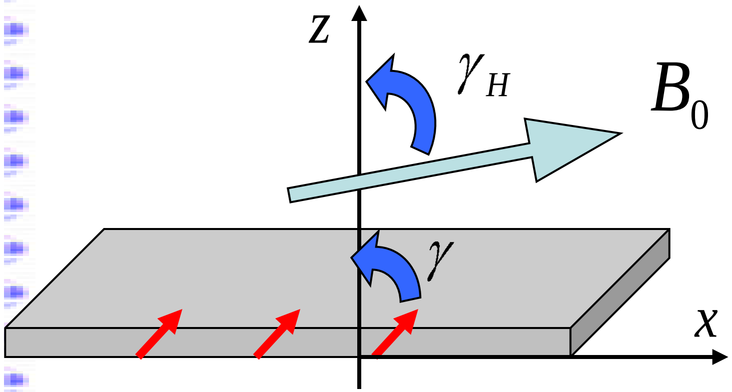


$$\varepsilon_{\text{int}}(\tilde{R}) \approx \frac{\phi_0^2}{8\pi^2}\left(-\frac{d_{eff}\tan^2\gamma}{\tilde{R}^2} + \frac{2}{\tilde{R}}\right) = 0$$

$$\boxed{\tan^2\gamma > \tan^2\gamma_c = \frac{1}{\dfrac{d}{2\lambda_{||}} - \tanh\dfrac{d}{2\lambda_{||}}}}$$

What provokes the vortex tilting?
Screening of the parallel component of the field!





Relation between the field and vortex axis orientation angles:

$$\gamma \approx \frac{4\pi d^2 B_x}{3\phi_0} = \frac{4\pi d^2 B_0 \sin\gamma_H}{3\phi_0}$$

Tilting of the vortex line in layered superconductor.

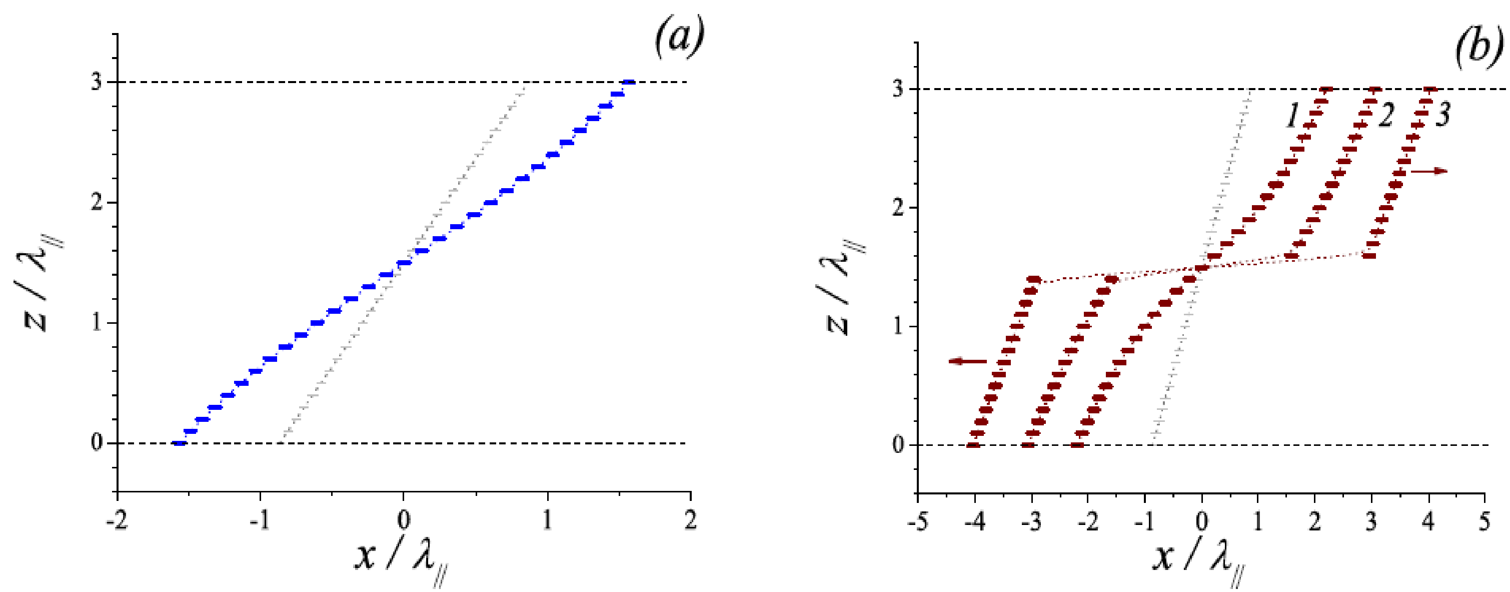
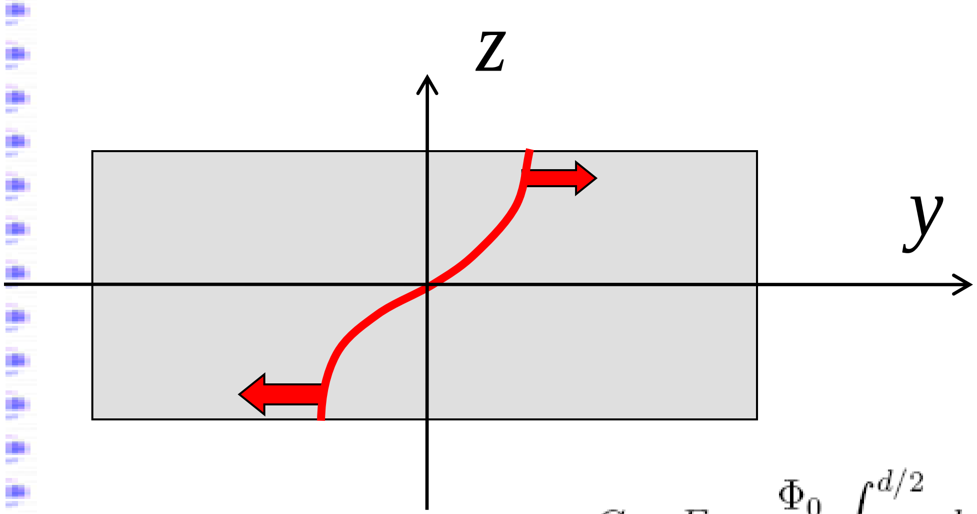


FIG. 2: Configurations of $N = 31$ pancakes in the finite stack in the presence of applied in-plane magnetic field H_a. Grey curve shows the initial distribution of pancakes. (a) The force-balanced (equilibrium) configuration of pancakes for $H_a = 0.2H_0 < H^*$. (b) Change of pancakes configurations in three sequential time points $t_1 < t_2 < t_3$ for $H_a = 0.22H_0 > H^*$. ($H_0 = \phi_0/2\pi\lambda_{ab}^2$), $\Lambda = 10\lambda_{ab}$, $s = 0.1\lambda_{ab}$.

$$\eta \frac{d\mathbf{r}_n}{dt} = \sum_{m \neq n} \mathbf{F}_n^m + \mathbf{F}_n^M$$

Tilting of the vortex line in superconductor with anisotropic effective mass.



$$\Gamma = \frac{\lambda_c}{\lambda_{ab}}$$

$$G = F_v - \frac{\Phi_0}{4\pi}\int_{-d/2}^{d/2} dz \left(1 - \frac{\cosh(z/\lambda_{ab})}{\cosh(d/2\lambda_{ab})}\right) y'(z) H_y$$

$$\delta F_v \simeq \frac{\Phi_0^2}{16\pi^2\Gamma\lambda_{ab}^2}\ln\left(L_c/r_c\right)\int_{-d/2}^{d/2} dz\sqrt{\Gamma^2 + y'^2(z)}$$

$$y'(z) = \frac{\Gamma B(z)}{\sqrt{1 - B^2(z)}}$$

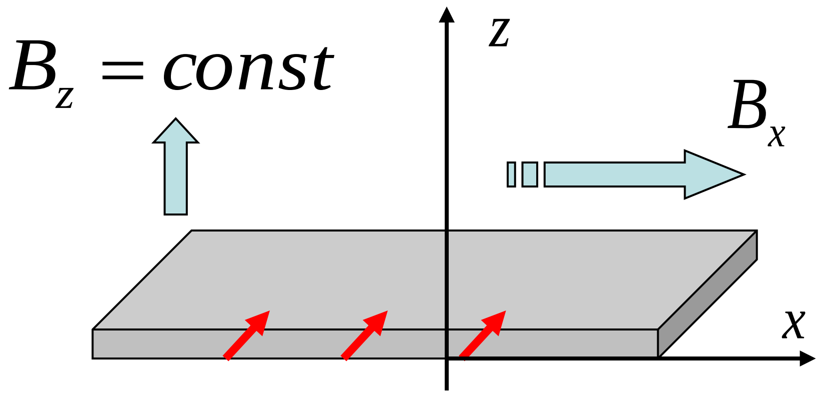
$$B(z) = b \times \left(1 - \frac{\cosh(z/\lambda_{ab})}{\cosh(d/2\lambda_{ab})}\right), \quad b = H_y/H_{ab}, \quad H_{ab} = \frac{\Phi_0}{4\pi\Gamma\lambda_{ab}^2}\ln(L_c/r_c)$$

Estimates and suggestions for experiment

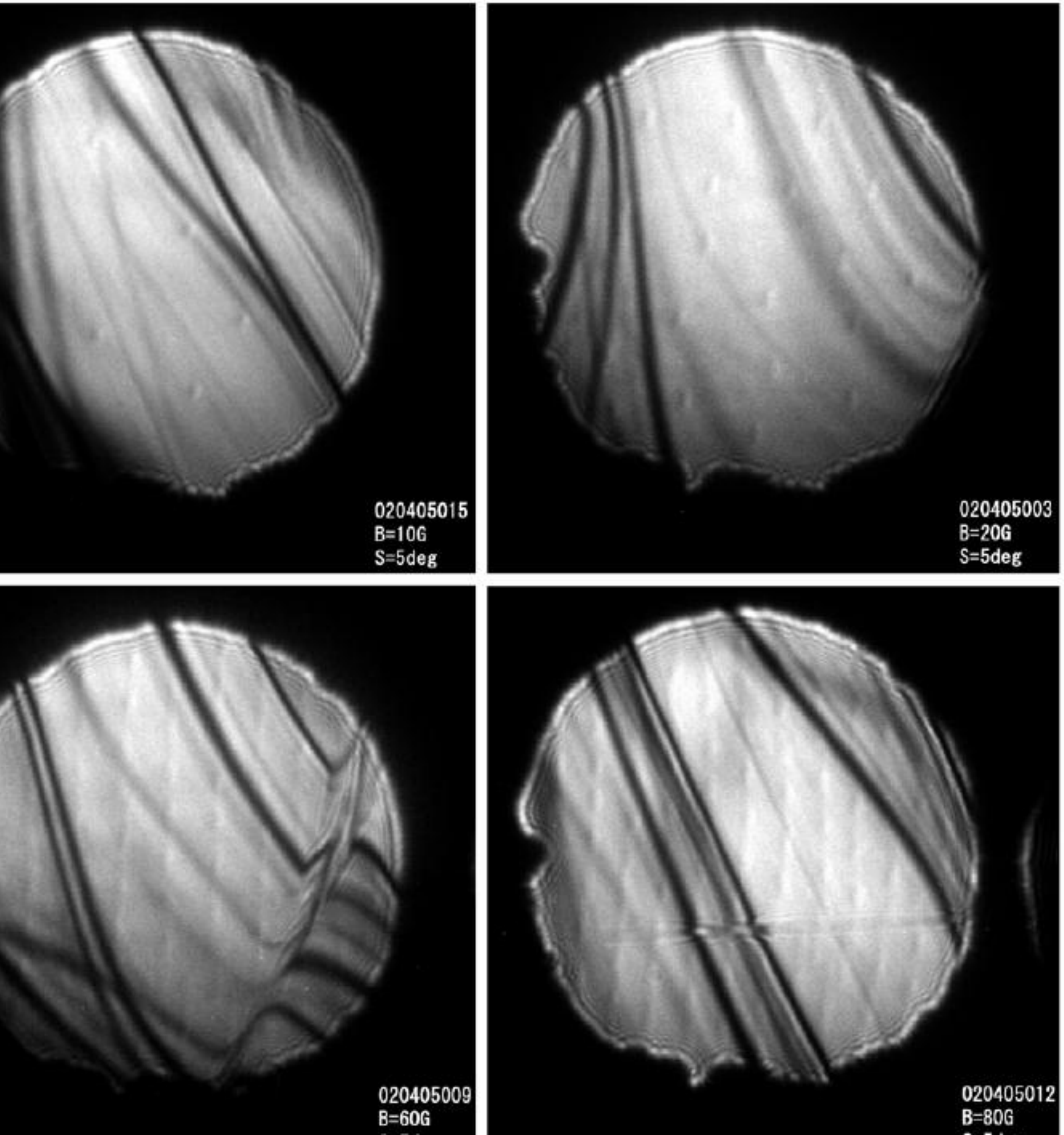
Example: $d \sim \lambda_{||} \sim 0.2\mu m$

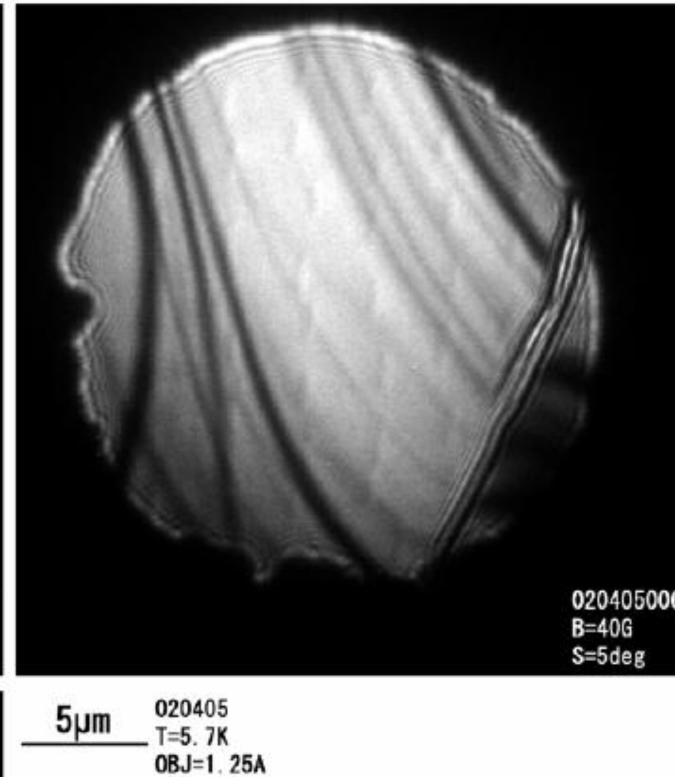
$$\tan\gamma_c \sim 1 \qquad \gamma_H \approx 85^\circ$$

Critical field of vortex chain formation: $B_0 \sim (10-100)\,G$



Chain structure Tonomura's group results



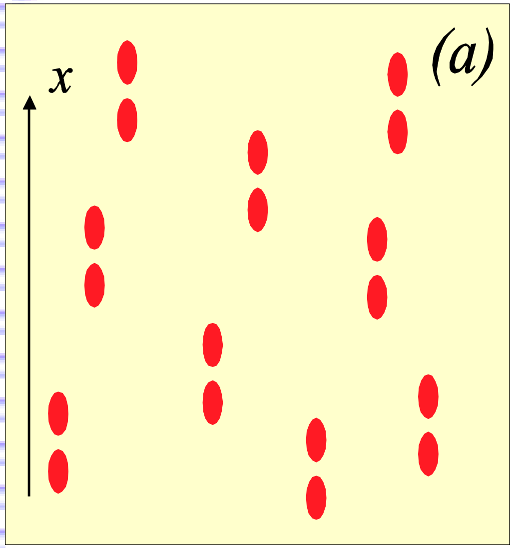


Thin YBaCuO films

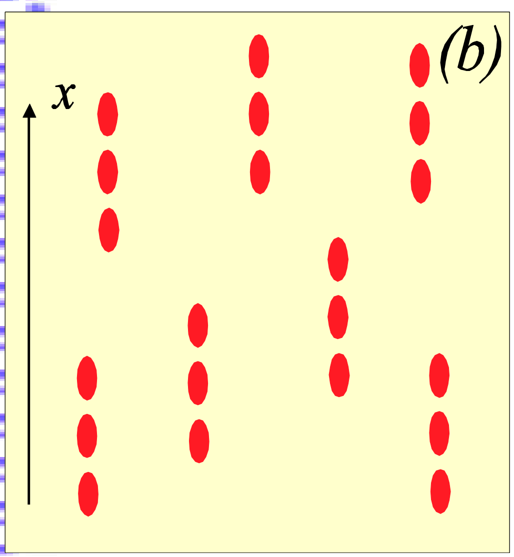
with thickness 0.5 μm

and λ~(0.15-0.2)μm.

Vortex molecules.



$N = 2$

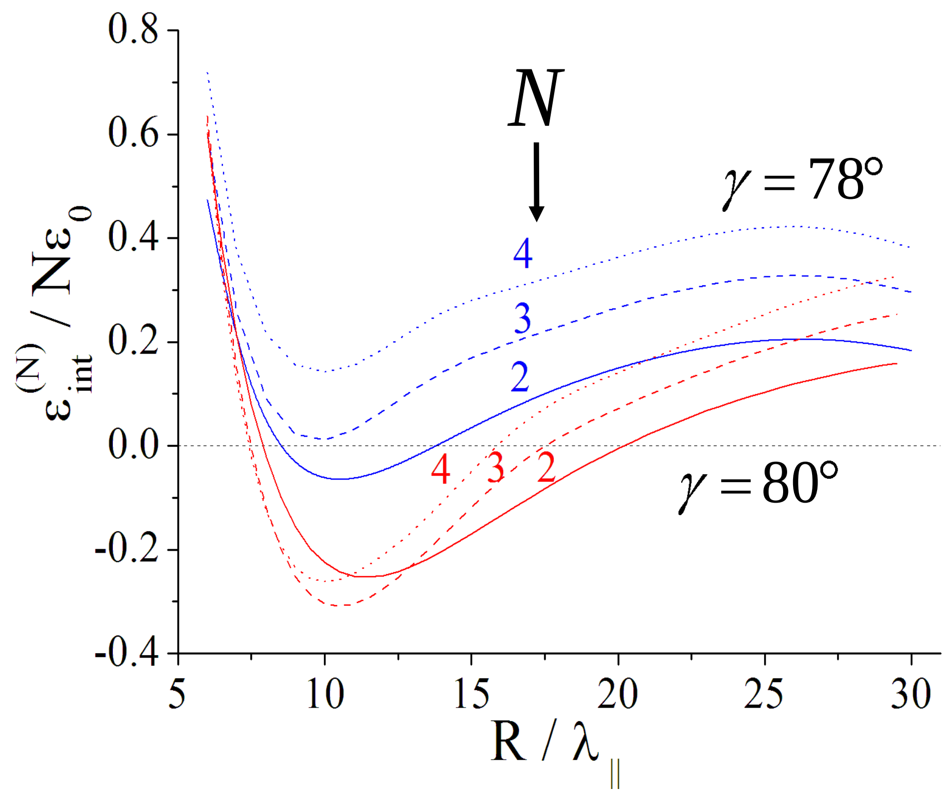


$N = 3$

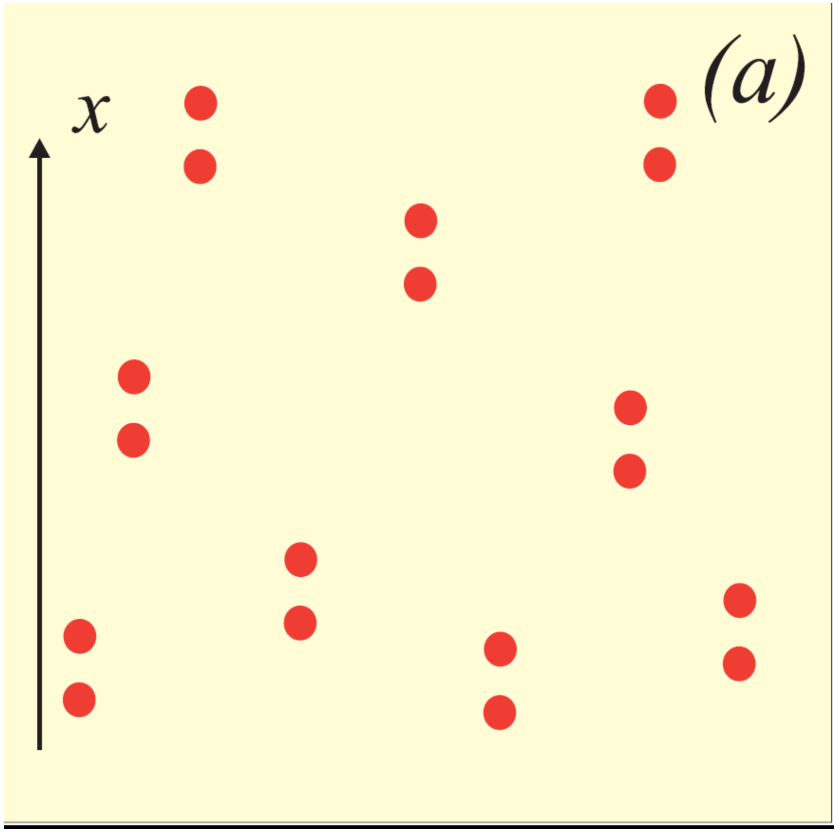
$$\varepsilon_{\text{int}}^{(N)} = \sum_{i>j} \varepsilon_{\text{int}}(R_{ij})$$

$$d = 3\lambda_{\parallel}$$

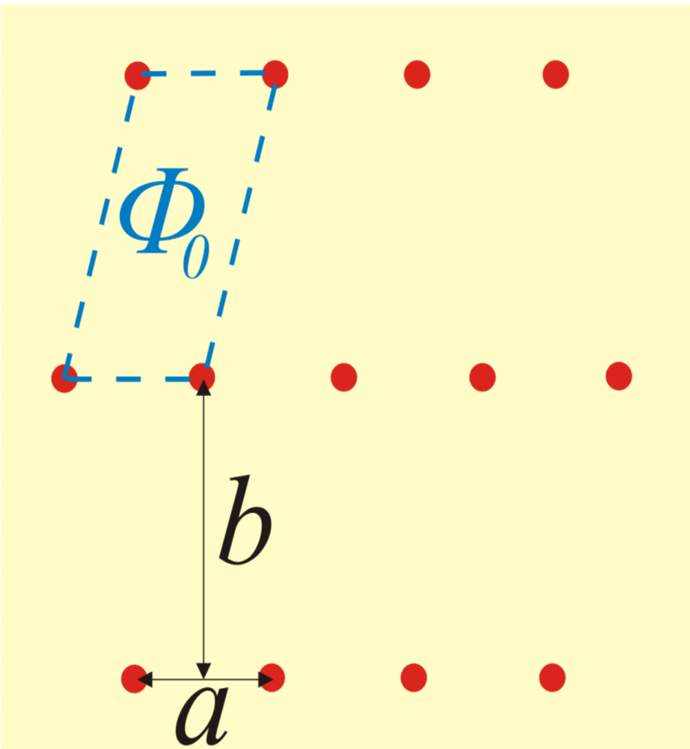
$$\varepsilon_0 = \frac{\phi_0^2}{8\pi^3 \lambda_{\parallel}}$$

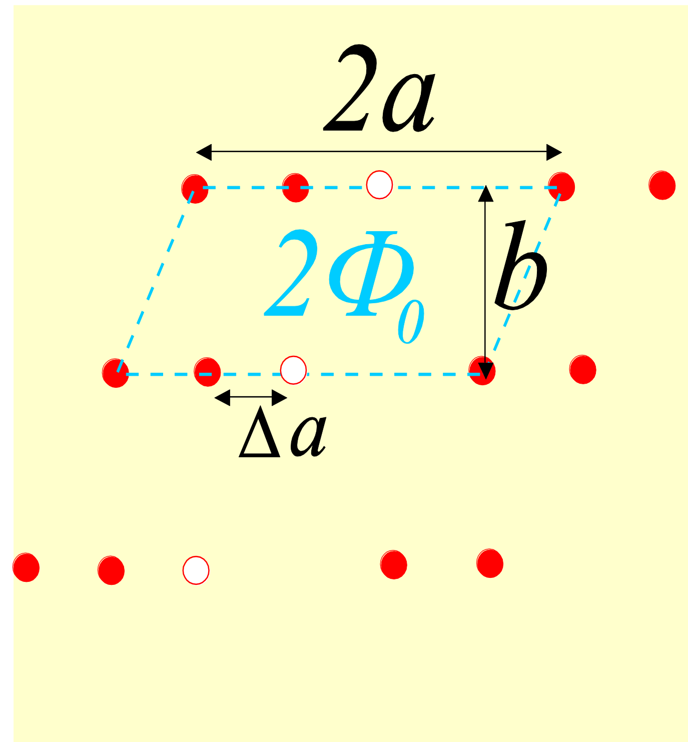


At very small tilting angle the vortices repeal each other.
At larger tilting the vortex pairs may appear.

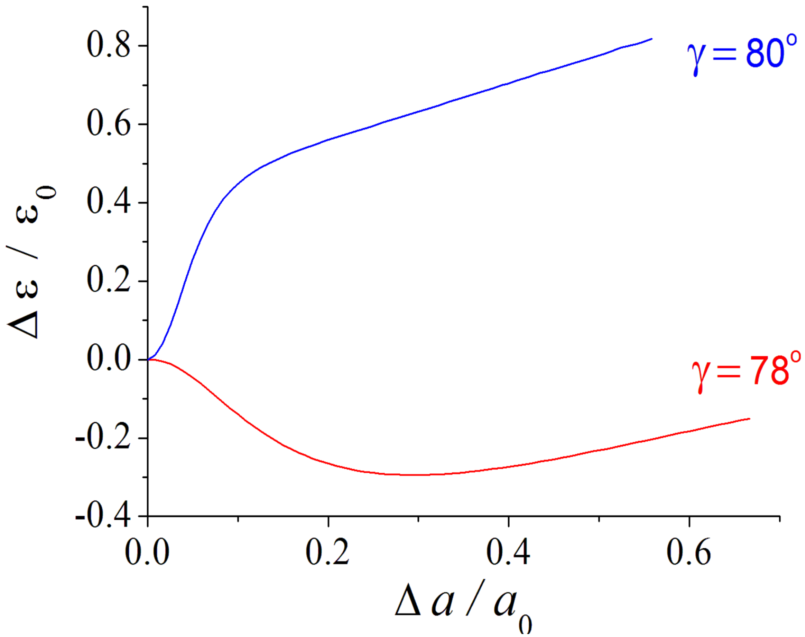


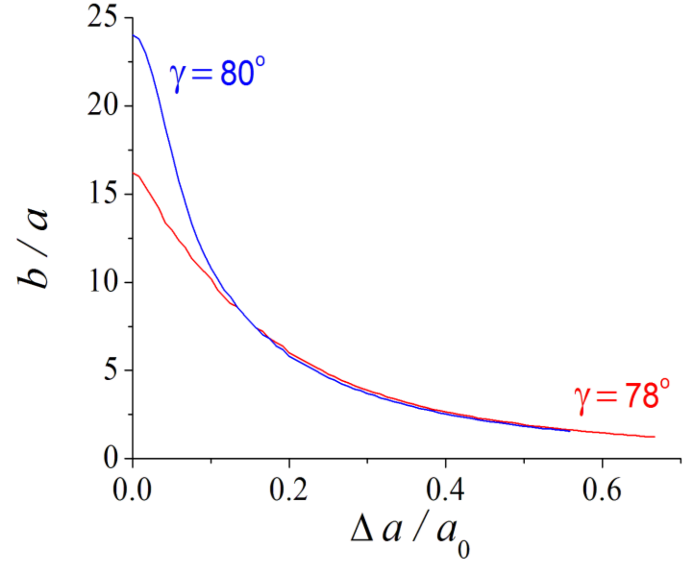
Transition s in the flux lattices with the change of vortex number in elementary cell.





Transition s in the flux lattices with the change of vortex number in elementary cell.





$$\Delta\varepsilon = \min_{\sigma} \varepsilon_{\text{int}}(\Delta a) - \min_{\sigma} \varepsilon_{\text{int}}(\Delta a = 0)$$

$$H_z a_0^2 \sqrt{3}/2 = \phi_0 \qquad d = 3\lambda_{ab}$$

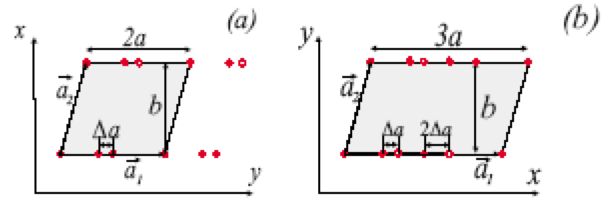


FIG. 11. (Color online) Vortex lattice with two (a) $M=2$ and (b) three $M=3$ vortices per a primitive cell.

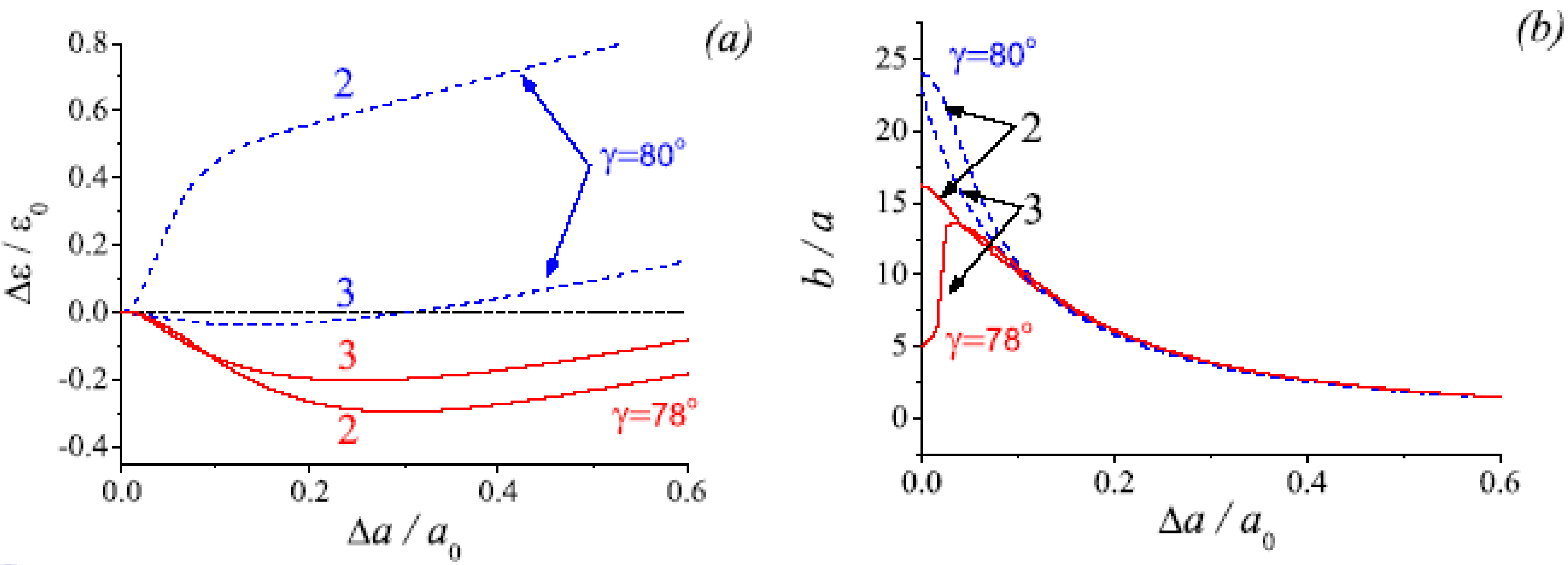
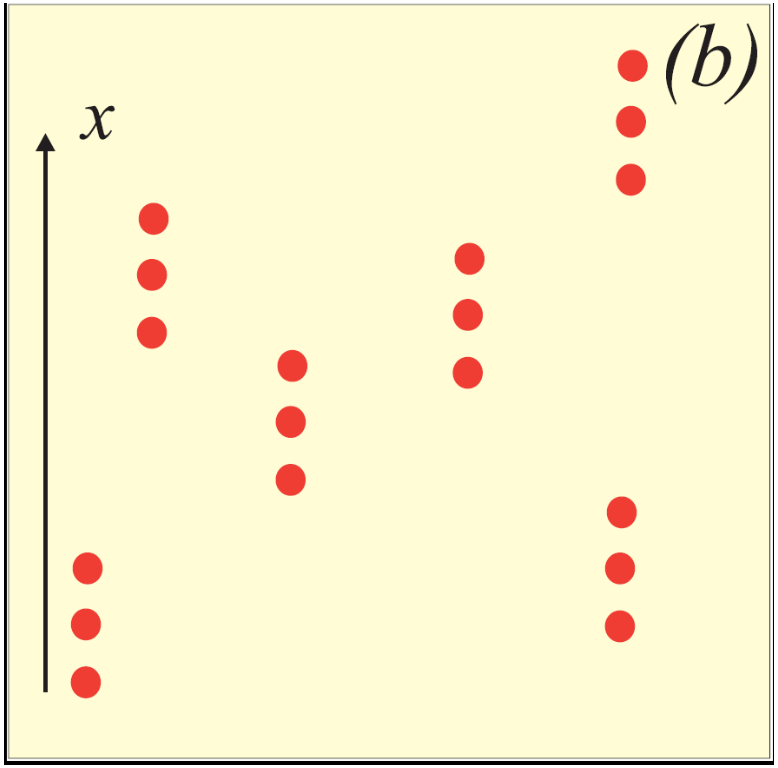


FIG. 12. (Color online) (a) The energy difference $\Delta\varepsilon_c$ vs the relative displacement Δa of vortex sublattices for different tilting angles $\gamma=78^\circ$ (solid line) and $\gamma=80^\circ$ (dashed line) and different number of flux quanta per unit cell $M=2,3$. (b) Lattice deformation ratio $\sigma=b/a$ vs the relative displacement Δa of vortex sublattices for different tilting angles $\gamma=78^\circ$ (solid line) and $\gamma=80^\circ$ (dashed line) and different number of flux quanta per unit cell $M=2,3$. Here we put $D=3\lambda_{ab}$, $a_0=60\lambda_{ab}$. The numbers near the curves denote the number M of vortices per unit cell.

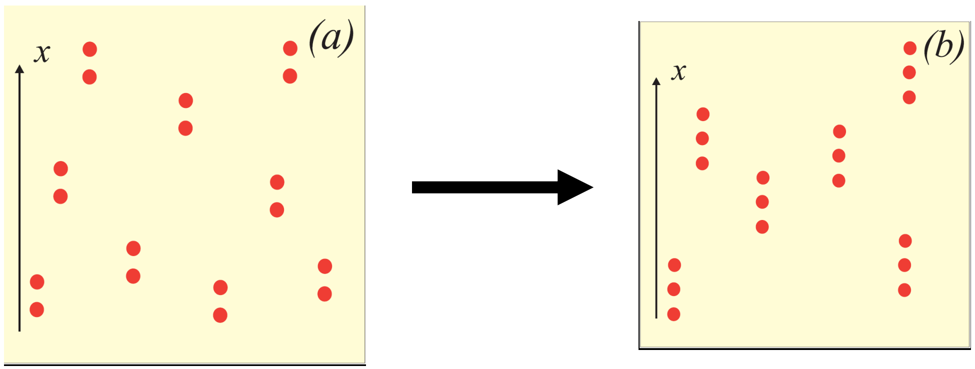
With the tilting increase the 3 vortex molecules, 4 vortex molecules etc. would emerge. Then the "polimerization" into a long chain occurs.



Crossover from vortex molecules to vortex chains.

Increase in the tilting angle of the field

It may be interesting to vary only the parallel component of the field. At constant perpendicular field
the vortex concentration is the same but their interaction will change! Increasing the parallel field we could see the transition from individual vortices to the molecules with 2, 3, 4 etc vortices.



Also we could vary the temperature at constant field. As λ depends on temperature, than near T_c the vortex repulsion will preveal and the vortex molecules will “evaporate”.

$$\tan\gamma > \left(\frac{\lambda_{II}}{d}\right)^{3/2}$$

Role of anisotropy

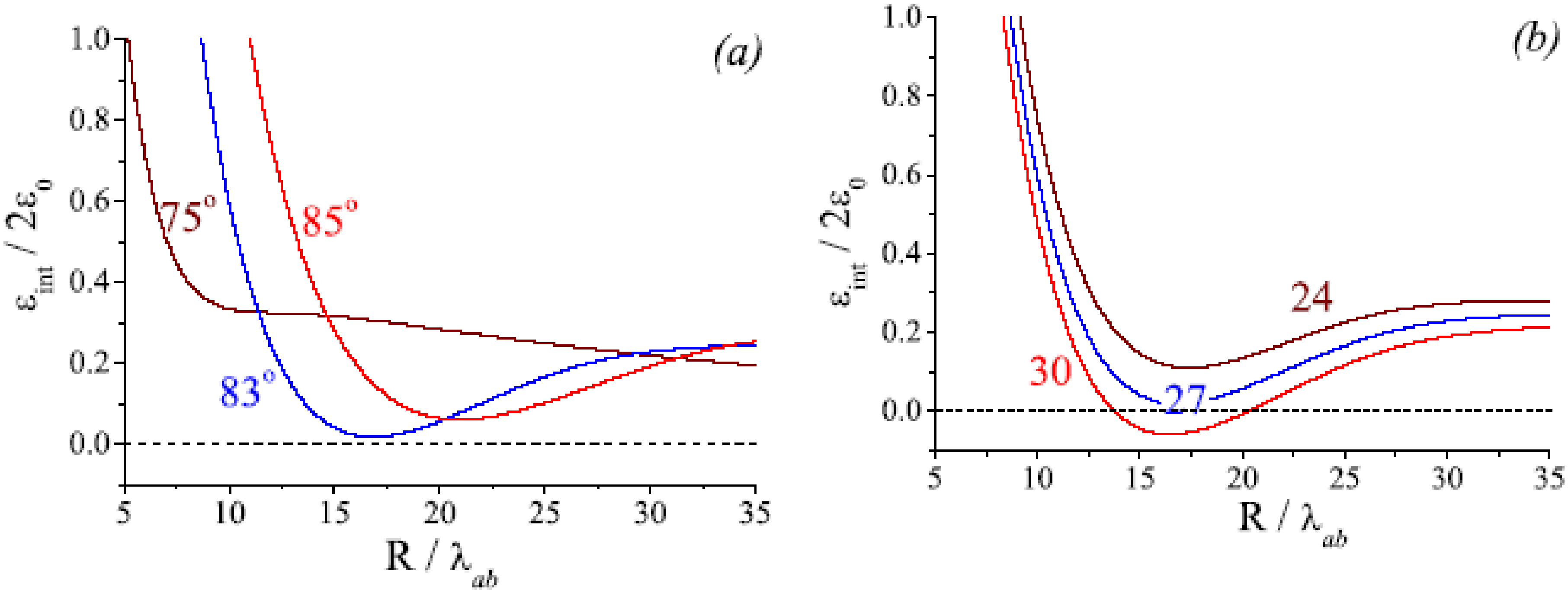


FIG. 5. (Color online) Typical plots of the interaction energy per vortex [Eqs. (38) and (39)] vs the distance R between two tilted vortices for an anisotropic film of the thickness $D=3\lambda_{ab}$. (a) Interaction energy for the anisotropy parameter $\Gamma=27$ and different tilting angles. The numbers near the curves denote the values of tilting angle γ. (b) Interaction energy for $\gamma=83°$ and different values of anisotropy parameter. The numbers near the curves denote the values of Γ.

Exact shape of the vortex line

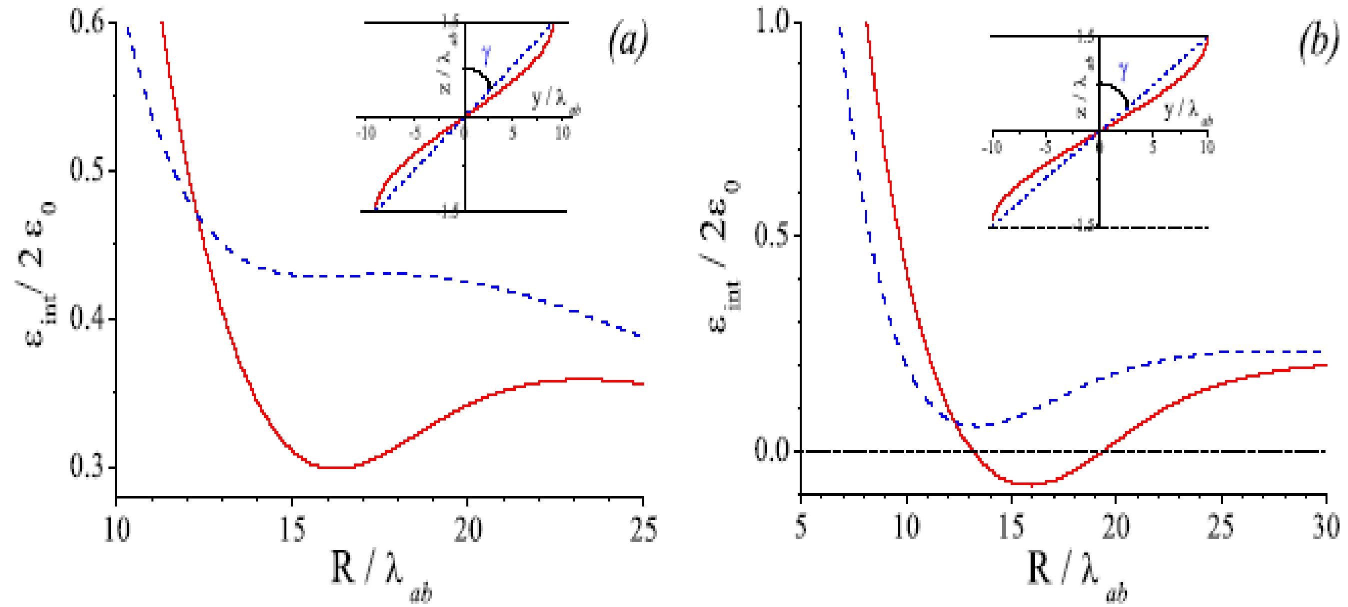


FIG. 8. (Color online) Comparison of the vortex-vortex interaction potentials for curved [Eq. (37)] (solid lines) and straight tilted [Eqs. (38) and (39)] (dashed lines) vortices for an anisotropic film of the thickness $D = 3\lambda_{ab}$ with different anisotropy parameters: (a) $\Gamma = 15$, $H_a = 0.91 H_{ab}$ ($\gamma = 80.6°$) and (b) $\Gamma = 27$, $H_a = 0.6 H_{ab}$ ($\gamma = 81.5°$). The shape of vortex lines is schematically shown in the insets.

Conclusions

•In bulk layered superconductors the tilted vortices attract each other at large distances and this interaction leads to the vortex chains formation.

•The very special situation for the tilted field pattern is realized in the thin films d~λ . The very rich new physics emerges!

•We have a crossover from the vortex attraction to vortex repulsion.

•Even if the interaction is attractive, the formation of the finite vortex chain is expected.

•The vortex forms ”molecules”: dimers, trimers etc.

•The structure of these “molecules” is controlled by the tilting angle and temperature.

References : Phys. Rev. B **79**, 094510 (2009).
Phys. Rev. B **82**, 104511 (2010).