Non-uniform (FFLO) states and quantum oscillations in superconductors and superfluid ultracold Fermi gases

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Outline

1. Singlet superconductivity destruction by the magnetic field:
   - The main mechanisms
   - Origin of FFLO state.

2. Exactly solvable models of FFLO state.

3. Experimental evidences of FFLO state.

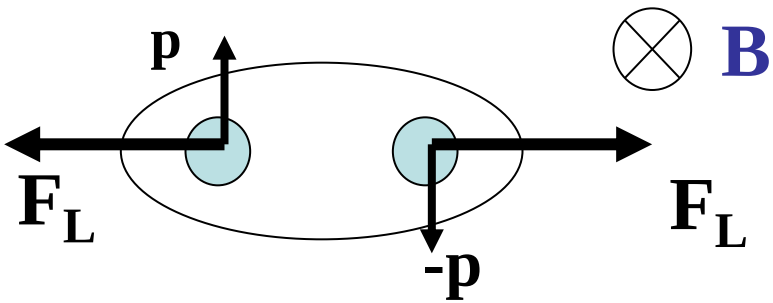
3. Vortices in FFLO state. Role of the crystal structure.

4. Quasi-2D superconductors: in-plane anisotropy of the critical field due to FFLO modulation.

5. Supefluid ultracold Fermi gases with imbalanced state populations: one more candidate for FFLO state?

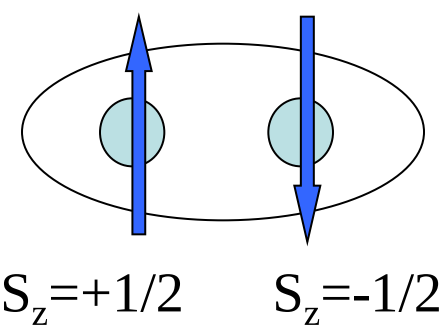
1. Singlet superconductivity destruction by the magnetic field.

- Orbital effect (Lorentz force)



Electromagnetic mechanism
(breakdown of Cooper pairs by magnetic field induced by magnetic moment)

- Paramagnetic effect (singlet pair)

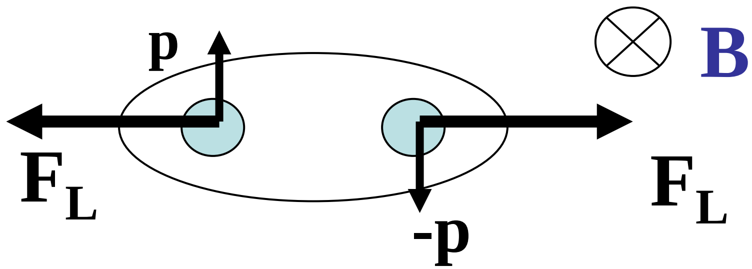


$\mu_B H \sim \Delta \sim T_c$

$I\left(\vec{S}\cdot\vec{s}\right) \approx T_c$

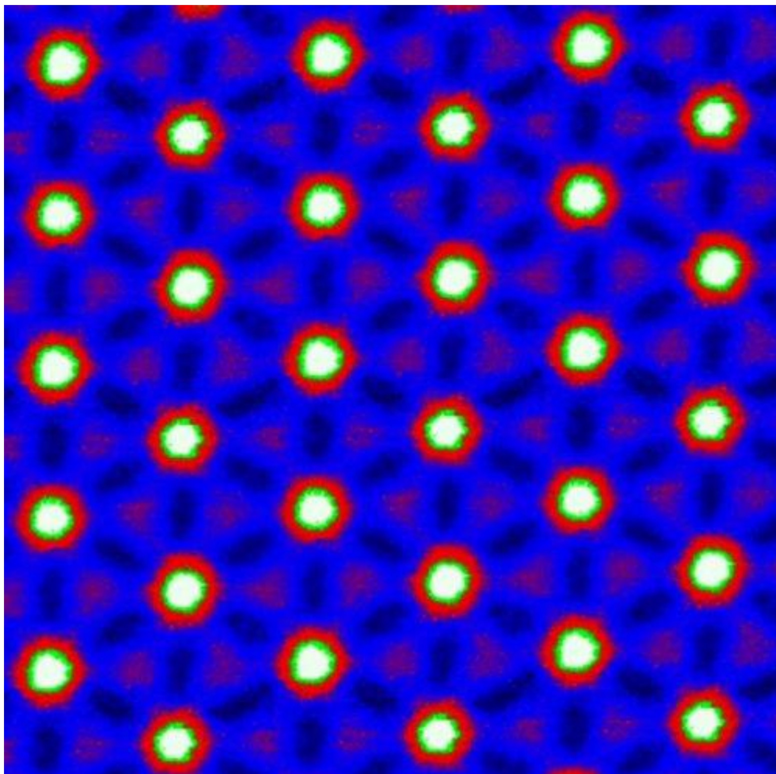
Exchange interaction

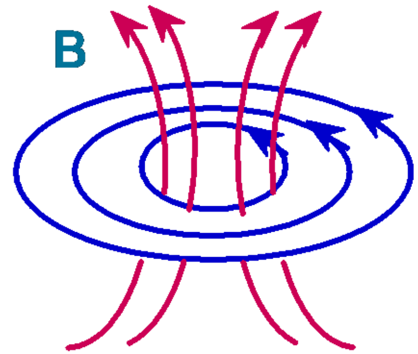
Orbital effect



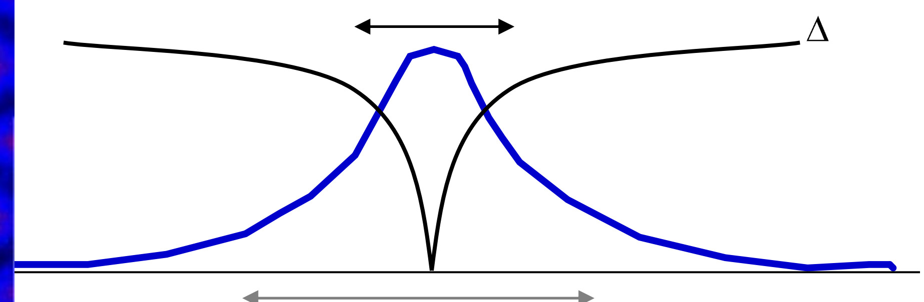
Vortex

Flux quantum





ξ(coherence length)



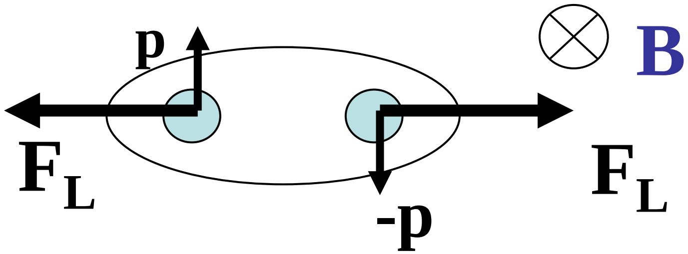
λ(penetration length)

Vortex lattice in $NbSe_2$
(STM)

$\Phi_0 = hc/2e = 2.07 \times 10^{-7} Oe \cdot cm^{2z}$

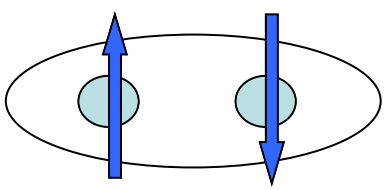
Superconductivity is destroyed by magnetic field

Orbital effect (Vortices)



$$H_{c2}^{orb} = \frac{\Phi_0}{2\pi\xi^2}$$

Zeeman effect of spin (Pauli paramagnetism)



$$\frac{1}{2}\chi_N H^2 = \frac{1}{2}N(0)\Delta^2$$

$$\chi_N = \frac{1}{2}(g\mu_B)^2 N(0)$$

$$\Rightarrow \quad H_{c2}^{P} = \frac{\sqrt{2}\Delta}{g\mu_B}$$

Maki parameter

$$\alpha \equiv \sqrt{2}\frac{H_{c2}^{orb}}{H_{c2}^{P}} \qquad \alpha \sim \frac{\Delta}{\varepsilon_F} << 1$$

Usually the influence of Pauli paramagnetic effect is negligibly small

Superconducting order parameter behavior under paramagnetic effect

Standard Ginzburg-Landau functional:

$$F = a|\Psi|^2 + \frac{1}{4m}|\nabla\Psi|^2 + \frac{b}{2}|\Psi|^4$$

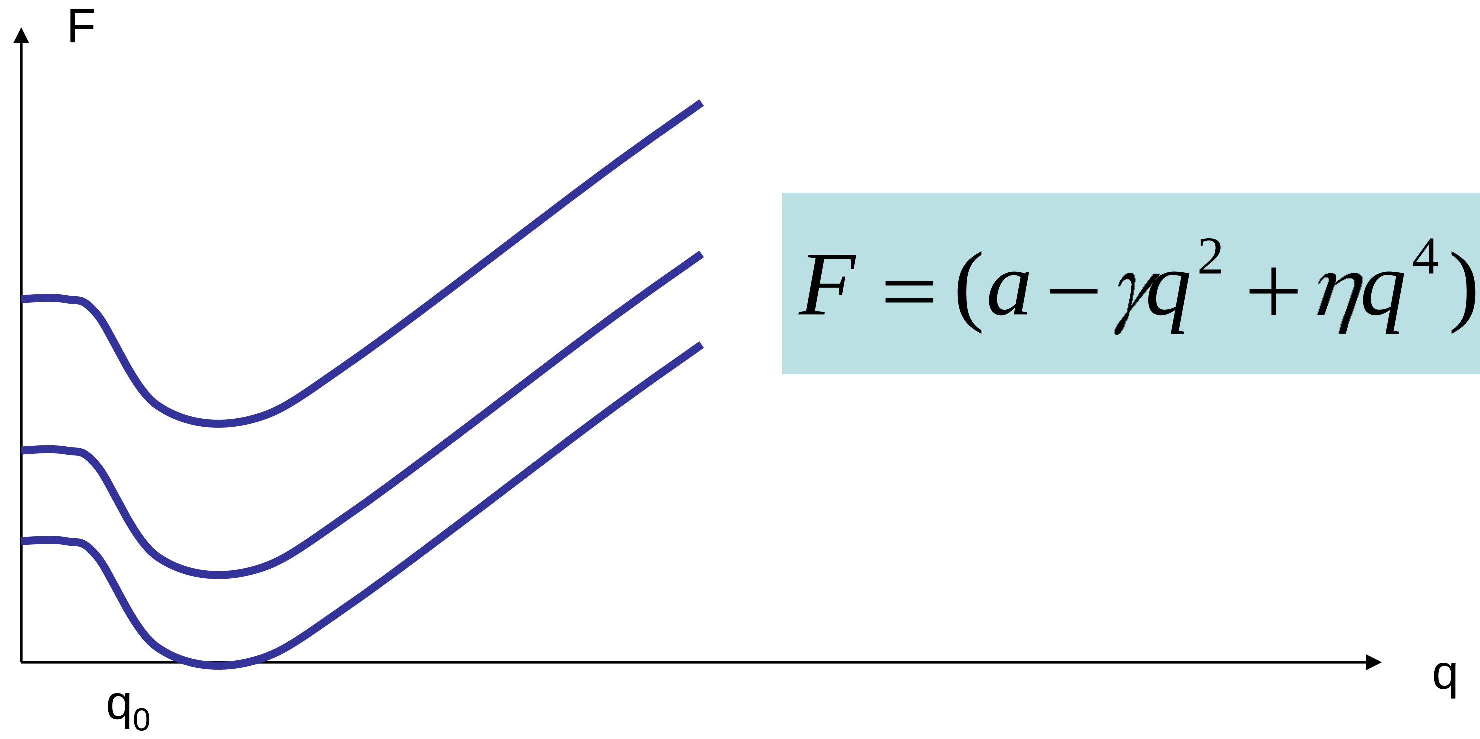
The minimum energy corresponds to Ψ=const

The coefficients of GL functional are functions of the Zeeman field h= $\mu_B H$ **!**

Modified Ginzburg-Landau functional ! :

$$F = a|\Psi|^2 - \gamma|\nabla\Psi|^2 + \eta|\nabla^2\Psi|^2 + \ldots$$

The **non-uniform** state $\Psi \sim \exp(iqr)$ will correspond to minimum energy and higher transition temperature



$$F = (a - \gamma q^2 + \eta q^4)\left|\Psi_q\right|^2$$

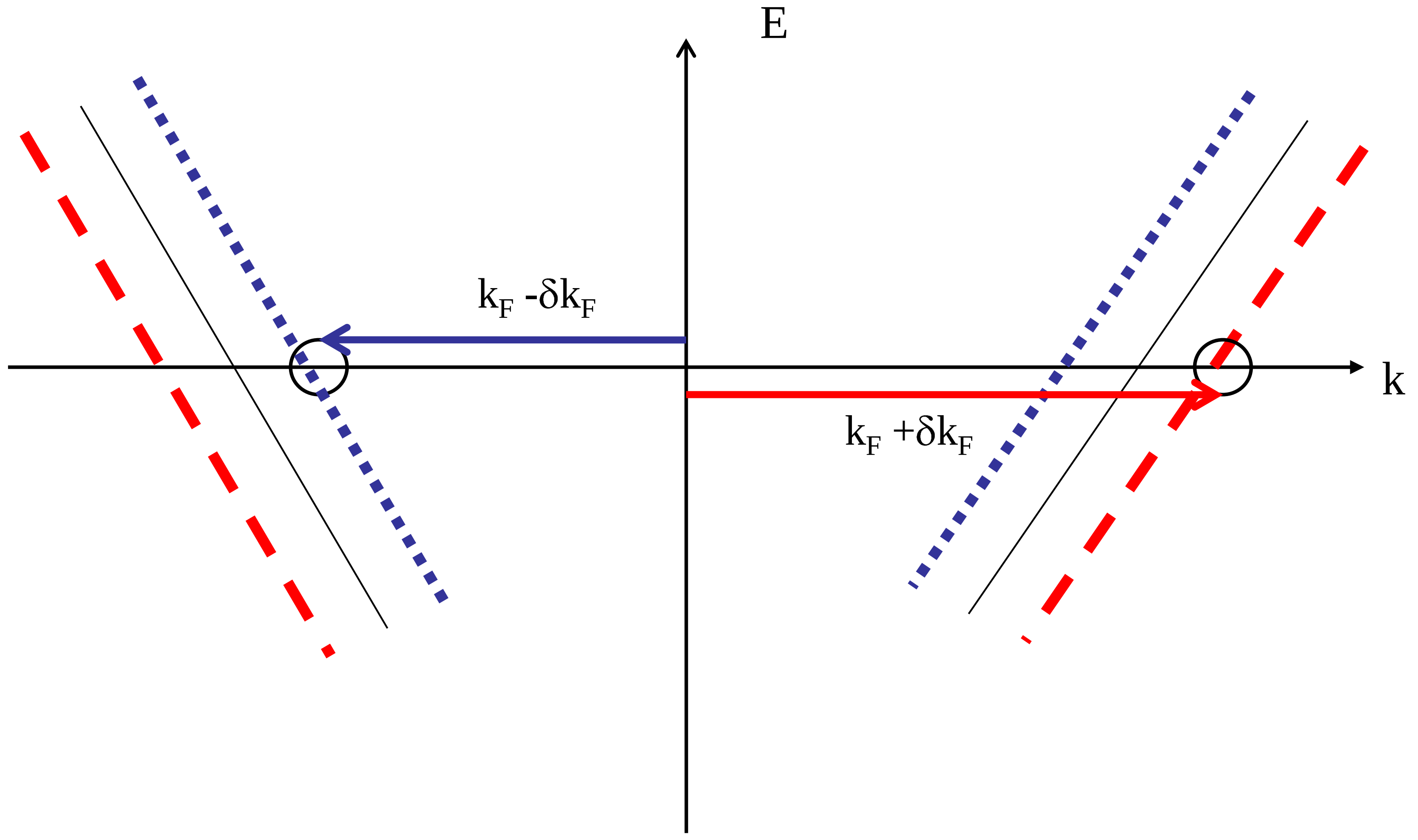
$\Psi \sim \exp(iqr)$ - **Fulde-Ferrell-Larkin-Ovchinnikov state (1964).**
Only in pure superconductors and in the rather narrow region.

P. Fulde and R. A. Ferrell. Phys. Rev. **135**, A550 (1964).
A. Larkin and Y. Ovchinnikov. Sov. Phys. JETP **20**, 762 (1965).

FFLO inventors

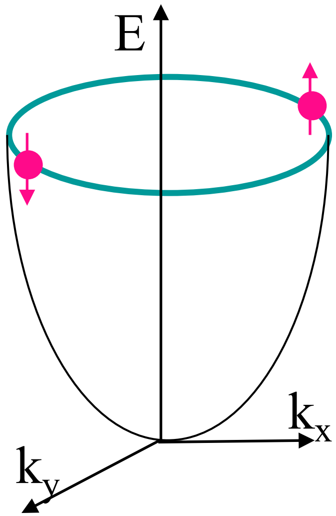
Fulde and Ferrell

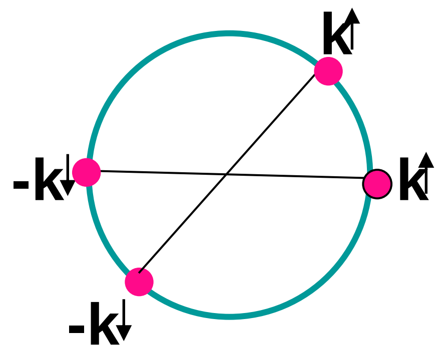
Larkin and Ovchinnikov

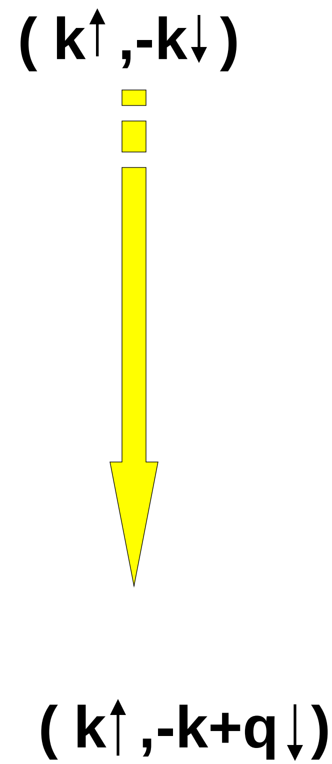


The total momentum of the Cooper pair is
-(k_F -δk_F)+ (k_F -δk_F)=2 δk_F

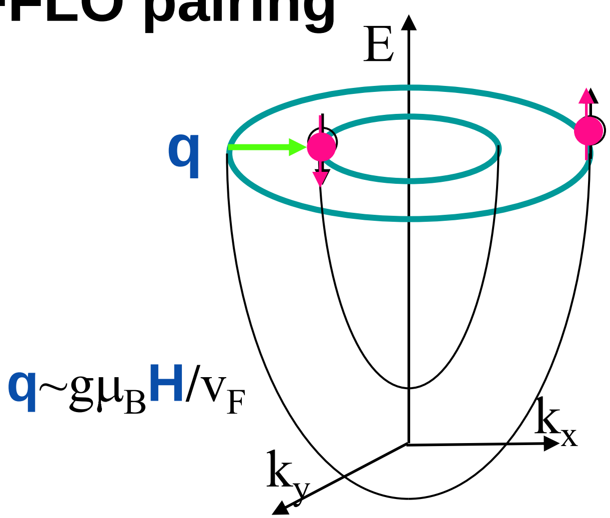
Conventional pairing

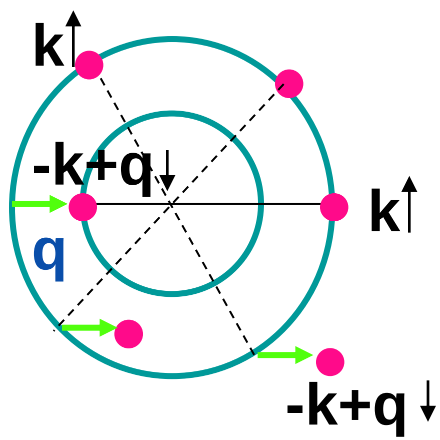






FFLO pairing





pairing between Zeeman split parts of the Fermi surface

⇨ **Cooper pairs have a single non-vanishing center of mass momentum**

Pairing of electrons with opposite spins and momenta unfavourable :

$$[\epsilon(\mathbf{k}) - \mu_B H^{\mathsf{eff}}] \neq [\epsilon(-\mathbf{k}) + \mu_B H^{\mathsf{eff}}]$$

But :

$$\boxed{[\epsilon(\mathbf{k}+\mathbf{q}) - \mu_B H^{\mathsf{eff}}] \approx [\epsilon(-\mathbf{k}+\mathbf{q}) + \mu_B H^{\mathsf{eff}}] \quad \text{if} \quad q \approx \frac{\mu_B H^{\mathsf{eff}}}{v_F}}$$

$$\rightarrow \Delta(\mathbf{r}) = \Delta \exp(i\mathbf{q}\cdot\mathbf{r})$$

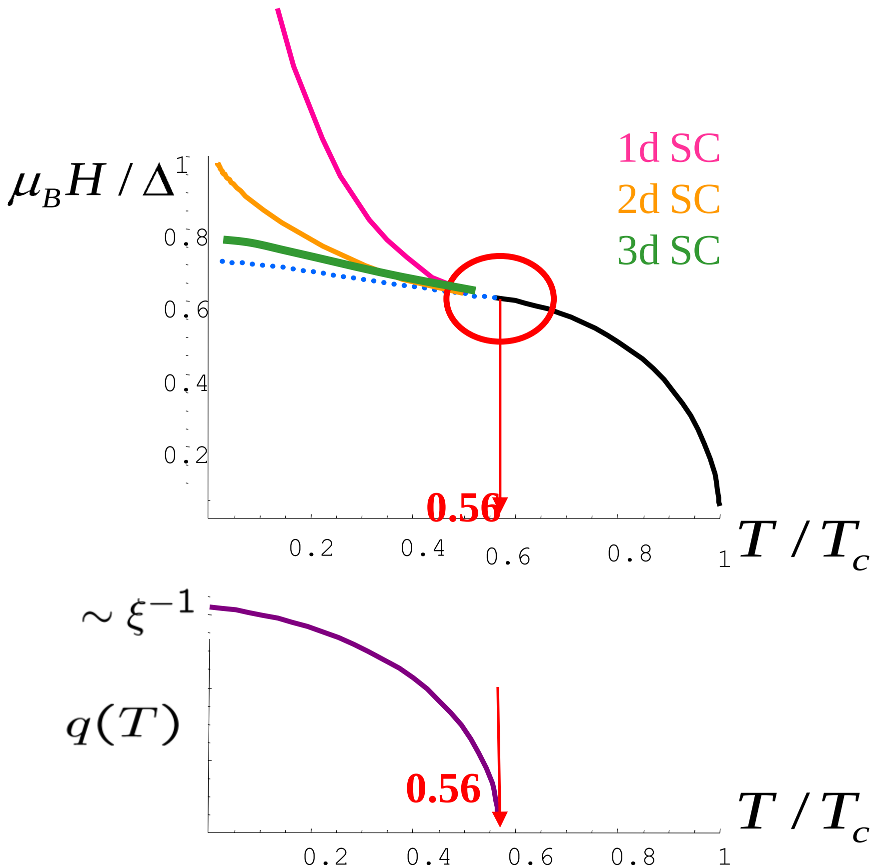
At $T = 0$, Zeeman energy compensation is exact in 1d, partial in 2d and 3d.

• the upper critical field is increased

• Sensivity to the disorder and to the orbital effect:

(clean limit)

$$\boxed{q(T) \gg \frac{1}{\ell_{\mathsf{imp}}}, \frac{1}{L_H}}$$

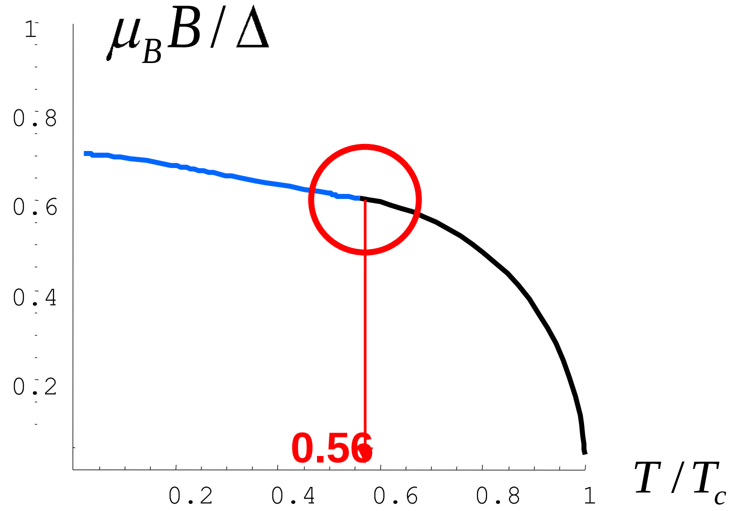


Modified Ginzburg-Landau functional :

$$F = a|\Psi|^2 - \gamma|\nabla\Psi|^2 + \eta|\nabla^2\Psi|^2 - \gamma'|\Psi|^4 + \beta|\nabla\Psi|^2|\Psi|^2$$
$$+\beta'\left[\Psi^{*2}(\nabla\Psi)^2 + \Psi^2(\nabla\Psi^*)^2\right] + \delta|\Psi|^6 + \dots$$

$$\tilde{\nabla} = \nabla - \frac{2ie}{\hbar c}\mathbf{A}$$

May be 1st order transition at $T < T^\star \approx 0.56 T_c$



2. Exactly solvable models of FFLO state.

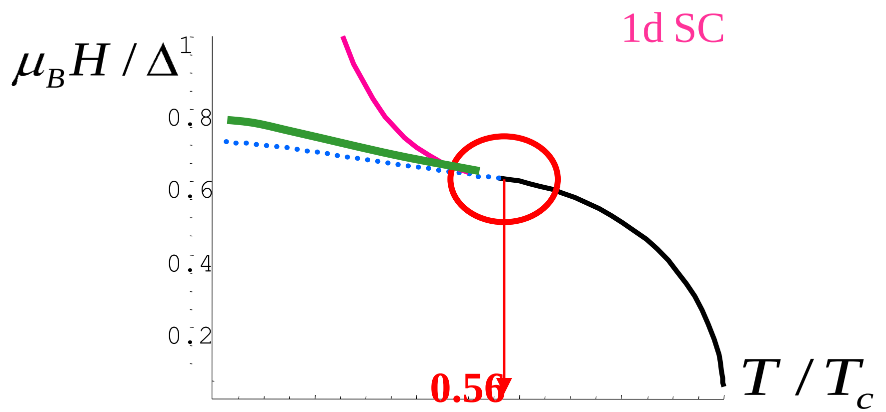
FFLO phase in the case of pure paramagnetic interaction and BCS limit

Exact solution for the 1D and quasi-1D superconductors ! (Buzdin , Tugushev 1983)

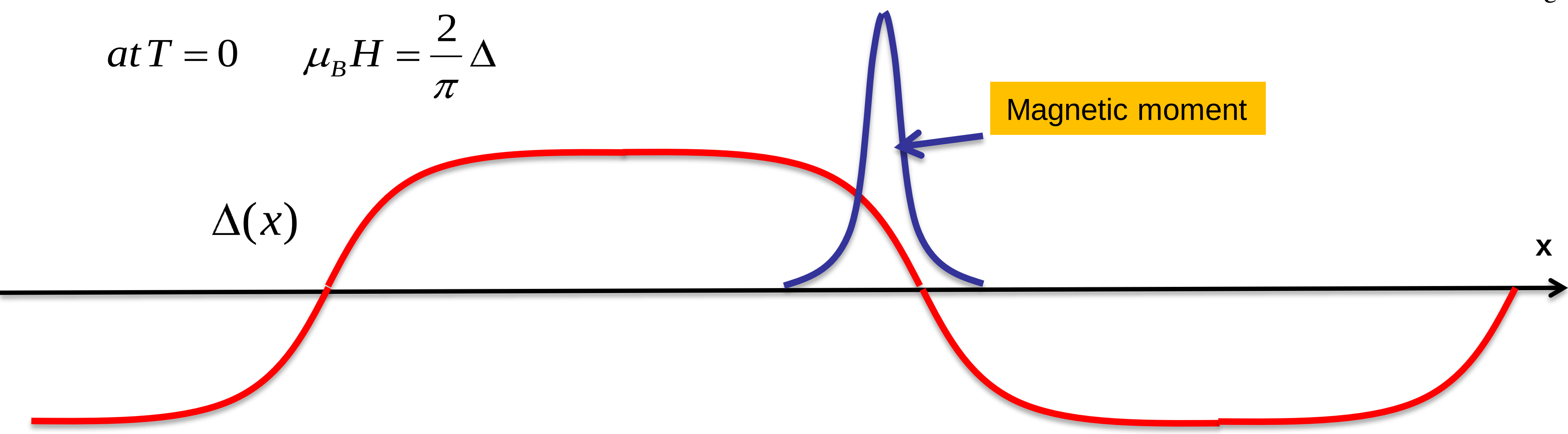
- The FFLO phase is the soliton lattice,

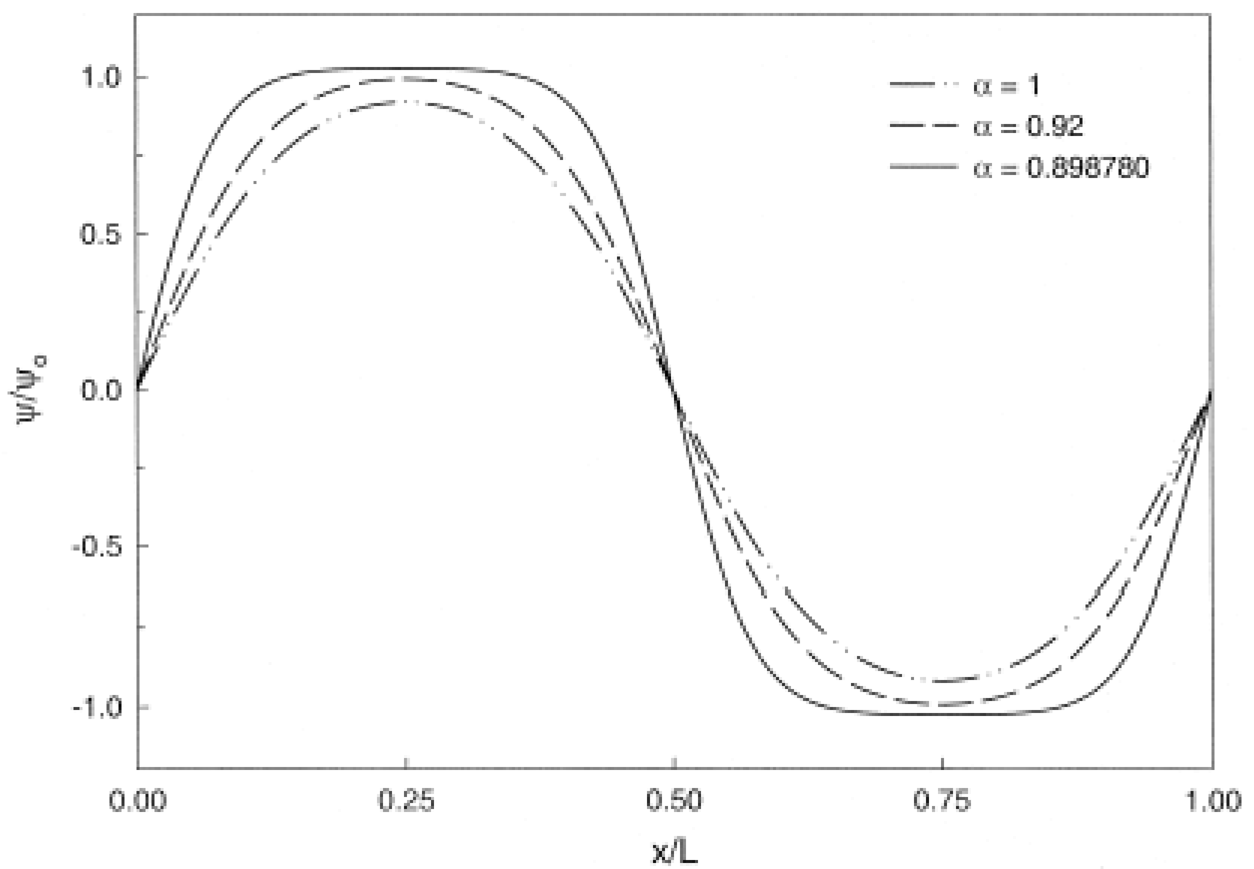
first proposed by **Brazovskii, Gordyunin and Kirova in 1980** for polyacetylene.

$$\boxed{\Delta(x) = \Delta_0 sn(x/\xi, k)}$$



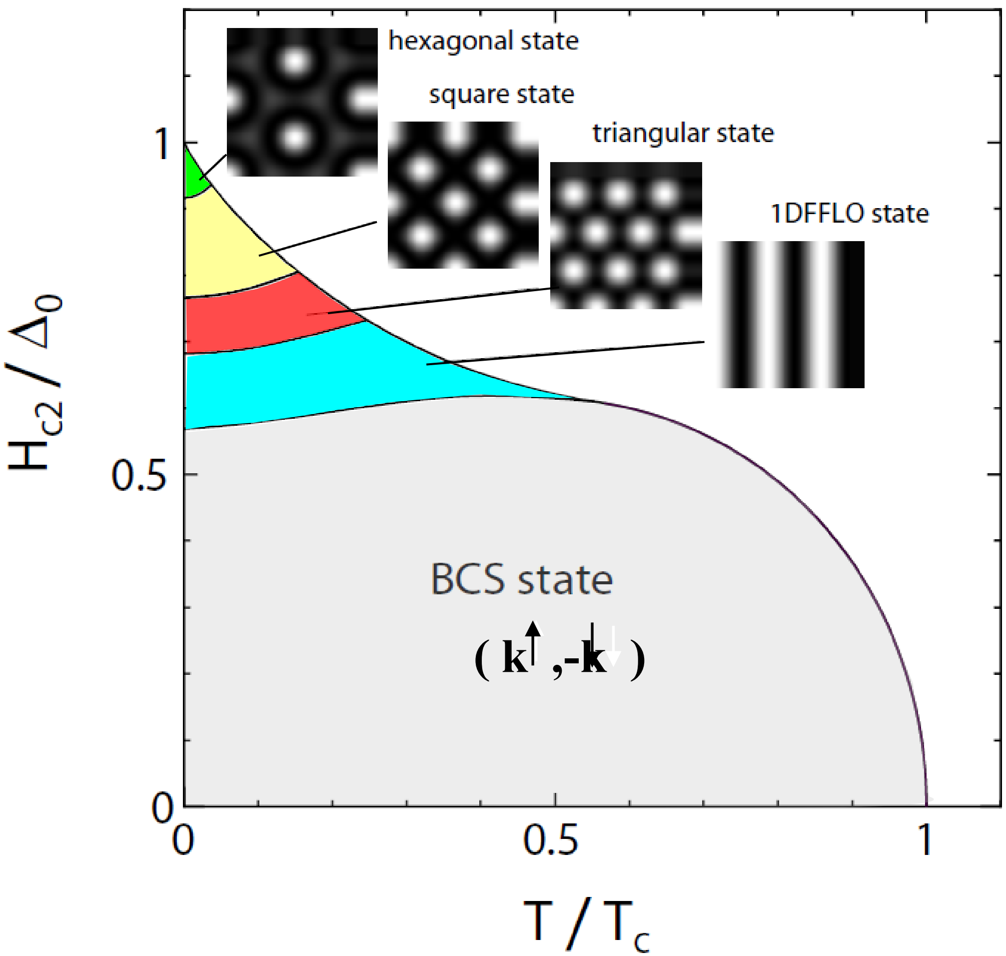
$$at\, T = 0 \qquad \mu_B H = \frac{2}{\pi}\Delta$$





α = 1
α = 0.92
α = 0.898780
1.0
0.5
0.0
-0.5
-1.0
ψ/ψ₀
0.00
0.25
0.50
0.75
1.00
x/L

In 2D superconductors



Y.Matsuda and H.Shimahara
J.Phys. Soc. Jpn (2007)

FFLO phase in the case of paramagnetic and orbital effect (3D BCS limit) – upper critical field

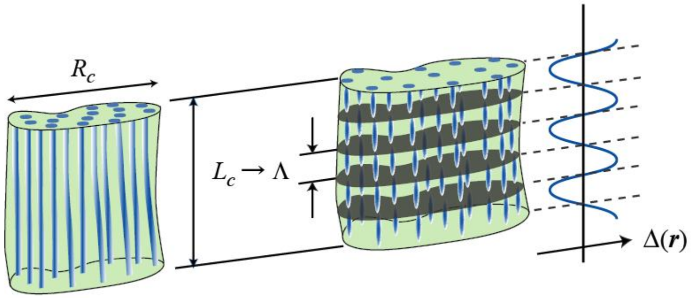
Note : The system with elliptic Fermi surface can be tranformed by scaling transformation to ihe isotropic one. Sure the direction of the magnetic field will be changed.

$$\boxed{\Delta(\mathbf{r}) \sim \exp(iQz)\exp(-\rho^2 eH/2\hbar c)}$$

Lowest m=0 Landau level solution, **Gruenberg and Gunter, 1966**

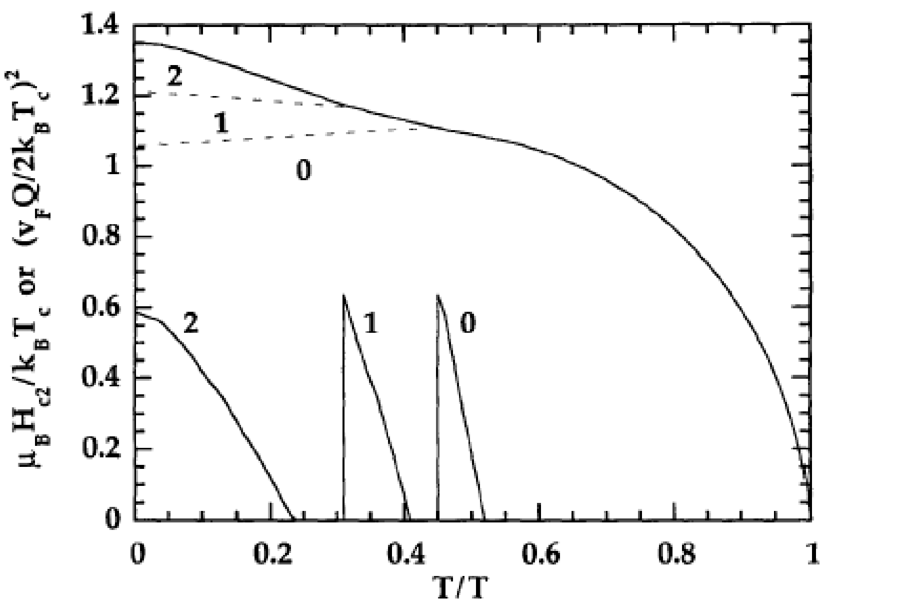
$$\boxed{\alpha \equiv \sqrt{2}\frac{H_{c2}^{orb}}{H_{c2}^{P}}}$$

FFLO exists for Maki parameter α>1.8.



For Maki parameter α>9 the highest Landau level solutions are realized – **Buzdin and Brison, 1996.**

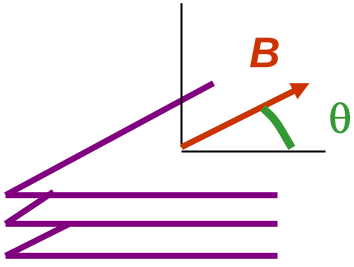
$$\Delta(\mathbf{r}) \sim e^{-im\varphi}\rho^m \exp(iQz)\exp(-\rho^2 eH/2\hbar c),$$

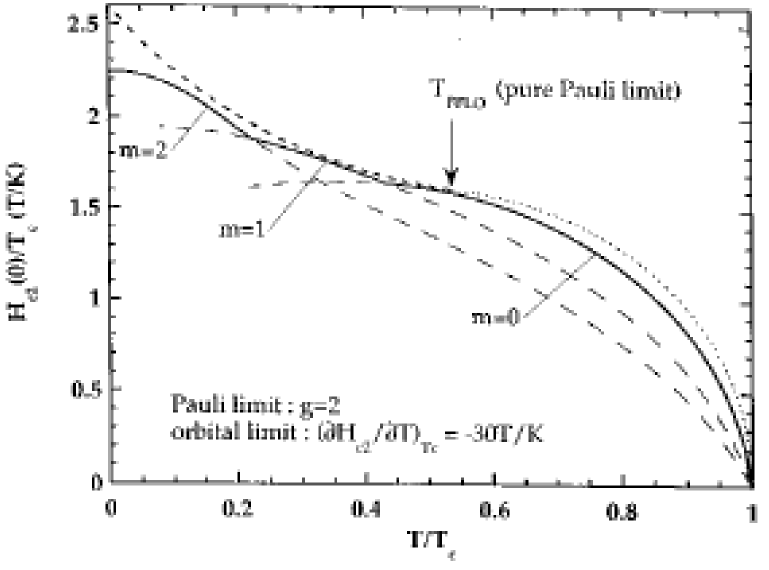


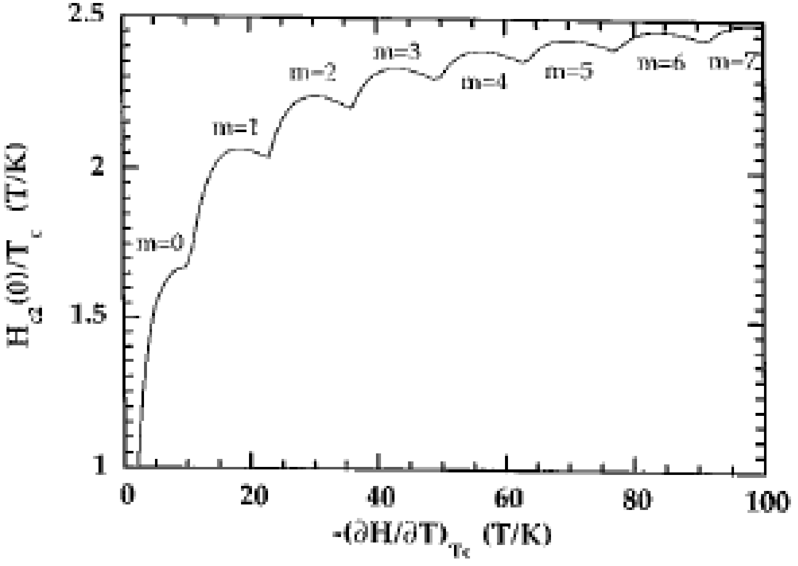
FFLO phase in 2D superconductors in the tilted magnetic field - upper critical field

Highest Landau level solutions are realized –
Bulaevskii, 1974; **Buzdin and Brison, 1996; Houzet and Buzdin, 2000.**

$$\Delta(\boldsymbol{r}) \sim e^{-im\varphi}\rho^m \exp(iQz)\exp(-\rho^2 eH/2\hbar c),$$







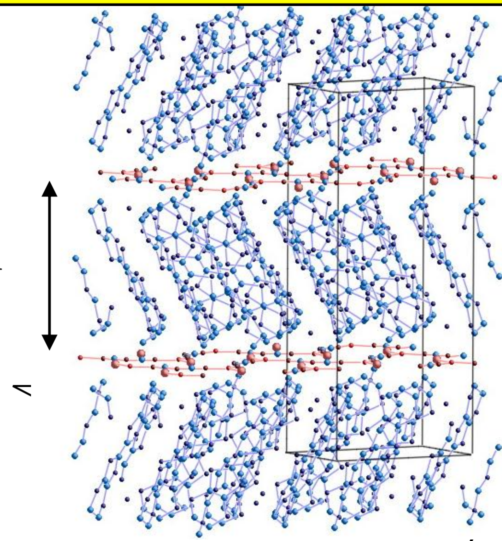
3. Experimental evidences of FFLO state.

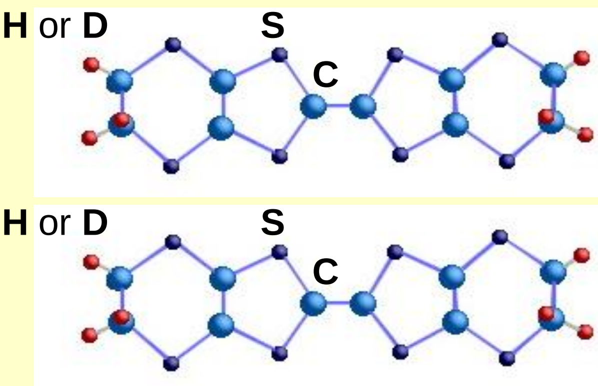
- Unusual form of $H_{c2}(T)$ dependence
- Change of the form of the NMR spectrum
- Anomalies in altrasound absorbtion
- Unusual behaviour of magnetization
- Change of anisotropy ….

Organic superconductor

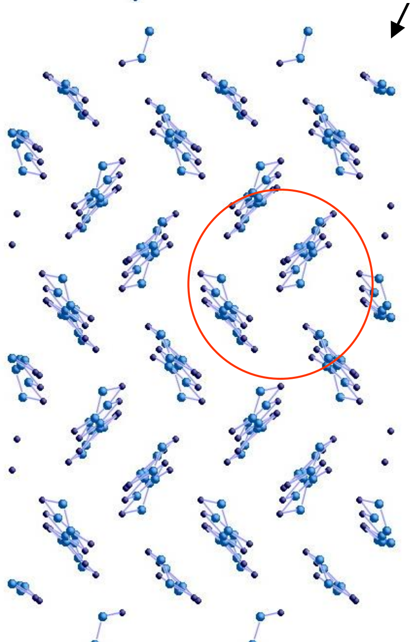
κ-(BEDT-TTF)$_2$Cu(NCS)$_2$ (T_c=10.4K)

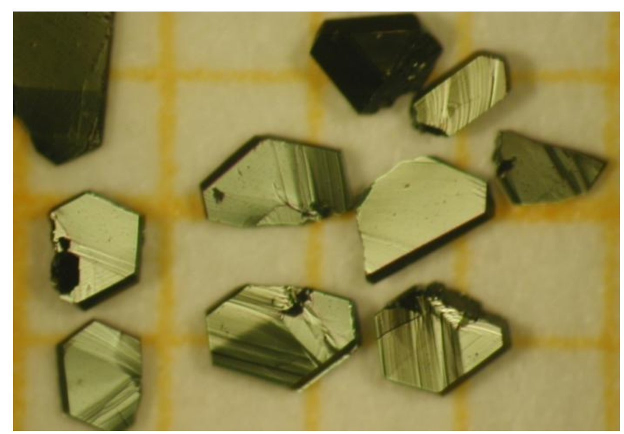
Layered structure ⇨ Suppression of orbital effects in ***H*** parallel to the planes

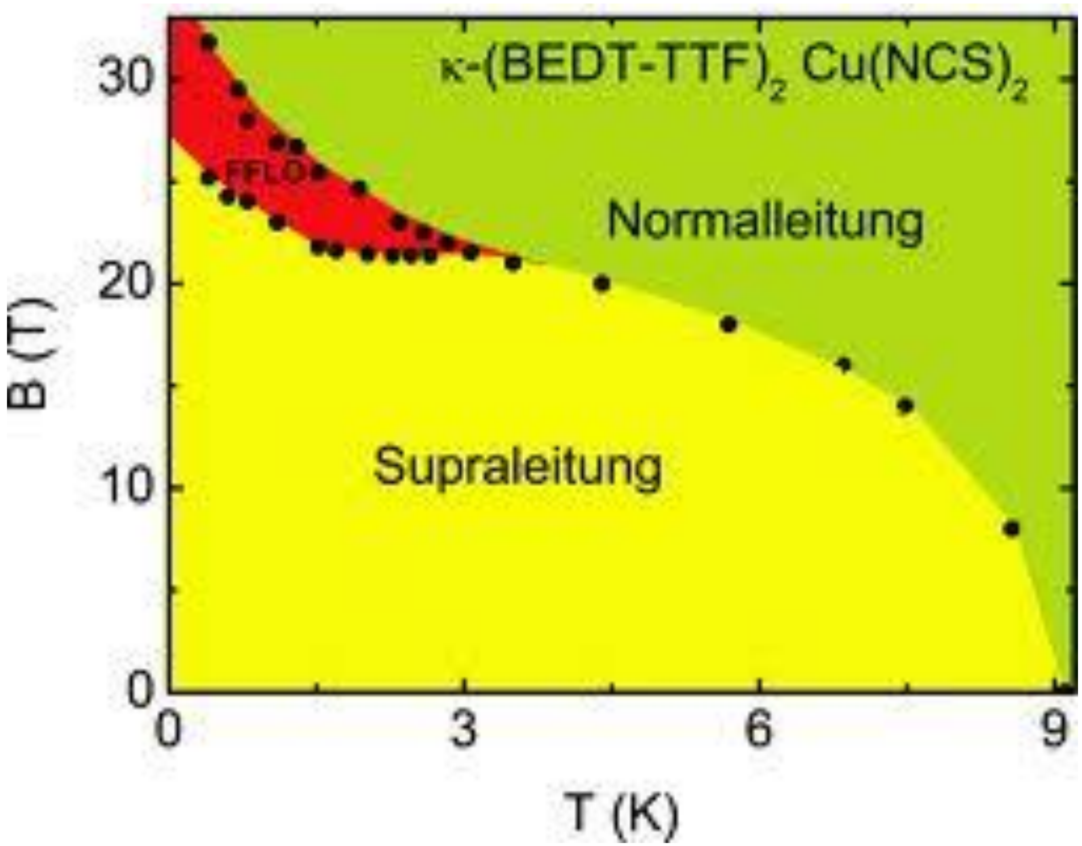




BEDT-TTF (donor molecule)



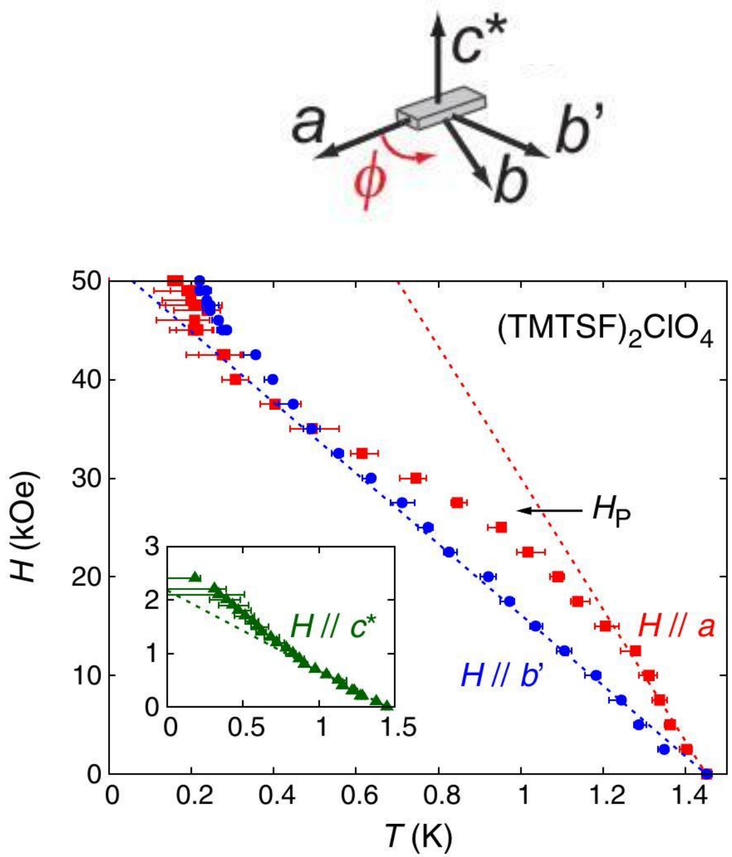


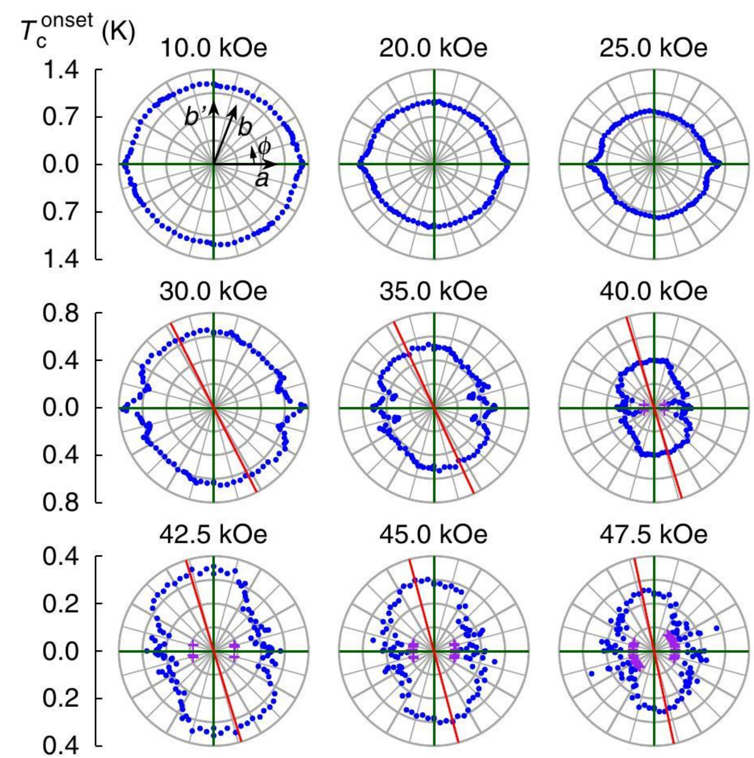


The Fulde-Ferrell-Larkin-Ovchinnikov State in the Organic Superconductor κ-(BEDT-TTF)$_2$Cu(NCS)$_2$ as Observed in Magnetic Torque Experiments

B. Bergk[a], A. Demuer[b], I. Sheikin[b], Y. Wang[c], J. Wosnitza[a], Y. Nakazawa[d], and R. Lortz[c]

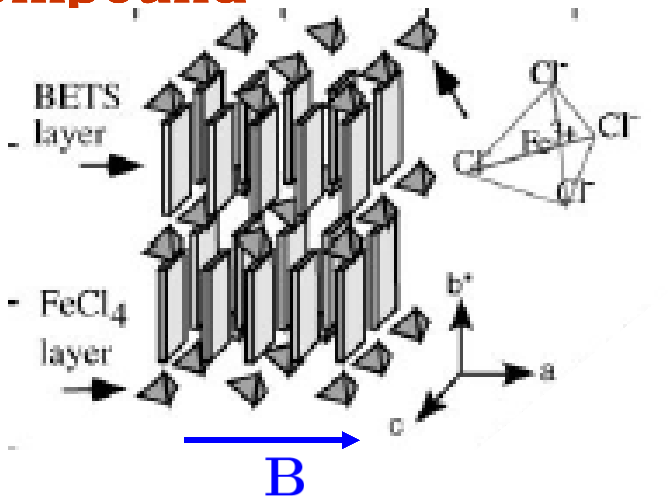
Anomalous in-plane anisotropy of the onset of SC in $(TMTSF)_2ClO_4$





S.Yonezawa, S.Kusaba, Y.Maeno, P.Auban-Senzier, C.Pasquier, K.Bechgaard, and D. Jerome, Phys. Rev. Lett. **100**, 117002 (2008)

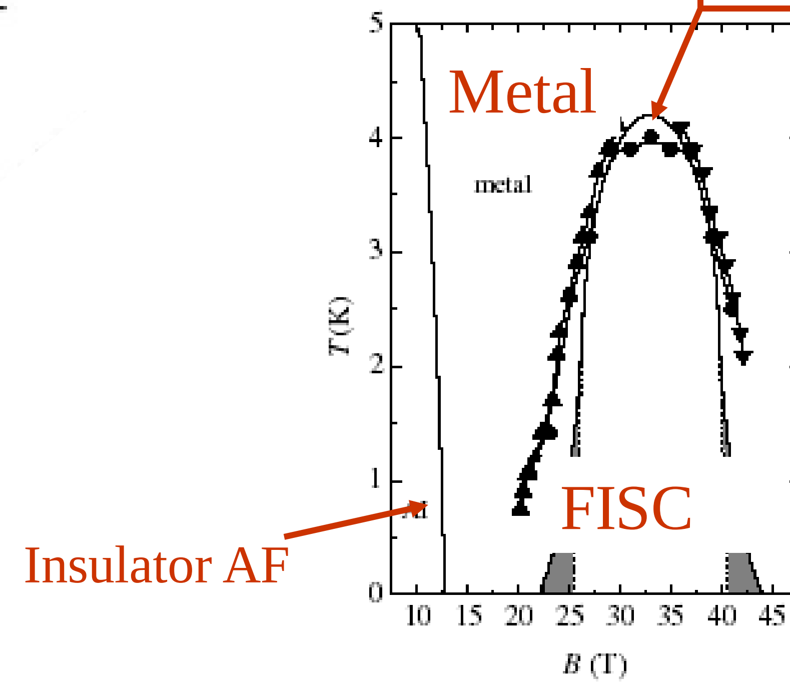
Field induced superconductivity (FISC) in an organic compound



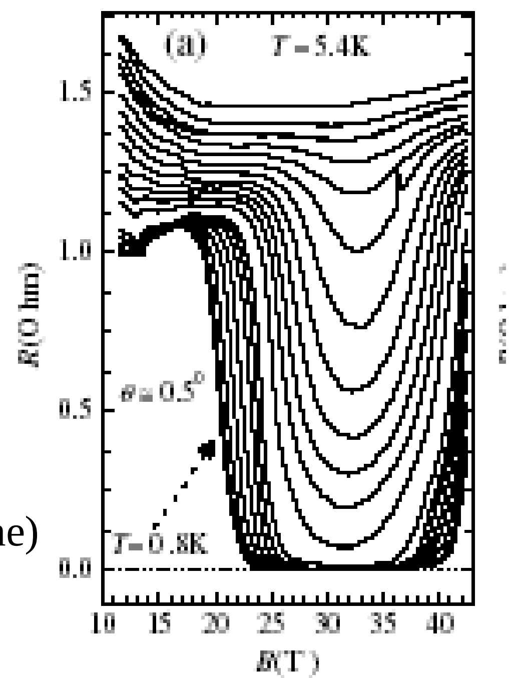
λ-$(BETS)_2FeCl_4$

$T_c^{max} \approx 4.2$K

$B_0 \approx 33$Tesla



FISC for $18\mathrm{T} < B < 45\mathrm{T}$



c-axis (in-plane) resistivity

S. Uji *et al.*, Nature **410** 908 (2001)

L. Balicas *et al.*, PRL **87** 067002 (2001)

Jaccarino-Peter effect

$$\mu_B H^{\text{eff}} = \mu_B H - J\langle S\rangle$$

Zeeman energy

Exchange energy between conduction electrons in the BETS layers and magnetic ions Fe^{3+} (S=5/2)

For some reason J > 0 : the paramagnetic effect is suppressed at $\boxed{\mu_B H_0 = J\langle S\rangle}$

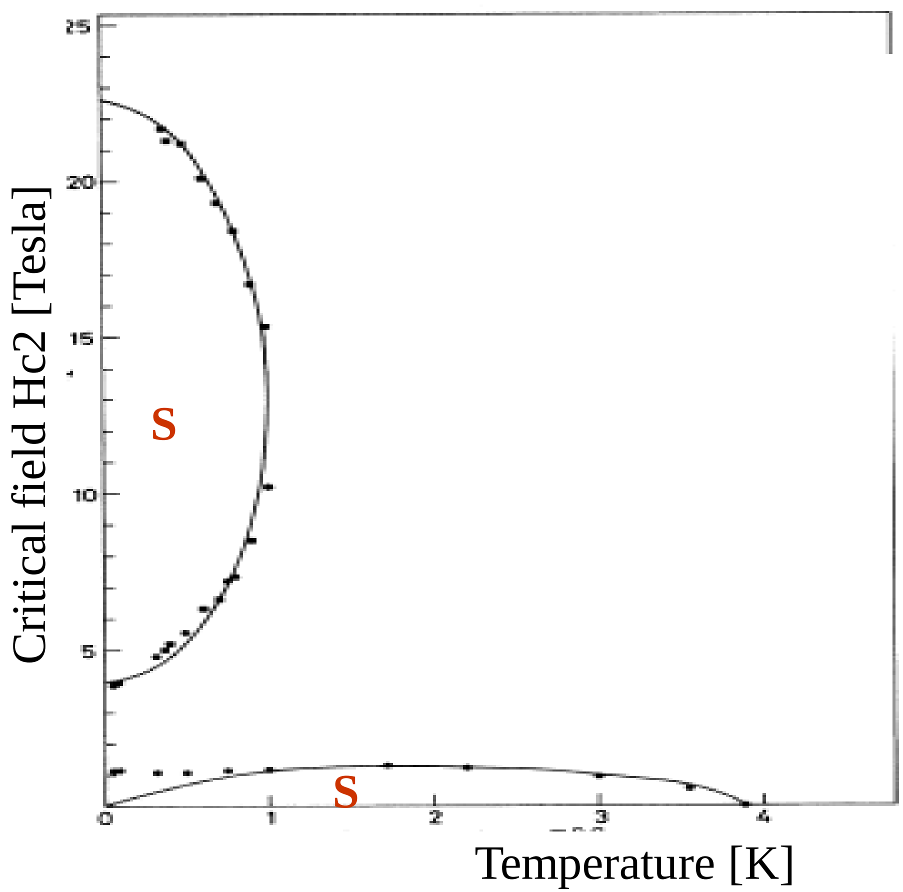
Other example:

Eu-Sn Molybdenum chalcogenide

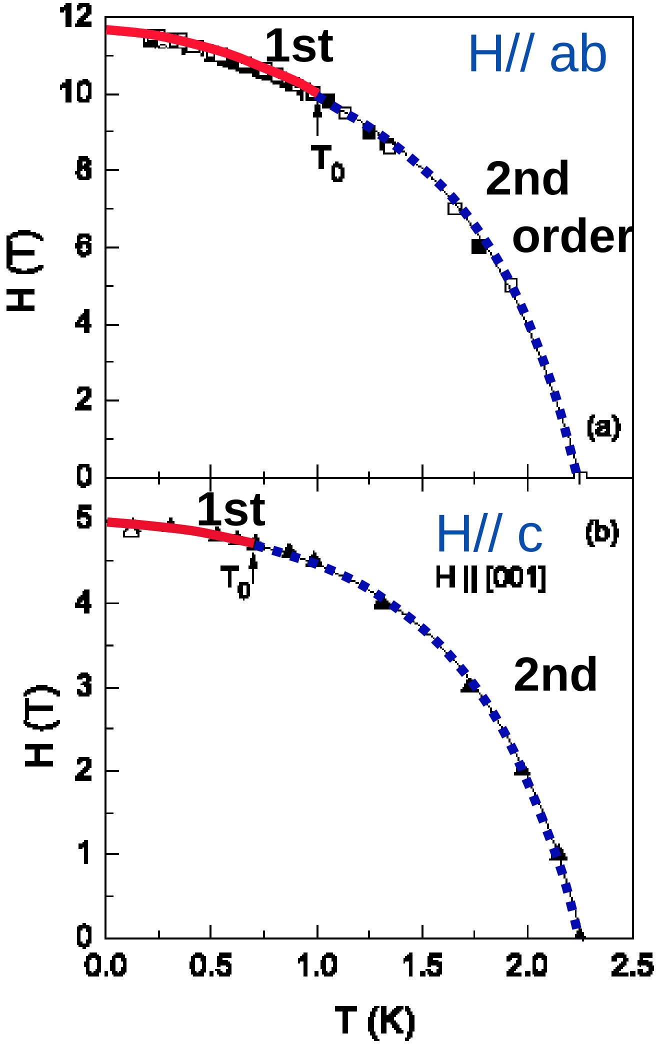
($Eu_{0.75}Sn_{0.25}Mo_6S_{7.2}Se_{0.8}$)

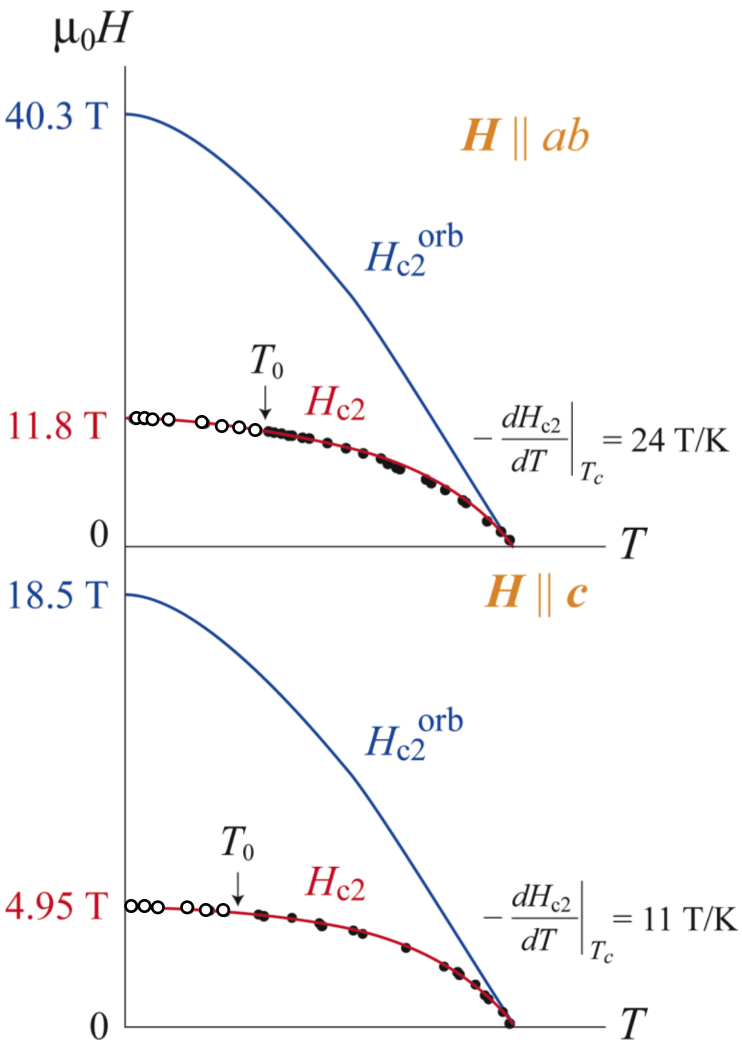
H. Meul et al, 1984

$T_c \approx 3.8\text{K}$



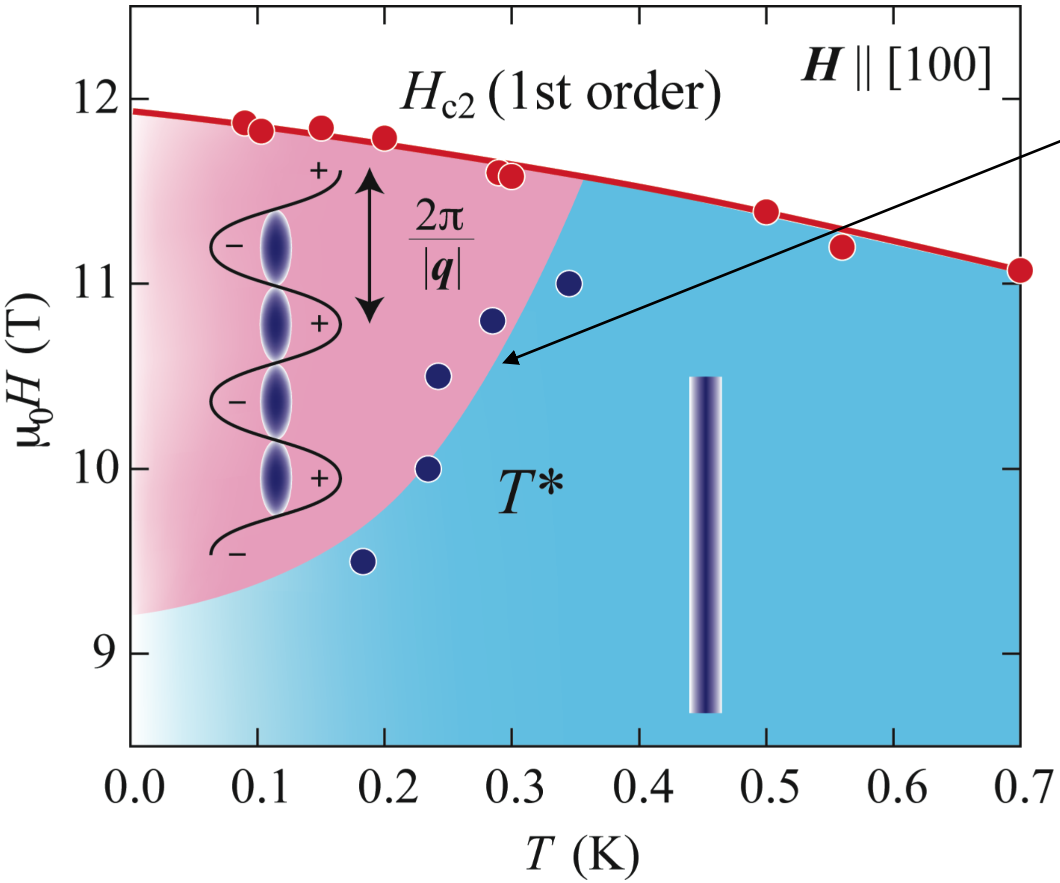
H-T phase diagram of $CeCoIn_5$





Pauli paramagnetically limited superconducting state

New high field phase of the flux line lattice in $CeCoIn_5$



This 2nd order phase transition is characterized by a structural transition of the flux line lattice

Ultrasound and NMR results are consistent with the FFLO state which predicts a segmentation of the flux line lattice

Proximity effect in a ferromagnet ?

In the usual case (normal proximity effect)

$$a\Psi - \frac{1}{4m}\nabla^2\Psi = 0, \text{ and solution for } T > T_c \text{ is } \Psi \propto e^{-qx}, \text{ where } q = \sqrt{4ma}$$

In **ferromagnet** (in presence of exchange field) the equation for superconducting order parameter is different

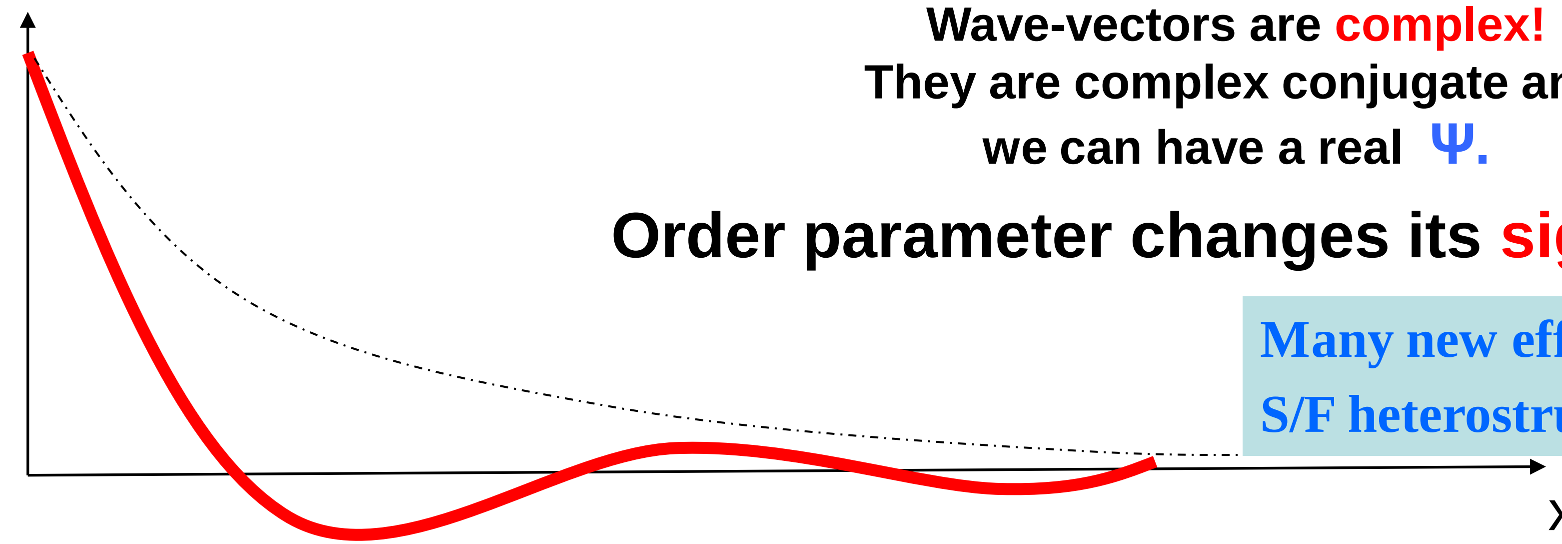
$$a\Psi + \gamma\nabla^2\Psi - \eta\nabla^4\Psi = 0$$

Its solution corresponds to the order parameter which decays with oscillations!

$\Psi \sim \exp[-(q_1 \quad iq_2\,)x]$

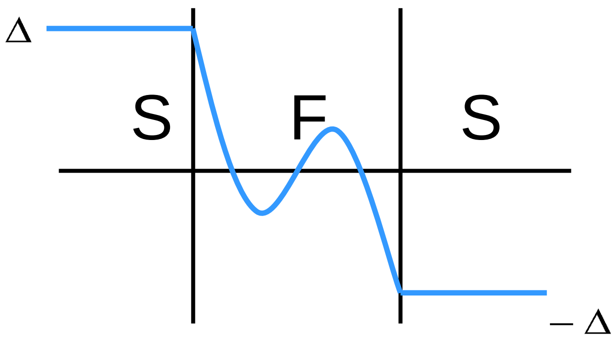
Wave-vectors are complex!
They are complex conjugate and we can have a real Ψ.

Order parameter changes its sign!

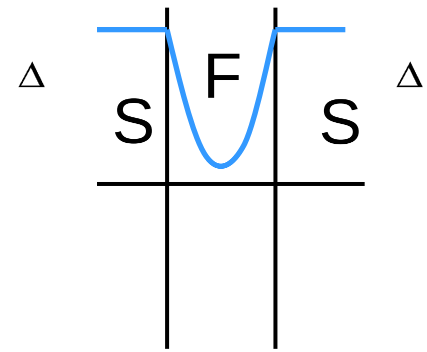


Many new effects in S/F heterostructures!

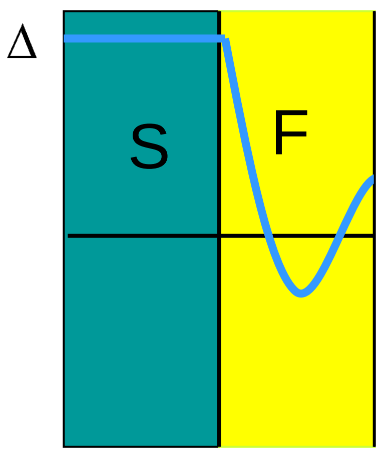
Remarkable effects come from the possible shift of sign of the wave function in the ferromagnet, allowing the possibility of a « π-coupling » between the two superconductors (π-phase difference instead of the usual zero-phase difference)



« π phase »



« 0 phase »



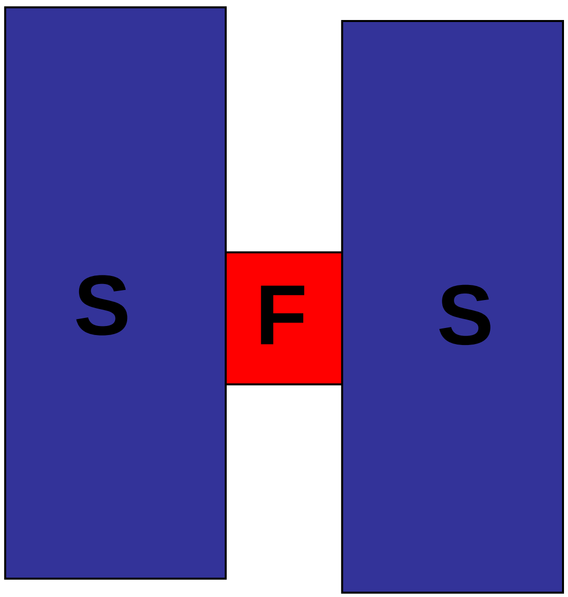
S/F bilayer

$$\xi_f = \sqrt{D_f / h} \propto (1-10)nm$$

h-exchange field,

D_f-diffusion constant

S-F-S Josephson junction in the clean/dirty limit

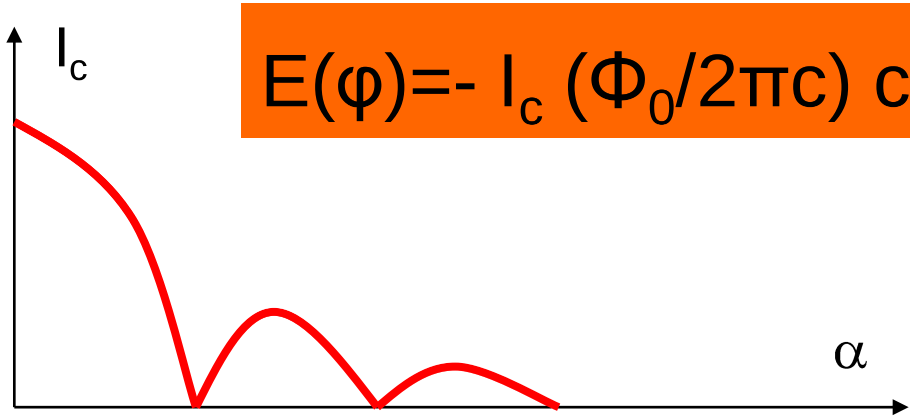


Damping oscillating dependence of the critical current I_c as the function of the parameter $\alpha = hd_F / v_F$ has been predicted.

(Buzdin, Bulaevskii and Panjukov, JETP Lett. 81)

h- exchange field in the ferromagnet,
d_F - its thickness

$E(\varphi) = -I_c (\Phi_0/2\pi c) \cos\varphi$

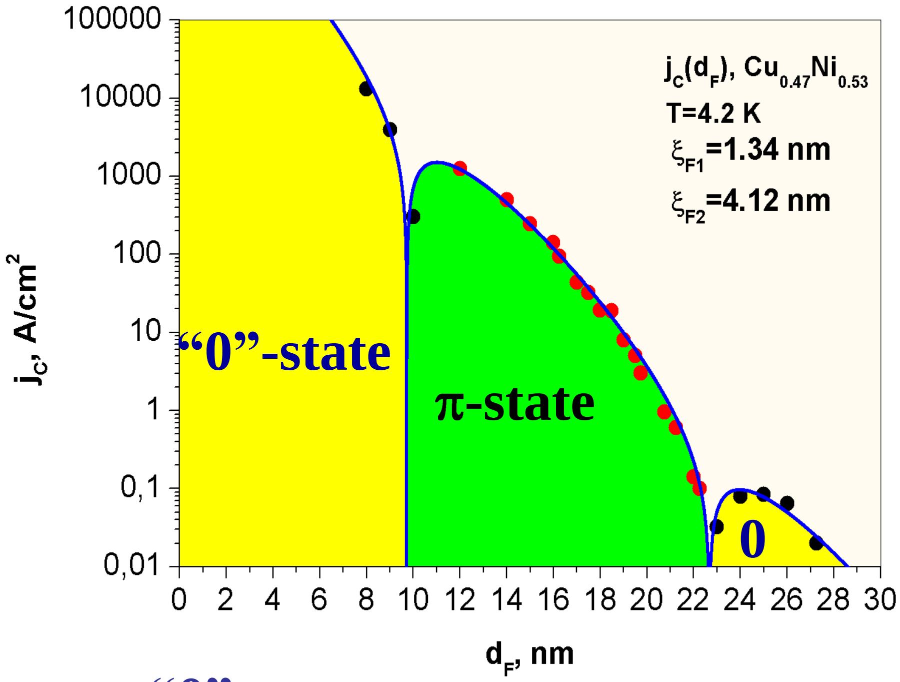


$J(\varphi) = I_c \sin\varphi$

Critical current density vs. F-layer thickness (V.A.Oboznov et al., PRL, 2006)

Collaboration with V. Ryazanov group from ISSP, Chernogolovka

$$I_c=I_{c0}\exp(-d_F/\xi_{F1})\,|\cos(d_F/\xi_{F2})+\sin(d_F/\xi_{F2})|$$



$d_F \gg \xi_{F1}$

Spin-flip scattering decreases the decaying length and increases the oscillation period.

$\xi_{F2} > \xi_{F1}$

$Nb\text{-}Cu_{0.47}Ni_{0.53}\text{-}Nb$ junctions

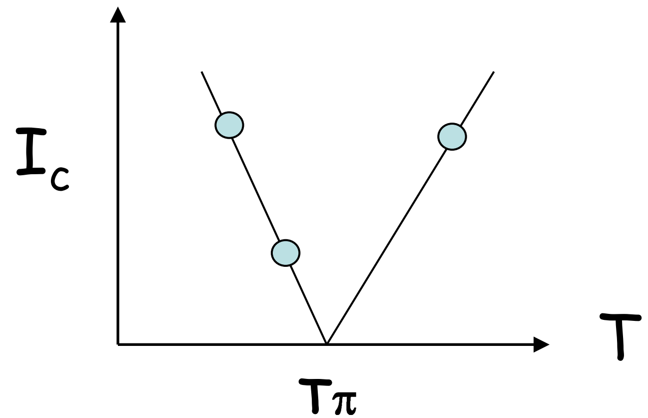
"0"-state

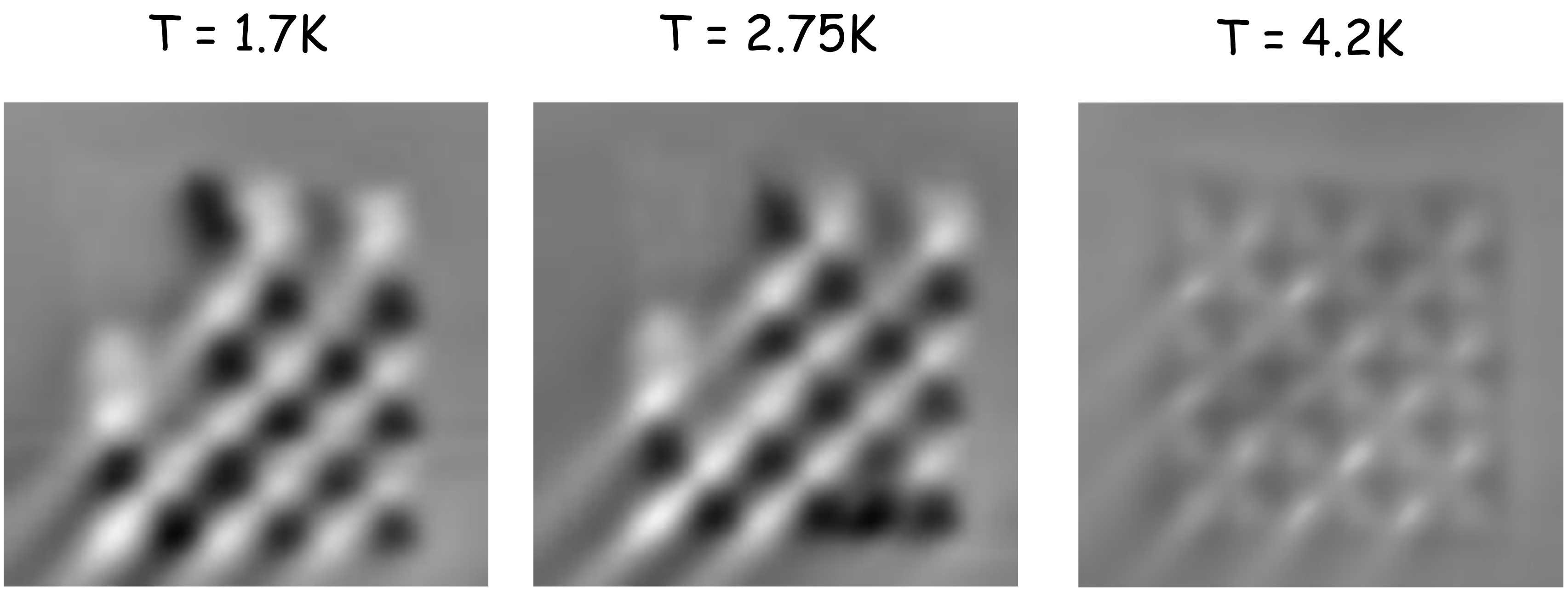
$I=I_c\sin\varphi$

π-state

$I=I_c\sin(\varphi+\pi)= - I_c\sin(\varphi)$

Scanning SQUID Microscope images
(Ryazanov et al., Nature Physics, 2008))

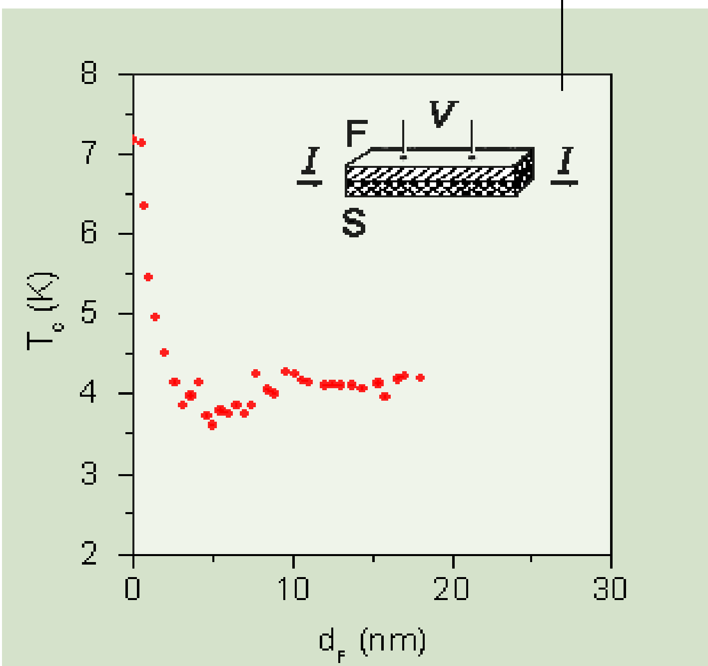


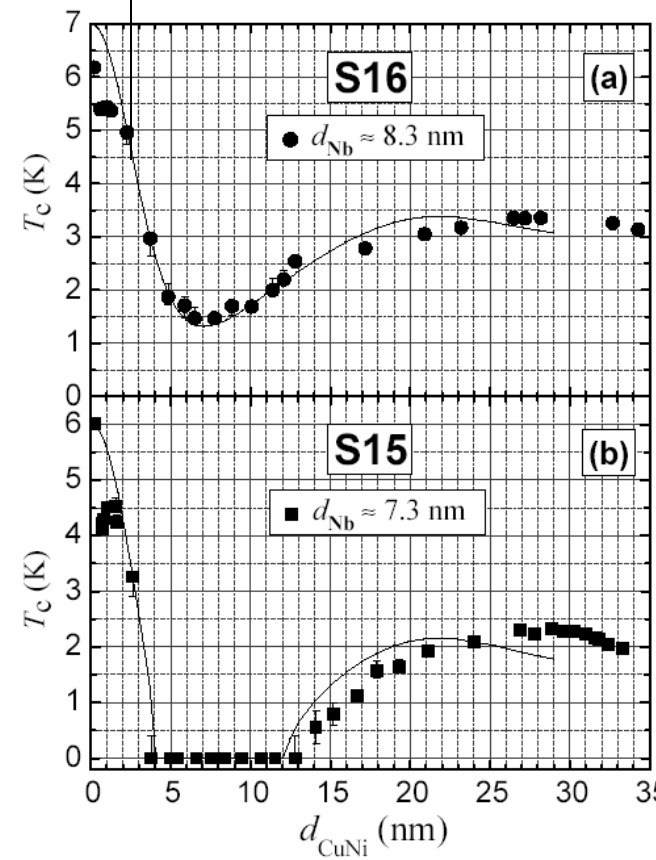


SF-bilayer T_c-oscillations

Ryazanov et al. JETP Lett. 77, 39 (2003) Nb-$Cu_{0.43}Ni_{0.57}$

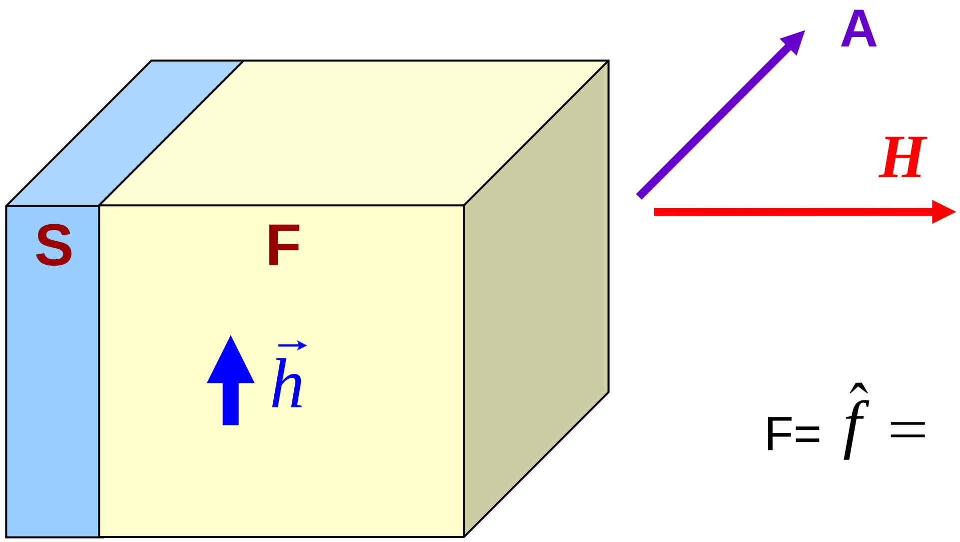
V. Zdravkov, A. Sidorenko et al
PRL (2006)
Nb-$Cu_{0.41}Ni_{0.59}$





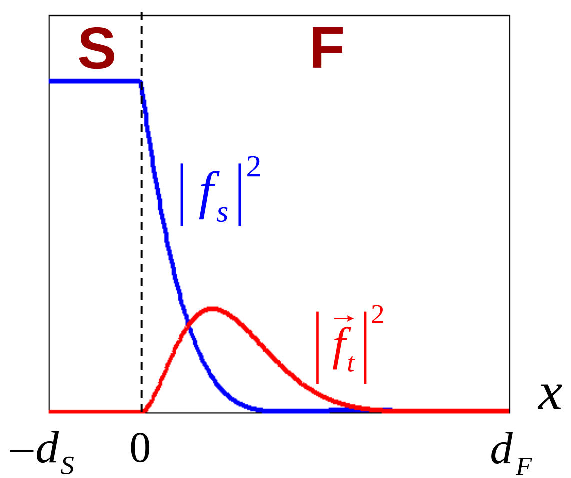
d_{Fmin} =(1/4) λ_{ex} largest T_c-suppression

Paramagnetic Meissner effect in dirty S/F bilayers



$$\vec{j} = -\frac{1}{4\pi}\lambda^{-2}\vec{A}$$

F= $\hat{f} = \; f_s + \vec{f}_t\hat{\vec{\sigma}} \;\; i\hat{\sigma}_y$



$$\boxed{\lambda^{-2} = \frac{16\pi^2 T_c}{d_0}\sum_{n=0}^{\infty}\int \sigma \; |f_s|^2 - |\vec{f}_t|^2 \;\; dx}$$

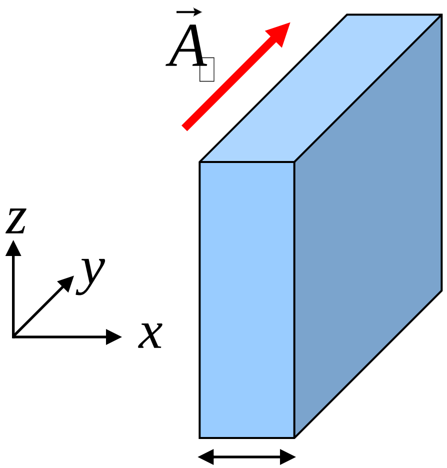
$$|\vec{f}_t|^2 > |f_s|^2 \quad \Rightarrow \quad \lambda^{-2}(x) < 0$$

Local paramagnetic Meissner effect

Anomalous screening for the long ranged triplet proximity effect?

Mironov, Melnikov, Buzdin, PRL (2012)

FFLO states in thin-film S/F systems



$d_0 \ll \lambda$

$$\boxed{\hat{f} = \hat{f}(x)\exp\left(i\vec{k}\vec{r}\right)}$$

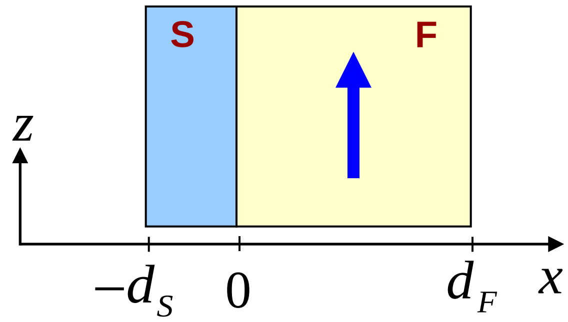
$$F_A = \left(\vec{A} - \frac{\Phi_0}{2\pi}\vec{k}\right)^2 \frac{S}{8\pi}\int \lambda^{-2}dx$$

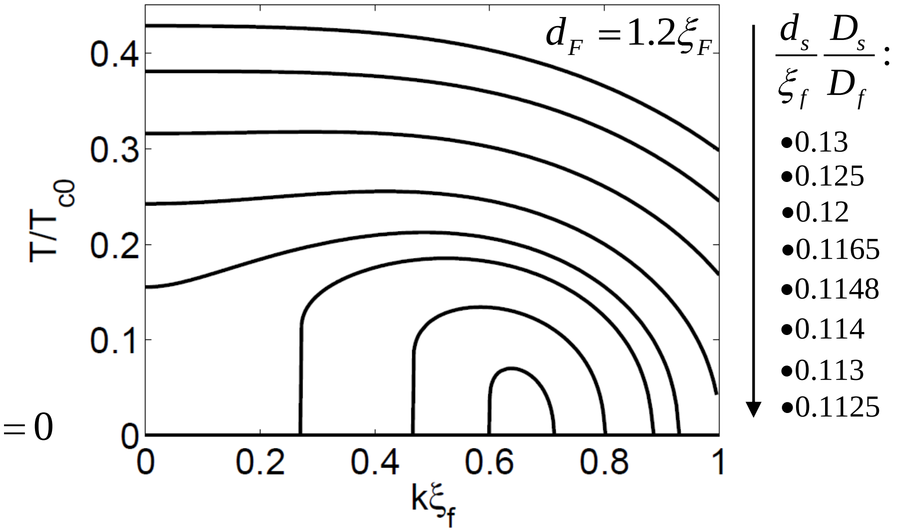
A hallmark of the instability: vanishing Meissner effect

$$\boxed{\lambda^{-2} = \frac{1}{d_0}\int \frac{e^2 n_s}{2m}dx < 0} \Longrightarrow$$ **FFLO state**

Note that previously the FFLO state in S/F systems was introduced by Proshin, Izyumov and Khusainov (JETP Lett., 2000) but on the basis of the erroneous boundary conditions and the modulation was considered only in the F layers.

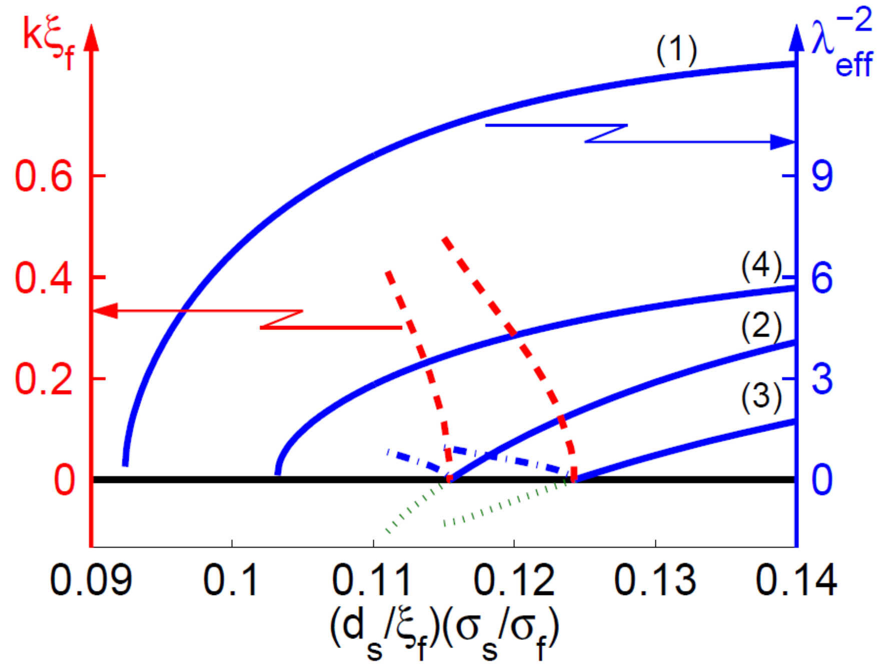
FFLO state in S/F bilayers





$$\frac{D}{2}\frac{\partial^2 \hat{f}}{\partial x^2} - \left(\omega_n + \frac{D}{2}k^2\right)\hat{f} - \frac{i}{2}\left\{\vec{h}\hat{\vec{\sigma}}\hat{f} + \hat{f}\vec{h}\hat{\vec{\sigma}}\right\} + \hat{\Delta} = 0$$

$$\Delta \ln\frac{T_c(k)}{T_{c0}} + \sum_{n=-\infty}^{\infty}\left(\frac{\Delta}{n+\frac{1}{2}} - \frac{f_{12}^S}{2\pi T_c(k)}\right) = 0$$



Stability of the FFLO state:

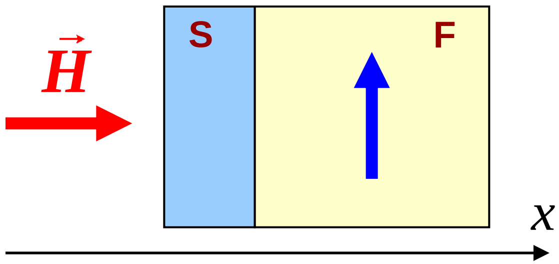
$$d_F \sim \xi_f$$

$$\frac{h}{T_{c0}}\xi_f \le d_S \le \frac{D_f}{D_s}\xi_f$$

$$\boxed{\frac{D_f}{D_s} >> \frac{h}{T_{c0}}}$$

$$\boxed{T_c << T_{c0}}$$

Upper critical field

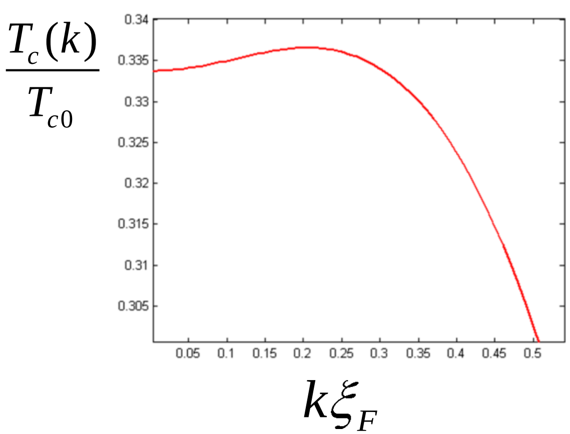


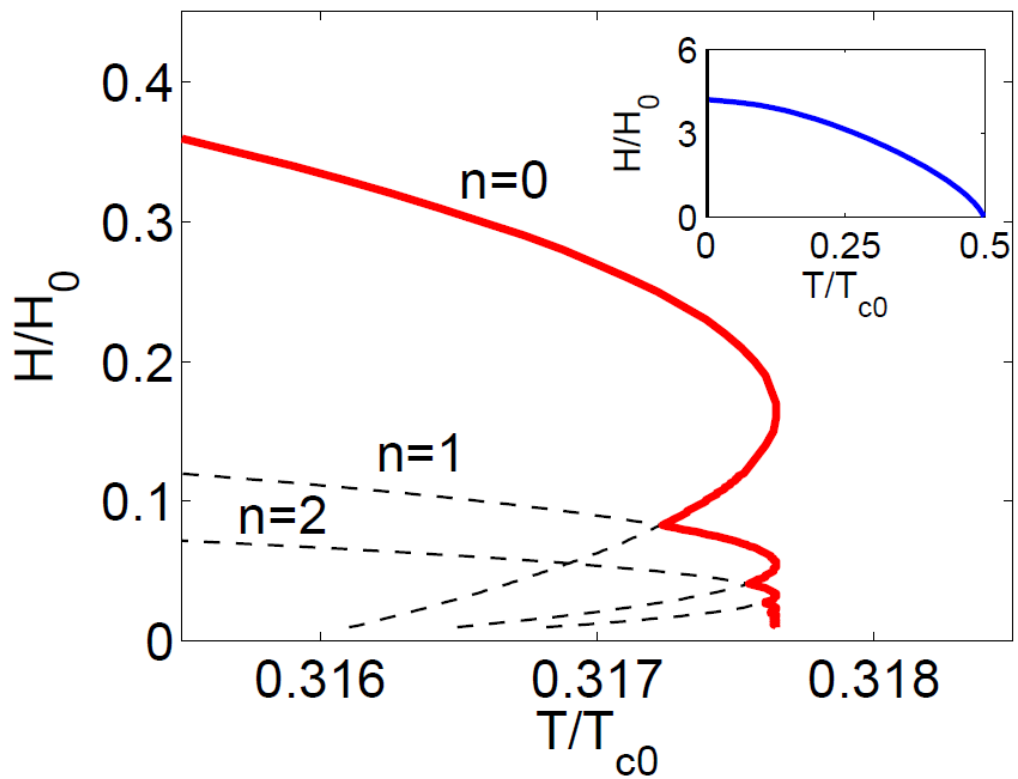
$$D\left[\partial_x^2 + \left(\partial_{\vec{r}_\parallel} - 2e\vec{A}_\parallel\right)^2\right] f_{12} - 2\left(\omega_n + ih\right) f_{12} + 2\Delta = 0$$

$$f_{12} = \chi_n(\vec{r}_\parallel)\varphi(x)$$

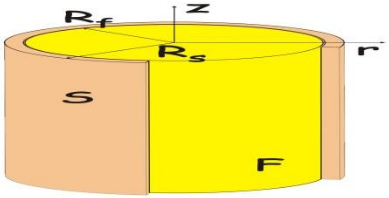
$$-\left(\partial_{\vec{r}_\parallel} - 2e\vec{A}_\parallel\right)^2 \chi_n(\vec{r}_\parallel) = 4eH\left(n + \frac{1}{2}\right)\chi_n(\vec{r}_\parallel)$$

$$\boxed{k^2 \to 4eH\left(n + \frac{1}{2}\right)}$$

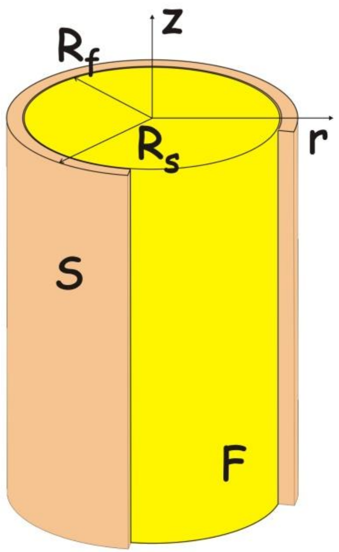




Proximity induced vortex states

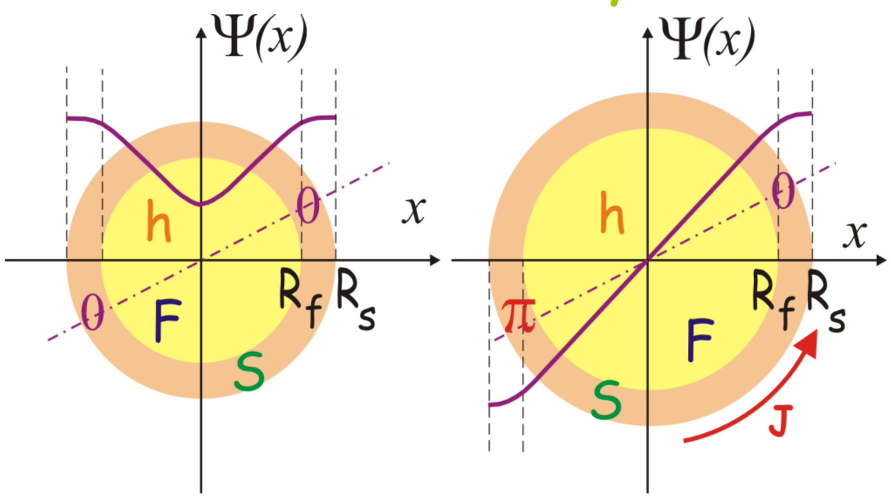


(A. Melnikov, A. Samokhvalov, A.B- PRB, 2007; PRB, 2009)



Thin-walled superconducting (S) shell around a ferromagnetic (F) cylinder

Commensurability effects

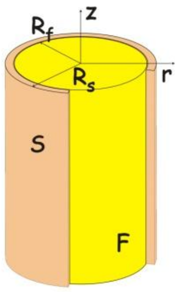


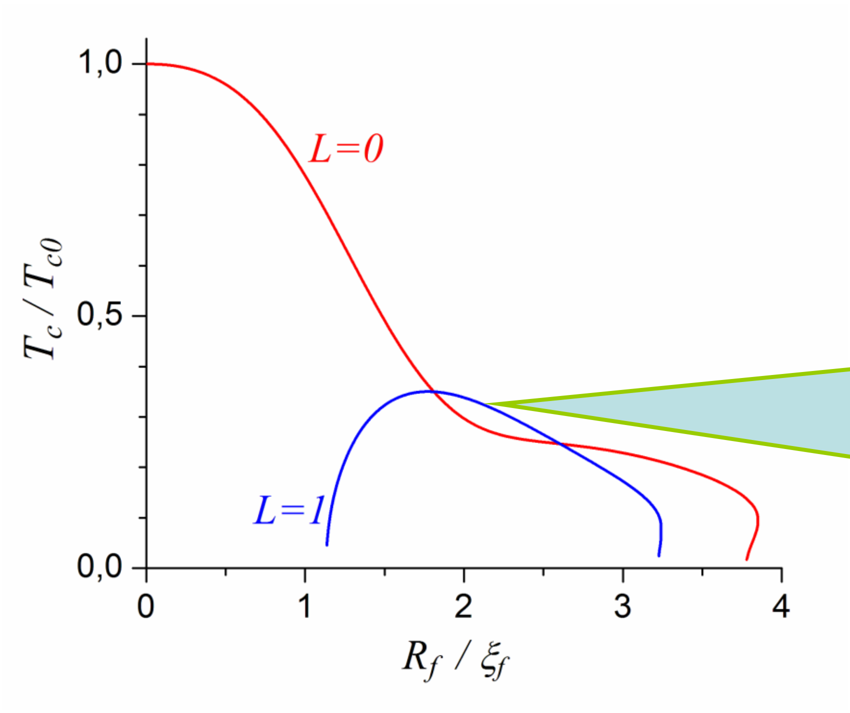
L = 0 phase
$2R_f < \xi_f$

L = 1 phase
$2R_f \sim \xi_f$

Interplay between the exchange effect and supercurrent energy can result in switching the states with different vorticity L

Critical temperature Tc of vortex states

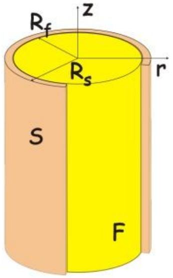


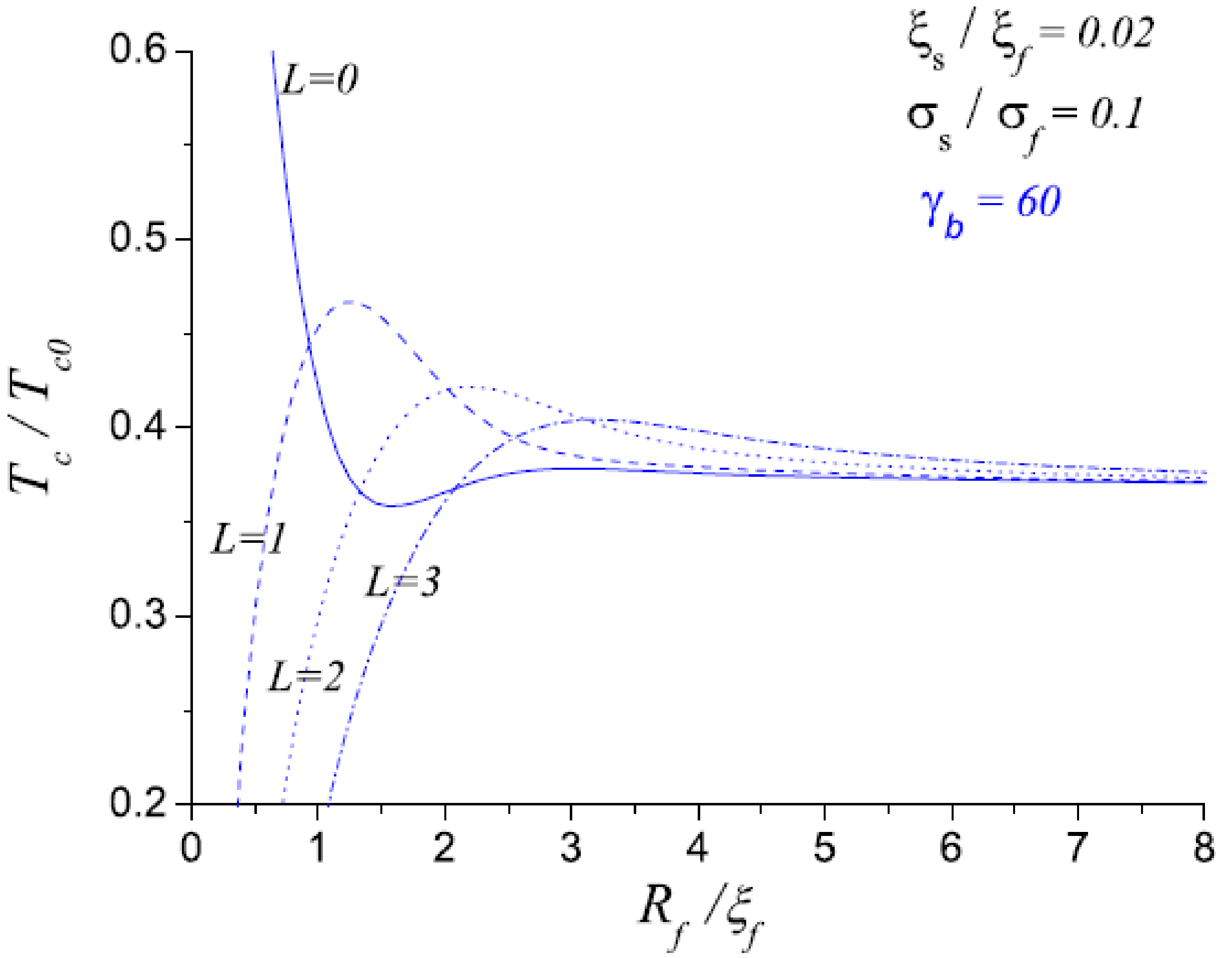


The penetration of Cooper pairs into the FM core and the phase shift of the pair wave function due to the exchange interaction can induce vortex states in the superconducting shell.

The dependence of the critical temperature Tc on the F core radius R_f for two values of the vorticity L = 0 and L = 1:
($d=0.5\xi_s$; $\sigma_s/\sigma_f = 2.5$; $\xi_s/\xi_f = 0.265$)

Cascades of the transitions



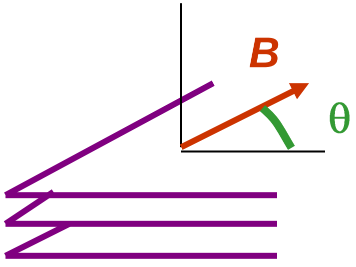


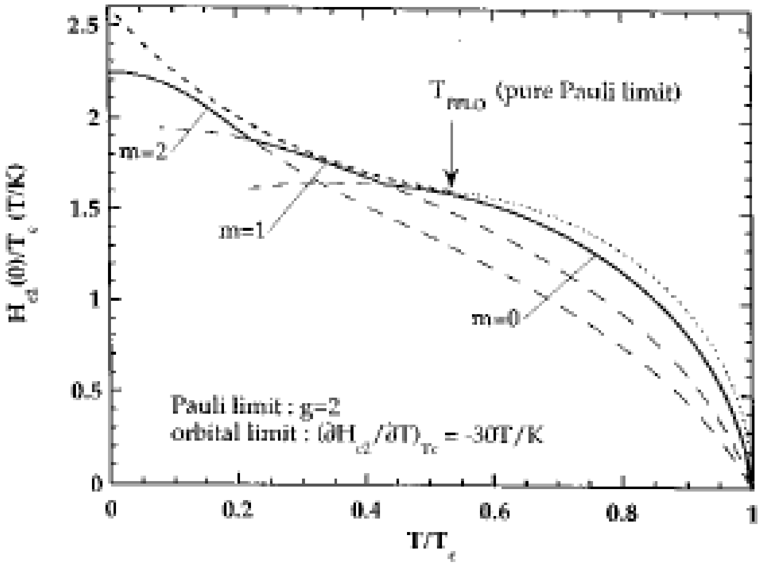
4. Vortices in FFLO state.

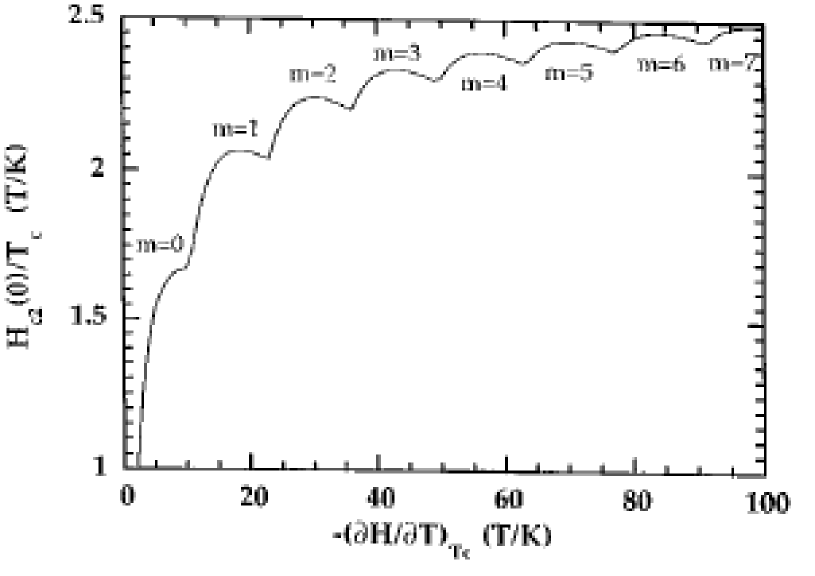
FFLO phase in 2D superconductors in the tilted magnetic field - upper critical field

Highest Landau level solutions are realized –
Bulaevskii, 1974; **Buzdin and Brison, 1996; Houzet and Buzdin, 2000.**

$$\Delta(\boldsymbol{r}) \sim e^{-im\varphi}\rho^m \exp(iQz)\exp(-\rho^2 eH/2\hbar c),$$







Exotic vortex lattice structures in tilted magnetic field

Generalized Ginzburg-Landau functional

Near the tricritical point, the characteristic length is $q(T)^{-1} \underset{T \to T^\star}{\longrightarrow} \infty$

Microscopic derivation of the Ginzburg-Landau functional :

$$\frac{\mathcal{F}}{N(0)T_c^2} = 0.86\,\frac{B - H^{\text{eff}}(T)}{H^{\text{eff},\star}}|\Delta|^2 + 3.0\,\frac{T - T^*}{T^*}\xi_0^2|\nabla\Delta|^2 + 0.15\,\frac{T - T^*}{T^*}|\Delta|^4$$

Instability toward FFLO state

Instability toward 1st order transition

Next orders are important :

$$\frac{\mathcal{F}}{N(0)T_c^2} = 0.86\,\frac{B - H^{\text{eff}}(T)}{H^{\text{eff},\star}}|\Delta|^2 + 3.0\,\frac{T - T^*}{T^*}\xi_0^2|\tilde{\nabla}\Delta|^2 + 3.1\,\xi_0^4|\tilde{\nabla}^2\Delta|^2 + 0.15\,\frac{T - T^*}{T^*}|\Delta|^4$$
$$+0.85\,\xi_0^2\left\{|\Delta|^2|\tilde{\nabla}\Delta|^2 + \frac{1}{8}\left[(\Delta^*\tilde{\nabla}\Delta)^2 + (\Delta\tilde{\nabla}\Delta^*)^2\right]\right\} + 0.011\,|\Delta|^6$$

Validity:

- large scale for spatial variation of Δ : vicinity of T^*

  small orbital effect, introduced with $\tilde{\nabla} = \nabla - \frac{2ie}{\hbar c}\mathbf{A}$
- we neglect diamagnetic screening currents (high-κ limit)

• 2nd order phase transition at

$$0.86\,\frac{B-H_{eff}(T)}{H^*_{eff}}\Delta - 3.0\,\frac{T-T^*}{T^*}\xi_0^2\tilde{\nabla}^2\Delta + 3.1\,\xi_0^4\tilde{\nabla}^4\Delta = 0$$

$\rightarrow$ higher Landau levels $\quad \tilde{\nabla}^2\Delta_N = -\frac{4eH_\perp}{\hbar c}(N+\frac{1}{2})\Delta_N$

• Near the transition: minimization of the free energy with solutions in the form

$$\boxed{\Psi_{\zeta=\rho+i\sigma}(x,y) = \frac{(2\sigma)^{\frac{1}{4}}}{(2^N N!)^{\frac{1}{2}}} e^{-\frac{\pi y^2 B_\perp}{\phi_0}} \sum_p \mathcal{H}_N\left(y\sqrt{\frac{2\pi B_\perp}{\phi_0}} + p\sqrt{2\pi\sigma}\right) e^{2i\pi p(x+iy)\sqrt{\frac{\sigma B_\perp}{\phi_0}} + i\pi p\zeta^2}}$$

gauge $\quad \mathbf{A} = (0, -yB_\perp, 0)$

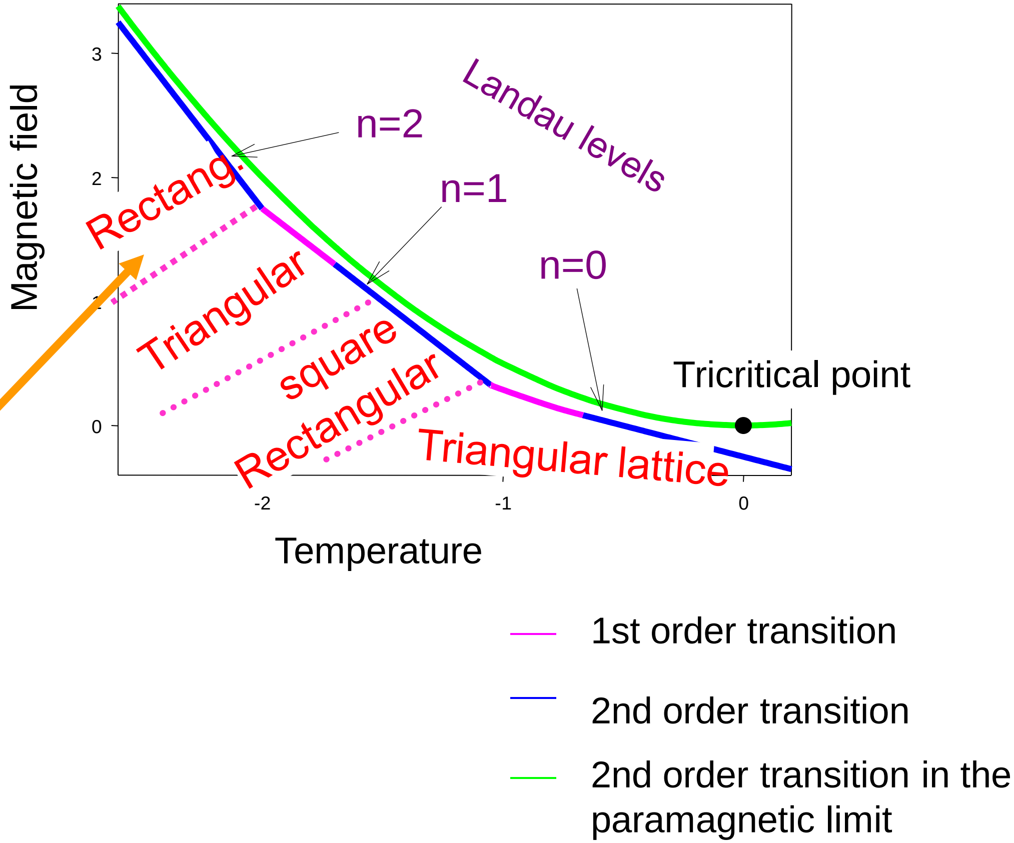
ζ Parametrizes all vortex lattice structures at a given Landau level N

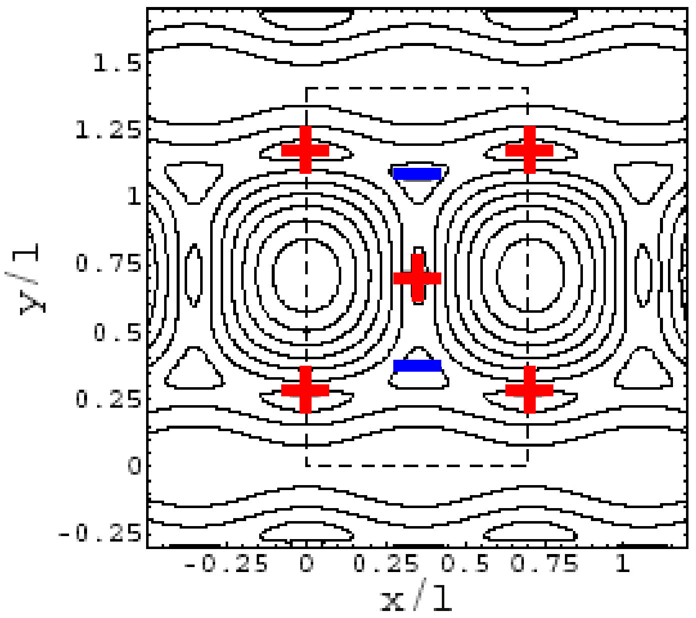
$$(\mathbf{r}_1, \mathbf{r}_2) = \left(\sqrt{\frac{\phi_0}{\sigma B_\perp}}, \zeta\sqrt{\frac{\phi_0}{\sigma B_\perp}}\right)$$ is the unit cell

All of them are decribed in the subset : $\quad \left[|\zeta| > 1; 0 < \rho < \frac{1}{2}\right]$

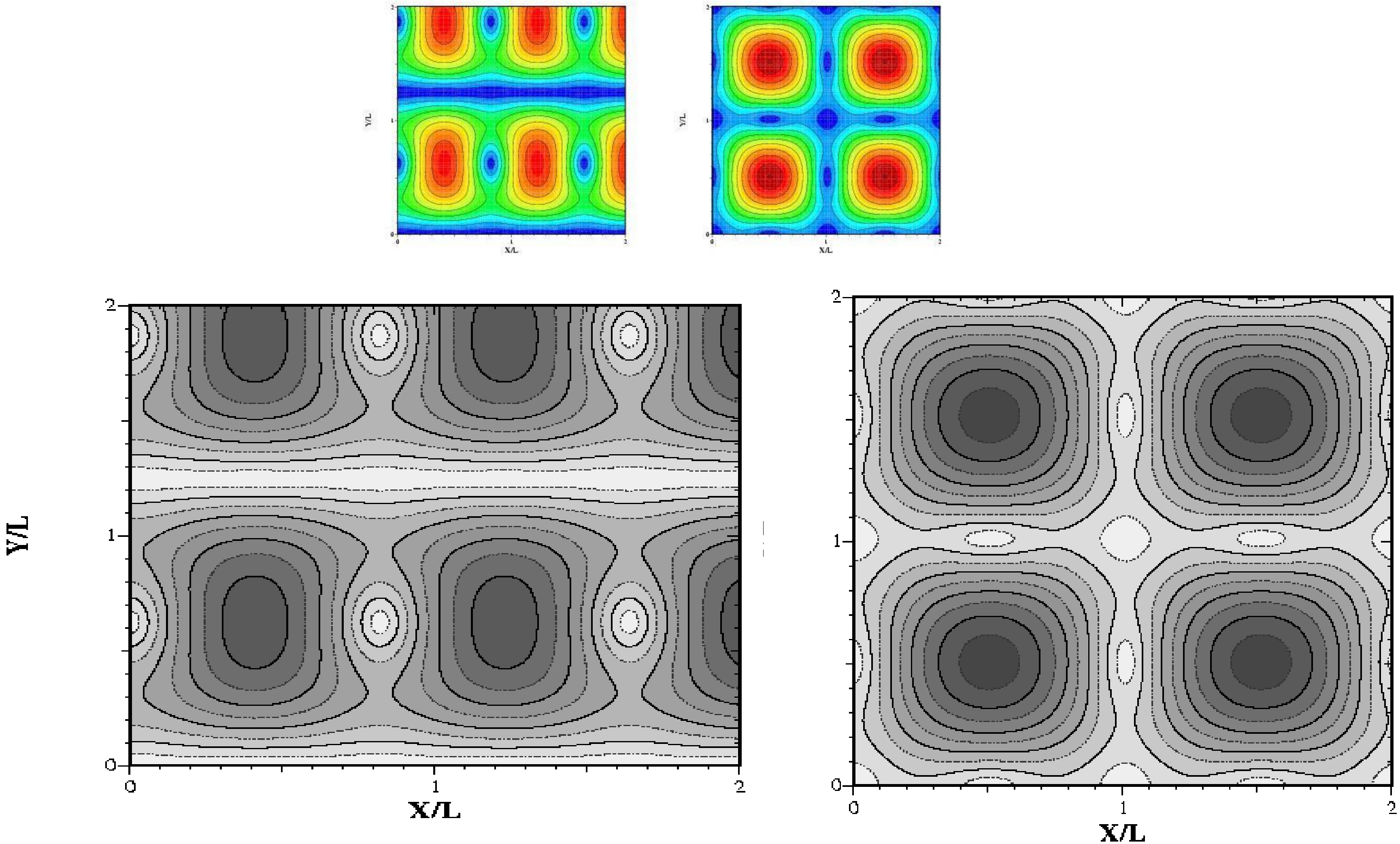
Analysis of phase diagram :

- cascade of 2nd and 1st order transitions between S and N phases
- 1st order transitions within the S phase
- exotic vortex lattice structures





At Landau levels n > 0, we find structures with several points of vanishment of the order parameter in the unit cell and with different winding numbers w = ± 1, ± 2 …

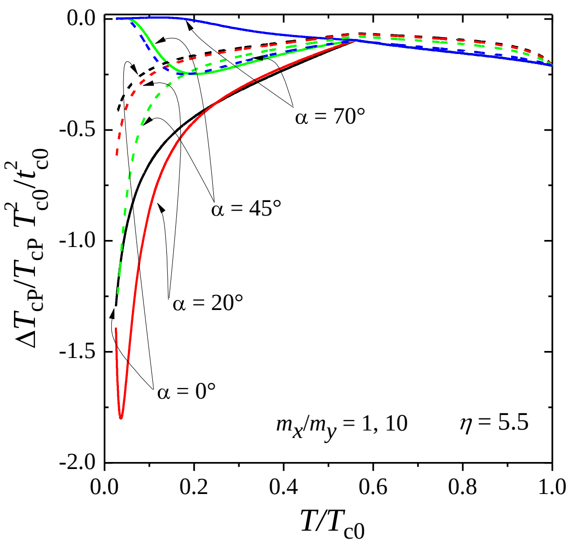


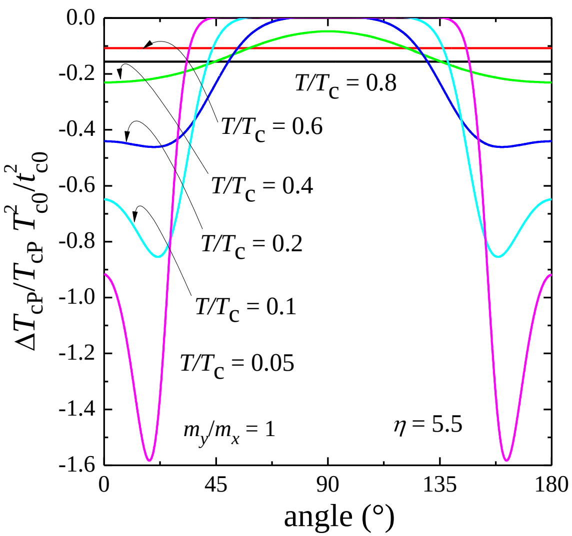
Order parameter distribution for the asymmetric and square lattices with Landau level n=1.
The dark zones correspond to the maximum of the order parameter and the white zones to its minimum.

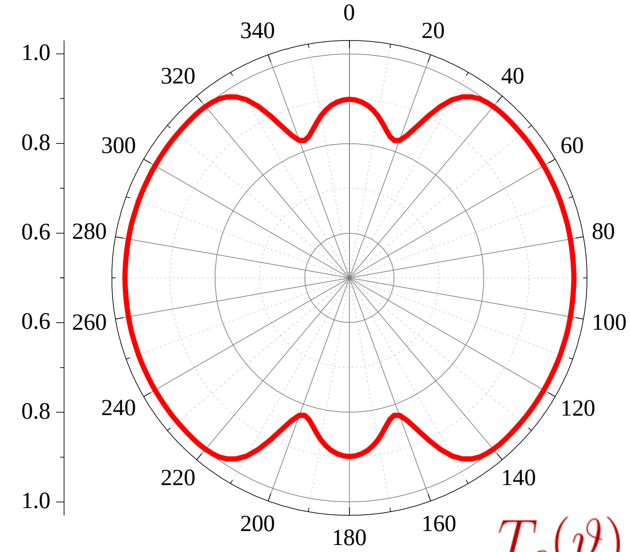
Phase diagram with FFLO

- Normalized correction of the superconducting onset temperature as a function of (*i*) the reduced temperature, (*ii*) of the magnetic field direction;
- Polar plots of $T_c(\vartheta)$.

$m_x = m_y$







$T_c(\vartheta)$

$$\eta = \hbar v_F \pi d / \phi_0 \mu_B$$

$$H \to H \sqrt{\sin^2(\vartheta) + \frac{m_x}{m_y} \cos^2(\vartheta)}$$

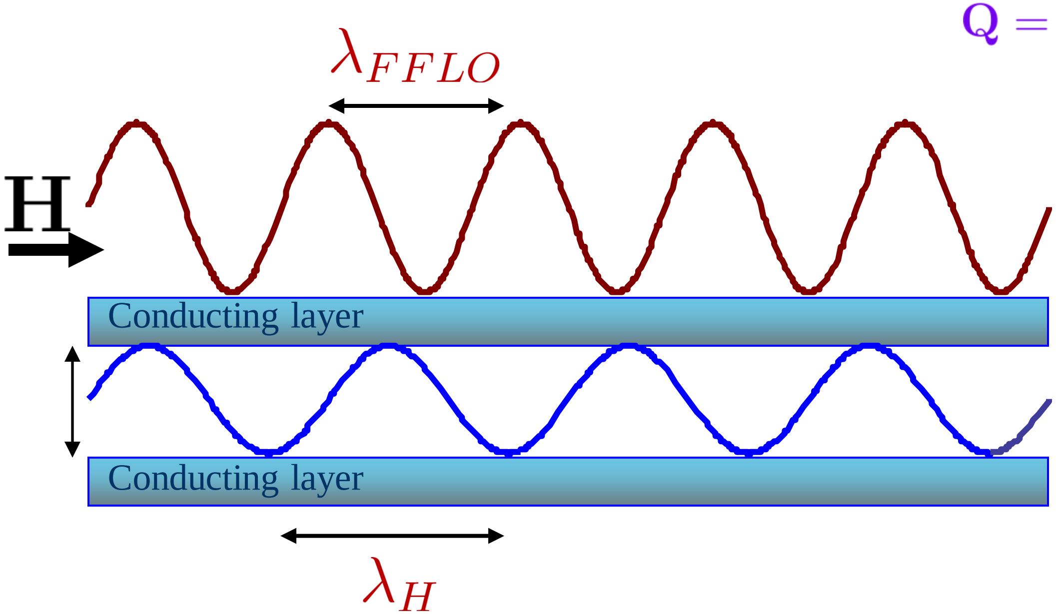
$$T/T_{c0} = 0.05$$

$$\eta = 5.5$$

Croitoru, Houzet, Buzdin, PRL, 2012

Resonance condition

- Vector potential of the parallel magnetic field results in a modulation of the interlayer coupling;



$$\mathbf{Q} = \frac{\pi d}{\phi_0} H \left(-\sin\alpha, \cos\alpha, 0\right)$$

$$\Delta\left(\mathbf{r}\right) = \Delta_0 \cos\left(\mathbf{q}.\mathbf{r}\right) e^{i\phi_n(r)}$$

$$t\cos(\mathbf{Q}.\mathbf{r})$$

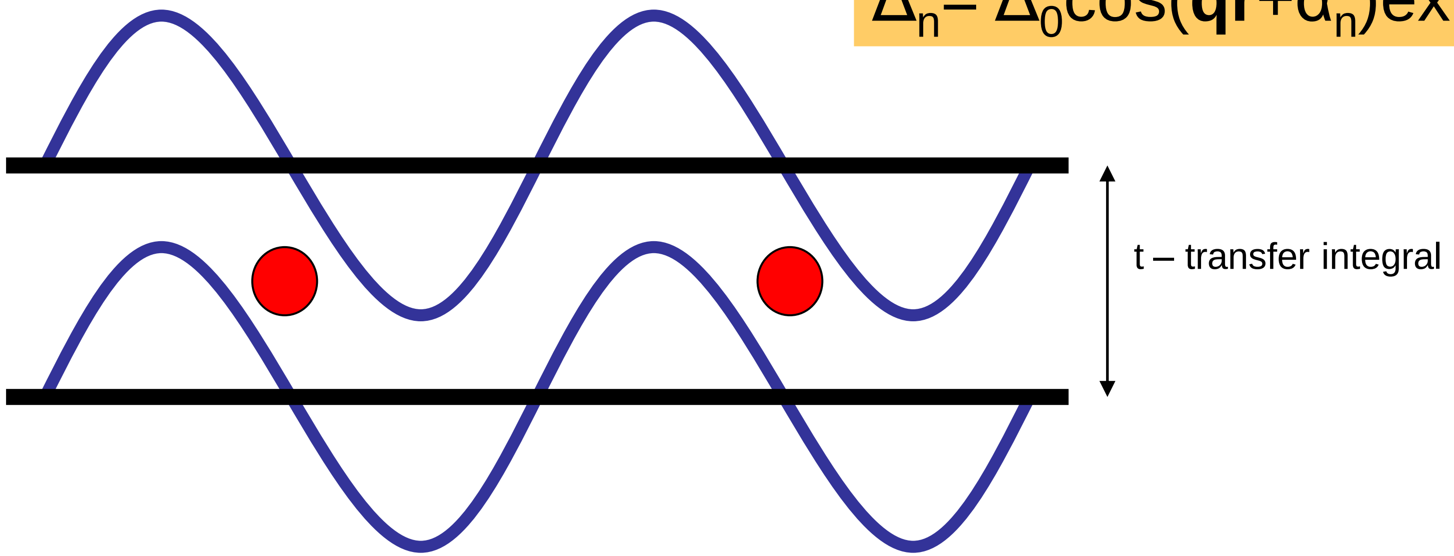
$$\Delta\left(\mathbf{r}\right) = \Delta_0 \cos\left(\mathbf{q}.\mathbf{r}\right) e^{i\phi_{n+1}(r)}$$

- The period of this modulation may interfere with the in-plane FFLO modulation leading to the anomalies in the critical field behavior;

Resonance condition: $|\mathbf{q} \pm 2\mathbf{Q}| = |\mathbf{q}|$ $\qquad$ $\mathbf{q}.\mathbf{Q} = \mp Q^2$

Intrinsic vortex pinning in LOFF phase for parallel field orientation

$\Delta_n = \Delta_0 \cos(\mathbf{qr}+\alpha_n)\exp(i\varphi_n(r))$



Josephson coupling between layers is modulated

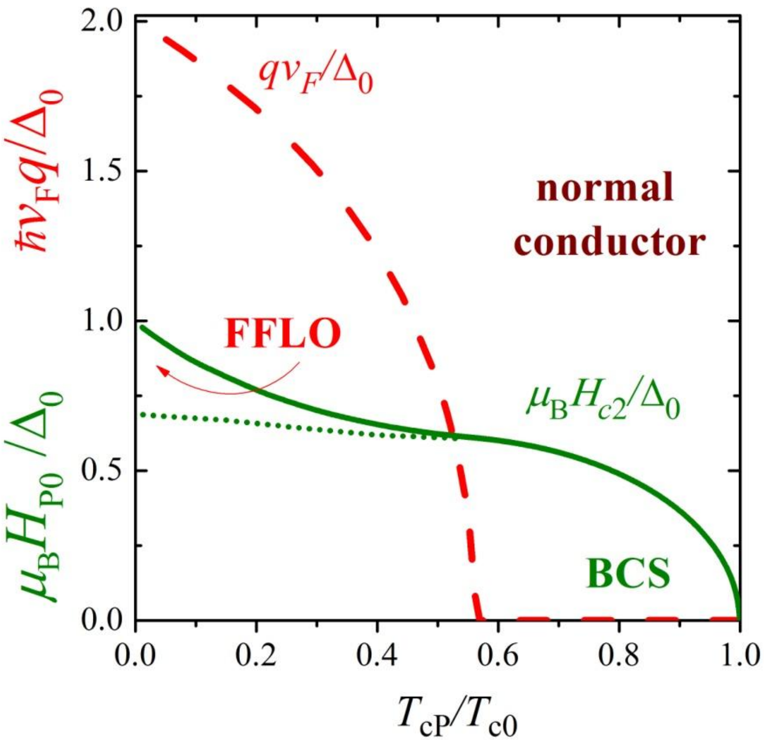
$F_{n,n+1}=[-I_0\cos(\alpha_n - \alpha_{n+1})+I_2\cos(\mathbf{qr})\cos(\alpha_n + \alpha_{n+1})]\ \cos(\varphi_n - \varphi_{n+1})$

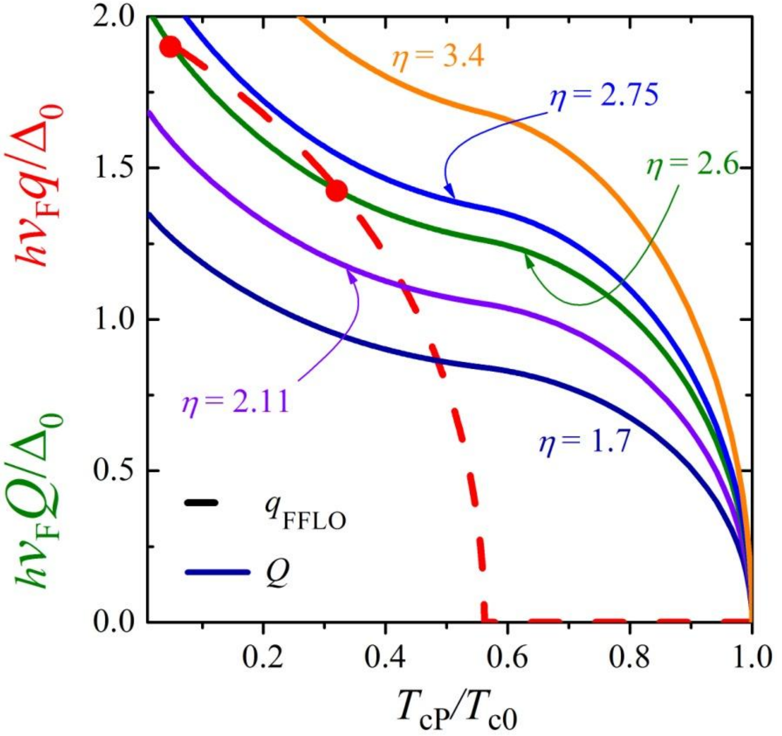
$\varphi_n - \varphi_{n+1} = 2\pi x H s/\Phi_0 + \pi n$

s-interlayer distance, x-coordinate along **q**

Resonance conditions

Absolute value of the wave vector **q** of the FFLO phase (dashed lines) and of the wave vectors **Q** (solid lines) versus the reduced temperature T_{cP}/T_{c0} calculated for several values of Fermi velocity $\eta = \hbar v_F \pi d/\phi_0 \mu_B$.





$$d = 1.6\ nm$$

$$v_F = 1.0 \times 10^5 m/s \Rightarrow \eta = 3.4$$

$$\mathbf{q}.\mathbf{Q} = \mp Q^2 \ \Rightarrow \ q = \mp Q$$

Orbital correction

Normalized correction of the superconducting onset temperature as a function of in-plane magnetic field **H** for several angles between magnetic field and the FFLO modulation vector **q**.

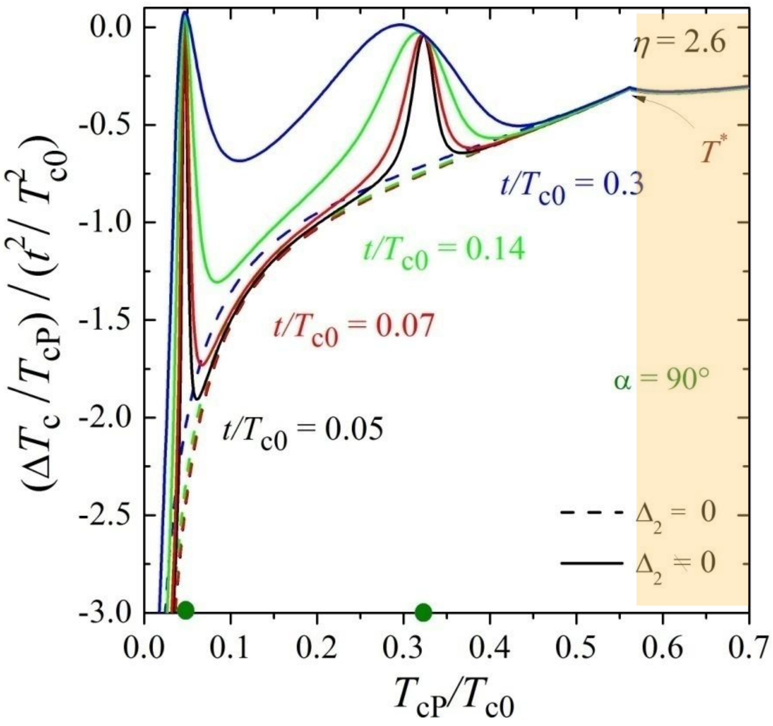
$$v_F = 0.75 \times 10^5 m/s$$

Resonance condition: $|\mathbf{q} \pm 2\mathbf{Q}| = |\mathbf{q}|$

$$\boxed{T_c = T_{cP}\left[1 - At^2(a - c_\pm)\right]}$$

$$\alpha = 90°$$

$$\mathbf{q}.\mathbf{Q} = \mp Q^2$$



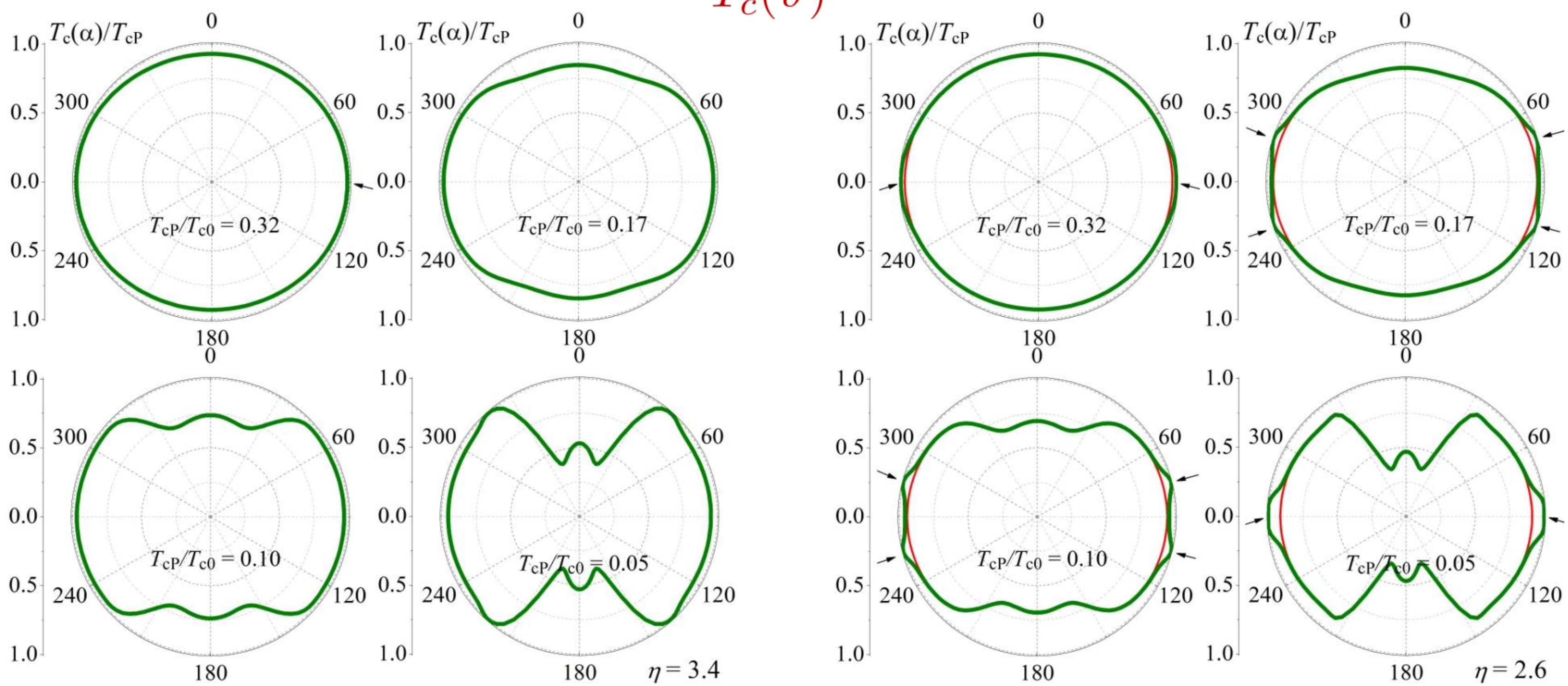
Croitoru, Buzdin, PRB, 2012

In-plane anisotropy of the onset of superconductivity

Normalized superconducting transition temperature, as $T_c\left(\alpha\right)/T_{cP}$ he angle between the directions of the applied magnetic field and the vector **q** for several values of T_{cP}/T_{c0}.

$T_c(\vartheta)$

Non-resonance case | **Resonance case**

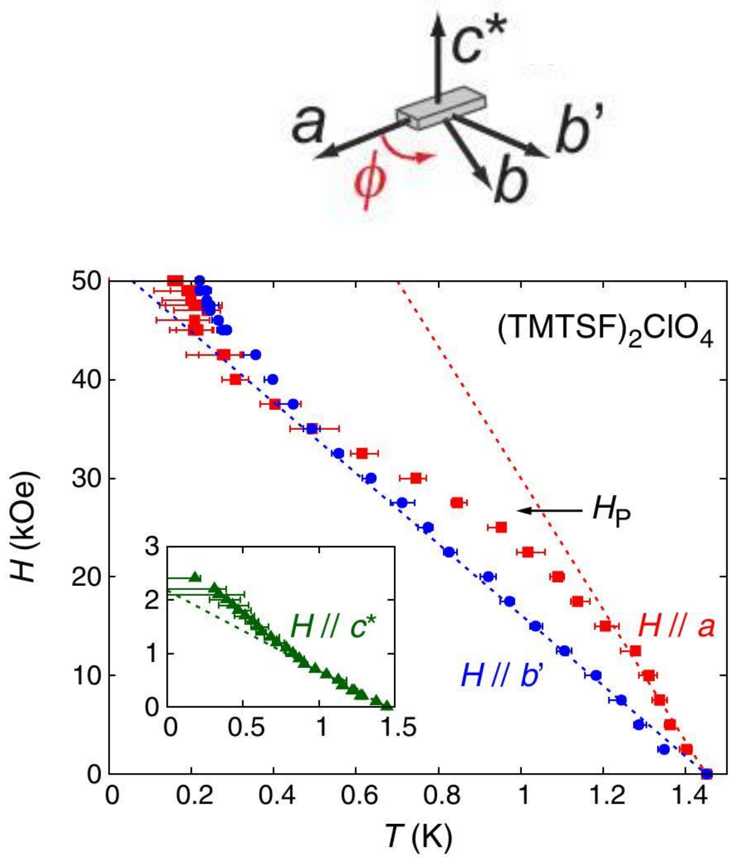


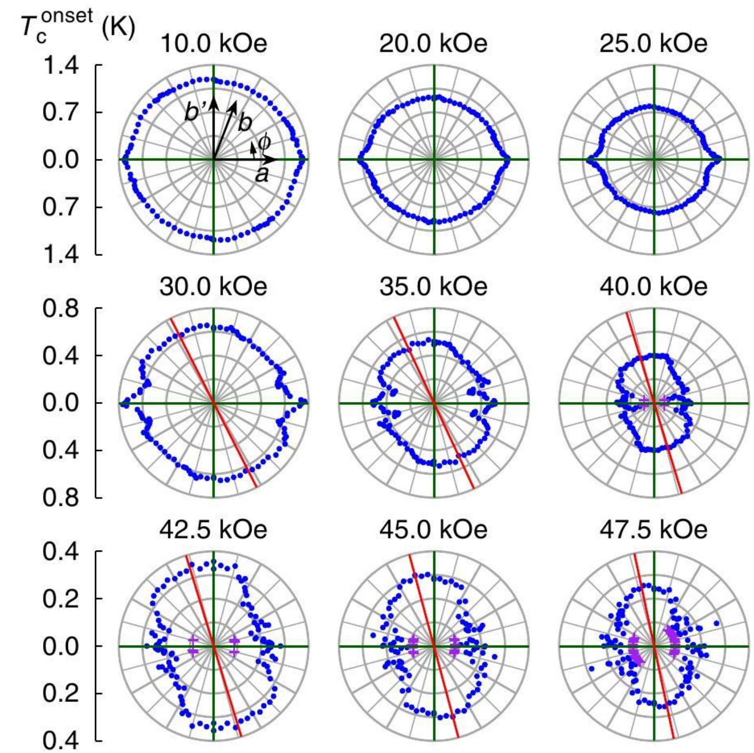
$v_F = 1.0 \times 10^5 m/s$ | $v_F = 0.75 \times 10^5 m/s$

$t/T_{c0} = 0.2$

Croitoru, Buzdin, PRB, 2012

Anomalous in-plane anisotropy of the onset of SC in $(TMTSF)_2ClO_4$





S.Yonezawa, S.Kusaba, Y.Maeno, P.Auban-Senzier, C.Pasquier, K.Bechgaard, and D. Jerome, Phys. Rev. Lett. **100**, 117002 (2008)

5. Supefluid ultracold Fermi gases with imbalanced state populations: one more candidate for FFLO state?

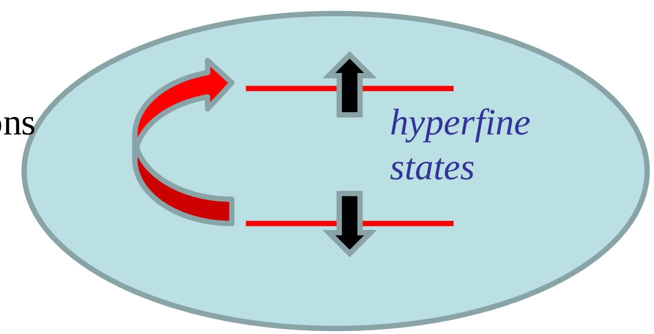
Massachusetts Institute of Technology:
M.W. Zwierlein, A. Schirotzek, C. H. Schunck, W.Ketterle (2006)

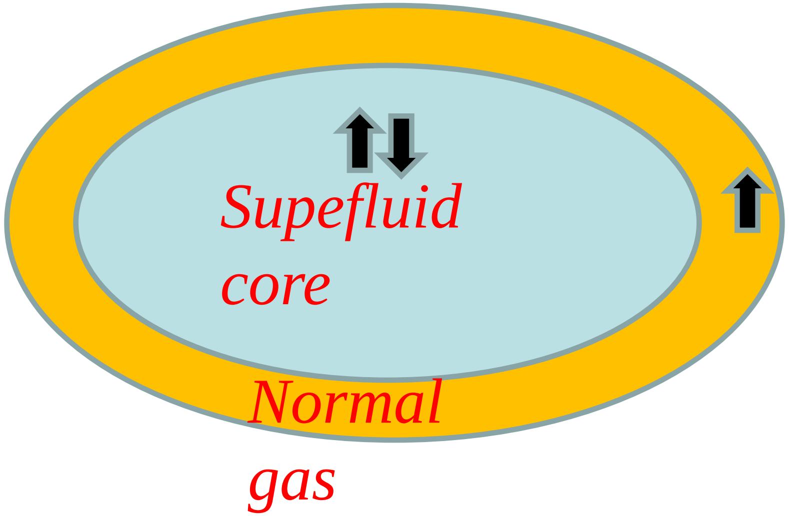
Rice University, Houston:
Guthrie B. Partridge, Wenhui Li, Ramsey I. Kamar, Yean-an Liao, Randall G. Hulet (2006)

Experimental system: Fermionic ^{6}Li atoms cooled in magnetic and optical traps (mixture of the two lowest hyperfine states with different populations)

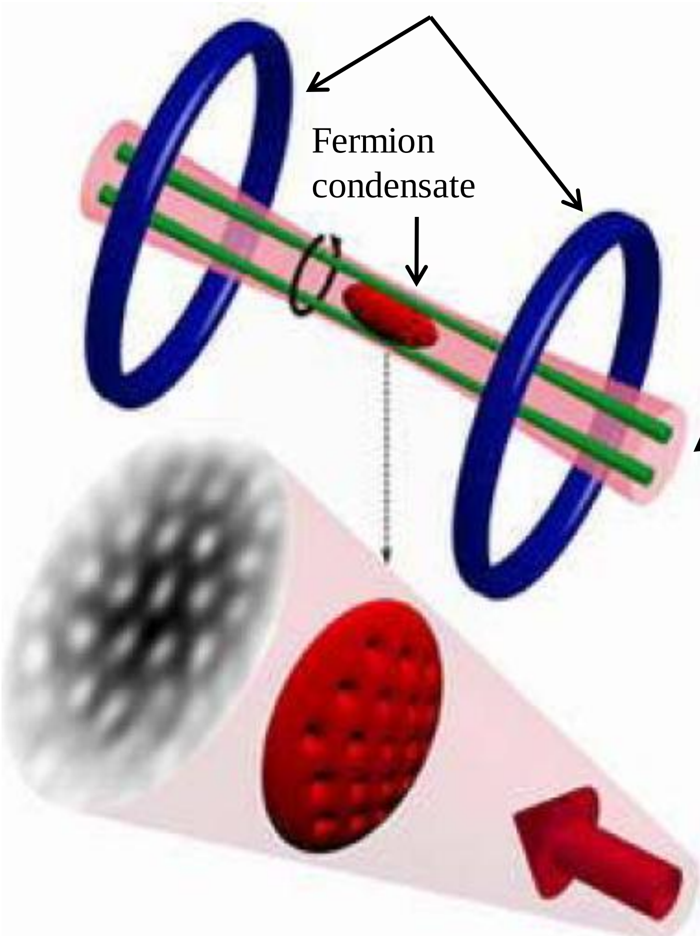
Experimental result: phase separation

rf transitions
76 MHz



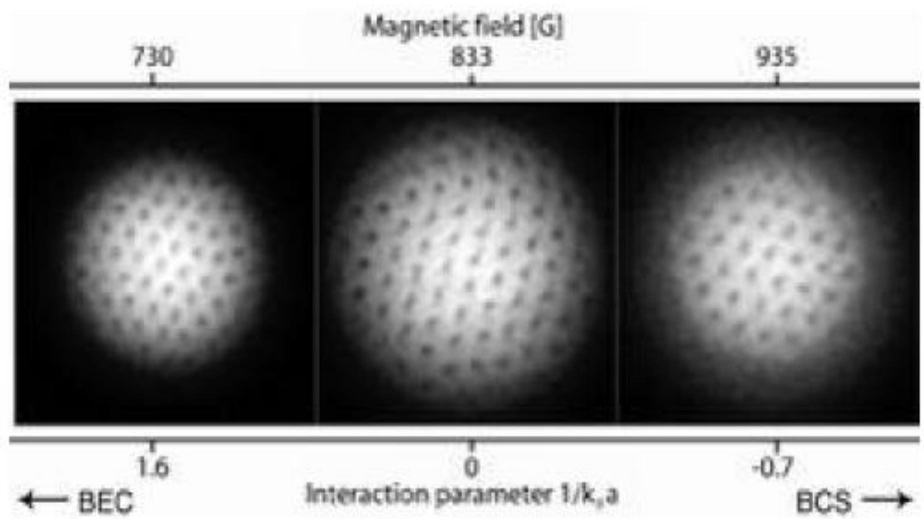


Rotating supefluid ultracold Fermi gases in a trap



Vortex as a test for superfluidity

Images of vortex lattices



MIT: Ketterle et al (2005)

Questions:

1. *What is the effect of confinement (finite system size) on FFLO states?*

2. *Effect of rotation on FFLO states in a trap (effect of magnetic field on FFLO state in a small superconducting sample).*

3. *Possible quantum oscillation effects.*

Model: Modified Ginzburg-Landau functional (2D)

$$F = \int ((a + V(\vec{r}))|\Psi|^2 - \beta |\vec{D}\Psi|^2 + \gamma |\vec{D}^2\Psi|^2) dxdy$$

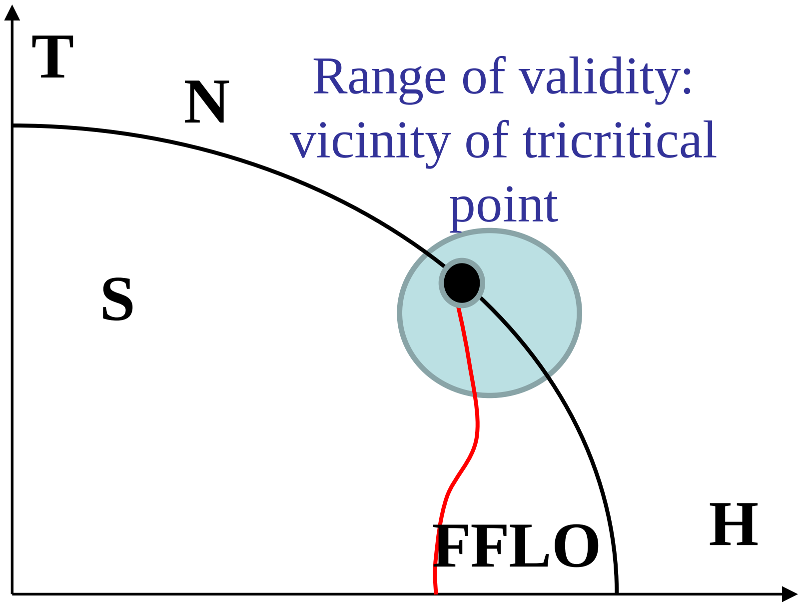
$a = \alpha(T - T_{c0})$

Trapping potential

FFLO instability

$$\vec{D} = \nabla - 2iM[\vec{\Omega}, \vec{r}]/\hbar$$

$$\vec{D} = \nabla - 2ie\vec{A}/\hbar c$$



Confinement mechanisms:

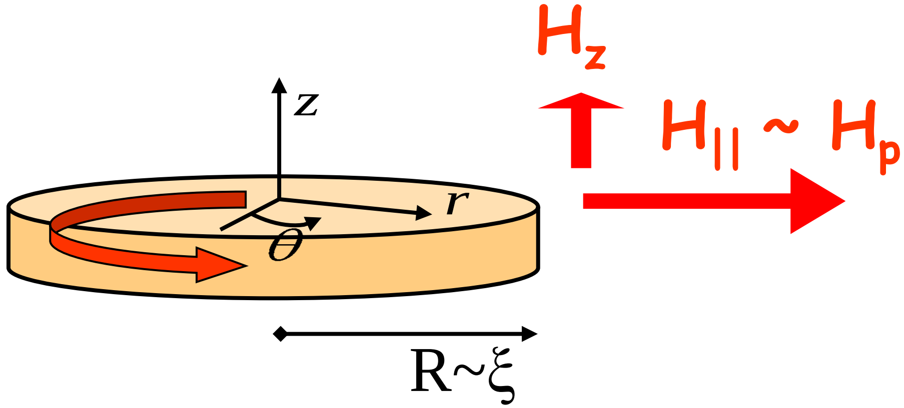
1. Zero trapping potential. Boundary condition at the system edge

$$\vec{n}\vec{D}\Psi = 0$$

2. nonzero trapping potential

$$V(\vec{r}) = M\omega^2 r^2 / 2$$

FFLO states in a 2D mesoscopic superconducting disk



Interplay between the system size, magnetic length, and FFLO length scale

Perpendicular magnetic field component $H_z = 0$

$$G = a|\Psi|^2 - \beta|\nabla\Psi|^2 + \gamma|\Delta\Psi|^2 \ ,$$

The critical temperature:

$$T_c = T_{c0} + \frac{\beta^2}{4\gamma\alpha} f(q) \qquad f(q) = 2q^2 - q^4$$

$$f\left(\frac{x_{Ln}}{k_0 R}\right) = \frac{2x_{Ln}^2}{(k_0R)^2} - \frac{x_{Ln}^4}{(k_0R)^4} \ .$$

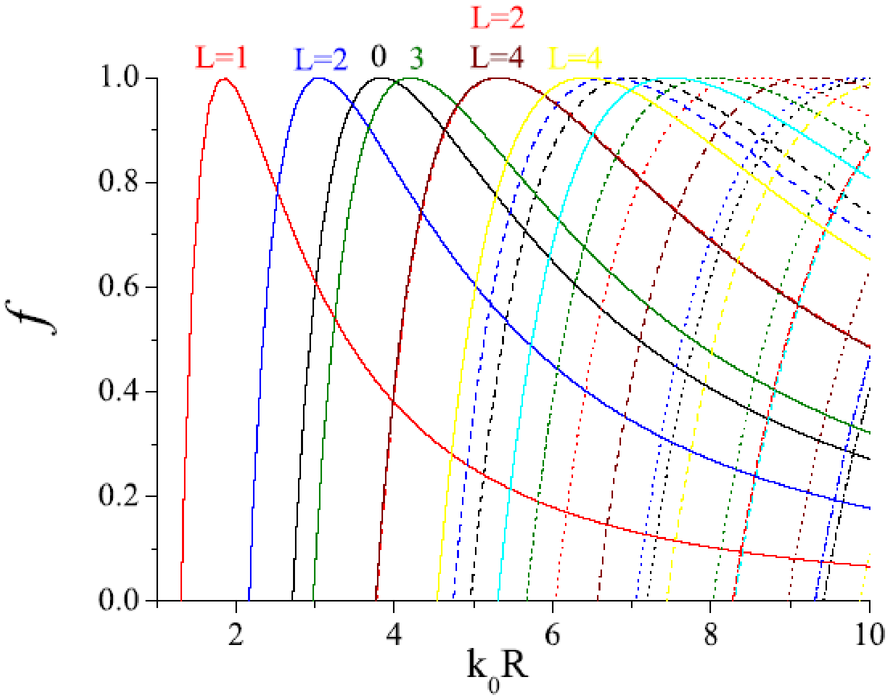
Eigenvalue problem:

$$-\Delta\Psi = k_0^2 q^2 \Psi \ , \qquad \frac{\partial\Psi}{\partial r}|_{r=R} = 0 \ .$$

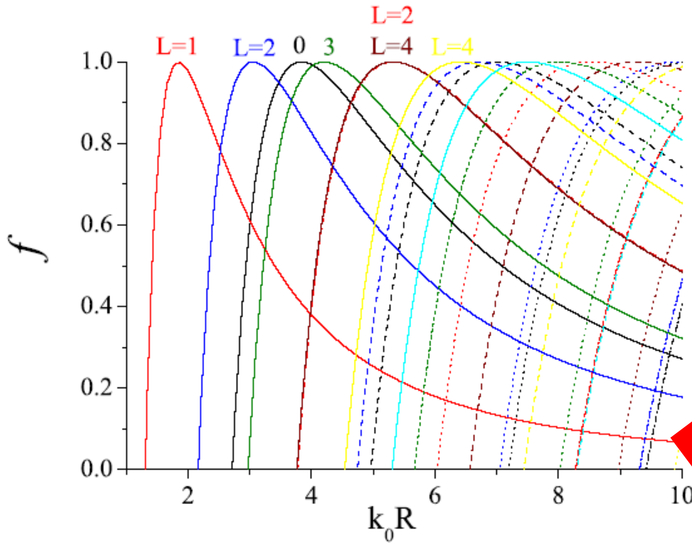
$k_0 = \beta/2\gamma$ Wave number of FFLO instability

$$\Psi = e^{iL\theta} J_L(q\rho) \qquad \frac{dJ_L(x)}{dx}|_{qk_0R} = 0$$

x_{Ln} a set of zeros of the derivative of the Bessel function of the L-th order



H - T Phase diagram: $H_z = 0$

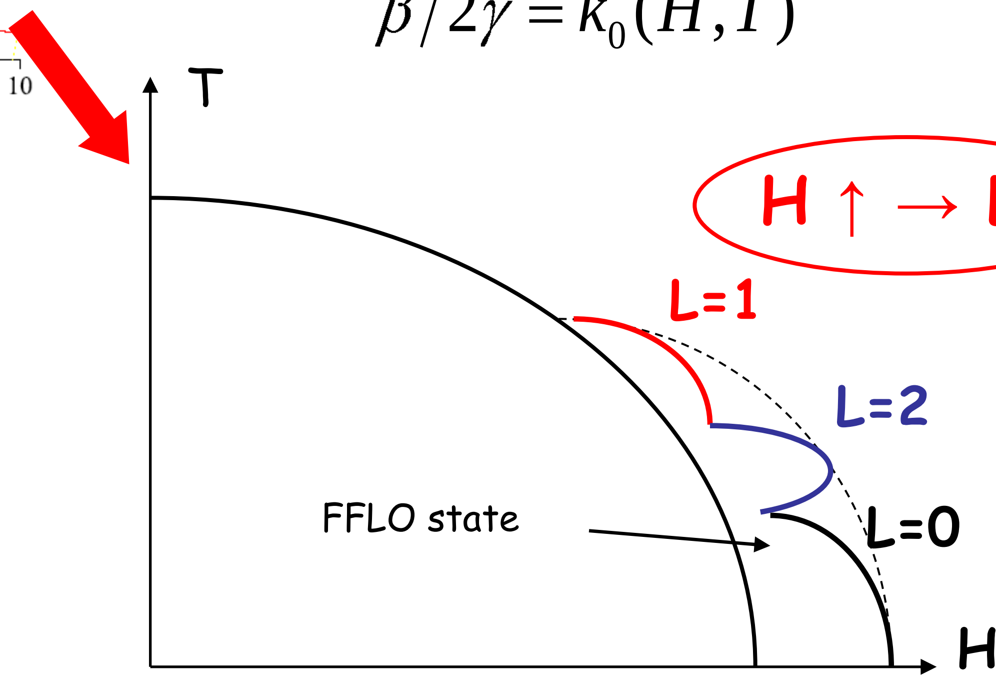


$$a = \alpha(T - T_{c0}),\ \beta = \beta(H/T),\ T_{c0} = T_{c0}(H)$$

$$\beta/2\gamma = k_0(H,T)$$

$$T_c = T_{c0} + \frac{\beta^2}{4\gamma\alpha} f(q)\ ,$$

$$\boxed{T_c = \max_L \{T_L\}\,.}$$



Tilted magnetic field: $H_z \neq 0$

$$T_c = T_{c0} + \frac{\beta^2}{4\gamma\alpha_0} f(q)\ , \qquad f(q) = 2q^2 - q^4\ .$$

$$\mathbf{H} = \mathbf{H}_{||} + H_z \mathbf{z}_0$$

$$G = a|\Psi|^2 - \beta|\mathbf{D}\Psi|^2 + \gamma|\mathbf{D}^2\Psi|^2\ ,$$

$$\mathbf{D} = \nabla + \frac{2\pi i}{\Phi_0}\mathbf{A}_{||} \qquad A_\theta = H_z r/2.$$

$$\beta = \beta(H_{||}, T)$$

Field induced superconductivity

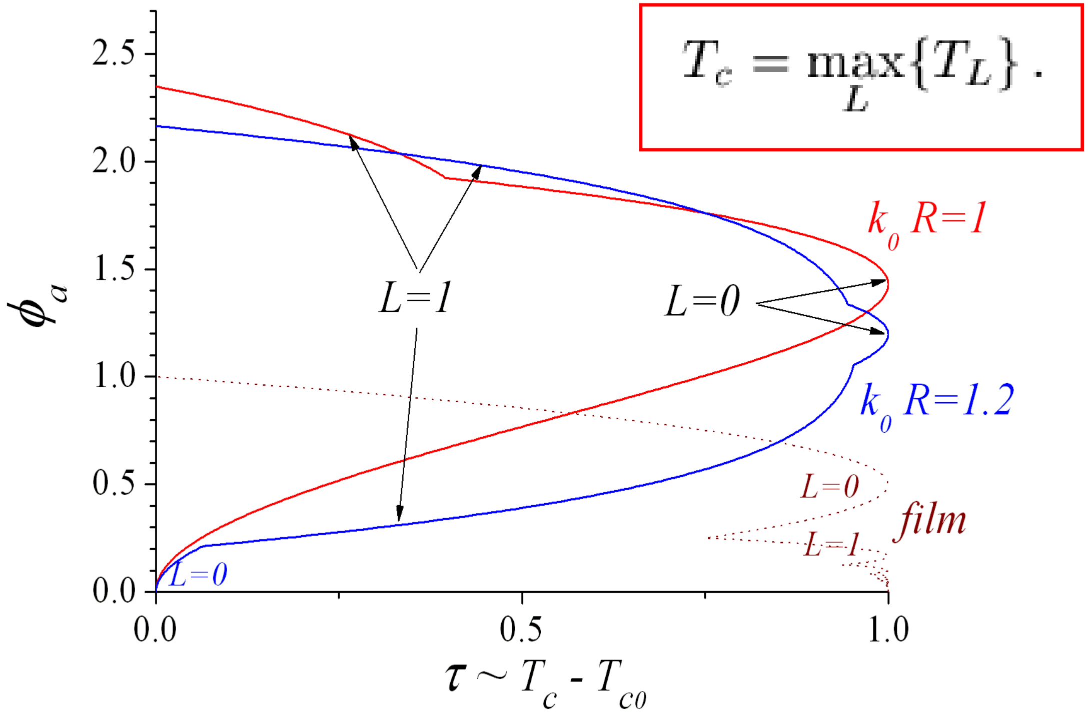
$$\boxed{T_c = \max_L \{T_L\}\ .}$$

Eigenvalue problem:

$$-\mathbf{D}^2\Psi = k_0^2 q^2 \Psi \qquad \left.\frac{\partial \Psi}{\partial r}\right|_{r=R} = 0\ .$$

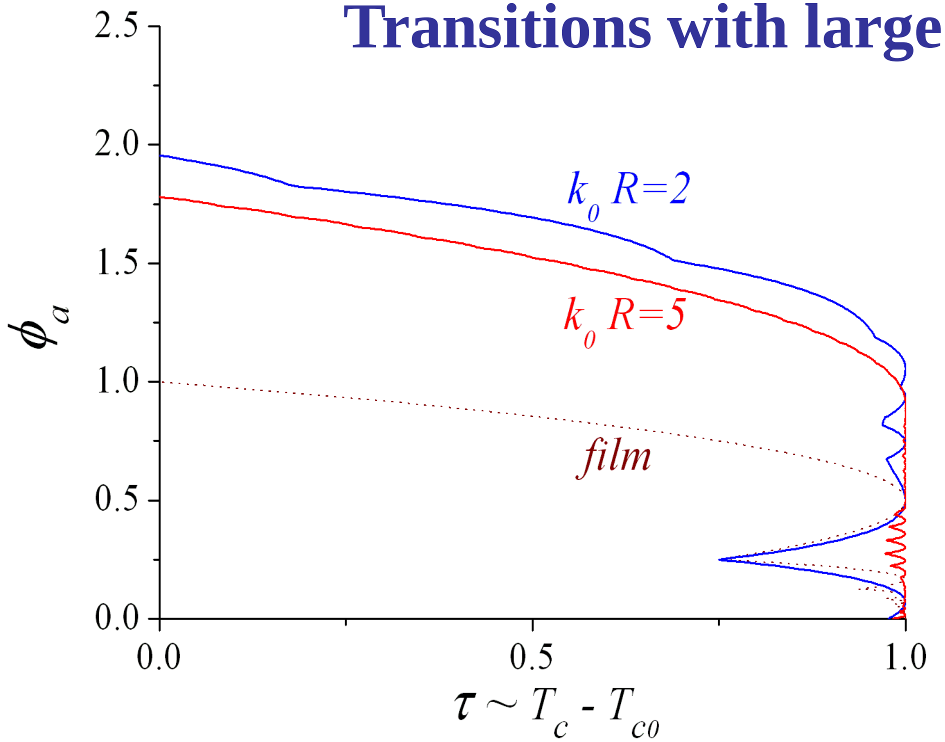
$$\psi(\rho) = \mathrm{e}^{-\phi/2}\phi^{|L|/2} F(a_L, b_L, \phi)\ ,$$

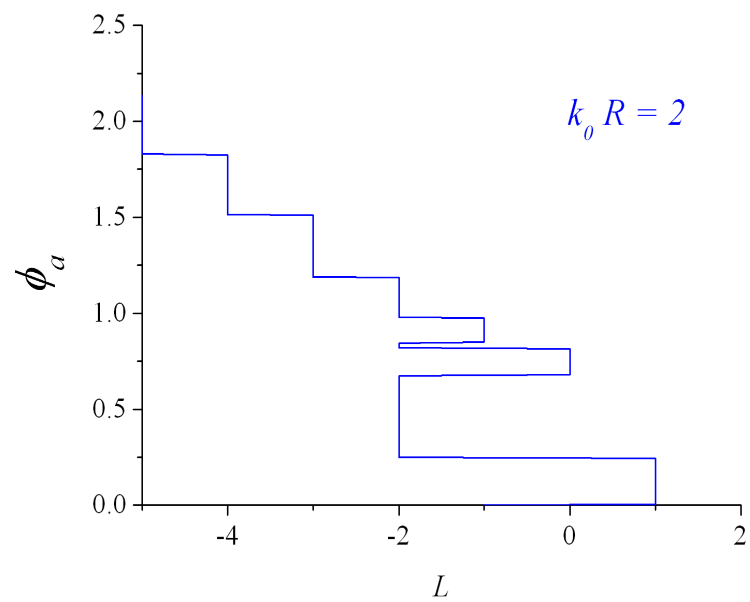
$$\phi_a = \frac{\pi R^2 H_z}{\Phi_0}$$



Tilted magnetic field: $H_z \neq 0$

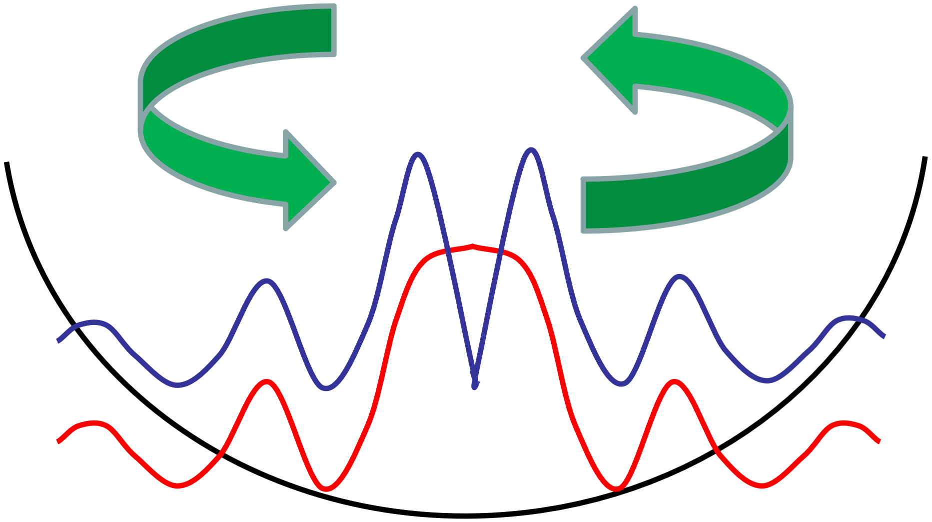
Transitions with large jumps in vorticity





$$\boxed{T_c = \max_L \{T_L\}\,.}$$

FFLO states in a trapping potential



Interplay between the rotation effect, confinement, and FFLO instability

FFLO states in a 2D system in a parabolic trapping potential (no rotation)

$$\Delta^4\Psi + 2\Delta^2\Psi + (\tau + v_0\rho^2)\Psi = 0$$

$$k_0 = \beta/2\gamma$$ FFLO length scale

$$\tau = a/\gamma k_0^4$$ Temperature shift

$$v_0 = M\omega^2/2\gamma k_0^6$$ $(\text{Trapping frequency})^2$

$$\vec\rho = k_0\vec r$$ Dimensionless coordinate

Fourier transform:

$$\Psi = \int e^{i\vec q\vec\rho}\psi_{\vec q}\, d^2\vec q$$

$$-v_0\frac{\partial^2}{\partial \vec q^{\,2}}\Psi + \left(q^4 - 2q^2\right)\Psi = -\tau\Psi$$

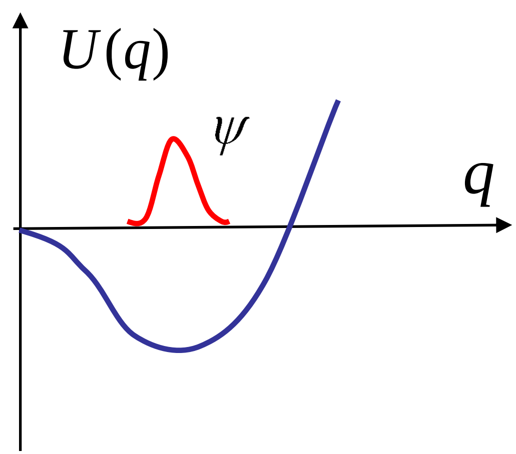
$$L = 0$$

$$q = 1 + x$$

$$U(q) = q^4 - 2q^2 \approx -1 + 4x^2$$

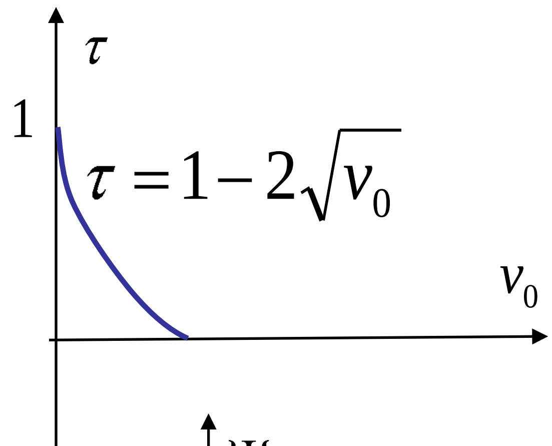
$$\psi_{\vec q} = e^{-\lambda x^2} \qquad \lambda = \frac{1}{\sqrt{v_0}}$$

$$v_0 \to 0$$



FFLO states in a 2D system in a parabolic trapping potential (no rotation)

Phase diagram

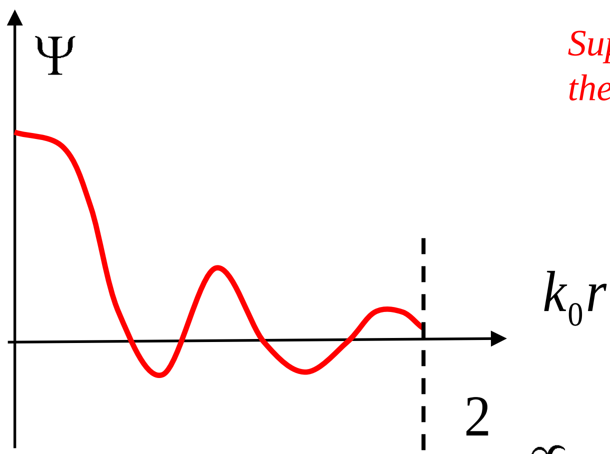


Condensate wave function

$$k_0 r >> 1$$

$$\Psi \propto \sqrt{\frac{\pi}{k_0 r}} \cos(k_0 r - \pi/4) e^{-k_0^2 r^2 \sqrt{v_0}/4}$$

Suppression of wave function oscillations by the increase in the trapping frequency



$$\frac{2}{v_0^{1/4}} \propto \frac{1}{\sqrt{\omega}}$$ *=Number of observable oscillations*

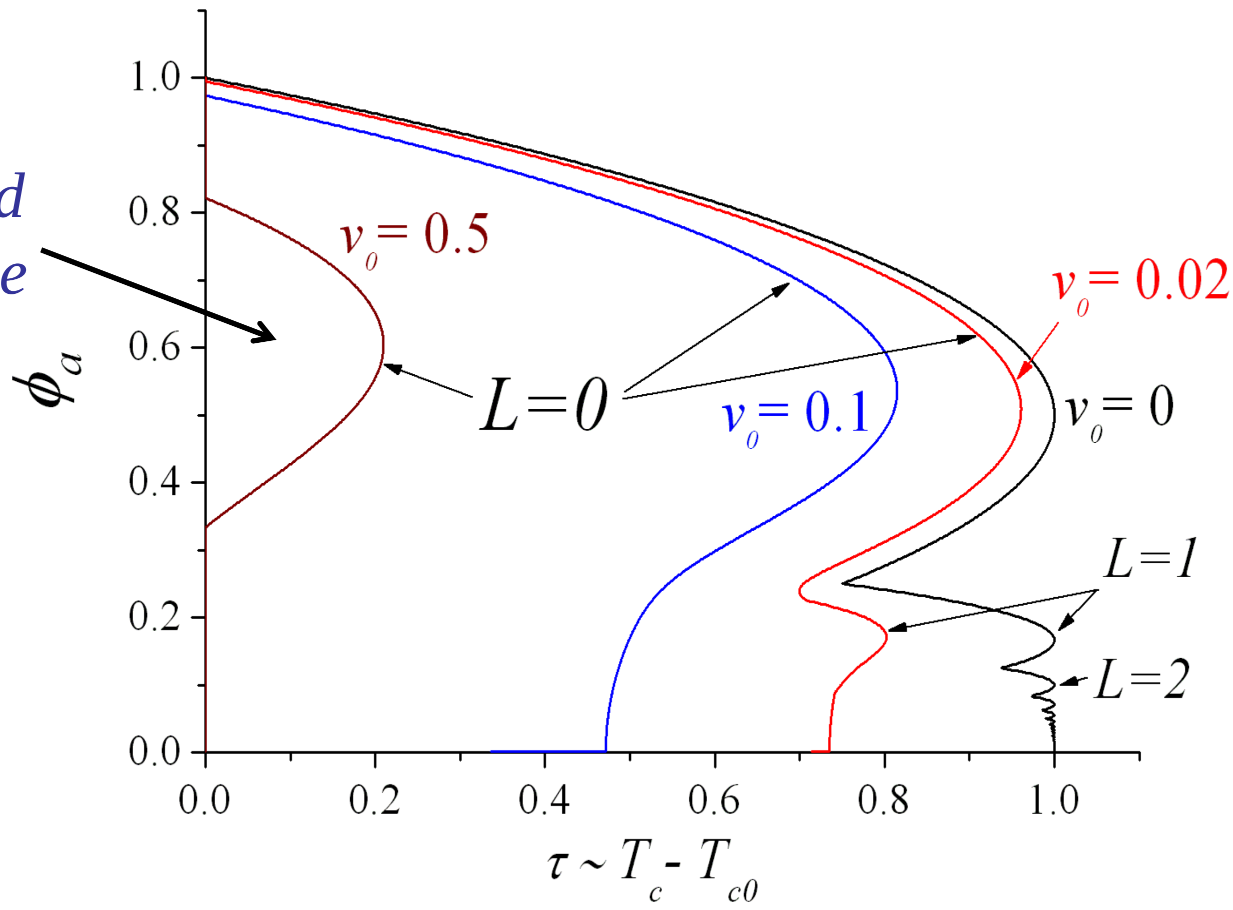
FFLO states in a rotating 2D gas in a parabolic trapping potential.

Suppression of quantum oscillations by the increase in the trapping frequency.

First-order perturbation theory:

$$\tau = \max_{L\geq 0}\left[(4\phi_a - v_0/\phi_a)(2L+1) + v_0 L/\phi_a - 4\phi_a^2(2L+1)^2\right]$$

rotation induced superfluid phase



Conclusions

- **There are strong experimental evidences of the existence of the the FFLO state in organic layered superconductors and in heavy fermion superconductor** $CeCoIn_5$

- **FFLO –type modulation of the superconducting order parameter plays an important role in uperconductor-ferromagnet heterostructures. The π-junction realization in S/F/S structures is quite a general phenomenon.**

- **The interplay between FFLO modulation and orbital effect results in new type of the vortex structures, non-monotonic critical field behavior in layered superconductor in tilted field.**

- **FFLO phase in ultracold Fermi gases with imbalanced state populations?**