Coexistence of Ferromagnetism and Superconductivity

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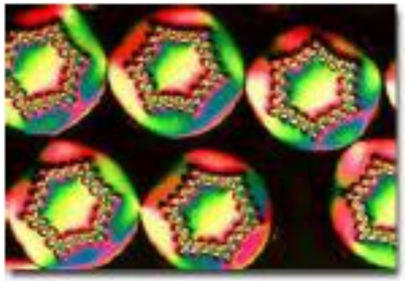
- Recall on magnetism and superconductivity coexistence
- Origin and the main peculiarities of the proximity effect in superconductor-ferromagnet systems.
- Josephson **π**-junction.
- Domain wall superconductivity. Spin-valve effet.
- φ-junctions.
- Possible applications

Supraconductivity

1913

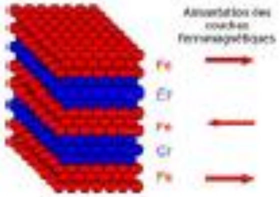
Since the discovery by Heike Kamerlingh Onnes
6 Nobel Prizes.
Many important applications.



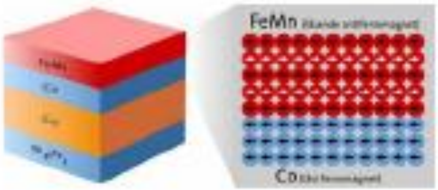
Magnetism

4 Nobel Prizes.

Recent Nobel Prize : A. Fert and P.Grunberg



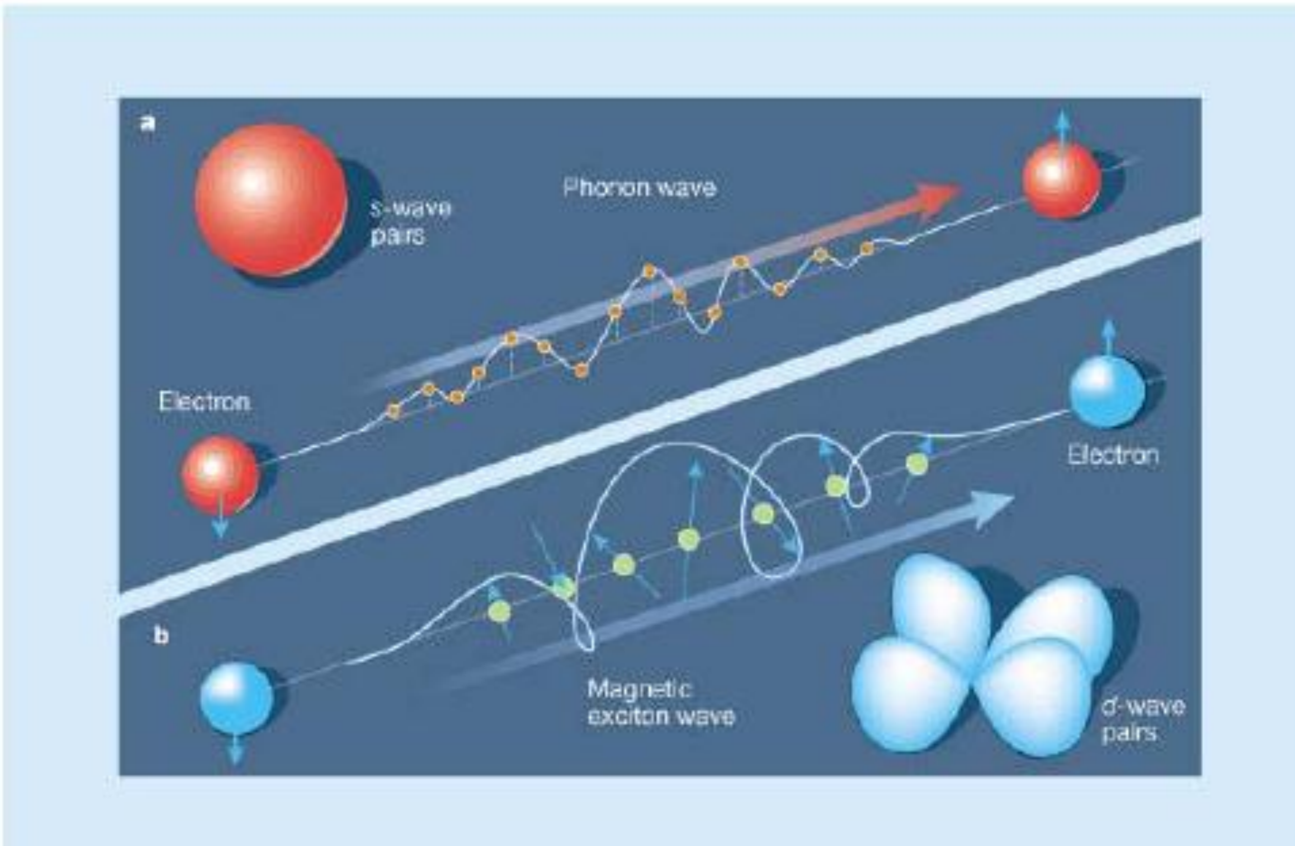
Spintronics



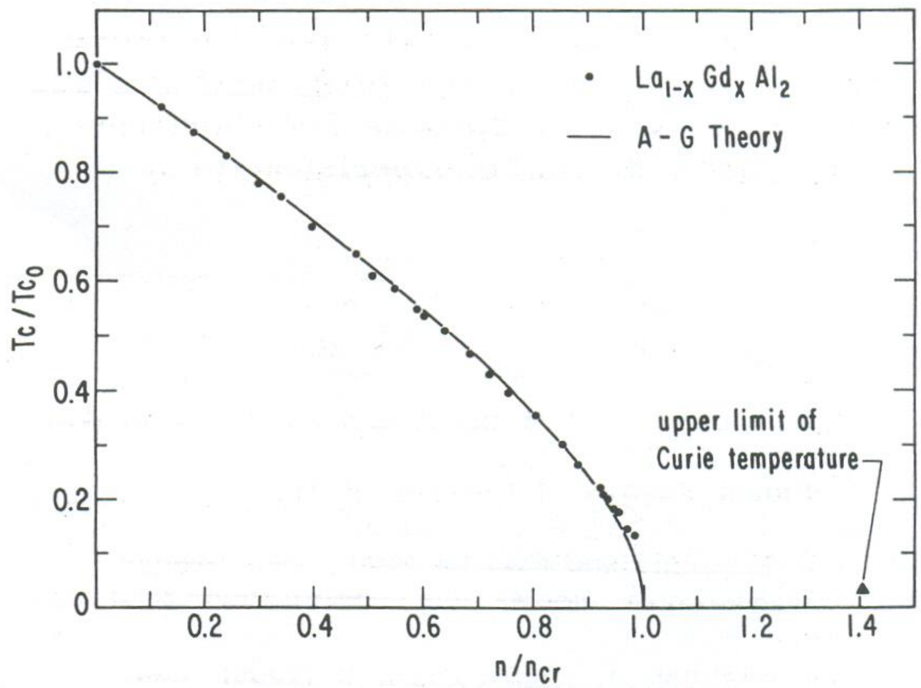
2007

Is it possible to couple magnetism and superconductivity?

Superconducting Cooper pairs



Magnetism and Superconductivity Coexistence



$$\frac{dT_c}{dx} \approx -\Theta_m x$$

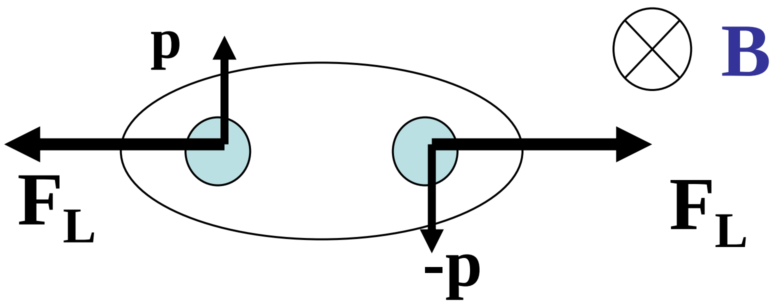
(Abrikosov and Gorkov, 1960)

The critical temperature variation versus the concentration n of the Gd atoms in $La_{1-x}Gd_xAl_2$ alloys (Maple, 1968). T_{c0}=3.24 K and n_{cr}=0.590 atomic percent Gd.

The earlier experiments (Matthias *et al*., 1958) demonstrated that the presence of the magnetic atoms is very harmful for superconductivity.

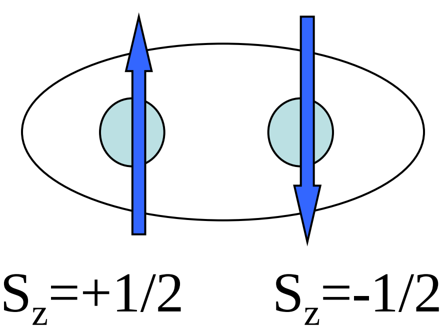
Antagonism of magnetism (ferromagnetism) and superconductivity

- Orbital effect (Lorentz force)



Electromagnetic mechanism (breakdown of Cooper pairs by magnetic field induced by magnetic moment)

- Paramagnetic effect (singlet pair)



$\mu_B H \sim \Delta \sim T_c$

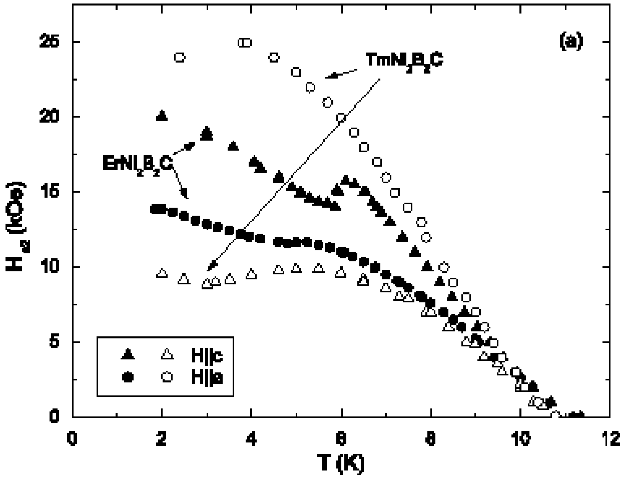
$I\left(\vec{S}\cdot\vec{s}\right) \approx T_c$

Exchange interaction

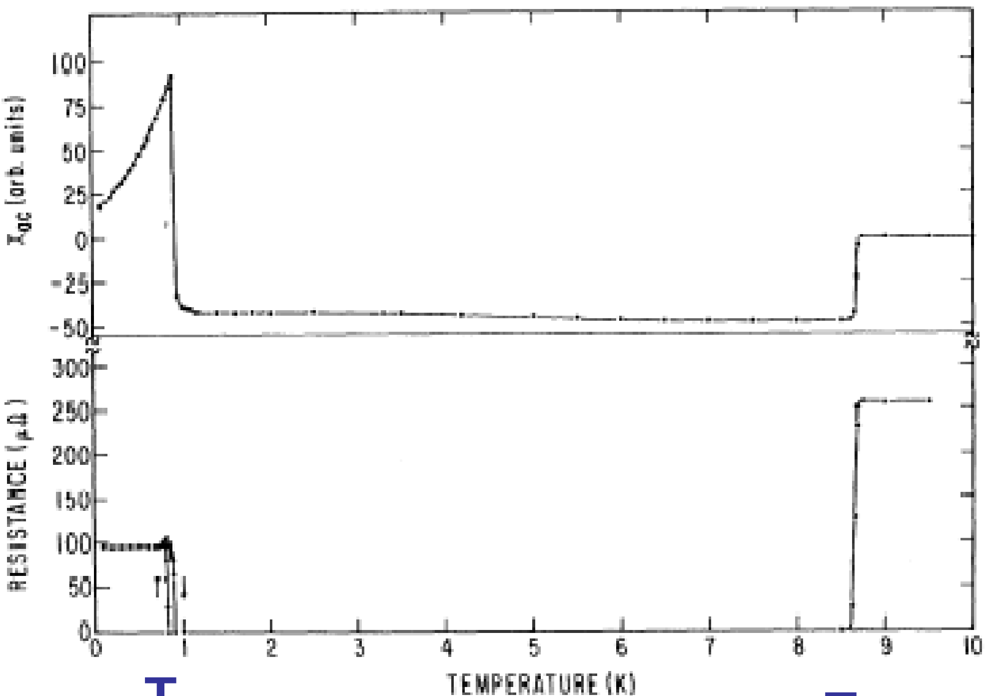
No antagonism between antiferromagnetism and superconductivity

	T_c (K)	T_N (K)
$NdRh_4B_4$	5.3	1.31
$SmRh_4B_4$	2.7	0.87
$TmRh_4B_4$	9.8	0.4
$GdMo_6S_8$	1.4	0.84
$TbMo_6S_8$	2.05	1.05
$DyMo_6S_8$	2.05	0.4
$ErMo_6S_8$	2.2	0.2
$GdMo_6Se_8$	5.6	0.75
$ErMo_6Se_8$	6.0	1.1
$DyNi_2B_2C$	6.2	11
$ErNi_2B_2C$	10.5	6.8
$TmNi_2B_2C$	11	1.5
$HoNi_2B_2C$	8	5

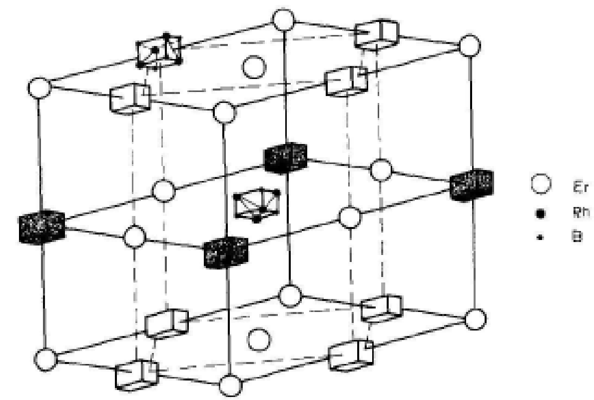
Usually $T_c > T_N$



FERROMAGNETIC CONVENTIONAL (SINGLET) SUPERCONDUCTORS



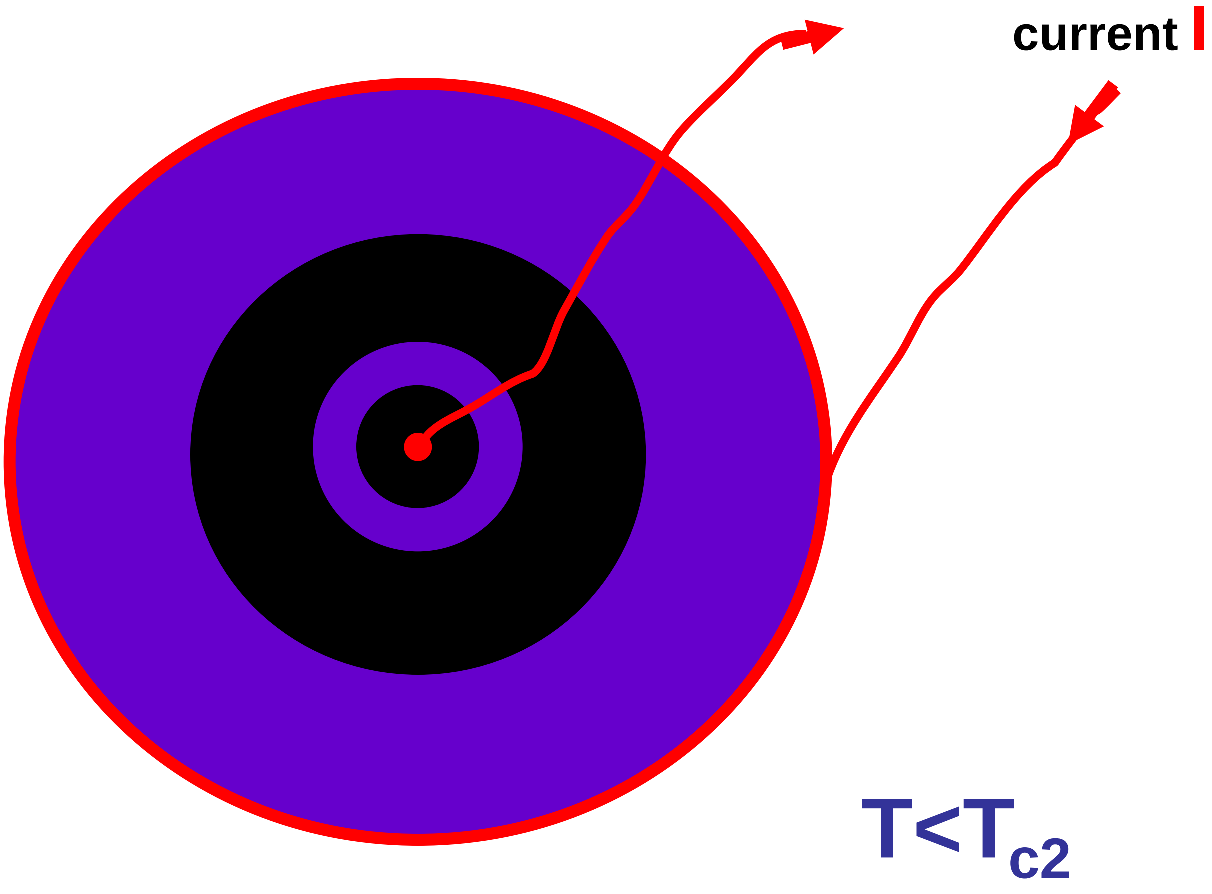
T_{c2} T_c



A. C. susceptibility and resistance versus temperature in $ErRh_4B_4$ (Fertig *et al.*,1977).

RE-ENTRANT SUPERCONDUCTIVITY in $ErRh_4B_4$, $HoMo_6S_8$

Auto-waves in reentrant superconductors?

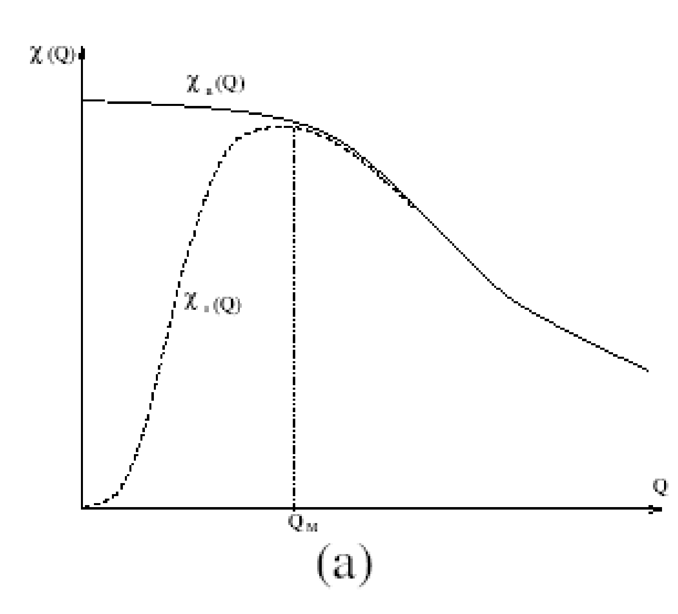


Coexistence phase

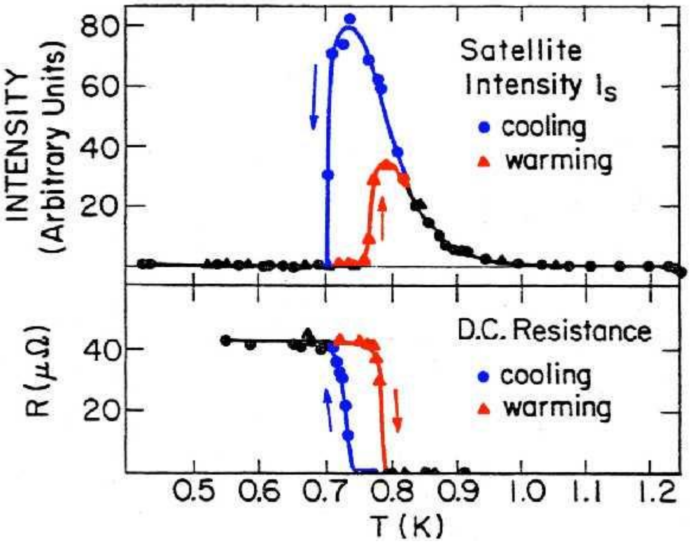
$$E_m = -\sum_Q \frac{\chi\,(Q)}{2}\left|h_Q\right|^2$$

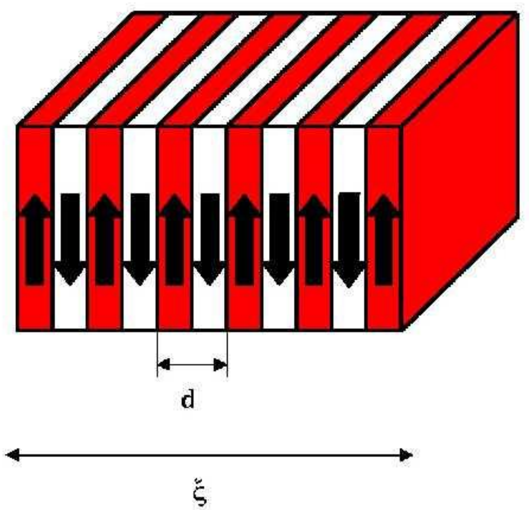
At T=0 and $Q\xi_0$>>1 following (**Anderson and Suhl, 1959**)

$$\frac{\chi_s(Q) - \chi(Q)}{\chi(0)} = \frac{\pi}{2Q\xi_0}$$

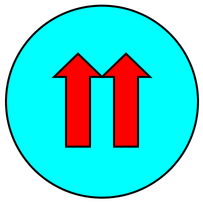


Intensity of the neutron Bragg scattering and resistance as a function of temperature in an $ErRh_4B_4$ (**Sinha *et al.*,1982**). The satellite position corresponds to the wavelength of the modulated magnetic structure around 92 Å.



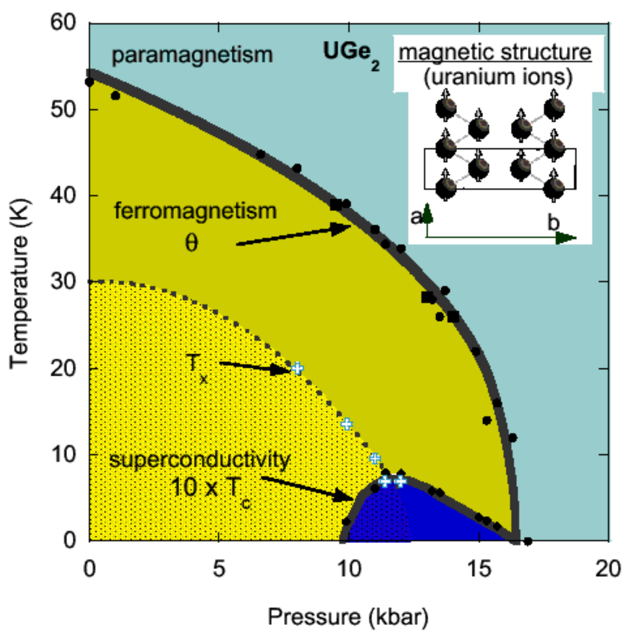


FERROMAGNETIC UNCONVENTIONAL (TRIPLET) SUPERCONDUCTORS

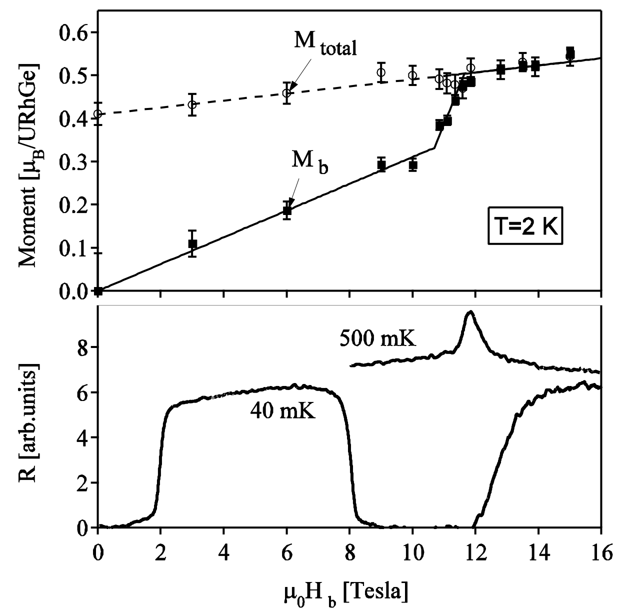


UGe_2 (Saxena *et al.*, 2000) and URhGe (Aoki *et al.*, 2001)

Triplet pairing

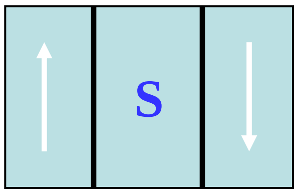


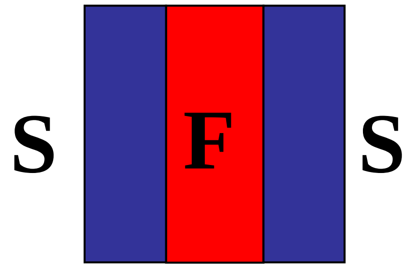
UGe_2

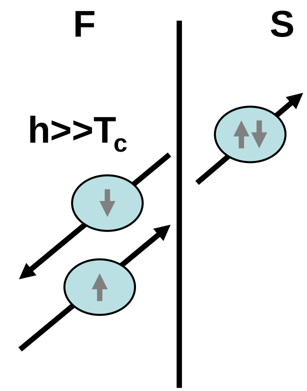


URhGe (a) The total magnetic moment M total and the component M_b measured for H// to the b axis . In (b), variation of the resistance at 40 mK and 500 mK with the field re-entrance of SC between 8-12 T (Levy et al 2005).

The coexistence of singlet superconductivity and ferromagnetism is basically impossible in the same compound but may be easily achieved in artificially fabricated superconductor/ferromagnet heterostructures.







Due to the proximity effect, the Cooper pairs penetrate into the F layer and we have the unique possibility to study the properties of superconducting electrons under the influence of the huge exchange field.

Varying in the controllable manner the thicknesses of the ferromagnetic and superconducting layers it is possible to change the relative strength of two competing ordering. Interesting effects at the nanoscopic scale.

The Josephson junctions with ferromagnetic layers reveal many unusual properties quite interesting for applications, in particular the so-called π-Josephson junction (with the π-phase difference in the ground state).

Superconducting order parameter behavior in ferromagnet

Standard Ginzburg-Landau functional:

$$F = a|\Psi|^2 + \frac{1}{4m}|\nabla\Psi|^2 + \frac{b}{2}|\Psi|^4$$

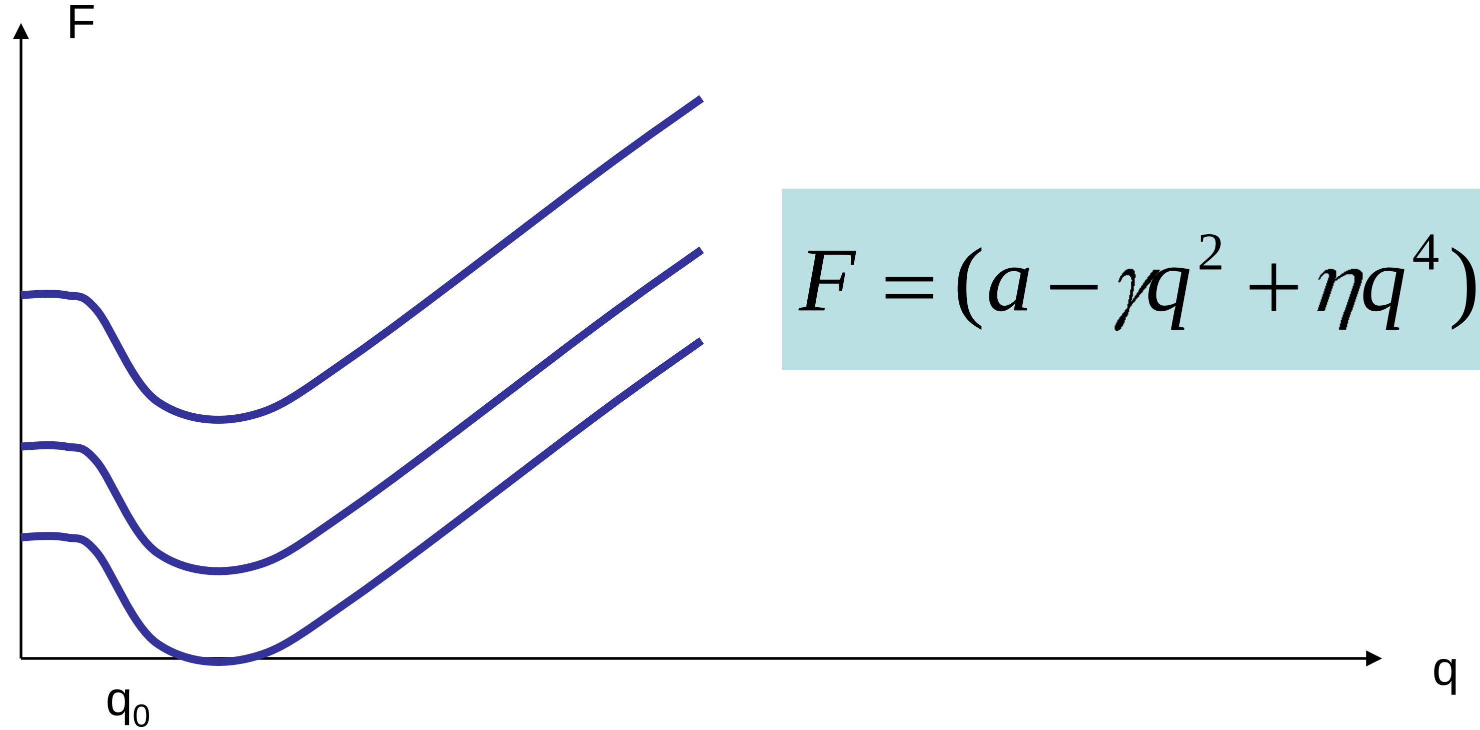
The minimum energy corresponds to Ψ=const

The coefficients of GL functional are functions of internal exchange field h !

Modified Ginzburg-Landau functional ! :

$$F = a|\Psi|^2 - \gamma|\nabla\Psi|^2 + \eta|\nabla^2\Psi|^2 + \ldots$$

The **non-uniform** state Ψ~exp(iqr) will correspond to minimum energy and higher transition temperature



$$F = (a - \gamma q^2 + \eta q^4)\left|\Psi_q\right|^2$$

Ψ~exp(iqr) - **Fulde-Ferrell-Larkin-Ovchinnikov state (1964).**
Only in pure superconductors and in the very narrow region.

Proximity effect in a ferromagnet ?

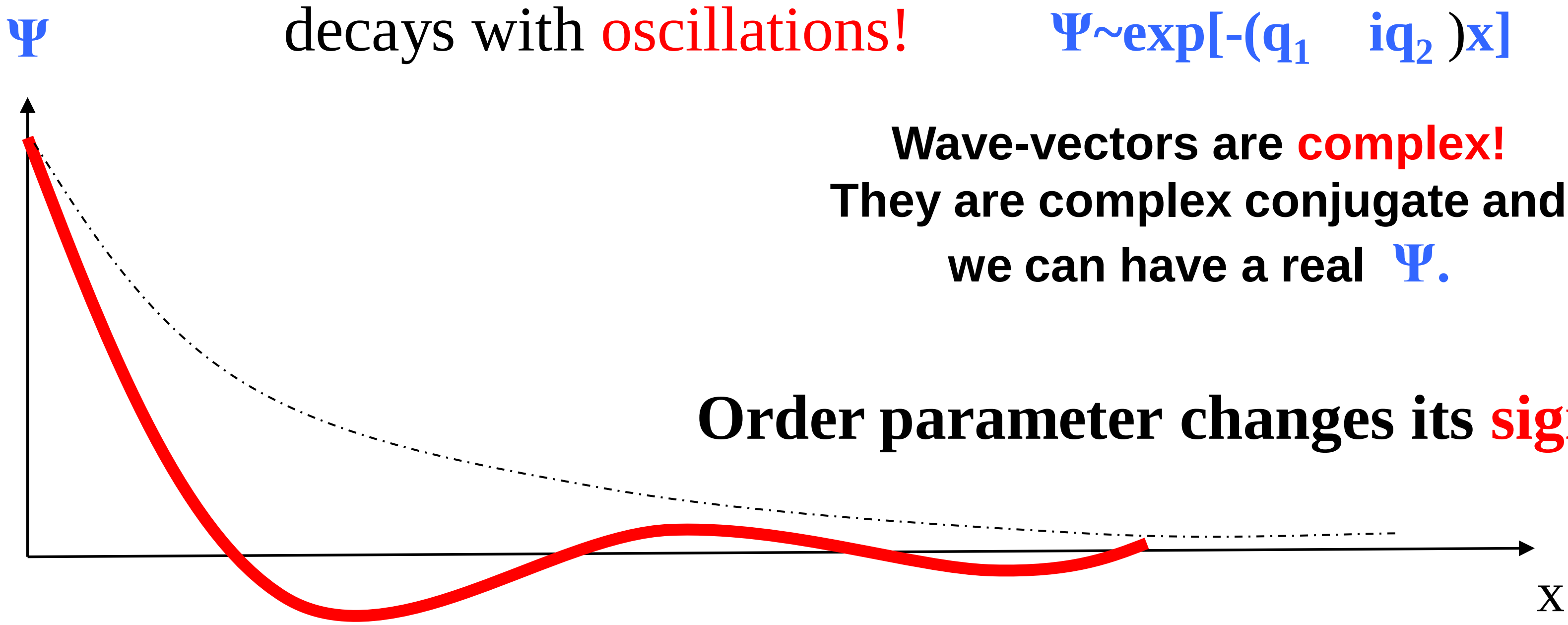
In the usual case (normal metal):

$$a\Psi - \frac{1}{4m}\nabla^2\Psi = 0, \text{ and solution for } \mathrm{T} > \mathrm{T_c} \text{ is } \Psi \propto e^{-qx}, \text{ where } \mathrm{q} = \sqrt{4\mathrm{ma}}$$

In **ferromagnet** (in presence of exchange field) the equation for superconducting order parameter is different

$$a\Psi + \gamma\nabla^2\Psi - \eta\nabla^4\Psi = 0$$

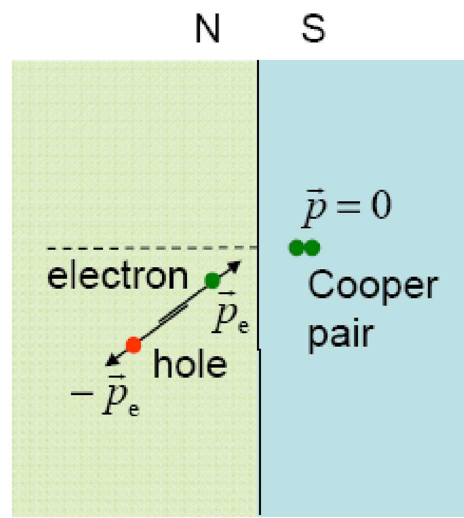
Its solution corresponds to the order parameter which decays with oscillations! $\Psi \sim \exp[-(q_1 \quad iq_2)x]$



Wave-vectors are complex!
They are complex conjugate and we can have a real Ψ.

Order parameter changes its sign!

Proximity effect as Andreev reflection

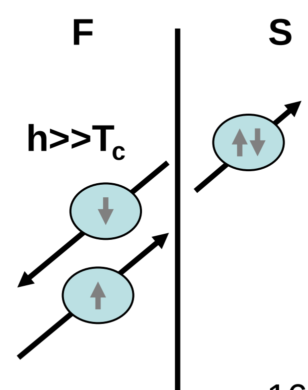


p_F

"retro"-reflection

Classical Andreev reflection

Quantum Andreev reflection



$$p_{F\uparrow} \neq p_{F\downarrow}$$

Theory of S-F systems in dirty limit

Analysis on the basis of the Usadel equations

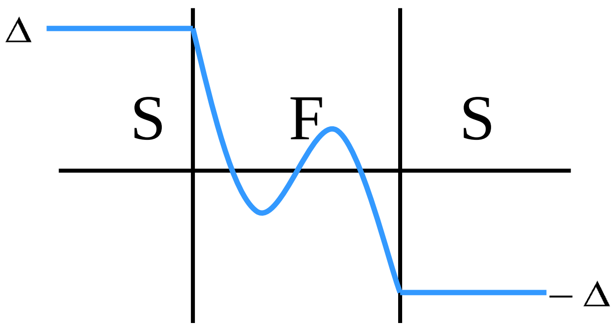
$$-\frac{D_f}{2}\vec{\nabla}^2 F_f(x,\omega,h) + (\omega + ih) F_f(x,\omega,h) = 0$$

$$G_f^{\,2}(x,\omega,h) + F_f(x,\omega,h) F_f^{\,*}(x,\omega,-h) = 1$$

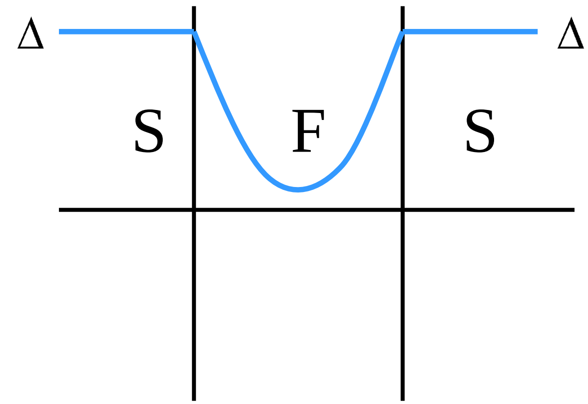
.

leads to the prediction of the oscillatory - like dependence of the critical current on the exchange field h and/or thickness of ferromagnetic layer.

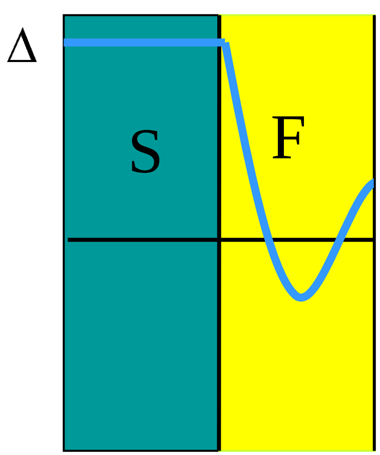
Remarkable effects come from the possible shift of sign of the wave function in the ferromagnet, allowing the possibility of a « π-coupling » between the two superconductors (π-phase difference instead of the usual zero-phase difference)



« π phase »



« 0 phase »



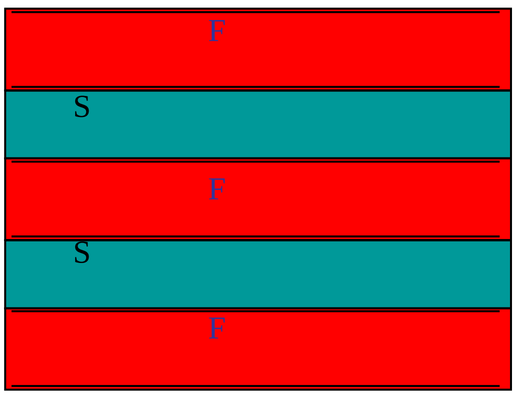
S/F bilayer

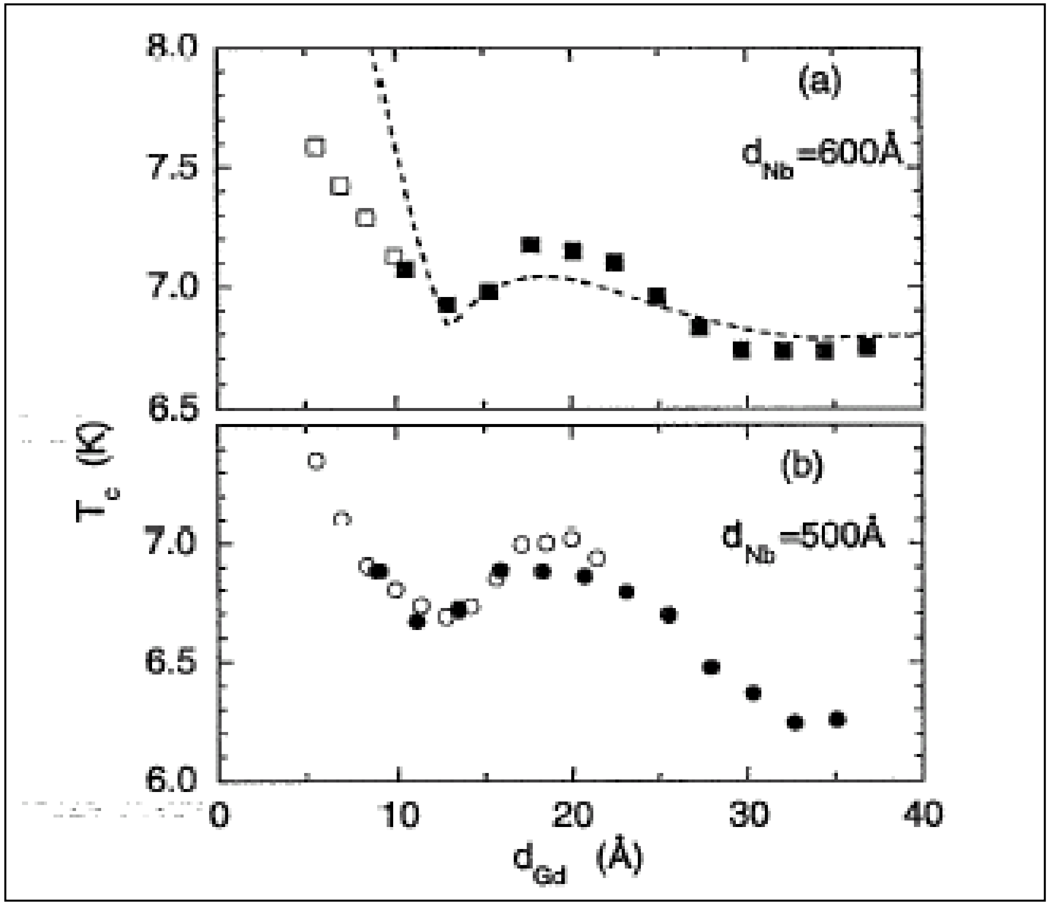
$$\xi_f = \sqrt{D_f / h}$$

h-exchange field,

D_f-diffusion constant

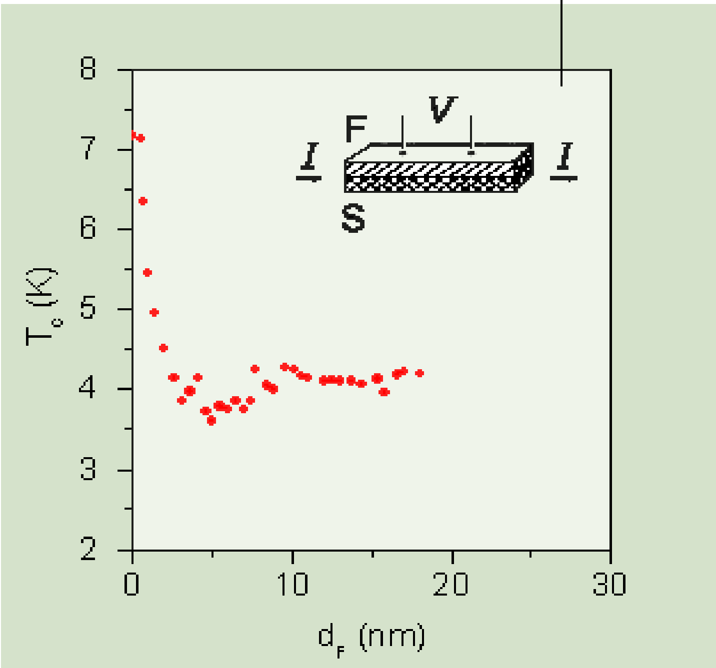
The oscillations of the critical temperature as a function of the thickness of the ferromagnetic layer in S/F multilayers has been predicted in 1990 and later observed on experiment by Jiang et al. PRL, **1995**, in Nb/Gd multilayers



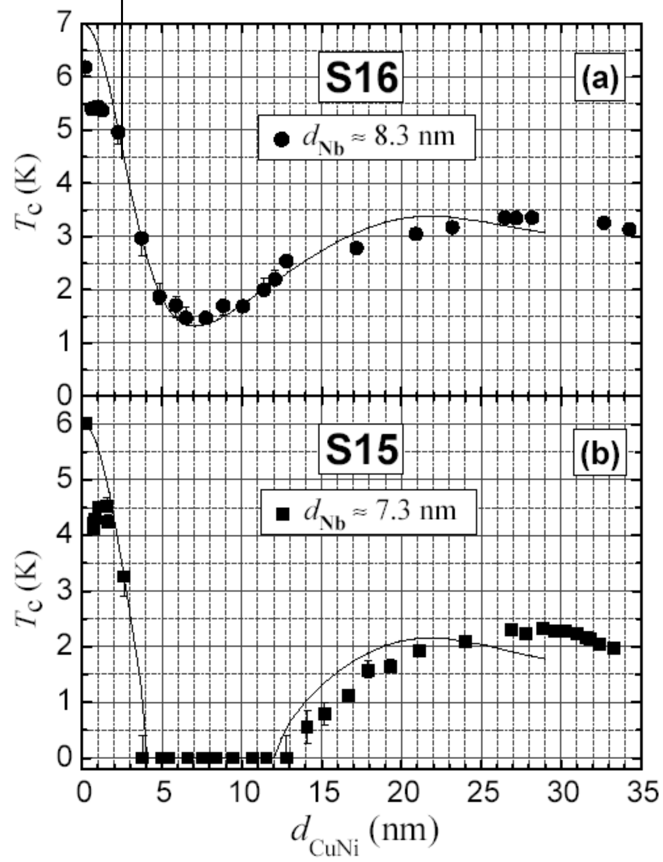


SF-bilayer T_c-oscillations

Ryazanov et al. JETP Lett. 77, 39 (2003) Nb-$Cu_{0.43}Ni_{0.57}$



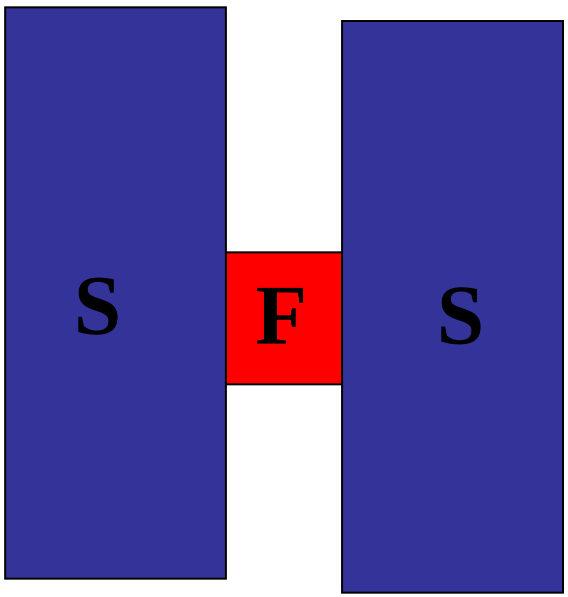
V. Zdravkov, A. Sidorenko et al
PRL (2007)
Nb-$Cu_{0.41}Ni_{0.59}$



d_{Fmin} =(1/4) λ_{ex} largest T_c-suppression

S-F-S Josephson junction in the clean limit

(Buzdin, Bulaevskii and Panjukov, JETP Lett. 81)

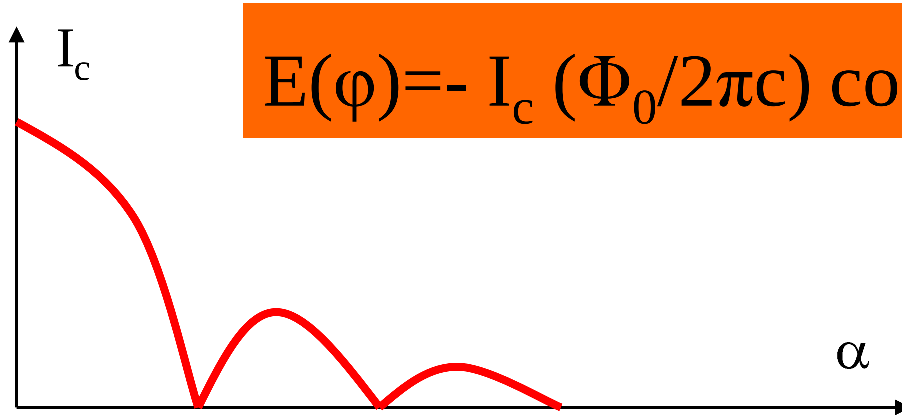


Damping oscillating dependence of the critical current I_c as the function of the parameter $\alpha = hd_F / v_F$ has been predicted.

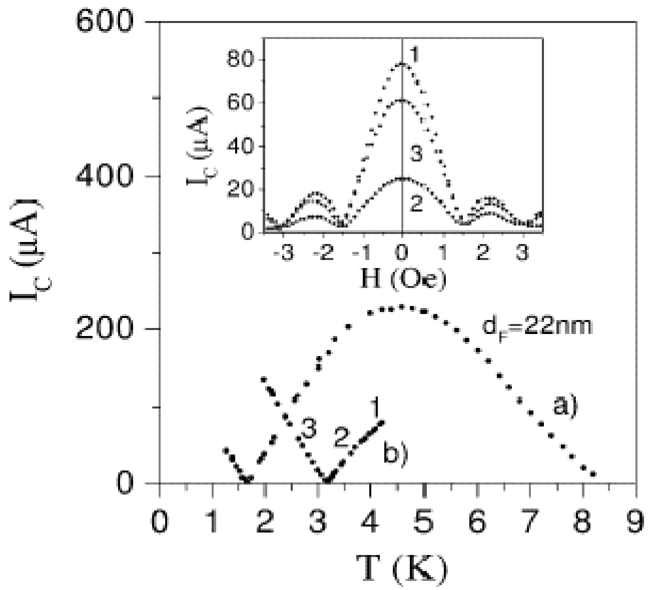
h- exchange field in the ferromagnet,
d_F - its thickness

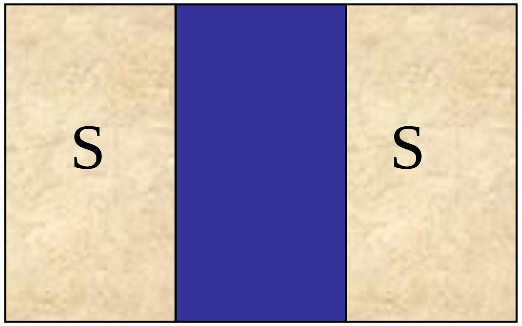
$$E(\varphi) = - I_c (\Phi_0 / 2\pi c) \cos\varphi$$

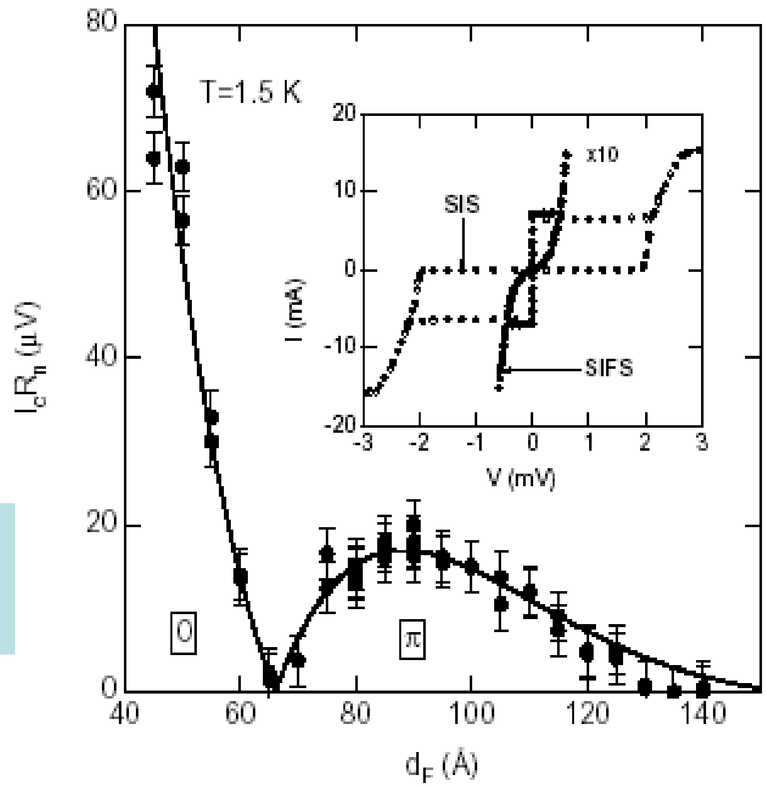
$$J(\varphi) = I_c \sin\varphi$$



The oscillations of the critical current as a function of temperature (for different thickness of the ferromagnet) in S/F/S trilayers have been observed on experiment by Ryazanov et al. 2000, PRL







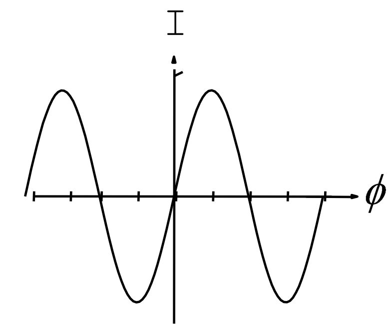
and as a function of a ferromagnetic layer thickness by Kontos et al. 2002, PRL

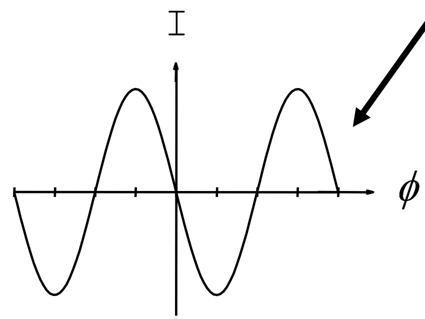
Phase-sensitive experiments

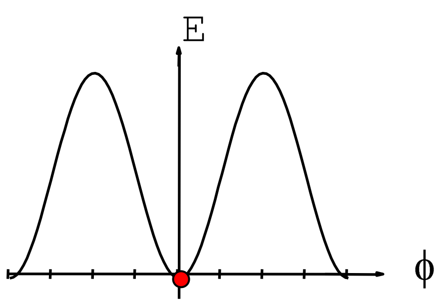
π-junction in one-contact interferometer

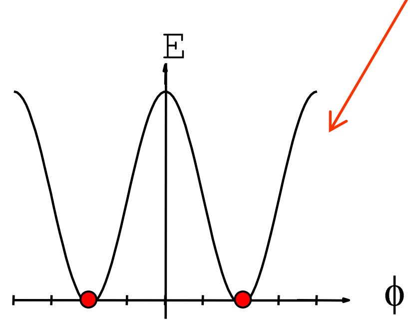
0-junction
minimum energy at 0

π-junction
minimum energy at π





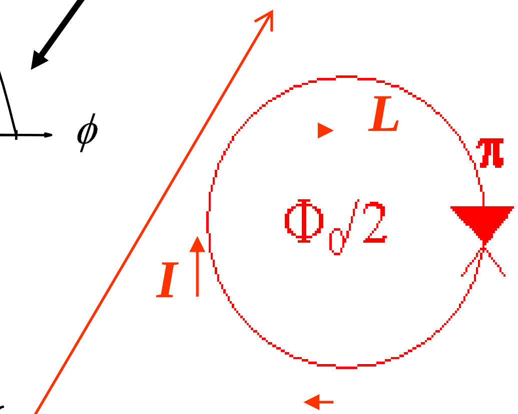




$I = I_c \sin(\pi+\phi) = -I_c \sin\phi$

$E = E_J[1-\cos(\pi+\phi)] = E_J[1+\cos\phi]$

$2\pi L I_c > \Phi_0/2$



$\phi = \pi = (2\pi/\Phi_0)\int A dl = 2\pi\,\Phi/\Phi_0$

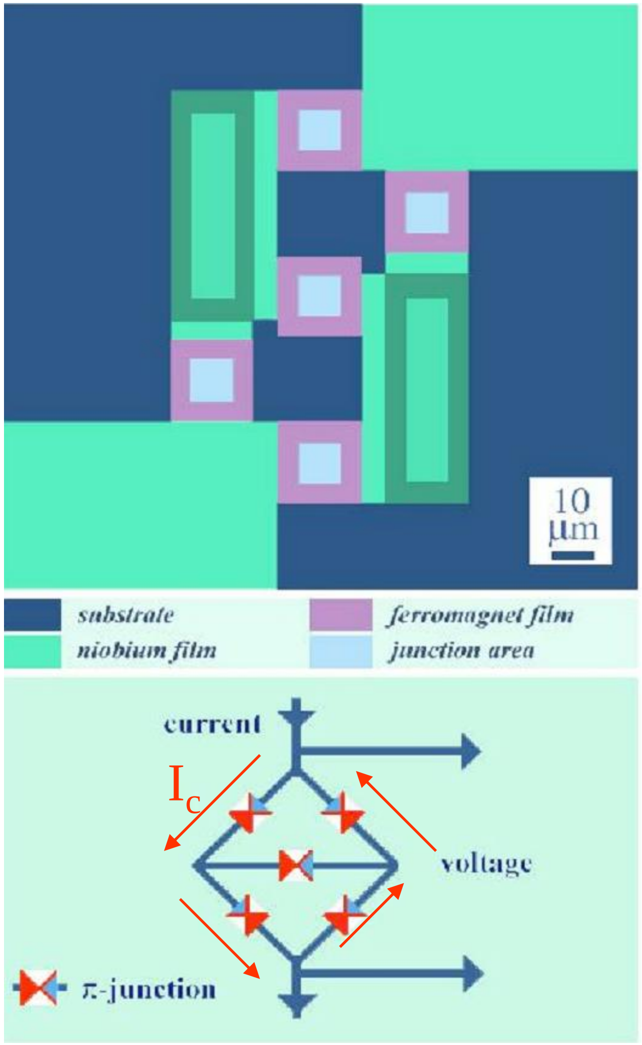
Spontaneous circulating current
in a closed superconducting loop
when $\beta_L > 1$ with NO applied flux

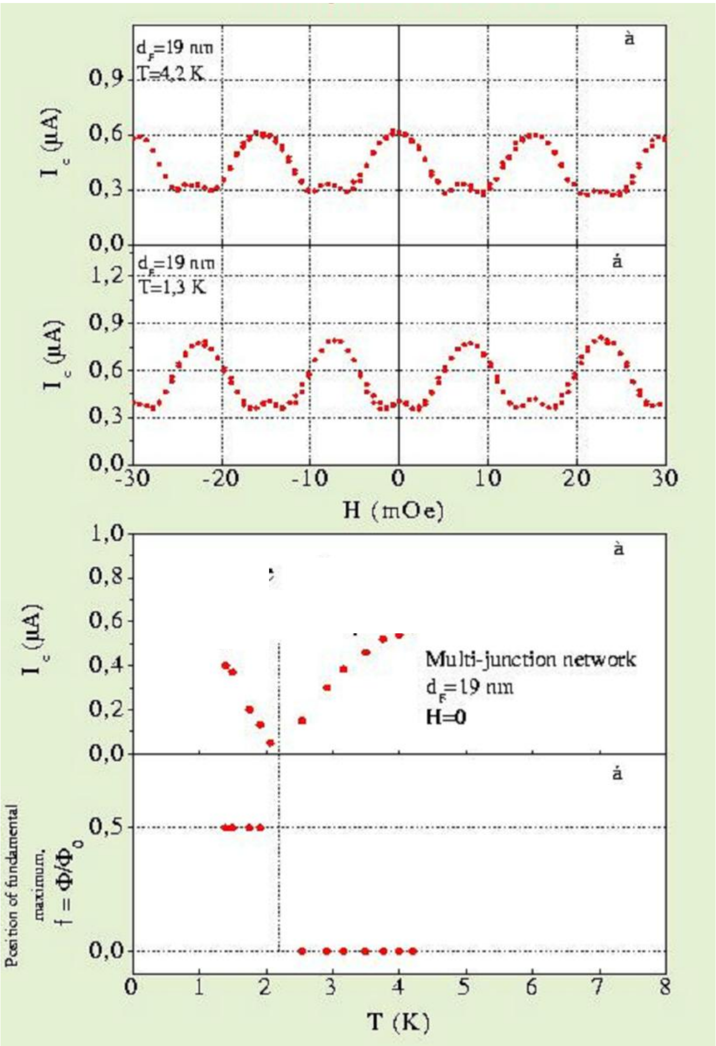
$\beta_L = \Phi_0/(4\pi L I_c)$

$\Phi = \Phi_0/2$

Bulaevsky, Kuzii, Sobyanin, JETP Lett. 1977

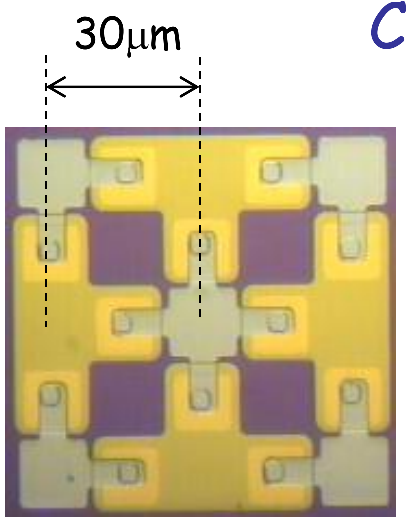
Current-phase experiment.
Two-cell interferometer



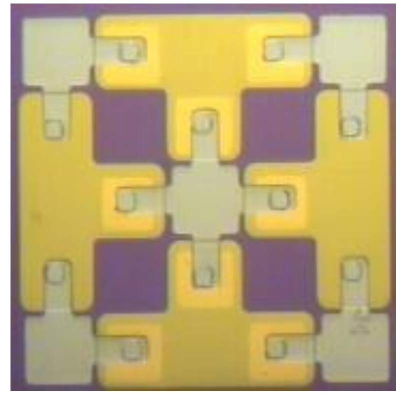


Cluster Designs (Ryazanov et al.)

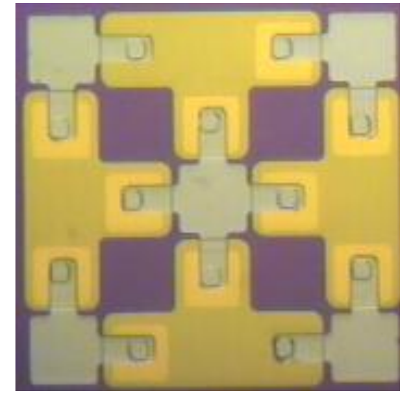
2 x 2



unfrustrated

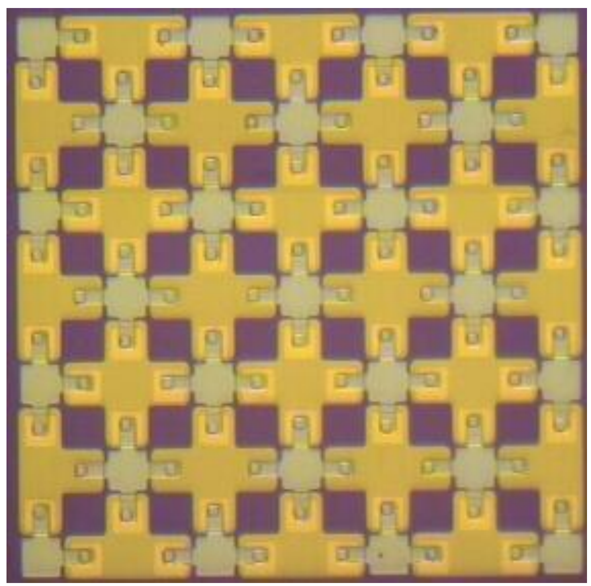


fully-frustrated

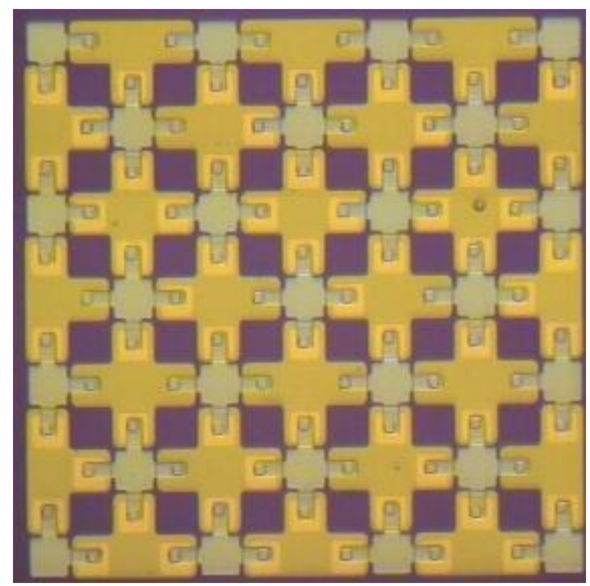


checkerboard-frustrated

6 x 6



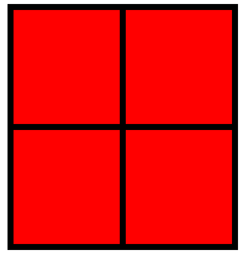
fully-frustrated

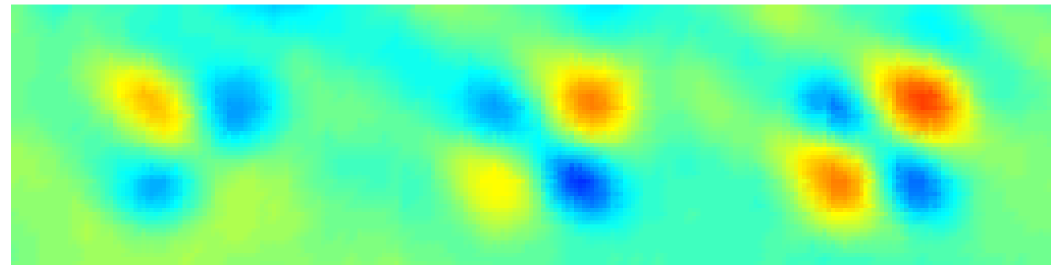


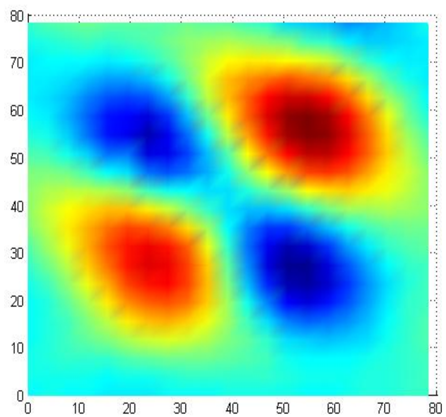
checkerboard-frustrated

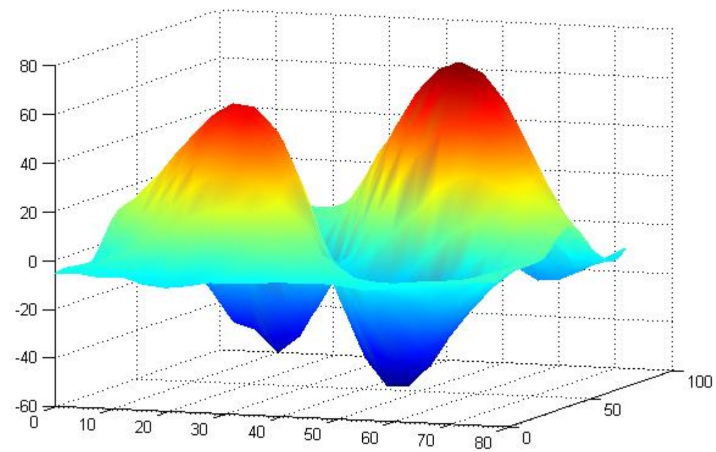
2 x 2 arrays: spontaneous vortices

Fully frustrated

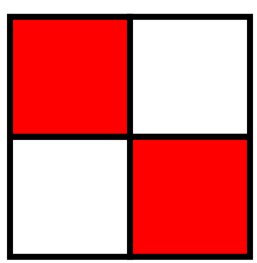


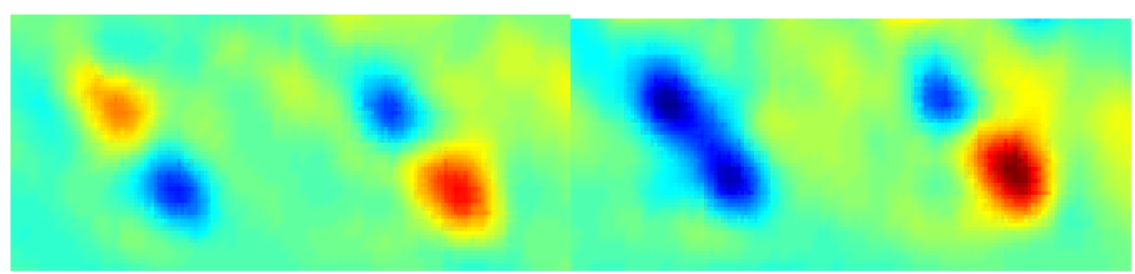






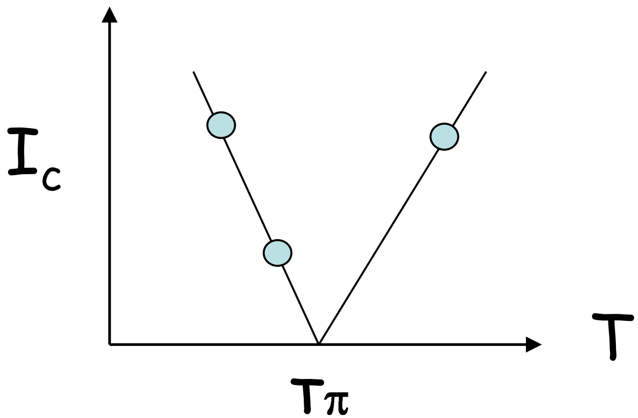
Checkerboard frustrated

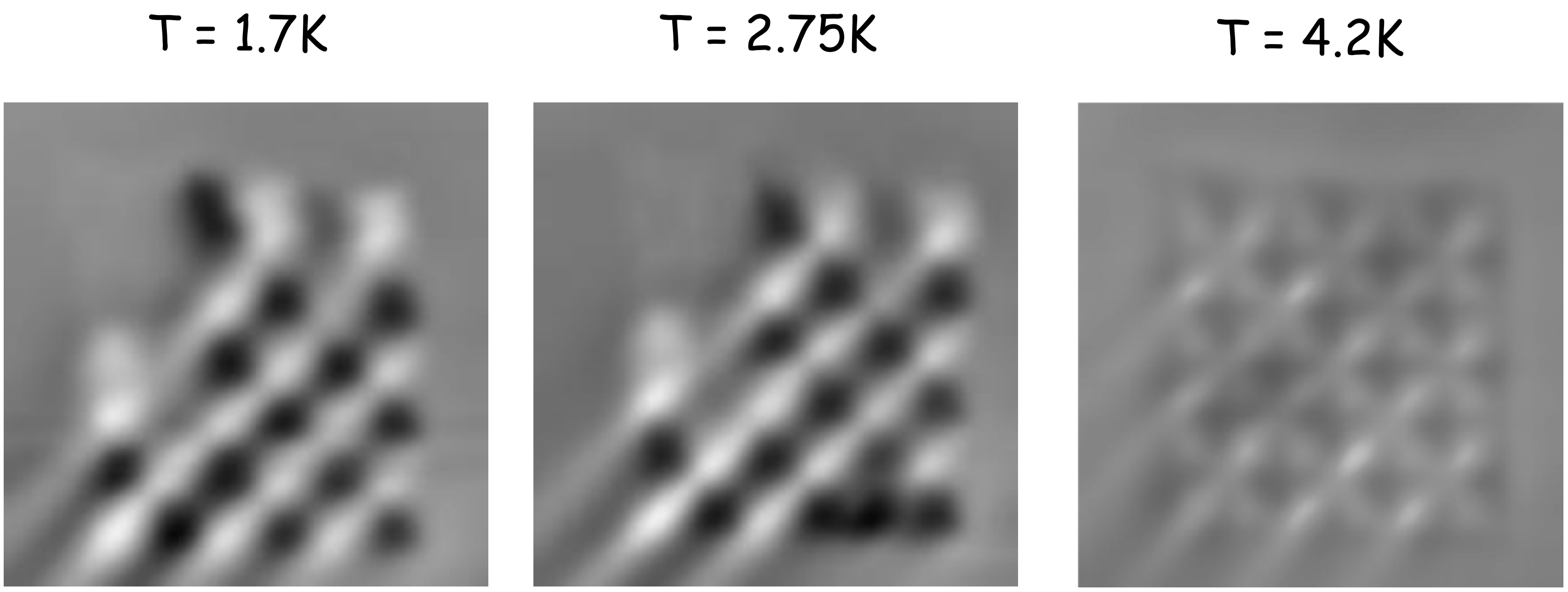




Scanning SQUID Microscope images

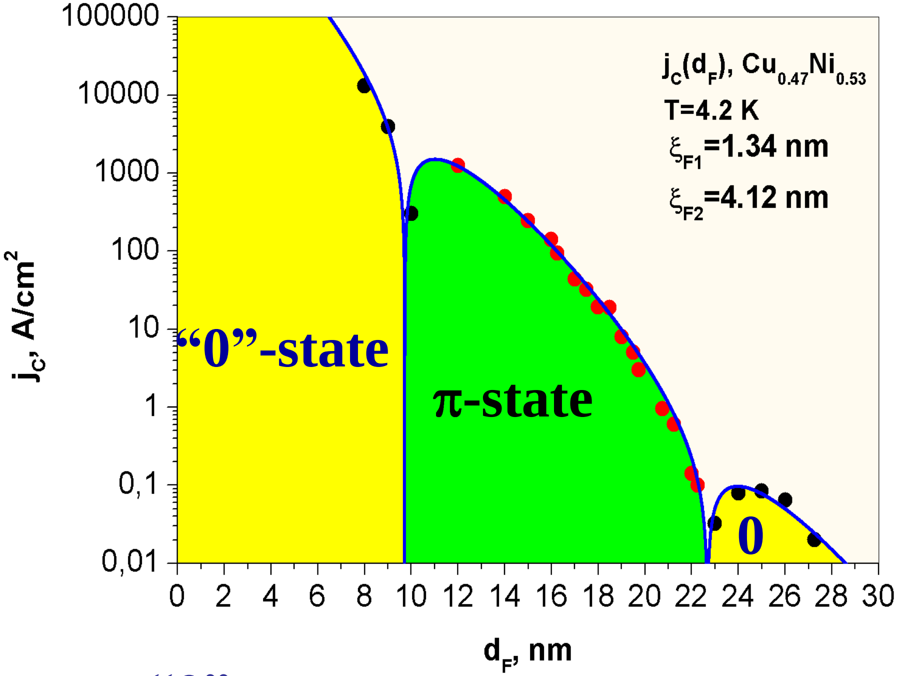
(Ryazanov et al.)





Critical current density vs. F-layer thickness (V.A.Oboznov et al., PRL, 2006)

$$I_c = I_{c0}\exp(-d_F/\xi_{F1})\,|\cos(d_F/\xi_{F2}) + \sin(d_F/\xi_{F2})|$$



$d_F \gg \xi_{F1}$

Spin-flip scattering decreases the decaying length and increases the oscillation period.

$\xi_{F2} > \xi_{F1}$

Nb-$Cu_{0.47}Ni_{0.53}$-Nb

“0”-state

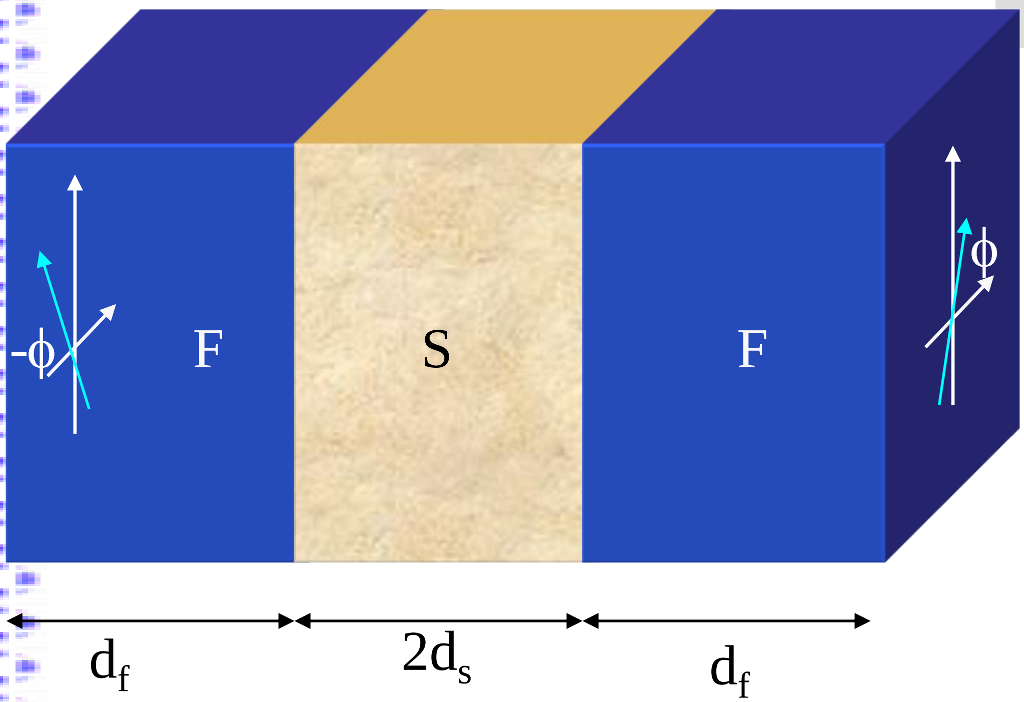
$I = I_c \sin\varphi$

π-state

$I = I_c \sin(\varphi + \pi) = - I_c \sin(\varphi)$

F/S/F trilayers, spin-valve effect

If d_s is of the order of magnitude of ξ_s, the critical temperature is controlled by the proximity effect.



Firstly the FI/S/FI trilayers has been studied experimentally in 1968 by Deutscher et Meunier.

In this special case, we see that the critical temperature of the superconducting layers is reduced when the ferromagnets are polarized in the same direction

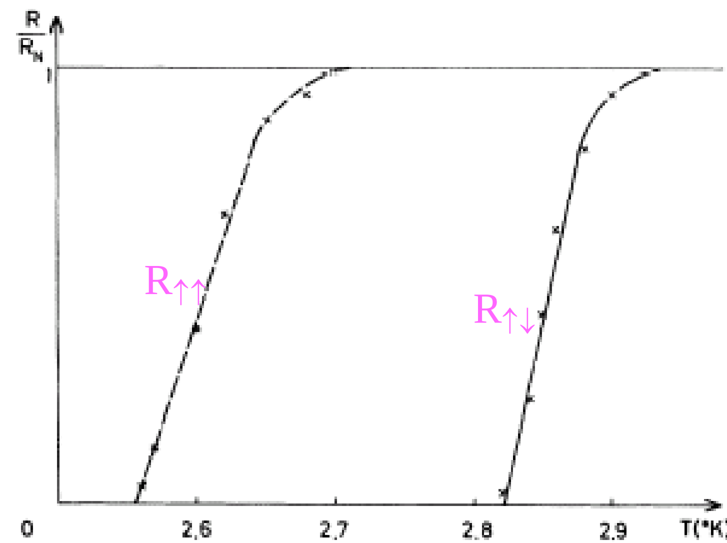


FIG. 1. Resistive measurements of the critical temperatures (R_N = resistance in the normal state) in zero field after the following: dashed line, application of 10 000 G ($T_{c\uparrow\uparrow}$) (all fields are applied parallel to the plane of the films); solid line, application of −10 000 G and subsequently +300 G to return the magnetization of the FeNi layer ($H_1 < 300\ \mathrm{G} < H_2$) ($T_{c\uparrow\downarrow}$).

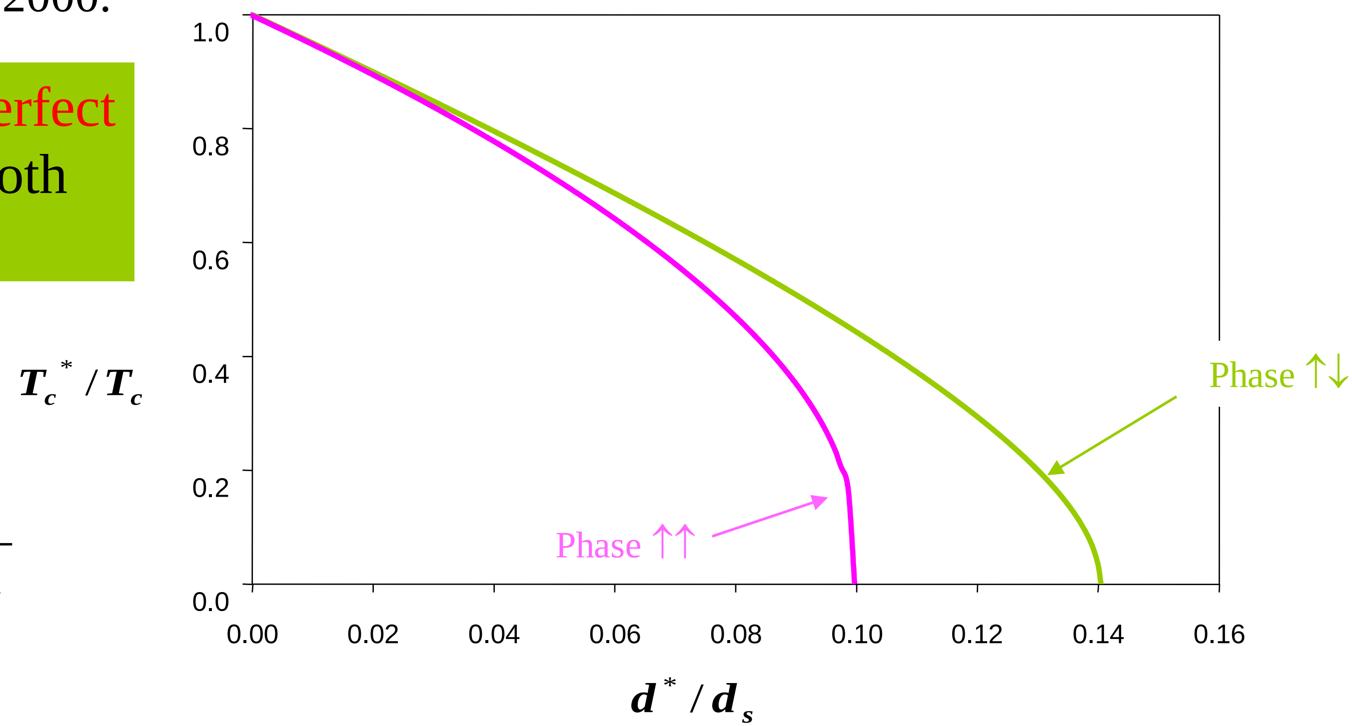
In the dirty limit, we used the quasiclassical Usadel equations to find the new critical temperature T^{*}_{c}.
We solved it self-consistently supposing that the order parameter can be taken as :

$$\boxed{\Delta = \Delta_0\left(1-\frac{x^2}{L^2}\right)}$$

with $L \gg d_S$

Buzdin, Vedyaev, Ryazhanova, Europhys Lett. 2000,
Tagirov, Phys. Rev. Let. 2000.

In the case of a perfect transparency of both interfaces



$$d^{*} = \gamma\sqrt{\frac{h}{D_n}\,\frac{D_s}{4\pi T_c}}$$

$$\ln\left(\frac{T_{c\uparrow\uparrow}^{\;*}}{T_c}\right) = \Psi\left(\frac{1}{2}\right) - \mathrm{Re}\,\Psi\left(\frac{1}{2} + \frac{d^{*}T_c}{d_s T_{c\uparrow\uparrow}^{\;*}}(1+i)\right)$$

$$\ln\left(\frac{T_{c\uparrow\downarrow}^{\;*}}{T_c}\right) = \Psi\left(\frac{1}{2}\right) - \Psi\left(\frac{1}{2} + \frac{d^{*}T_c}{d_s T_{c\uparrow\downarrow}^{\;*}}\right)$$

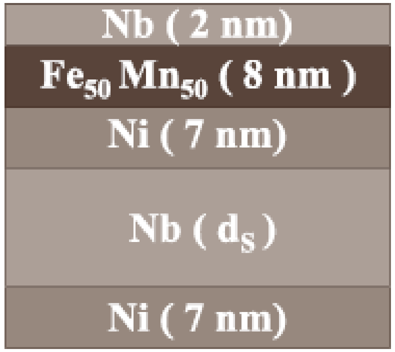
Recent experimental verifications

AF

F1

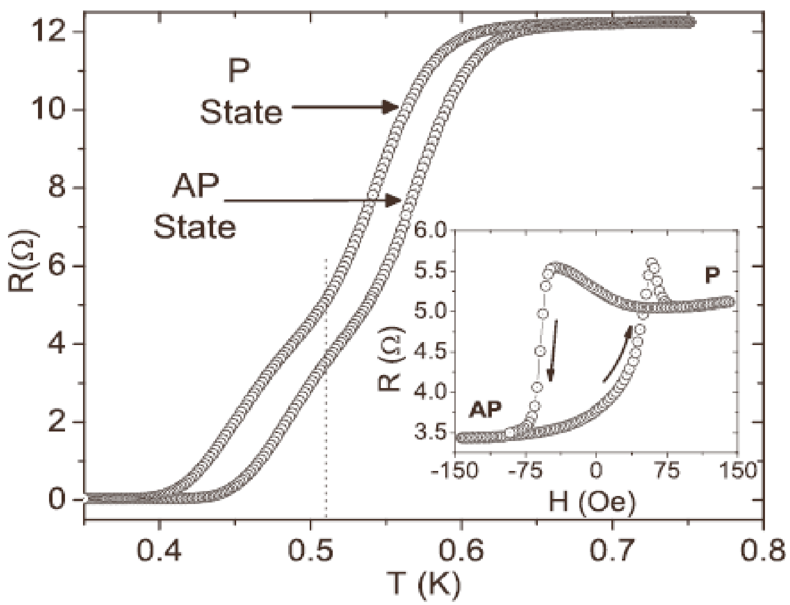
S

F2



F layer with fixed magnetization

« free » F layer



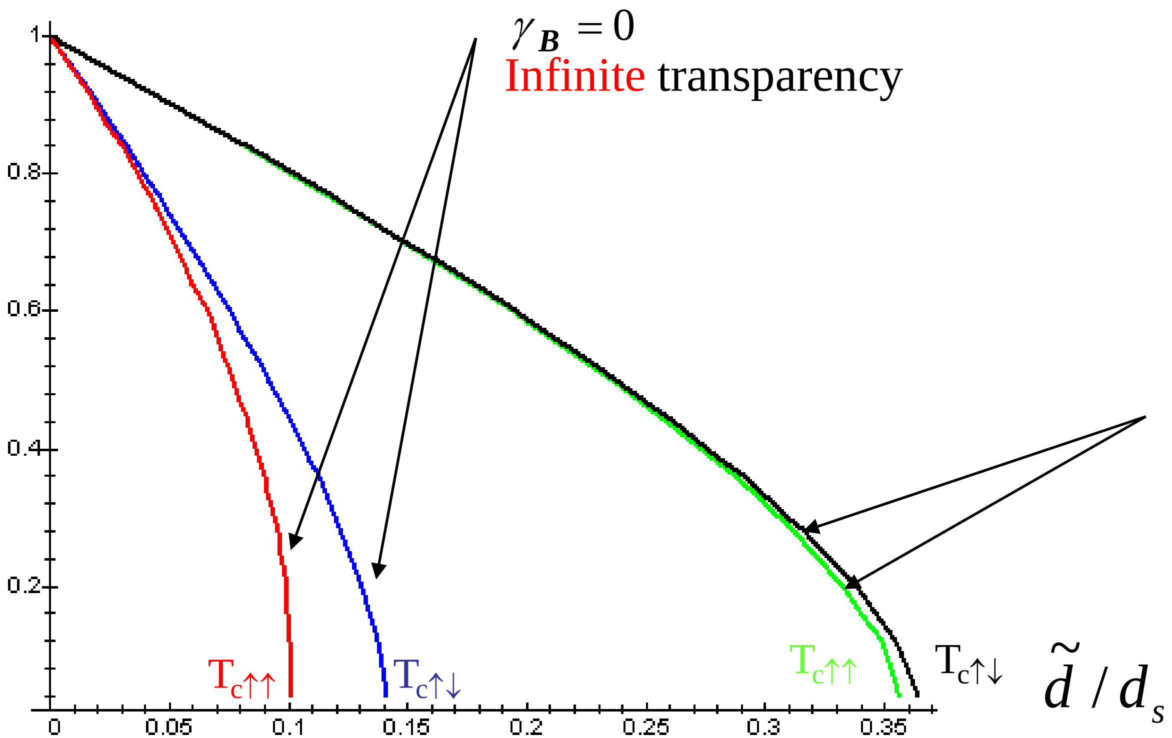
CuNi/Nb/CuNi

Gu, You, Jiang, Pearson, Bazaliy, Bader, 2002

Ni/Nb/Ni

Moraru, Pratt Jr, Birge, 2006

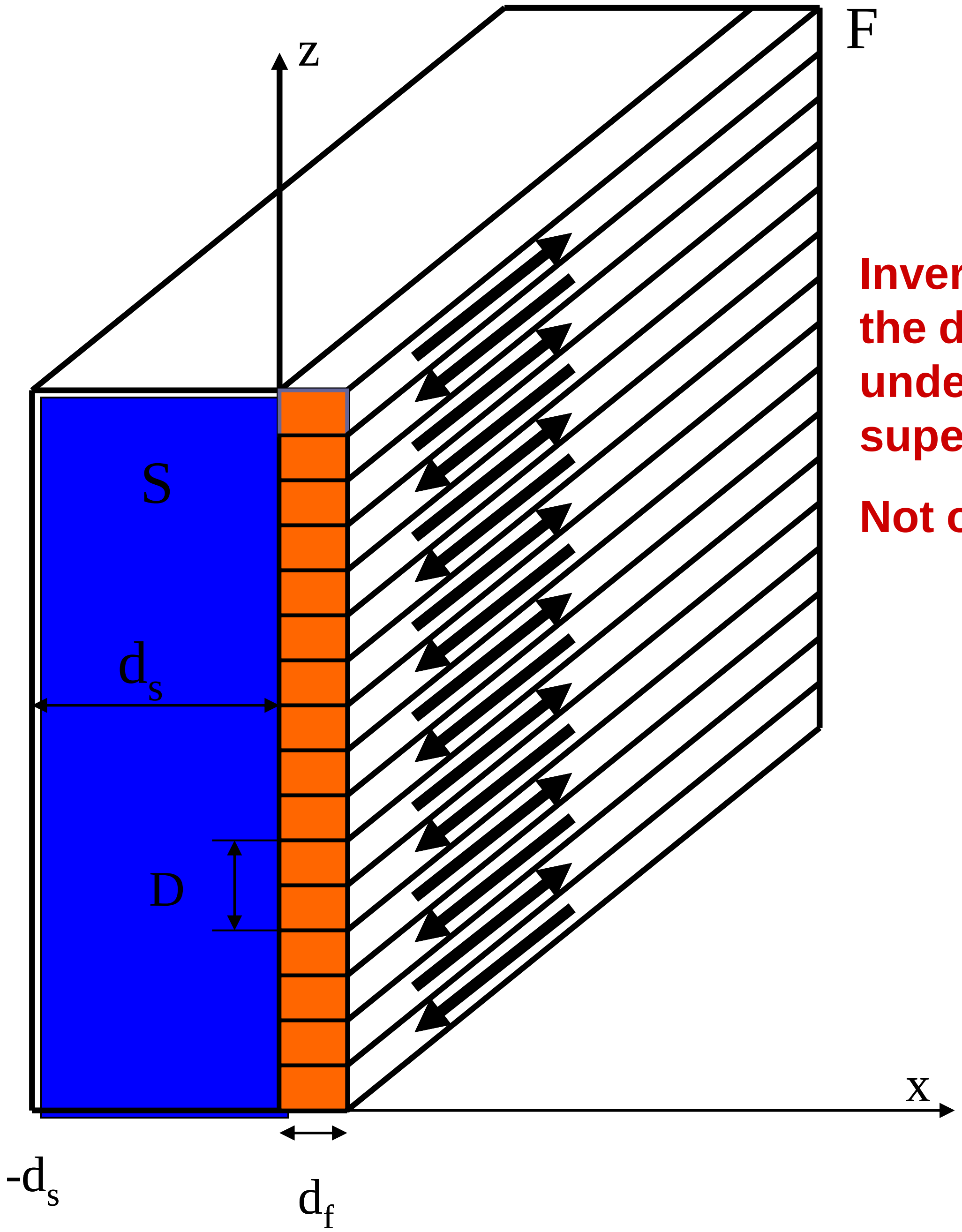
Evolution of the difference between the critical temperatures as a function of interfaces' transparency



$$\ln\left(\frac{T_{c\uparrow\uparrow}^{*}}{T_c}\right) = \Psi\left(\frac{1}{2}\right) - \mathrm{Re}\,\Psi\left(\frac{1}{2} + \frac{\tilde{d}T_c}{d_s T_{c\uparrow\uparrow}^{*}}(1+i)\right)$$

$$\ln\left(\frac{T_{c\uparrow\downarrow}^{*}}{T_c}\right) = \Psi\left(\frac{1}{2}\right) - \Psi\left(\frac{1}{2} + \left(\frac{\tilde{d}T_c}{d_s T_{c\uparrow\downarrow}^{*}}\right)\right)$$

$$\tilde{\boldsymbol{d}} = \frac{\boldsymbol{D_s}}{4\pi \boldsymbol{T_c}} \frac{\gamma\sqrt{\dfrac{\boldsymbol{h}}{\boldsymbol{D_n}}}}{1 + (1+\boldsymbol{i})\gamma_{\boldsymbol{B}}\gamma\sqrt{\dfrac{\boldsymbol{h}}{\boldsymbol{D_n}}}}$$



Inverse effect: appearence of the dense domain structure under the influence of superconductivity.

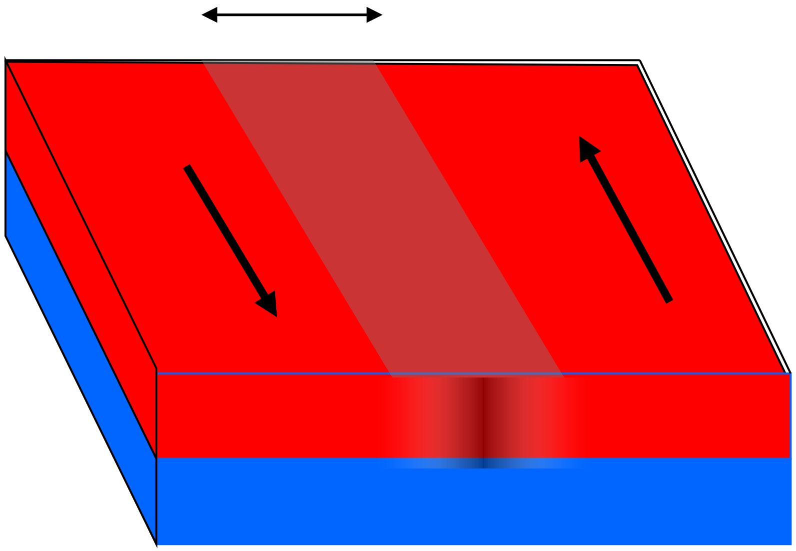
Not observed yet.

D<<ξ

Similar physics in F/S bilayers

In practice, magnetic domains appear quite easily in ferromagnets

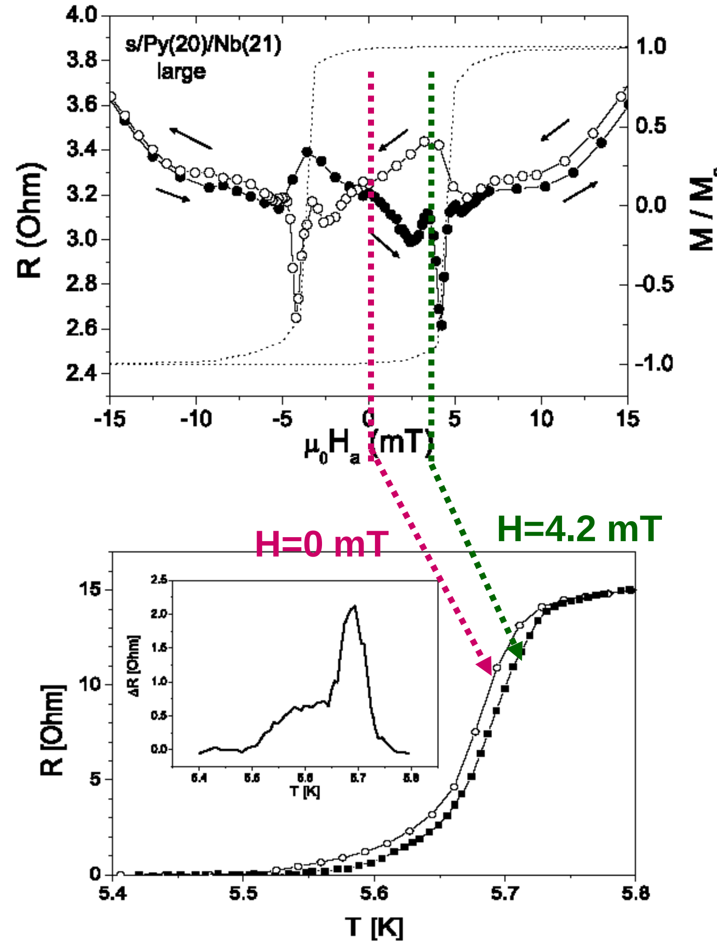
w: width of the domain wall



$Ni_{0.80}Fe_{0.20}$/Nb (20nm)

Thin films : Néel domains

Rusanov et al., PRL, 2004

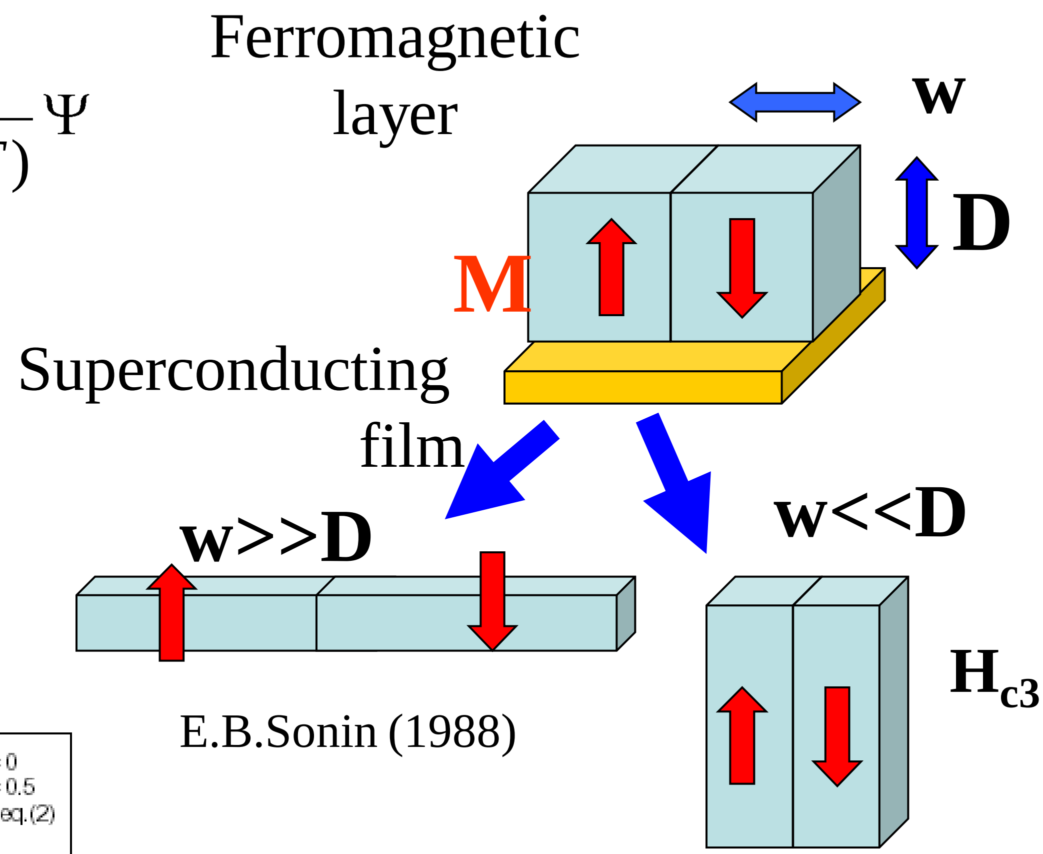


Localized (domain wall) superconducting phase.

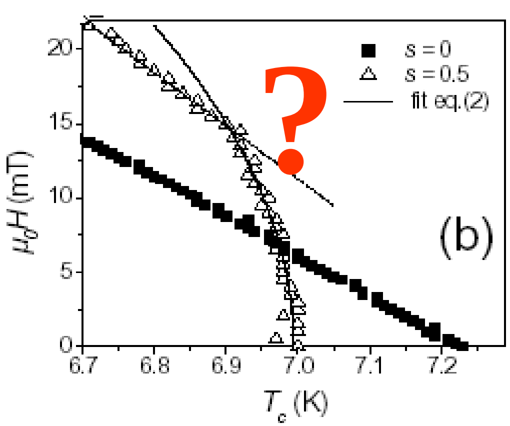
Theory - Houzet and Buzdin, Phys. Rev. B (2006).

Domain wall superconductivity in purely electromagnetic model

$$-\left(\nabla + \frac{2\pi i}{\Phi_0}\vec{A}(\vec{r})\right)^2 \Psi = \frac{1}{\xi^2(T)}\Psi$$



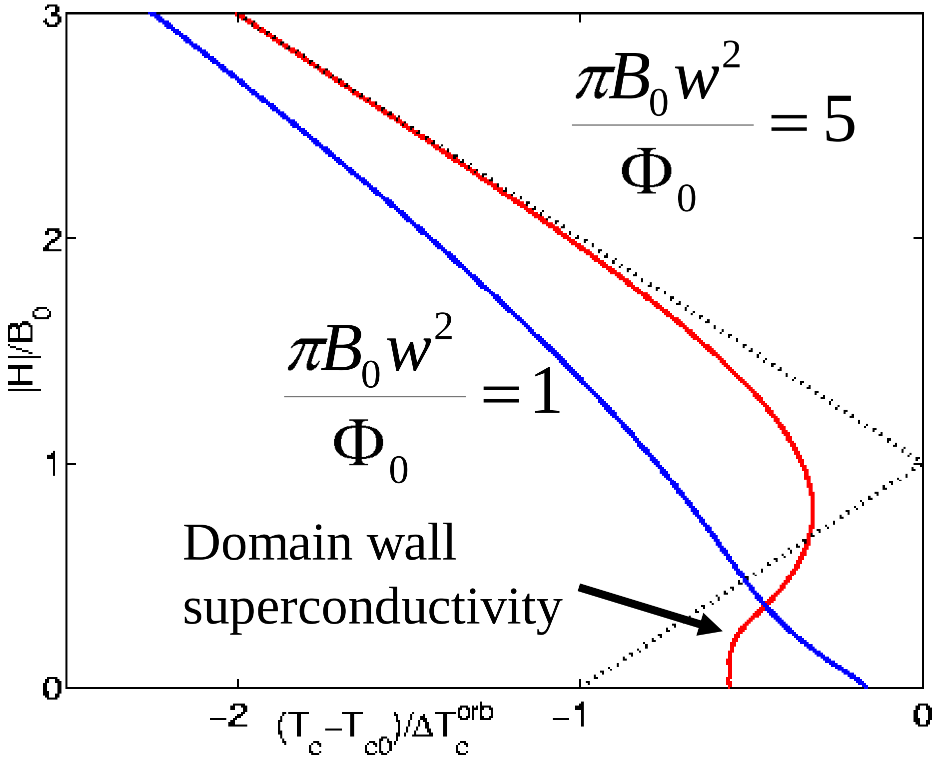
E.B.Sonin (1988)

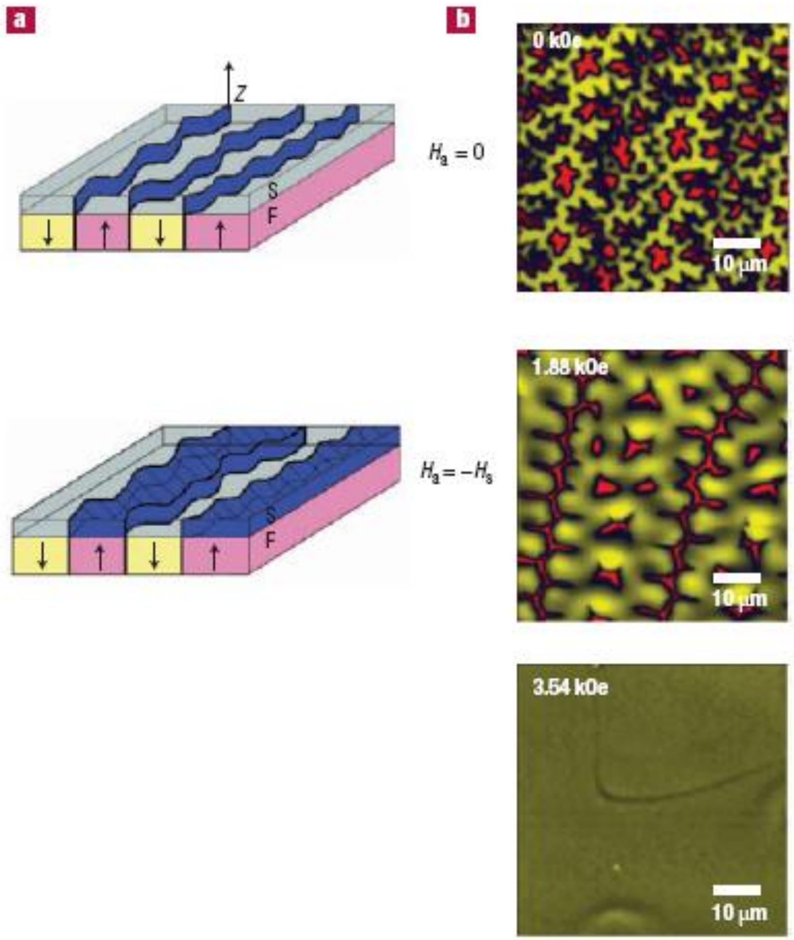


Pb-Co/Pt

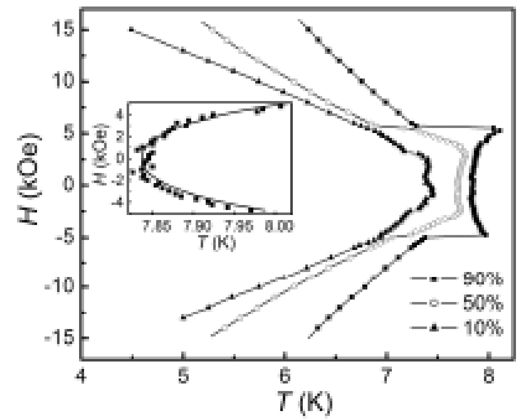
Superconducting nucleus in a periodic domain structure in an external field

$H \neq 0$





Nb/$BaFe_{12}O_{19}$

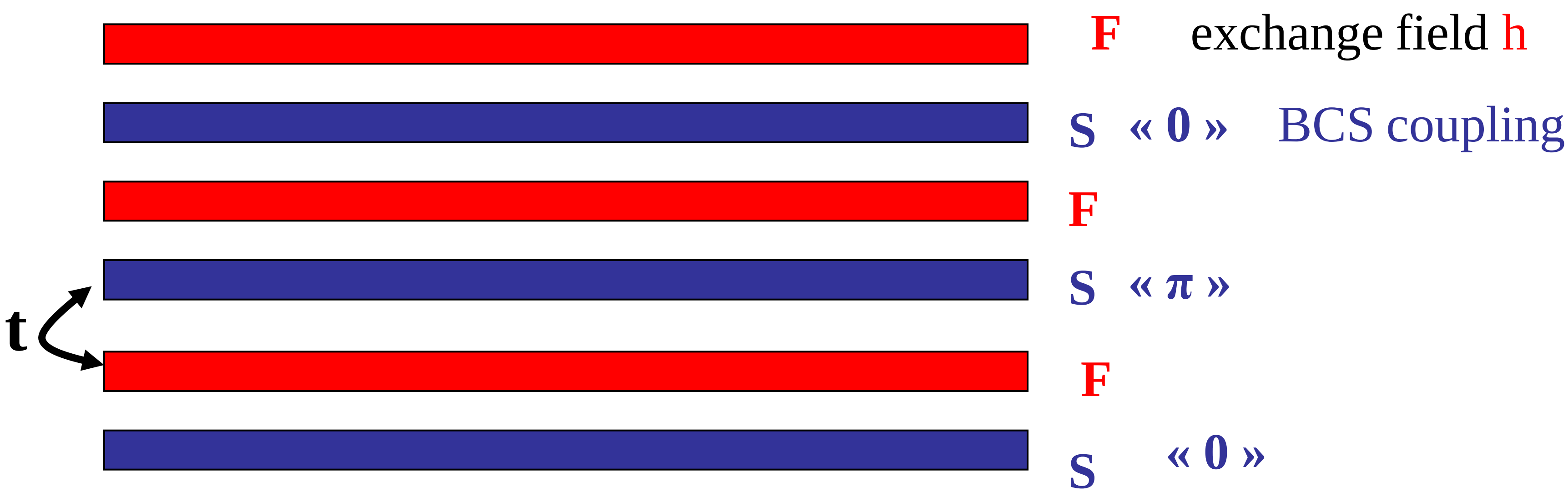


Z. YANG et al, Nature Materials, 2004

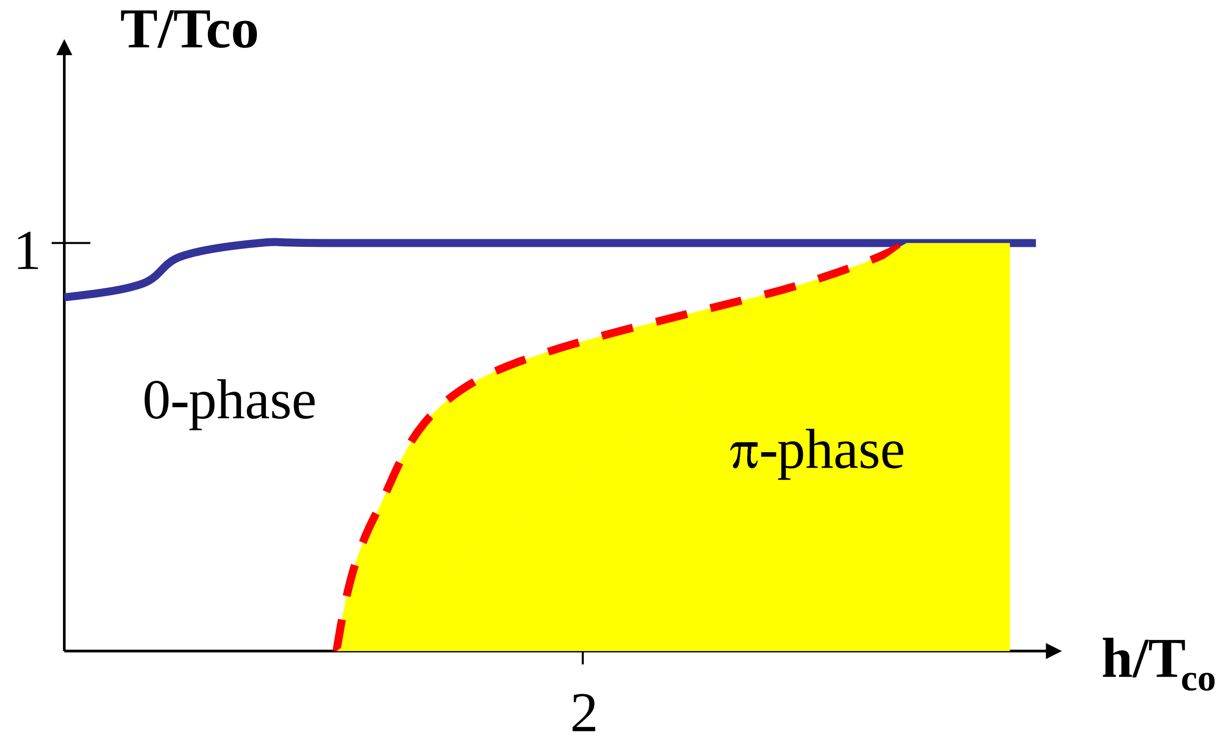
Atomic layered S-F systems

(Andreev et al, PRB 1991, Houzet et al, PRB 2001, Europhys. Lett. 2002)

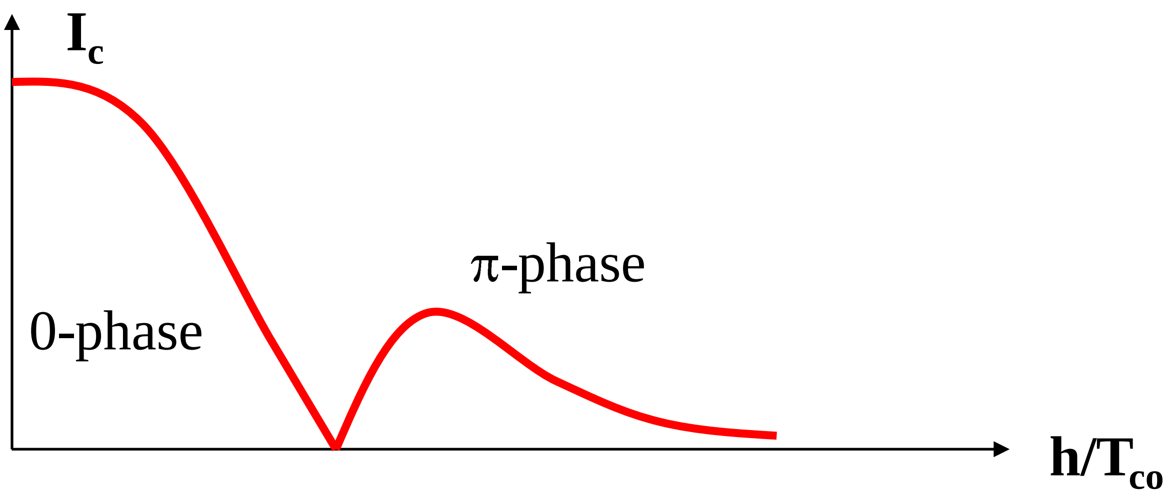
Magnetic layered superconductors like $RuSr_2GdCu_2O_8$



Also even for the quite small exchange field ($h>T_c$) the **π-phase** must appear.



The limit **t** <<$\mathbf{T_{co}}$



Different mechanism for the φ_0 - Josephson junction realization.

Recently the broken inversion symmetry (BIS) superconductors (like $CePt_3Si$) have attracted a lot of interest.

Very special situation is possible when the weak link in Josephson junction is a non-centrosymmetric magnetic metal with broken inversion symmetry !

Suitable candidates : MnSi, FeGe.

Josephson junctions with time reversal symmetry: $j(-\varphi)= - j(\varphi)$;

i.e. higher harmonics can be observed $\sim j_n \sin(n\varphi)$ –the case the π junctions.

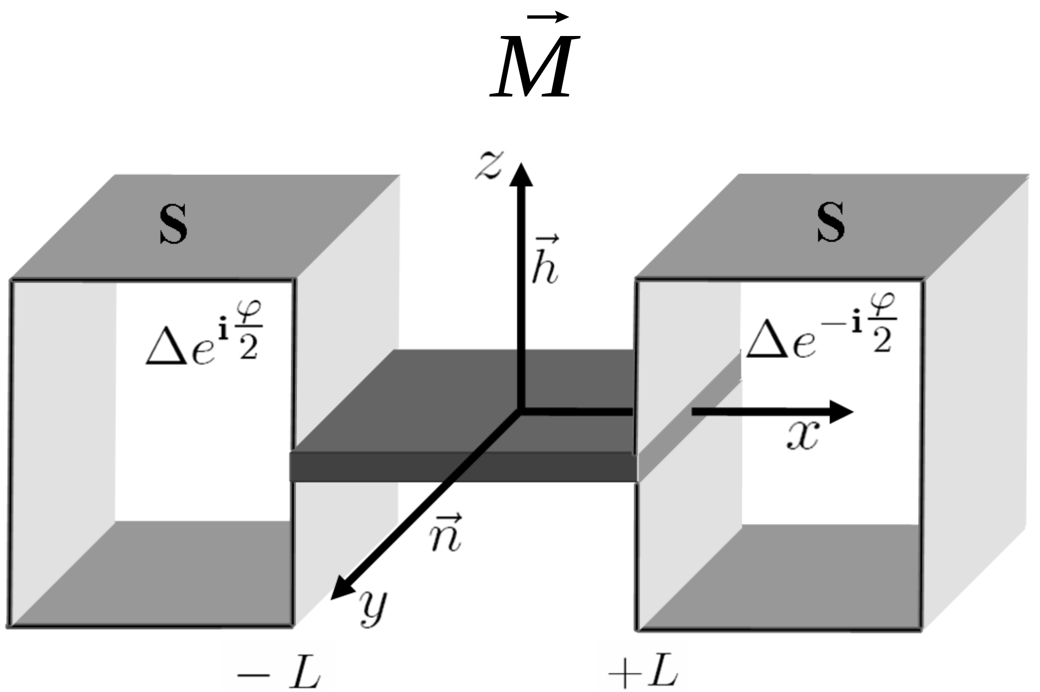
Without this restriction a more general dependence is possible

$$j(\varphi)= j_0 \sin(\varphi+ \varphi_0).$$

Rashba-type spin-orbit coupling $\alpha(\vec{\sigma}\times\vec{p})\cdot\vec{n}$

$\vec{n}$ is the unit vector along the asymmetric potential gradient.

Geometry of the junction with BIS magnetic metal



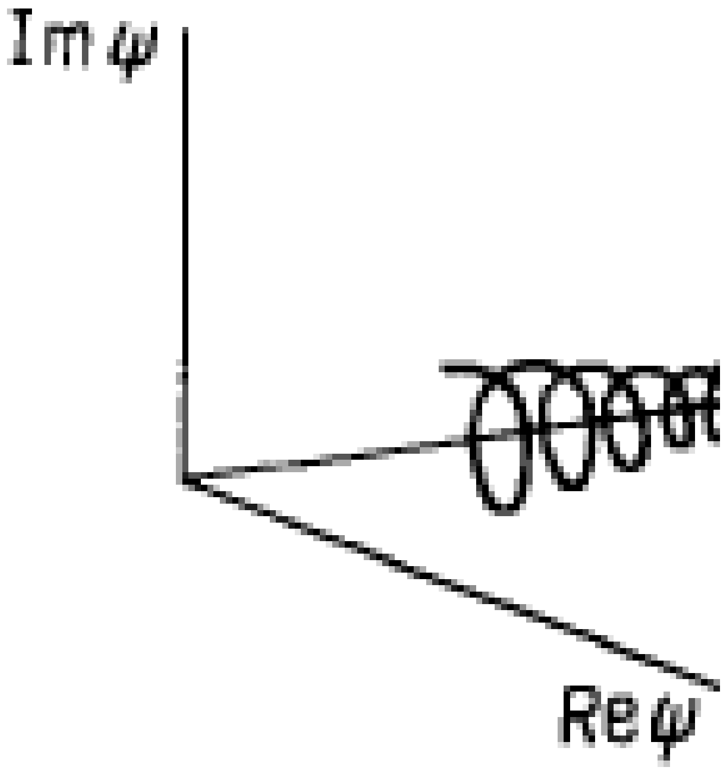
$$F = a|\Psi|^2 + \gamma|\vec{D}\Psi|^2 + \frac{b}{2}|\Psi|^4 - \varepsilon\vec{n}\,[\,\times(\Psi(\vec{D}\Psi)^* + \Psi^*(\vec{D}\Psi))]$$

$$D_i = -i\partial_i - 2eA_i$$

$$a\Psi - \gamma\frac{\partial^2\Psi}{\partial x^2} + 2i\varepsilon h\frac{\partial\ \Psi}{\partial x} = 0,$$

$$\Psi \propto \exp(i\tilde{\varepsilon}x)\exp\left(-x\sqrt{\frac{a-a_c}{\gamma}}\right), \qquad where\ \ \tilde{\varepsilon} = \frac{\varepsilon h}{\gamma}$$

$$\Psi \propto \exp(i\tilde{\varepsilon}x)\exp\left(-x\sqrt{\frac{a-a_c}{\gamma}}\right), \qquad where\ \ \tilde{\varepsilon} = \frac{\varepsilon h}{\gamma}$$



In contrast with a π junction it is not possible to choose a real Ψ function !

φ_0 Josephson junction

$$j(\varphi) = j_c \sin(\varphi + \varphi_o)$$

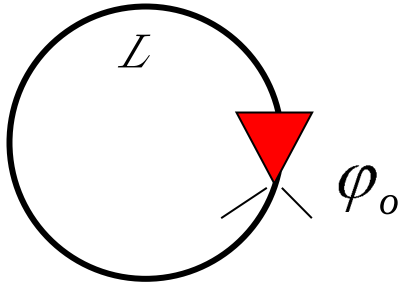
where $$\varphi_o = \frac{2\varepsilon hL}{\gamma}$$

The phase shift φ_0 is proportional to the length and the strength of the BIS magnetic interaction.

The φ_0 Junction is a natural phase shifter.

Energy $E_J(\varphi) \sim -j_c \cos(\varphi + \varphi_0)$

Spontaneous flux (current) in the superconducting ring with φ_0 - junction.



$$E(\varphi) = \frac{j_c}{2e}\left(-\cos(\varphi + \varphi_0) + \frac{k\varphi^2}{2} \right)$$

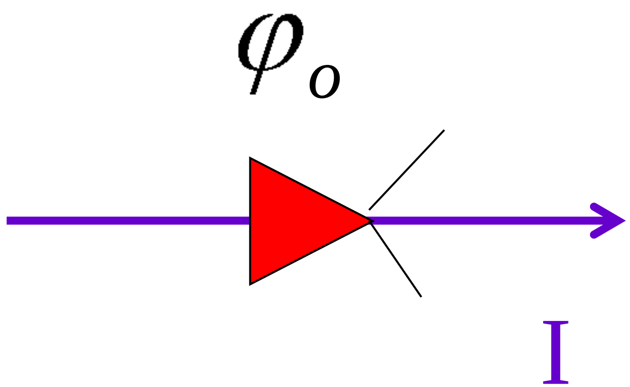
$$k = \frac{c\Phi_0}{2\pi L j_c}$$

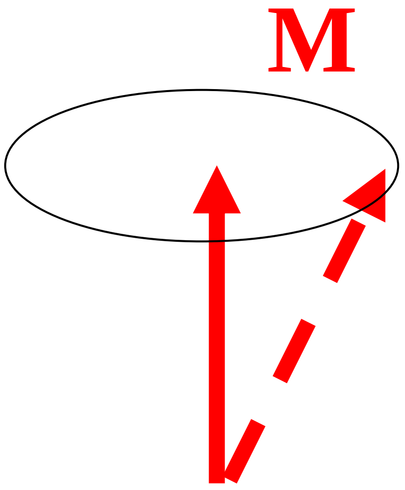
In the k<<1 limit the junction generates the flux $\Phi = \Phi_0(\varphi_0/2\pi)$

$$\varphi_o = \frac{2\varepsilon h L}{\gamma}$$

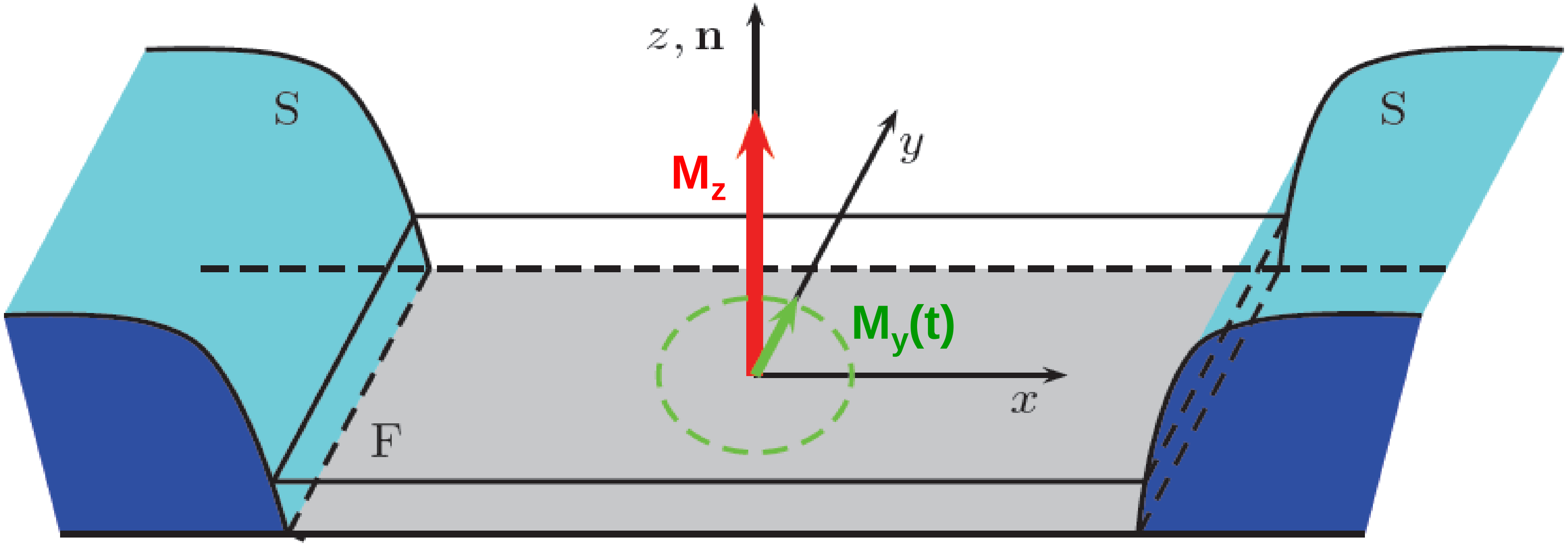
Very important : The φ_0 junction provides a mechanism of a direct coupling between supercurrent (superconducting phase) and magnetic moment (z component).

Applying to the φ_0 - junction the current (phase difference) we can generate the magnetic moment rotation.





Magnetic moment precession – voltage-biased φ_0- juncti $\varphi(t) = \omega_J t$



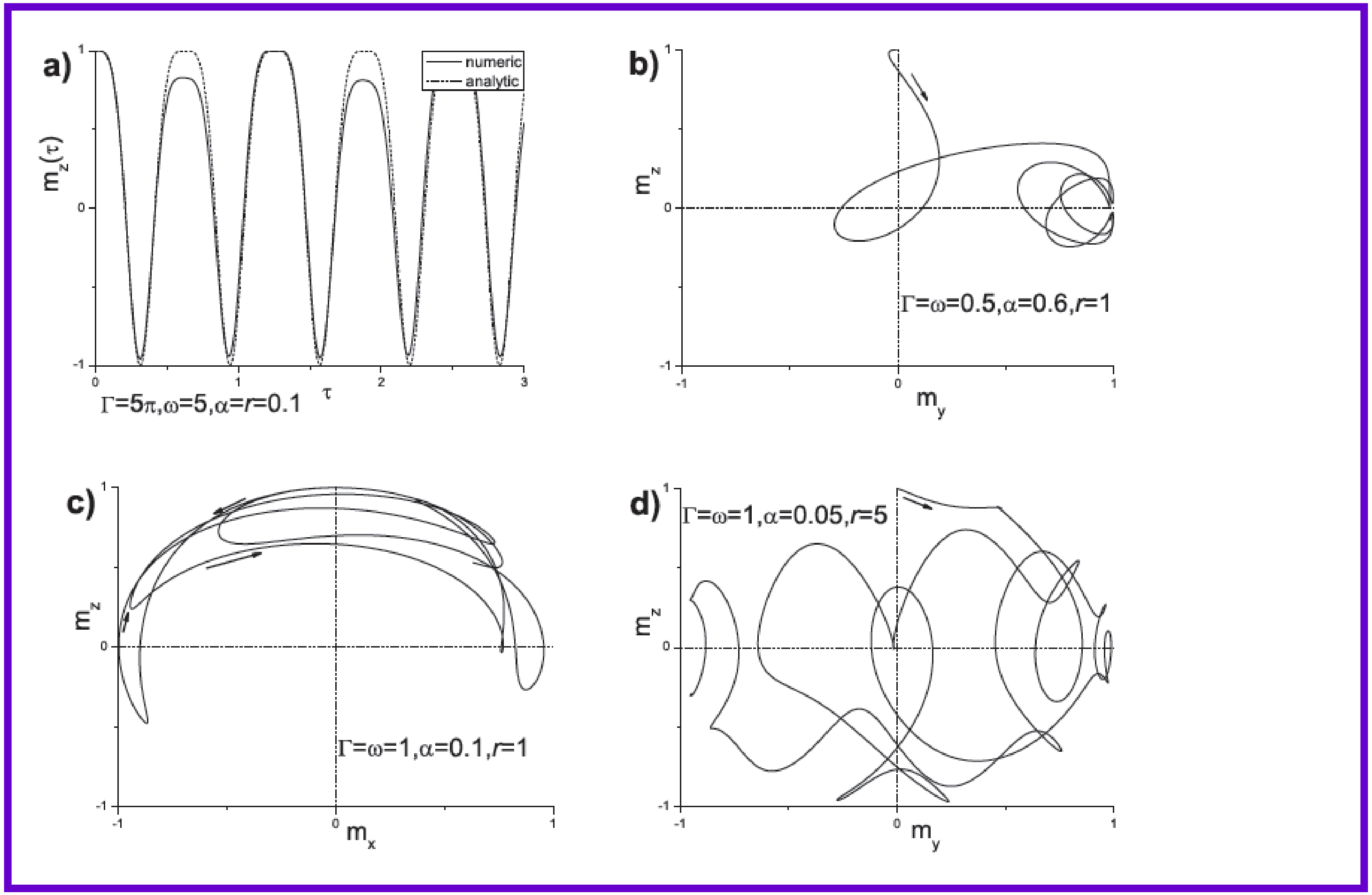
$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \wedge \mathbf{H}_{\text{eff}} + \frac{\alpha}{M_0}\left(\mathbf{M} \wedge \frac{d\mathbf{M}}{dt}\right),$$

where $\mathbf{H}_{\text{eff}} = -\delta F/\mathcal{V}\delta\mathbf{M}$ is the effective magnetic field

$$\boxed{\mathbf{H}_{\text{eff}} = \frac{K}{M_0}\left[\Gamma \sin\left(\omega_J t - r\frac{M_y}{M_0}\right)\hat{\boldsymbol{y}} + \frac{M_z}{M_0}\hat{\boldsymbol{z}}\right]}$$

$$\boxed{r = x v_{\text{so}}/v_F}$$

Complicated regime of the magnetic dynamics :

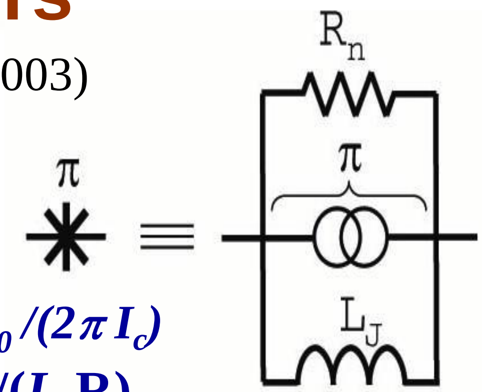


For more details – see (F. Konschelle and A. Buzdin, PRL, 2009).

Complementary Josephson logic

RSFQ-logic using π-shifters

A.V.Ustinov, V.K.Kaplunenko. Journ. Appl. Phys. **94**, 5405 (2003)

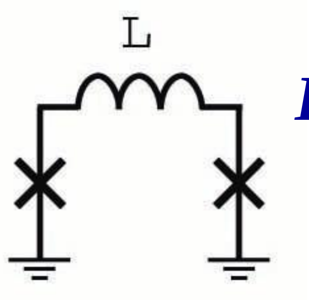


$L_J = \Phi_0/(2\pi I_c)$

$\tau \sim 1/(I_c R)$

RSFQ- logic: Rapid Single Quantum logic

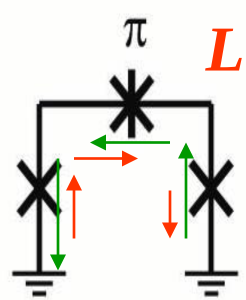
Conventional RSFQ-cell



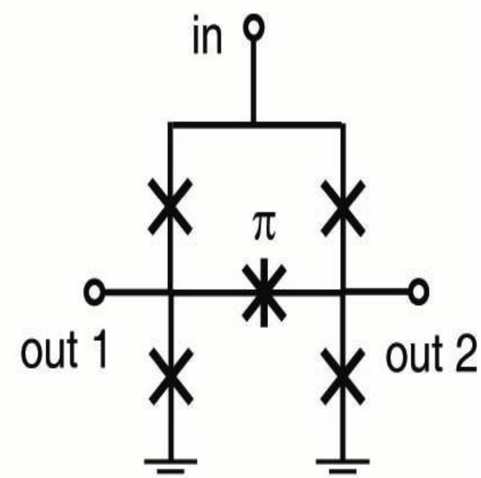
$LI_c > \Phi_0$

Fluxon memorizing cell

RSFQ -*π*– cell



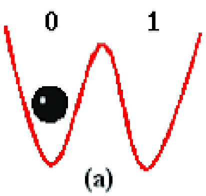
$L \rightarrow 0$



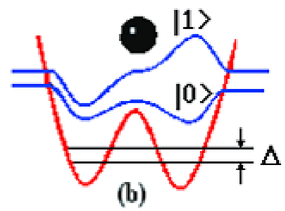
π-RSFQ –***Toggle Flip-Flop***

To operate at 20 GHz clock rate
$I_c R$~50 μV has to be
We have $I_c R$ > 0.1 μV for the present

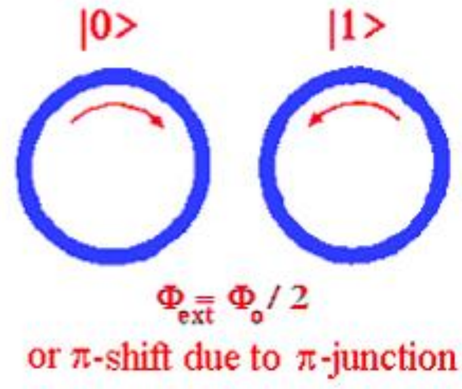
Superconducting phase qubit

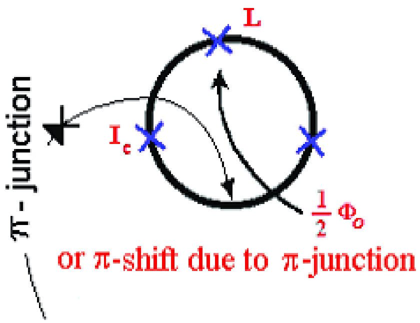


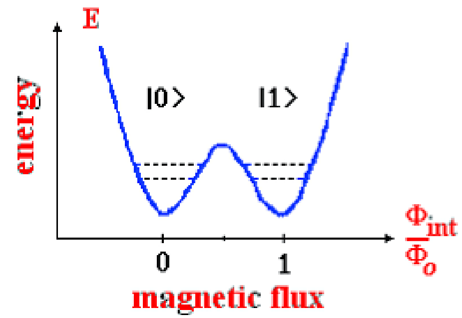
Digital bit

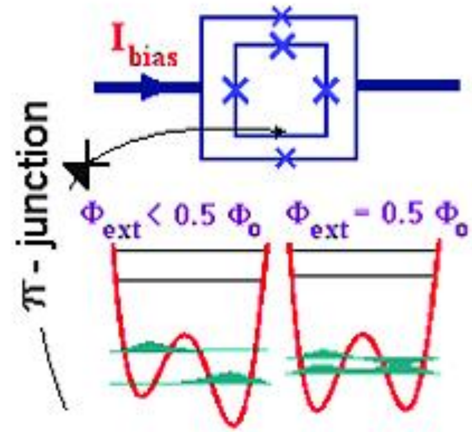


Quantum bit









qubit operation

Conclusions

- **Superconductor-ferromagnet heterostructures permit to study superconductivity under huge exchange field ($h \gg T_c$).**

- **The π-junction realization in S/F/S structures is quite a general phenomenon.**

- **Domain wall superconductivity. Spin - valve effects.**

- **Inversion of the proximity effect in atomic F/S/F structures.**

- **The BIS magnets provide a mechanism of the realization of the novel φ_0 - junctions with the very special properties.**
- **In these φ_0 - junctions a direct (linear) coupling between superconductivity and magnetism is realized. They are the natural phase shifter**

Refs.: **Magnetic superconductors-** M. Kulic and A. Buzdin in **Superconductivity,** Springer, 2008 (eds. Benneman and Ketterson).
S/F proximity effect - A. Buzdin, Rev. Mod. Phys. (2005).