

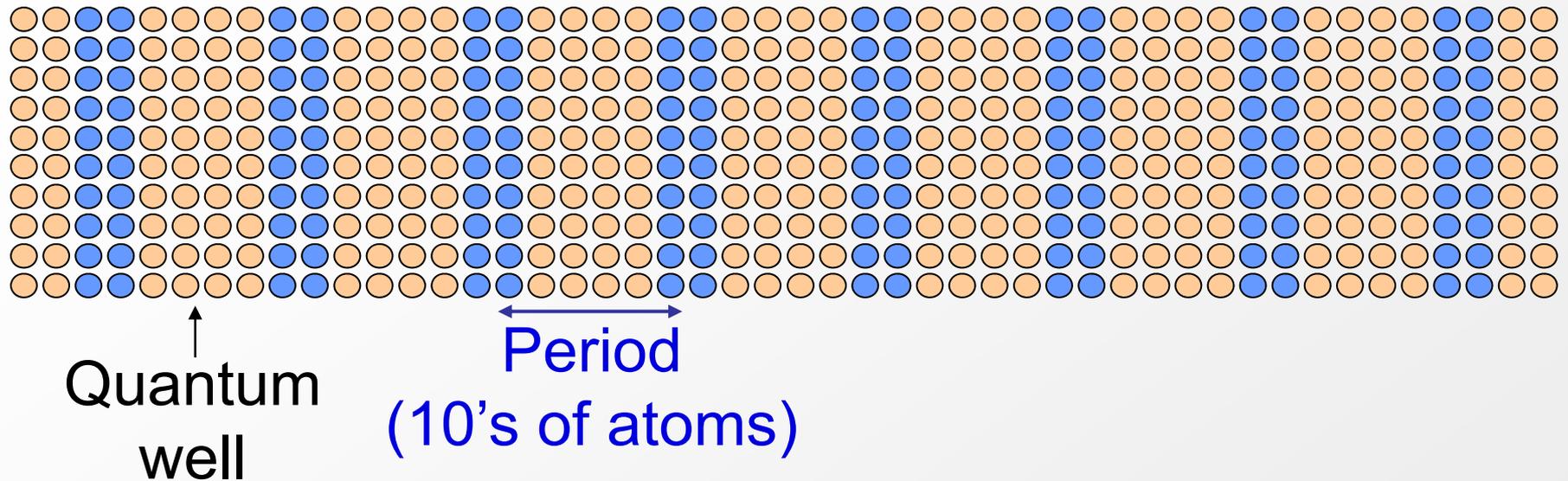
УПРАВЛЕНИЕ ВЫСОКОЧАСТОТНОЙ ДИНАМИКОЙ ЗАРЯДА В ПОЛУПРОВОДНИКОВЫХ СВЕРХРЕШЕТКАХ.

Александр Баланов

Collaborators

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O. Makarovsky (University of Nottingham, UK)
M. Gaifullin (Loughborough University, UK)
L. Eaves (University of Nottingham, UK)
F.V. Kusmartsev (Loughborough University, UK)

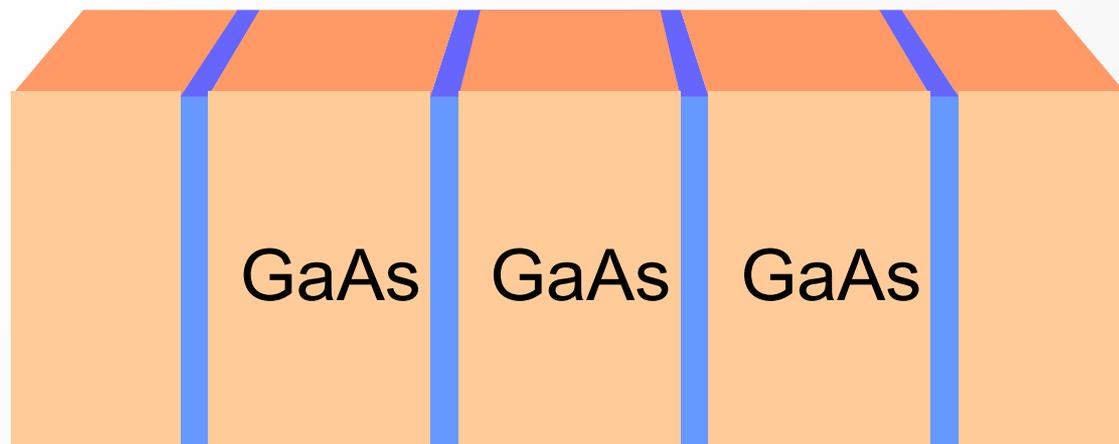
SEMICONDUCTOR SUPERLATTICE



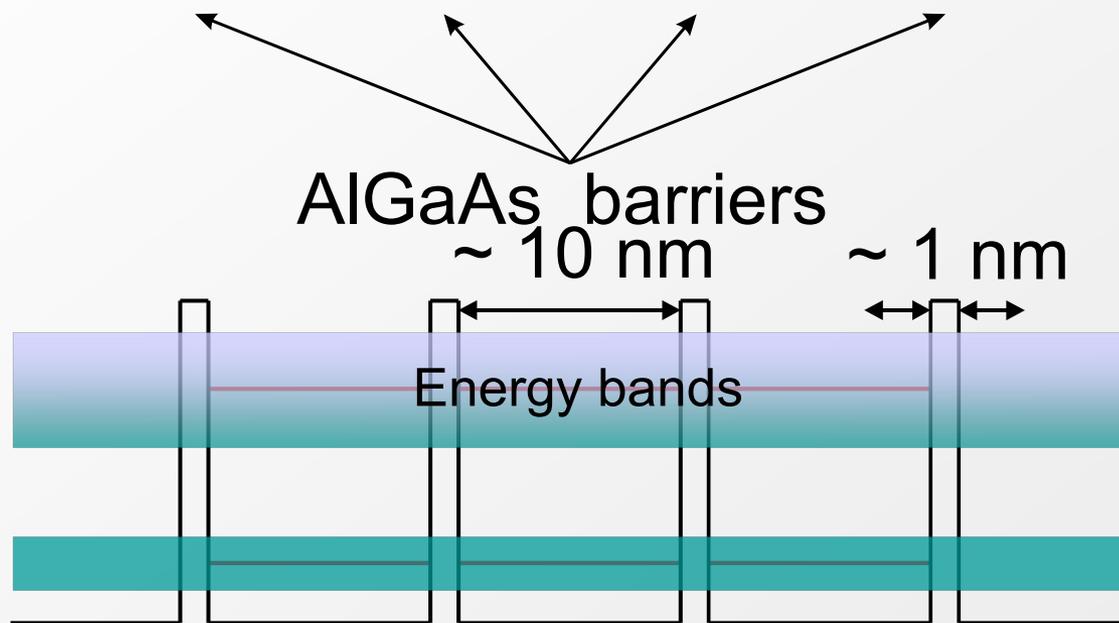
Chain of quantum wells forms a 1D periodic structure similar to a crystal lattice

But since the lattice period is much longer than for any natural crystal, this type of structure is known as a **“Superlattice”**

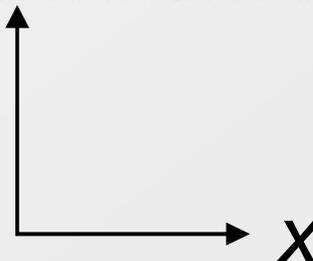
ENERGY BAND STRUCTURE



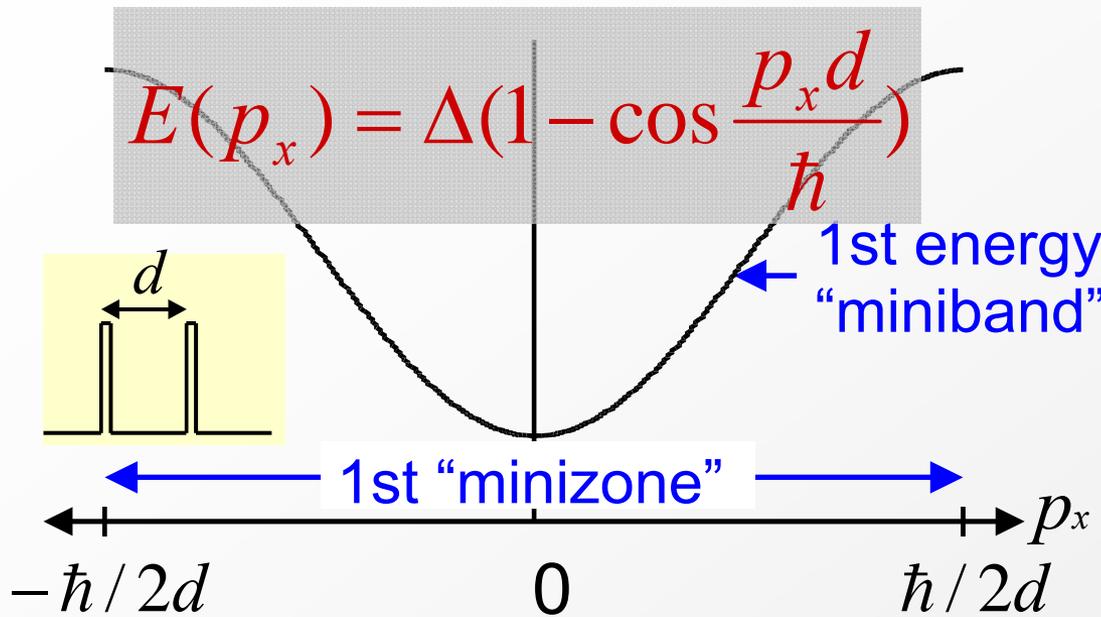
Superlattices usually contain ~ 10 - 100 quantum wells



Potential energy of conduction electron

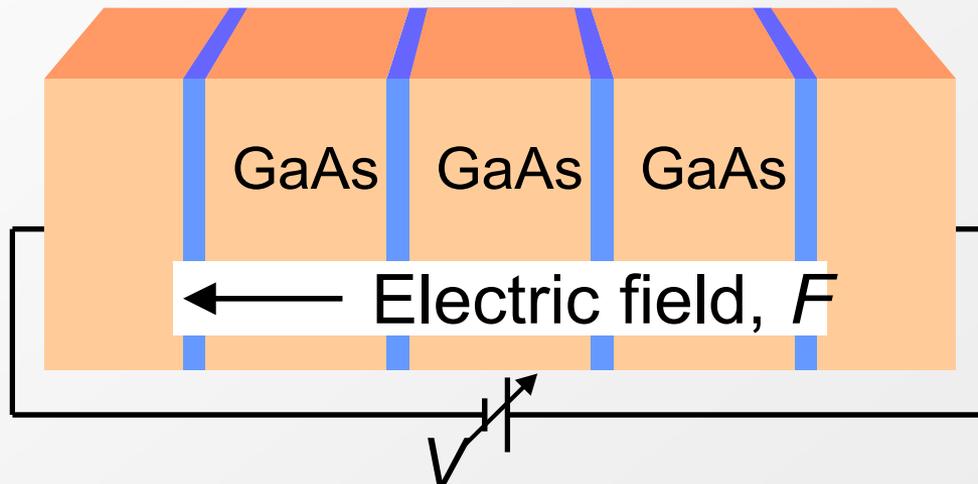


ELECTRON MOTION IN 1st MINIBAND



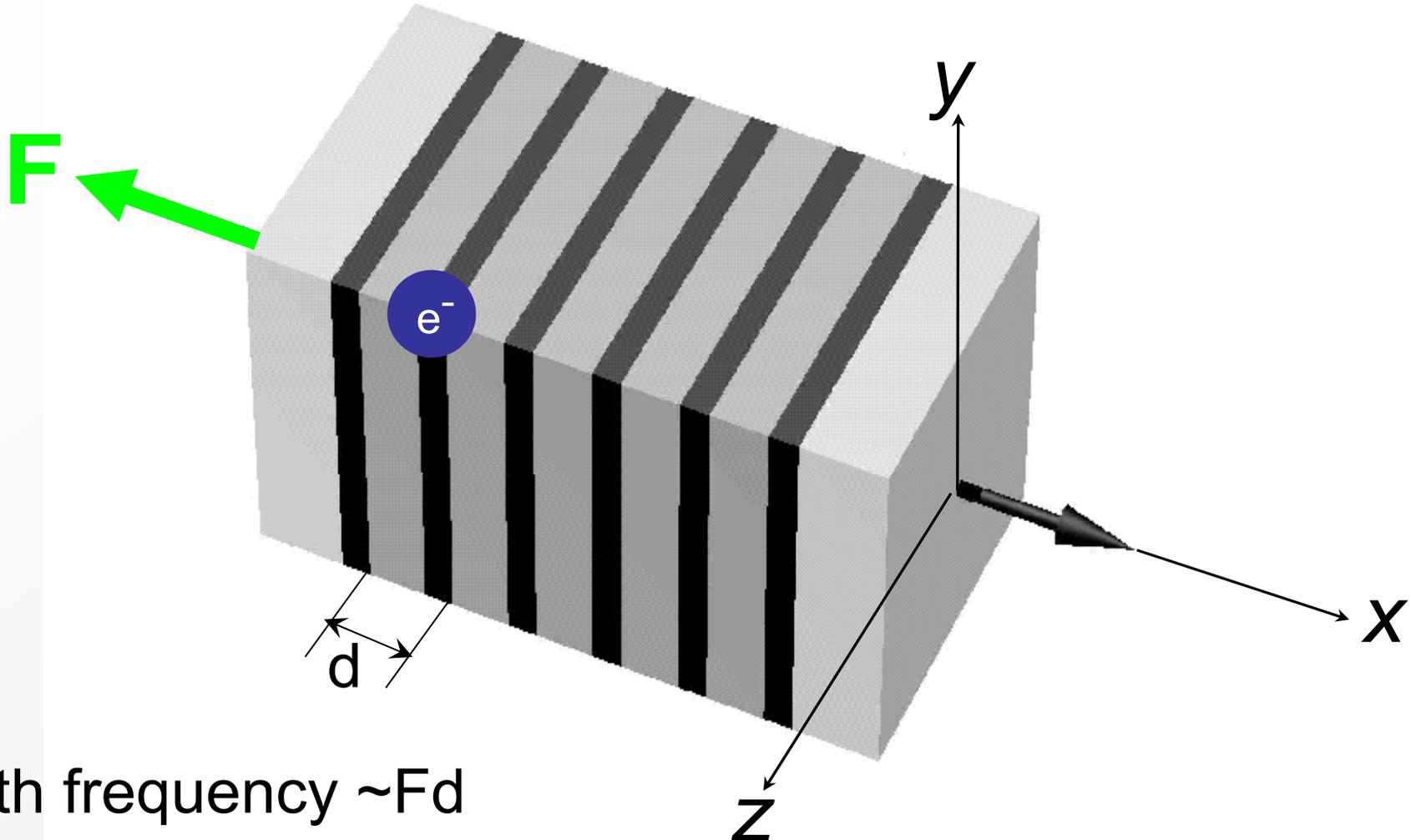
$$eF = \frac{dp_x}{dt}$$

$$v_x = \frac{dE}{dp_x}$$



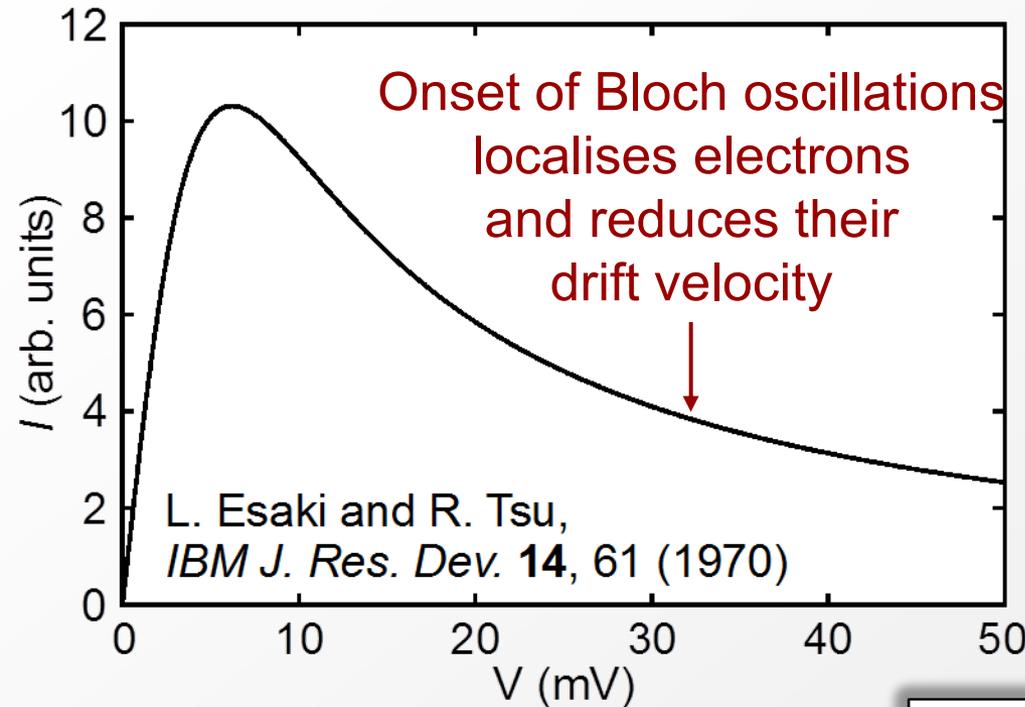
Bloch oscillations

A constant **electric field** gives regular Bloch oscillations



Drift velocity of electron, v_d

$I \sim v_d$

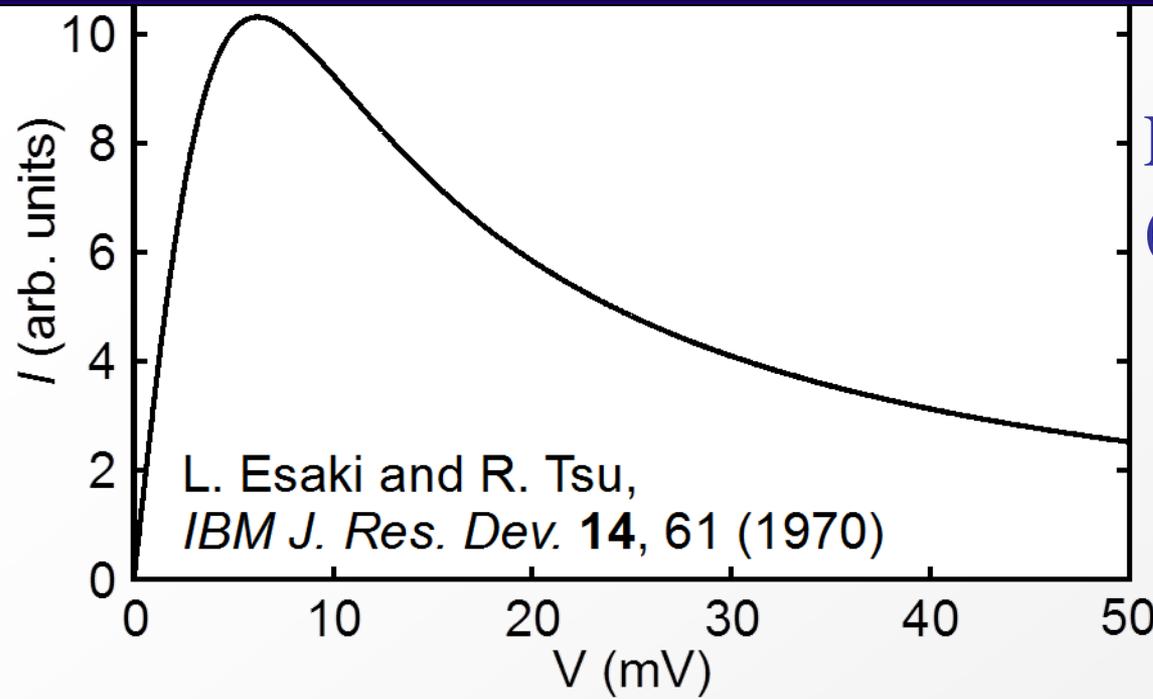


$$eF = \frac{dp_x}{dt}$$

$$v_x = \frac{dE}{dp_x}$$

$$E(p_x) = \Delta \left(1 - \cos \frac{p_x d}{\hbar} \right)$$

$$v_d(F) = \frac{1}{\tau} \sum \int_0^{\infty} v_x(t) \exp(-t/\tau) dt$$

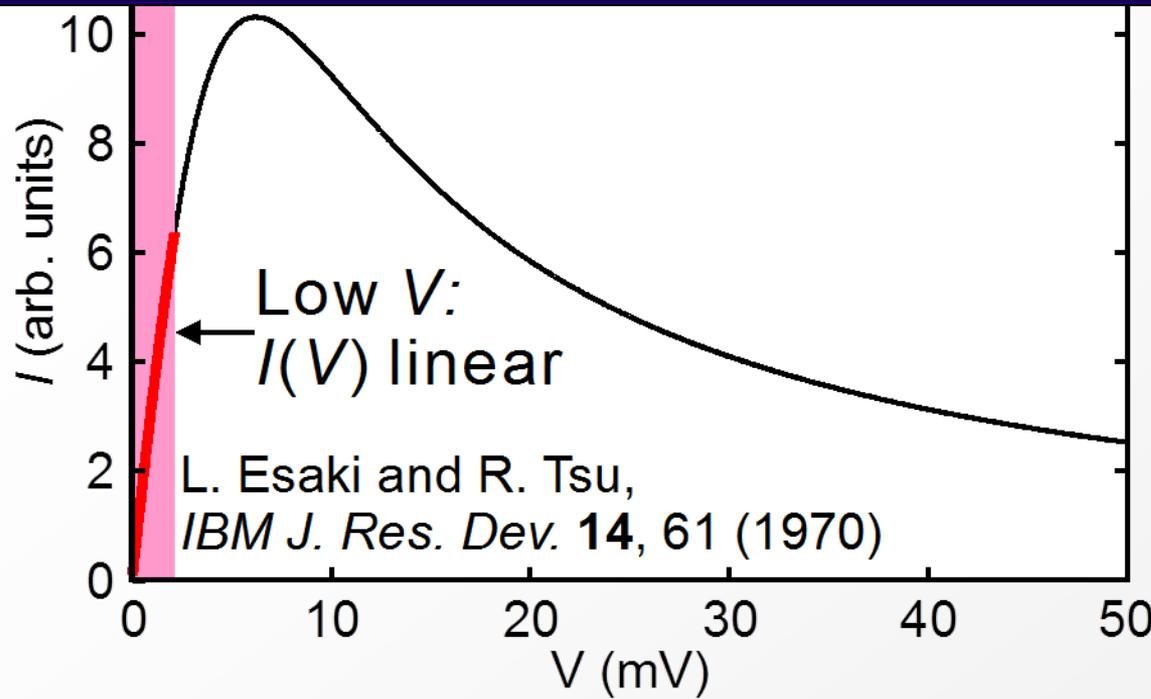


Esaki, Tsu I-V curve
(*IBM J. Res. Dev* 1970)

$$eF = \frac{dp_x}{dt}$$

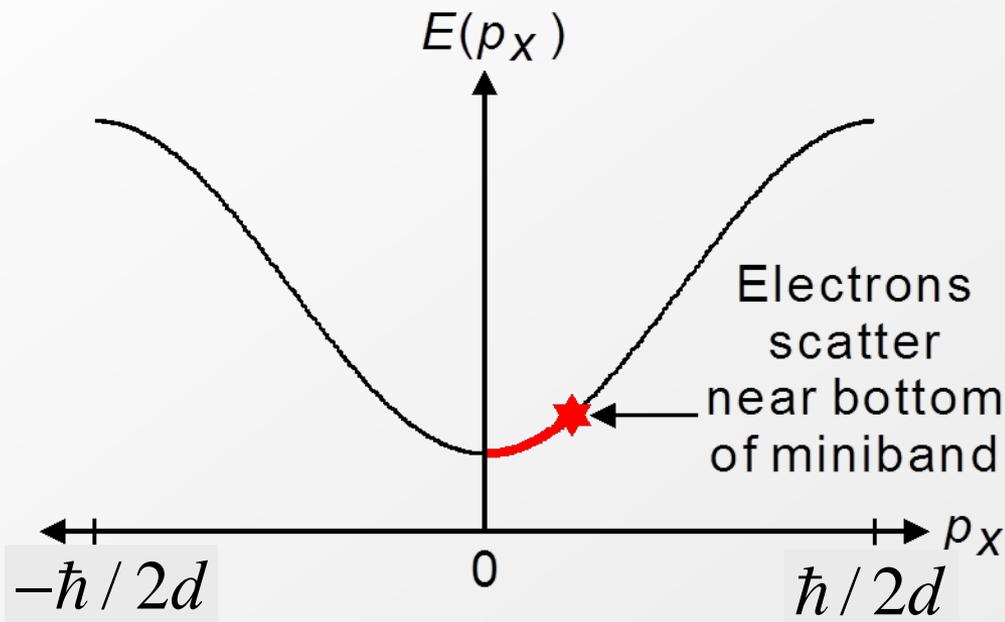
$$v_x = \frac{dE}{dp_x}$$

$$E(p_x) = \Delta \left(1 - \cos \frac{p_x d}{\hbar} \right)$$

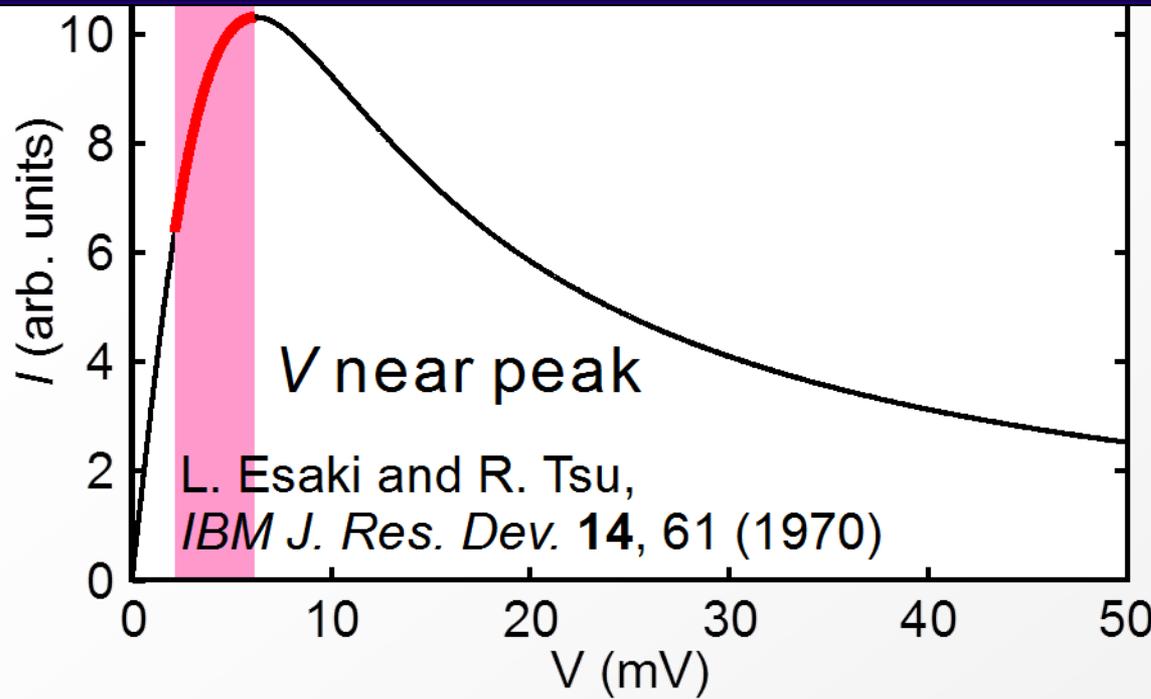


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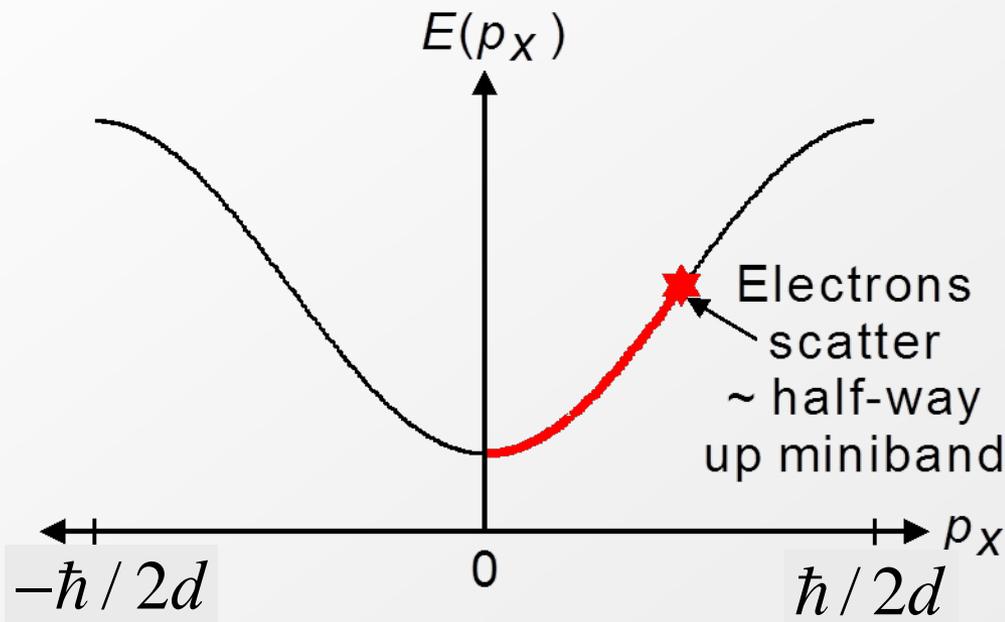


$$E(p_x) = \Delta \left(1 - \cos \frac{p_x d}{\hbar} \right)$$

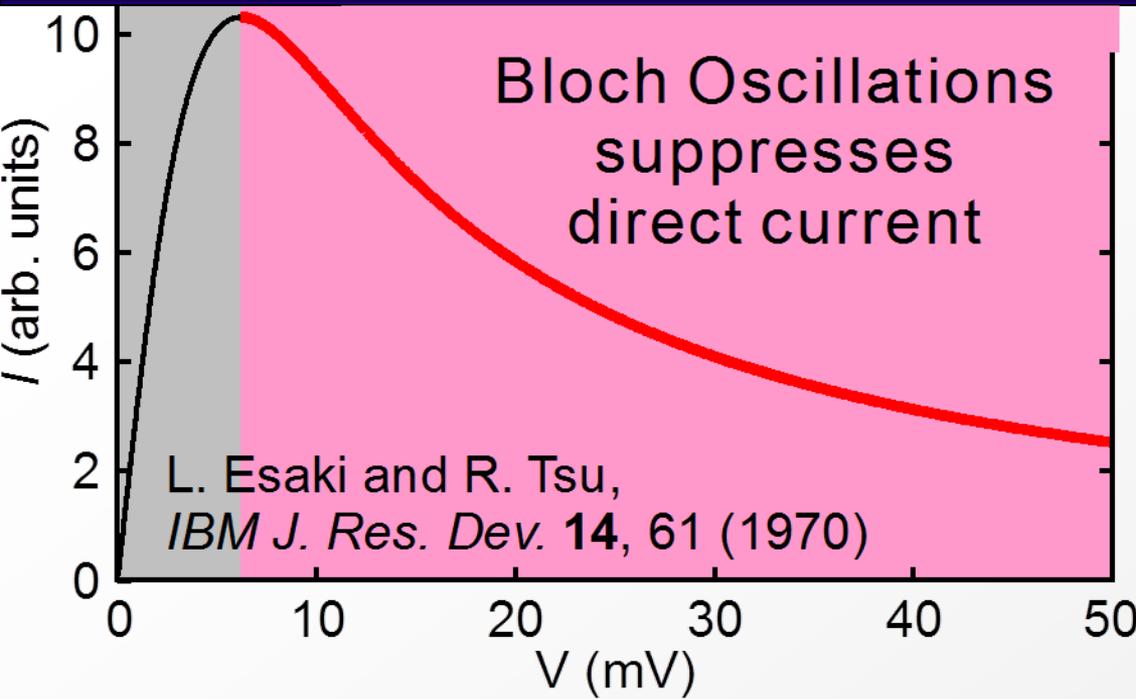


$$eF = \frac{dp_x}{dt}$$

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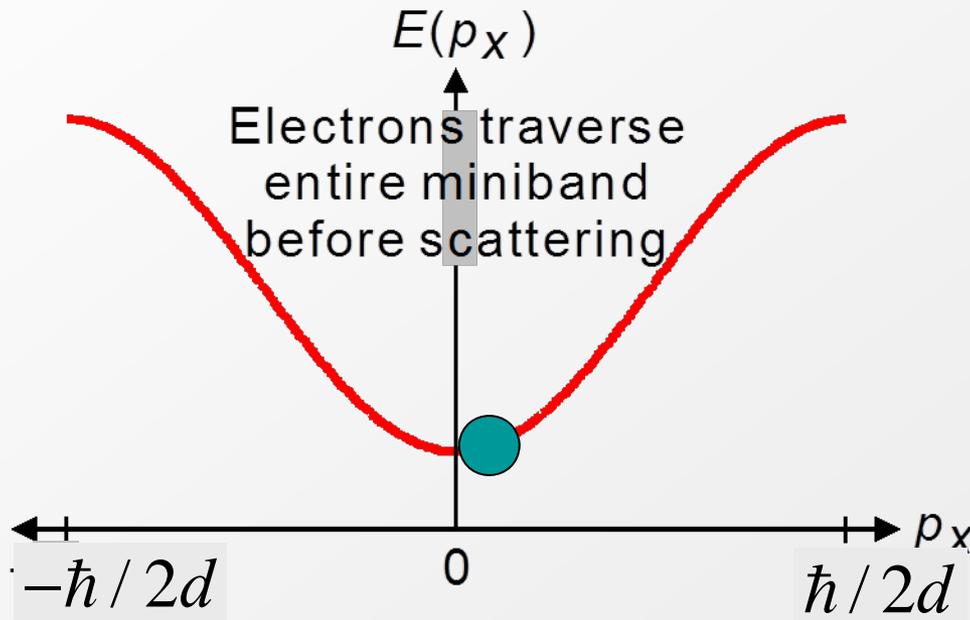


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$$eF = \frac{dp_x}{dt}$$

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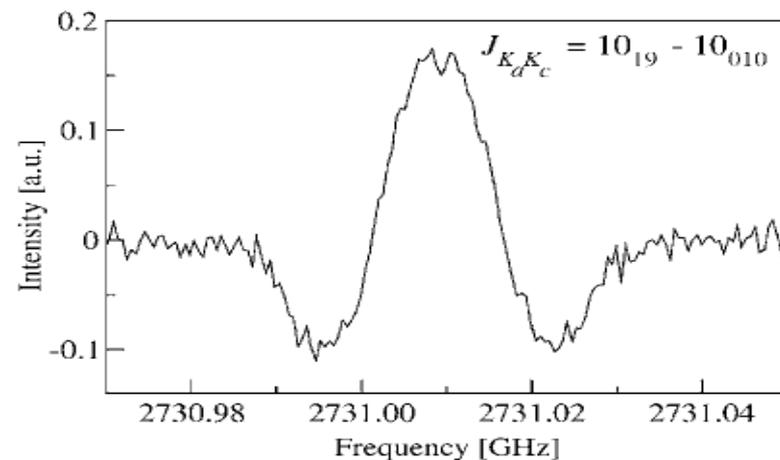
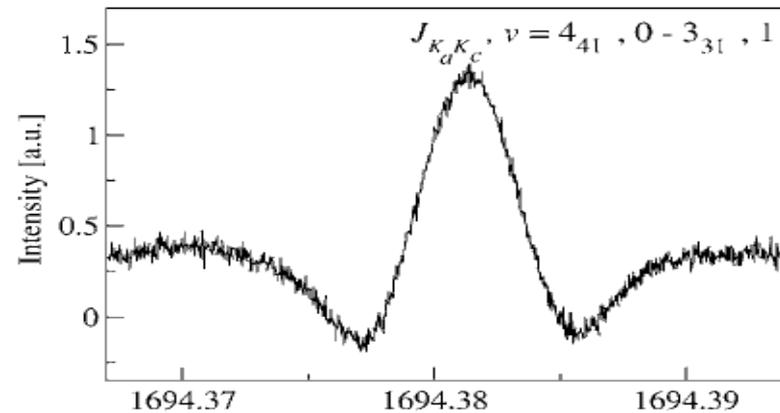


$$E(p_x) = \Delta \left(1 - \cos \frac{p_x d}{\hbar} \right)$$

THz applications of SL

- C.P. Edres et al, Rev. Sci. Instr. 78 043106 (2007)

Frequency multiplier in
high-resolution THz
spectroscopy
(room temperature)

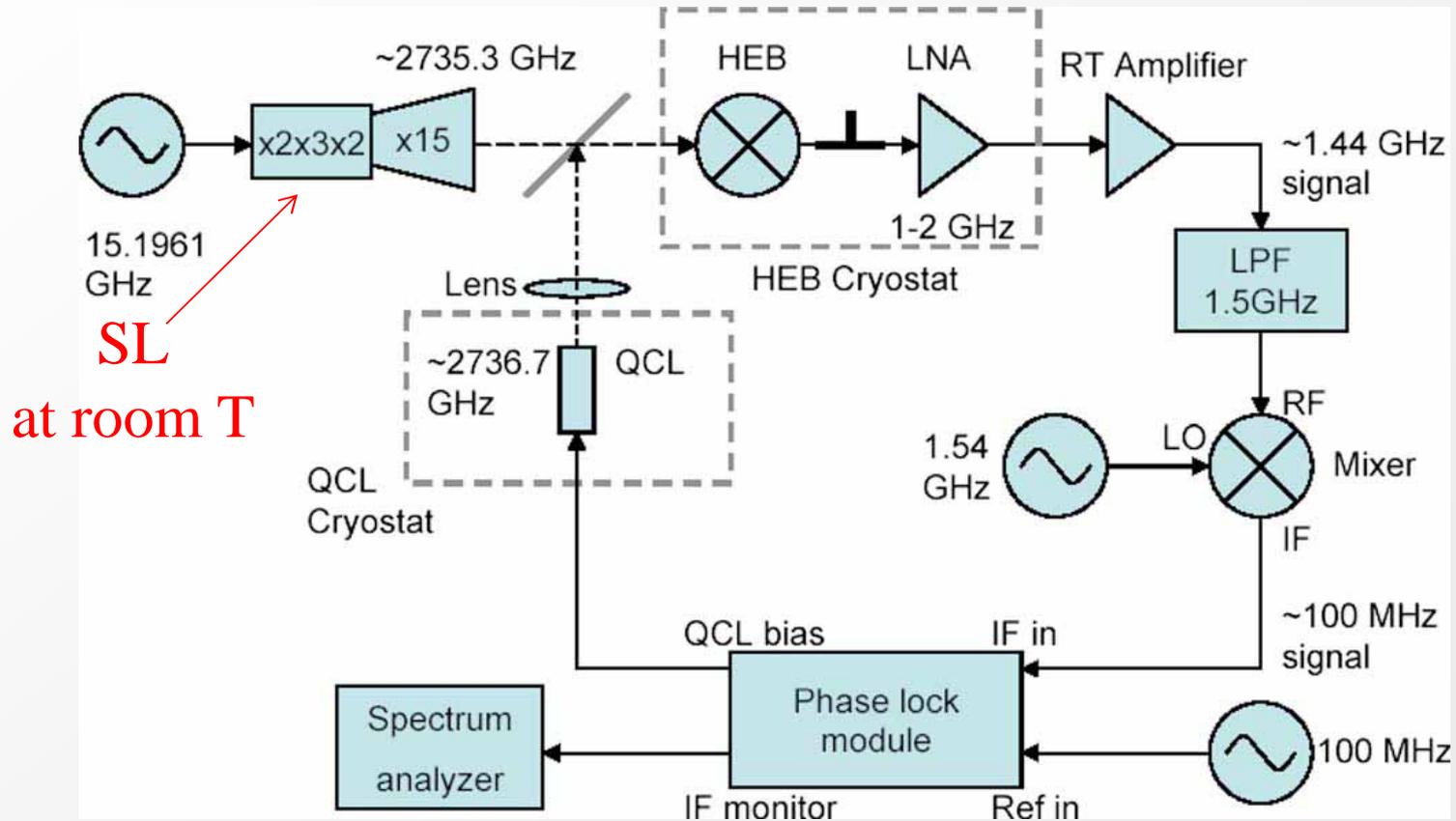


Spectra of ND₂H (upper panel) and D₂O (lower panel)

THz applications of SL

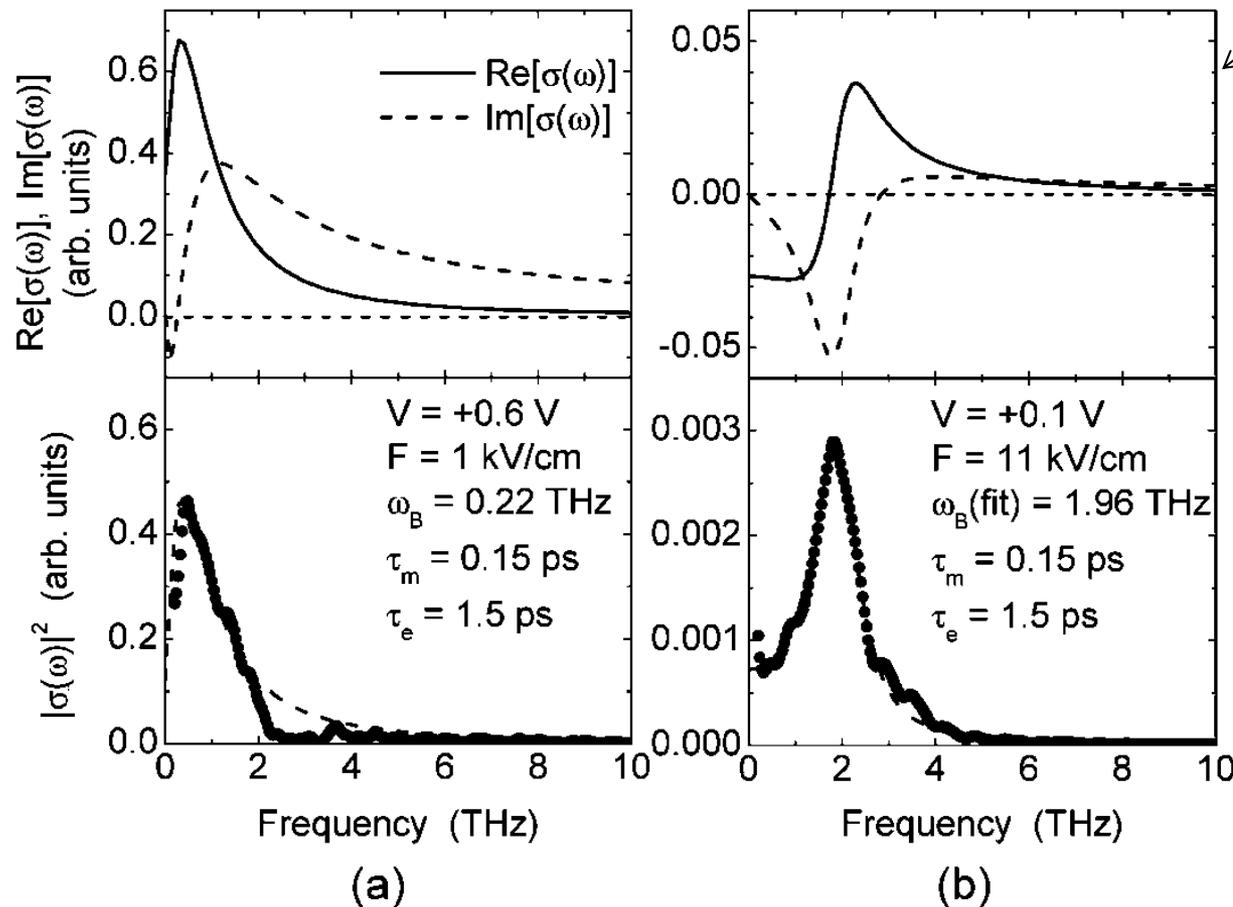
- P. Khosropanahet al, Opt. Lett. 34, 2958 (2009)

Phase locking of a 2.7 THz quantum cascade laser



THz applications of SL

- Y. Shimada et al, Phys. Rev. Lett. 90 046806 (2003)

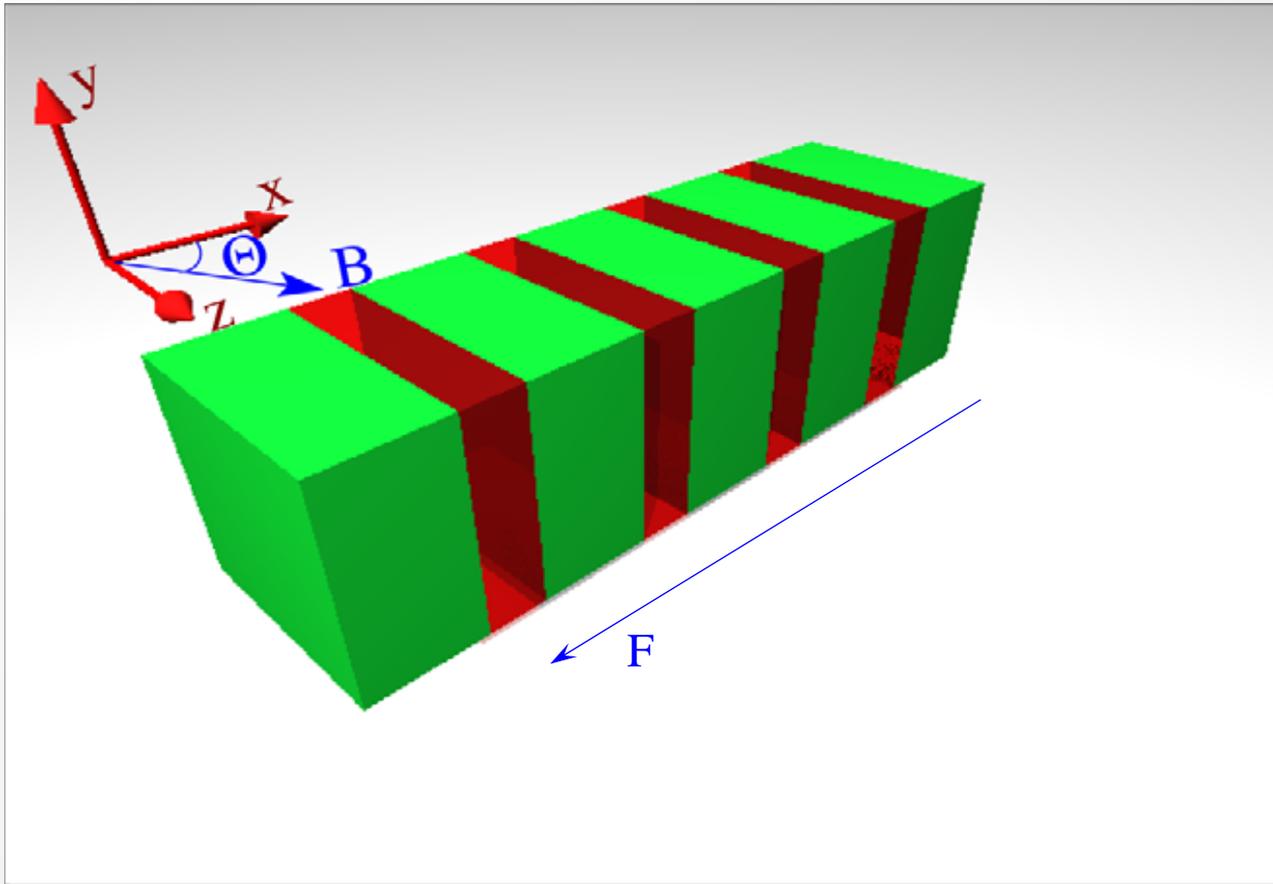


← absorption:

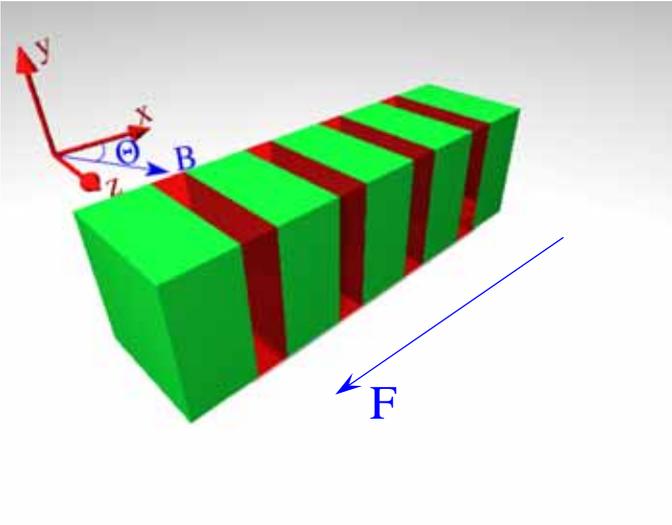
S.A. Ktitorov, G.S. Simin, V.Ya. Sindalovskii, Sov. Phys. Solid State 13 (1971) 1872.

THz emission under optical injection ($T \sim 10 \text{ K}$)

How to control electron dynamics? using magnetic field



MODEL EQUATIONS I:



Electric field: $\mathbf{F}(-F, 0, 0)$

Magnetic field: $\mathbf{B}(B\cos\Theta, 0, B\sin\Theta)$

Dispersion relation:

$$E(\mathbf{p}) = \frac{\Delta}{2} \left(1 - \cos \frac{p_x d}{\hbar}\right) + \frac{1}{2m^*} (p_y^2 + p_z^2)$$

Electron impulse: $\mathbf{p}(p_x, p_y, p_z)$

Balance equation: $\dot{\mathbf{p}} = -e\mathbf{F} - e(\nabla_{\mathbf{p}} E \times \mathbf{B})$

$$\dot{p}_x = eF - \omega_{\parallel} p_y \tan \Theta$$

$$\dot{p}_y = m^* \omega_{\parallel} \tan \Theta \frac{d\Delta}{2\hbar} \sin\left(\frac{p_x d}{\hbar}\right) - \omega_{\parallel} p_z$$

$$\dot{p}_z = \omega_{\parallel} p_y$$

Cyclotron frequency:

$$\omega_c = eB / m^*$$

$$\omega_{\parallel} = \omega_c \cos \Theta$$

MODEL EQUATIONS II:

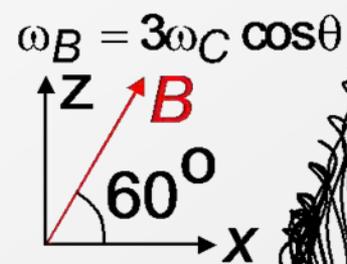
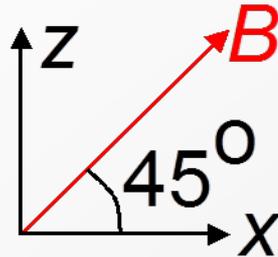
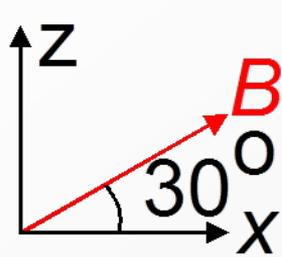
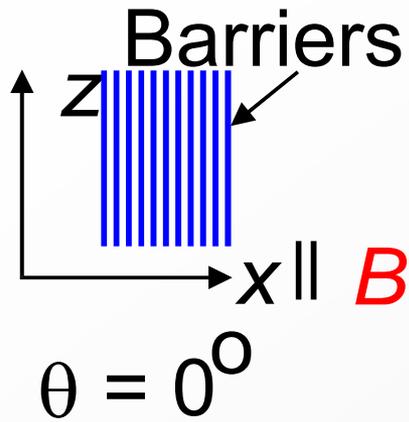
$$\ddot{p}_z + \omega_{\parallel}^2 p_z = -\frac{m^* \omega_c^2 \Delta \sin 2\Theta}{4\hbar} \sin\left(\frac{d \tan \Theta}{\hbar} p_z - \omega_b t\right)$$

All other states can be expressed in terms of p_z :

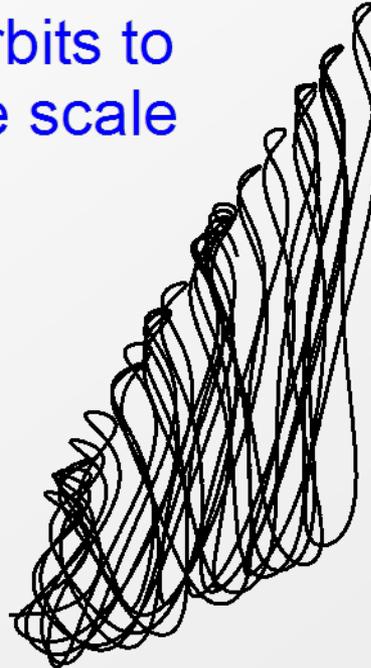
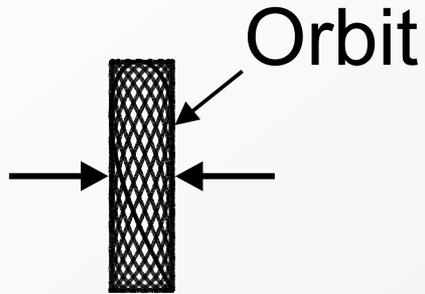
$$p_x = eFt - \dot{p}_z \tan \Theta, \quad p_y = \omega_{\parallel}^{-1} \dot{p}_z$$

$$\dot{x} = \frac{d\Delta}{2\hbar} \sin\left(\frac{d \tan \Theta}{\hbar} p_z - \omega_b t\right), \quad \dot{y} = \frac{\dot{p}_z}{\omega_{\parallel} m^*}, \quad \dot{z} = \frac{\dot{p}_z}{m^*}$$

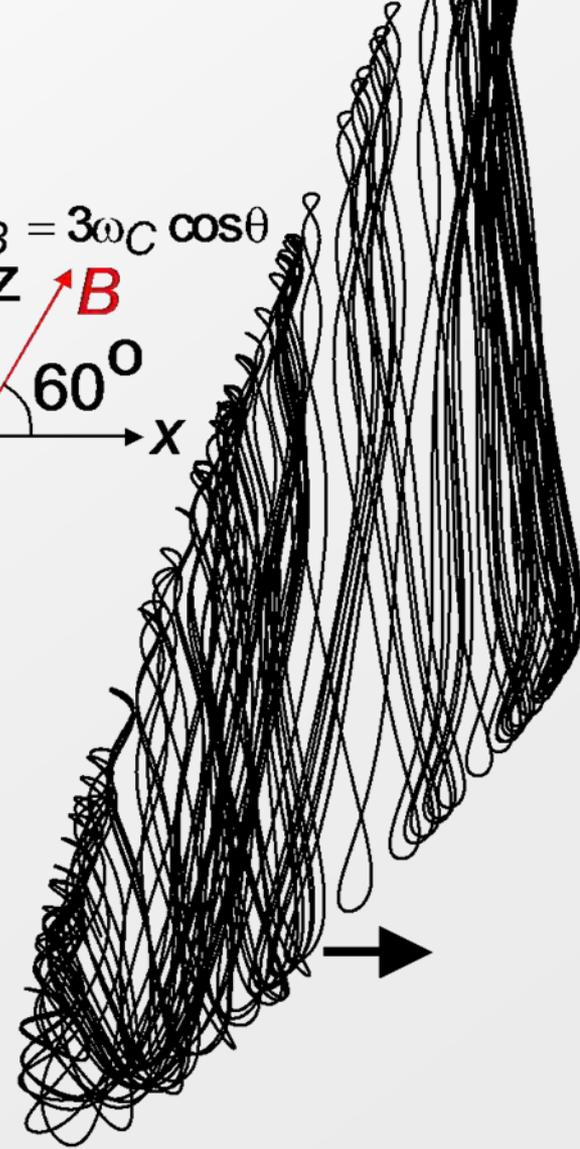
ELECTRON ORBITS:



All orbits to same scale



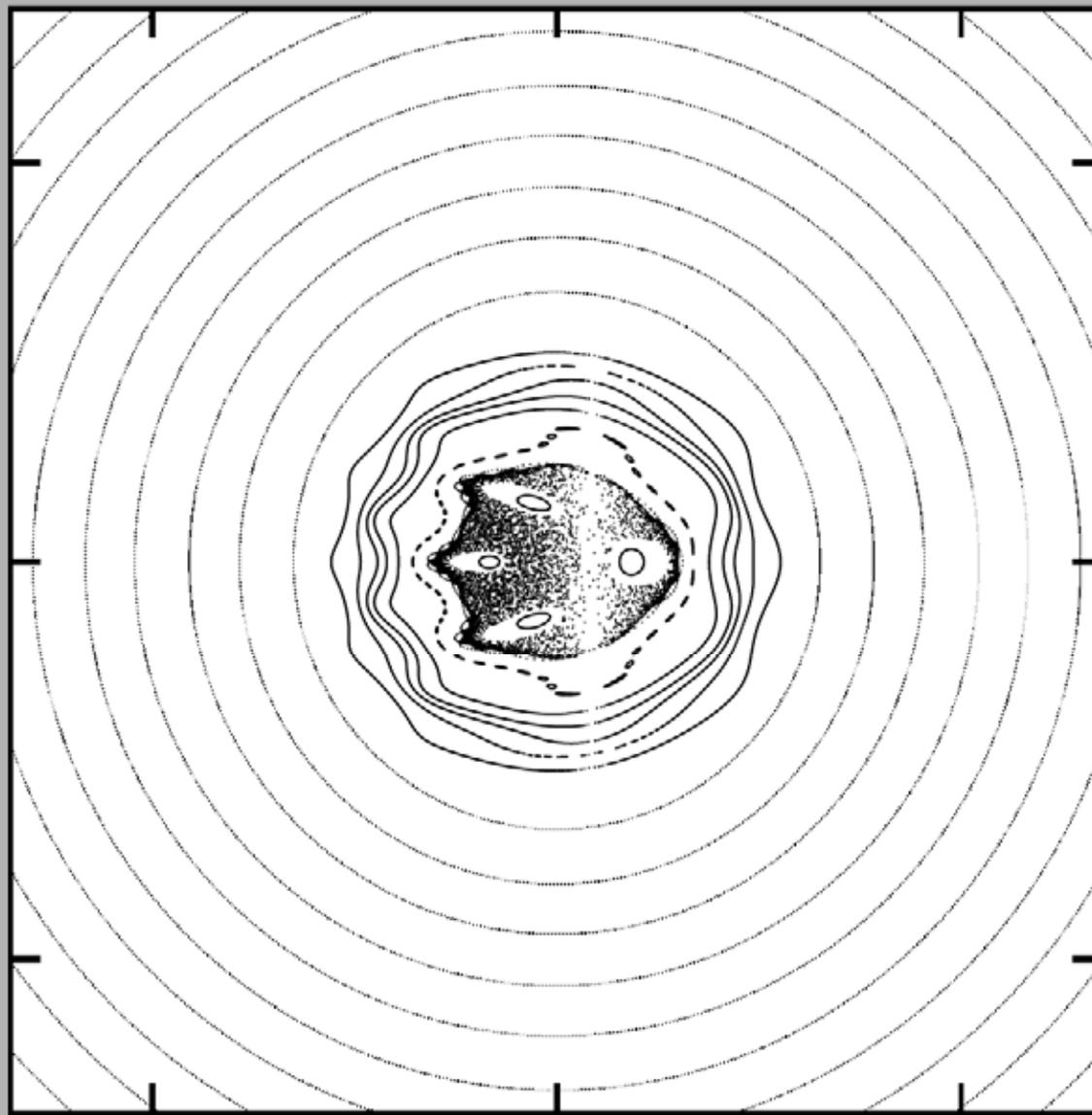
10 SL periods



$$\ddot{p}_z + \hat{\omega}_c^2 p_z = -\frac{m^* \omega_c^2 \Delta \sin 2\Theta}{4\hbar} \sin\left(\frac{d \tan \Theta}{\hbar} p_z - \omega_b t\right)$$

Cyclotron frequency $\sim B$
 Bloch frequency $\sim F$

p_z (arb. units)



p_y (arb. units)

Stochastic web

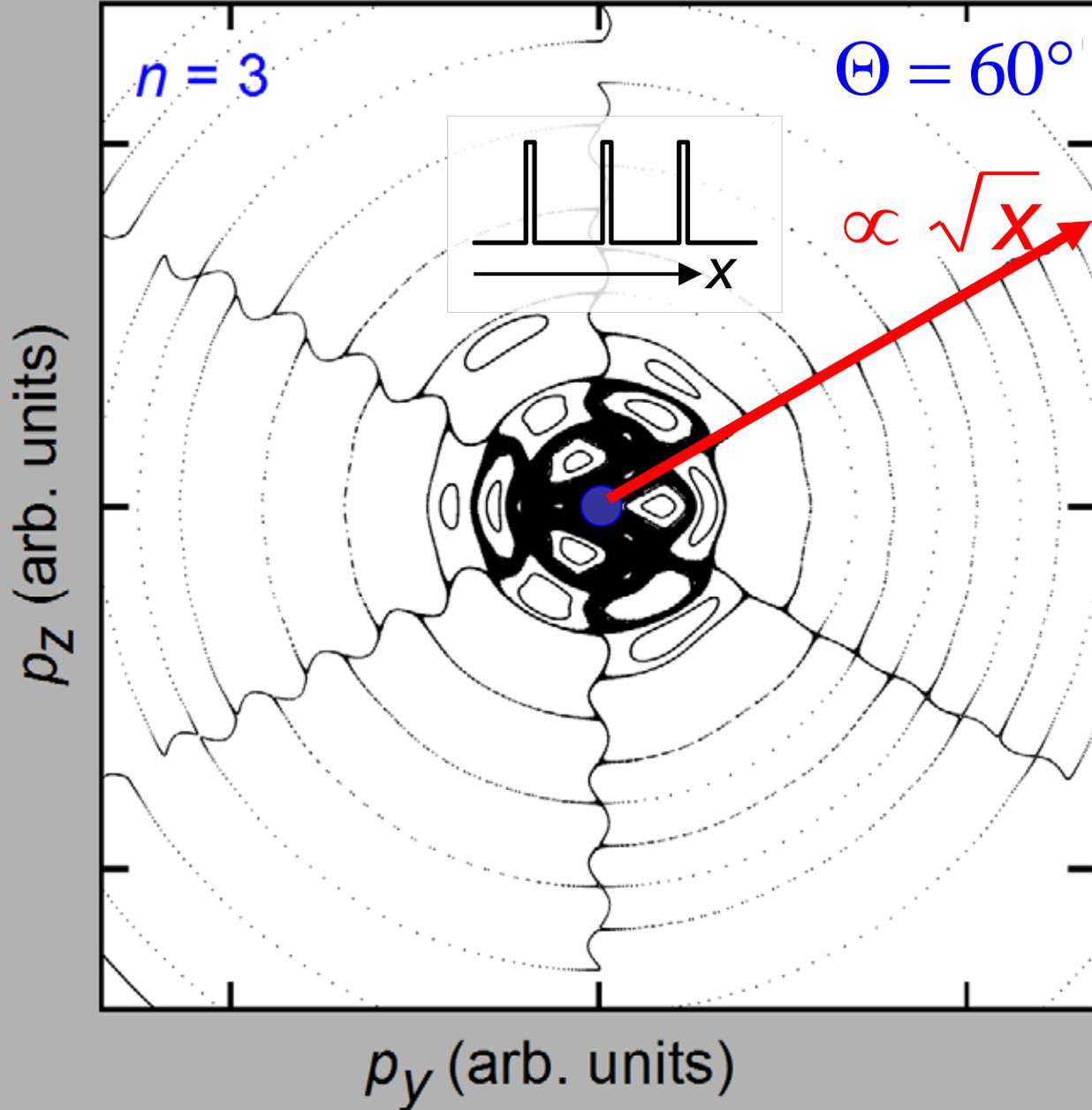
broken
when

$$\omega_b / \omega_c \cos \Theta \neq n$$

integer 

at *most*
field values

Off resonance chaotic orbits are localised  current low



Stochastic web

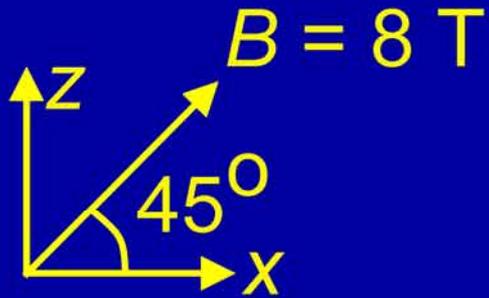
formed
when

$$\omega_b / \omega_c \cos \Theta = n$$

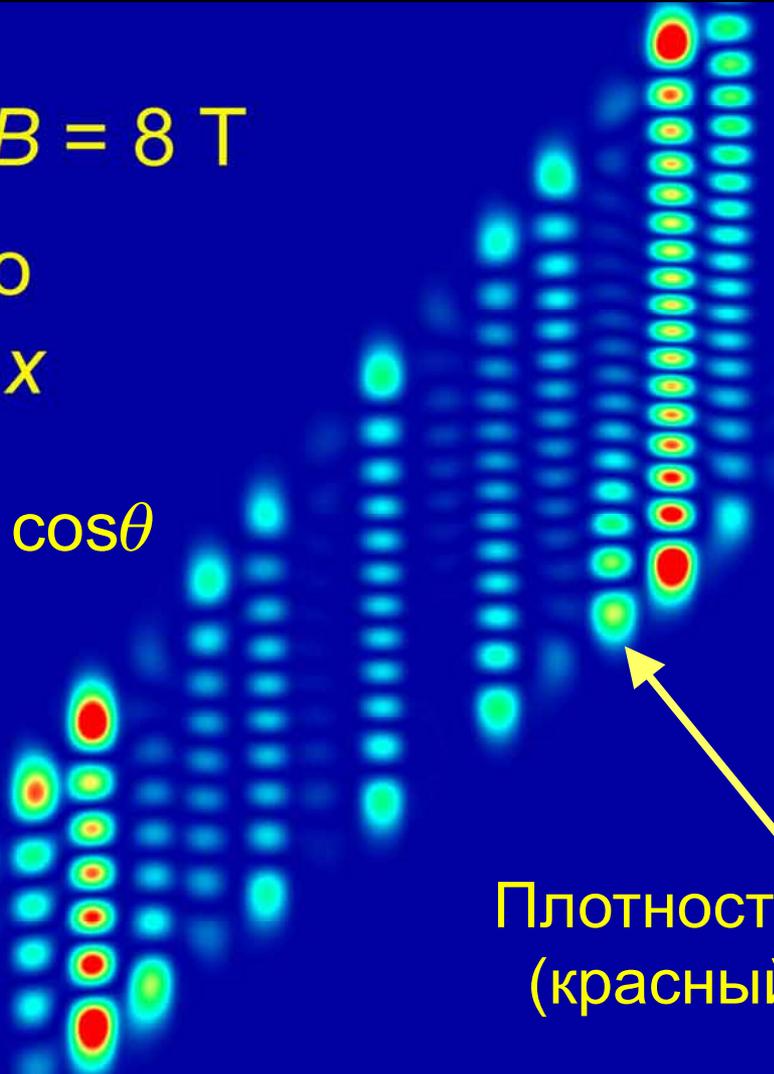
\uparrow $\sim F$ \uparrow $\sim B$ integer \uparrow

i.e. at *discrete*
field values

On resonance chaotic orbits are *unbounded* \rightarrow current high

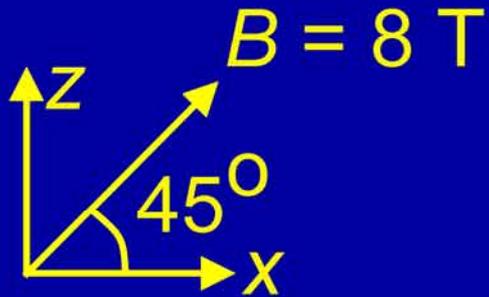


$$\omega_B = \omega_C \cos\theta$$

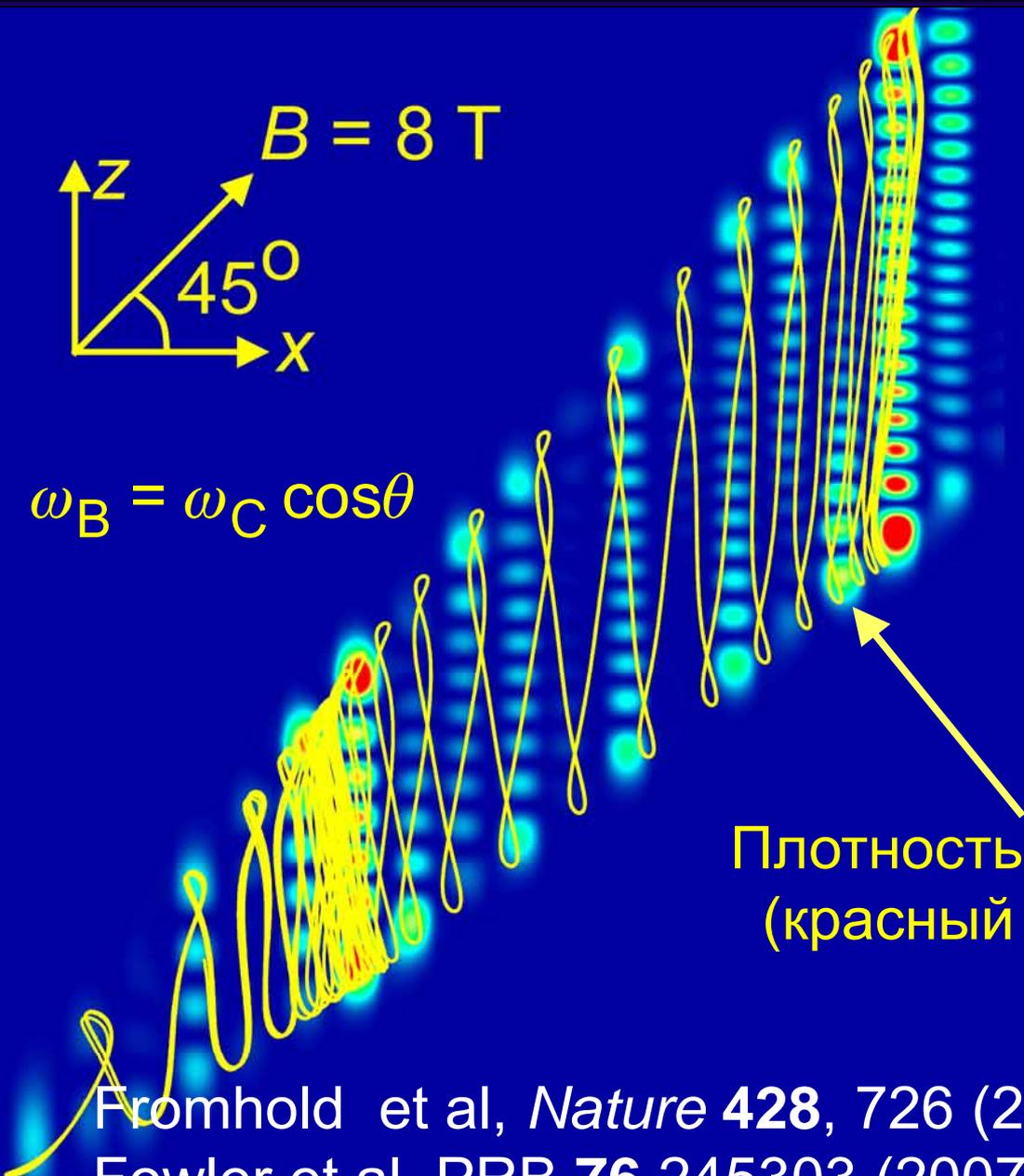


Плотность вероятности
(красный -- высокая)

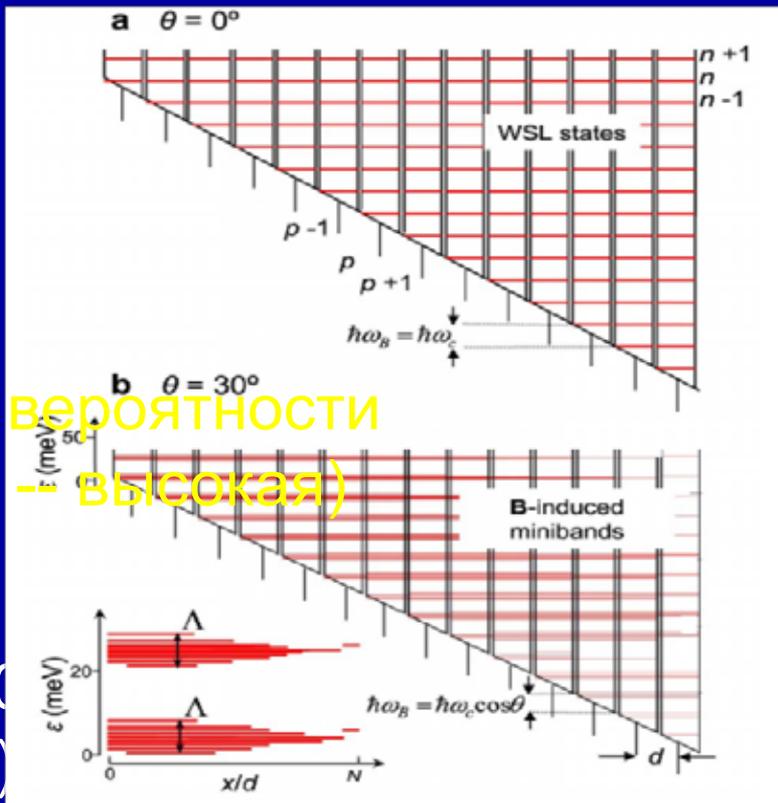
Fromhold et al, *Nature* **428**, 726 (2004),
Fowler et al, *PRB* **76** 245303 (2007)



$$\omega_B = \omega_C \cos\theta$$



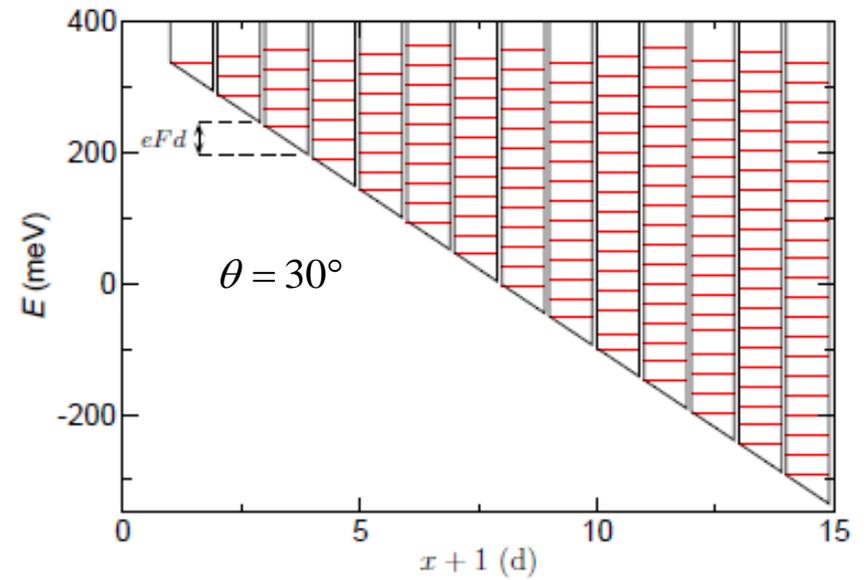
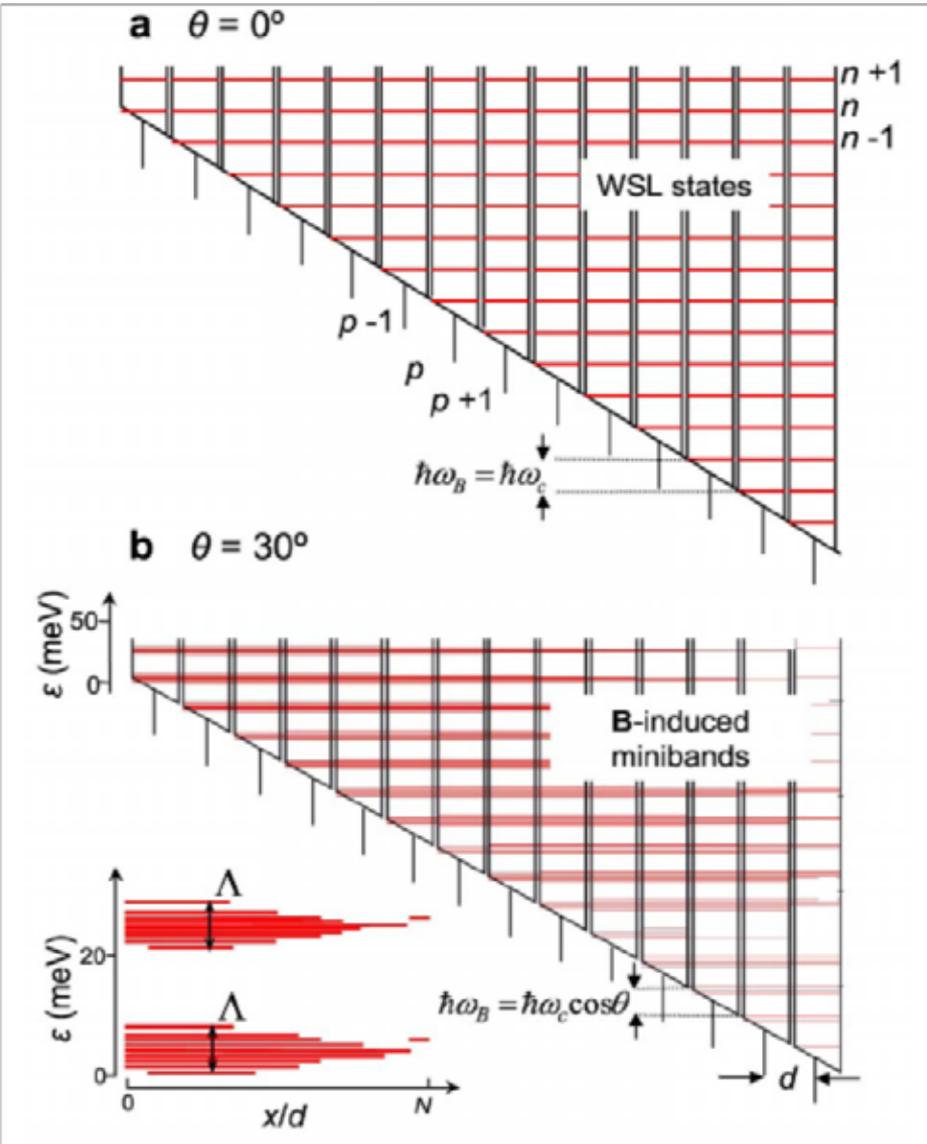
При больших значениях наклонного **B**-поля, собственные значения – **не** Ванье-Штарка и Ландау уровни



Fromhold et al, *Nature* **428**, 726 (2004)
Fowler et al, *PRB* **76** 245303 (2007)

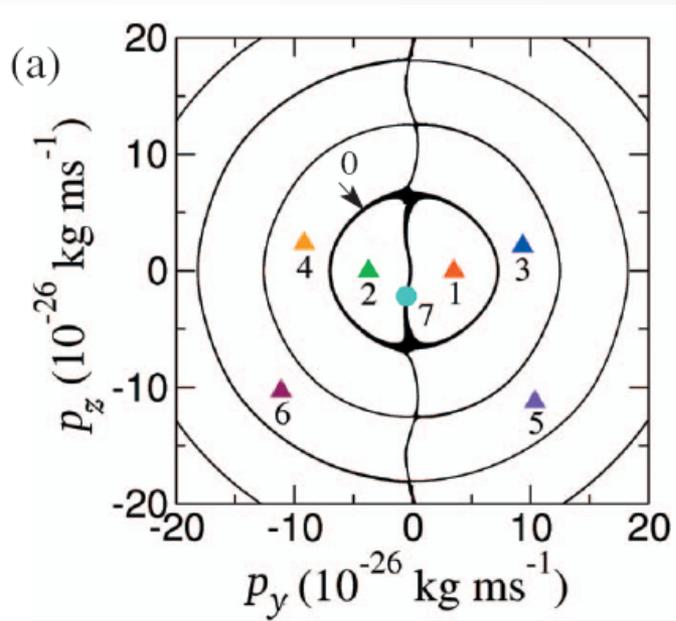
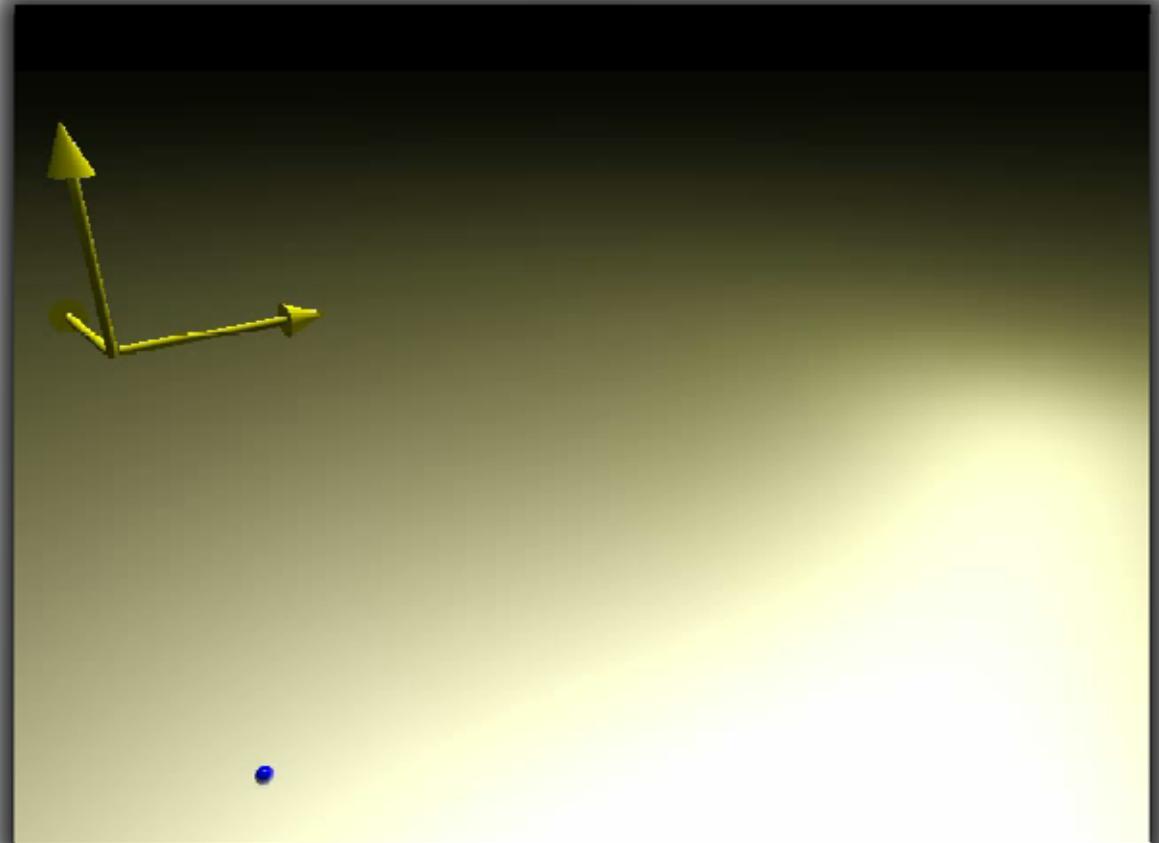
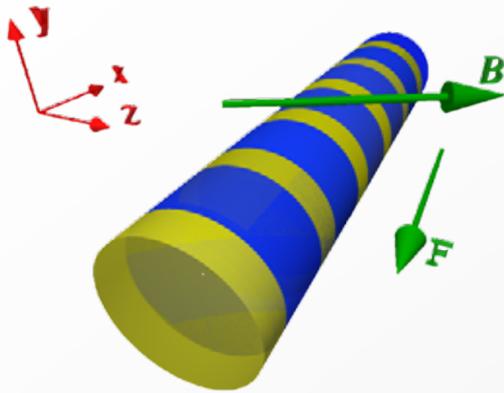
on resonance

out of resonance



726 (2004),
(2007)

Chaotic electron's trajectory at cyclotron-Bloch resonance



$$\omega_b / \omega_c \cos \Theta = 1$$

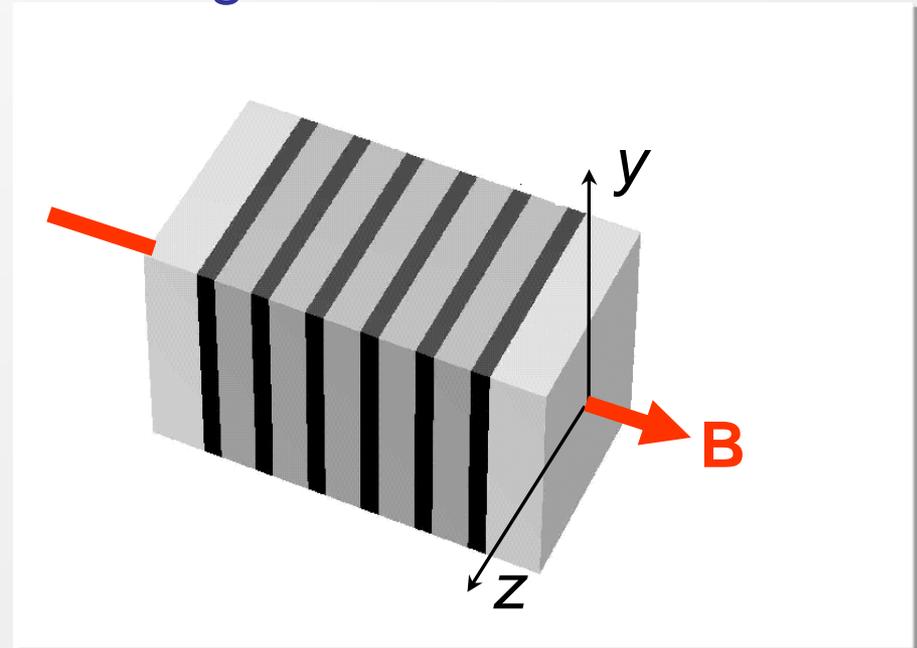
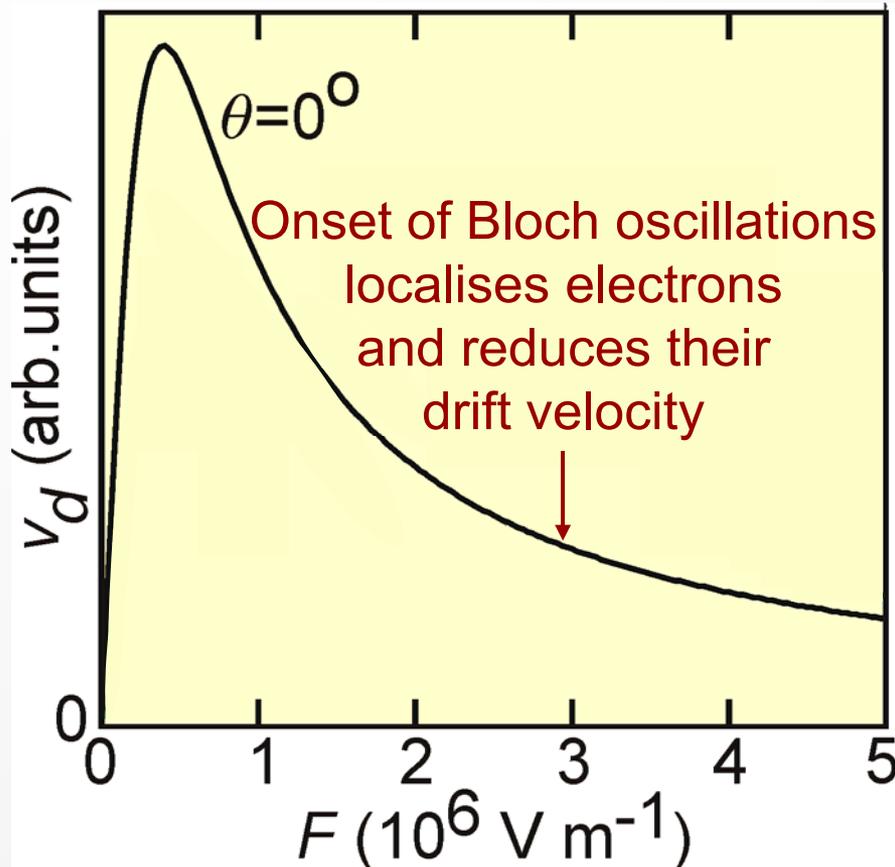
ELECTRON DRIFT VELOCITY

$$v_d(F) = \frac{1}{\tau} \sum \int_0^{\infty} v_x(t) \exp(-t / \tau) dt$$

175 fs



Summation over all starting velocities

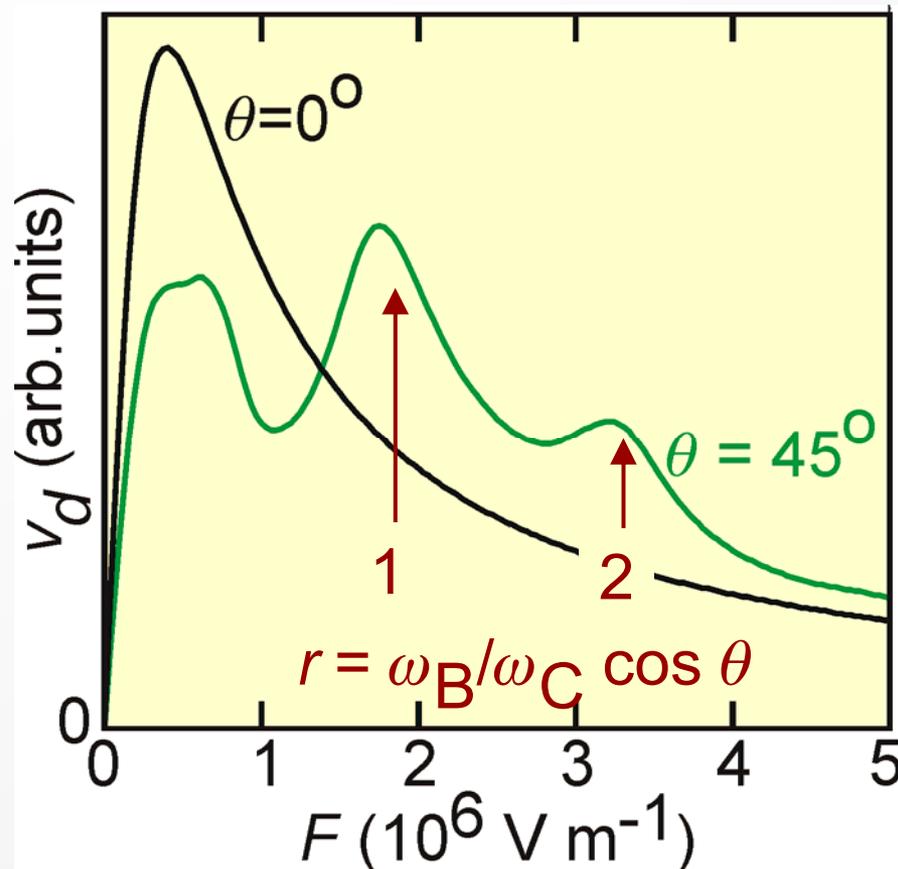


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Fromhold et al., *Phys. Rev. Lett.* **87**, 046803 (2001); *Nature* **428**, 726 (2004)
 Fowler et al., *Phys. Rev. B.* **76**, 245303 (2007)

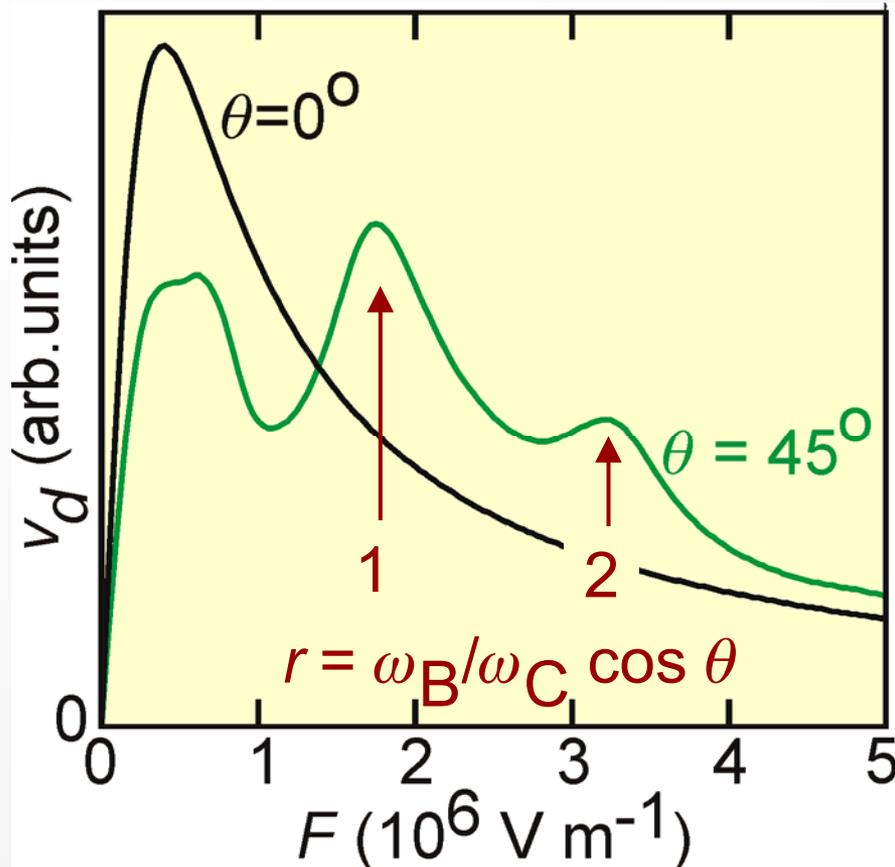
Balanov et al., *Phys. Rev. E.* **77**, 026209 (2008)

Related work on "Ultrafast Fiske Effect" by Kosevich et al., *Phys. Rev. Lett.* **96**, 137403 (2006)

ELECTRON DRIFT VELOCITY

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175 fs



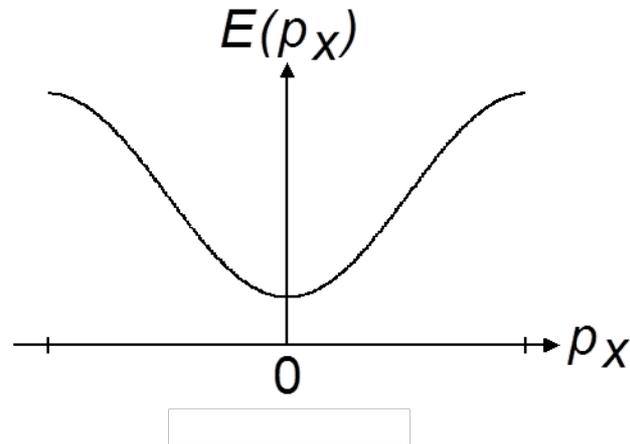
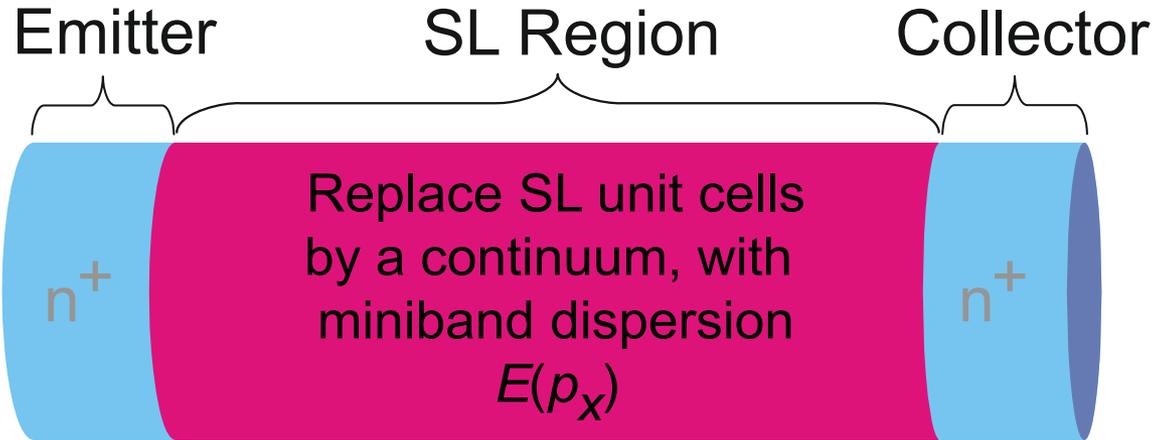
$v_d(F)$ curves were used as a basis for calculating $I(V)$

by self-consistent solution of

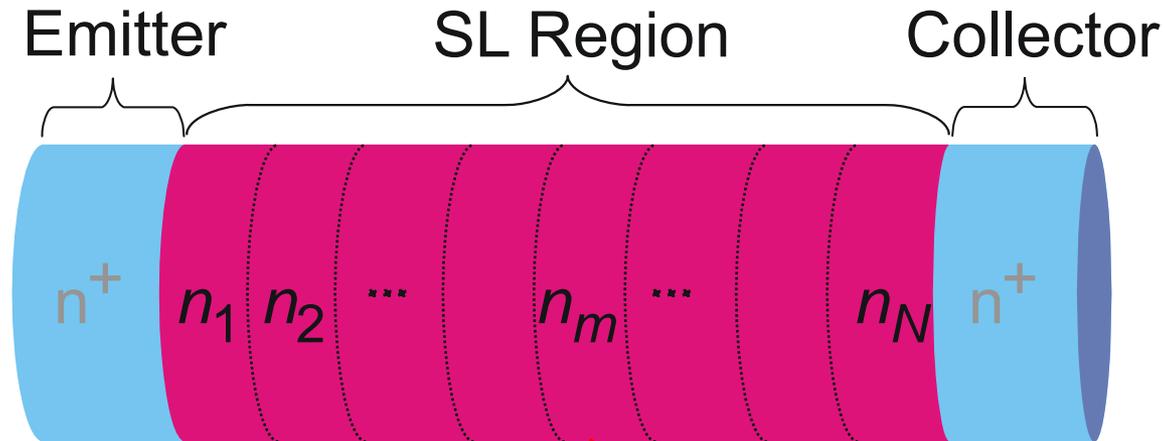
- Current continuity equation
- Poisson's equation

throughout the device

Semiclassical transport model

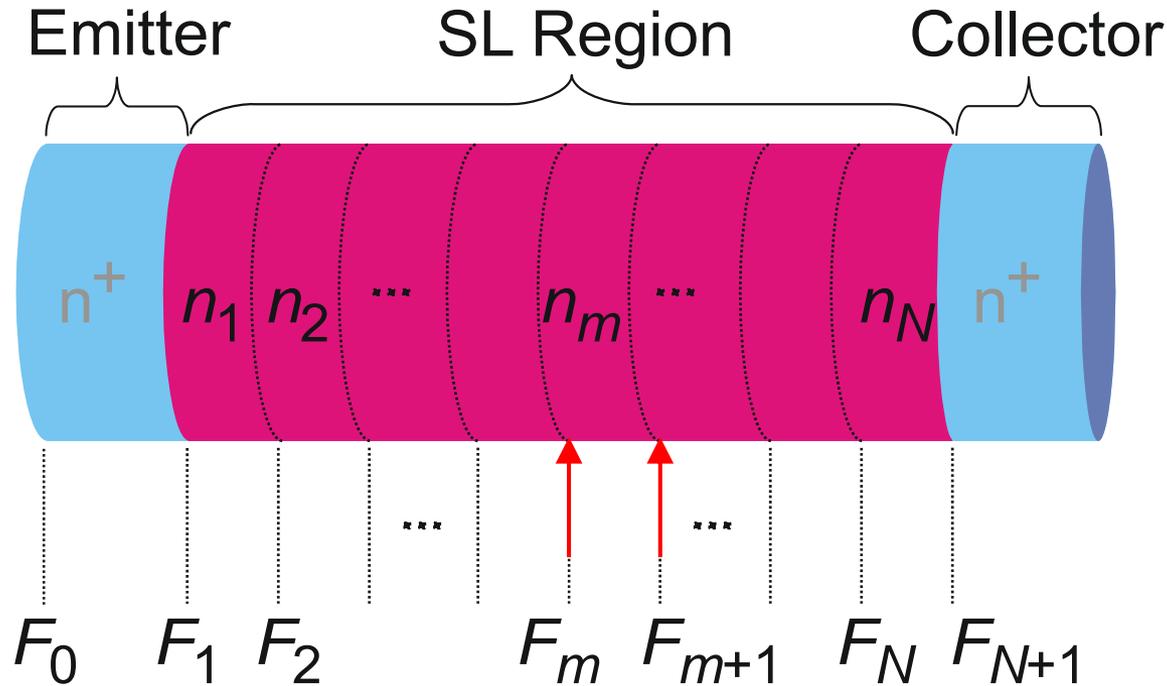


Semiclassical transport model



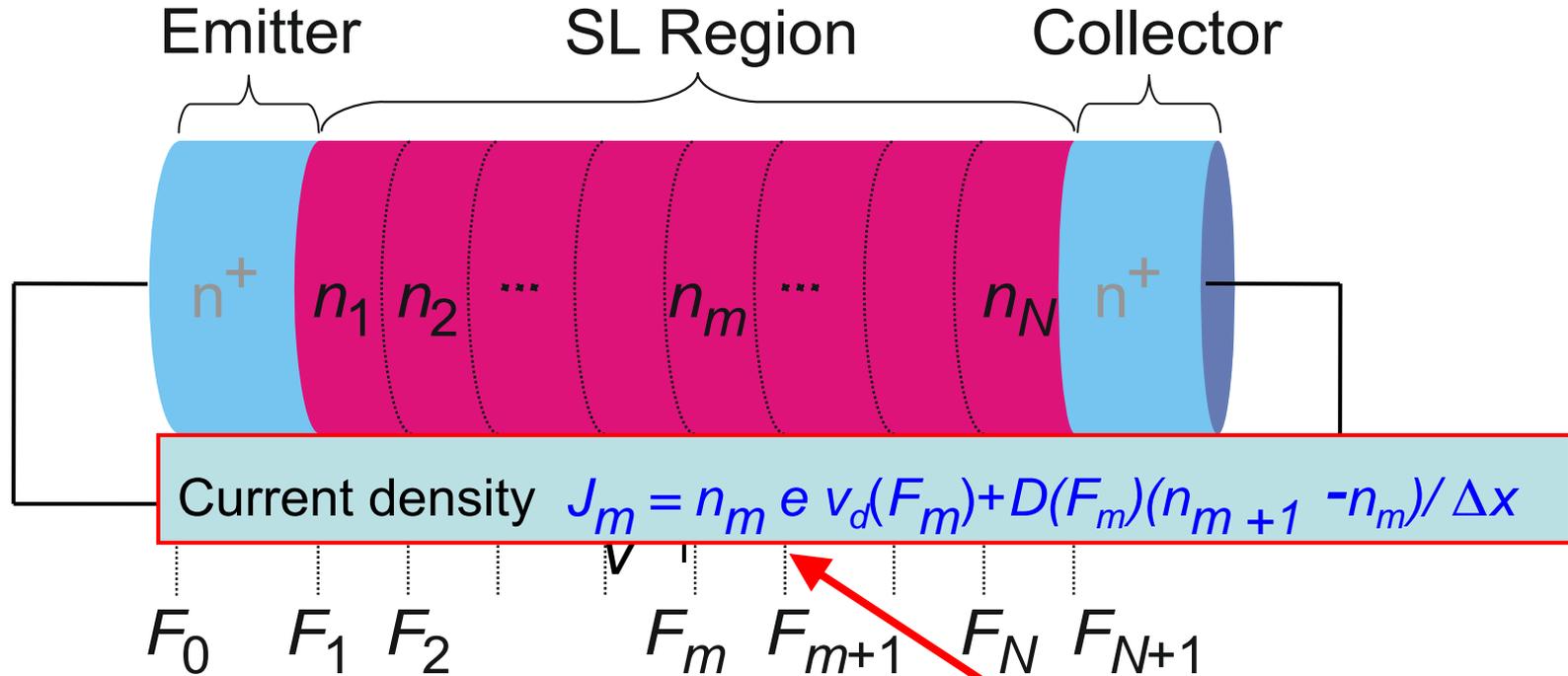
- Model SL continuum by ~ 500 layers, each ~ 0.25 nm thick
- In m^{th} layer, electron density is n_m

Semiclassical transport model



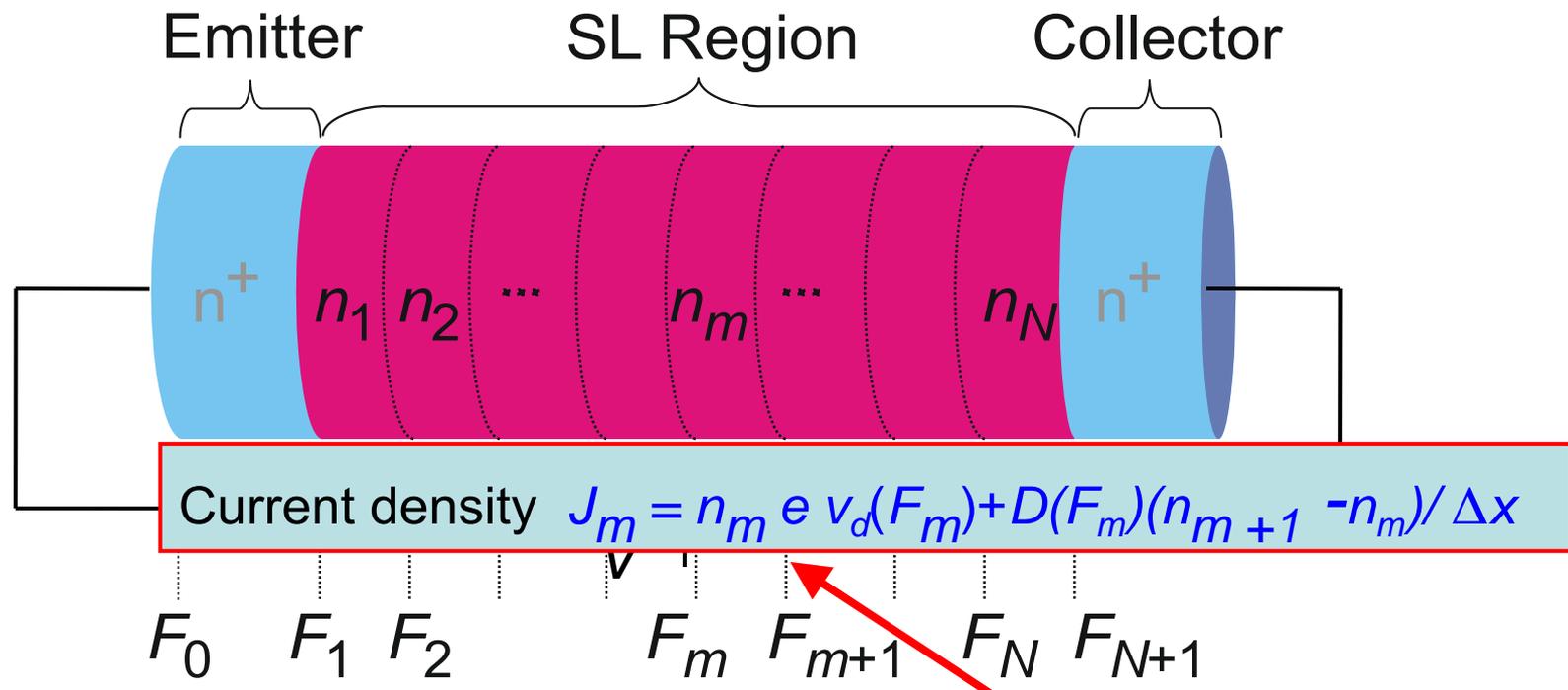
- At edges of m^{th} layer, electric field values are F_m and F_{m+1}

Semiclassical transport model

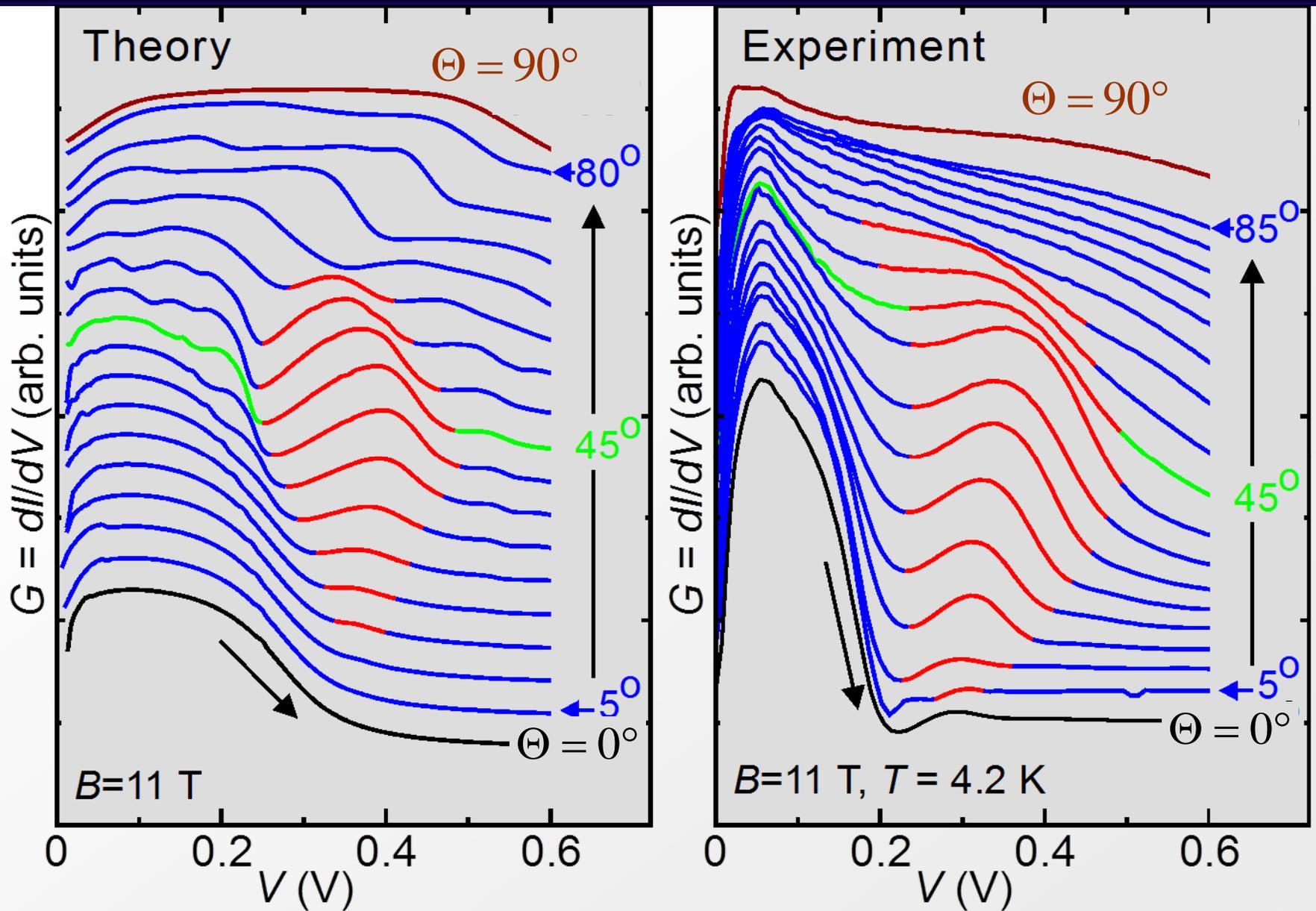


- Solve charge conservation equations $dn_m/dt = (J_m - J_{m+1}) / e\Delta x$,
- and Poisson equations $F_{m+1} - F_m = (n_m - n_D) e\Delta x / \epsilon_0\epsilon_r$
- self-consistently requiring sum of voltage drops = V

Semiclassical transport model

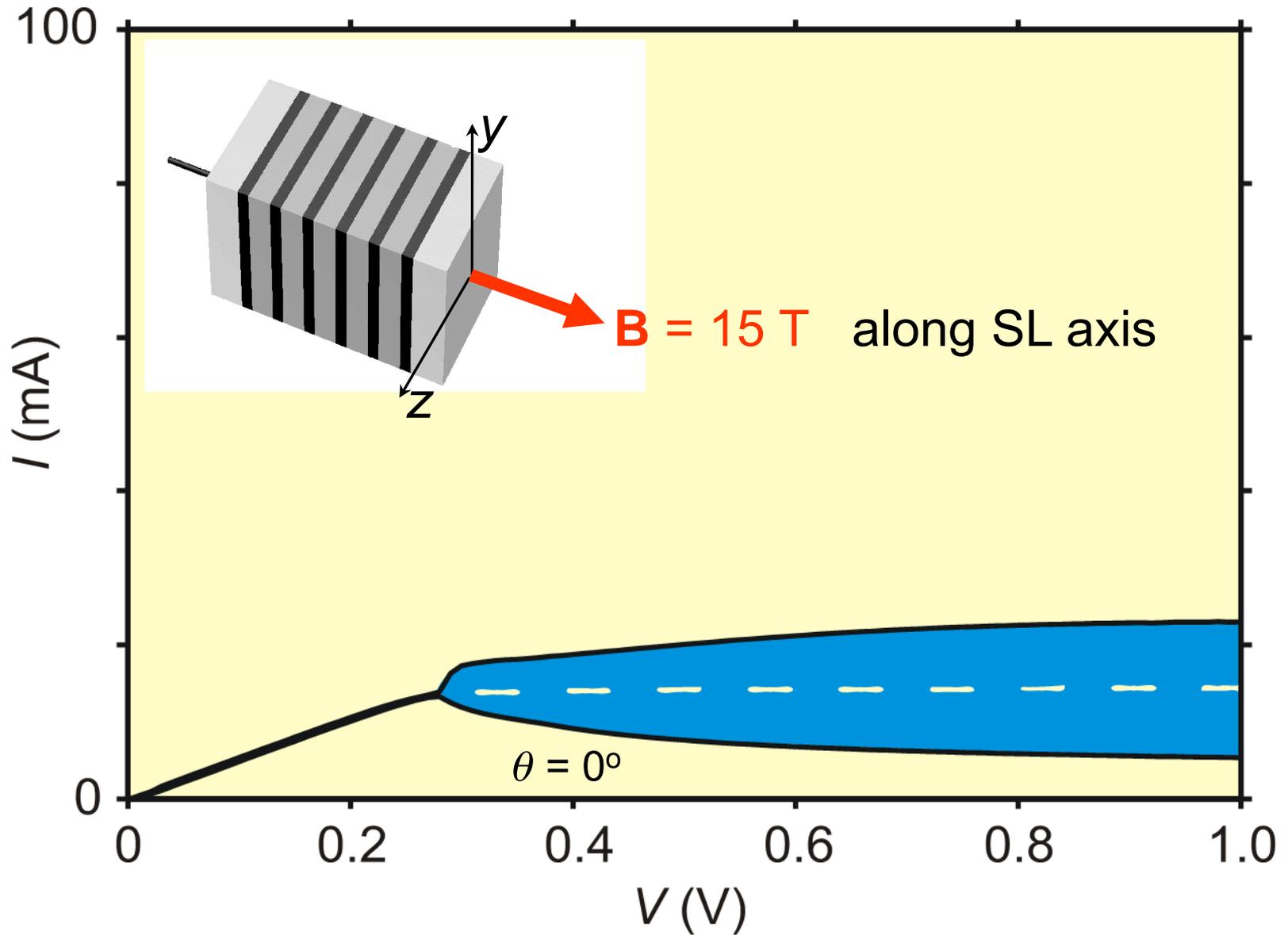


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- and Poisson equations $F_{m+1} - F_m = (n_m - n_D) e\Delta x / \epsilon_0\epsilon_r$
- self-consistently requiring sum of voltage drops = V
- Calculate current $I(t) = \frac{A}{N+1} \sum_{m=0}^N J_m$

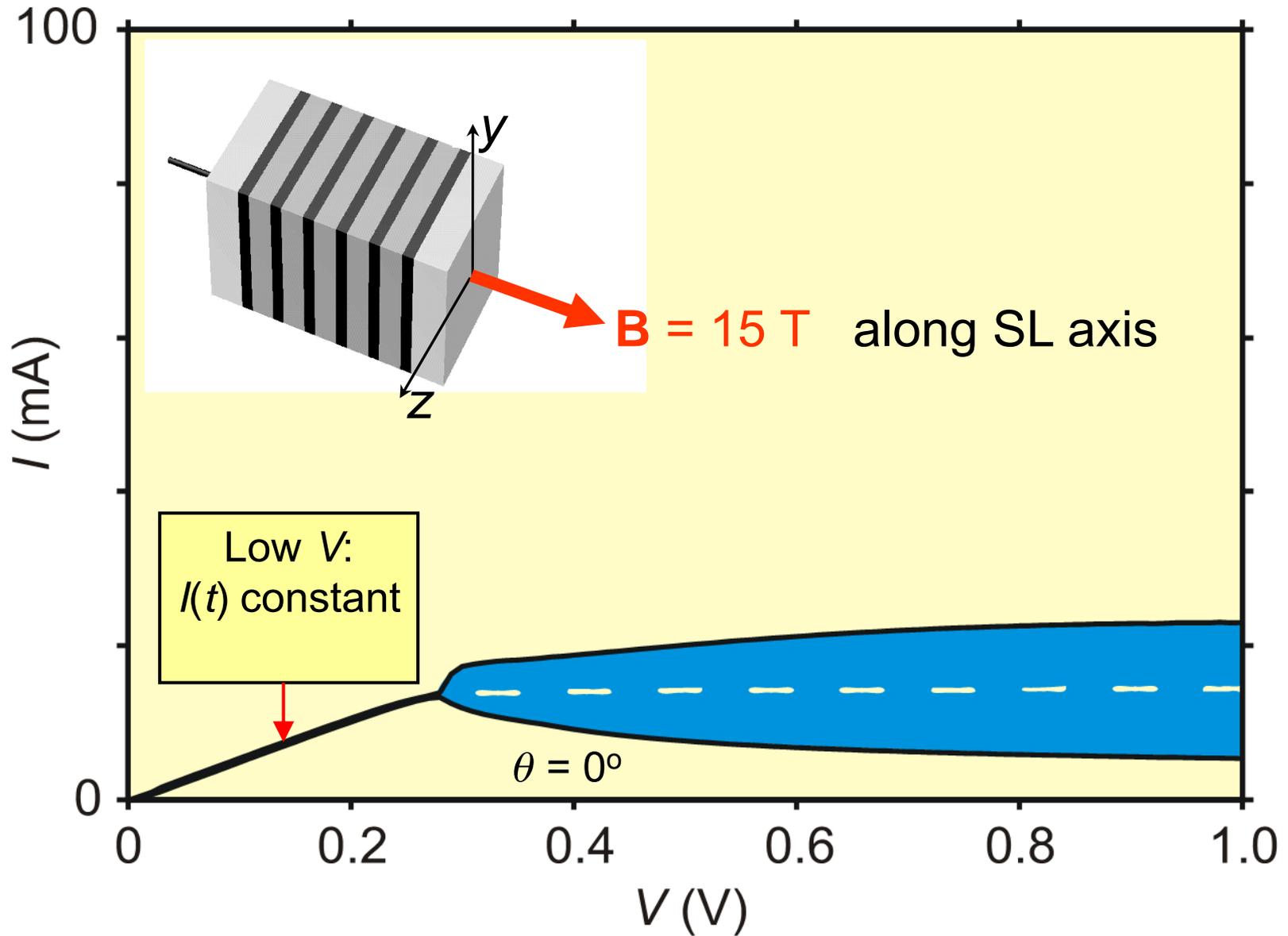


Fromhold et al, *Nature* **428**, 726 (2004)

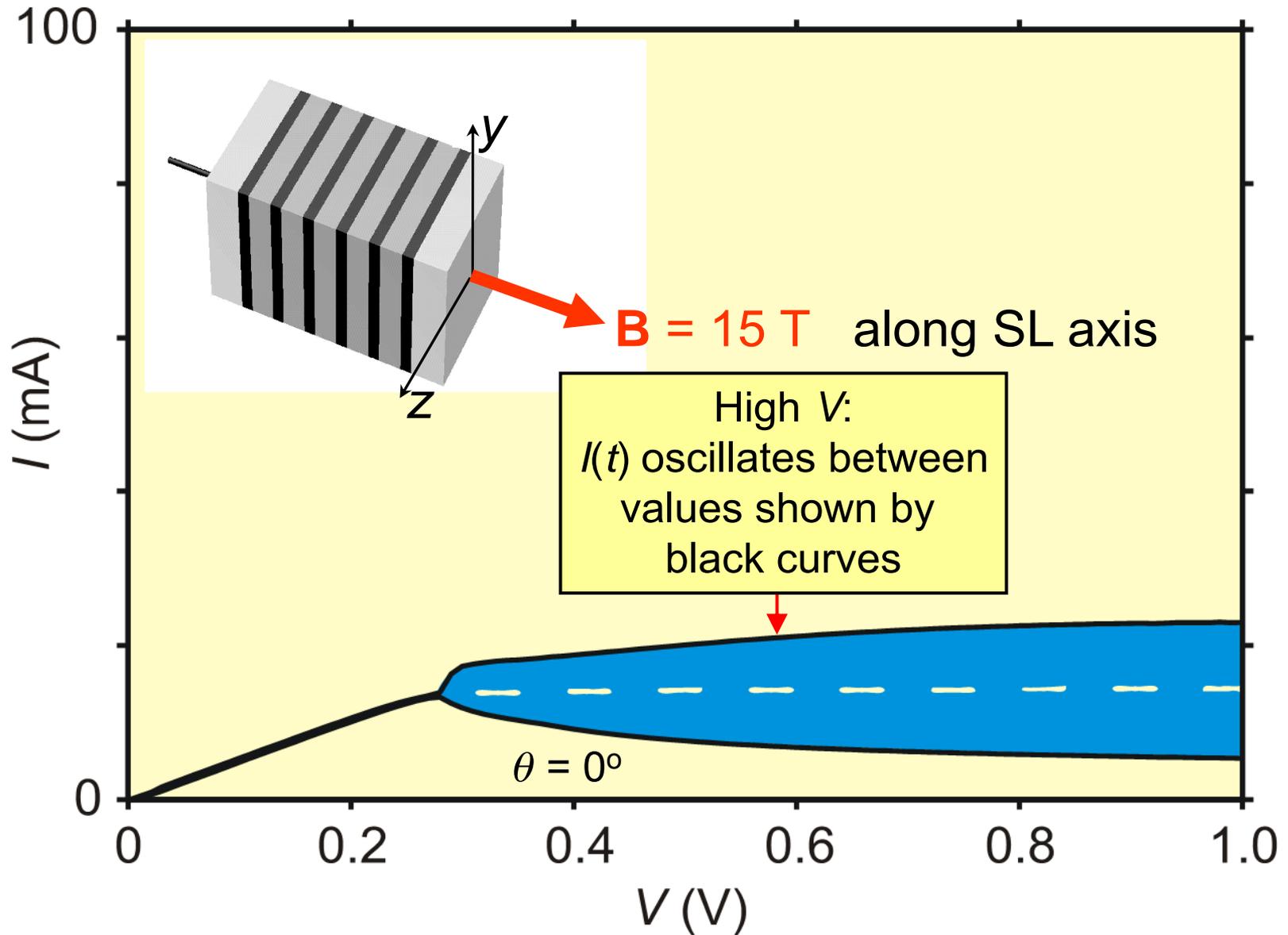
Theory



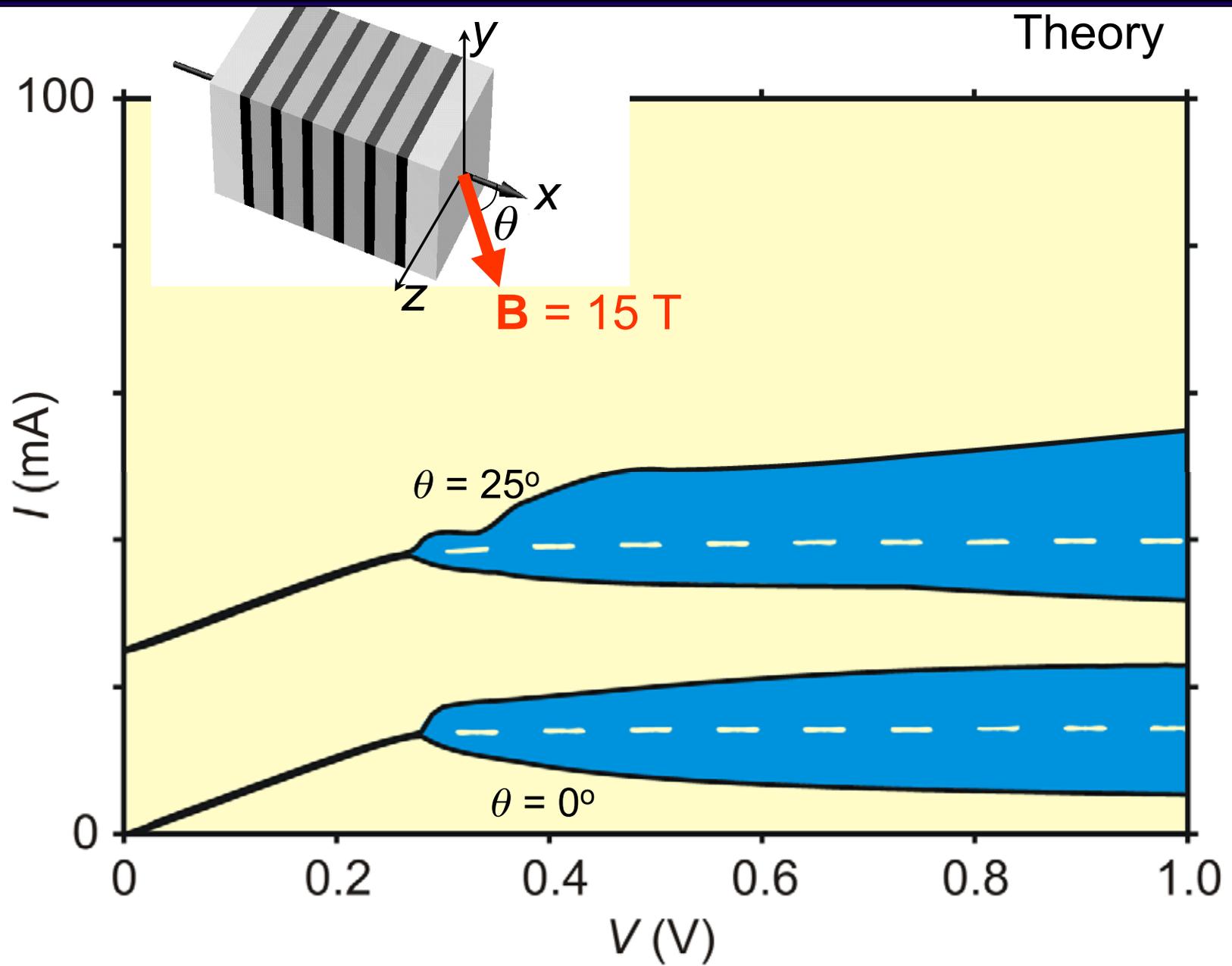
Theory

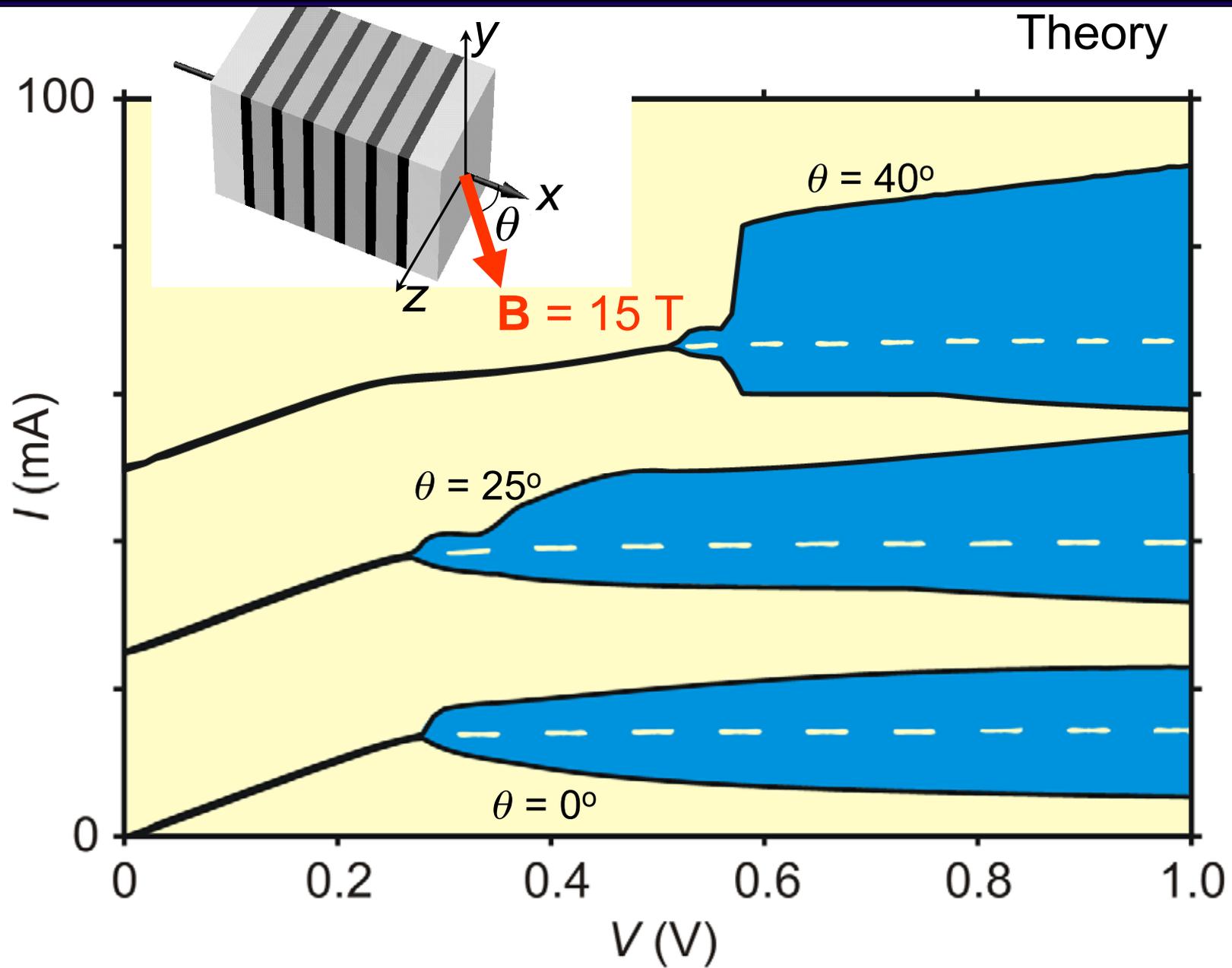


Theory

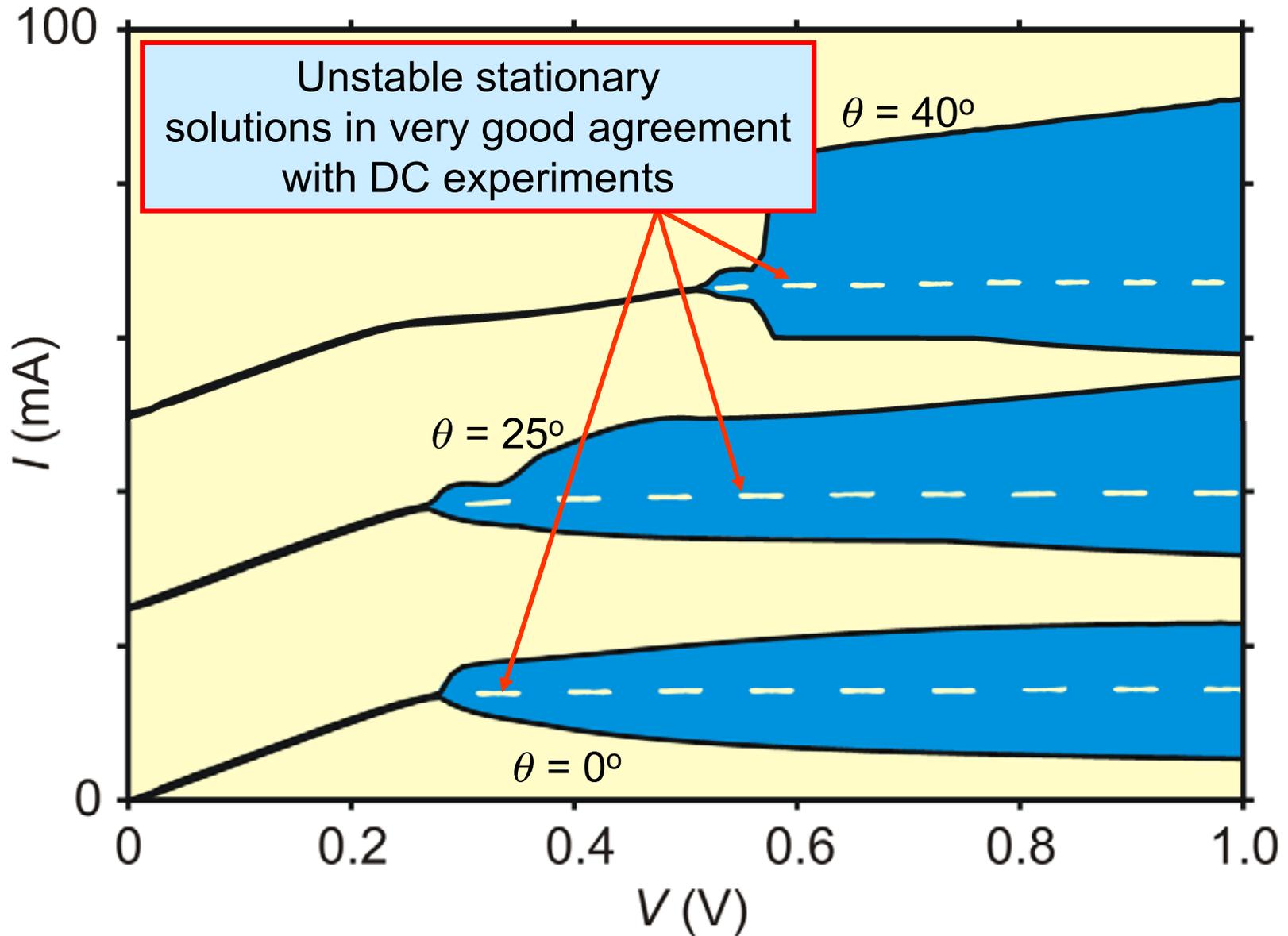


Theory



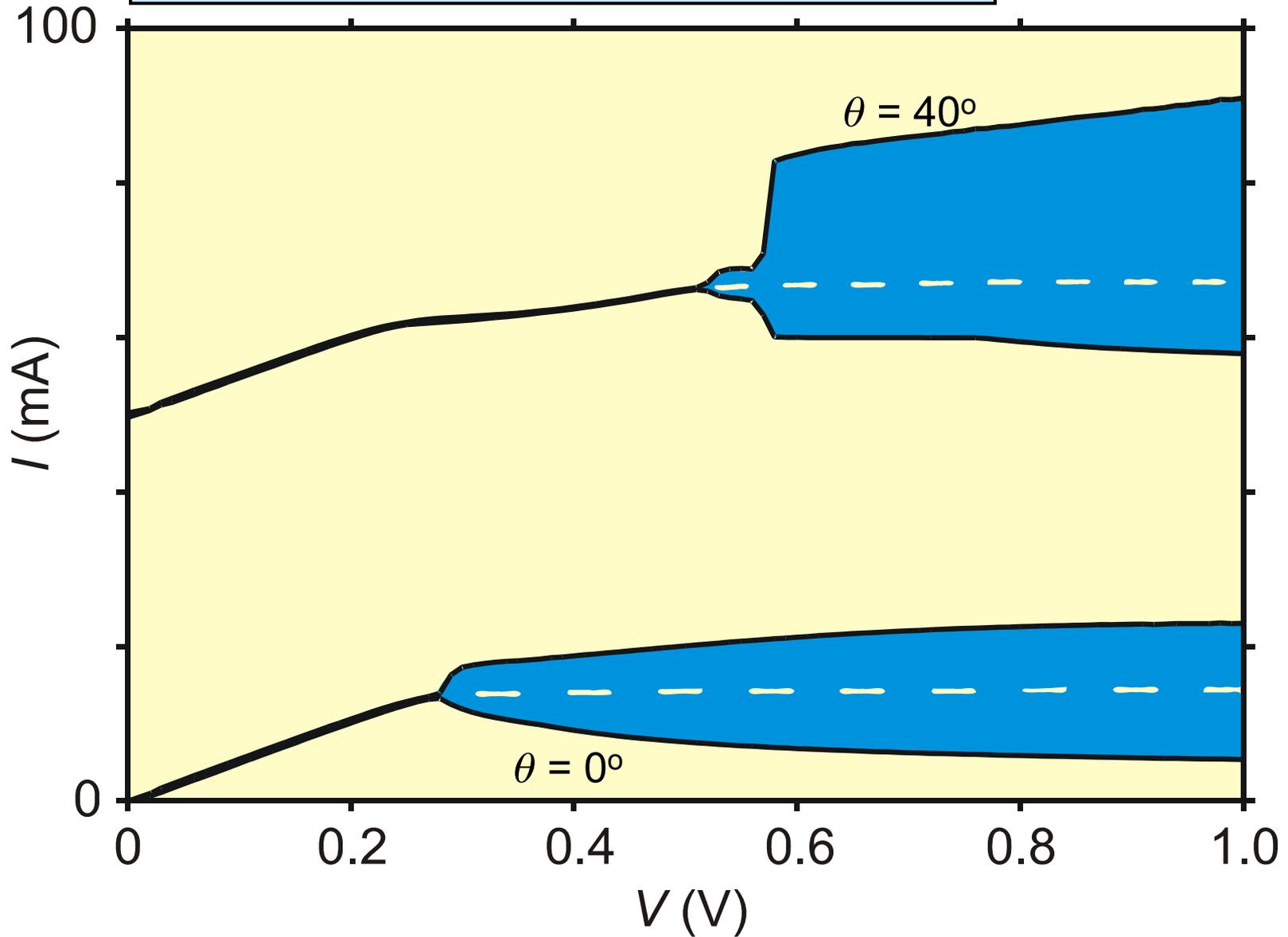


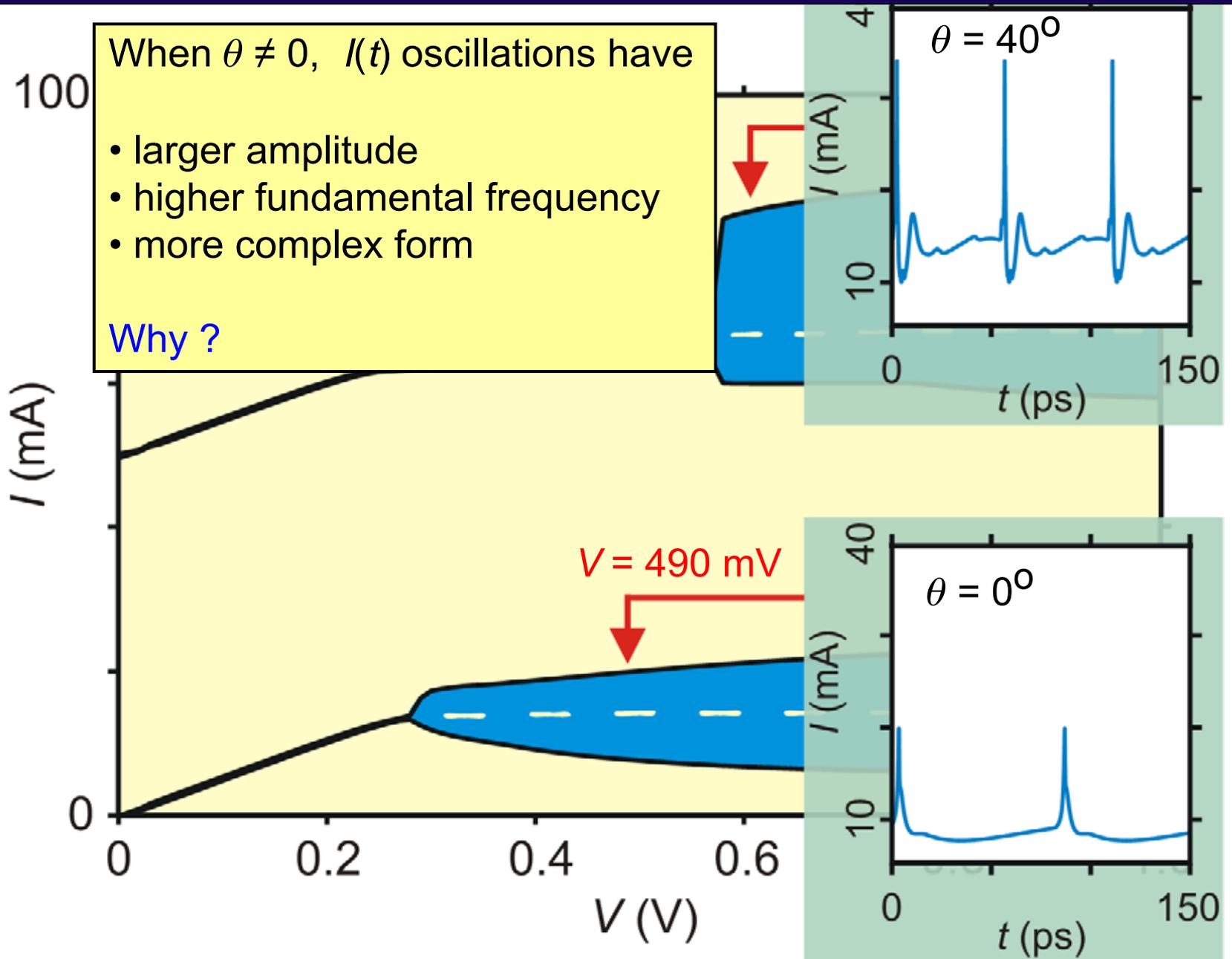
Theory

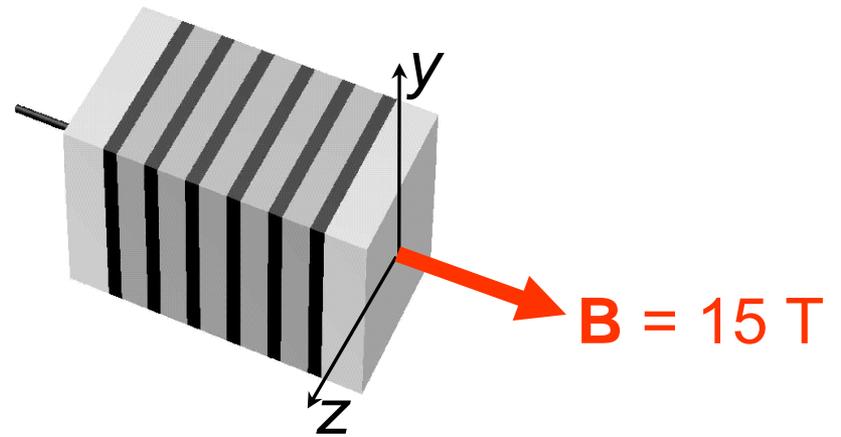
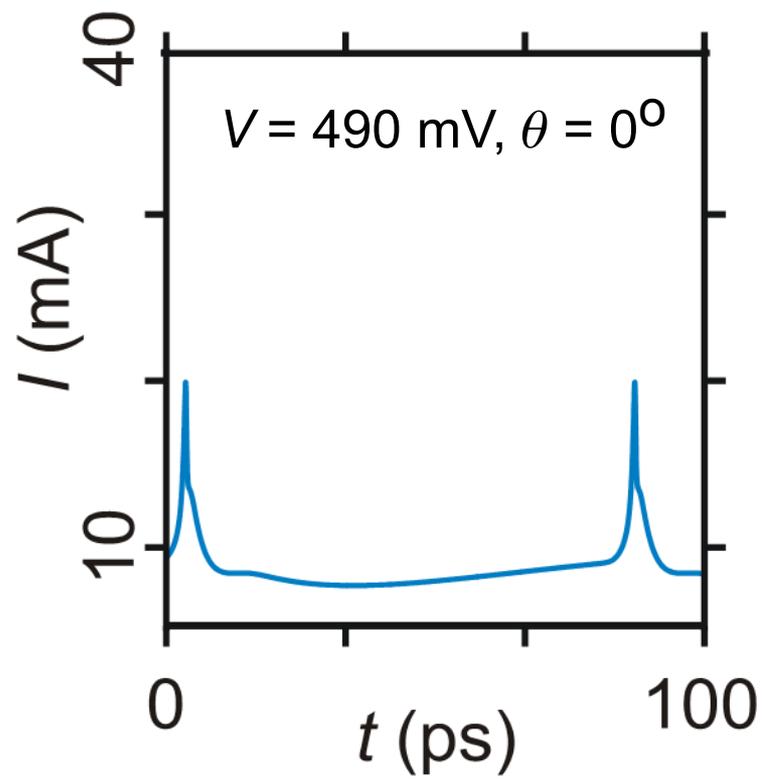


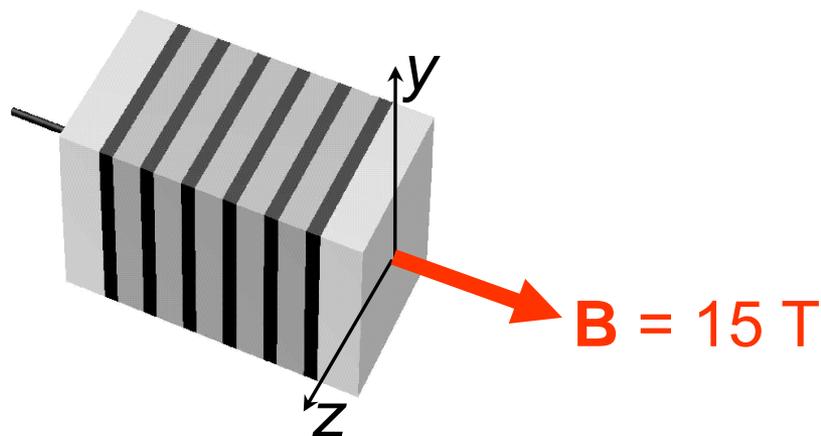
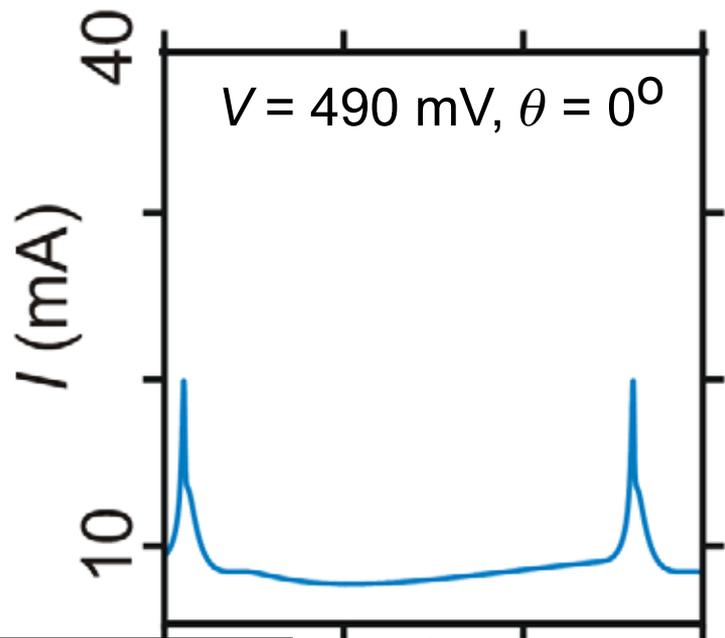
Consider current oscillations at high V

Theory



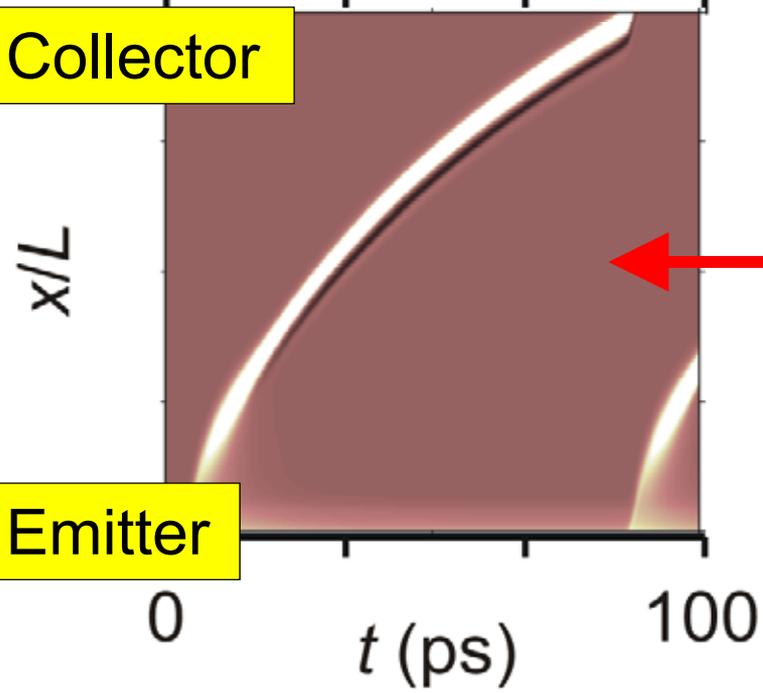




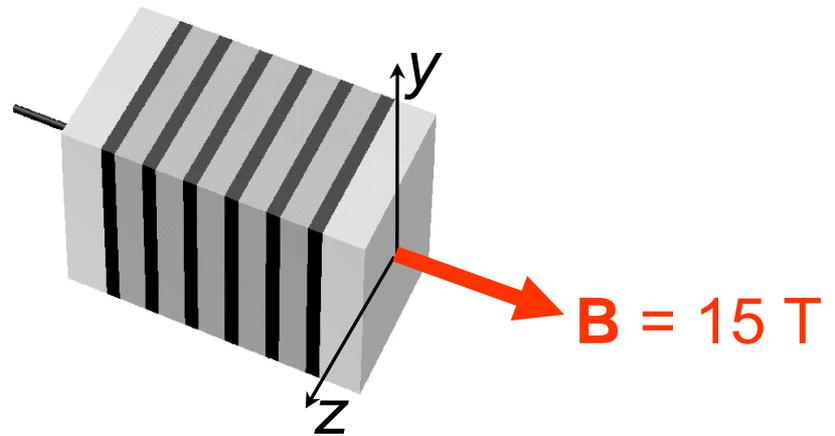
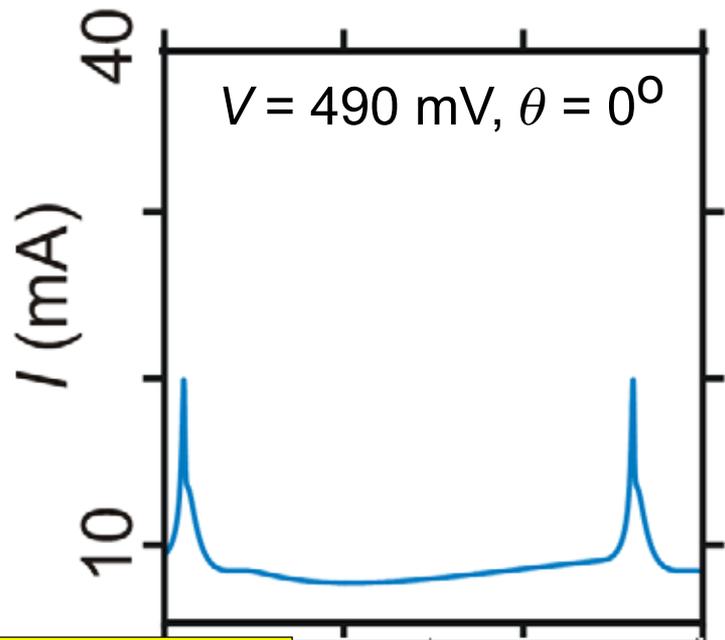


Collector

Emitter



Electron density calculated versus t and position x through SL from emitter to collector



Collector

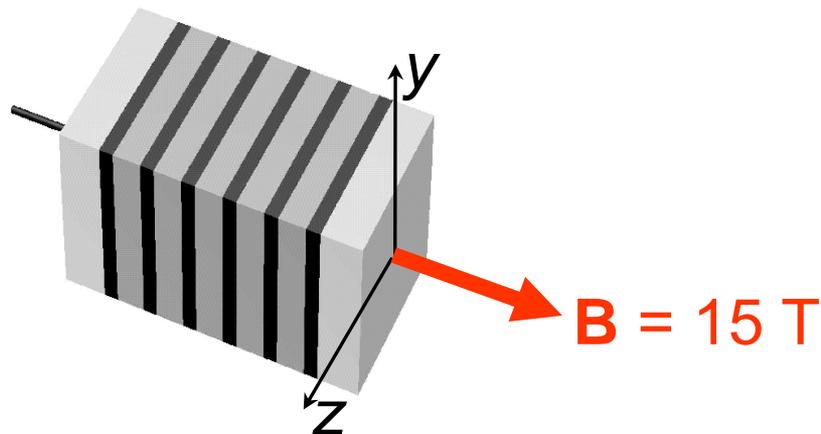
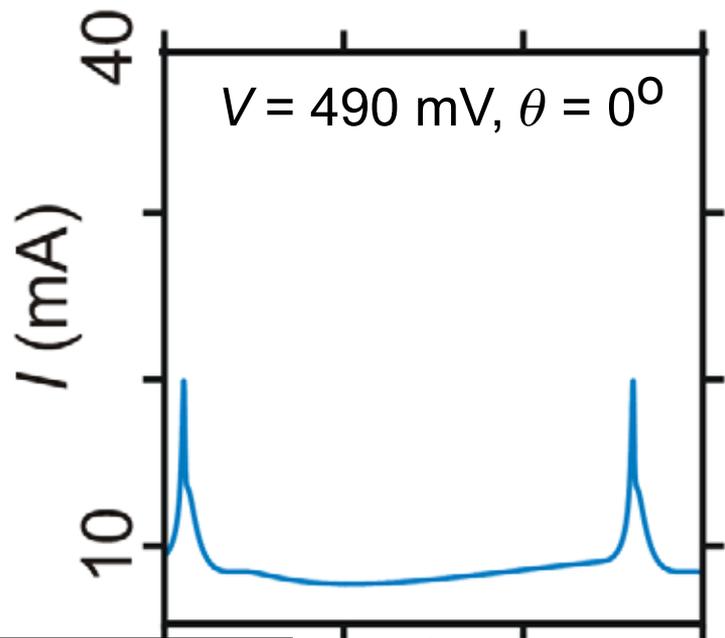
x/L

Emitter

0 $t \text{ (ps)}$ 100

Electron density calculated versus t and position x through SL from emitter to collector

White areas show charge domains, where electron density is high



Collector

x/L

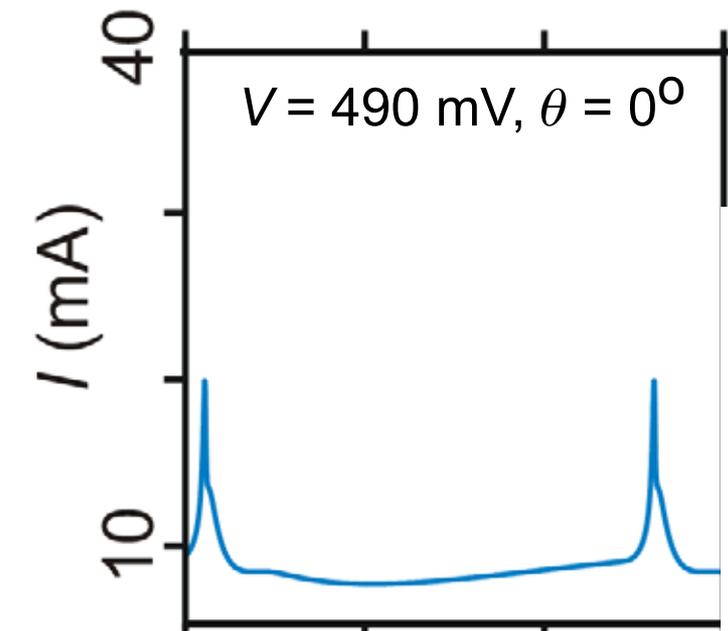
Emitter

0 $t \text{ (ps)}$ 100

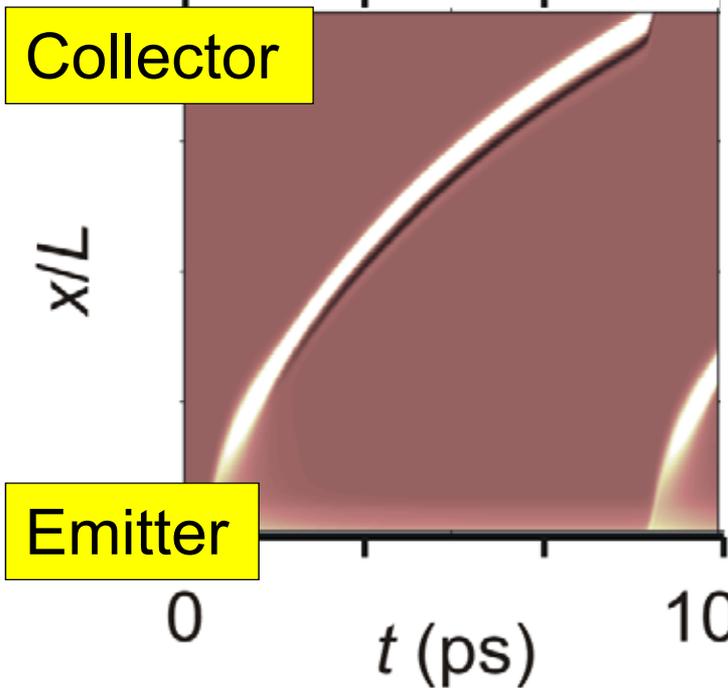
Electron density calculated versus t and position x through SL from emitter to collector

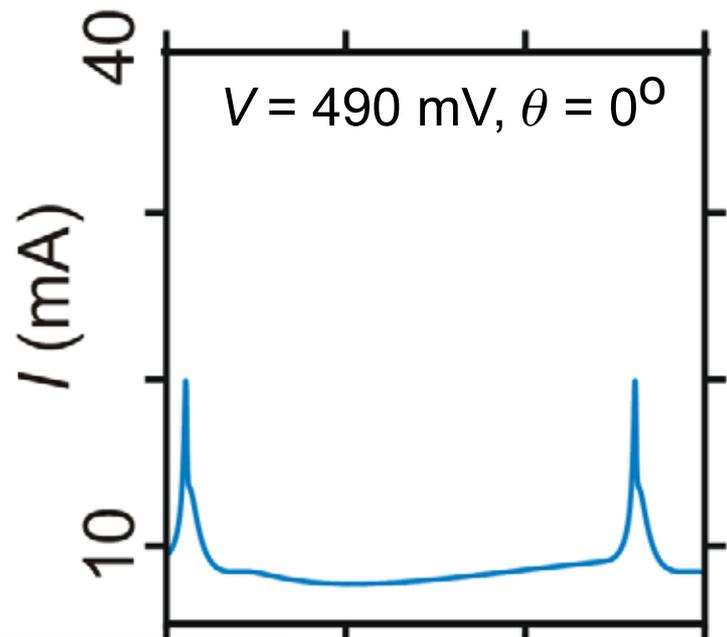
White areas show charge domains, where electron density is high

Domains propagate through SL



↑ Electron density (log scale)



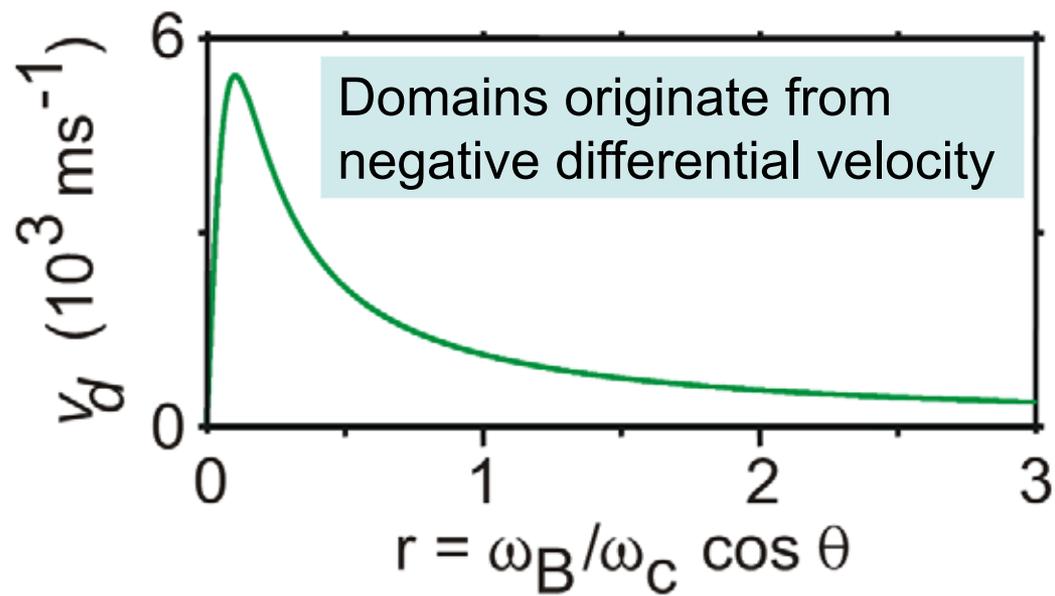


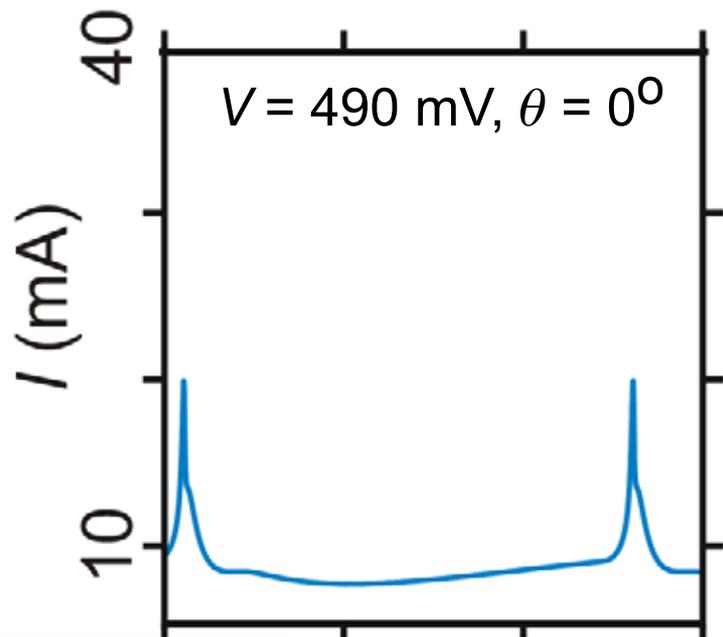
Collector

x/L

Emitter

t (ps)



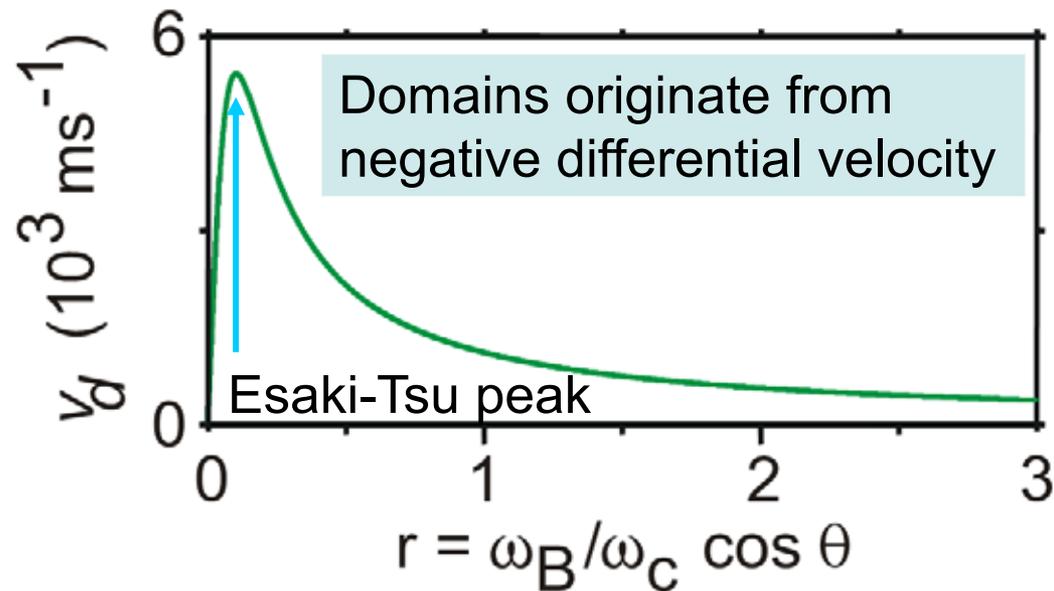


Collector

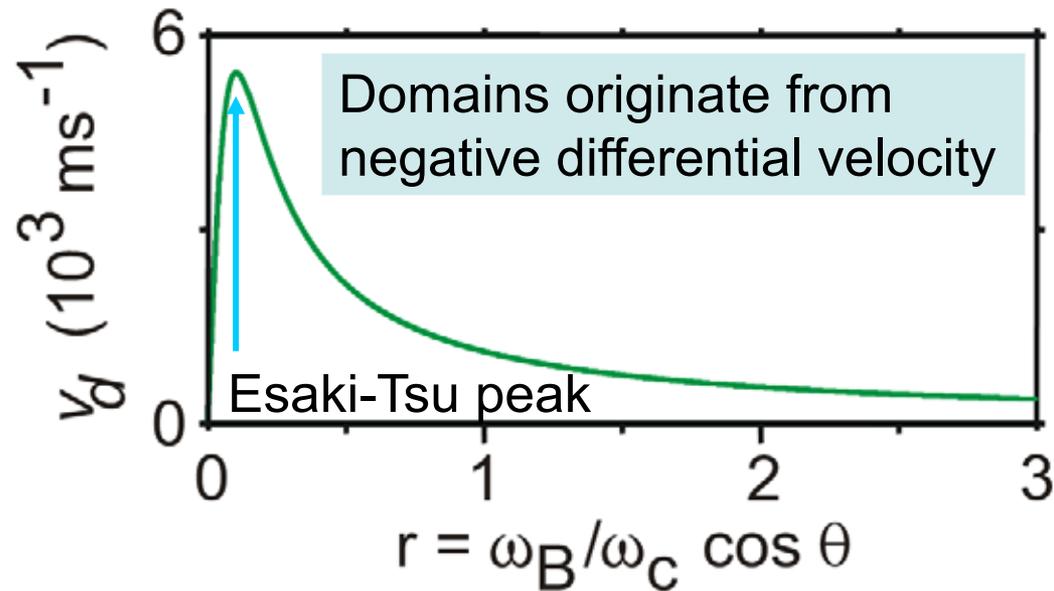
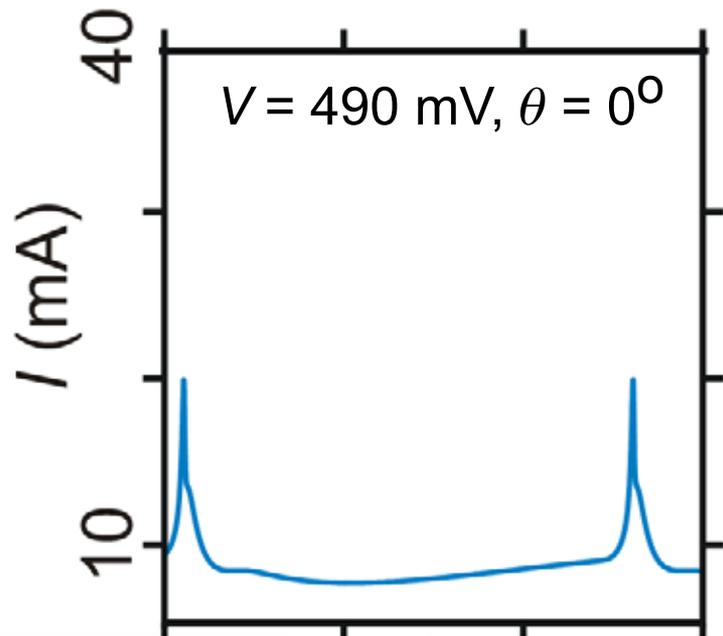
x/L

Emitter

$t \text{ (ps)}$



Electrons are accelerated by applied V , attaining maximum speed along dotted curve where F coincides with Esaki-Tsu peak in $v_d(F)$



Collector

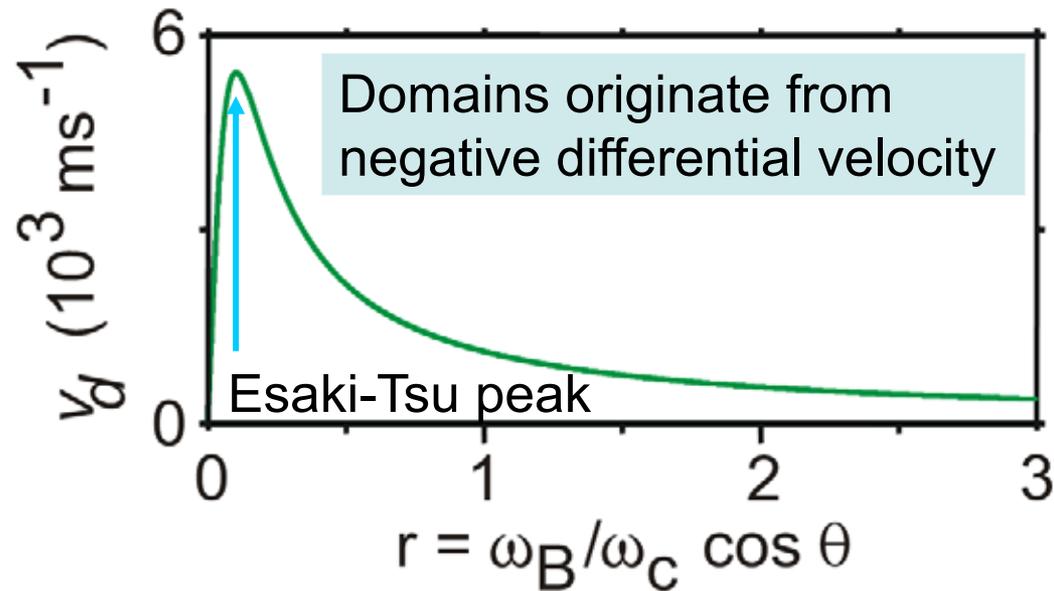
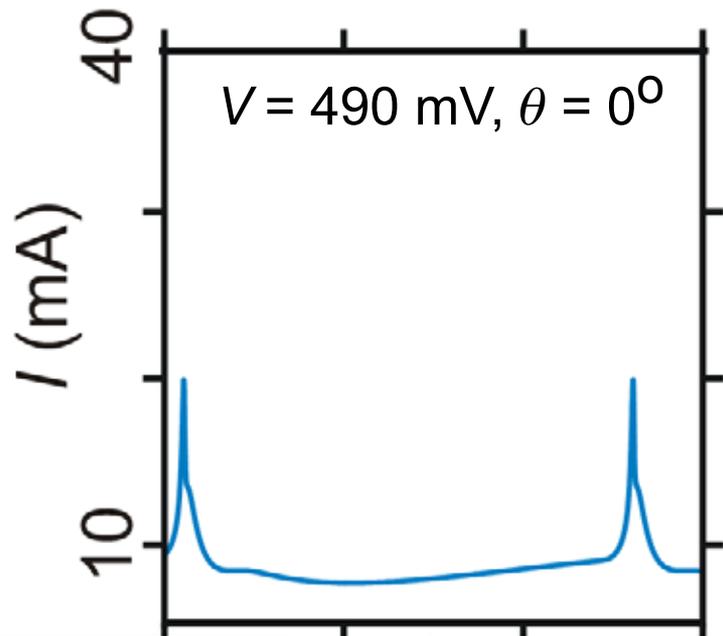
x/L

Emitter

$t \text{ (ps)}$

Electrons are accelerated by applied V , attaining maximum speed along dotted curve where F coincides with Esaki-Tsu peak in $v_d(F)$

As electrons cross this locus, they slow and accumulate, forming the domain



Collector

x/L

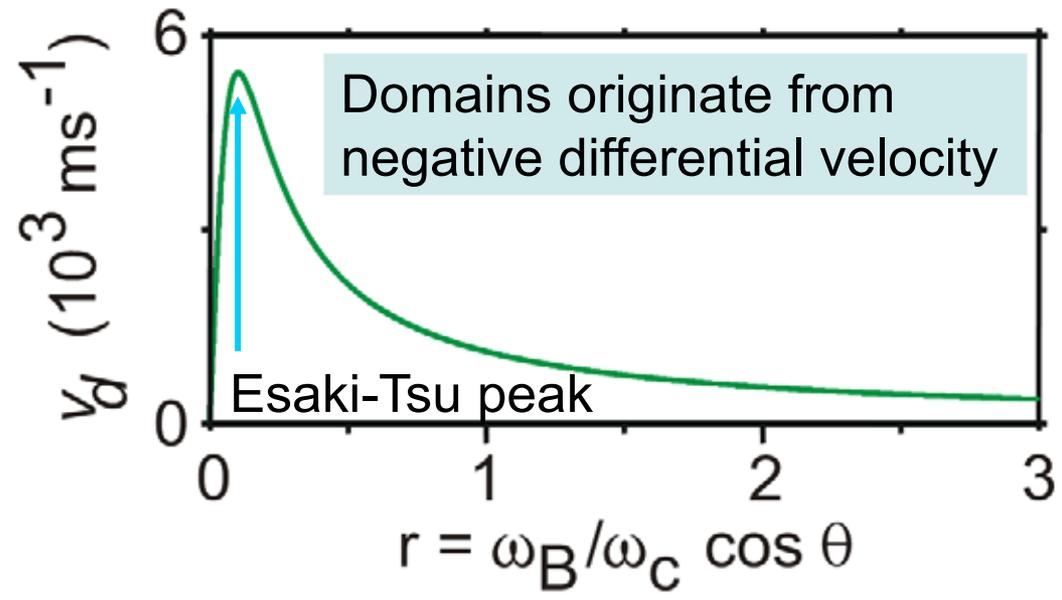
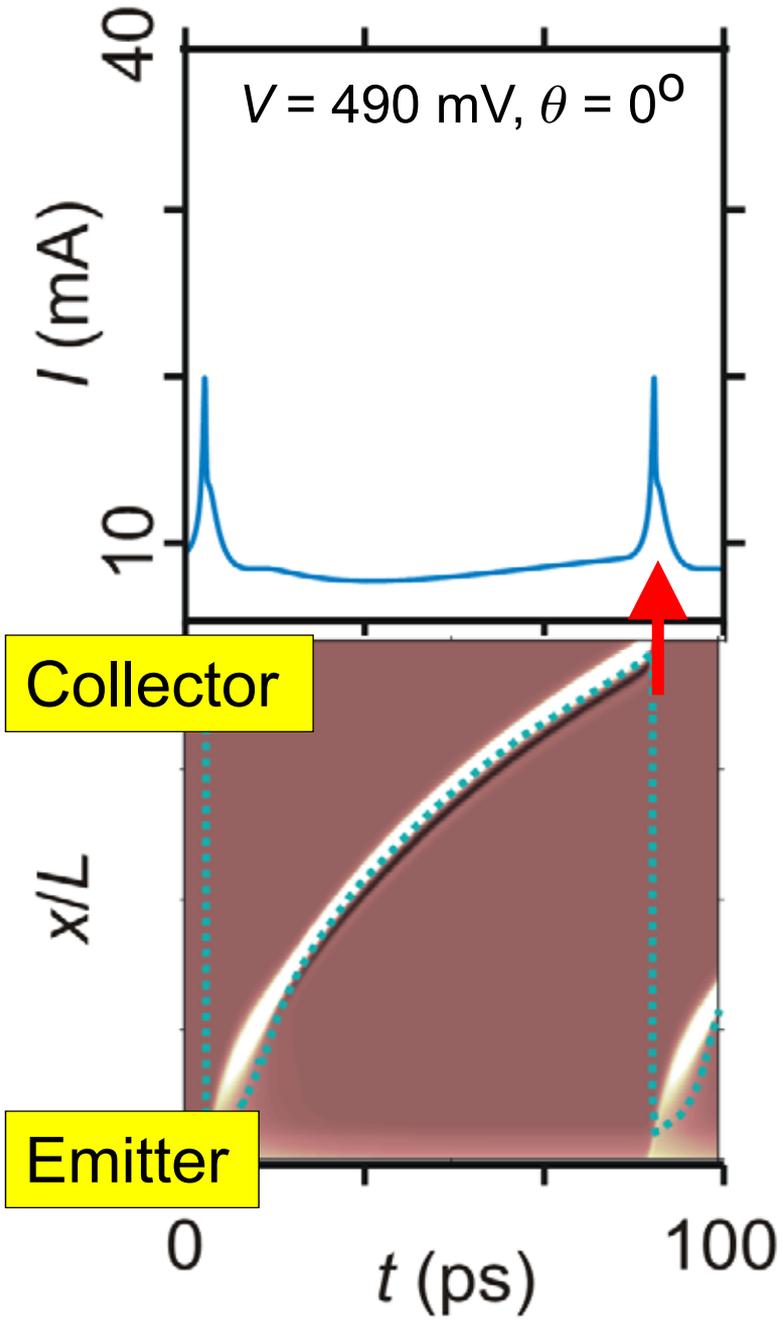
Emitter

$t \text{ (ps)}$

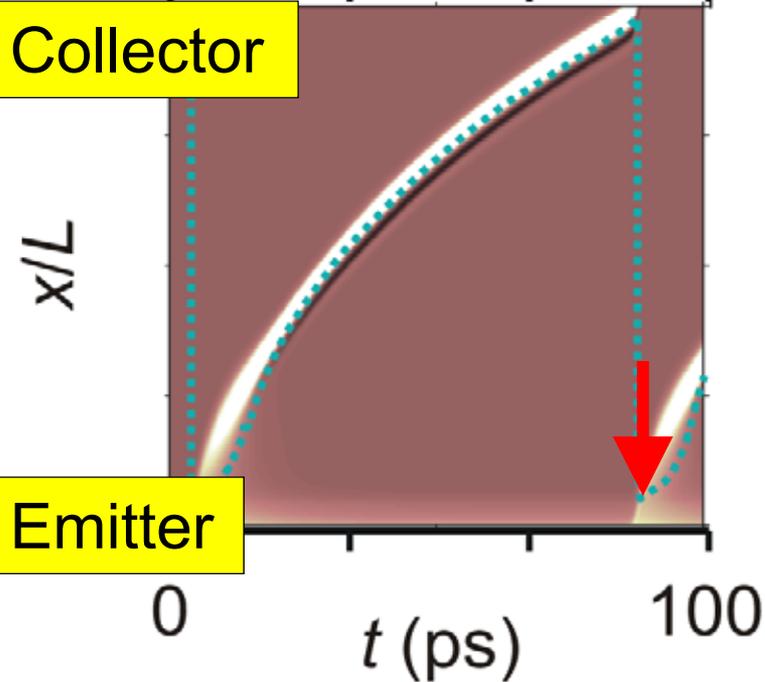
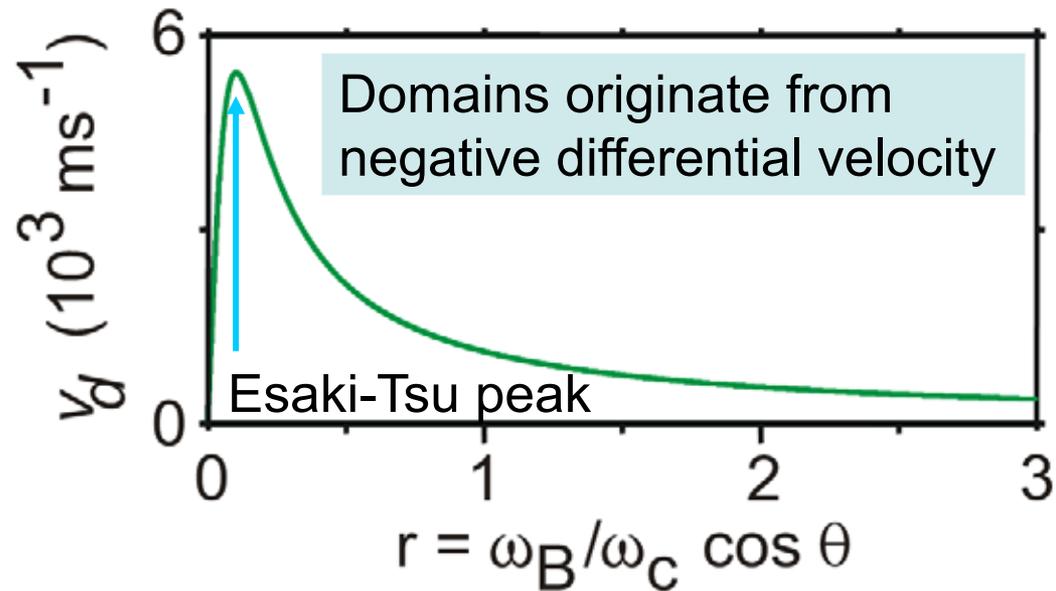
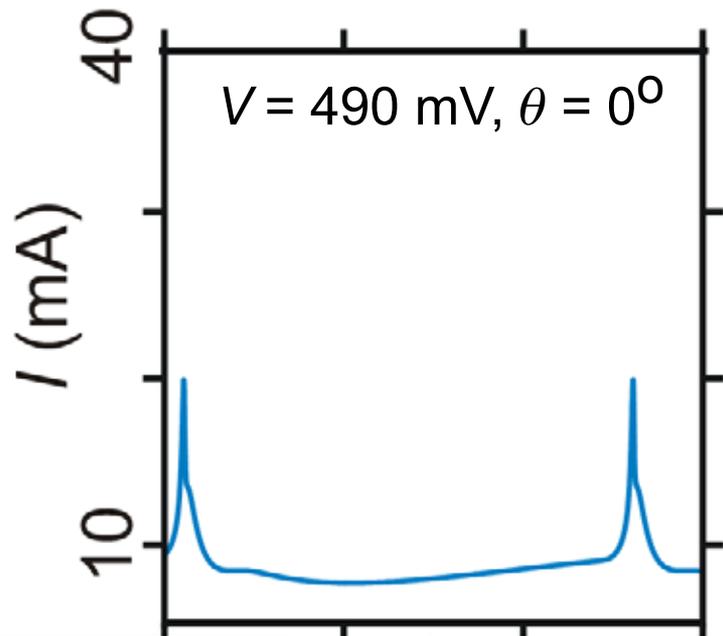
Electrons are accelerated by applied V , attaining maximum speed along dotted curve where F coincides with Esaki-Tsu peak in $v_d(F)$

As electrons cross this locus, they slow and accumulate, forming the domain

As the domain propagates, it grows to keep V constant

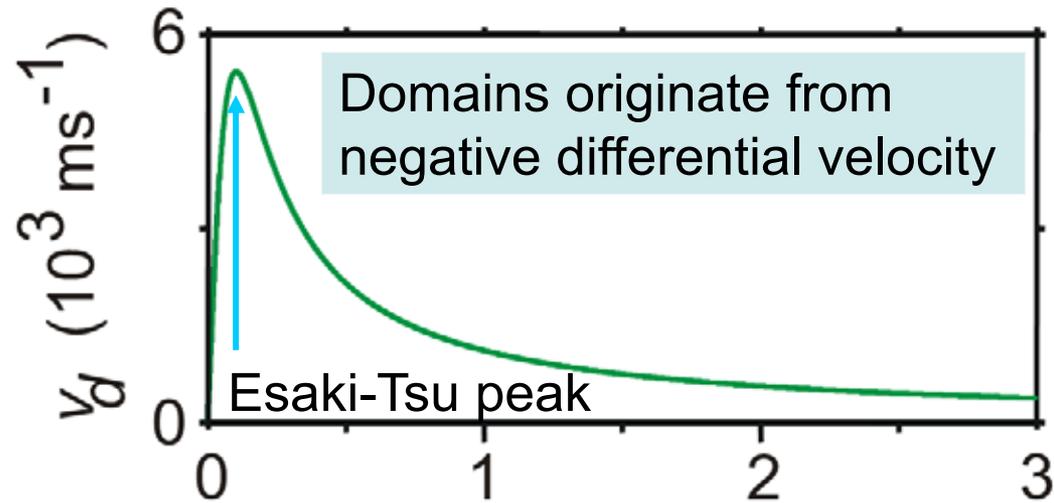
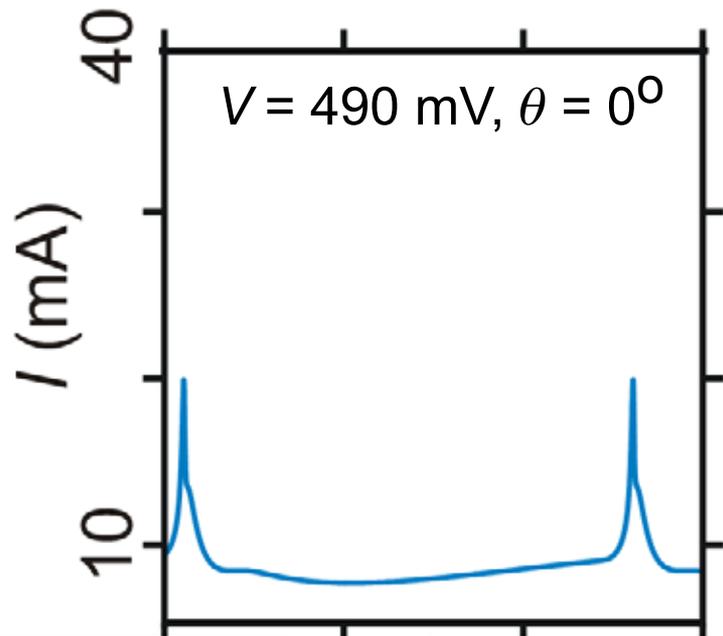


When domain reaches collector, it produces a peak in $I(t)$



When domain reaches collector, it produces a peak in $I(t)$

Another domain forms near the emitter and the cycle repeats



Collector

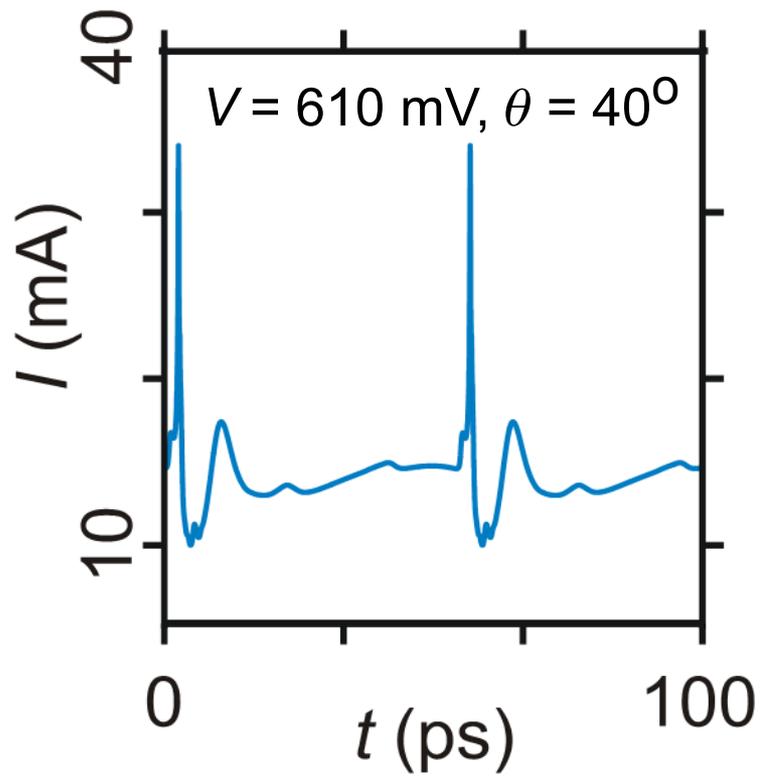
x/L

Emitter

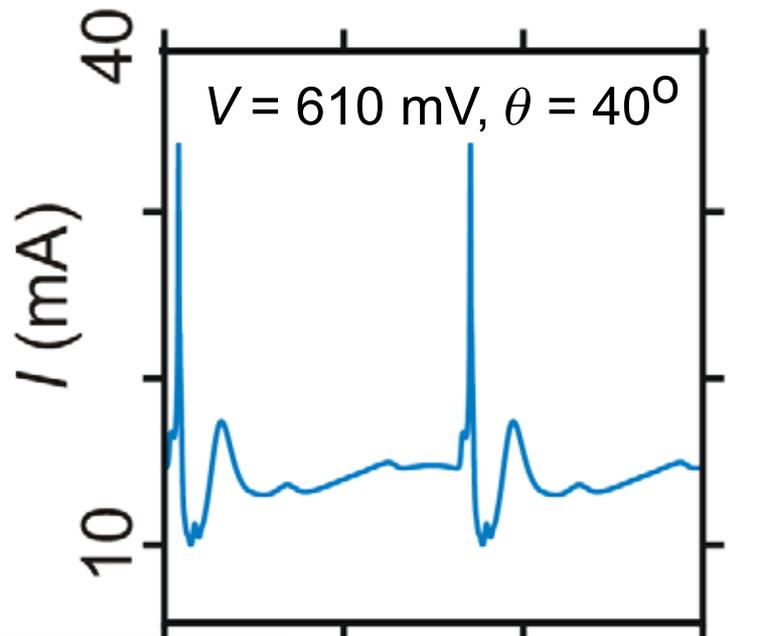
$t \text{ (ps)}$

For reviews see, e.g.,
 A. Wacker, *Phys. Rep.* **357**, 1 (2002);
 L. Bonilla and H. Grahn,
Rep. Prog. Phys. **68** 577, (2005);
 E. Schöll, *Nonlinear Spatio-temporal
 Dynamics and Chaos in
 Semiconductors* (CUP, 2001)

Propagating domains can generate
 electromagnetic radiation at frequencies
 $> 100 \text{ GHz}$ [e.g. Schomberg, Renk et al.,
Appl. Phys. Lett. **74**, 2179 (1999)]



When **B** tilted, $I(t)$ oscillations are stronger and richer



When \mathbf{B} tilted, $I(t)$ oscillations are stronger and richer

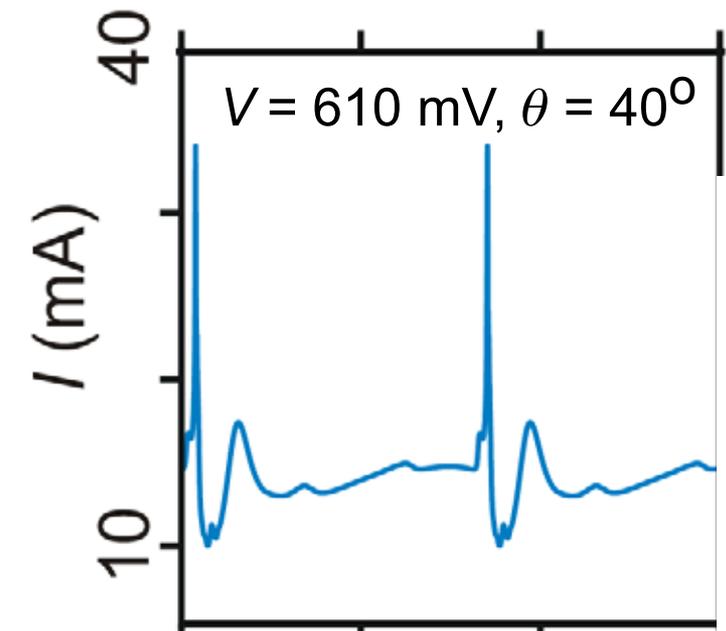
Collector

x/L

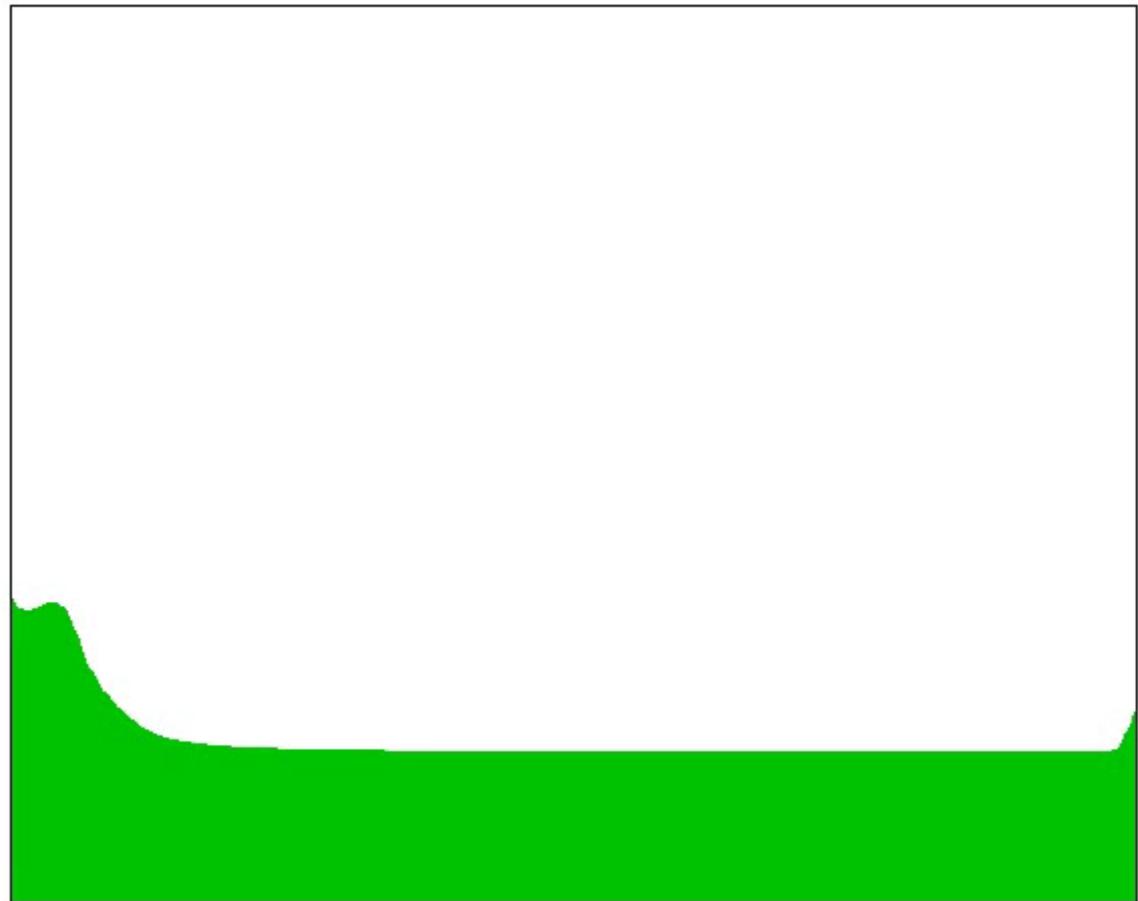
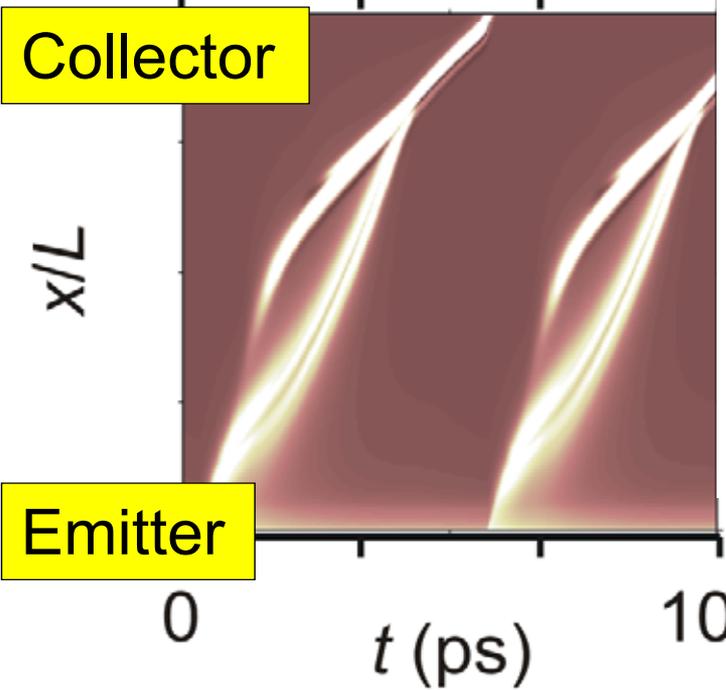
Emitter

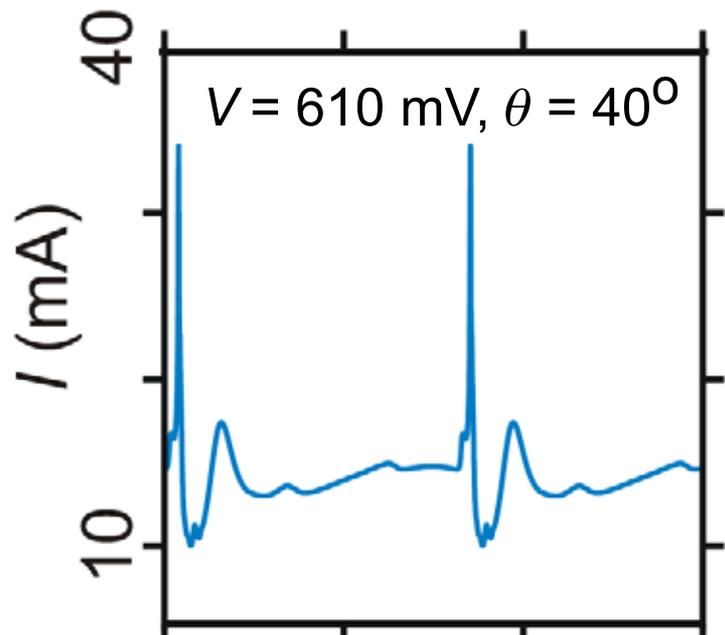
0 t (ps) 100

Because there are more domains (4 here) with complex dynamics

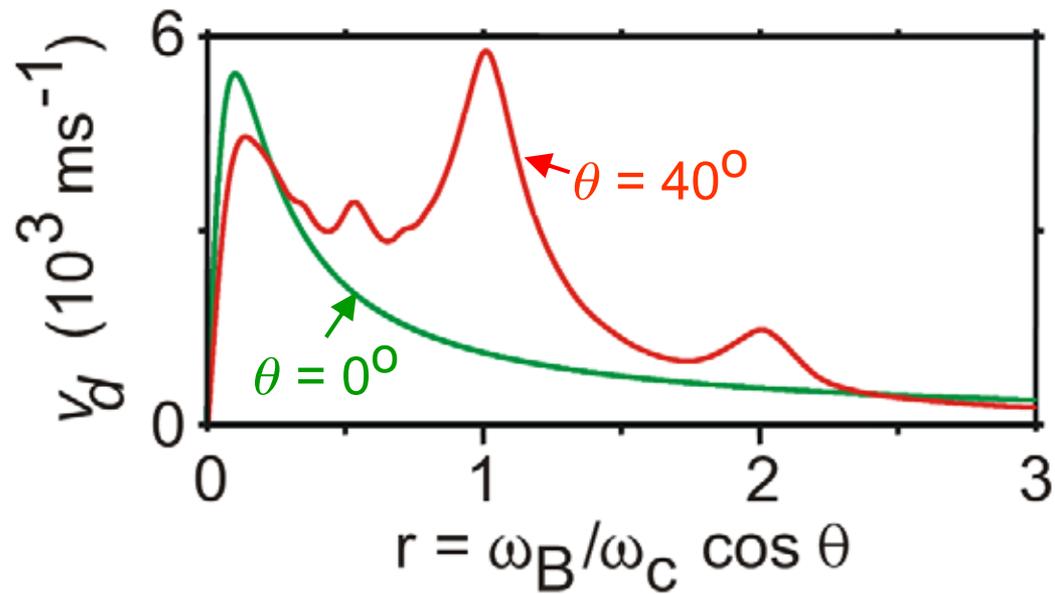
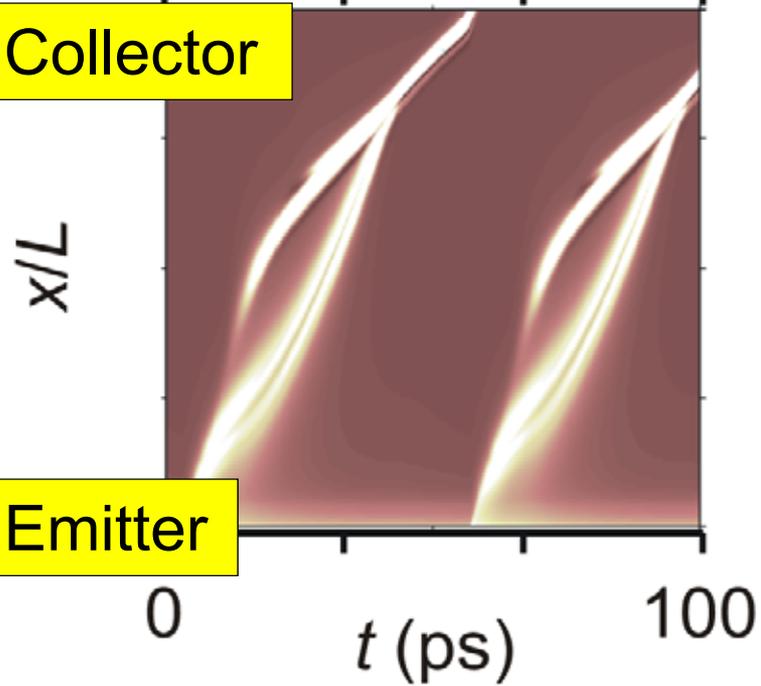


↑ Electron density (Log scale)

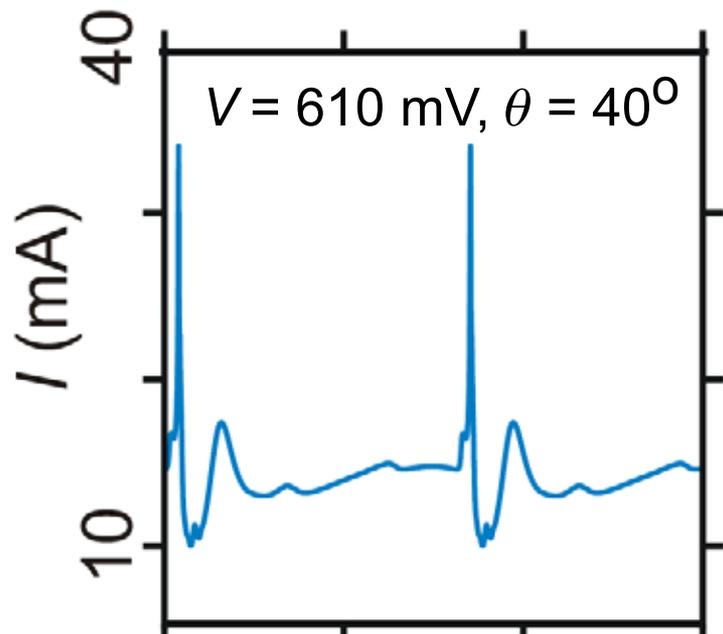




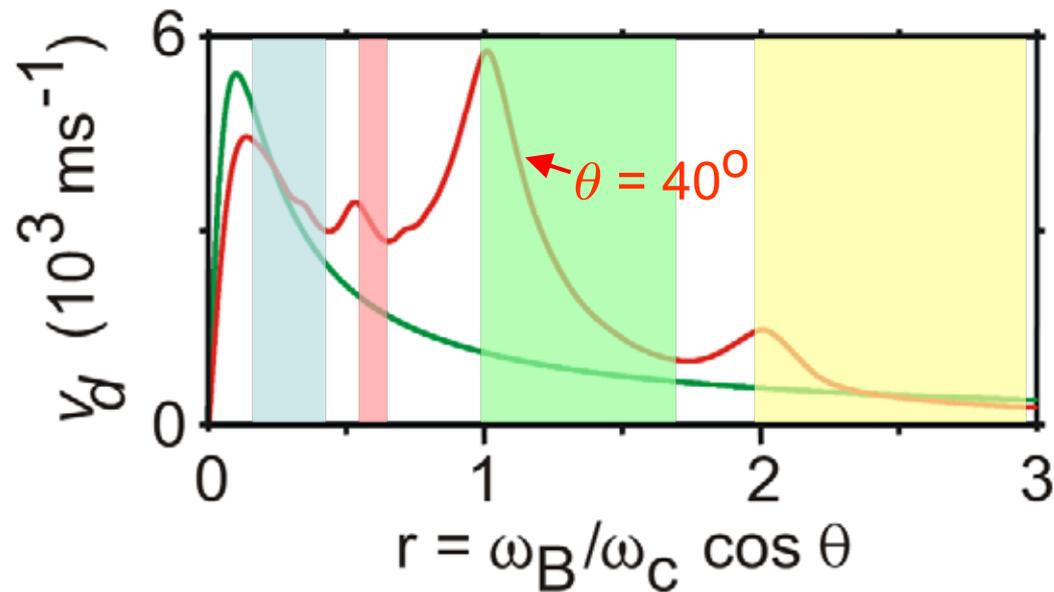
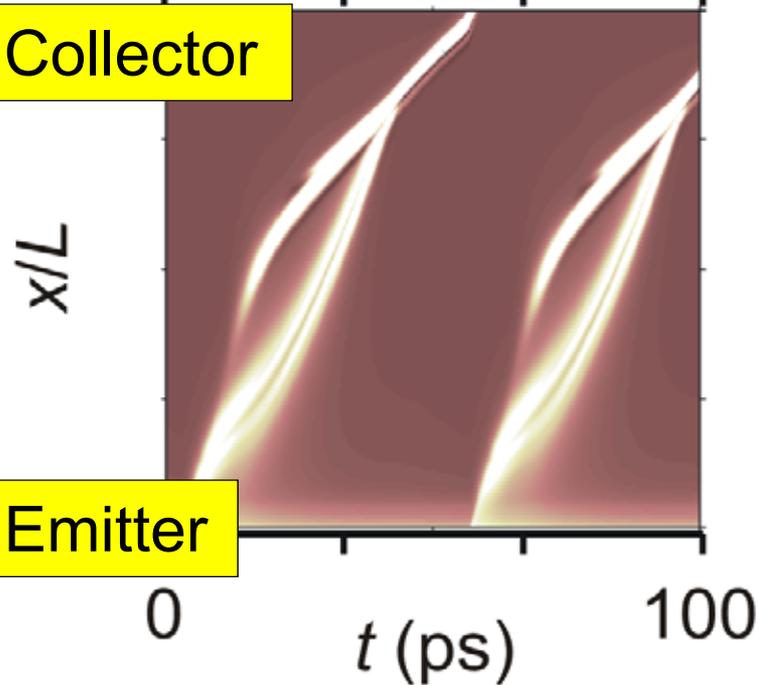
Collector



When $\theta = 0^\circ$, there is only one region of negative differential velocity

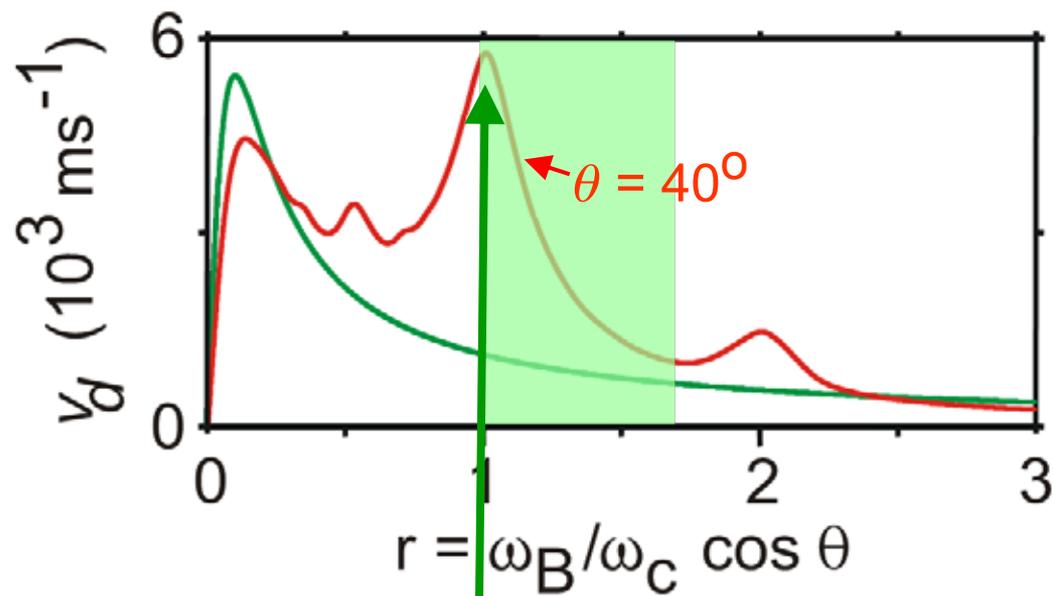
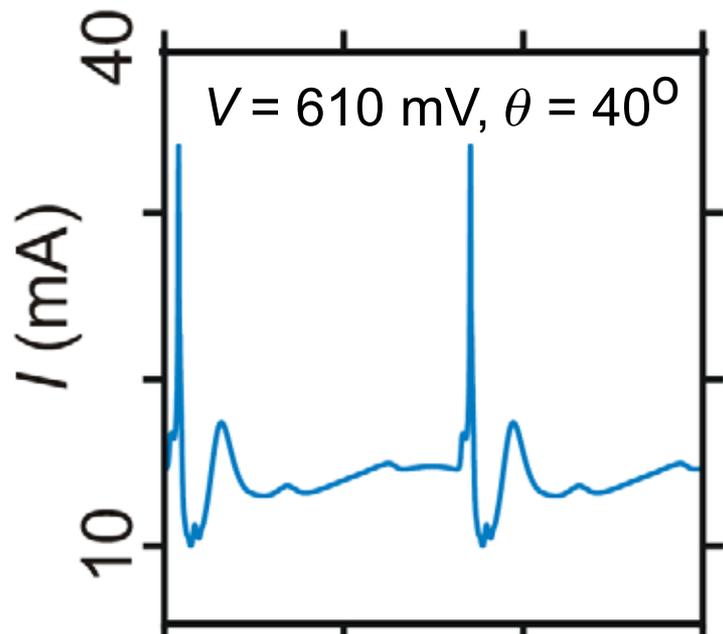


Collector



When $\theta = 0^\circ$, there is only one region of negative differential velocity

But when $\theta = 40^\circ$, there are 4 and each creates a charge domain



Collector

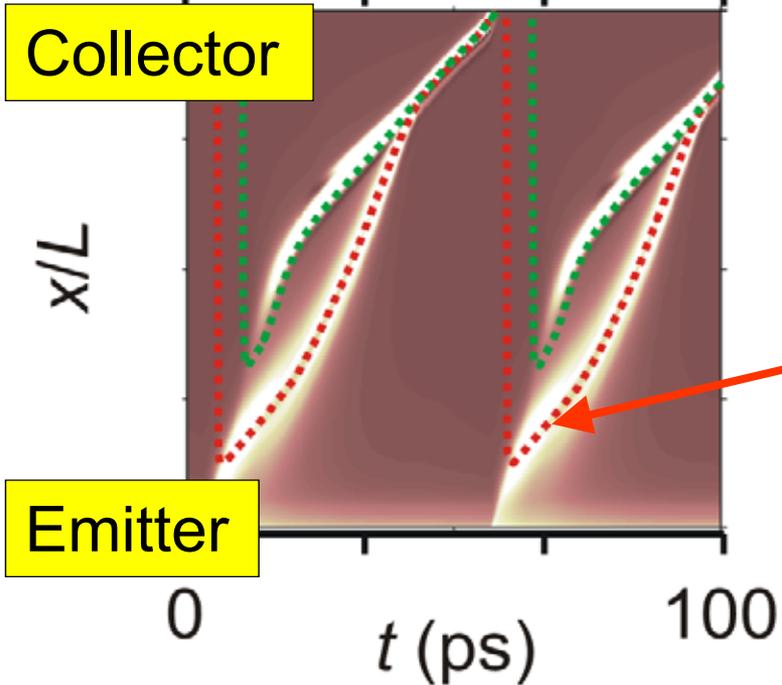
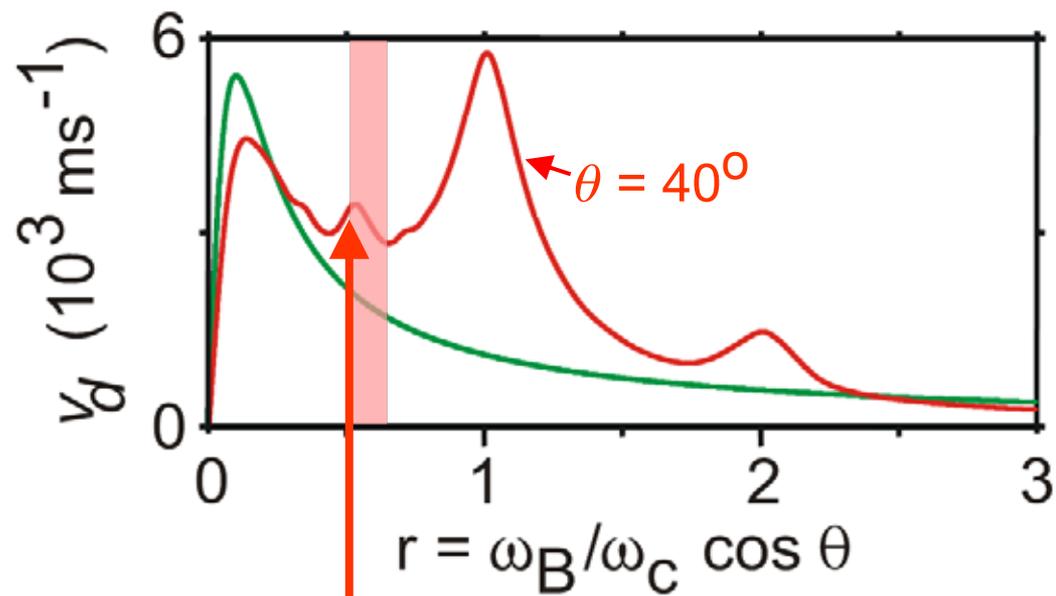
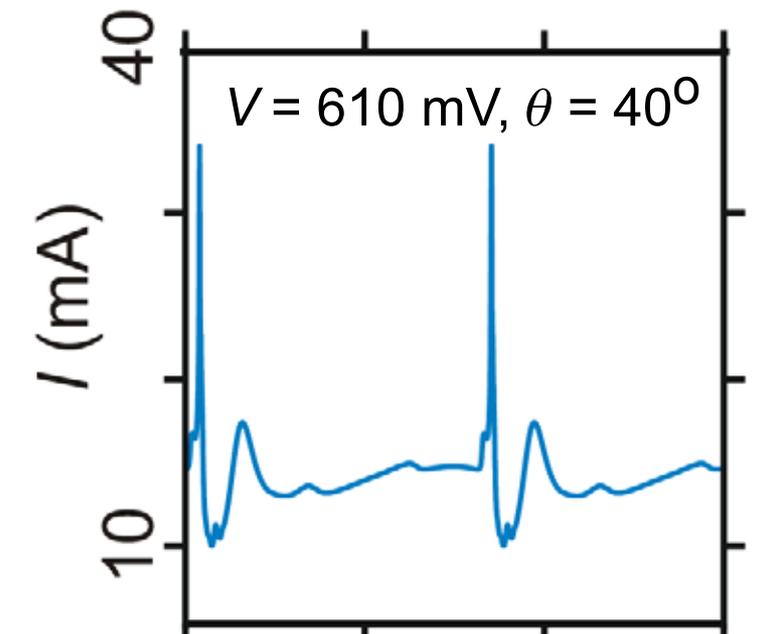
x/L

Emitter

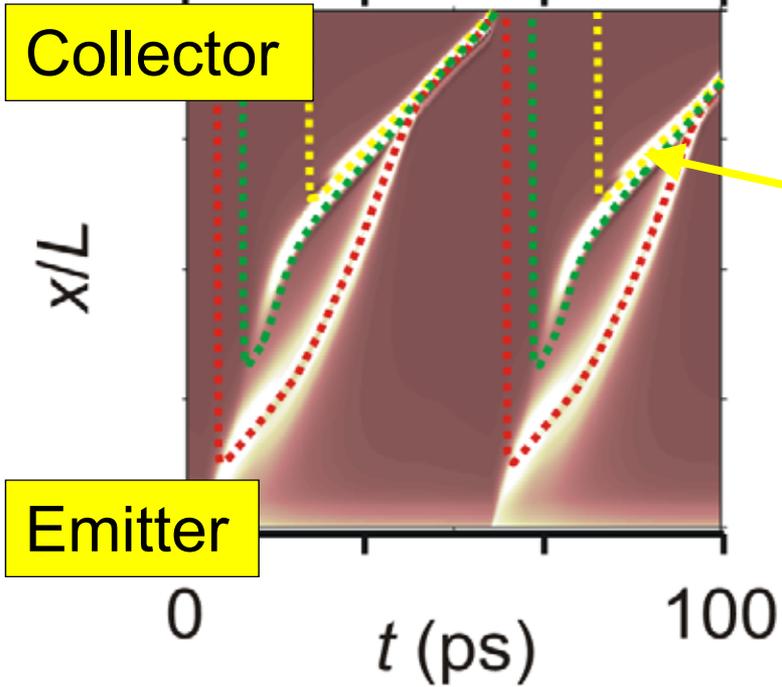
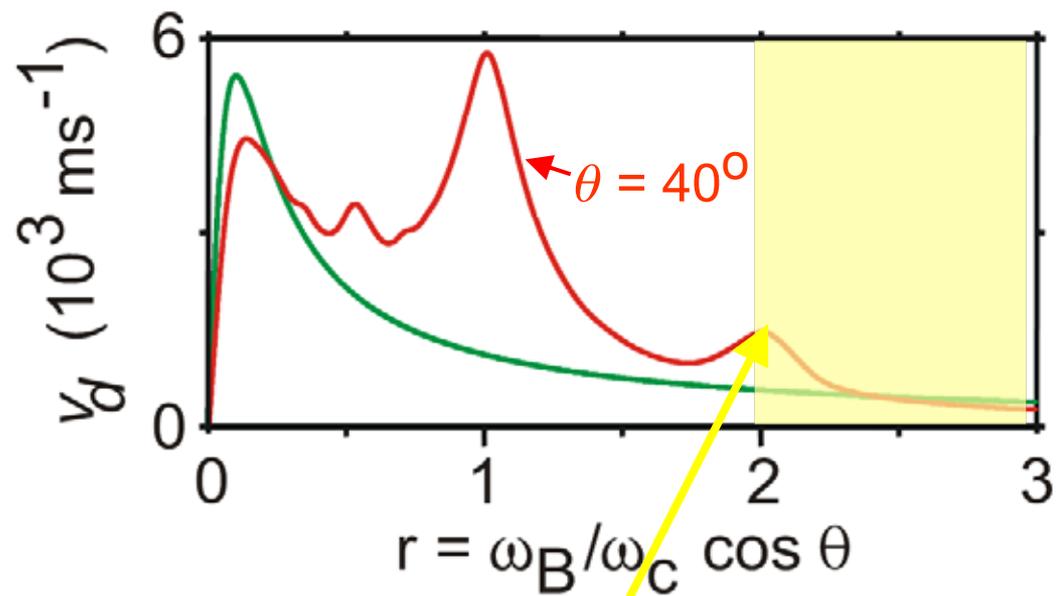
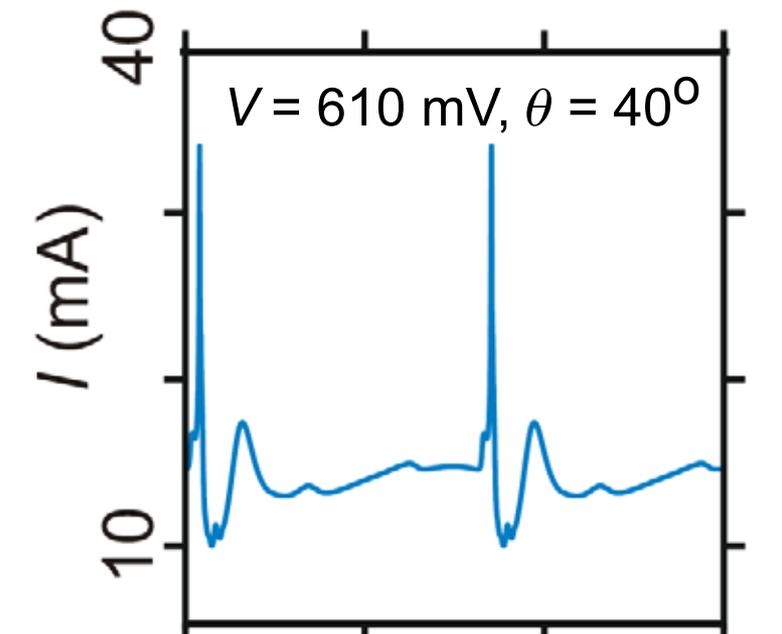
t (ps)

Strongest domain triggered by negative differential velocity region after $r = 1$ peak

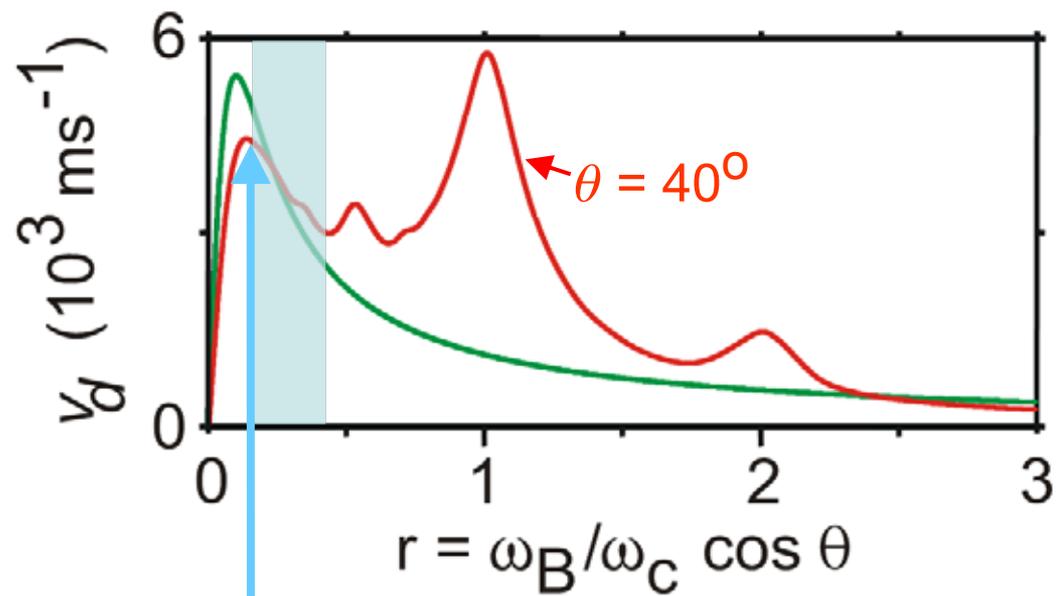
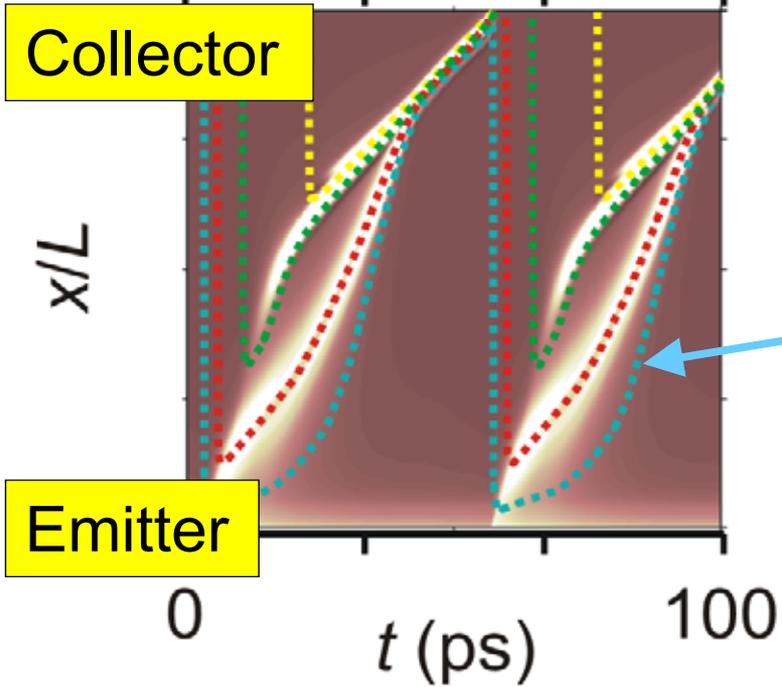
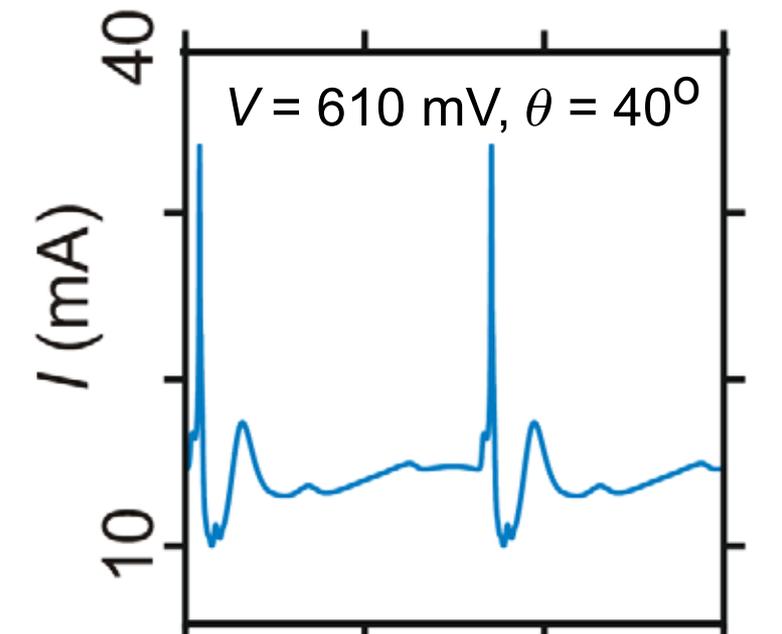
whose locus is shown dotted



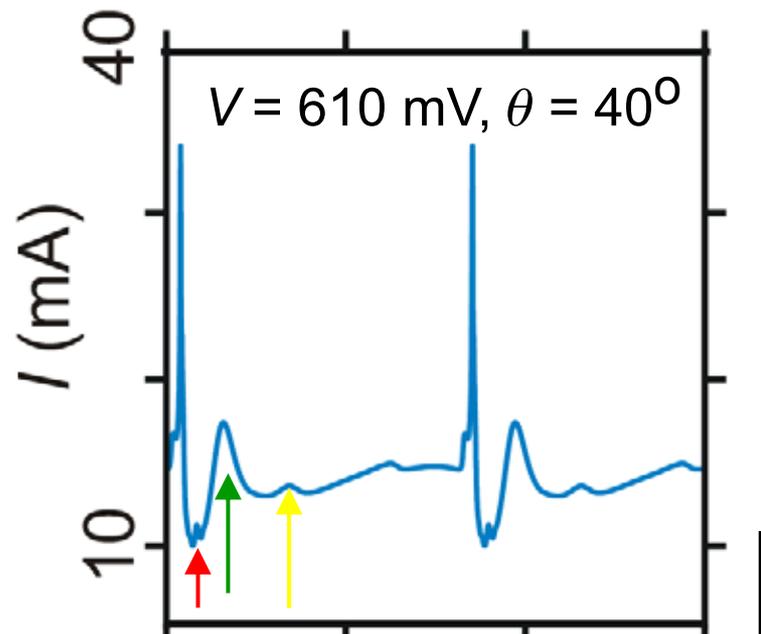
$r = 0.5$ peak
and locus



$r = 2$ peak
and locus



Esaki-Tsu-like peak and locus



Formation of $r = 0.5, 1$ and 2 domains creates additional peaks in $I(t)$

Collector

x/L

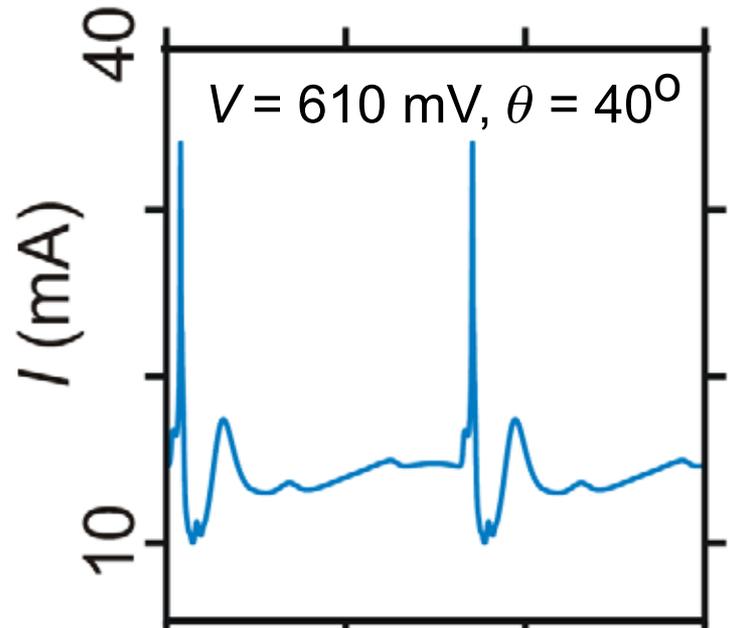
Emitter

0

100

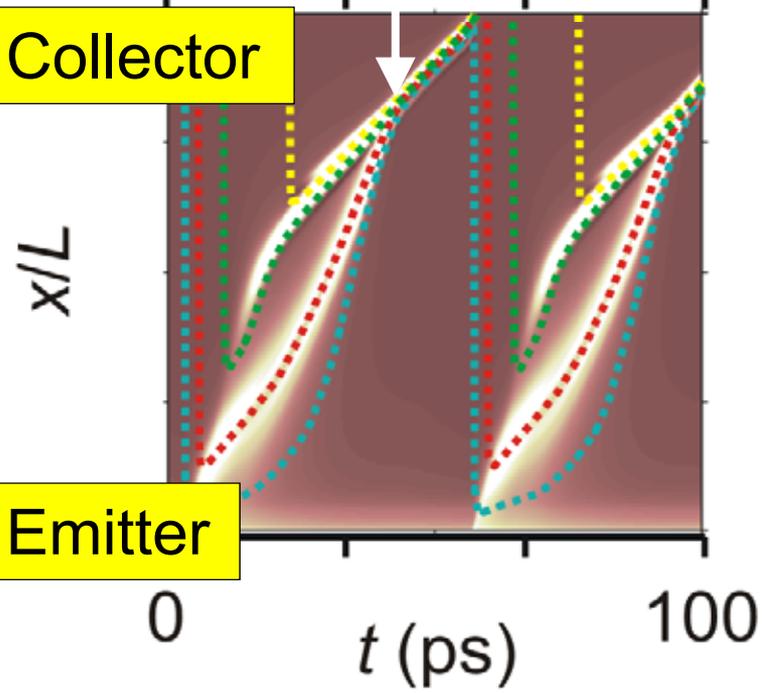
$t \text{ (ps)}$

$V = 610 \text{ mV}, \theta = 40^\circ$

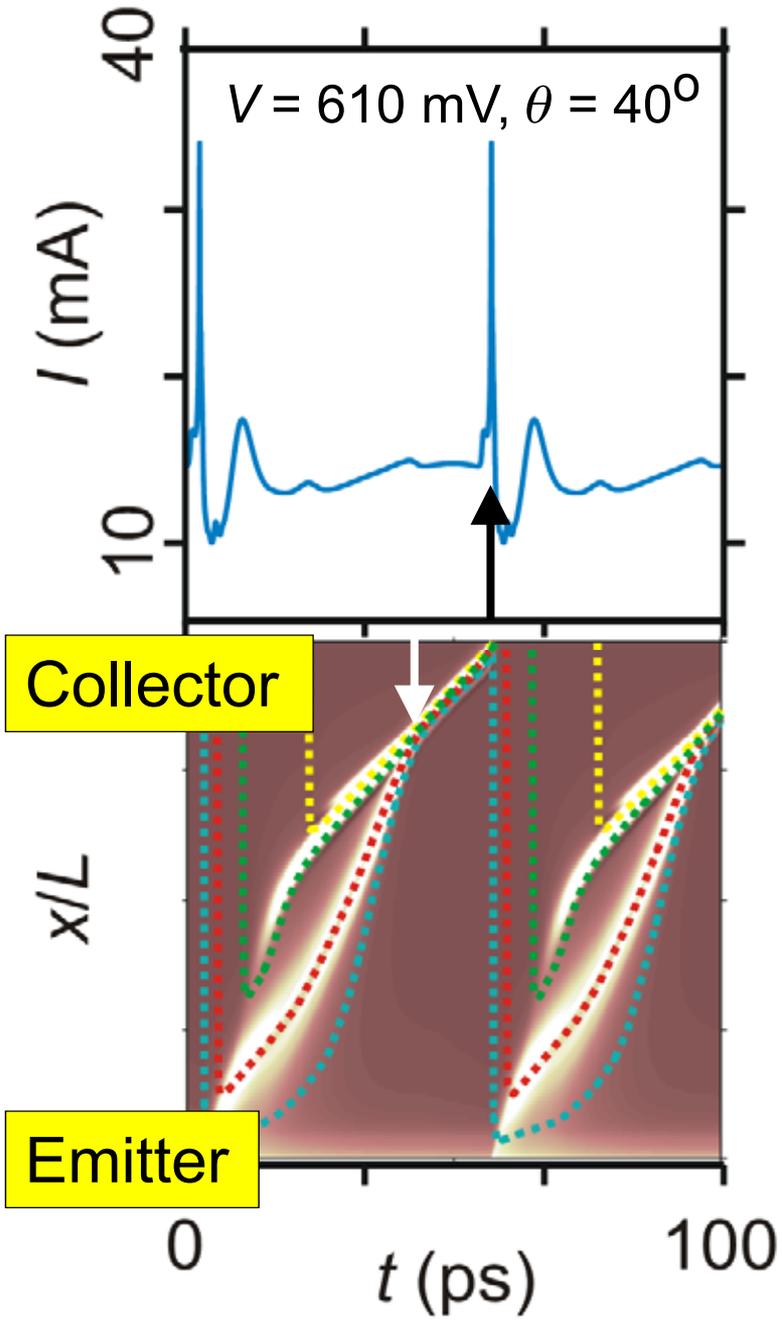


Collector

Emitter

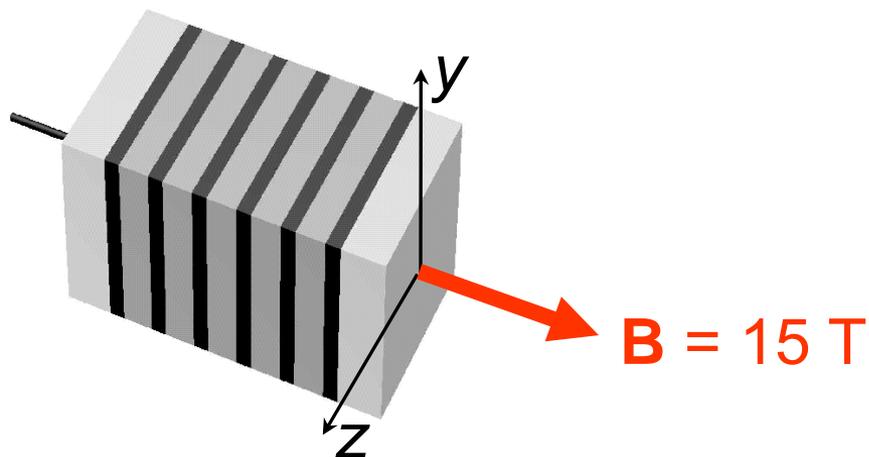
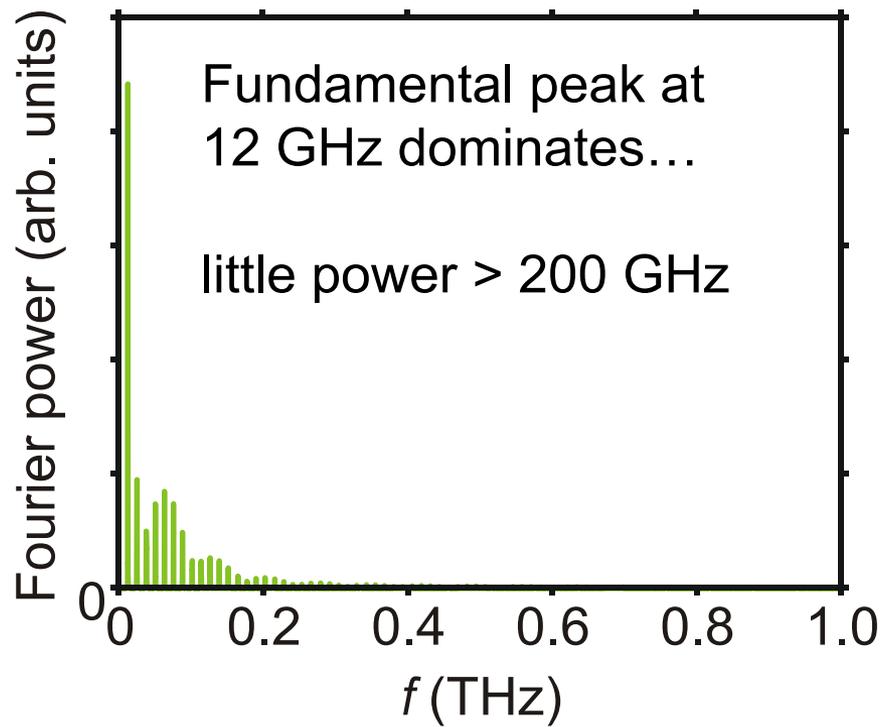
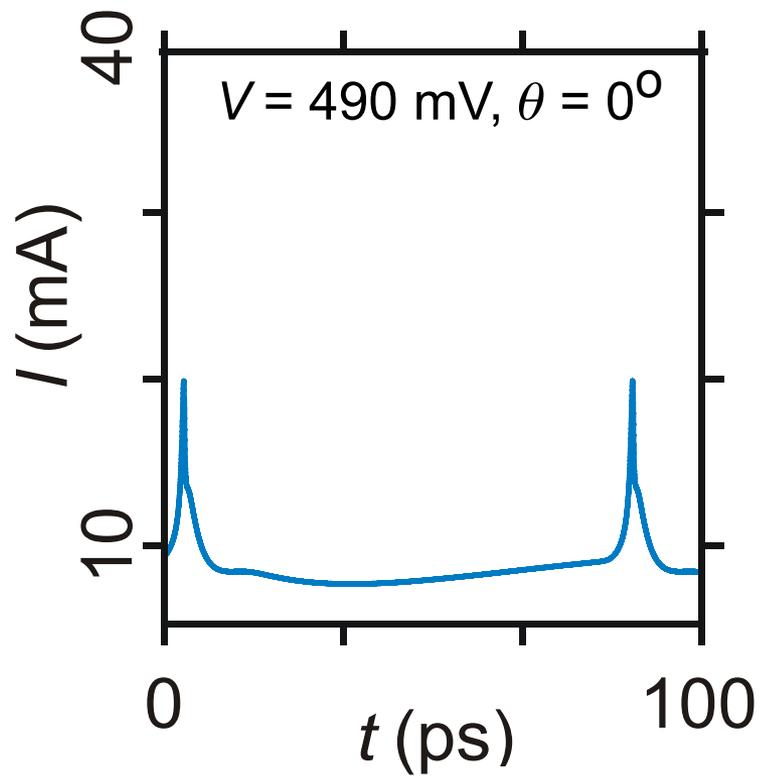


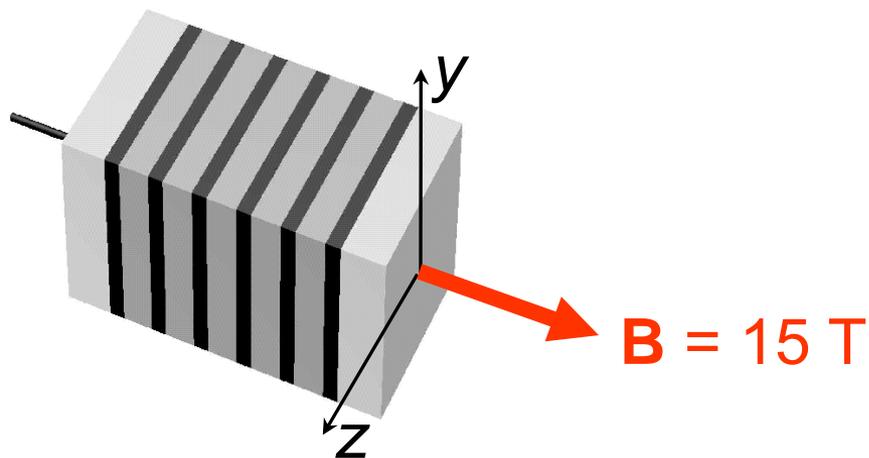
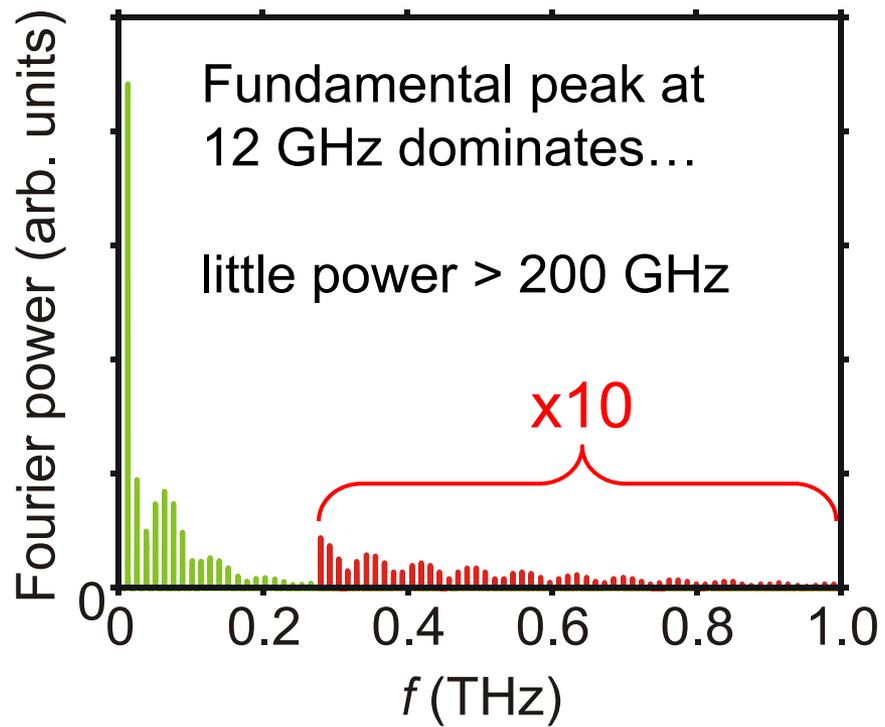
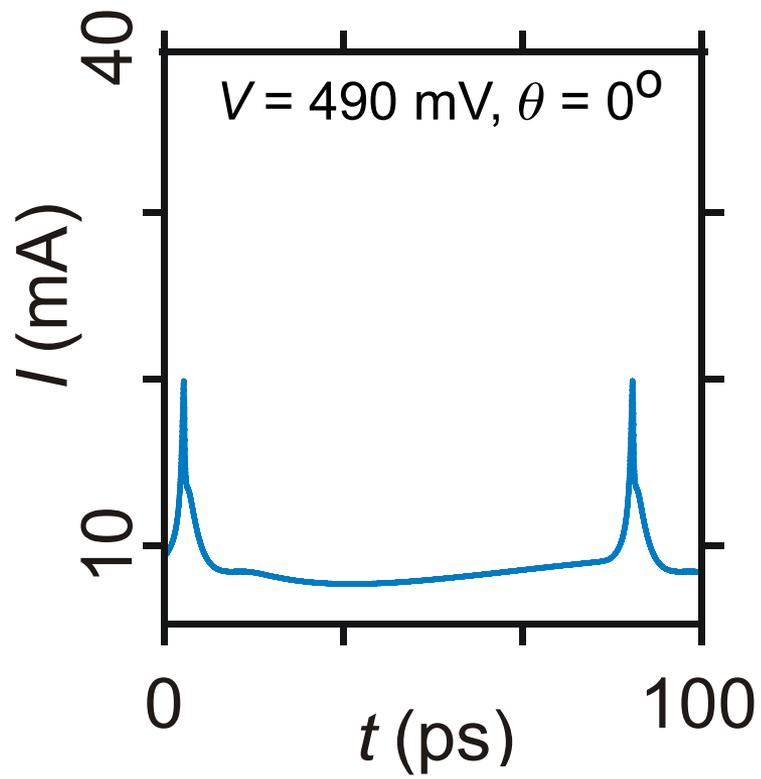
Eventually, the domains merge into one "superdomain"...

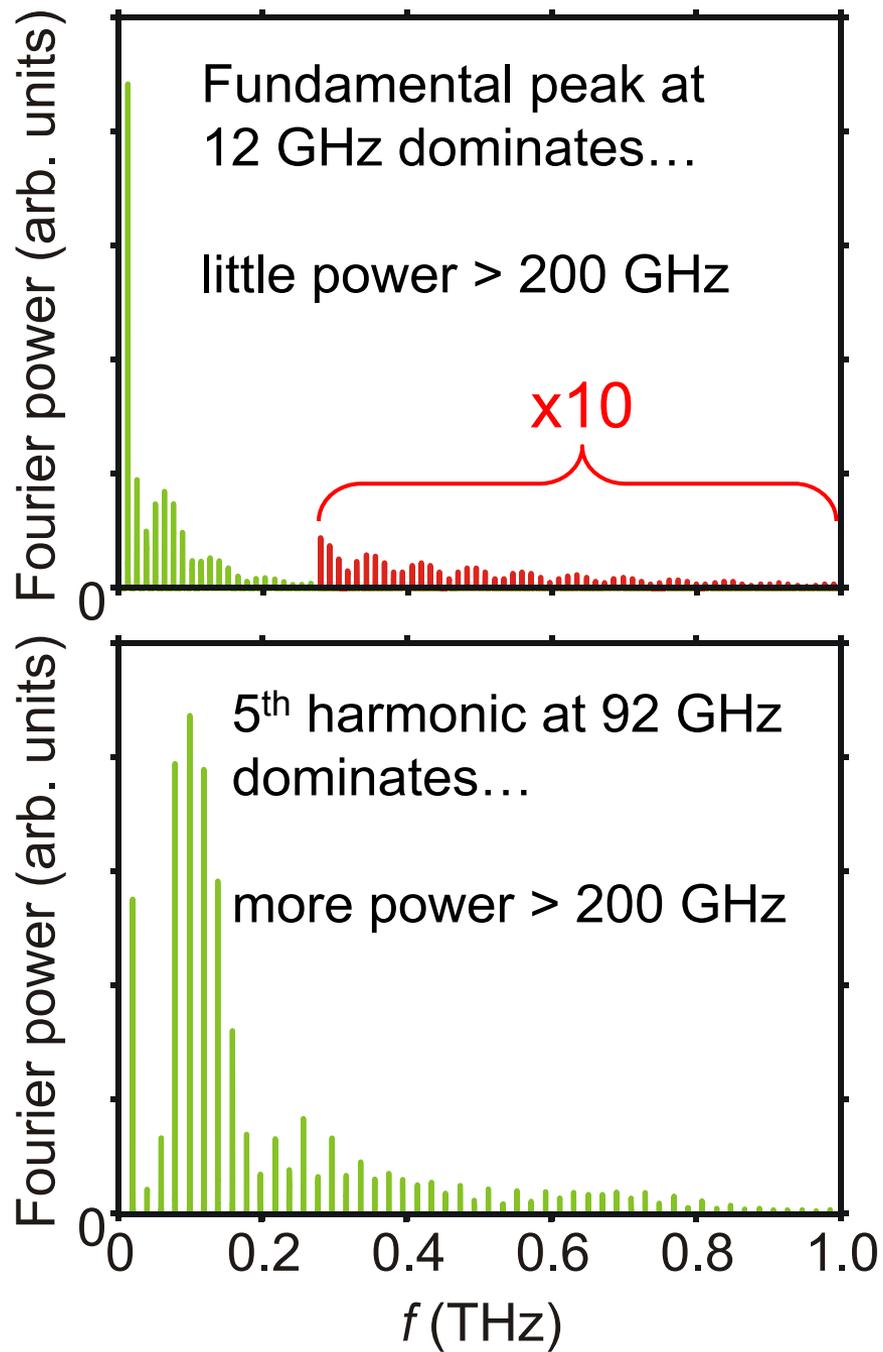
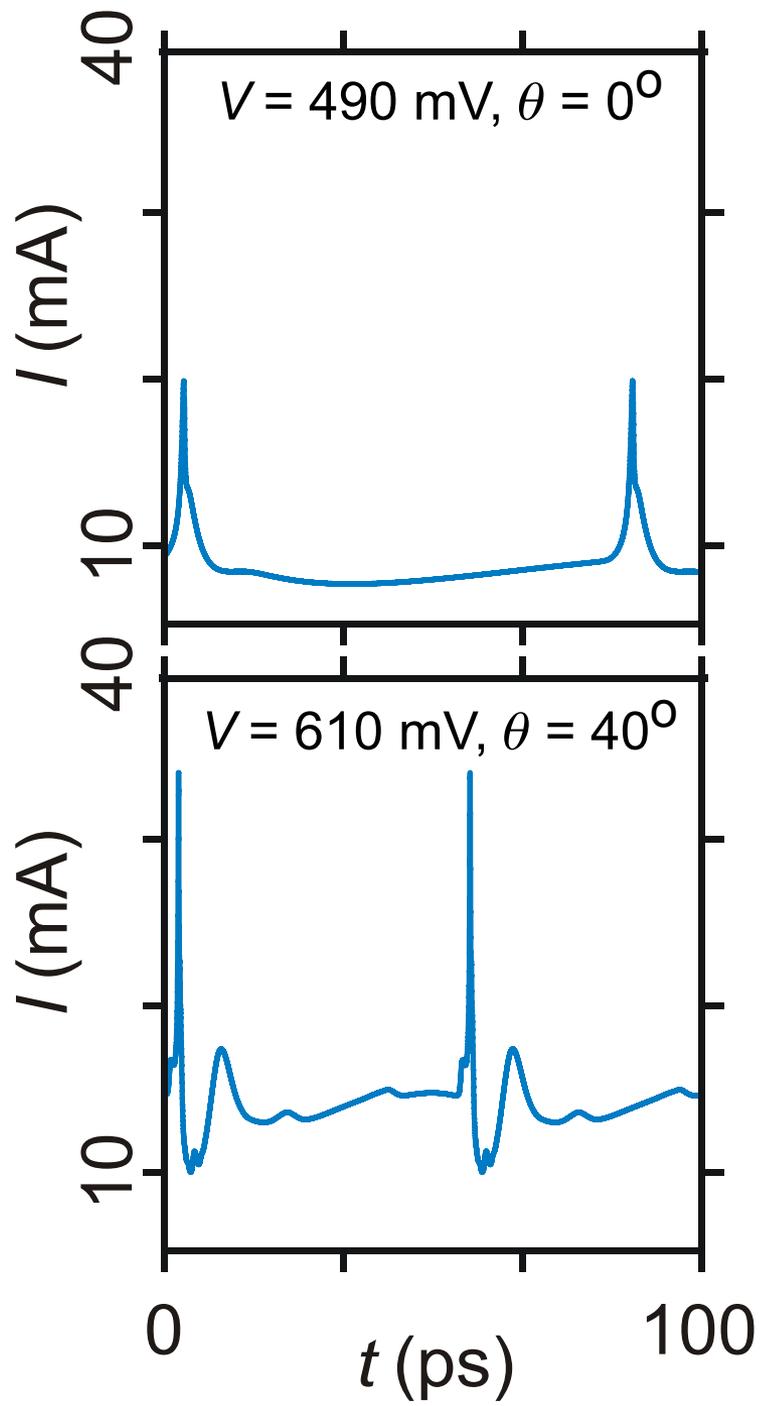


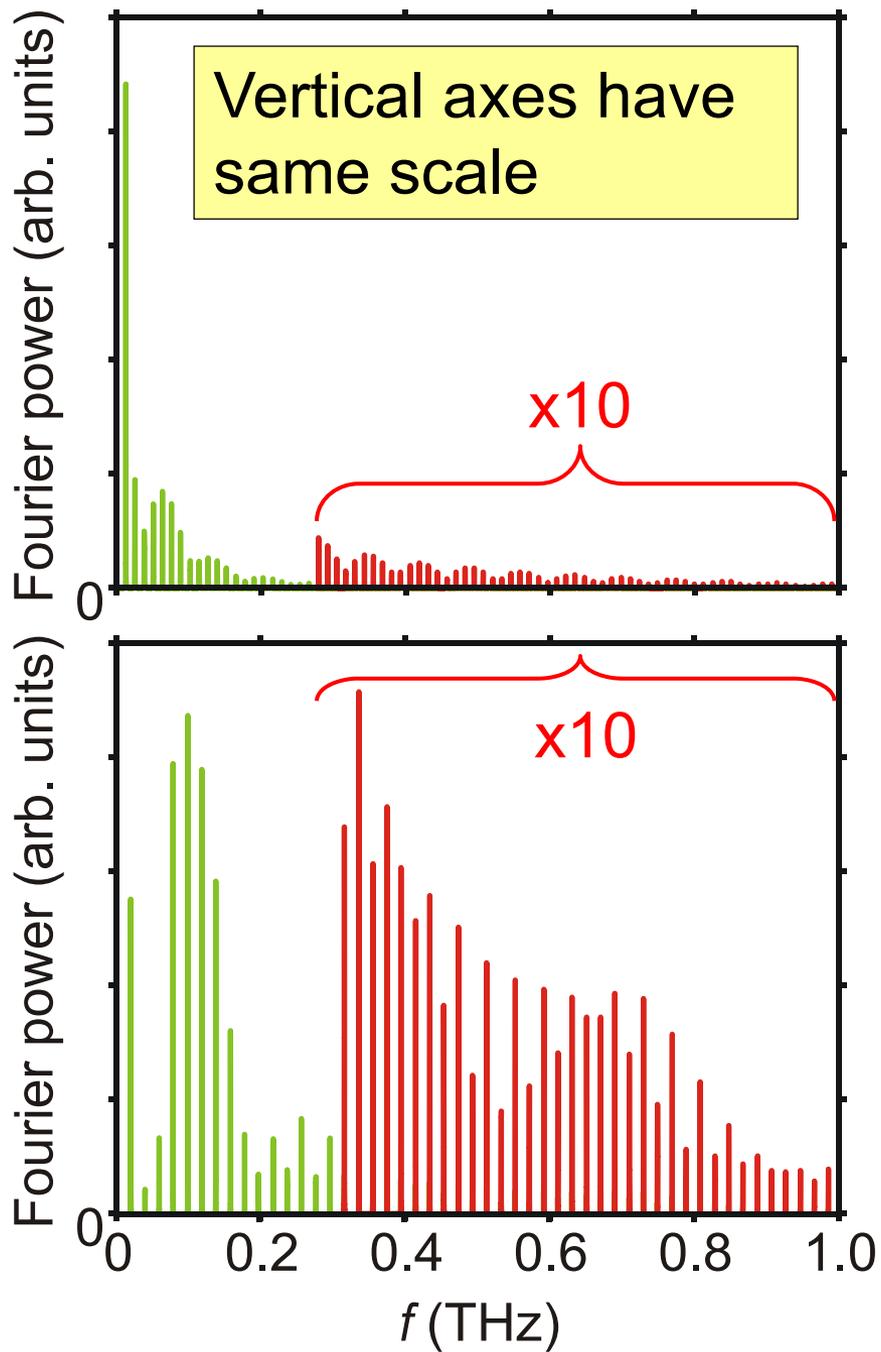
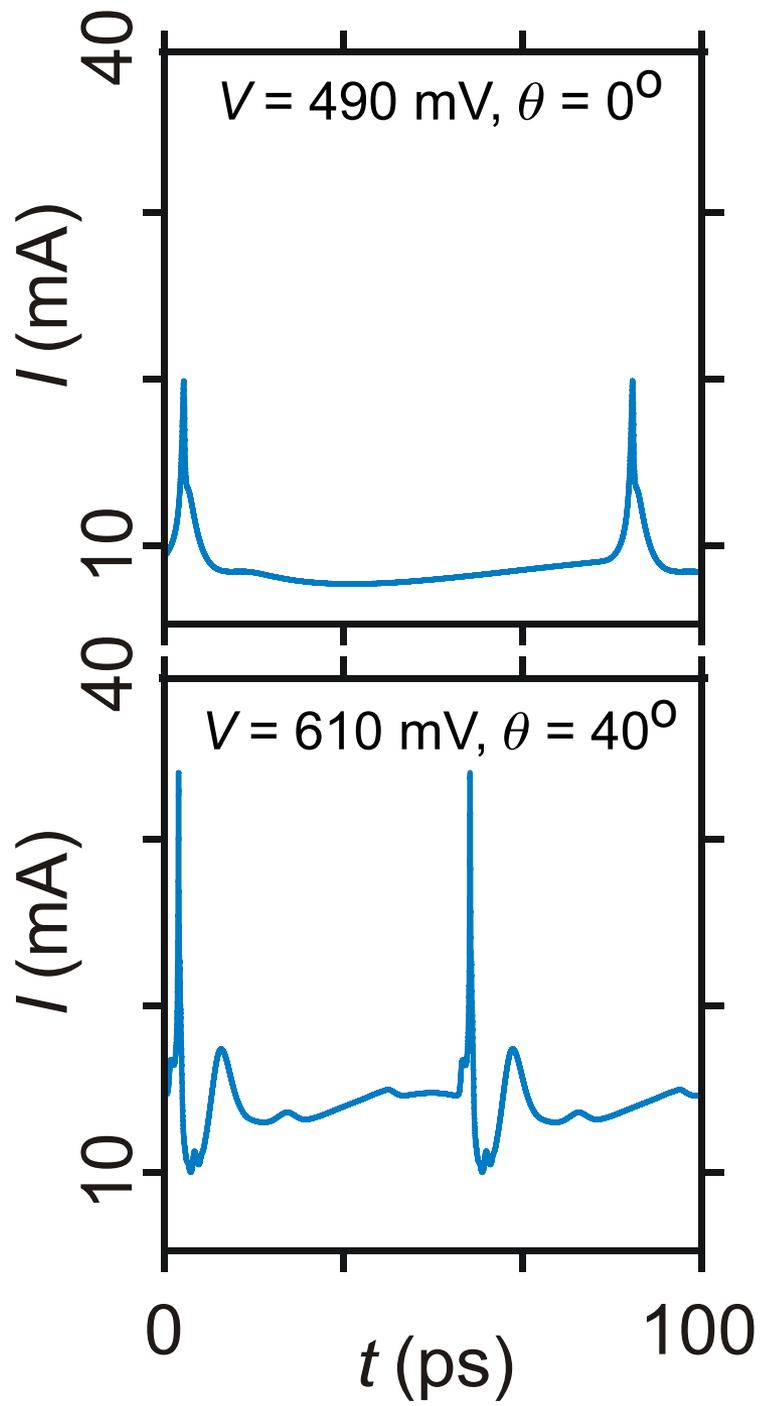
Eventually, the domains merge into one fast “superdomain”...
...which creates a large $I(t)$ peak when it arrives at the collector

Let's now compare Fourier power spectra of $I(t)$ for $\theta = 0^\circ$ and $\theta = 40^\circ$

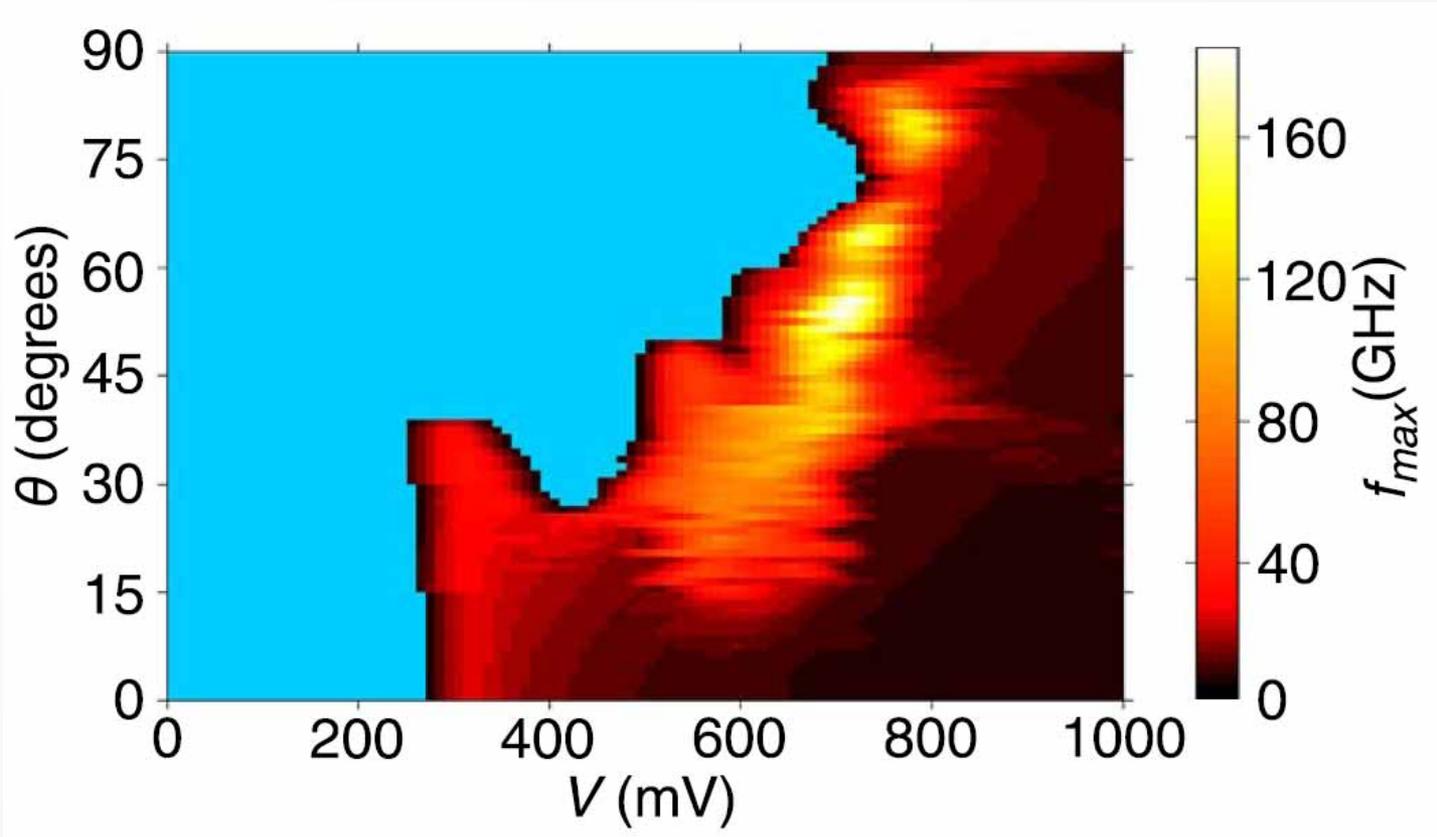






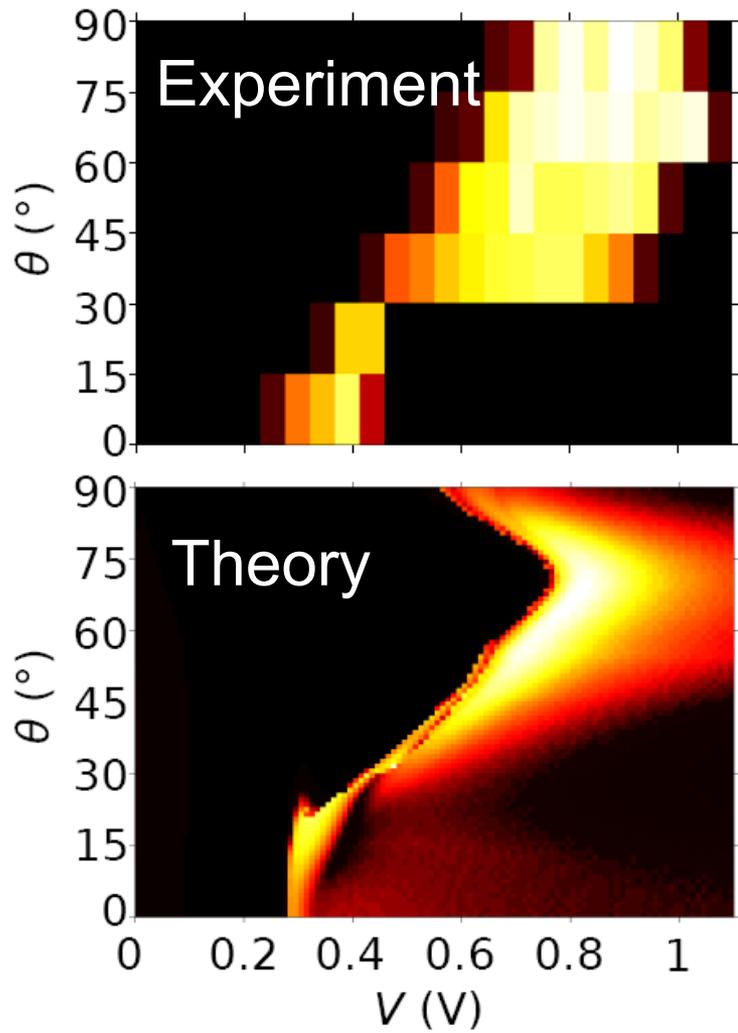


Frequency of the largest peak in the spectrum of current oscillations

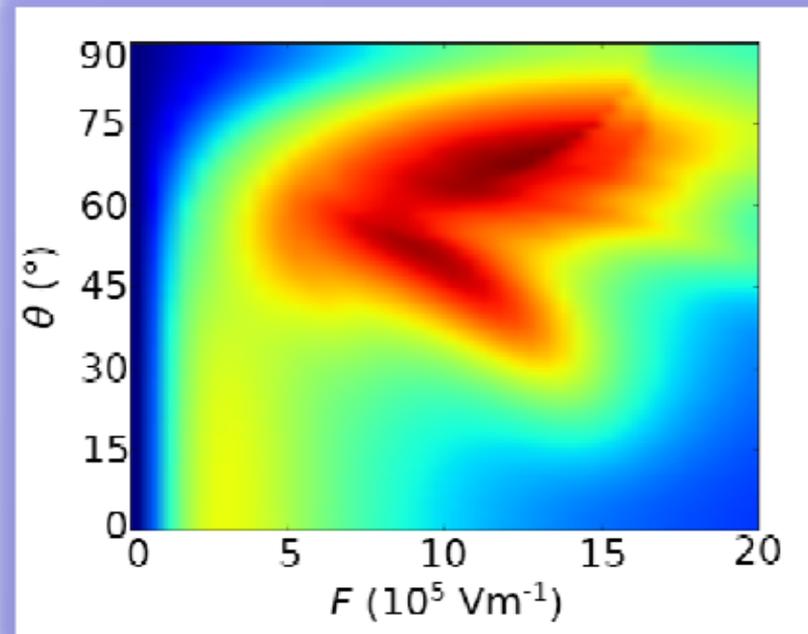


M. Greenaway et al, PRB 80 205308 (2009)

Total power of current oscillations



drift velocity (theory)



N. Alexeeva et al, 2011

Effect of temperature on drift velocity of electrons v_d

$$\dot{p}_x(t) = eF - \omega_{\perp} p_y(t)$$

$$\dot{p}_y(t) = \frac{d\Delta m^* \omega_{\perp}}{2\hbar} \sin\left(\frac{p_x(t)d}{\hbar}\right) - \omega_{\parallel} p_z(t)$$

$$\dot{p}_z(t) = \omega_{\parallel} p_y(t),$$

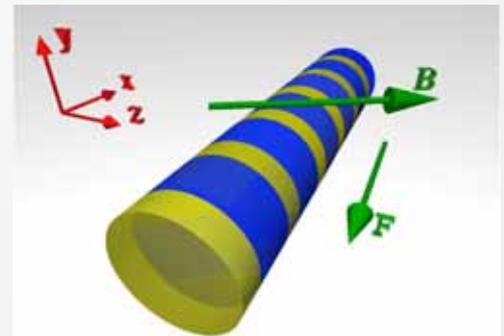
$$v_x(t) = \dot{x}(t) = v_0 \sin\left(\frac{p_x(t)d}{\hbar}\right)$$

Maximal miniband velocity of electron $v_0 = \Delta d / (2\hbar)$

Drift velocity for particular initial momentum $\mathbf{P}(P_x, P_y, P_z)$

$$u_d(\mathbf{P}) = \nu \int_0^{\infty} v_x(t) e^{-\nu t} dt.$$

$\nu = 1/\tau$ - scattering rate



Cyclotron frequencies

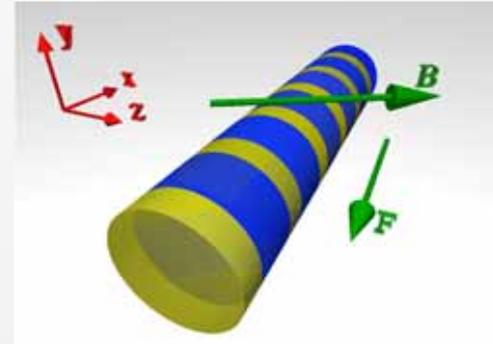
$$\omega_{\parallel} = eB \cos \theta / m^*$$

$$\omega_{\perp} = eB \sin \theta / m^*$$

Effect of temperature on drift velocity of electrons v_d

Drift velocity for particular initial momentum $\mathbf{P}(P_x, P_y, P_z)$

$$u_d(\mathbf{P}) = \nu \int_0^{\infty} v_x(t) e^{-\nu t} dt.$$



We assume the Boltzmann statistics for electron momenta

$$f(\mathbf{P}) = \frac{1}{Z} e^{-\frac{\Delta}{2k_B T} \left(1 - \cos \frac{P_x d}{\hbar}\right) - \frac{P_y^2 + P_z^2}{2m^* k_B T}}$$

$$Z = (2\pi)^2 m^* k_B T \frac{\hbar}{d} I_0 \left(\frac{\Delta}{2k_B T} \right) e^{-\frac{\Delta}{2k_B T}}$$

Averaged drift velocity of electrons

$$v_d = \int_{-\pi\hbar/d}^{\pi\hbar/d} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\mathbf{P}) u_d(\mathbf{P}) dP_x dP_y dP_z$$

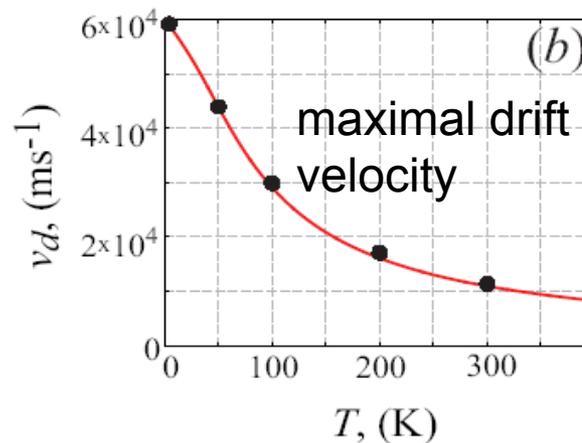
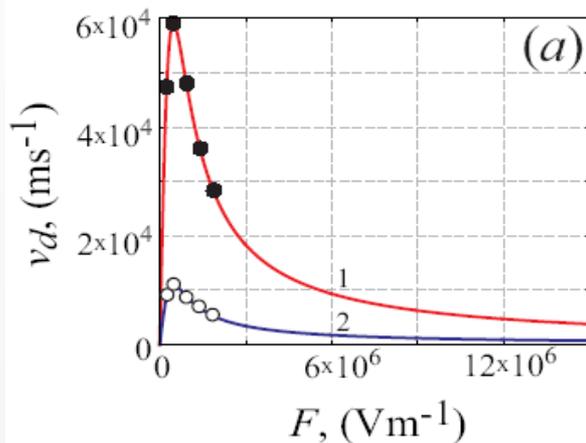
analytical approach vs numerical simulation for B=0

F.G. Bass et al, JETP Letters 31, 314 (1980):

$$v_d = v_0 \frac{I_1(\Delta/2k_B T)}{I_0(\Delta/2k_B T)} \exp \left[-m^* k_B T \left(\frac{\omega_{\perp} d}{\omega_{\parallel} \hbar} \right)^2 \right] \sum_{n=-\infty}^{\infty} I_n \left[m^* k_B T \left(\frac{\omega_{\perp} d}{\omega_{\parallel} \hbar} \right)^2 \right] \frac{\nu(\omega_B - n\omega_{\parallel})}{\nu^2 + (\omega_B - n\omega_{\parallel})^2}$$

If there is no magnetic field, $\mathbf{B}=0$, the formula becomes exact. It coincides with one derived, e.g. in Y.A. Romanov, Optika i Spektroskopiya 33, 917 (1972).

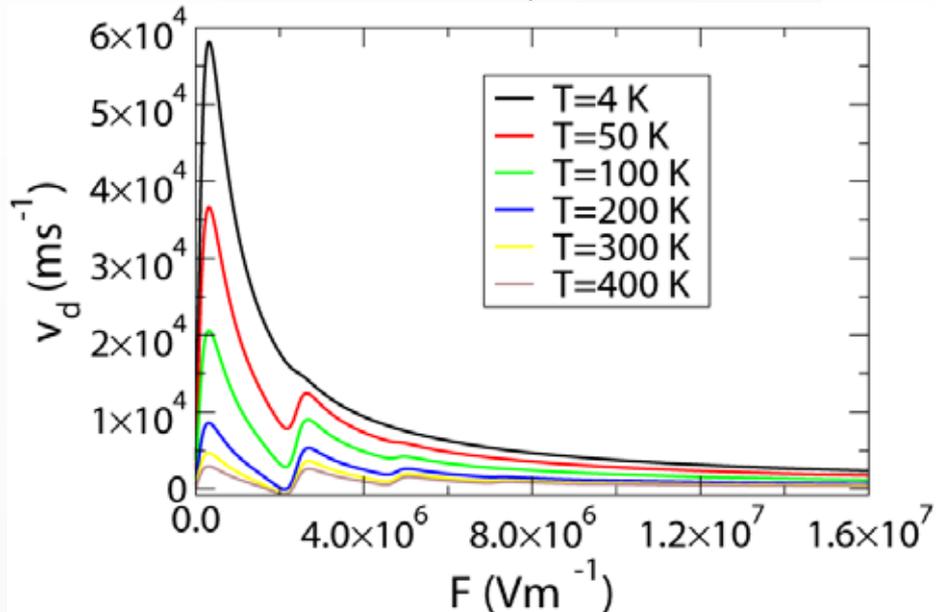
$$v_d = v_0 \frac{I_1(\Delta/2k_B T)}{I_0(\Delta/2k_B T)} \frac{\nu \omega_B}{\nu^2 + \omega_B^2}$$



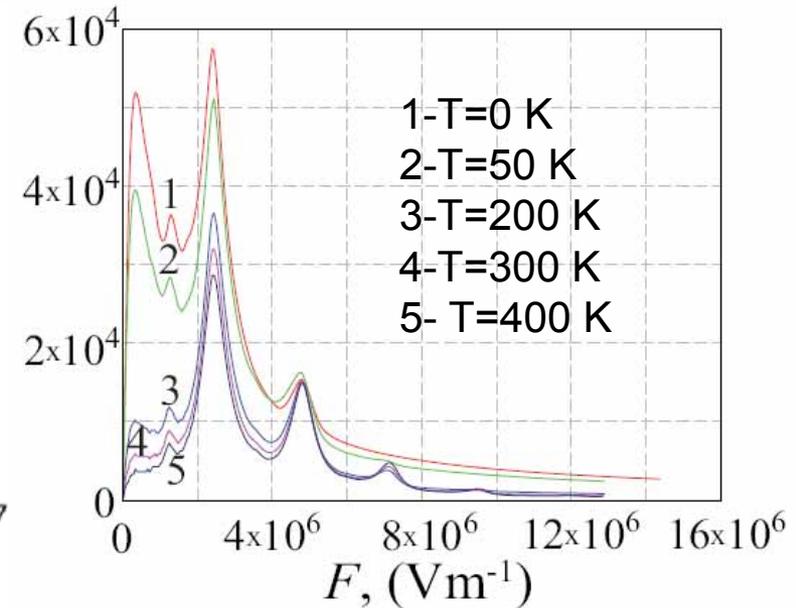
solid lines – analytics
symbol – numerical simulation

Effect of temperature on drift velocity of electrons

analytics

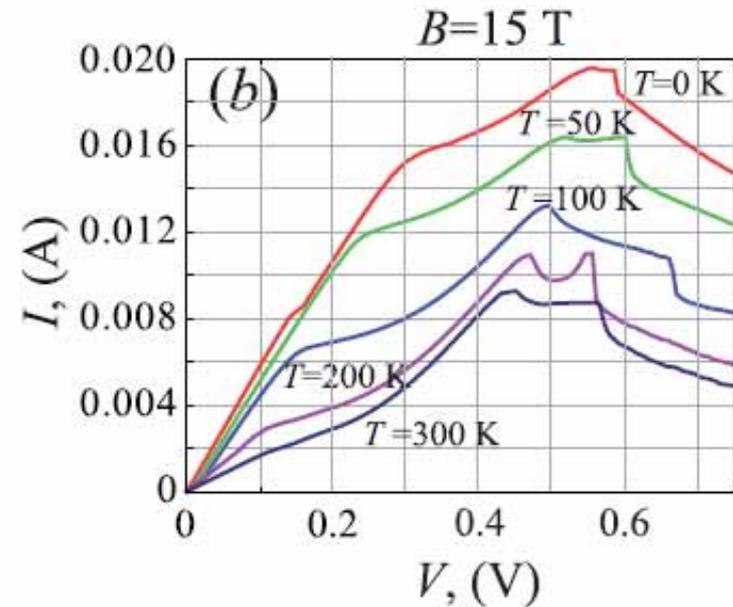
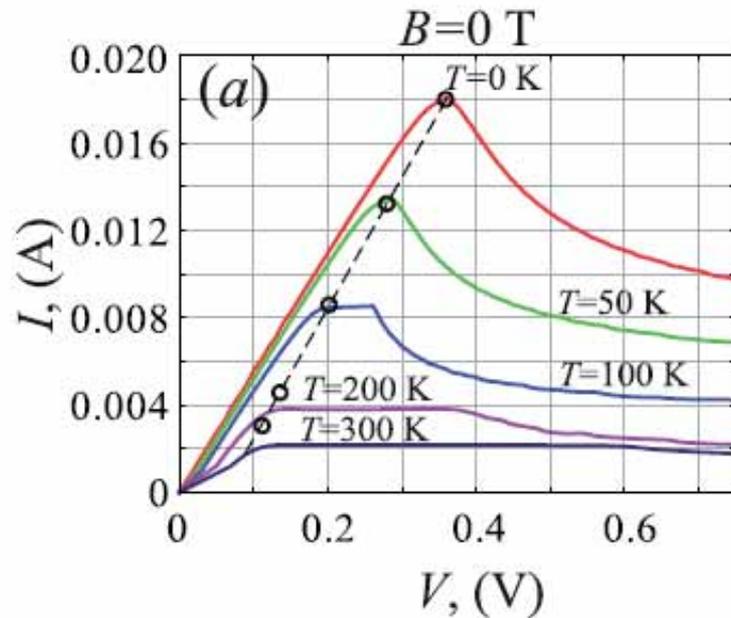


numerics



$B=15\text{ T}, \theta=40^\circ \rightarrow \omega_{\parallel} \gg \omega_{\perp} (!)$

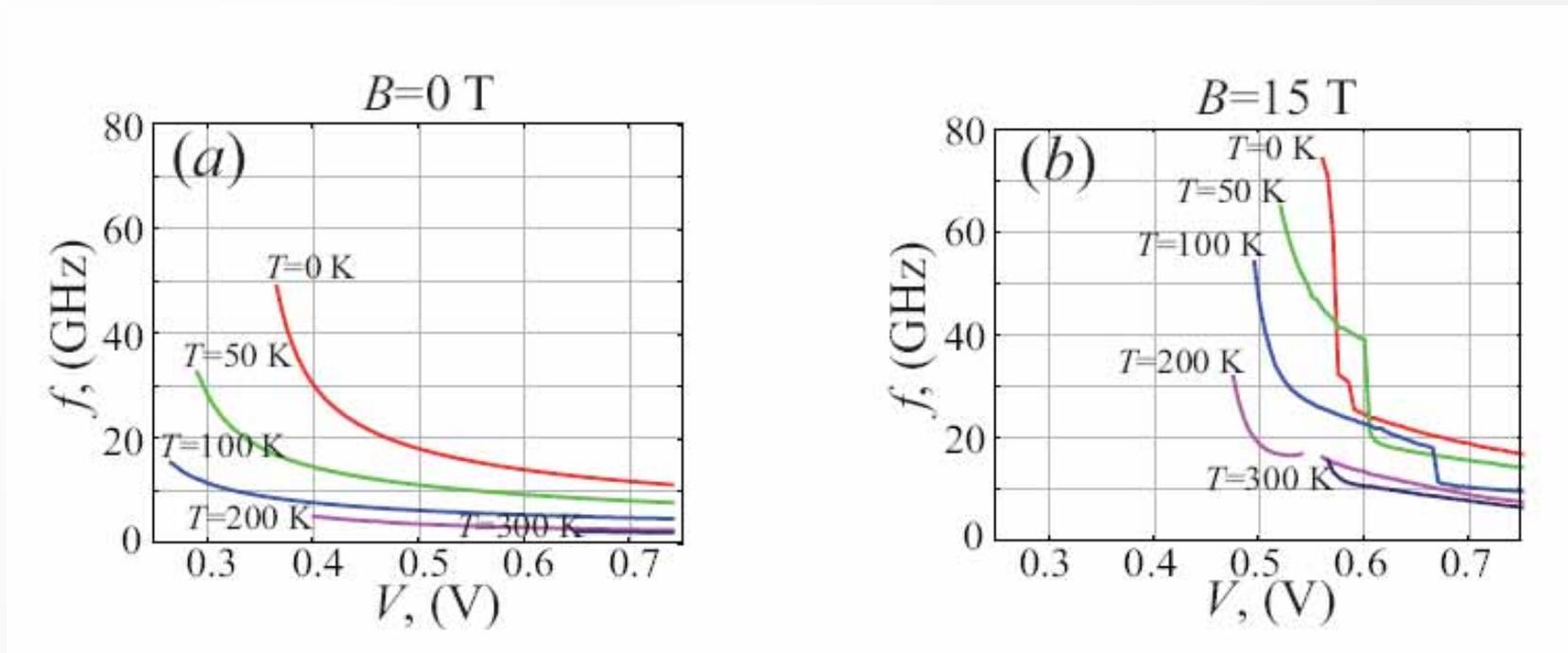
Effect of temperature on electric current: I-V curves



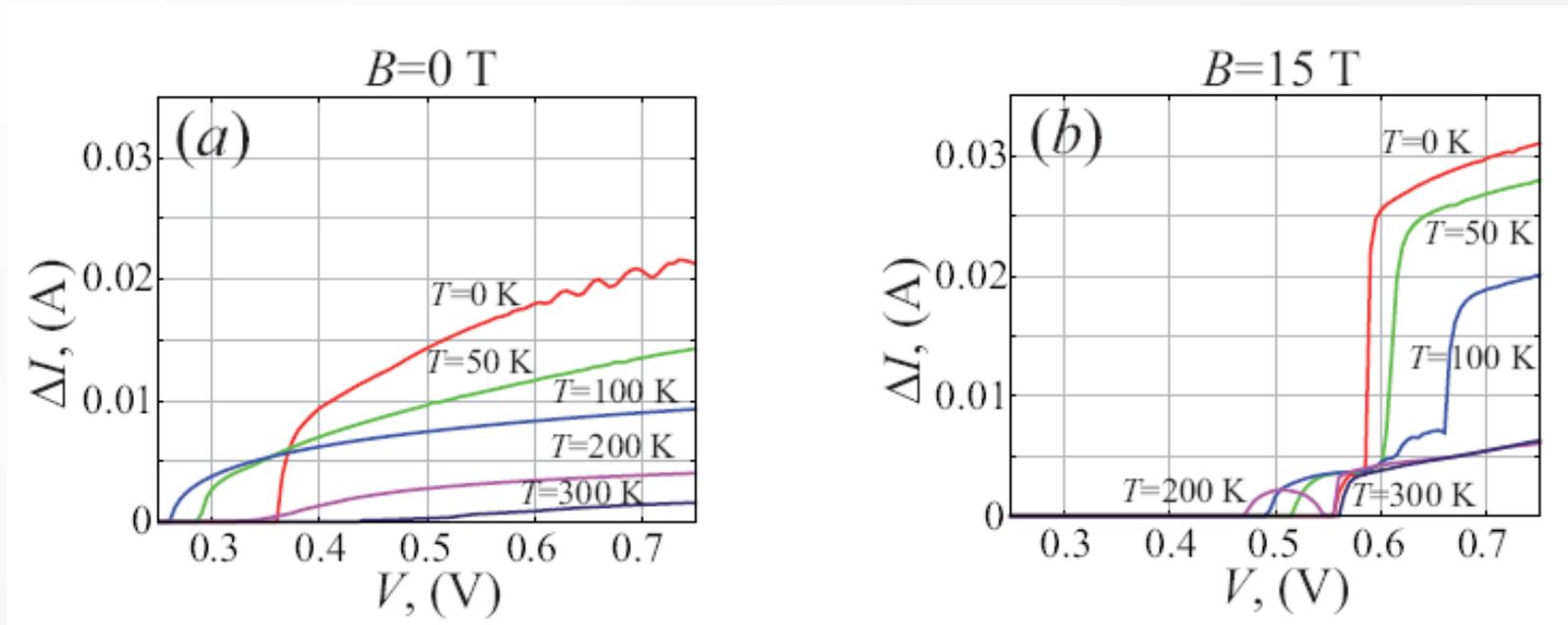
$$I_{th}(T) = I_{th}^0 \frac{I_1(\Delta/2k_B T)}{I_0(\Delta/2k_B T)}$$

$$V_{th}(T) = V_{th}^0 + R(I_{th}(T) - I_{th}^0)$$

Effect of temperature on electric current: frequency



Effect of temperature on electric current: amplitude



Conclusions

- A tilted magnetic field strongly affects, and can significantly enhance, the transport characteristics of SLs – even at room temperature.
- Increasing T quickly suppresses the $B = 0$ Esaki-Tsu peak in the $v_d(F)$ curve, but has a much smaller effect when a tilted magnetic field is applied. Increasing T can enhance the drift velocity peaks caused by the Bloch-cyclotron resonances
- At the presence of magnetic field electrons demonstrate much higher mobility, which amplifies DC- and AC-components of the current through SL, and also improves its frequency characteristics.

Related systems

- Miniband electrons driven by an acoustic wave

Greenaway et al., *Phys. Rev. B* **81**, 235313 (2010)

- Semiclassical predictions of very high NDV now supported by recent wavepacket and charge domain studies

- Ultracold atoms in an optical lattice with a tilted harmonic trap

Scott et al., *Phys. Rev. A* **66**, 023407 (2002)

- Intrinsically low scattering rates
- Could using Bose-Fermi interactions to control scattering rate ?

[Ponomarev, Madroñero, Kolovsky, Buchleitner, *Phys. Rev. Lett.* **96**, 050404 (2006)]

- Optical analogue

Wilkinson & Fromhold, *Optics Letters* **28**, 1034 (2003)