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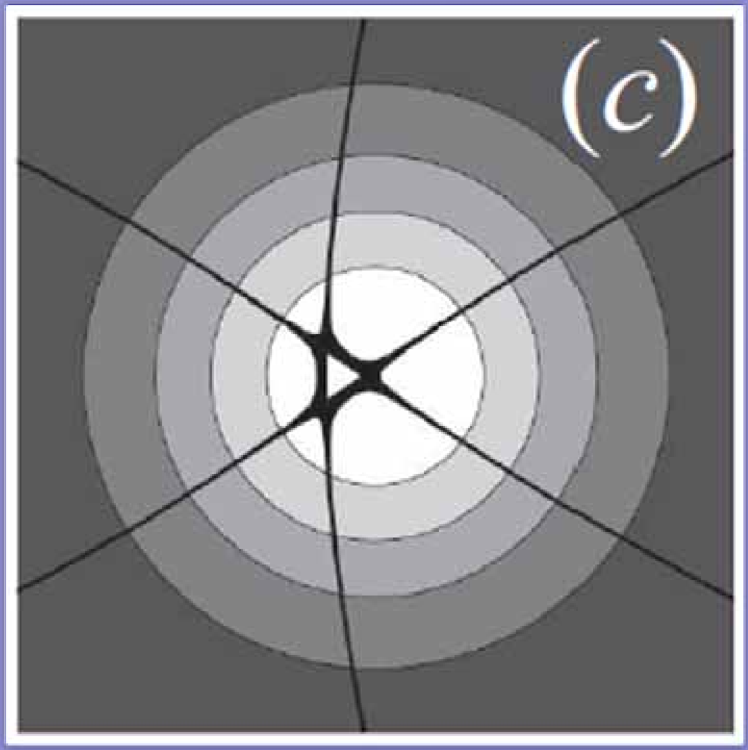


Формирование и эволюция доменов заряда в полупроводниковой сверхрешетке в присутствии внешнего наклонного магнитного поля, эффекты температуры

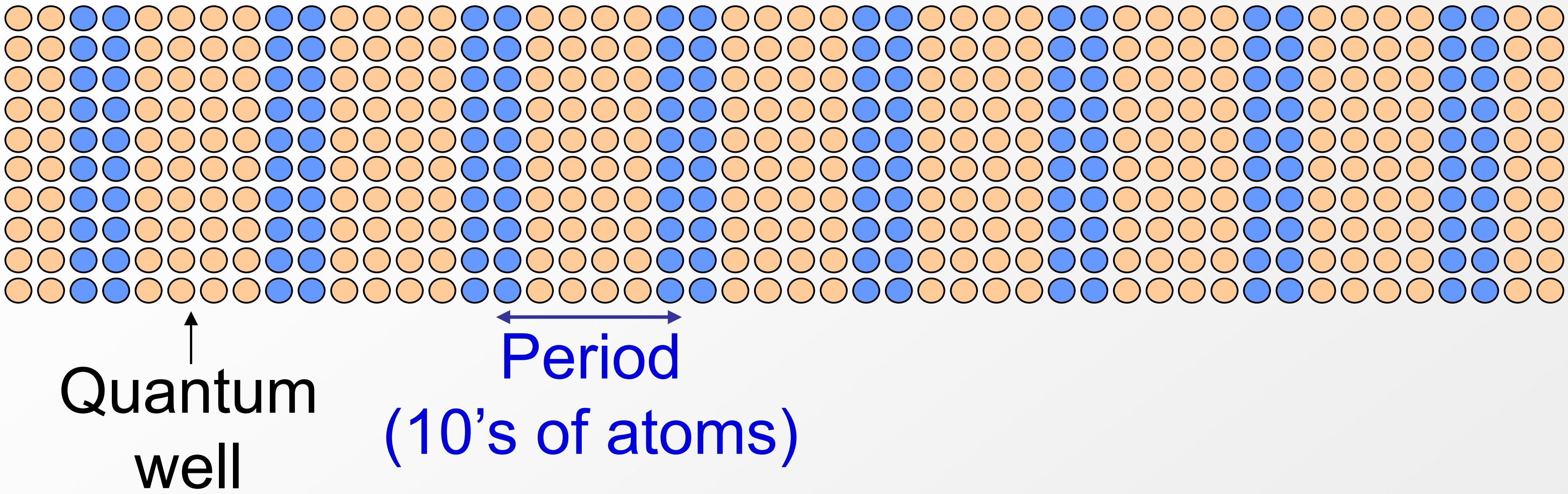
Александр Баланов

a.balanov@lboro.ac.uk

Collaborators

M.T. Greenaway (University of Nottingham, UK)
T.M. Fromhold (University of Nottingham, UK)
A.O. Selskii (Saratov State University, Russia)
A.A. Koronovskii (Saratov State University, Russia)
A.E. Khramov (Saratov State University, Russia)
K.N. Alekseev (Loughborough University, UK)
A.V. Shorokhov (Mordovian State University, Russia)
N.N. Khvastunov (Mordovian State University, Russia)
N. Alexeeva (Nottingham/Loughborough UK)
A. Patanè (University of Nottingham, UK)
O. Makarovsky (University of Nottingham, UK)
M. Gaifullin (Loughborough University, UK)
L. Eaves (University of Nottingham, UK)
F.V. Kusmartsev (Loughborough University, UK)

SEMICONDUCTOR SUPERLATTICE

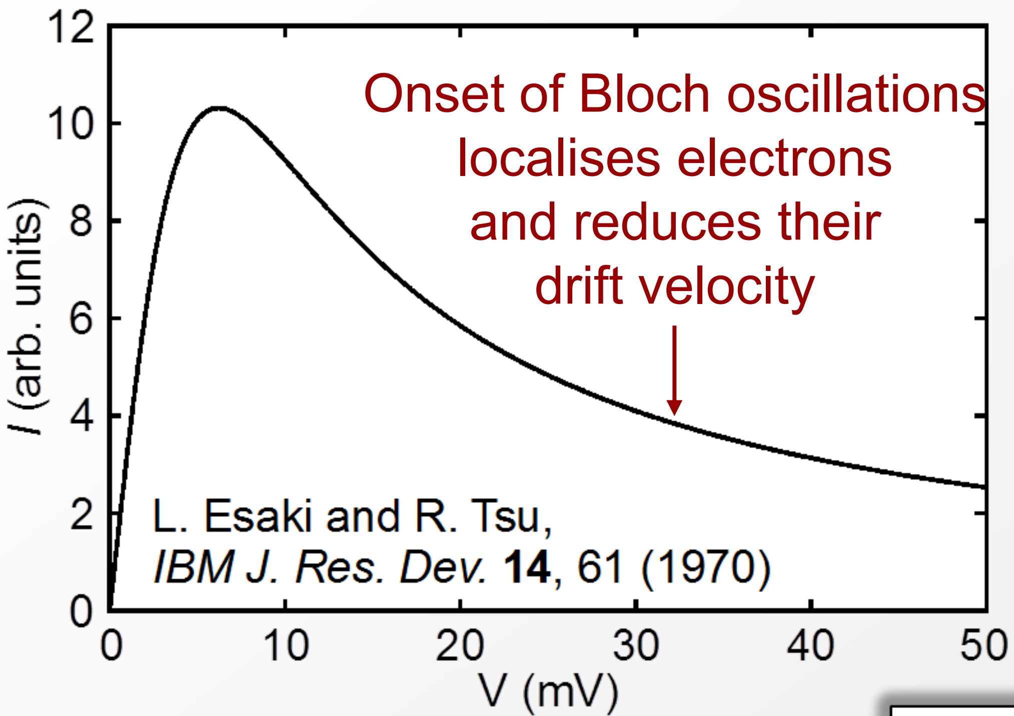


Chain of quantum wells forms a 1D periodic structure similar to a crystal lattice

But since the lattice period is much longer than for any natural crystal, this type of structure is known as a "Superlattice"

Drift velocity of electron, v_d

$I \sim v_d$



$$eF = \frac{dp_x}{dt}$$

$$v_x = \frac{dE}{dp_x}$$

$$E(p_x) = \Delta\left(1 - \cos\frac{p_x d}{\hbar}\right)$$

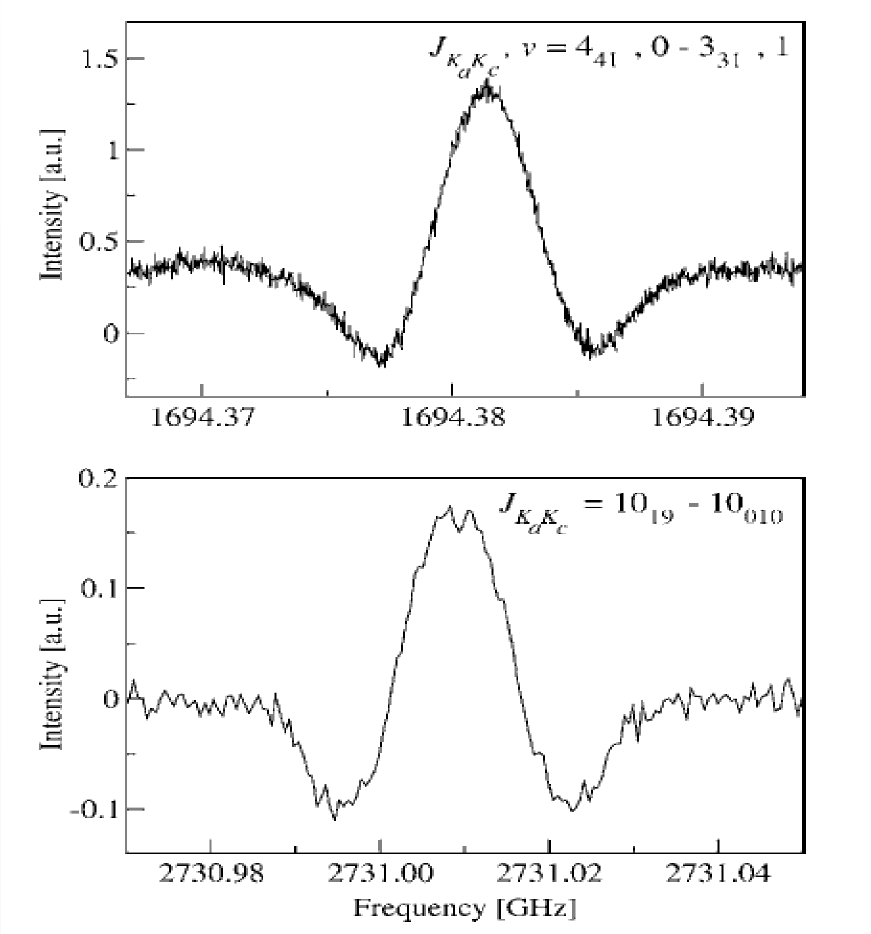
$$v_d(F) = \frac{1}{\tau}\sum\int_0^\infty v_x(t)\exp(-t/\tau)\,dt$$

a.balanov@lboro.ac.uk

THz applications of SL

- C.P. Edres et al, Rev. Sci. Instr. 78 043106 (2007)

Frequency multiplier in high-resolution THz spectroscopy
(room temperature)

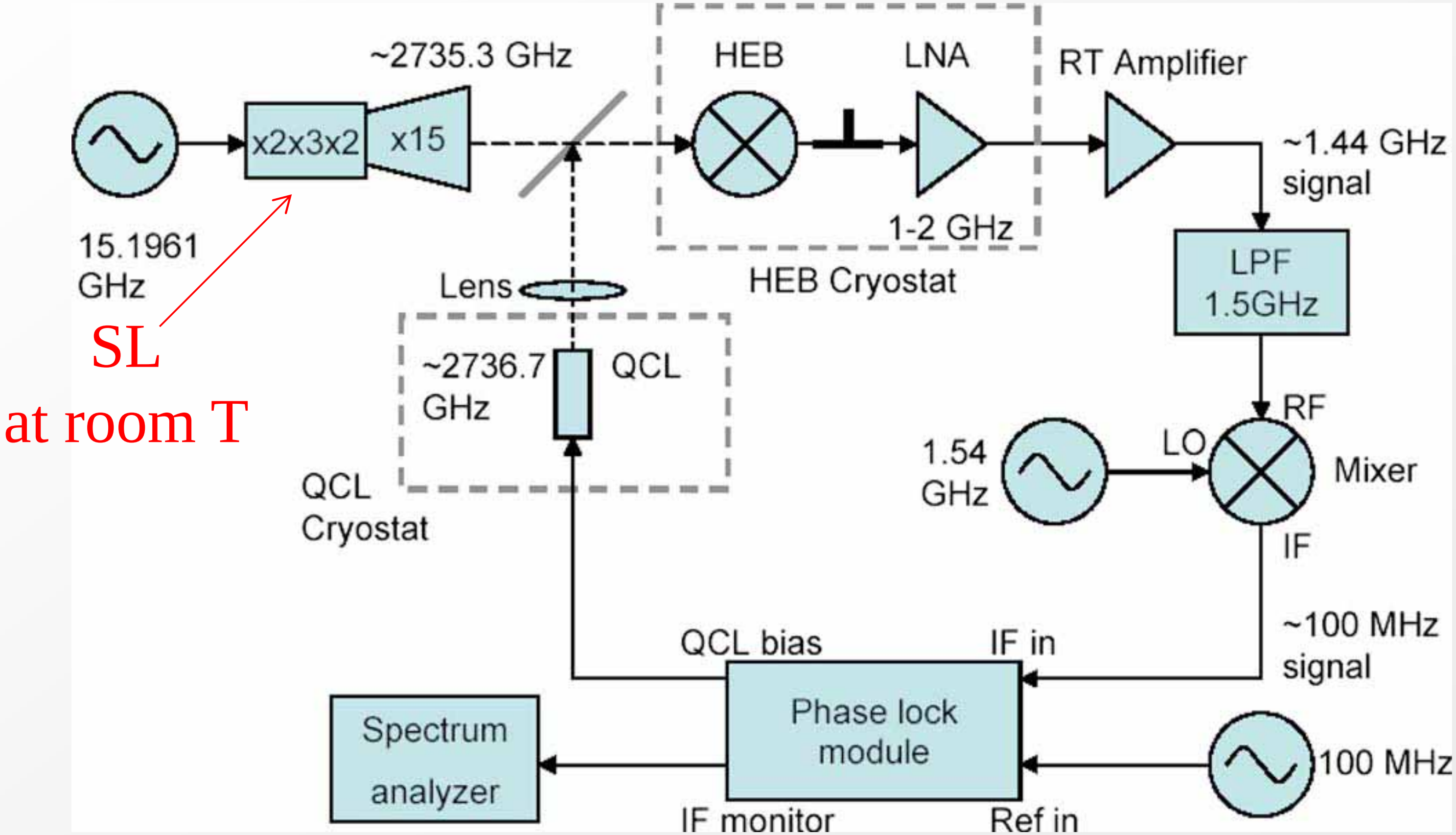


Spectra of ND_2H (upper panel) and D_2O (lower panel)

THz applications of SL

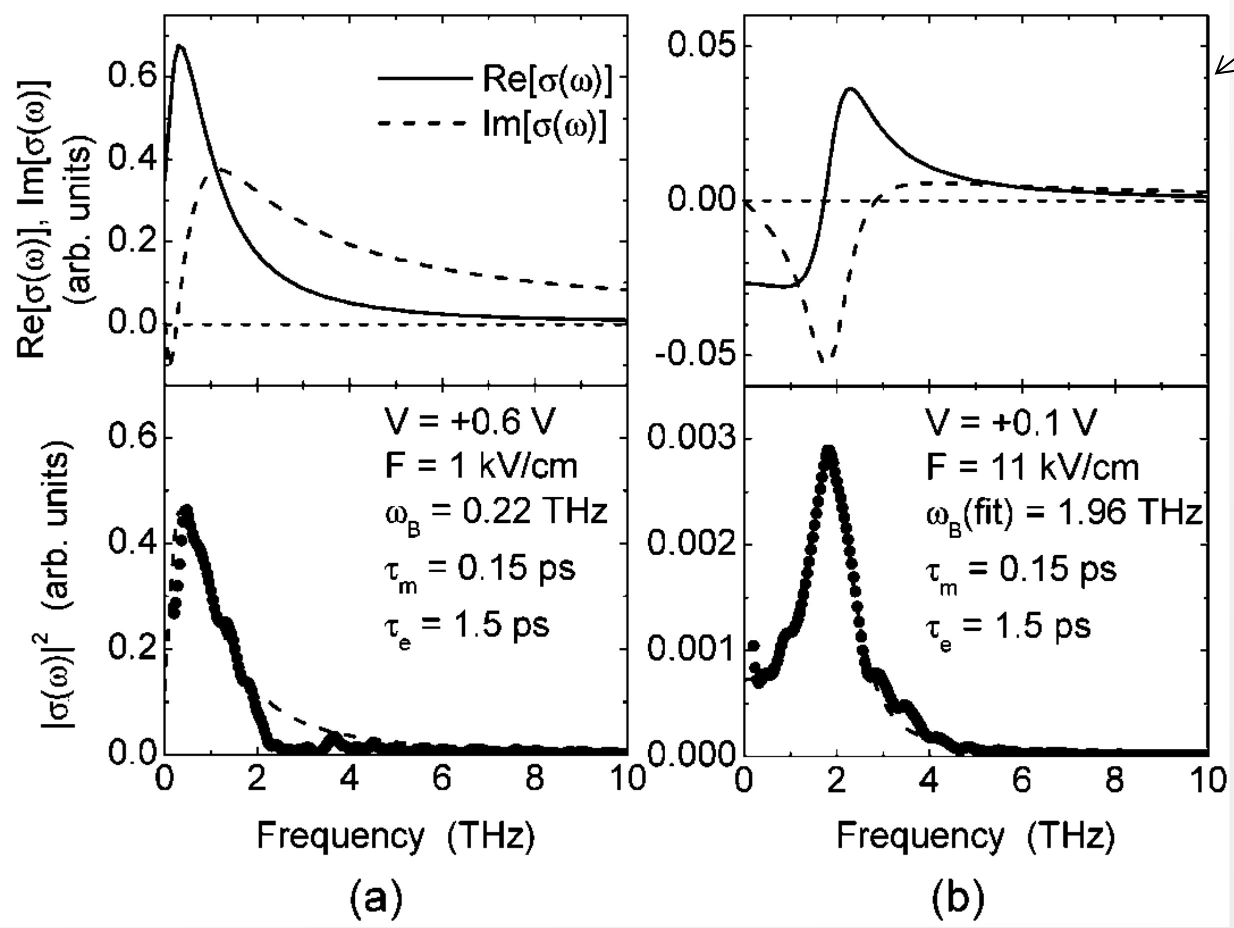
- P. Khosropanahet al, Opt. Lett. 34, 2958 (2009)

Phase locking of a 2.7 THz quantum cascade laser



THz applications of SL

- Y. Shimada et al, Phys. Rev. Lett. 90 046806 (2003)

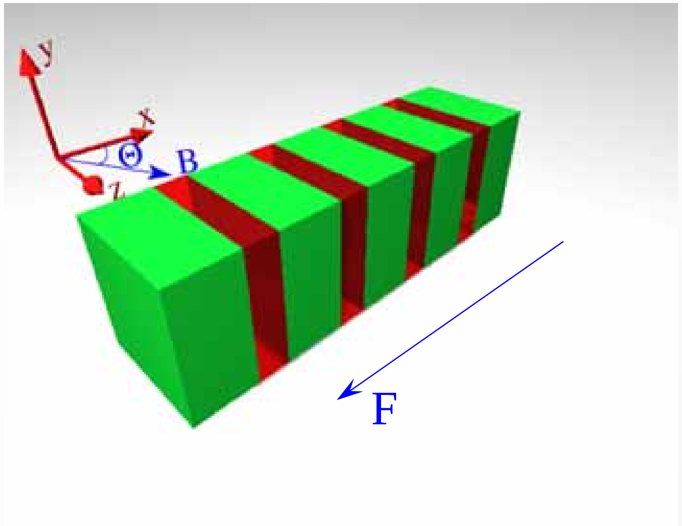


absorption:

S.A. Ktitorov, G.S. Simin, V.Ya. Sindalovskii, Sov. Phys. Solid State 13 (1971) 1872.

THz emission under optical injection (T~10K)

MODEL EQUATIONS I:



Electric field: $\mathbf{F}(-F, 0, 0)$

Magnetic field: $\mathbf{B}(B\cos\Theta, 0, B\sin\Theta)$

Dispersion relation:

$$E(\mathbf{p}) = \frac{\Delta}{2}\left(1 - \cos\frac{p_x d}{\hbar}\right) + \frac{1}{2m^*}\left(p_y^2 + p_y^2\right)$$

Electron impulse: $\mathbf{p}(p_x, p_y, p_x)$

Balance equation: $\dot{\mathbf{p}} = -e\mathbf{F} - e(\nabla_{\mathbf{p}} E \times \mathbf{B})$

$$\dot{p}_x = eF - \omega_{\|} p_y \tan\Theta$$

$$\dot{p}_y = m^* \omega_{\|} \tan\Theta \frac{d\Delta}{2\hbar}\sin\left(\frac{p_x d}{\hbar}\right) - \omega_{\|} p_z$$

$$\dot{p}_z = \omega_{\|} p_y$$

Cyclotron frequency:

$$\omega_c = eB / m^*$$

$$\omega_{\|} = \omega_c \cos\Theta$$

MODEL EQUATIONS II:

$$\ddot{p}_z + \omega_{\|}^{2} p_z = -\frac{m^* \omega_c^2 \Delta \sin 2\Theta}{4\hbar} \sin\left(\frac{d \tan\Theta}{\hbar} p_z - \omega_b t\right)$$

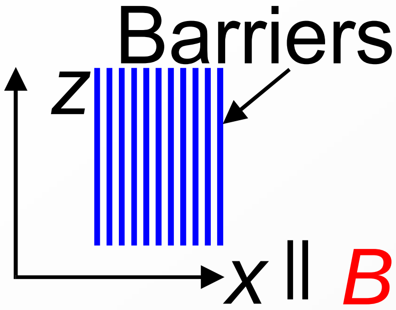
All other states can be expressed in terms of p_z:

$$p_x = eFt - \dot{p}_z \tan\Theta, \quad p_y = \omega_{\|}^{-1} \dot{p}_z$$

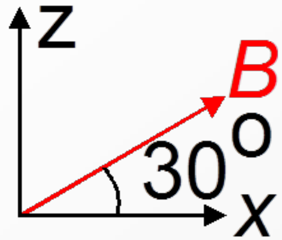
$$\dot{x} = \frac{d\Delta}{2\hbar} \sin\left(\frac{d \tan\Theta}{\hbar} p_z - \omega_b t\right), \quad \dot{y} = \frac{\dot{p}_z}{\omega_{\|} m^*}, \quad \dot{z} = \frac{\dot{p}_z}{m^*}$$

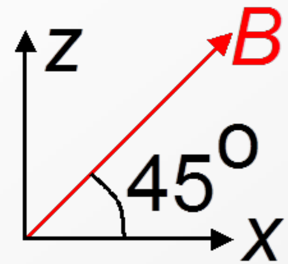
a.balanov@lboro.ac.uk

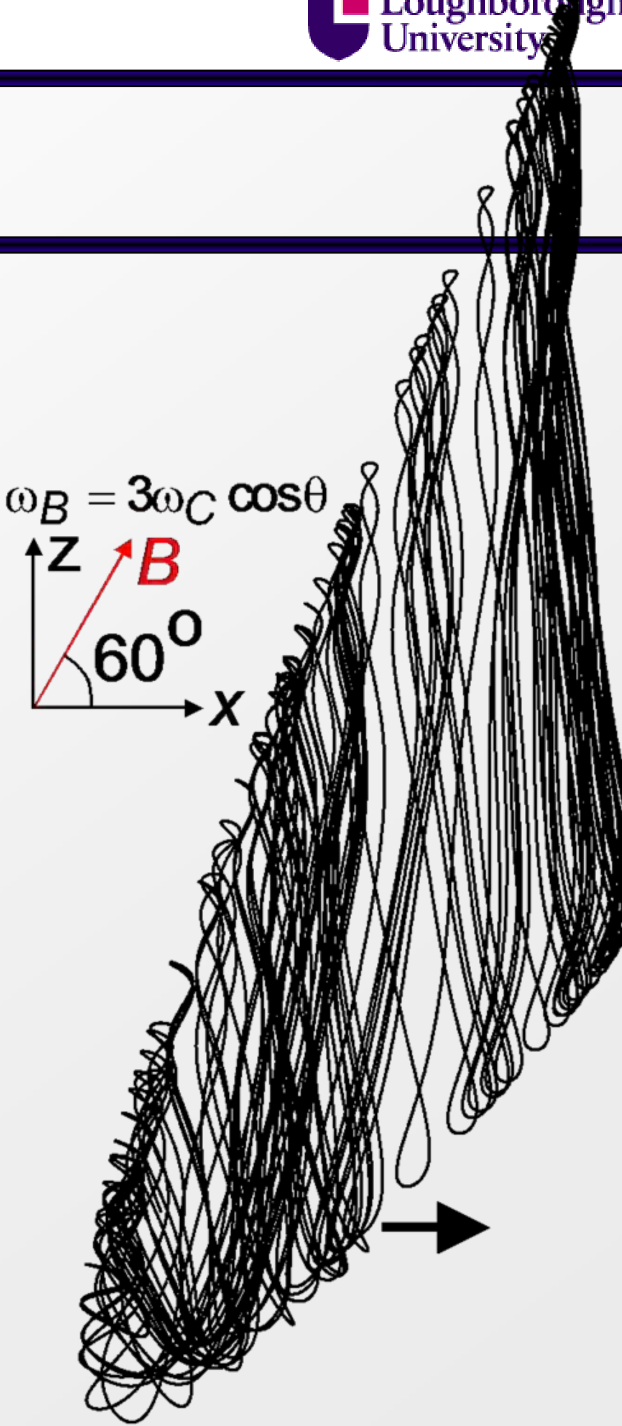
ELECTRON ORBITS:



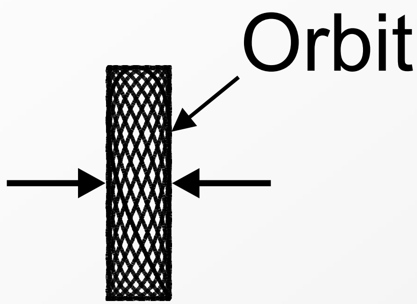
$\theta = 0^o$

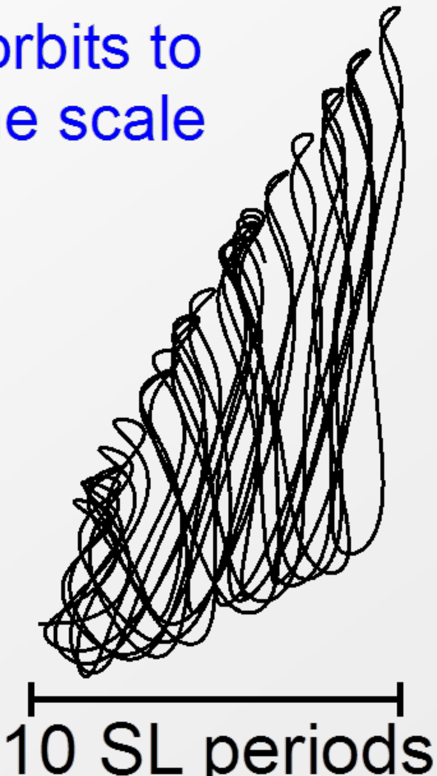






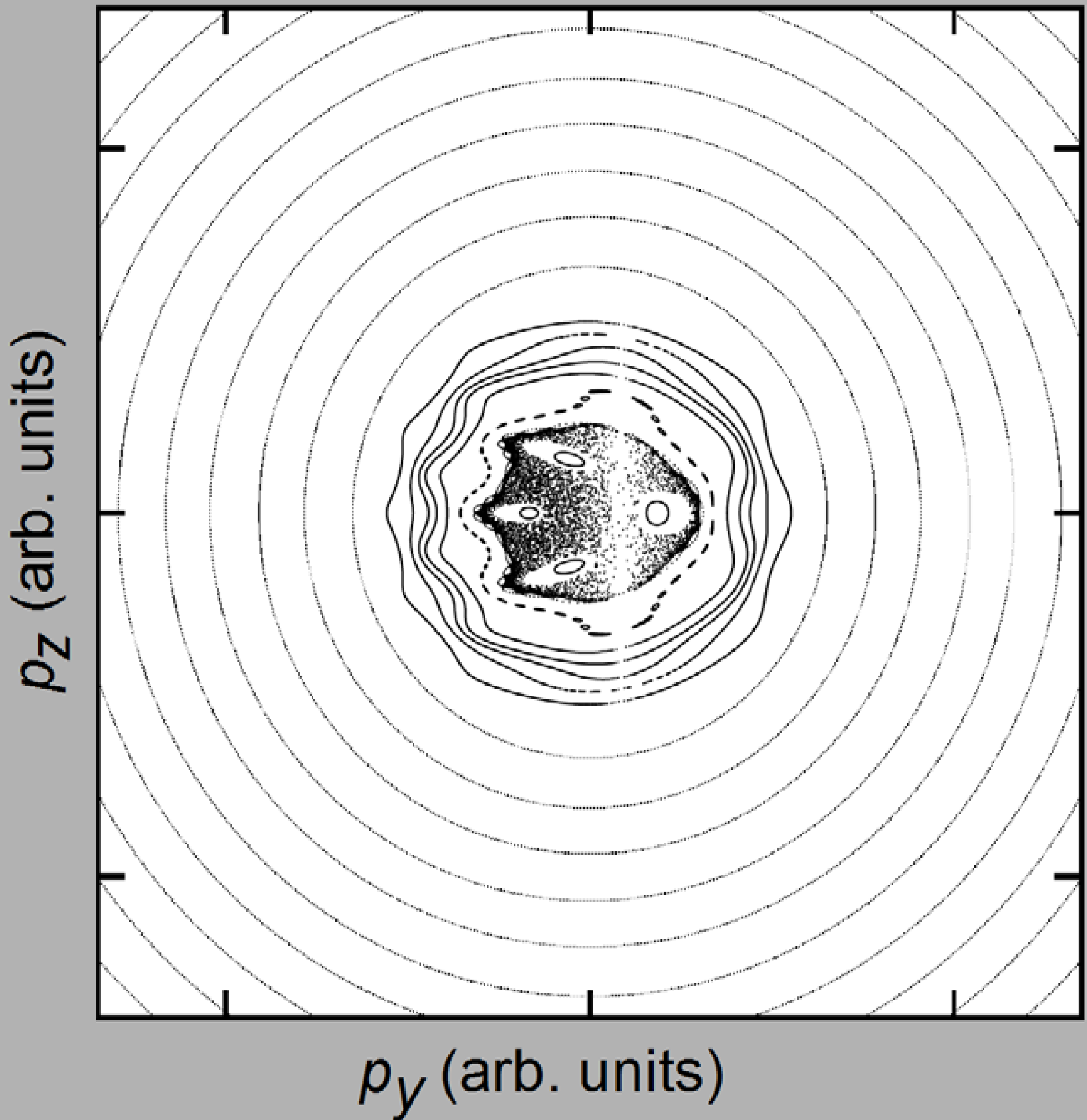
All orbits to same scale





$$\ddot{p}_z + \hat{\omega}_c^2 p_z = -\frac{m^* \omega_c^2 \Delta \sin 2\Theta}{4\hbar} \sin\left(\frac{d \tan\Theta}{\hbar} p_z - \omega_b t\right)$$

Cyclotron frequency ~ *B*
Bloch frequency ~ *F*



Stochastic web

broken
when

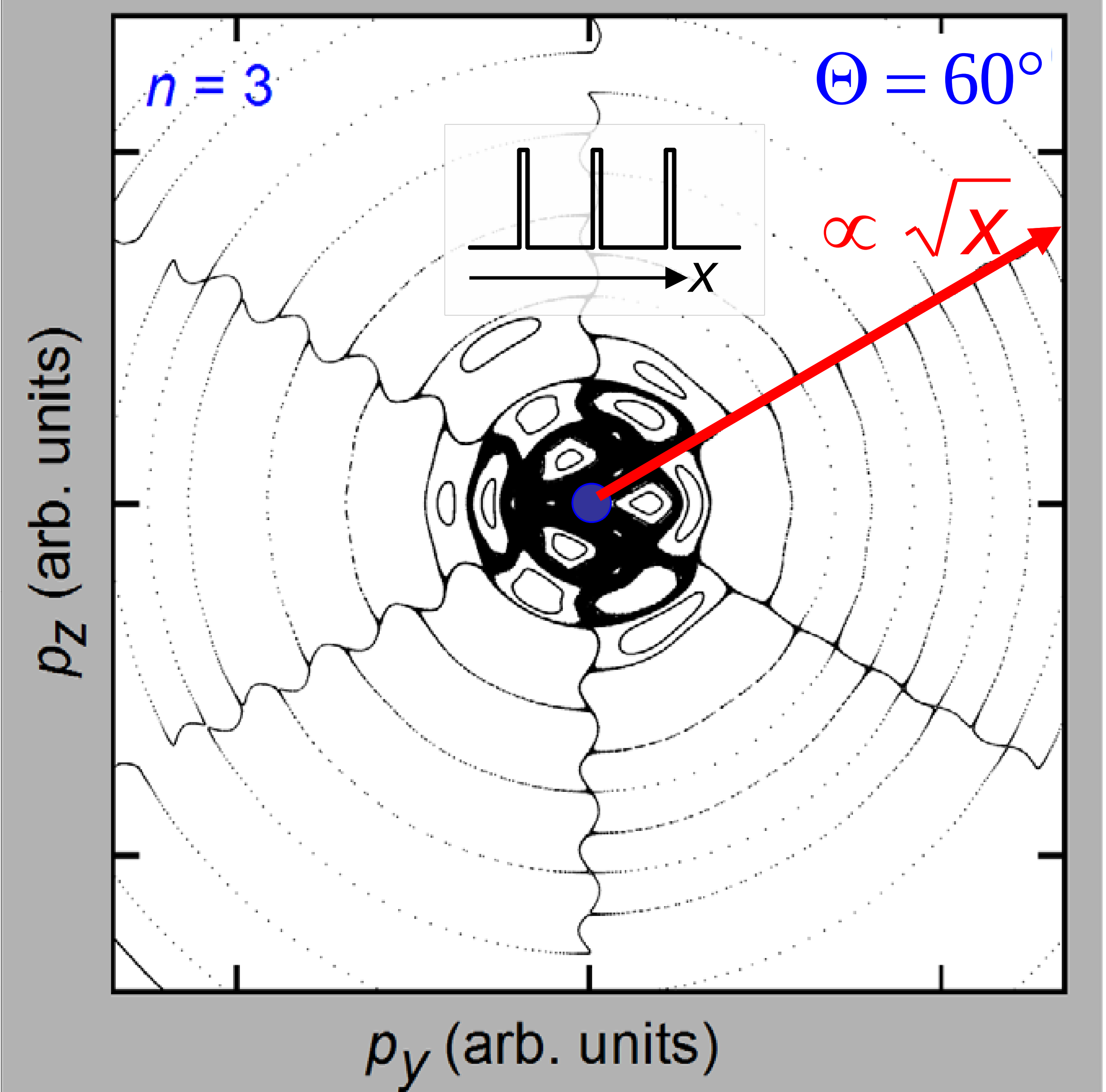
$$\omega_b / \omega_c \cos\Theta \neq n$$

integer

at *most*

field values

Off resonance chaotic orbits are localised → current low



Stochastic web

formed when

$$\omega_b / \omega_c \cos\Theta = n$$

~ F ~B integer

i.e. at *discrete* field values

On resonance chaotic orbits are *unbounded* → current high

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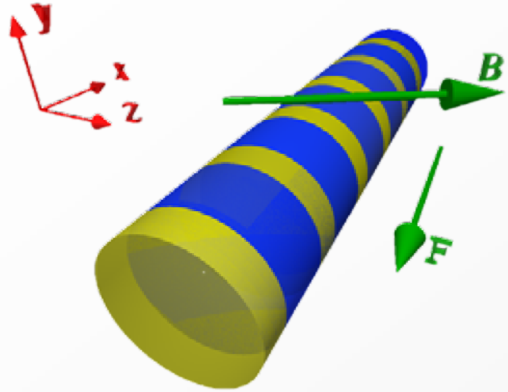
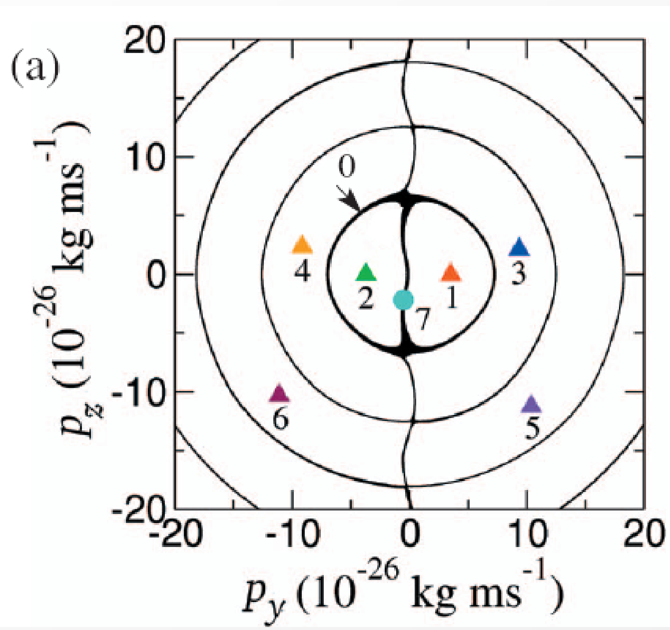
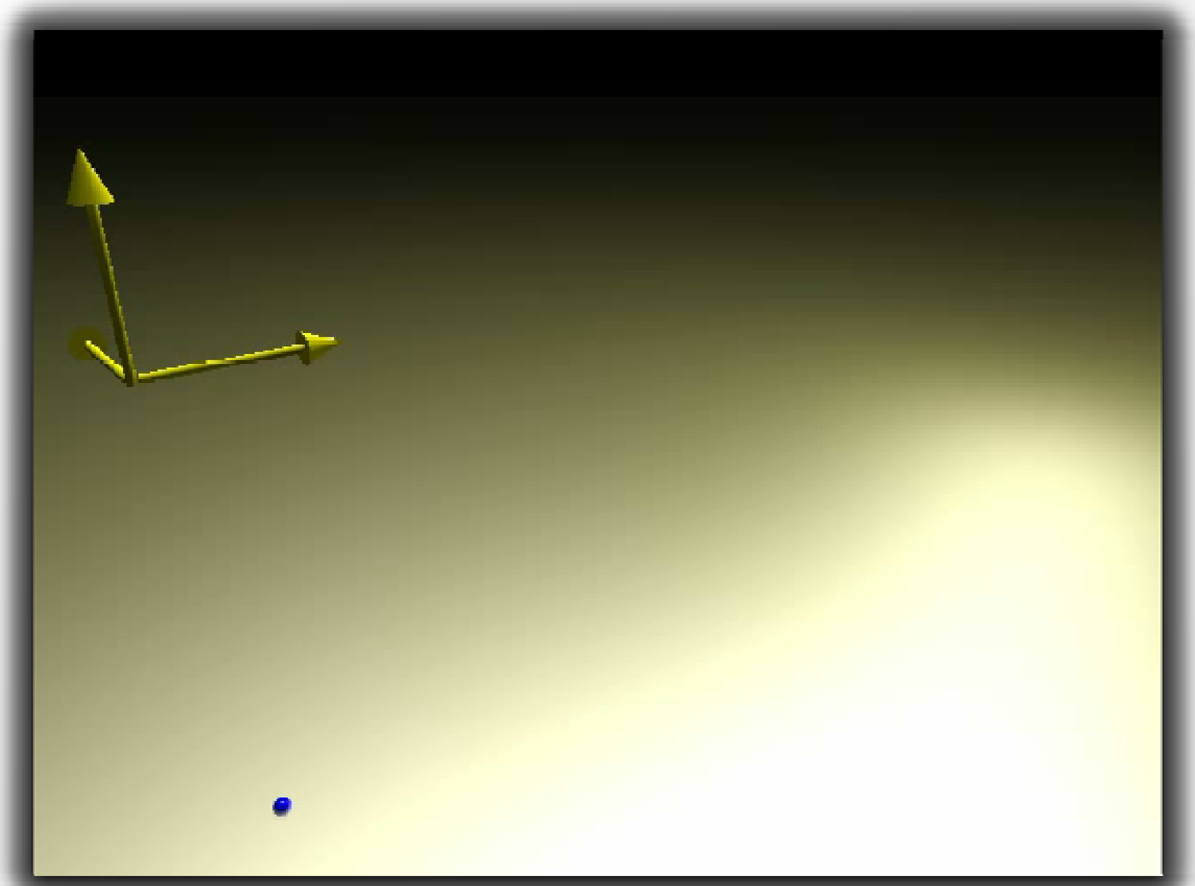
Chaotic electron's trajectory at cyclotron-Bloch resonance







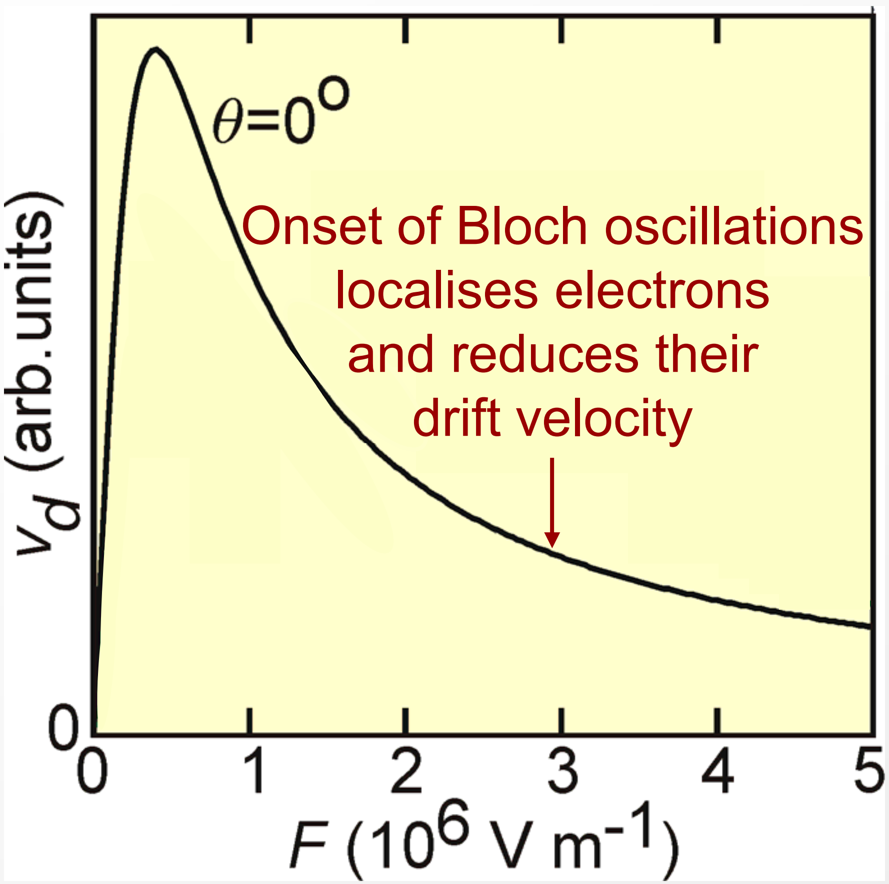
$$\omega_b / \omega_c \cos\Theta = 1$$

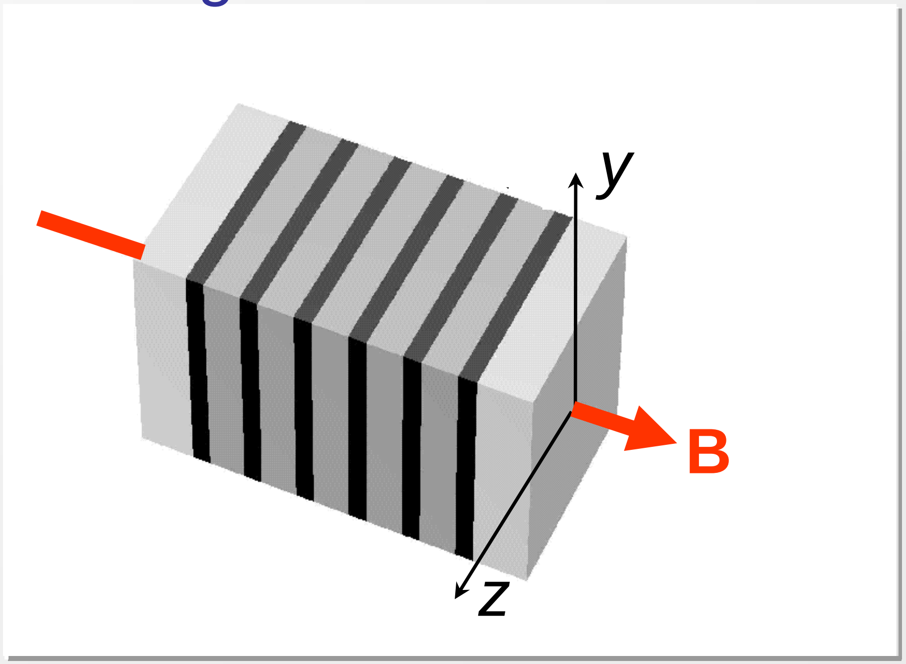
ELECTRON DRIFT VELOCITY

175 fs

$$v_d(F) = \frac{1}{\tau} \sum \int_0^\infty v_x(t) \exp(-t/\tau) dt$$

Summation over all starting velocities



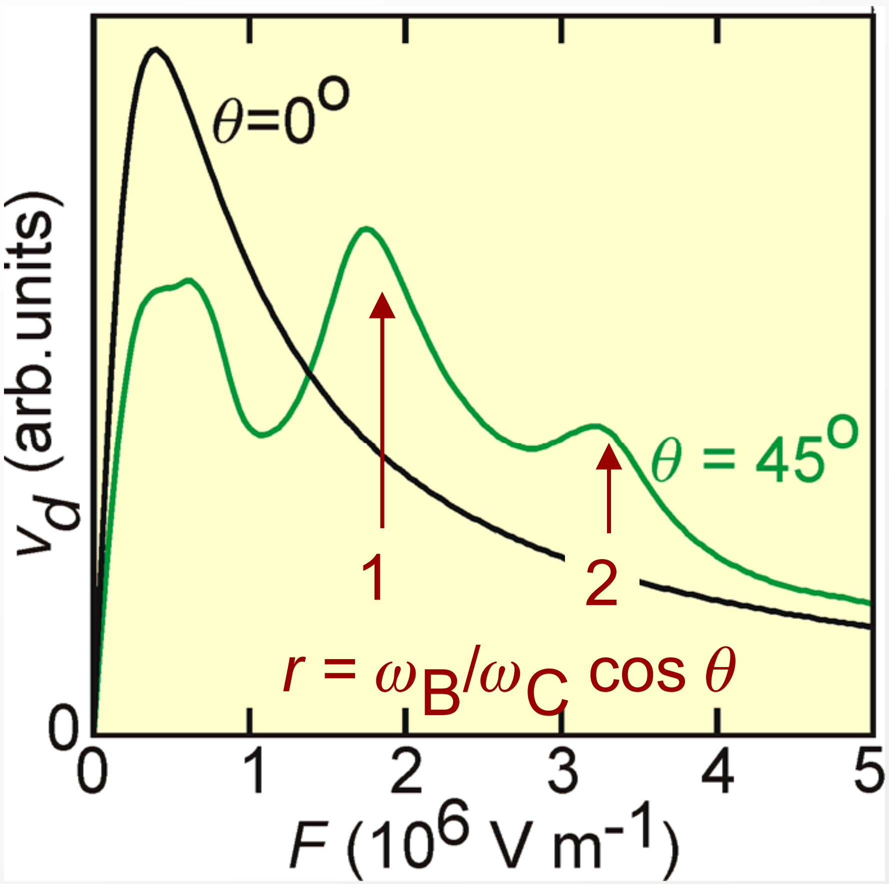


ELECTRON DRIFT VELOCITY

175 fs

$$v_d(F) = \frac{1}{\tau}\sum\int_0^{\infty} v_x(t)\exp(-t/\tau)dt$$

Summation over all starting velocities



Fromhold et al., *Phys. Rev. Lett.* **87**, 046803 (2001); *Nature* **428**, 726 (2004)
Fowler et al., *Phys. Rev. B*. **76**, 245303 (2007)

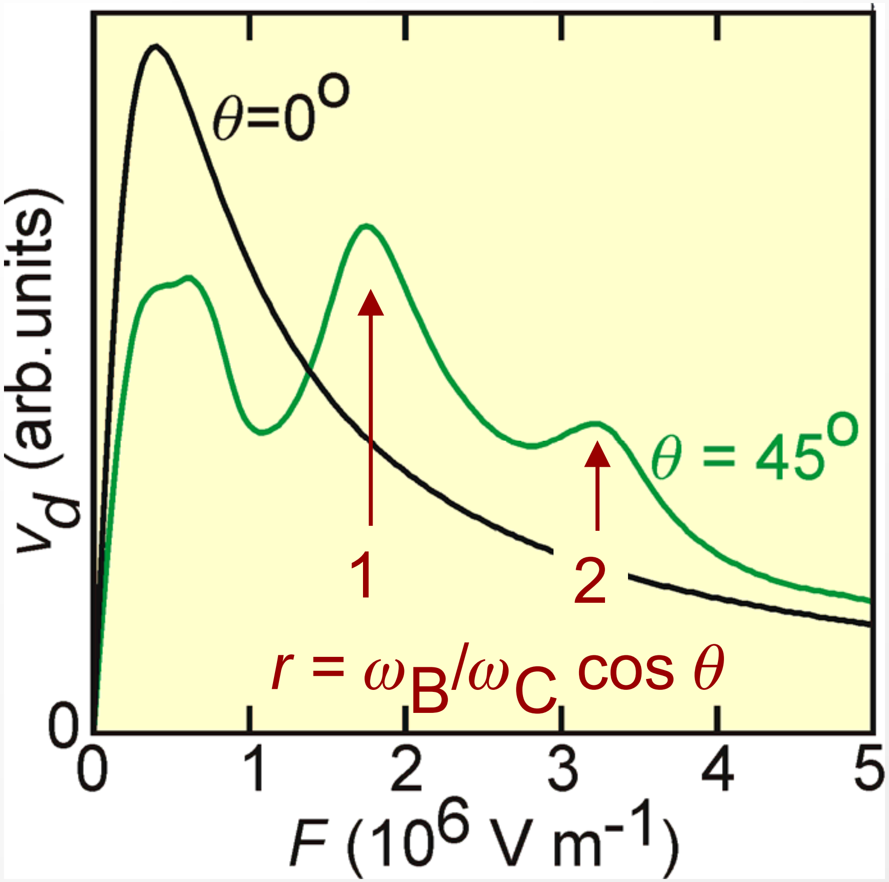
Balanov et al., *Phys. Rev. E*. **77**, 026209 (2008)

Related work on "Ultrafast Fiske Effect" by Kosevich et al., *Phys. Rev. Lett.* **96**, 137403 (2006)

ELECTRON DRIFT VELOCITY

175 fs

$$v_d(F) = \frac{1}{\tau}\sum\int_0^\infty v_x(t)\exp(-t/\tau)dt$$



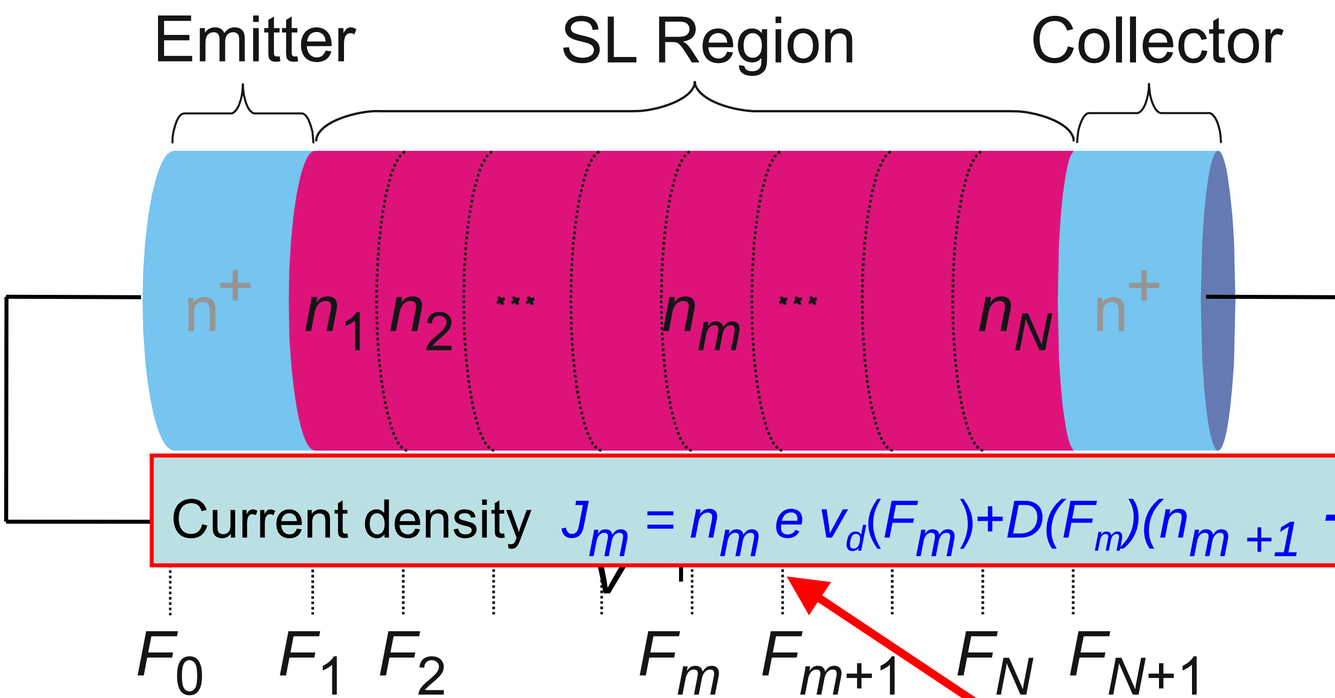
$v_d(F)$ curves were used as a basis for calculating $I(V)$

by self-consistent solution of

- Current continuity equation
- Poisson's equation

throughout the device

Semiclassical transport model



Current density $J_m = n_m e\, v_d(F_m) + D(F_m)(n_{m+1} - n_m)/\Delta x$

- Solve charge conservation equations $dn_m/dt = (J_m - J_{m+1})/e\Delta x$,
- and Poisson equations $F_{m+1} - F_m = (n_m - n_D)\, e\Delta x / \varepsilon_0 \varepsilon_r$
- self-consistently requiring sum of voltage drops = V
- Calculate current $I(t) = \frac{A}{N+1} \sum_{m=0}^{N} J_m$

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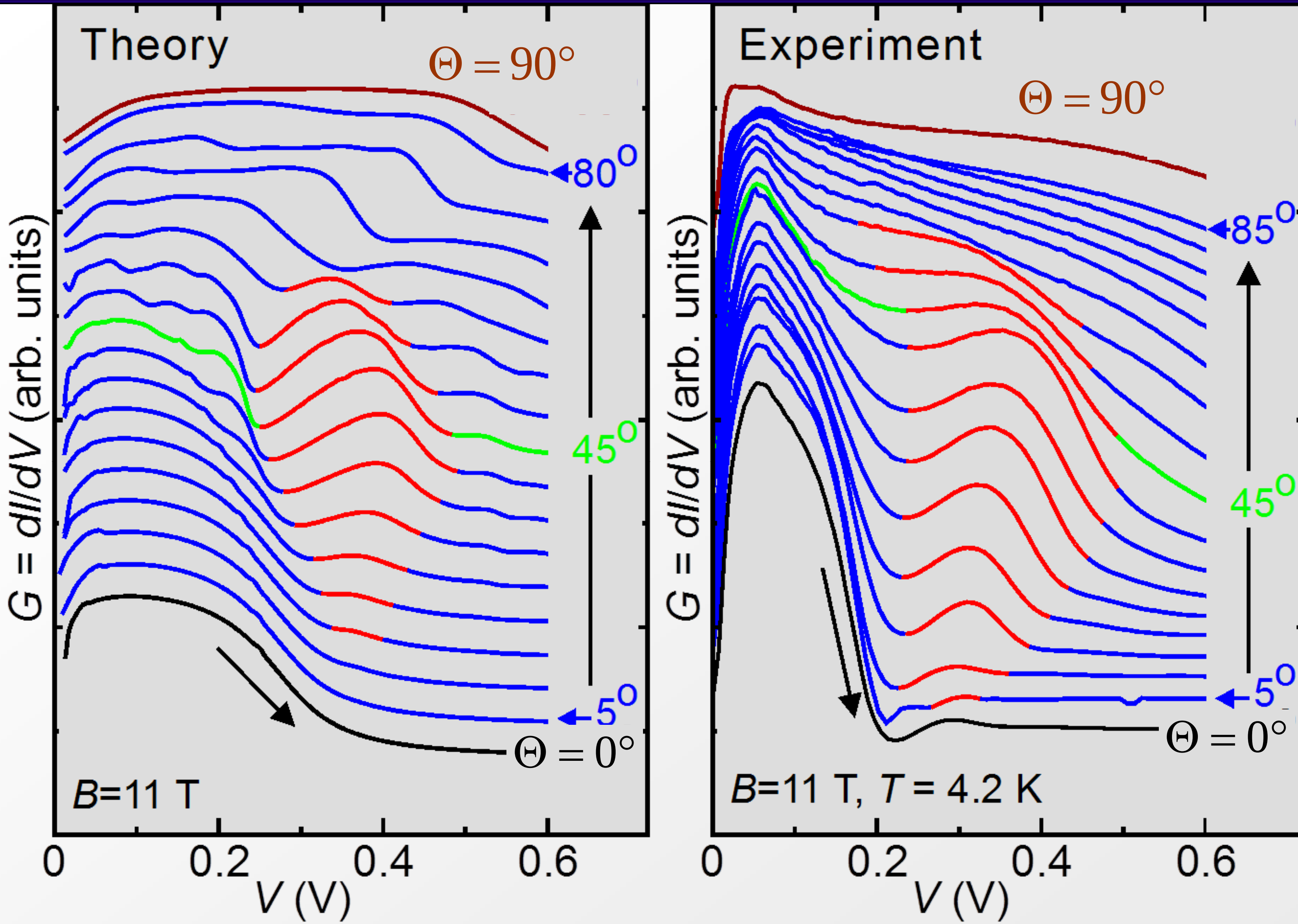
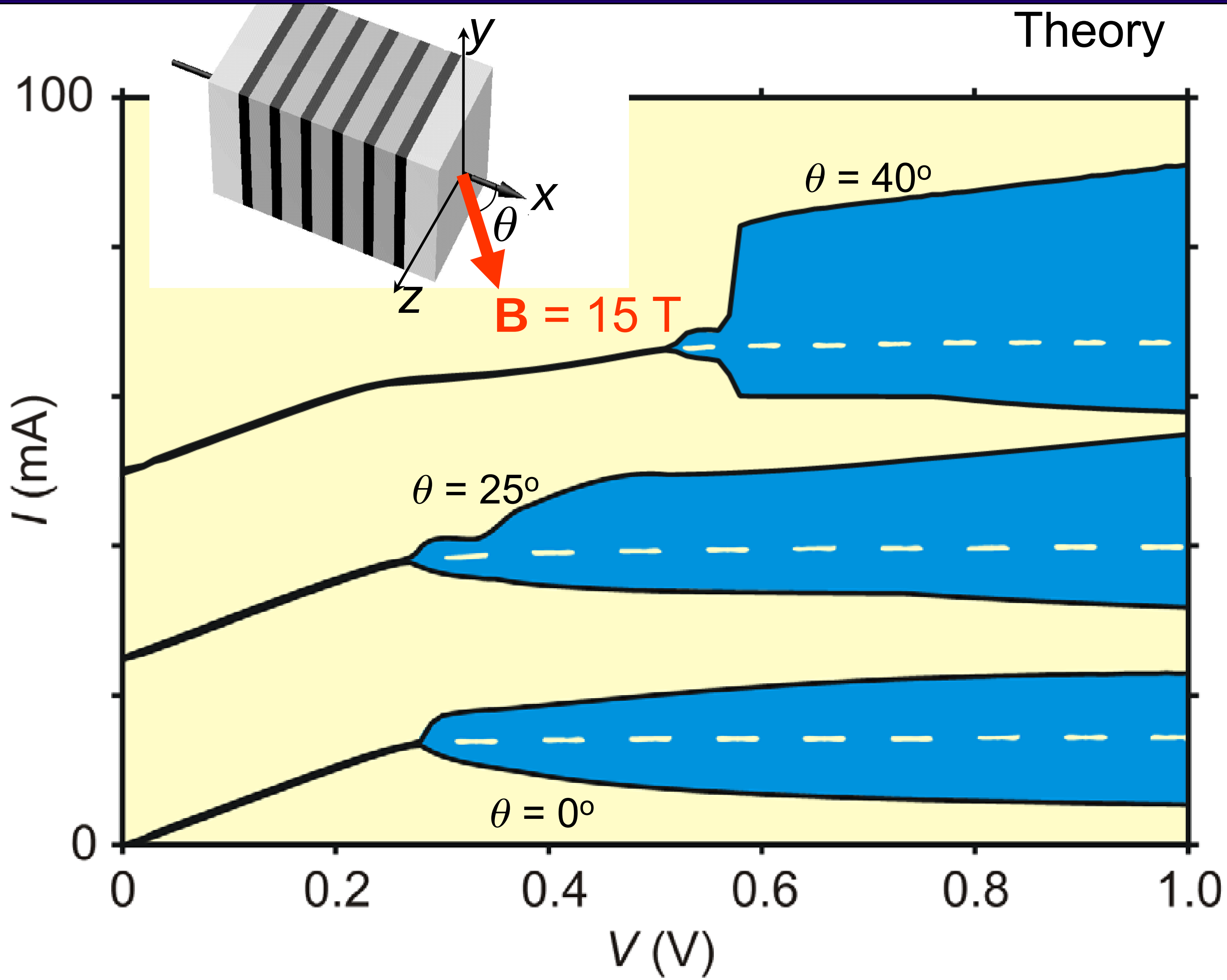
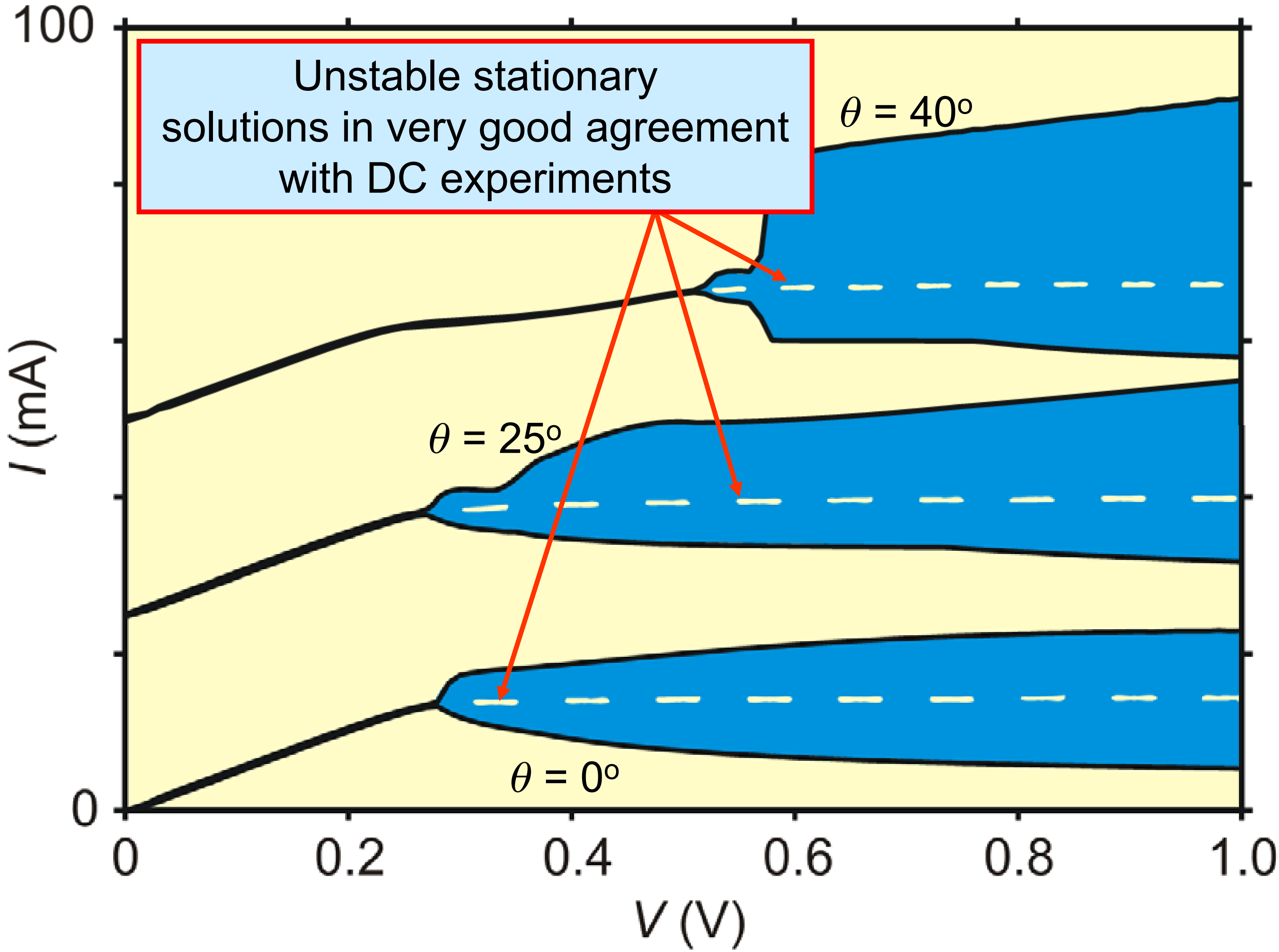


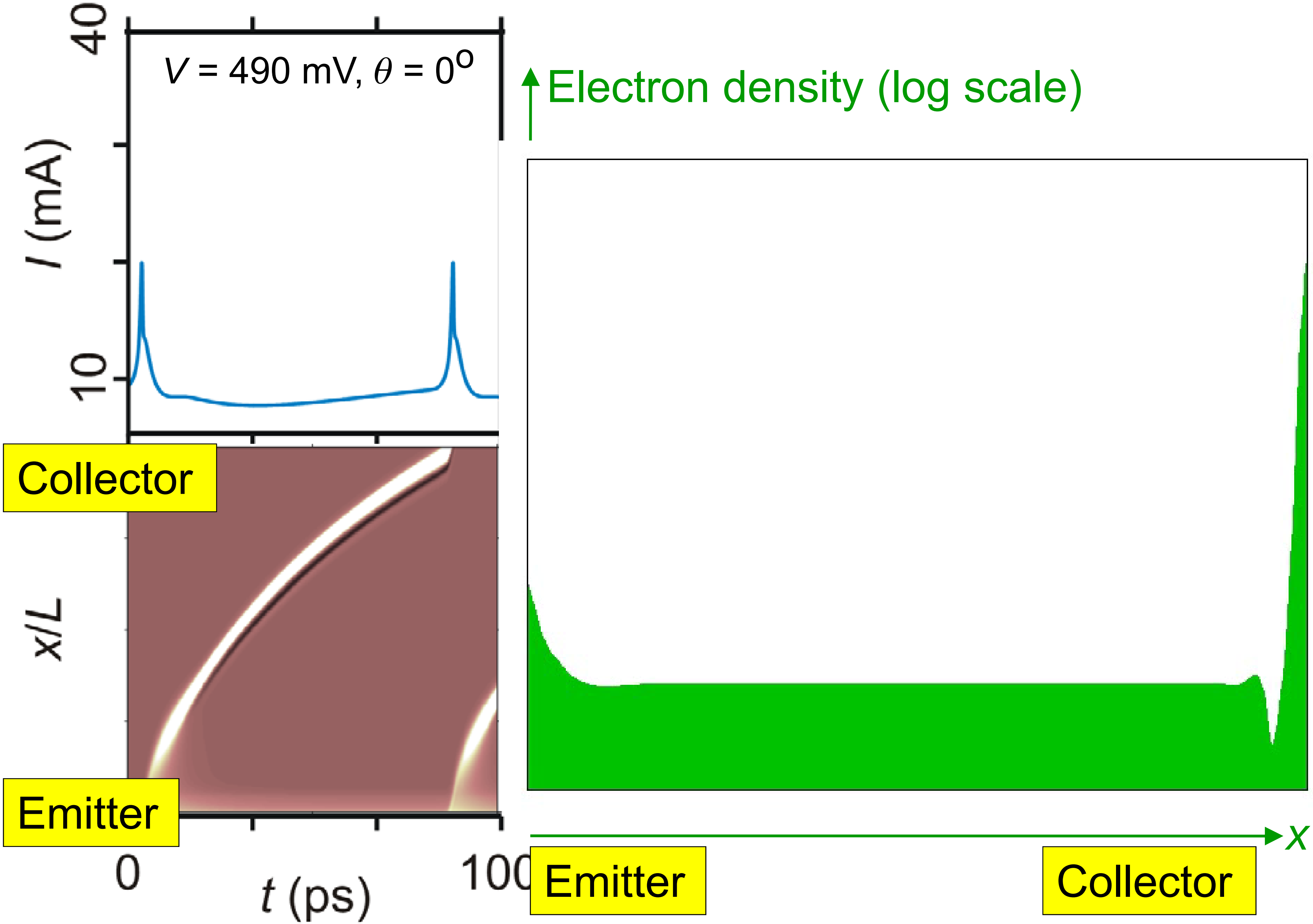
Fromhold et al, *Nature* **428,** 726 (2004)



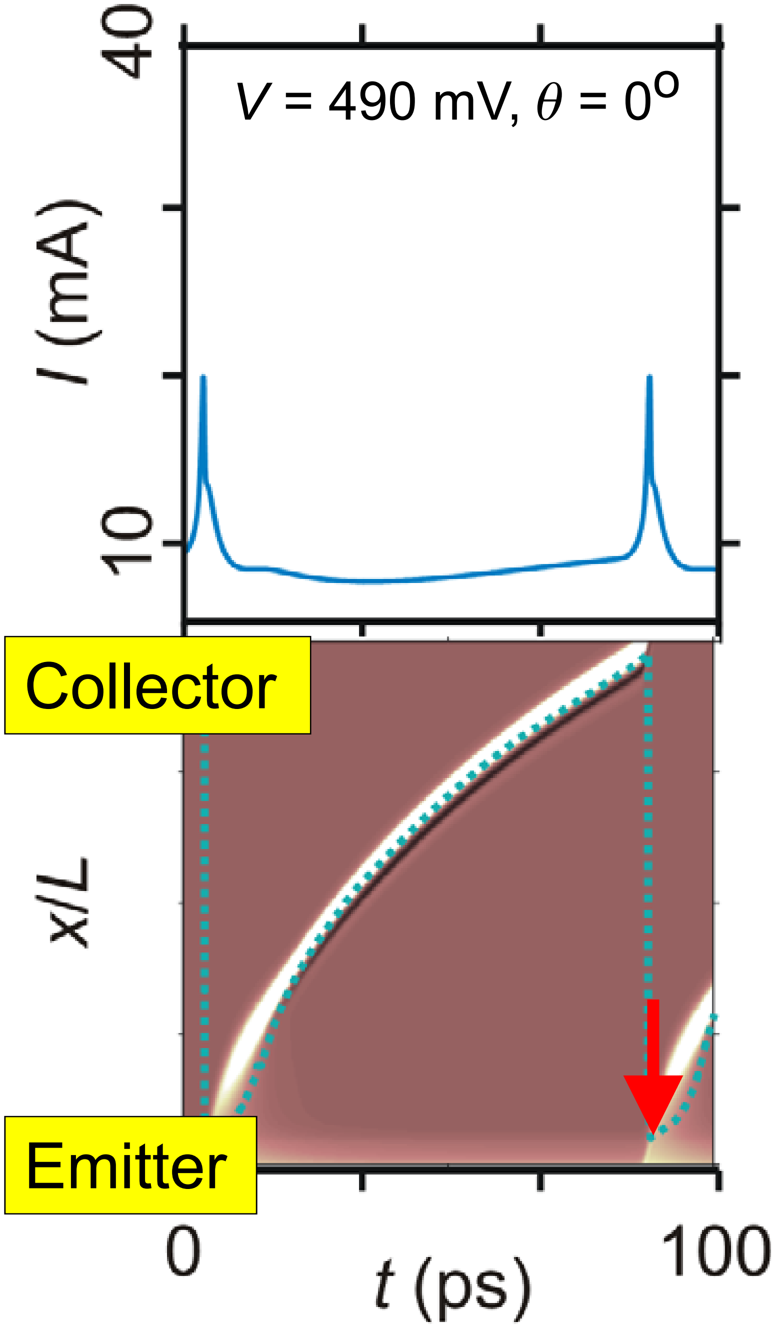
Theory
100
0
I (mA)
0
0.2
0.4
0.6
0.8
1.0
V (V)
y
x
z
θ
B = 15 T
θ = 40°
θ = 25°
θ = 0°

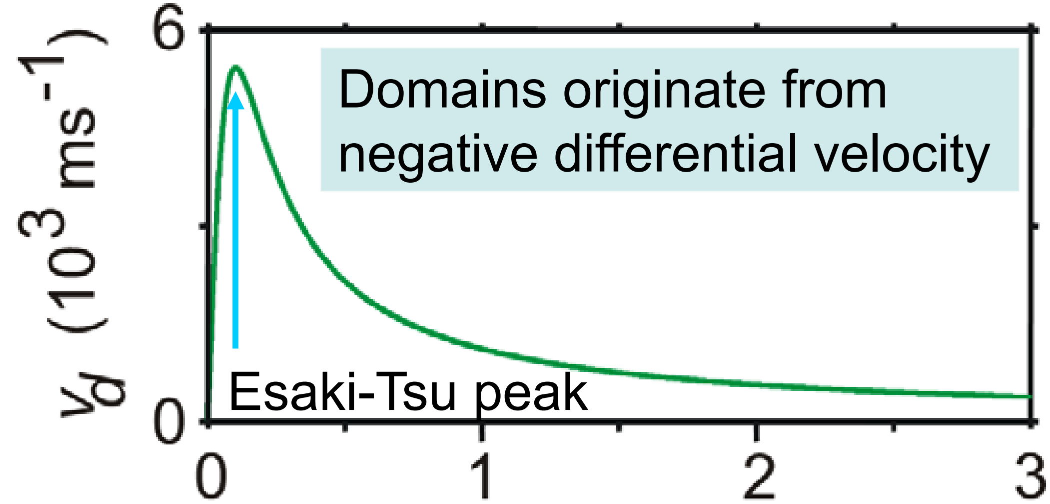
Theory


Unstable stationary solutions in very good agreement with DC experiments
θ = 40°
θ = 25°
θ = 0°
100
0
I (mA)
0
0.2
0.4
0.6
0.8
1.0
V (V)



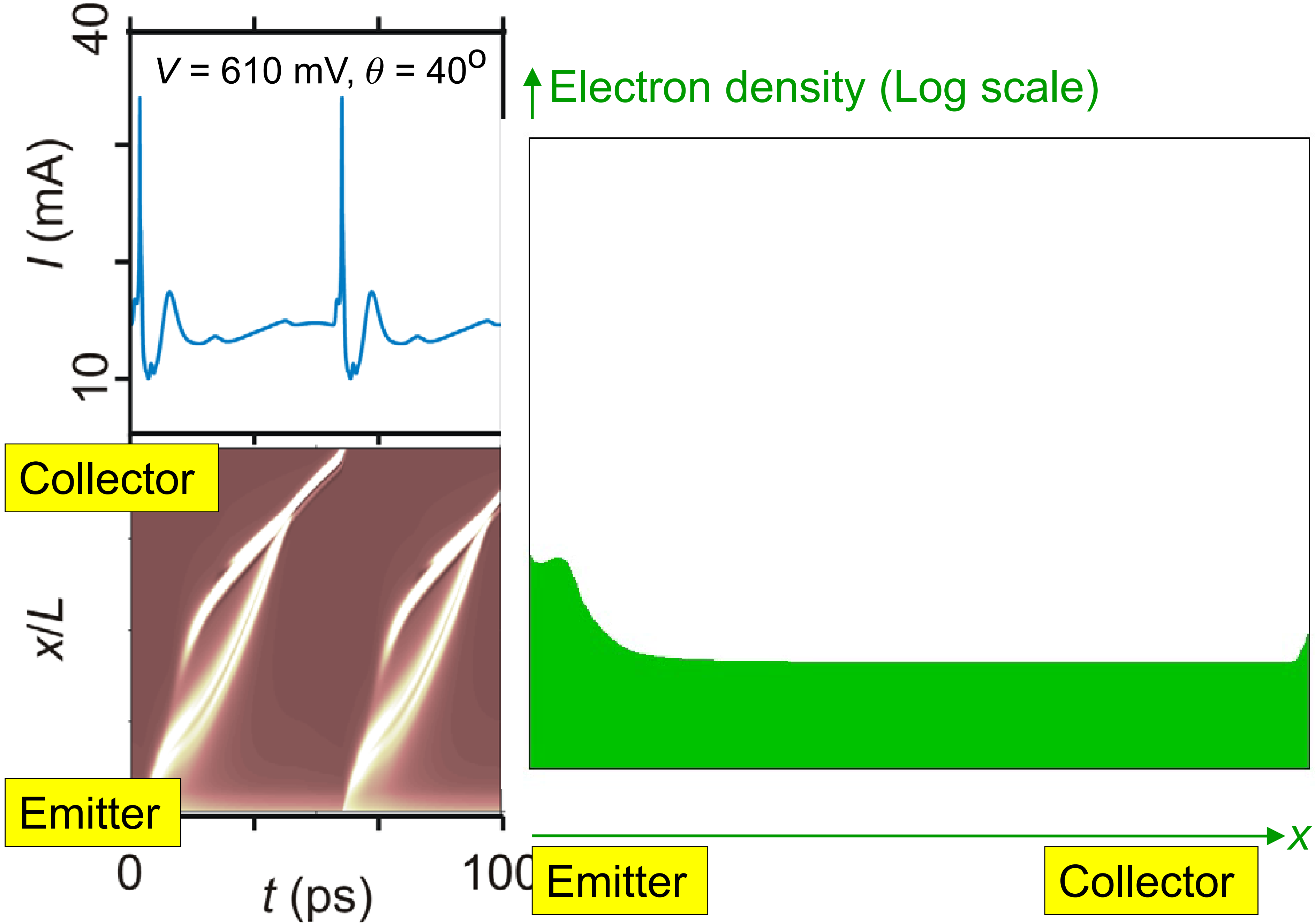
V = 490 mV, θ = 0°
40
I (mA)
10
Collector
x/L
Emitter
0
t (ps)
100
Electron density (log scale)
x
Emitter
Collector



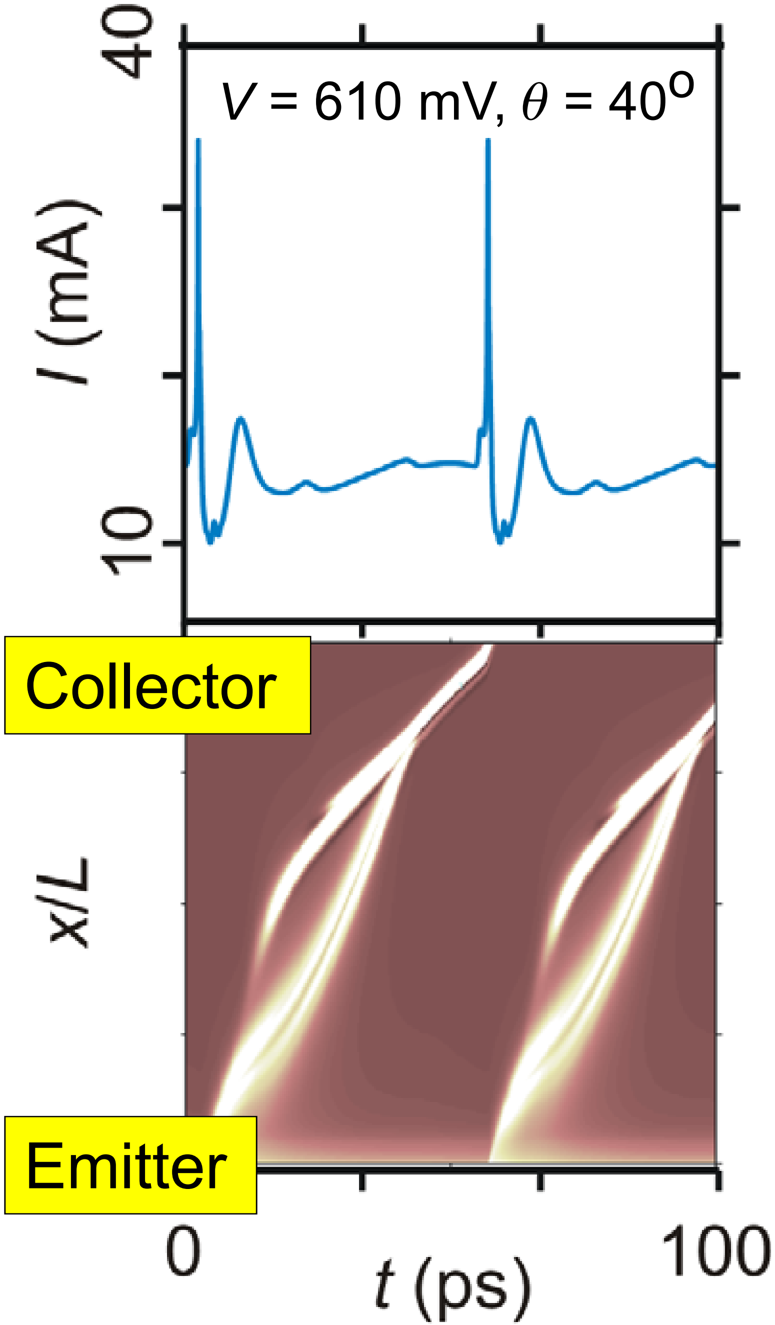


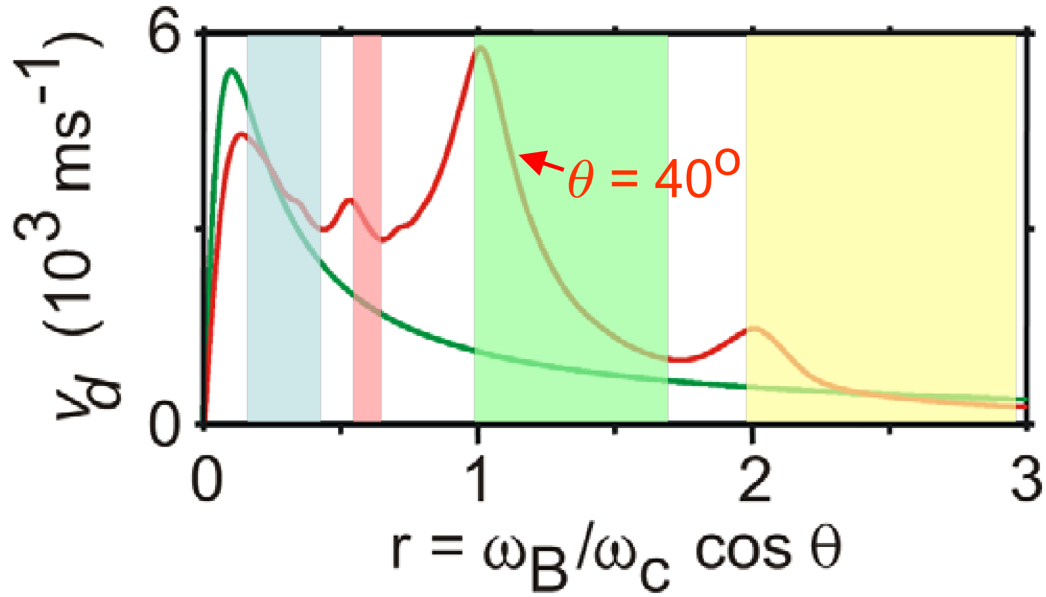
For reviews see, e.g.,
A. Wacker, *Phys. Rep.* **357**, 1 (2002);
L. Bonilla and H. Grahn,
Rep. Prog. Phys. **68** 577, (2005);
E. Schöll, *Nonlinear Spatio-temporal Dynamics and Chaos in Semiconductors* (CUP, 2001)

Propagating domains can generate electromagnetic radiation at frequencies > 100 GHz [e.g. Schomberg, Renk et al., *Appl. Phys. Lett.* **74**, 2179 (1999)]



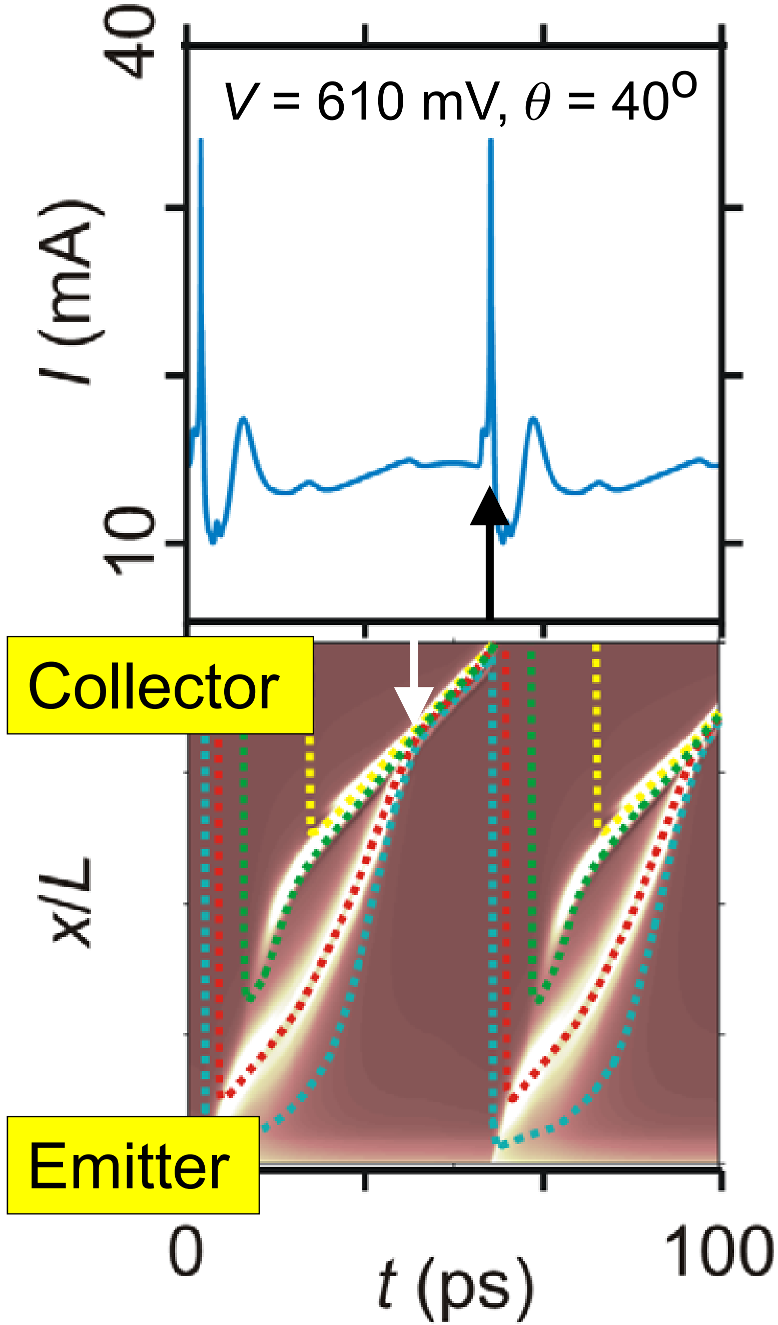
40
10
I (mA)
V = 610 mV, θ = 40°
x/L
Collector
Emitter
0
t (ps)
100
Electron density (Log scale)
x
Emitter
Collector





When $\theta = 0^o$, there is only one region of negative differential velocity

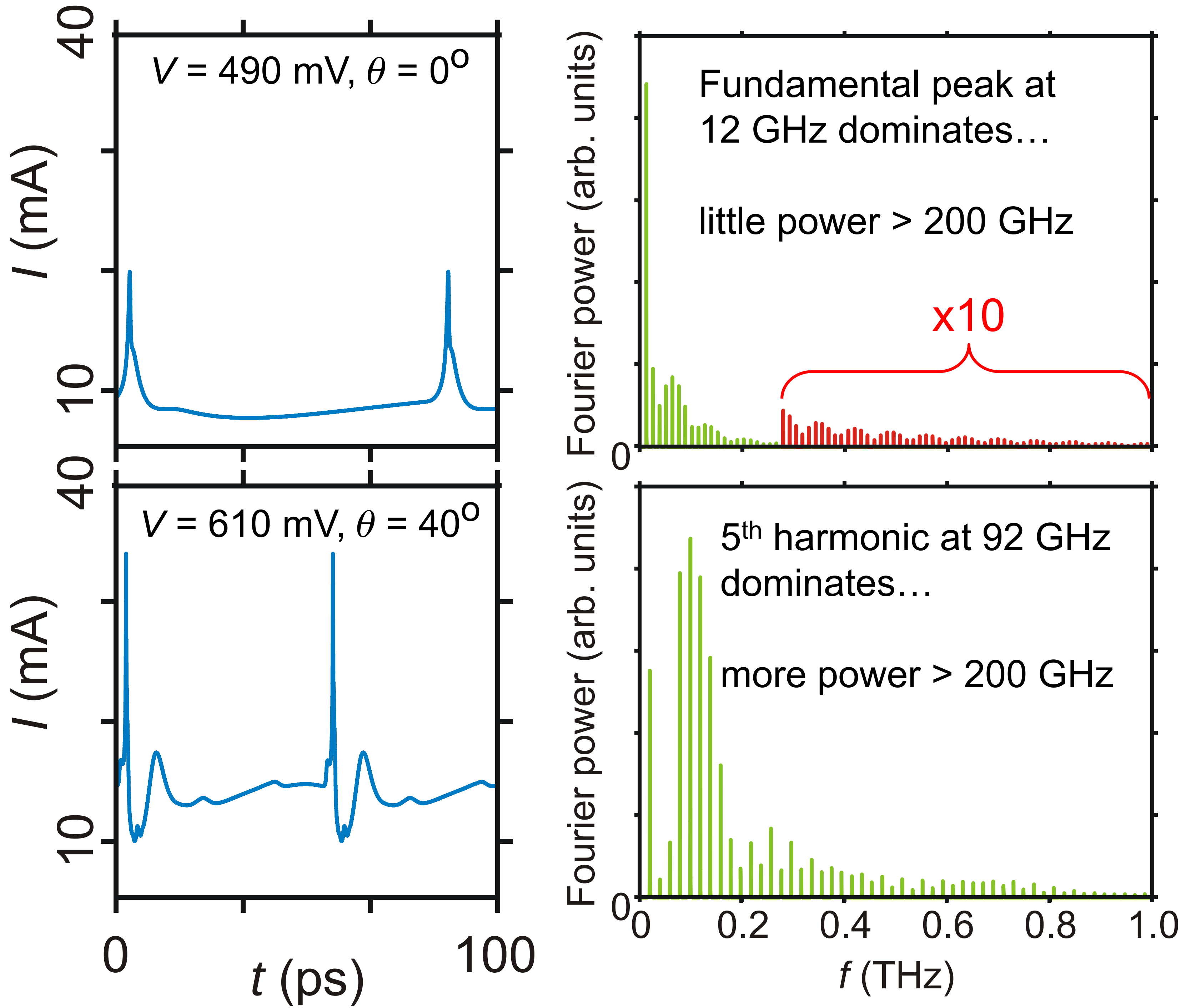
But when $\theta = 40^o$, there are 4 and each creates a charge domain



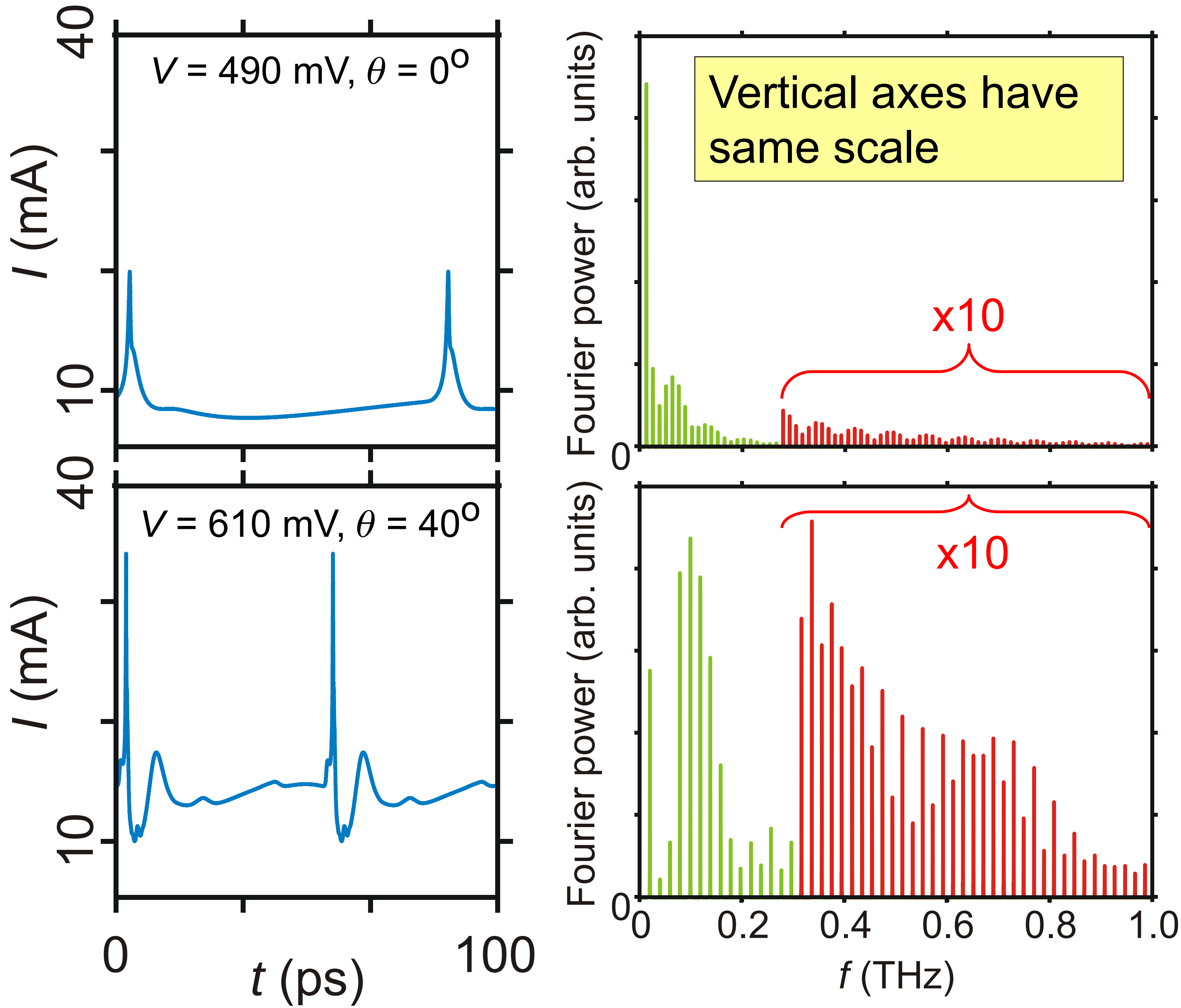
Eventually, the domains merge into one fast "superdomain"…

…which creates a large $I(t)$ peak when it arrives at the collector

Let's now compare Fourier power spectra of $I(t)$ for $\theta = 0^o$ and $\theta = 40^o$



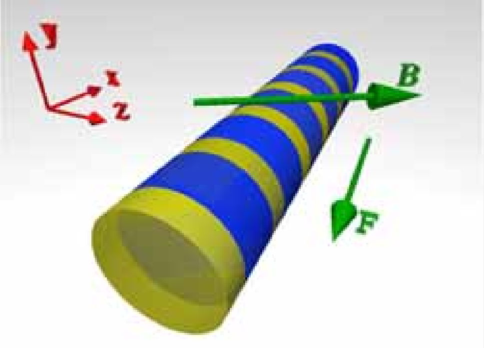
40
V = 490 mV, θ = 0°
I (mA)
10
40
V = 610 mV, θ = 40°
I (mA)
10
0
t (ps)
100
Fourier power (arb. units)
Fundamental peak at
12 GHz dominates…
little power > 200 GHz
x10
0
Fourier power (arb. units)
5th harmonic at 92 GHz
dominates…
more power > 200 GHz
0
0
0.2
0.4
0.6
0.8
1.0
f (THz)



40
V = 490 mV, θ = 0°
I (mA)
10
40
V = 610 mV, θ = 40°
I (mA)
10
0
t (ps)
100
Fourier power (arb. units)
Vertical axes have same scale
x10
0
Fourier power (arb. units)
x10
0
0
0.2
0.4
0.6
0.8
1.0
f (THz)

Effect of temperature on drift velocity of electrons v_d

$$\dot{p}_x(t) = eF - \omega_\perp p_y(t)$$
$$\dot{p}_y(t) = \frac{d\Delta m^* \omega_\perp}{2\hbar} \sin\left(\frac{p_x(t)d}{\hbar}\right) - \omega_\| p_z(t)$$
$$\dot{p}_z(t) = \omega_\| p_y(t),$$



Cyclotron frequencies

$\omega_\| = eB\cos\theta/m^*$,

$\omega_\perp = eB\sin\theta/m^*$

$$v_x(t) = \dot{x}(t) = v_0 \sin\left(\frac{p_x(t)d}{\hbar}\right)$$

Maximal miniband velocity of electron $v_0 = \Delta d/(2\hbar)$

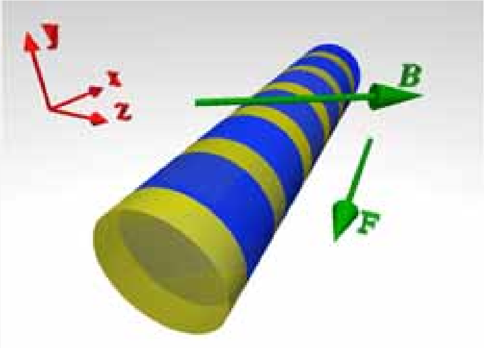
Drift velocity for particular initial momentum $\mathbf{P}(P_x, P_y, P_z)$

$$u_d(\mathbf{P}) = \nu \int_0^\infty v_x(t) e^{-\nu t} dt.$$

$\nu = 1/\tau$ - scattering rate

Effect of temperature on drift velocity of electrons v_d

Drift velocity for particular initial momentum $\mathbf{P}(P_x, P_y, P_z)$



$$u_d(\mathbf{P}) = \nu \int_0^\infty v_x(t) e^{-\nu t} dt.$$

We assume the Boltzmann statistics for electron momenta

$$f(\mathbf{P}) = \frac{1}{Z} e^{-\frac{\Delta}{2k_BT}\left(1-\cos\frac{P_x d}{\hbar}\right)-\frac{P_y^2+P_z^2}{2m^* k_B T}}$$

$$Z = (2\pi)^2 m^* k_B T \frac{\hbar}{d} I_0\left(\frac{\Delta}{2k_BT}\right) e^{-\frac{\Delta}{2k_BT}}$$

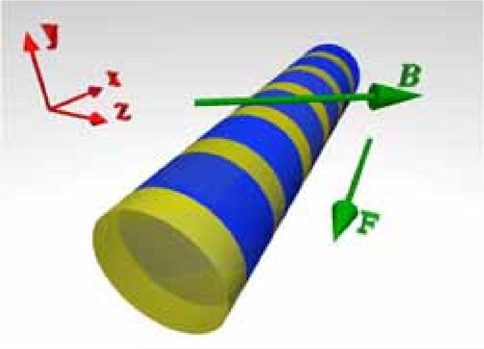
Averaged drift velocity of electrons

$$v_d = \int_{-\pi\hbar/d}^{\pi\hbar/d} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\mathbf{P}) u_d(\mathbf{P}) dP_x dP_y dP_z$$

A. Selskii et al, in preparation

Effect of temperature on v_d: analytical approach

$$\dot{p}_x(t) = eF - \omega_\perp p_y(t)$$
$$\dot{p}_y(t) = \frac{d\Delta m^* \omega_\perp}{2\hbar} \sin\left(\frac{p_x(t)d}{\hbar}\right) - \omega_\| p_z(t)$$
$$\dot{p}_z(t) = \omega_\| p_y(t),$$



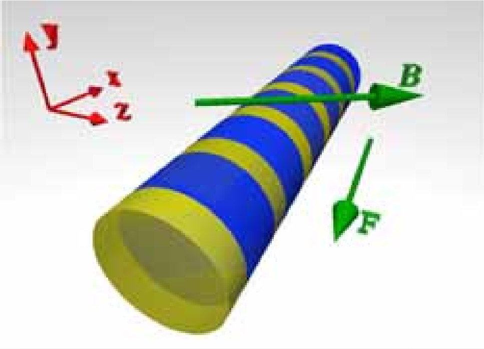
If $\omega_{||} \gg \omega_\perp$

$$\dot{p}_x(t) = eF - \omega_\perp p_y$$
$$\dot{p}_y(t) = -\omega_\| p_z(t)$$
$$\dot{p}_z(t) = \omega_\| p_y(t),$$

F.G. Bass et al, JETP Letters 31, 314 (1980).

Effect of temperature on v_d: analytical approach

$$\dot{p}_x(t) = eF - \omega_\perp p_y$$
$$\dot{p}_y(t) = -\omega_\| p_z(t)$$
$$\dot{p}_z(t) = \omega_\| p_y(t),$$



Solution!

$$p_x(t) = p_\| + eFt - \frac{\omega_\perp}{\omega_\|} p_\perp \sin(\omega_\| t + \varphi_0) + \frac{\omega_\perp}{\omega_\|} p_\perp \sin\varphi_0,$$
$$p_y(t) = p_\perp \cos(\omega_\| t + \varphi_0),$$
$$p_z(t) = p_\perp \sin(\omega_\| t + \varphi_0),$$

where $p_\perp = \sqrt{P_y^2 + P_z^2}$, and $\varphi_0 = atan(P_z/P_y)$.

$$p_x(0) = p_\|$$

Effect of temperature on v_d: analytical approach

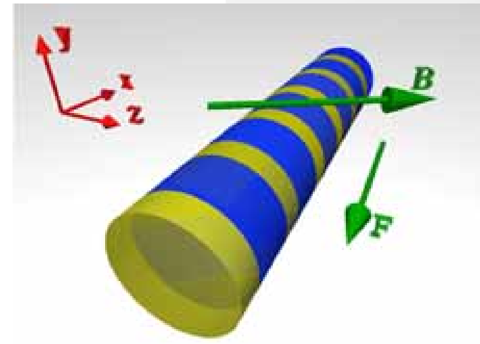
$$p_x(t) = p_\parallel + eFt - \frac{\omega_\perp}{\omega_\parallel} p_\perp \sin(\omega_\parallel t + \varphi_0) + \frac{\omega_\perp}{\omega_\parallel} p_\perp \sin \varphi_0,$$

$$p_y(t) = p_\perp \cos(\omega_\parallel t + \varphi_0),$$

$$p_z(t) = p_\perp \sin(\omega_\parallel t + \varphi_0),$$

where $p_\perp = \sqrt{P_y^2 + P_z^2}$, and $\varphi_0 = atan(P_z/P_y)$.

$$p_x(0) = p_\parallel$$



$\omega_B = eFd/\hbar$ - Bloch frequency

$$v_x(t) = \frac{v_0}{2i}\left[e^{i\frac{p_\parallel d}{\hbar}} \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} J_k\left(\frac{\omega_\perp}{\omega_\parallel}\frac{d}{\hbar}p_\perp\right) e^{i(\omega_B - k\omega_\parallel)t} J_n\left(\frac{\omega_\perp}{\omega_\parallel}\frac{d}{\hbar}p_\perp\right) e^{i\varphi_0(n-k)} \right.$$

$$\left. - e^{-i\frac{p_\parallel d}{\hbar}} \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} J_k\left(\frac{\omega_\perp}{\omega_\parallel}\frac{d}{\hbar}p_\perp\right) e^{-i(\omega_B - k\omega_\parallel)t} J_n\left(\frac{\omega_\perp}{\omega_\parallel}\frac{d}{\hbar}p_\perp\right) e^{-i\varphi_0(n-k)} \right]$$

Effect of temperature on v_d: analytical approach

$$v_x(t) = \frac{v_0}{2i}\left[e^{i\frac{p_\parallel d}{\hbar}} \sum_{n=-\infty}^{\infty}\sum_{k=-\infty}^{\infty} J_k\left(\frac{\omega_\perp}{\omega_\parallel}\frac{d}{\hbar}p_\perp\right) e^{i(\omega_B - k\omega_\parallel)t} J_n\left(\frac{\omega_\perp}{\omega_\parallel}\frac{d}{\hbar}p_\perp\right) e^{i\varphi_0(n-k)}\right.$$
$$\left. - e^{-i\frac{p_\parallel d}{\hbar}} \sum_{n=-\infty}^{\infty}\sum_{k=-\infty}^{\infty} J_k\left(\frac{\omega_\perp}{\omega_\parallel}\frac{d}{\hbar}p_\perp\right) e^{-i(\omega_B - k\omega_\parallel)t} J_n\left(\frac{\omega_\perp}{\omega_\parallel}\frac{d}{\hbar}p_\perp\right) e^{-i\varphi_0(n-k)}\right]$$

$$u_d(p_\parallel, p_\perp,, \varphi_0) = A(p_\perp, \varphi_0)e^{i\frac{p_\parallel d}{\hbar}} - B(p_\perp, \varphi_0)e^{-i\frac{p_\parallel d}{\hbar}}$$

$$A(p_\perp, \varphi_0) = \sum_{n=-\infty}^{\infty}\sum_{k=-\infty}^{\infty} J_k\left(\frac{\omega_\perp}{\omega_\parallel}\frac{d}{\hbar}p_\perp\right) J_n\left(\frac{\omega_\perp}{\omega_\parallel}\frac{d}{\hbar}p_\perp\right) e^{i\varphi_0(n-k)} \frac{\nu}{\nu - i(\omega_B - k\omega_\parallel)},$$

$$B(p_\perp, \varphi_0) = \sum_{n=-\infty}^{\infty}\sum_{k=-\infty}^{\infty} J_k\left(\frac{\omega_\perp}{\omega_\parallel}\frac{d}{\hbar}p_\perp\right) J_n\left(\frac{\omega_\perp}{\omega_\parallel}\frac{d}{\hbar}p_\perp\right) e^{-i\varphi_0(n-k)} \frac{\nu}{\nu + i(\omega_B - k\omega_\parallel)}.$$

Effect of temperature on v_d: analytical approach

$$u_d(p_\parallel, p_\perp,, \varphi_0) = A(p_\perp, \varphi_0)e^{i\frac{p_\parallel d}{\hbar}} - B(p_\perp, \varphi_0)e^{-i\frac{p_\parallel d}{\hbar}}$$

$$v_d = \int_{-\pi\hbar/d}^{\pi\hbar/d}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u_d(p_\parallel, p_\perp,, \varphi_0) f(p_\parallel, P_y, P_z) dp_\parallel dP_y dP_z$$

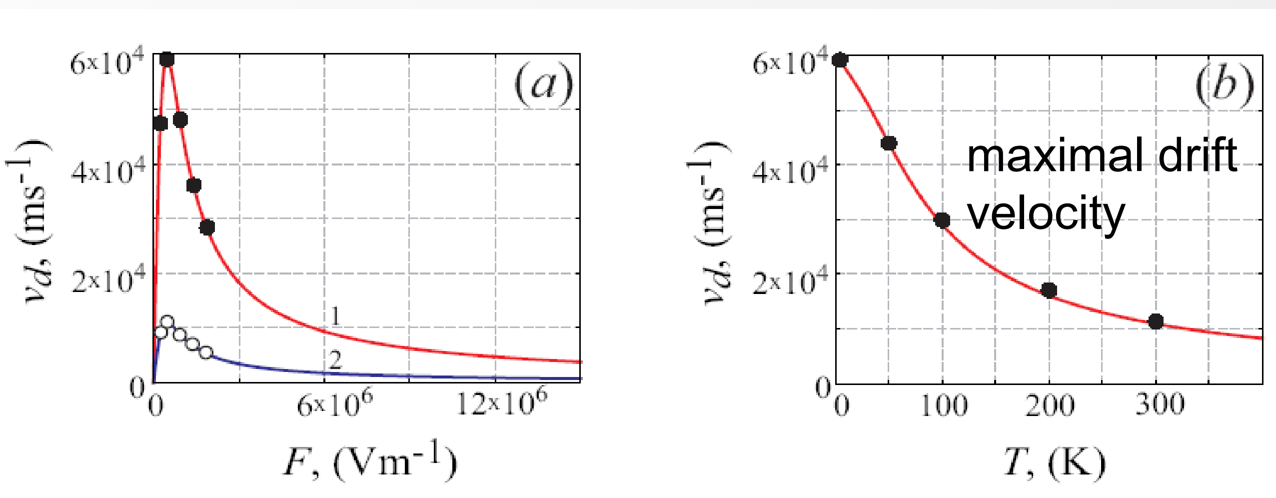
$$v_d = v_0 \frac{I_1(\Delta/2k_BT)}{I_0(\Delta/2k_BT)} \exp\left[-m^* k_B T \left(\frac{\omega_\perp}{\omega_\parallel}\frac{d}{\hbar}\right)^2\right] \sum_{n=-\infty}^{\infty} I_n \left[m^* k_B T \left(\frac{\omega_\perp}{\omega_\parallel}\frac{d}{\hbar}\right)^2\right] \frac{\nu(\omega_B - n\omega_\parallel)}{\nu^2 + (\omega_B - n\omega_\parallel)^2}$$

analytical approach vs numerical simulation for B=0

$$v_d = v_0 \frac{I_1(\Delta/2k_BT)}{I_0(\Delta/2k_BT)} \exp\left[-m^* k_B T \left(\frac{\omega_\perp}{\omega_\parallel}\frac{d}{\hbar}\right)^2\right] \sum_{n=-\infty}^{\infty} I_n\left[m^* k_B T\left(\frac{\omega_\perp}{\omega_\parallel}\frac{d}{\hbar}\right)^2\right]\frac{\nu(\omega_B - n\omega_\parallel)}{\nu^2+(\omega_B-n\omega_\parallel)^2}$$

If there is no magnetic filed, **B**=0, the formula becomes exact. It coincides with one derived,e.g in Y.A. Romanov, Optika i Spektroskopiya 33, 917 (1972).

$$v_d = v_0 \frac{I_1(\Delta/2k_BT)}{I_0(\Delta/2k_BT)} \frac{\nu\omega_B}{\nu^2+\omega_B^2}$$

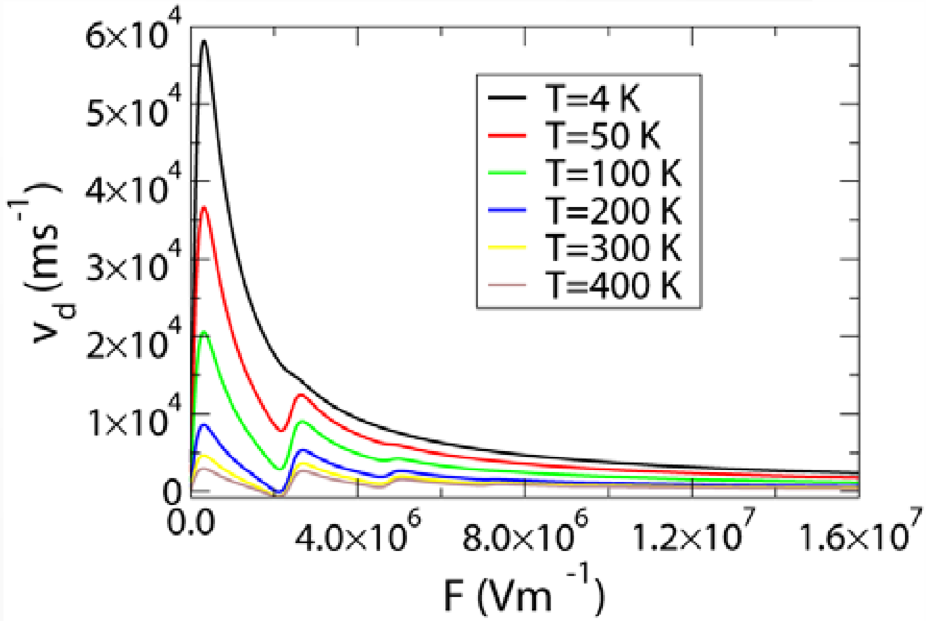


solid lines – analytics

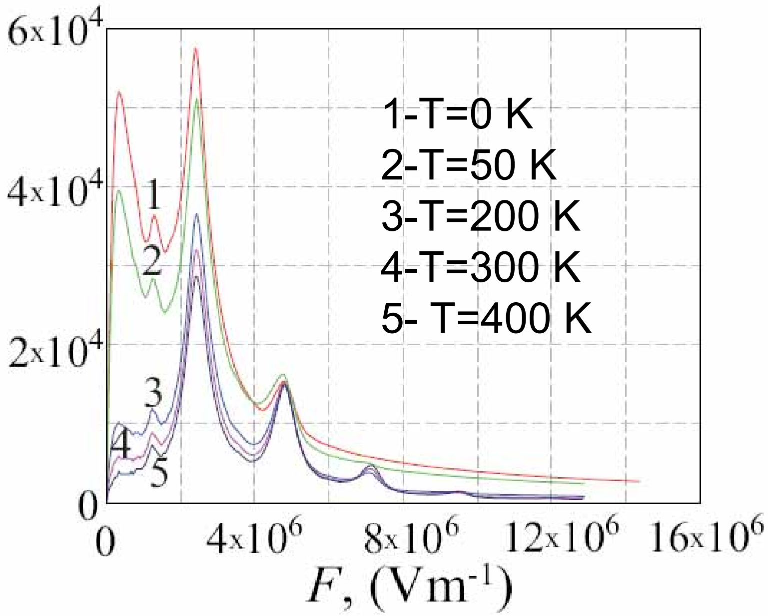
symbol – numerical simulation

Effect of temperature on drift velocity of electrons

analytics



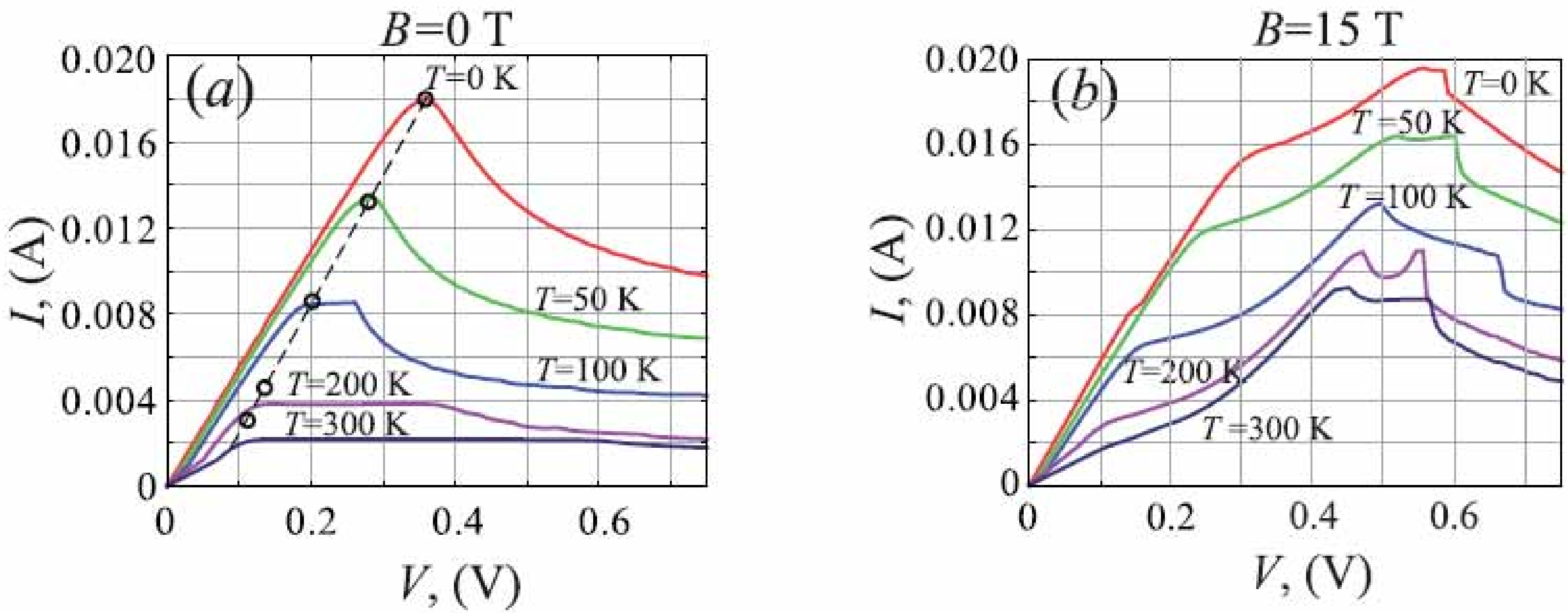
numerics



B=15 T, θ=40° $\rightarrow$ $\omega_{||} \not\gg \omega_{\perp}$ (!)

A. Selskii et al, (2011)

Effect of temperature on electric current: I-V curves

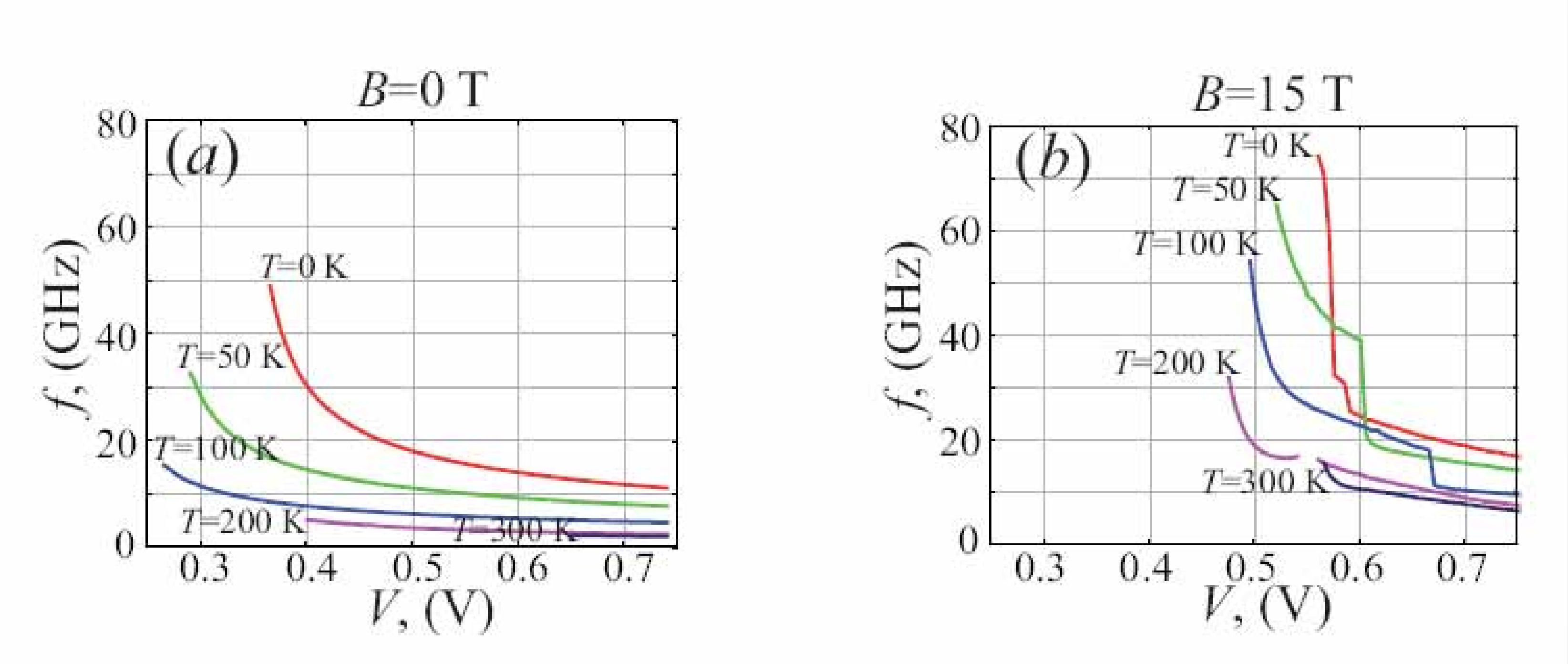


$$I_{th}(T) = I_{th}^{0}\frac{I_1(\Delta/2k_BT)}{I_0(\Delta/2k_BT)}$$

$$V_{th}(T) = V_{th}^{0} + R(I_{th}(T) - I_{th}^{0})$$

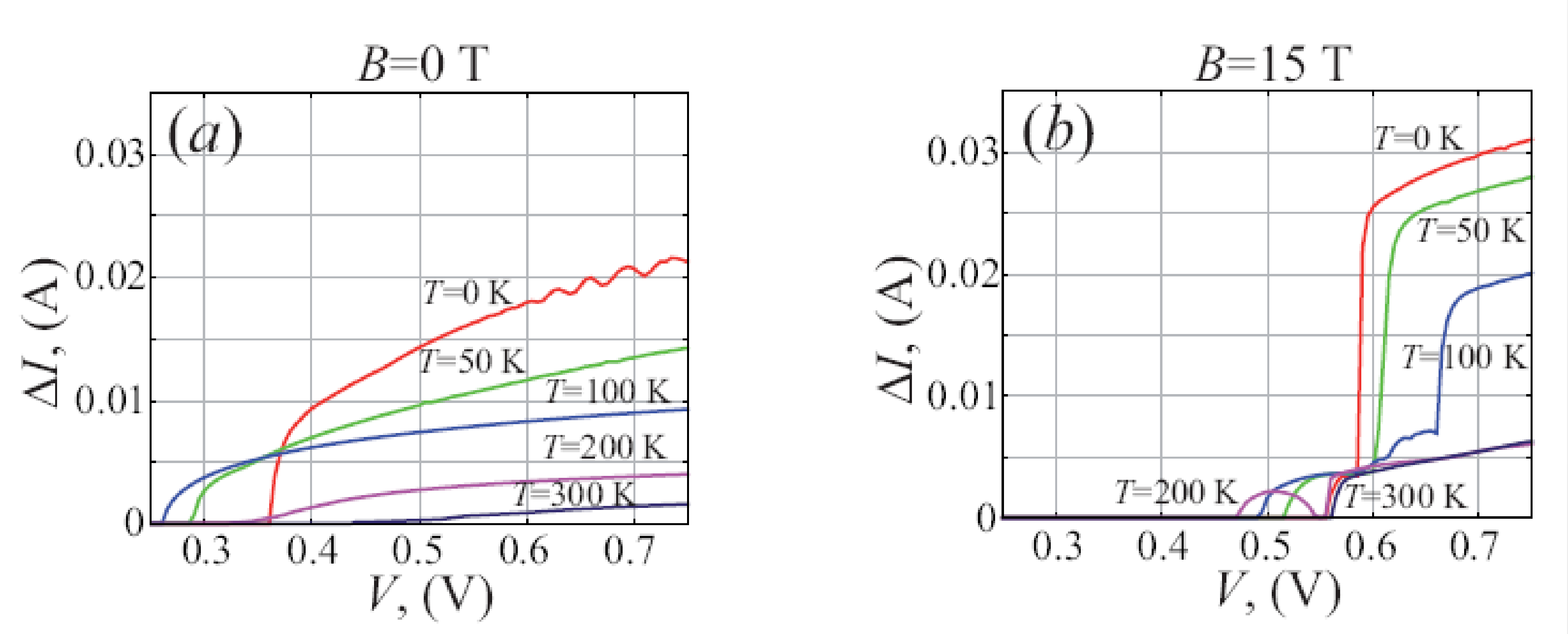
A. Selskii et al, (2011)

Effect of temperature on electric current: frequency



A. Selskii et al, (2011)

Effect of temperature on electric current: amplitude



A. Selskii et al, (2011)

Conclusions

- A tilted magnetic field strongly affects, and can significantly enhance, the transport characteristics of SLs – even at room temperature.

- Increasing T quickly suppresses the B = 0 Esaki-Tsu peak in the vd(F) curve, but has a much smaller effect when a tilted magnetic field is applied. Increasing T can enhance the drift velocity peaks caused by the Bloch-cyclotron resonances

- At the presence of magnetic field electrons demonstrate much higher mobility, which amplifies DC- and AC-components of the current through SL, and also improves its frequency characteristics.

a.balanov@lboro.ac.uk

Related systems

- Miniband electrons driven by an acoustic wave

  Greenaway et al., *Phys. Rev*. B **81**, 235313 (2010)

  - Semiclassical predictions of very high NDV now supported by recent wavepacket and charge domain studies

- Ultracold atoms in an optical lattice with a tilted harmonic trap

  Scott et al., *Phys. Rev. A.* **66**, 023407 (2002)

  - Intrinsically low scattering rates
  - Could using Bose-Fermi interactions to control scattering rate ?

  [Ponomarev, Madroñero, Kolovsky, Buchleitner, *Phys. Rev. Lett.* **96,** 050404 (2006)]

- Optical analogue

  Wilkinson & Fromhold, *Optics Letters* **28**, 1034 (2003)